A STUDY FOR ORBIT REPRESENTATION
AND SIMPLIFIED ORBIT DETERMINATION
METHODS

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STATEMENT OF ORIGINAL AUTHORSHIP

The work contained in this thesis has not been previously submitted for a degree or diploma at any other higher education institute. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

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Abstract

ABSTRACT

This research effort is concerned with the methods of simplified orbit determination and orbit representation and their applications for Low Earth Orbit (LEO) satellite missions, particularly addressing the operational needs of the FedSat mission.

FedSat is the first Australian-built satellite in over thirty years. The microsatellite is approximately 50cm cubed with a mass of 58 kg. The satellite was successfully placed into a low-earth near-polar orbit at an altitude of 780km by the Japanese National Space Development Agency (NASDA) H-IIA launch vehicle on 14, December 2002. Since then, it has been streaming scientific data to its ground station in Adelaide almost daily. This information is used by Australian and international researchers to study space weather, to help improve the design of space computers, communication systems and other satellite technology, and for research into navigation and satellite tracking.

This research effort addresses four practical issues regarding the FedSat mission and operations.

First, unlike most satellite missions, the GPS receiver onboard FedSat operates in a duty-cycle mode due to the limitations of the FedSat power supply. This causes significant difficulties for orbit tracking, Precise Orbit Determination and scientific applications. A covariance analysis was performed before the mission launch to assess the orbit performance under different operational modes. The thesis presents the analysis methods and results.

Second, FedSat supports Ka-band tracking experiments that require a pointing accuracy of 0.03 degree. The QUT GPS group is obligated to provide the GPS precise orbit solution to meet this requirement. Ka-band tracking requests satellite orbital position at any instant time with respect to any of the observation stations. Because orbit determination and prediction software only provide satellite orbital data at a discrete time point, it is necessary to find a way to represent the satellite orbit as a continuous trajectory with discrete observation data, able to obtain the position of the satellite at the time of interest. For this purpose, an orbit interpolation algorithm using...
the Chebyshev polynomial was developed and applied to Ka-band tracking applications. The thesis will describe the software and results

Third, since the launch of FedSat, investigators have received much flight GPS data. Some research was invested in the analysis of FedSat orbit performance, GPS data quality and the quality of the onboard navigation solutions. Studies have revealed that there are many gross errors in the FedSat onboard navigation solution (ONS). Although the 1-sigma accuracy of each component is about 20 m, there are more than 11 % positioning errors that fall outside +/-50m, and 5% of the errors are outside the 100m bound. The 3D RMS values would be 35m, 87m, and 173m for the above three cases respectively. The FedSat ONS uncertainties are believed to be approximately three times greater than those from other satellite missions.

Due to the high percentage of outlier solutions, it would be dangerous to use these without first applying data detection and exclusion procedures. Therefore, this thesis presents two simplified orbit determination methods that can improve the ONS. One is the “geometric method”, which makes use of delta-position solutions derived from carrier phase differences between two epochs to smooth the code-based navigation solutions. The algorithms were tested using SAC-C GPS data and showing some improvement. The second method is the “dynamic method”, which uses orbit dynamics information for orbit improvements.

Fourth, the FedSat ground tracking team at Adelaide use the NORAD TLE orbit for daily FedSat tracking. Research was undertaken to convert an orbit trajectory into these Two Line Elements (TLE). Algorithms for the estimation of TLE solutions from the FedSat onboard GPS navigation solutions are outlined. Numerical results have shown the effects of the unmodelled forces/perturbations in the SPG4 models for the FedSat orbit determination would reach a level of ±1000m. This only includes the orbit representation errors with TLE data sets. The total FedSat orbit propagation should include both the orbit propagation and orbit representation terms. The analysis also demonstrates that the orbit presentation error can be reduced to ±200m and ±100m levels with the EGM4x4 and EGM10x10 gravity field models respectively. This can meet the requirements for Ka-band tracking. However, a simplified tracking program based on numerical integration has to be developed to replace the SPG4 models.
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Chapter 1

Chapter 1 Introduction

This research is concerned with the methods of simplified orbit determination and orbit representation, as well as their applications for Low Earth Orbiter (LEO) satellite missions. In this Chapter we clarify several concepts relevant to the topic: orbit determination, orbit integration, orbit estimation, orbit presentation, orbit interpolation, and so on.

1.1 Orbit determination

Orbit Determination is the process to estimate the position and velocity (state vector) of a satellite at a specific epoch based on models of the forces acting on the satellite, integration of satellite orbital motion equations and measurements to the satellites. Orbit Determination (OD) is generally divided into two categories: preliminary orbit determination and precise orbit determination (POD). Simplified Orbit Determination (SOD) is a new concept introduced in this thesis to describe improved preliminary orbit determination methods based on fast or real time orbit knowledge. These three OD concepts are briefly described below.

Preliminary orbit determination is a geometric method to estimate orbit elements from a minimal set of observations before the orbit is known from other sources. Traditionally, and still typically used, ground-based satellite observations of angles, distance or velocity measurements, which depend on the satellite’s motion with respect to the centre of the Earth. They may be used to deduce the orbit elements. With the Global Positioning System (GPS), satellite positions at different epochs can be computed. An initial orbit can be determined from two positions or three sets of angles [1]. Based on the formulation of the unperturbed two-body problem, a variety of different analytical orbit determination methods have been developed. They are generally divided into Laplacian or Gaussian type methods. Laplacian orbit determination is designed to derive the initial position and velocity at a time instant from different combinations of observations. Gaussian orbit determination, on the other hand, was originally designed for determining the orbit parameters from three sets of widely spaced direction observations. It is now also used to find the orbit from two
positions. This method is useful if both range and angle measurements are available. The accuracy of preliminary orbit solutions will vary from mission to mission, depending on the measurements used and the orbit characteristics. In general, the accuracy varies from tens of metres to thousand of metres.

**Precise orbit determination** is a dynamic, or combined geometric and dynamic method, a process completed with two distinct procedures: orbit integration and orbit improvement. *Orbit integration* yields a nominal orbit trajectory, while *orbit improvement* estimates the epoch state with all the measurements collected over the data arc in a batch estimation process [2]. Simply stated, the goal of POD is to determine the satellite orbit that best fits or matches a set of tracking data over comparatively long arcs. Tracking observations include any observable quantities that are a function of the position and/or velocity of a satellite at a point in time. Examples include range, range-rate (or Doppler), and azimuth and elevation from ground stations of known locations. Other data types can include range and/or range-rate from other satellites, such as from GPS satellites. In theory, the six satellite orbit parameters (position and velocity, or six orbit elements) can be determined from a geometric computation based on very few observations. Because actual observation data includes effects of un-modelled or poorly modelled forces, as well as random and systematic noises, it is often necessary to obtain far more observations than the theoretical minimum [3]. A primary goal of POD is to compute an orbit solution that uses as much of the information in the tracking arcs as possible, while not being overly influenced by noise or spurious data. In general, the better the quality of tracking data processed, the more reliable the orbit solution. With GPS measurements, POD can meet some classes of missions requiring orbit accuracies ranging from 1 metre down to a few centimetres.

**Simplified orbit determination (SOD)** refers to those orbit determination techniques between pure geometric and pure dynamic methods, aiming to meet the requirements for real time or near real time orbit knowledge at accuracies ranging from a few hundred metres down to a few metres. They may be also called “improved preliminary orbit determination or simplified POD”. With GPS measurements, there are a number of SOD options. The simplest method is to create six orbit elements with onboard (x, y, z) navigation solutions at two consecutive epochs. Another method is to use the navigation solutions obtained over a period to create mean orbit elements, such as Two Line
Elements for real time applications. How to improve or make use of GPS navigation solutions generated on a spacecraft to create orbit elements/orbit solutions will be discussed in one of the latter Chapters.

1.2 Orbit representation

Orbit Representation is a means of representing a satellite orbit as a continuous trajectory with discrete observation data at the time of interest. The simplest orbit representation is the “osculating Keplerian elements” method, which describes an orbit as an ellipse. The most typical example is the satellite almanacs published by NASA for almost all spacecrafts in orbit. Figure 1.1 illustrates the concepts of the Keplerian elements with respect to the earth-centred inertial coordinate system.

![Image of Six Keplerian Elements](image)

Figure 1.1. Six Keplerian Elements

As shown in Figure 1.1, the six Keplerian orbital elements include: a (semi-major axis), e (eccentricity), i (inclination), Ω (longitude of the ascending node), ω (angle of perigee), and M₀ (mean anomaly).

The Keplerian elements used by most satellite tracking software are the mean orbit elements rather than the osculating parameters. The difference between these two is quite a complex topic, which involves orbital perturbations and long and short-term periodic variations. Because of computational expediency, the Keplerian elements
defined by The Northern American Aerospace Defence Command (NORAD) are the mean values. This has resulted in most tracking programs using the Space Command Simplified General Perturbation (SGP/SGP4) orbit propagation algorithms in order to maintain compatibility with the Keplerian elements. The NORAD elements sets are “mean” values obtained by removing periodic variations in a particular way. The general perturbations element sets generated by NORAD can be used to compute position and velocity of earth-orbiting objects. These element sets are periodically refined so as to maintain a reasonable periodical capability on all space objects, and provided to users with a mean of propagating these element sets in time to obtain a position and velocity of the space object. The Two Line Elements (TLE) is also called Keplerian Elements in NASA/NORAD format [4]. The GPS system uses two types of representations for the GPS satellite orbits, which are known as almanac and broadcast ephemerides. Both parameters sets are transmitted as part of the GPS navigation message and enable a GPS receiver to compute positions of the GPS satellites with different levels of accuracy. Almanac data are mainly used to determine the constellation of visible satellites above the horizon for mission planning, and to determine approximate Doppler shifts for improved tracking signal acquisition. The ephemerides parameters, on the other hand, provide a much more accurate description of the spacecraft trajectory that is essential for the computation of precise user-position fixes. The GPS ephemerides are represented in the form of Keplerian elements with additional perturbation parameters. Forces of gravitational and non-gravitational origin perturb the motion of the GPS satellites, causing the orbits to deviate from a Keplerian ellipse in inertial space. As shown in Figure 1.2, in addition to the six Keplerian orbital elements a (semi-major axis), e (eccentricity), i (inclination), \( \Omega \) (longitude of the ascending node), \( \omega \) (angle of perigee), and \( M_0 \) (mean anomaly), there are 9 additional parameters used to characterise the periodic and secular perturbations over a certain period, including: \( \Delta n \) (correction to mean motion), \( \text{di/dt} \) (the rate of change of inclination), \( \Omega_0 \) (the rate of change of the right ascension of the ascending node); \( C_{rc} \), \( C_{ri} \) (Amplitude of (co)sine harmonic corrections term to the orbit radius); \( C_{uc} \), \( C_{us} \) (Amplitude of (co)sine harmonic corrections term to the argument of latitude); \( C_{ic} \), \( C_{is} \) (Amplitude of (co)sine harmonic corrections term to the inclination). These parameters must be continually determined and updated through the analysis of tracking data, involving a three-step process:
• An off-line orbit determination is performed through the analysis of tracking to generate a reference orbit for each satellite. This is an initial estimate of the satellite trajectory computed from about one week's tracking data collected by the five Control Segment monitor stations.

• An on-line daily updating of the reference orbit within a Kalman filter as new tracking data are added. This provides the current estimates of the satellite trajectory, which is used to predict the future orbit.

• The ephemeris is derived by extrapolating the estimated orbit for 1 to 14 days into the future. To obtain the necessary broadcast information, curve fits are made to 4 to 6 hour portions of the extrapolated ephemeris, and hourly orbit parameters determined at the central epoch of the fitted curve. This implies use of each set of ephemerides parameters outside the fitting period may cause the error to grow, or the solution to fail. To ensure the accuracy of positioning, each set of broadcast ephemerides should be used only within the period of up to 30 or 60 minutes with respect to the reference time [5].

Figure 1.2. GPS broadcast orbit representation

Both TLE and GPS ephemeris methods belong to the class of analytical type. Polynomial approximation is another class of orbit representation methods. Lagrange and Chebyschev polynomial functions are two popular examples. In these methods, the x, y, z states at the data points over an arc are used to solve set of coefficients for each component, allowing no loss of orbit accuracy over the represented data arc.
1.3 FedSat missions and orbit determination problems

FedSat is the first Australian-built satellite in over thirty years. The microsatellite is approximately 50cm cubed, with a mass of 58 kg. Figure 1.3 shows the dimensions and some features of the satellite. It is a 3-axis stabilised spacecraft that was successfully placed into a low-earth near-polar orbit at an altitude of 780km by the Japanese National Space Development Agency (NASDA) H-IIA launch vehicle on 14, December 2002. Figure 1.4 shows FedSat flying in a sun-synchronised orbit. Since the launch, FedSat has been delivering scientific data to its ground station in Adelaide almost daily. This information is used by Australian and international researchers to study space weather, to help improve the design of space computers, communication systems and other satellite technology, and to research topics in navigation and satellite tracking.

FedSat flies a dual-frequency GPS receiver, known as the “BlackJack”, which was supplied under a collaborative agreement between NASA and CSIRO. The onboard GPS receiver, as shown in Figure 1.5, provides raw GPS measurements for the following purposes:

- To compute the state (position and velocity) of the satellite onboard, referred to as the “onboard navigation solutions” (ONS).
- To provide timing outputs for other onboard satellite electronics.
- To provide raw data for precise real time orbital knowledge for tracking purpose.
- To provide raw data for post-processing precise orbit determination.
- To provide raw GPS data for atmospheric occultation studies.

While the first three applications are engineering in nature, the last two are scientific-driven applications. Of these applications, the most restrictive engineering requirement is for the Ka-band tracking, which requires pointing accuracy of 0.03 degrees. This should not be a problem if the GPS receiver onboard FedSat would operate continuously. The problem is that under normal operational circumstances, the onboard GPS receiver operates 20 to 30 minutes per orbit only, which allows effective collection of GPS data 10 to 20 minutes per orbit. This so-called duty cycle operation mode is necessary due to the limitations of the FedSat power supply. However, it causes significant difficulties for orbit tracking, POD and scientific applications. In this
research, the author has investigated how to contribute to solutions of the above problems before and after the launch of the satellite. The methodology used and the software developed, however, are applicable for other satellite missions flying GPS receivers.

Figure 1.3. FedSat

Figure 1.4. FedSat flying a sun synchronized orbit
1.4 Scope of this research

The overall objective of this research is to address some of the orbit determination and orbit representation problems in the FedSat mission, with focus on the engineering needs for FedSat orbits. Yet, the methods and algorithms developed and tested in this thesis are applicable to any GPS-based LEO satellite missions. The particular objectives of the thesis include:

- Orbit accuracy analysis for its dependencies on FedSat operational modes. This work was conducted in early 2001, in conjunction with other CRCSS staff at QUT. The work was focused on the requirement analysis and simulation studies, using Orbit Performance Analysis and Simulation Study software (OPASS) developed by QUT GPS group, including the author’s contribution.

- Development of FedSat orbit interpolation software based on Chebyshev polynomial functions for precise pointing purposes, in order to provide continuous orbit solutions for Ka-band ground tracking. Algorithms and software have been developed, and extensively tested before the software was handed to the UTS Ka-band Earth Station group.

- Analysis of FedSat in-orbit performance of onboard navigation solutions after the launch of the satellite. A geometric SOD method for improved FedSat orbit GPS navigation solutions was developed and tested.

- Estimation of TLE solutions from the FedSat onboard GPS navigation solutions. Numerical methods to create TLE orbits from GPS orbit solutions were developed and tested. This allows the FedSat tracking to be done autonomously using only GPS data. It also allows for evaluation of the NORAD
FedSat TLE accuracy. The research experimentally answered the question: what is the NORAD TLE orbit accuracy?

The scope of this research is limited to engineering aspects of FedSat orbit problems. The issues of precise orbit determination and automation of FedSat data processing were addressed by other researchers at QUT.

1.5 Outline of the thesis

Chapter 1 briefly reviews some relevant concepts which are closely related to the research topic. It provides essential knowledge and background information for the research topic.

Chapter 2 introduces the fundamentals of satellite orbital motion in order to provide useful information concerning satellite orbit modelling, solutions, and estimation methods. Also GPS-based LEO orbit determination and FedSat observation techniques are emphasised.

Chapter 3 presents the results of studies on accuracy dependencies of the FedSat orbit propagation on operational modes. This part of work was conducted before the launch of FedSat.

Chapter 4 describes the orbit representation algorithms based on Chebyshev polynomials to precisely provide continuous FedSat orbit solutions for Ka-band ground tracking. In this chapter orbit interpolation with polynomials is briefly introduced. The algorithms based on Chebyshev polynomials for orbit representation are described and the software aspects described. Finally the results of experiments using the software are demonstrated and its accuracy is analysed.

Chapter 5 the problems of FedSat onboard navigation solutions and the characteristics of the FedSat orbital performance are discussed. An experimental analysis of the FedSat GPS measurement quality and the quality of onboard navigation solutions are presented.
Finally the algorithms and phase smoothing method for improvement of the FedSat navigation solution are introduced.

Chapter 6 presents the method of orbit representation using Two-line Elements. FedSat ground tracking at Adelaide use NORAD Two Line Elements for daily tracking, and it was necessary to evaluate the accuracy of the NORAD FedSat TLE. In this Chapter, the concepts of TLE and the SGP4 model are introduced. The algorithms for the estimation of TLE elements are presented. The FedSat orbit accuracy achieved with different Earth Gravity Models was analysed schemes and the results summarised.

Chapter 7 Summarises the research work carried out, and highlights major research findings.
Chapter 2

Fundamentals of satellite orbits

2.1 Time and coordinate systems

Time system

Several time systems are involved in the orbit determination problems for FedSat mission. From the measurement systems, satellite laser ranging measurements are usually time-tagged in UTC (Coordinated Universal Time) and GPS measurements are time-tagged in GPS System Time (referred to here as GPS-ST). Although both UTC and GPS-ST are based on atomic time standards, UTC is loosely tied to the rotation of the Earth through the application of “leap seconds” to keep UT1 and UTC within a second. GPS-ST is continuous to avoid complications associated with a discontinuous time scale like UTC [6]. Leap seconds are introduced on January 1 or July 1, as required. The relation between GPS-ST and UTC is

\[ \text{GPS – ST} = \text{UTC} + n \] (2.1)

where \( n \) is the number of leap seconds since January 6, 1980. For example, the relation between UTC and GPS-ST in mid-July, 1999, was GPS-ST = UTC + 13 sec. The independent variable of the near-Earth satellite equations of motion (Equation 2.1) is typically TDT (Terrestrial Dynamical Time), which is an abstract, uniform time scale implicitly defined by the equations of motion. This time scale is related to the TAI (International Atomic Time) by the relation

\[ \text{TDT} = \text{TAI} + 32.184s \] (2.2)

The planetary ephemerides are usually given in TDB (Barycentric Dynamical Time) scale, which is also an abstract, uniform time scale used as the independent variable for the ephemerides of the Moon, Sun, and planets. The transformation from the TDB time to the TDT time with sufficient accuracy for most application has been given by Moyer [7]. For a near-Earth satellite like FedSat, it is unnecessary to distinguish between TDT and TDB. New time systems are under discussion by the International Astronomical Union.
Chapter 2

Coordinate systems

The inertial reference system adopted for Equation 2.1 for the dynamic model is a geocentric inertial coordinate system which is defined by the mean equator and vernal equinox at Julian epoch 2000.0. The Jet Propulsion Laboratory (JPL) DE-405 planetary ephemeris [8], which is based on this inertial coordinate system, has been adopted for the positions and velocities of the planets with the coordinate transformation from barycentric inertial to geocentric inertial. In this coordinate system, the X-axis points towards a fixed direction commonly referred to as the ‘First point of Aries’. The Z-axis is parallel to earth’s spin axis and Y-axis completes the right hand coordinate systems shown in Figure 2.1.

Tracking station coordinates, atmospheric drag perturbations, and gravitational perturbations are usually expressed in the Earth-fixed, geocentric, rotating system, which can be transformed into the ICRF reference frame by considering the precession and nutation of the Earth, its polar motion, and the UT1 transformation. The 1976 International Astronomical Union (IAU) precession [9, 10] and the 1980 IAU nutation formula [11,12], with the correction derived from VLBI analysis [13], are used as the model of precession and nutation of the Earth. Polar motion and UT1-TAI variations are derived from Lageos (Laser Geodynamics Satellite) laser ranging analysis [14, 15]. In this system, the zₙ axis points along Earth’s spin axis, the xₙ axis is perpendicular to zₙ and lies in the Greenwich Meridian (0° longitude) and yₙ competes the right hand system, as shown in Figure 2.2.

Figure 2.1. Earth Centred Inertial (ECI) frame
In celestial mechanics one is concerned with the motions of celestial bodies under the influence of mutual mass attraction. The simplest form is the motion of two bodies the so-called (Two-body problem). For artificial satellites the mass of the smaller body (satellite) usually is neglected, compared with the mass of the central body (the Earth). The “two-body” problem can be formulated in the following way:

Given at any time the positions and velocities of two particles of known mass moving under their mutual gravitational force, calculate their positions and velocities at any other time.

Under the assumption that the mass distribution of bodies is homogeneous, and thus generates the gravitational field effect of a point mass the orbital motion for the two-body problem can be described empirically by Kepler's laws and can also be derived analytically from Newtonian Mechanics [16]. The solution to the two-body problem in celestial mechanics is based on Newton’s second law of Gravitation and his law of Universal Gravitation:

\[
\ddot{\mathbf{r}} = -\frac{\mu}{r^3} \mathbf{r}
\]  

(2.3)

where \( \mu = G(M + m) \), \( G = 6.673.10^{-10} \text{m}^3\text{kg}^{-1}\text{s}^{-2} \) is the gravitational constant, M and m are the masses of the Earth and satellite respectively [17].
In order to uniquely define the position and velocity of a satellite at any instant of time, six parameters are needed. They can be the three position components and three velocity components. In addition to this, there are other forms of orbit representation, which have more geometrical significance. One such representation is the form of Kepler Elements. As shown in Figure 1.1, the six components of the Keplerian elements are:

- Semi-major axis \(a\),
- Eccentricity \(e\),
- Inclination \(i\),
- Argument of Perigee \(\omega\),
- Right Ascension of ascending node \(\Omega\), and
- Mean Anomaly \(M\).

which uniquely define the position and velocity at a given time \(T_0\), commonly known as epoch time. From the definition of the orbital elements, it may be observed that: \(\Omega\) and \(I\) define the orientation of orbit plane in the ECI frame; \(\omega\) defines the location of the perigee point in the orbit plane with reference to the ascending node; \(a\) and \(e\) define the shape of the orbit. If \(e = 0\), the orbit is circular; and if \(0 < e < 1\) the orbit is elliptical. \(M\) defines the location of the satellite in the orbit plane [18]. The integration of Equation (2.3) formally gives the solution:

\[
\begin{align*}
\dot{r}(t) &= \ddot{r}(t,a,e,i,\Omega,\omega,M) \\
\ddot{r}(t) &= \dddot{r}(t,a,e,i,\Omega,\omega,M)
\end{align*}
\]  

(2.4) (2.5)

with the Keplerian elements being free selectable integration constants [16].

### 2.3 Perturbed orbits and solutions

In reality a number of additional forces act on the near Earth satellite. To distinguish them from the central force (central body acceleration) these are generally referred to as perturbing forces. The satellite experiences additional accelerations because of these forces, which can be combined into a resulting perturbing acceleration vector \(\vec{k}\). The extended equations of motion are:

\[
\ddot{r} = -\frac{\mu}{r^3}r + k
\]  

(2.6)
where
\[
\vec{k} = \vec{r}_E + \vec{r}_s + \vec{r}_m + \vec{r}_o + \vec{r}_D + \vec{r}_{sp} + \vec{r}_A
\]  
(2.7)

The perturbing forces and the corresponding accelerations are:

- Due to the non-spherically and inhomogeneous mass distribution within the Earth (the central body), $\vec{r}_E$
- Due to other celestial bodies, mainly the Sun ($\vec{r}_s$) and the Moon ($\vec{r}_m$)
- Earth and oceanic tides, ($\vec{r}_o$)
- Atmospheric drag $\vec{r}_D$
- Direct and Earth-reflected solar radiation pressure, ($\vec{r}_{sp}$), ($\vec{r}_A$) [16].

Figure 2.3. Perturbing forces acting on a satellite

Detailed explanations for each force can be founded in textbooks by Seeber [16]. Figure 2.3 is a graphical description of the perturbing forces. The resulting total acceleration depends on the location $\vec{r}$ of the satellite, i.e. a quantity which first has to be determined from the solution of the differential Equation (2.6) as a function of time. Consequently, the integration of the Equation (2.6) is a challenge. Basically, there are two types of solutions to Equation (2.6): analytical methods and numerical methods. An analytical method for such a complex problem in celestial mechanics starts with reasonable simplifications and to correct the resulting "error" in a separate second step.
Chapter 2

Analytical solutions

There are many options for simplifications, thus leading to different analytical solutions. What is of most interests to the FedSat mission is the Spacecraft General Propagation (SGP) models, which are used by NORAD to maintain general perturbation element sets for all space objects being tracked. These element sets are periodically redefined so as to maintain a reasonable prediction capability. A SGP model is used to propagate these element sets in time to obtain a position and velocity of the space object at a time interest. The SGP4 model is an example of an analytical solution for satellite motion. This model was developed by Ken Cranford in 1970 [19] and is used for near-Earth satellites. This model was obtained by simplification of the more extensive analytical theory of Lane and Cranford (1969) [20], which uses the solution of Brouwer (1995) [21] for its gravitational model, and a power density function for its atmospheric model [22].

In general, an analytical solution for the satellite motion problem regards the integration constants of the undisturbed case to be considered as time dependent functions, which may be described as follows:

\[
\begin{align*}
\mathbf{r}(t) &= \mathbf{r}(t, a(t), e(t), i(t), \Omega(t), \omega(t), \bar{M}(t)) \\
\dot{\mathbf{r}}(t) &= \mathbf{r}(t, a(t), e(t), i(t), \Omega(t), \omega(t), \bar{M}(t))
\end{align*}
\] (2.8a) (2.8b)

Numerical Solutions

It is not possible to have an analytical solution without simplifications. For precise orbit determination, however, numerical integration is the only choice. Given a satellite epoch state \((\mathbf{r}_0, \dot{\mathbf{r}}_0)\) as well as the physical parameters \(\mathbf{u}_0\), the orbit trajectory is the predicted numerically by double integration of the force models [23]:

\[
\mathbf{r}(t) = \int\int \dot{\mathbf{r}}(t) dt + \mathbf{r}_0 + t \dot{\mathbf{r}}_0
\] (2.9)

The predicted orbit accuracy depends on the following factors:

- The epoch state \((\mathbf{r}_0, \dot{\mathbf{r}}_0)\). A filtering process will estimate the correction to the epoch state (and possibly to selected physical parameters) that brings the model trajectory into better agreement with the tracking data.
- The force (acceleration) model \( \mathbf{r}^\prime (t) \), including those physical parameters. Where precision prediction of orbits is required, sophisticated and high accuracy models for the perturbing accelerations must be found. This is especially true in the case of a low altitude satellite, which is sensitive to the uncertainties of gravity models and atmospheric variations. While the computation speed is of concern, the orbit software that has long-term numerical stability is highly desirable.

Feng (2001) proposed a new Integral Equation method for the numerical treatment of Equation (2.9), which is a comparative simple, but precise method for orbit integration \[24\].

### 2.4 Orbit estimation

An enduring technique for estimating celestial orbits is the method of least squares, first employed by Gauss in 1795. Let \( \mathbf{Z} \) be a vector of observations \((z_i, \cdots z_n)^T\) made over an interval of time, or a “tracking arc”. The objective is to find that trajectory, among all possible trajectories, satisfying the dynamical constraint Equation (2.6), which minimises the mean square difference between the actual observations \( z_i \), and the theoretical observations \( \bar{z}_i \) derived from the solution trajectory. That is, the trajectory \( \mathbf{r}(t) \) that minimises the functional is required \[23\]:

\[
J = \sum_{i=1}^{n} (z_i - \bar{z}_i[r(t)])^2
\]  

(2.10)

As this is a nonlinear problem, one reformulates it as requiring the computation of a linear correction to the nominal trajectory \( \mathbf{r}_n(t) \) given by Equation \( \mathbf{r}_n(t) = \int \dot{\mathbf{r}}_n(t) dt + \mathbf{r}_{on} + \mathbf{r}_{on} \). First, theoretical observations \( \bar{z}_i \) are computed from the nominal trajectory, and then the differences \( \delta z_i = z_i - \bar{z}_i \) are formed. These pre-fit residuals become the observations to be used in a linear adjustment of the nominal trajectory. (Strictly speaking, this is still not a linear problem, but if the nominal trajectory is sufficiently close to the true trajectory, it will be in the “linear regime,” where a linear correction is adequate, if not perfect. If greater accuracy is needed, a
linear correction to the new solution can be computed, and so on for multiple iterations, until the solution converges.). The familiar linear equation can be written as follows:

$$\delta x = Ax + n$$  \hspace{1cm} (2.11)  

where $x$ is the vector of parameters to be estimated, $n$ is the vector of random measurement noise on the observations $\delta x$, and $A$ is a matrix of partial derivatives of the observations with respect to the elements $x$. Here $x$ includes, as a minimum, adjustments to the six epoch state parameters, but may also include adjustments to various dynamic, geometric, and clock parameters. Equation (2.11) is called the linear equation and $A$ is the design matrix of observations. The element $a_{ij}$ of $A$ are given by the following equation:

$$a_{ij} = \frac{\partial z_i}{\partial x_j}$$  \hspace{1cm} (2.12)  

where, for simplicity, $z_i$ now represents the differential element $\delta z_i$. This partial derivative relates an observation $z_i$ at one time to a state parameter $x_j$ at a possibly remote reference time. The $A$ matrix, thus, contains the state transition information from the reference epoch to all times in the data arc and must, therefore, embody the dynamical constraint of Equation (2.6). To compute the $a_{ij}$, one writes:

$$\frac{\partial z_i}{\partial x_j} = \frac{\partial z_i}{\partial x_{ci}} \frac{\partial x_{ci}}{\partial x_j}$$  \hspace{1cm} (2.13)  

where $x_{ci}$ represents the satellite state at the time of observation $z_i$. This explicitly introduces the current state $x_{ci}$ and its relation to both the current observation $z_i$ and the state variables $x_j$. The partial $\frac{\partial z_i}{\partial x_{ci}}$ contains no dynamical information and can be computed directly. The partial $\frac{\partial x_{ci}}{\partial x_j}$ relates the satellite state at the observation time to the epoch state and, thus, embodies the dynamical constraint. To determine that partial derivatives, the equation of motion $r_n(t) = \int \int \dot{r}_n(t)dt + \dot{r}_n^{on} + r_n^{on}$ is differentiated with respect to the epoch state parameters, producing a set of linear second-order differential equations in $\frac{\partial x_{ci}}{\partial x_j}$. These variational equations are then integrated numerically to obtain the partial derivatives and, thus, the final design matrix.
The well-known least squares solution to the linear Equation (2.11) is given by:

$$\hat{x} = (A^T R_n^{-1} A)^{-1} A^T R_n^{-1} z$$  \hspace{1cm} (2.14)

where  $$R_n = E(n,n^T)$$  \hspace{1cm} (2.15)

is the covariance matrix associated with the measurement noise vector n. This is known as the batch least squares solution because it requires that all observations over a data arc be collected and processed as a “batch”. In practice, when many parameters are estimated, Equation (2.14) will require large matrix inversions, which can cause numerical instability. Most orbit estimators today employ more stable techniques [23].

### 2.5 GPS-based LEO positioning and orbit determination

The techniques involved in these applications can be classified into direct GPS-based orbit determination, and differential GPS precise orbit determination (POD). As illustrated in Figure 2.4, in the former case only flight GPS measurements are used for orbit tracking and/or autonomous navigation, achieving orbit accuracy of metres to hundreds of metres. In the latter case, the data collected by a global GPS tracking network of tens of stations are processed along with the flight data to achieve orbit accuracies of better than 10 centimetres. Figure 2.5 illustrates the concepts of GPS-based satellite POD using global differential GPS networks.

The potential of GPS to provide accurate and autonomous satellite orbit solutions was noted early in its development. Early studies of direct GPS based-tracking can be found in [25], which addressed the applications from near-Earth to beyond geosynchronous
Chapter 2

orbit [26] which examined GPS tracking of the space shuttle, and [27] which focused on autonomous near-Earth navigation. The recent studies and applications include those in [28], which reported the BIRD satellite mission as a milestone towards GPS-based autonomous navigation and [29] which examined the GPS receiver architecture and expected performance for autonomous navigation in High Earth Orbits. The first reported results from relative GPS navigation were those of the ETS-VII autonomous rendezvous using relative GPS navigation [30], which achieved an accuracy of relative navigation of 10m in position and 3cm/s in velocity. Further information can be found at the website [31], which is a GPS mission directory listing, chronologically, the space missions that have included a GPS receiver (or receivers) for any number of reasons. It is rather a comprehensive directory of missions that have been cited in scientific literature, although it may not be an all-inclusive listing of spaceborne GPS missions. Mission descriptions, the function of the GPS receivers, the model of GPS receivers and references citing the mission are given.

Direct GPS orbit determination can meet the most demanding of the accuracy needs for spacecraft tracking and navigation for the dynamically unpredictable vehicles. The orbit computation may be conducted on-board the spacecraft in real time, or at a ground-station in near-real time. There are two ways to obtain the orbit solutions: GPS navigation solutions (point positioning) and orbit filter/improvement solutions. GPS standard point positioning is as accurate in low orbit as on the ground: theoretically 10 to 20 metres (with zero Selective Availability). Any orbit below 3000km is considered Low Earth Orbit (LEO). For LED orbits, signals from 10 or more GPS satellites may be received by an up-looking antenna with the current GPS constellation, reaching the receiver with nearly uniform power levels and geometric distribution above the horizon. Above 3000km altitude, the conditions for receiving GPS signals become less favourable [29, 23]. Received signal power typically decreases because the transmitted power of some signals drops off as a result of the attenuation pattern of the transmitting GPS satellite, and the ranges to many of the visible satellites having increased. As a result, the number of visible or receivable satellites by an up-looking antenna drops dramatically in orbits above 3000km. The flight data from an on-board receiver would have difficulty to generating single-point solution above 3000km. The flight data from an on-board receiver would have difficulty generating single-point solutions under such
circumstances. In any orbit, there is also another problem: some on-board GPS receivers cannot operate all of the time due to restrictions of onboard power supply. As a result, continuous supply of navigation solutions are not always possible.
Chapter 3

Accuracy dependencies of FedSat orbit propagation on operational modes

3.1 Introduction

FedSat is a low Earth orbiting microsatellite which conducts space science, communications, Earth remote sensing and engineering experiments. The satellite flies the BlackJack spaceborne GPS receiver to compute its position and velocity for routine tracking operations, platform engineering needs (time keeping), as well as scientific experiments such as precise orbit determination (POD) and GPS occultation studies. Of these applications, the most challenging engineering need is to support by the Ka-band tracking, requiring a pointing accuracy of 0.03 degrees. As shown in Figure 3.1, the allowed orbit error $\Delta r$ is approximately expressed as the function of the pointing error $\Delta \beta$, elevation angle $\beta$, and altitude of the orbit:

$$\Delta r = \Delta \beta \cdot \frac{\pi}{180} \cdot \rho(\beta)$$  \hspace{1cm} (3.1)
Figure 3.2 plots the allowed orbit error against the elevation angle, assuming the pointing error is 0.01 degree. It is observed that the most restrictive orbit accuracy is required when the satellite passes over the zenith $\beta=90$ degrees. In the case of the FedSat orbit, the requirement will be $\Delta r=3\times139$ m. Only errors in along-track and cross-track will affect the pointing accuracy. The accuracy requirement for each direction is:

$$\Delta A = \Delta C = 3 \times 139 / \sqrt{2} \approx 300m$$

Unlike many other GPS-based LEO satellites, the FedSat flight GPS receiver only operates in duty-cycle modes, eg, 2-by-10 minutes per orbit period (100.9 minutes) or 20 minutes per orbit, due to the strict limitation of the power supply. It may occasionally be operate continuously for precise orbit experiments. Pre-launch testing revealed that it takes 4 to 5 minutes for the GPS receiver to start normal operation after power up. This means that there will be observations of 2-by-5 minutes or 1-by-15 minutes per orbit available for orbit determination.

Knowing the achievable orbit accuracy under specific operation conditions and processing modes is of primary concern to the operators. Earlier covariance analysis have confirmed the adequacy of the 2-by-10 and 1-by-20 operational cases [32,33]. It is necessary now to investigate the achievable orbit accuracy in the cases of 2-by-5 and 1-by-15 minute operations. Because a filtering process is unlikely to work onboard the
FedSat, the alternation is to use the flight GPS data downloaded each day to compute and predict the orbit into the future 24 to 48 hours for real time operational use. Hence, the effect of duty-cycle operation on the predicted orbit accuracy is of more interest.

In this Chapter, the structure of the simulation software is described and the dependence of FedSat orbit accuracies on different operation scenarios through covariance analysis and numerical studies is investigated. Results show that under the assumption of the expected GPS standalone positioning performance (3D positional RMS accuracy for about 10m to 15m), the effective data set of 2-by-5 minute per orbit for 24 hours can still result in quality predicted orbits for 48 hours. Longer prediction may still be possible. The predominant errors in the predicted orbit are due to the uncertainties of the atmospheric force, which alone will reach 80 metres after 72 hours of prediction. The second largest modelling error, Solar Radiation Pressure, will cause the orbit errors of less than 10 metres. Considering all the effects, including the atmospheric drag, the accuracy requirements of 300 metres in each component can be satisfied over a two day prediction.

3.2 Software structure for Orbit performance analysis And simulation studies (OPASS)

Figure 3.3 gives the block diagram for the Orbit Performance Analysis and Simulation Study software (OPASS) [33]. The system requires inputs of onboard GPS data or solutions. To begin with, it must create a nominal satellite orbit with an orbit integrator. Next, the system forms the observation equation for point positioning results every epoch, or for raw flight measurements from which point positioning solutions or kinematic GPS solutions can be produced. In the latter case, a global differential tracking network of ground receivers is assumed. After that, a dynamic filtering process is performed to produce the filtered orbit solutions. To improve the orbit states on-line, this sequential filter updates the current state estimate and the processes each new measurement until the end of the data arc. The system finally creates smoothed solutions for all the epoch states and predicts the orbits hours to days into the future. A detailed description of each module is given in [34]. The concept of orbit improvement based on GPS navigation solutions is illustrated in Figure 3.4.
OPASS supports the following studies:

- Selection of an Earth Gravity Model (EGM), and its spherical harmonics orders and degrees, for different FedSat orbit requirements. This is based on an analysis of the numerical effects of the most recent EGMs, such as EGM96, EGM99 and JGM-3, on the FedSat orbit solutions, and the orbit errors of different EGMs.
- Post-fitted and predicted orbit accuracy of various levels achievable against different tracking scenarios: duty-cycle or continuous, different accuracies of GPS positioning solutions (e.g., 100 metres, metres, decimetres and centimetres).
- Stability of orbit filtering solution, accounting for choice of sample rates, length of data arcs or moving orbit windows, effects of acceleration model errors including those physical parameters, and external observation outliers as well.

These are fundamental matters that need to be tested in order to establish a working prototype for routine operations of FedSat orbit production. Additional development of the well-tested OPASS will lead to an orbit determination prototype for FedSat engineering applications that can operate in real time.

![Block Diagram of OPASS Software](image)

Figure 3.3. Block Diagram of OPASS Software
3.3 Results of simulation and covariance analysis

3.3.1 Cases of study
FedSat would obtain its state (position and velocity) at any time in a variety of ways, depending on the availability of GPS solutions and measurements, scenarios for onboard GPS operations, and the requirements for time latency. The accuracy performance of FedSat orbit solutions obtained in the following cases is of great concern to the FedSat mission.

**Case I**: Duty-Cycle Operation Mode 1: 2-by-5 minutes per orbit (100.9 minutes). The FedSat orbit can be derived through a filtering process over all the “broken” tracking arcs across 24 h, and then predicted 48 h into the future. The overall orbit accuracy achieved over the filtering period and the prediction period can be evaluated by imposing GPS point positioning errors or simulated random errors at the same level. For this case, three options were considered:

a) With Selective Availability (SA) on, adding real GPS SPS point positioning errors.

b) With SA off, adding random noise at the level typically expected of a ground receiver.

c) With SA off, adding random noise 50% higher than the level typically found in a ground receiver.
**Case II:** Duty-Cycle Operation Mode 2: 1-by-15 minutes per orbit (100.9 minutes). This may be another option for GPS onboard operation, which consumes the same power as case I.

The process and analysis are the same as for Case I. It is necessary to determine which produces better orbit accuracy. The same three options as for Case I are investigated.

Table 3.1 lists the FedSat orbit characteristics and force models introduced into this simulation study. To reflect the effects of duty-cycle operations more effectively under the assumption SA is present. 3D orbit errors obtained from GPS 30-sec point positioning solutions over 24 h at a known GPS tracking station were added to a 24-h reference orbit at 30 second rate to generate 3D orbit position measurements. Statistical details of these errors are presented in Table 3.2a, where the final column $(\text{RMS}=\sqrt{\text{Mean}^2 + \text{STD}^2})$ represents the total error. The 3D RMS of 60m is considered typical using standalone GPS with Selective Availability on.

Since SA becomes zero, the standalone GPS positioning 3D RMS accuracy has been improved by four to six times. (The RMS accuracy also consists of both Mean and STD errors). Based on the statistical analysis of GPS navigations conducted by some investigators [34,35], the noise terms for each component are obtained by adding both systematic errors and random terms. Table 3.2b summarises all the error strategies, and also lists the computation schemes for each case of study, including information about the length of data arcs, number of measurement epochs and error models.

### 3.3.2 Results for filtered and predicted Orbits

Figures 3.5 to 3.15 show the filtered and predicted position errors in altitude, along-track and cross-track and 3D rms for the Schemes Ia through to Ic, and IIa to IIc, computed against the reference trajectory obtained with the same force models but without initial epoch state errors.

(1) The expected duty-cycle GPS operation mode effectively 2-by-5 minute per orbit can result in fairly stable filtered and predicted orbit solutions. With SA on,
the overall 3D orbit uncertainties for a 24-h data arc and a 48-h predicted arc are 20 m and 40 m respectively. With the Selective Availability turned off, the 3D RMS accuracy of orbit positional errors are reduced to 5 m and 10 m for 24-h filtered orbit and 48 predicted orbit respectively.

(2) The alternative duty-cycle GPS operation mode effectively 1-by-15 minutes per orbits results in much worse filtered and predicted orbit solutions. With SA on, the overall 3D orbit uncertainties for a 24-h data arc and a 48-h predicted arc are 40 m and 130 m respectively. With Selective Availability zero, the 3D RMS accuracy of orbit positional errors are reduced to 15 m and 30 m for 24-h filtered orbit and 48 predicted orbit respectively.

(3) The alternative operation mode (1-by-15 min data) results in large along-track orbit error, showing a stronger error growth when it is predicted into future. This is because the second operational mode has a disadvantage in controlling the effects of systematic errors in measurements and models. However, the predicted error over this operational mode over 48 hours is still less than 40 m.

(4) The orbit accuracy can be further improved for all these six cases studied if the systematic errors in GPS point positioning solutions can be reduced. This is the one of our research objectives.

Table 3.1. Summary of FedSat orbit characteristics and force models

<table>
<thead>
<tr>
<th>Feature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbit Altitude</td>
<td>802.92km</td>
</tr>
<tr>
<td>Inclination</td>
<td>98.673 deg</td>
</tr>
<tr>
<td>Numerical eccentricity</td>
<td>0.002 (assumed)</td>
</tr>
<tr>
<td>Right ascension of ascending node</td>
<td>140 deg</td>
</tr>
<tr>
<td>Argument of perigee</td>
<td>0 deg (assumed)</td>
</tr>
<tr>
<td>Earth Gravity Model (tested)</td>
<td>JGM3 50x50 / EGM96_100x100</td>
</tr>
<tr>
<td>Atmospheric model (tested)</td>
<td>J70 (modified Jacchia 1970 model)</td>
</tr>
<tr>
<td>Solar and Lunar gravity (used)</td>
<td>Standard model</td>
</tr>
<tr>
<td>Solar Radiation Pressure (used)</td>
<td>Direct</td>
</tr>
<tr>
<td>Tidal potential model (used)</td>
<td>2nd degree Legendre polynomial for the Sun &amp; Moon</td>
</tr>
</tbody>
</table>
Table 3.2a. Statistics of (x y z) errors added to a 24-h reference orbit at 30-sec rate to generate 3D orbit position measurements: Case I: 2-by-5 minutes per orbit

<table>
<thead>
<tr>
<th></th>
<th>Case Ia (SA ON)</th>
<th>Case Ib (SA OFF)</th>
<th>Case Ic (SA OFF)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean std rms</td>
<td>mean std rms</td>
<td>mean std rms</td>
</tr>
<tr>
<td>x (m)</td>
<td>-6.646 29.66 30.39</td>
<td>3.08 5.43 6.26</td>
<td>5.08 8.14 9.60</td>
</tr>
<tr>
<td>y (m)</td>
<td>-0.422 32.87 32.83</td>
<td>2.93 5.55 6.24</td>
<td>4.93 8.28 9.66</td>
</tr>
<tr>
<td>z (m)</td>
<td>1.371 20.98 21.02</td>
<td>-2.96 4.71 5.56</td>
<td>-4.96 7.06 8.62</td>
</tr>
<tr>
<td>3D(m)</td>
<td>6.799 48.96 49.43</td>
<td>5.18 9.06 10.44</td>
<td>6.64 13.60 16.11</td>
</tr>
</tbody>
</table>

Table 3.2b. Statistics of (x y z) errors added to a 24-h reference orbit at 30-sec rate to generate 3D orbit position measurements: Case II: 1-by-15 minutes per orbit

<table>
<thead>
<tr>
<th></th>
<th>Case IIa (SA ON)</th>
<th>Case IIb (SA OFF)</th>
<th>Case IIc (SA OFF)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean std rms</td>
<td>mean std rms</td>
<td>mean std rms</td>
</tr>
<tr>
<td>x (m)</td>
<td>-3.81 37.02 37.22</td>
<td>3.10 4.98 5.86</td>
<td>5.10 7.46 9.04</td>
</tr>
<tr>
<td>y (m)</td>
<td>-0.55 39.34 39.34</td>
<td>2.98 5.36 6.14</td>
<td>4.99 8.04 9.46</td>
</tr>
<tr>
<td>z (m)</td>
<td>-1.68 23.70 23.76</td>
<td>-2.94 4.64 5.49</td>
<td>-4.94 6.96 8.53</td>
</tr>
<tr>
<td>3D(m)</td>
<td>4.20 58.69 59.14</td>
<td>5.21 8.66 10.11</td>
<td>8.67 13.00 15.63</td>
</tr>
</tbody>
</table>

3.4 Acceleration model errors

In the above studies, modelling and computational errors have been assumed to be zero. In the section, the effects of modelling errors on the orbit solutions are studied, with emphasis on the atmospheric errors.

Figure 3.11 shows the x, y, z orbit errors of the earth gravity model EGM96 with degree and order of 70x70 referred to EGM96_70x70, plotted against the solutions with EGM96-100x100. Within 24 hours the errors reach about 0.60m. Figure 3.12 shows the x, y, z orbit errors of EGM96_80x80, plotted against the solutions with EGM96-100x100. Within 48 hours the errors reduced to 0.06m. Therefore, EGM96_80x80 is appropriate for FedSat prediction.
Figure 3.13 compares the effects of solar radiation pressure on the Topex orbit and FedSat orbit. It is evident that the 3D RMS errors of the FedSat predicted orbit due to un-modelled solar radiation pressure are less than 10m over 7 days.

The most uncertain error source for FedSat is the atmosphere drag. Figure 3.14 shows the total effects over the prediction of 7 days. The atmospheric model is a modified Jacchia 1970 model and the subroutine, developed Mike P. Hickey, Universities Space Research Association, NASA/Marshall Space Flight Centre.

It is believed that the approximate model has uncertainty of about 10 to 15 % of the total effect. Figure 3.15 illustrates the 20% of the total errors, showing that the orbit errors at the end of the third day are still within 80m.

### 3.5 Conclusions and remarks

1. The duty-cycle GPS operation mode – effectively 2-by-5 minute per orbit can result in stable filtered and predicted orbit solutions. With the conservative assumption of RMS errors of 9m (mean =5m and std=7.5m) on each component, the orbit uncertainties for a 24-h data arc and a 48-h predicted arc are 5m and 10m respectively.

2. The alternative duty-cycle GPS operation mode effectively 1-by-15 minute per orbit would result in worse filtered and predicted orbit solutions. With the conservative assumption of RMS errors of 9m (mean = 5m and std =7.5m) on each component, the orbit uncertainties for a 24-h data arc and a 48-h predicted arc are 15m and 30m respectively.

3. The predominant errors in the predicted orbit are the uncertainty of the atmospheric forces, which alone will reach 80 metres within 72 hours of prediction. Other force errors, such as Solar Radiation Pressure, will result in orbit errors of less than 10 metres.

In summary, under the assumption of expected GPS standalone positioning accuracy of about 10m to 15m, the effective data set of 2-by-5 minute per orbit for 24 hours can still result in a good quality predicted orbit for 48 hours. Longer prediction may also still be
acceptable. Considering all the effects, including the atmospheric drag, the accuracy requirements of 100 metres in each component can be satisfied with a 48-hour prediction.

Figure 3.5. Filtered and predicted position errors in altitude, along-track and cross-track and 3D RMS for Scheme Ia, computed against the reference trajectory obtained with the same force models but without initial epoch state errors.

Figure 3.6. Filtered and predicted position errors in altitude, along-track and cross-track and 3D RMS for Scheme Ib, computed against the reference trajectory obtained with the same force models but without initial epoch state errors.
Figure 3.7. Filtered and predicted position errors in altitude, along-track and cross-track and 3D RMS for Scheme Ic, computed against the reference trajectory obtained with the same force models but without initial epoch state errors.

Figure 3.8. Filtered and predicted position errors in altitude, along-track and cross-track and 3D RMS for Scheme IIa, computed against the reference trajectory obtained with the same force models but without initial epoch state errors.
Figure 3.9. Filtered and predicted position errors in altitude, along-track and cross-track and 3D RMS for Scheme IIb, computed against the reference trajectory obtained with the same force models but without initial epoch state errors.

Figure 3.10. Filtered and predicted position errors in altitude, along-track and cross-track and 3D RMS for Scheme IIc, computed against the reference trajectory obtained with the same force models but without initial epoch state errors.
Figure 3.11. shows the x, y, z orbit errors of the earth gravity model EGM96 with degree and order 70x70—called EGM96_70x70, plotted against the solutions with EGM96-100x100.

Figure 3.12. shows the x, y, z orbit errors of EGM96_80x80, plotted against the solutions with EGM96-100x100.
Figure 3.13. Comparison of the effects of solar radiation pressure on the Topex orbit and FedSat orbit. It is evident that 3D RMS errors of FedSat predicted orbit due to unmodelled solar radiation pressure are less than 10m over 7 days.

Figure 3.14. shows the total effects over the prediction of 7 days. The atmospheric model is a modified Jacchia 1970.
Figure 3.15. Illustration of the 20% of the total atmospheric errors, showing that the orbit errors at the end of the third day are still within ± 80m.

**Some Remarks**

Post-launch FedSat in-orbit performance demonstrates that the FedSat 3D RMS orbits reaches 35 metres, which is about 3 times the assumed value above. Therefore, a more realistic estimation of the orbit errors should also be enlarged at least three times.
Chapter 4

Orbit representation with Chebyshev polynomials

4.1 Overview

The first Australian scientific satellite FedSat was successfully launched on 14 December 2002. FedSat is a low Earth orbiting (LEO) microsatellite, flying a sun-synchronous orbit. In order to track FedSat from a ground observation station, it is necessary to have real time access to the positions of FedSat at any instant of time. However, the output of the orbit determination and prediction software only provides satellite orbit data at discrete time points, typically every 60 seconds. Therefore, one needs to represent satellite orbit as a function of time so that it is possible to obtain the position of the satellite at the time of interest. These can be achieved by orbit interpolation using polynomials within the ephemeris sample interval representing satellite orbit as a continuous form.

The simplest polynomial can be expressed as the analytic function:

\[ C = A_0 + A_1 T + A_2 T^2 + \ldots + A_N T^N, \]

where \( C \) represents the X, Y, Z coordinate value; \( T \) is time interval with respect to the reference time; \( A_0 \) through to \( A_N \) are coefficients of the polynomial to be adjusted to fit the source (ephemeris) data. However, this form of polynomial may fail as the order of the polynomial is increased, or lose accuracy as the order of the polynomial is decreased because of a dynamic range problem [36]. Chebyshev polynomial algorithms avoid these limitations, while using a relatively simple recursive algorithm for computing the function's value, even at the limits of interval. In this study, Chebyshev polynomial interpolation is used for the representation of the satellite orbit. Experimental results are based on the precise ephemeris of the TOPEX/Poseidon and CHAMP missions. The comparisons between the precise ephemeris and the interpolated results permit an evaluation of the accuracy of Chebyshev polynomial interpolation.
4.2 The method of Chebyshev polynomial interpolation

A Chebyshev polynomial is a formula that allows for wave shaping to take place. Assume that these points fitting are equidistant. Chebyshev polynomial $T(x)$ of degree $n$ is a special orthogonal polynomials function defined on the domain $[-1, 1]$, given by [37]:

$$T_n(x) = \cos(nx, \arccos x)$$ (4.1)

for $|x| \leq 1$, and may be recursively decomposed by:

$$T_1 = 1$$
$$T_2 = x(t)$$
$$T_3 = 2x(t)T_2 - T_1$$

$$\cdots$$

$$T_n = 2x(t)T_{n-1} - T_{n-2}$$ (4.2)

The property which makes these polynomials so well suited for the approximation of orbit function is their behaviour within the interval $[-1, -1]$. The absolute value of each $T_n(x)$ is always less than or equal to one for $-1 \leq x \leq 1$. Therefore, given an approximation:

$$f(t) = \sum_{i=1}^{n} c_i T_i(x)$$ (4.3)

of a function $f(t)$ over the finite time interval $[a, b]$ that is mapped to $[-1,1]$ by the following transformation:

$$x = 2 \frac{x-a}{b-a}$$ (4.4)

In other words, the domain $[-1, 1]$ is the standard range. The Chebyshev polynomials interpolation can be applied to any arbitrary time interval other than $[-1, 1]$ by mapping its domain $[-1, 1]$ onto the range of interest [38].

A satellite position $X_s$ is a continuous function of time. For three components of $x(t)$, $y(t)$, and $z(t)$, the function (4.3) is:

$$X_s(t) = \begin{bmatrix} x_s(t) \\ y_s(t) \\ z_s(t) \end{bmatrix} = \begin{bmatrix} T(t)C_x \\ T(t)C_y \\ T(t)C_z \end{bmatrix}$$ (4.5)
where T is 1-by-n vector $T(t) = [T_1, T_2, \ldots, T_n]$, composed by 1 to $n$ degree Chebyshev polynomials. $C_x$ is the $n$-by-$1$ vector $ix$, which is to be estimated with known $x$ at the sample points, as for the $C_y$ and $C_z$.

For an orbit to be fitted, there are sample values at the points of K, where K>n. The linear equations for the x component are:

$$
\begin{bmatrix}
  x_1(t_1) \\
  x_2(t_2) \\
  \vdots \\
  x_K(t_K)
\end{bmatrix}
= 
\begin{bmatrix}
  T_1(t_1) & T_2(t_1) & \cdots & T_n(t_1) \\
  T_1(t_2) & T_2(t_2) & \cdots & T_n(t_2) \\
  \vdots & \vdots & \ddots & \vdots \\
  T_1(t_K) & T_2(t_K) & \cdots & T_n(t_K)
\end{bmatrix}
\begin{bmatrix}
  C_{x_1} \\
  C_{x_2} \\
  \vdots \\
  C_{xn}
\end{bmatrix}
$$

which may be written as:

$$
X_s = T_x C_x
$$

The least-squares estimation of the Equation (4.7) is given by:

$$
C_x = (T_x^T T_x)^{-1} T_x^T X_s(t)
$$

Similarly one can obtain the solution for $C_y$ and $C_z$:

$$
C_y = (T_y^T T_y)^{-1} T_y^T Y_s(t)
$$

and

$$
C_z = (T_z^T T_z)^{-1} T_z^T Z_s(t)
$$

Theoretically the optimal degree of the polynomials may exist for each component in the fitting. In practice, after testing with a certain orbit, the same order may be selected for all three components, thus $T_x=T_y=T_z$.

After obtaining the coefficients, one can compute the position at time epoch using the following equations:

$$
X_s(t) = T_x(t) C_x
$$

$$
Y_s(t) = T_y(t) C_y
$$

$$
Z_s(t) = T_z(t) C_z
$$

For some applications, it is necessary to compute the velocities of the spacecraft at any time from the Chebyshev approximation of the spacecraft position. The derivative of a given Chebyshev approximation is therefore required:

$$
f'(t) = \sum_{i=1}^{i=n} c_i \dot{T}_i(x)
$$

where

$$
\dot{T}_i = 0
$$
\[
\begin{align*}
\dot{T}_2 &= x(t) \\
\dot{T}_1 &= 4x(t) \dot{T}_2 \\
\dot{T}_n &= 2 \frac{n-1}{n-2} x(t) T_{n-1} - \frac{n-1}{n-3} T_{n-2}
\end{align*}
\] (4.11)

Replacing \(T_n\) with \(\dot{T}_n\) in Equation (4.10), the velocity components can be obtained [39].

### 4.3 The software structure for FedSat mission

![Block diagram of the orbital interpolation software](image)

In the FedSat mission, Ka-band tracking relies on precise orbit determination (POD) and prediction solutions from GPS flight measurements. The POD solutions are
provided by the QUT POD software, which produces FedSat orbit data at 60-second intervals. To support the engineering needs, a software module was designed to read the standard QUT orbit text file and estimate the Chebyshev polynomial coefficients. These were then used to interpolate the position and velocity at any time instant, to determine tracking parameters such as elevation, azimuth, and satellite latitude, longitude and altitude and Doppler rates. Figure 4.1 show the flow chart of the main program structure.

4.4 Experimental results

The purpose of the experimental studies was to analyse the performance of the Chebyshev polynomial interpolation for FedSat orbital representation. The experimental results are based on 48-hour precise orbit ephemeris of the TOPEX/Poseidon and CHAMP missions. Both of TOPEX/Poseidon and CHAMP are LEO satellites. Their orbit altitudes are 1340 km and 450 km respectively. In order to improve the proposed method and to analyse the accuracy of interpolation, interpolated results were compared with precise orbit ephemeris. The residuals shown in the following figures illustrate the interpolated results of TOPEX, CHAMP satellite position and velocity respectively (are all within millimetres). This internal agreement demonstrates the excellent fitting behaviour of the method.

![Figure 4.2. Positional residuals of the TOPEX orbit approximation at millimetre levels.](image-url)
Figure 4.3. Velocity residuals of TOPEX orbit approximation, compared with the interpolated velocity and real velocities at sample points, showing the agreement at the 10\textsuperscript{th} of millimetre levels.

Figure 4.4 Positional residuals of the CHAMP orbit approximation at sub-millimetre levels.
Figure 4.5. Velocity residuals of CHAMP orbit approximation, compared with the interpolated velocity and real velocities at sample points, showing the agreement at the 10\textsuperscript{th} of millimetre levels.

The computed coordinates at the data points 30 seconds were compared, which were not used in the Chebyshev approximation. Figure 4.6 illustrates the interpolating errors for positional components, while Figure 4.7 illustrates velocity components for the CHAMP data. The interpolation errors are at the millimetres and sub-millimetre per second levels respectively.

Figure 4.6. Interpolation errors of CHAMP positions at millimetre level
Figure 4.7. Interpolation errors of CHAMP velocity at the levels of sub-millimetres per second

Table 4.1. An example of FedSat POD outputs

<table>
<thead>
<tr>
<th>#</th>
<th>2003</th>
<th>3</th>
<th>24</th>
<th>2</th>
<th>23.0000000</th>
<th>60.0000000</th>
<th>52722</th>
<th>0.0849884259259</th>
<th>2881</th>
<th>QUT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

* 2003 3 24 2 23.0000000
SV01 -3799.711688 5879.271609 -1619.047212 2.28714483 -0.48088349 -7.17147490
* 2003 3 24 2 3 23.0000000
SV01 -3655.415402 5838.494761 -2045.915917 2.52058682 -0.87821627 -7.05289140
* 2003 3 24 2 4 23.0000000
SV01 -3497.507293 5773.917581 -2464.848247 2.74072991 -1.27395298 -6.9070004
* 2003 3 24 2 5 23.0000000
SV01 -3326.810224 5685.685259 -2874.224362 2.94675487 -1.66644729 -6.73445923
* 2003 3 24 2 6 23.0000000
SV01 -3144.194077 5574.041877 -3272.463427 3.13791393 -2.05463256 -6.53591030
* 2003 3 24 2 7 23.0000000
SV01 -2950.571393 5439.327627 -3658.029693 3.31353346 -2.43518224 -6.3127407
SV01 4052.595191 -5823.880152 1073.863096 -1.99432359 -0.02554888 7.28461625
* 2003 3 26 1 58 23.0000000
SV01 3925.203737 -5813.604247 1508.563730 -2.25012483 0.36836317 7.20069617
* 2003 3 26 1 59 23.0000000
SV01 3782.822230 -5779.659660 1937.384773 -2.49382722 0.76312641 7.08869216
* 2003 3 26 2 0 23.0000000
SV01 3626.205635 -5722.045248 2358.653532 -2.72447359 1.15708098 6.94903431
* 2003 3 26 2 1 23.0000000
SV01 3456.14390 -5640.859302 2770.726464 -2.94117275 1.54856764 6.78226396
* 2003 3 26 2 2 23.0000000
SV01 3273.560326 -5536.299633 3171.995797 -3.14310338 1.93592961 6.58903162

Table 4.1 is an example of FedSat POD orbit data in SP3 format, providing coordinates at 60 seconds for 48 hours. Table 4.2 is an example of Chebyshev polynomial coefficients obtained with a data arc of typically 25 minutes. It is observed that the
optimal degree of the polynomial is different from one component to another, ranging from 20 to 22. Therefore, in practice, a fixed degree of 21 has been chosen for the coordinates in the computations.
Table 4.2: Chebyshev polynomial interpolation coefficients (with 25 data points)

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>vx</th>
<th>vy</th>
<th>vz</th>
</tr>
</thead>
<tbody>
<tr>
<td>sigma</td>
<td>0.000406</td>
<td>0.000594</td>
<td>0.000673</td>
<td>0.000005</td>
<td>0.000005</td>
<td>0.000004</td>
</tr>
<tr>
<td>degree</td>
<td>22</td>
<td>21</td>
<td>22</td>
<td>21</td>
<td>22</td>
<td>21</td>
</tr>
</tbody>
</table>

co[01]: -1272268.398346 3308035.720845 -5096030.949427 31.2518467452 -37.2849276325 -32.0794206125
c0[02]: 2700159.454615 -3221417.739043 -2771661.499406 1.1365292226 -6.7480067086 9.0268936850
c0[03]: 98206.834622 -583027.841541 779923.443592 0.8698490067 0.8720293658 0.762349936

c0[04]: -75155.030737 75343.327146 65866.641023 -0.0004672761 0.0890873473 -0.1065143462
c0[05]: -40.354283 7697.043134 -9202.885409 0.0069385953 -0.061319579 -0.0049278626
c0[06]: 599.448128 -529.809801 -425.690688 -0.000089592 -0.0004124798 0.0004963482

c0[07]: -7.763985 -35.670790 42.859593 -0.000240780 0.0000234690 0.0000066223

c0[08]: -2.089718 2.020468 0.588549 0.0000011903 0.0000003373 -0.0000019837

c0[09]: 0.101384 0.017225 -0.171757 0.000002725 -0.000005105 0.000002469

c0[10]: -0.09749 -0.049693 -0.019509 -0.000001044 0.0000001372 0.0000003877

c0[11]: -0.019066 0.019384 0.053973 0.000001298 0.0000001866 -0.000000784

c0[12]: 0.039646 0.012829 -0.087857 0.000000386 0.000000187 -0.000000914

c0[13]: -0.011291 0.018740 0.025138 0.000000246 0.000000370 0.000000574

c0[14]: 0.061978 -0.004866 -0.090421 -0.000000091 0.000000152 0.000000138

c0[15]: -0.015505 0.025949 0.035530 0.000000048 -0.000000049 -0.000000233

c0[16]: 0.052441 -0.00673 -0.085323 -0.000000012 -0.000000466 -0.000000201

c0[17]: -0.011443 0.016783 0.025005 0.000000058 -0.000000221 -0.000000118

c0[18]: 0.035235 -0.001498 -0.056878 0.000000045 -0.000000426 -0.000000097

c0[19]: -0.006244 0.009579 0.014311 0.000000045 -0.000000111 0.000000006

c0[20]: 0.016385 0.00097 -0.026675 -0.000000006 -0.000000170 -0.000000093

c0[21]: -0.002739 0.04809 0.04895 0.000000040 -0.000000079 -0.000000064

c0[22]: 0.004170 -0.007691 -0.000000099
4.5 Summary

The orbit interpolation algorithm using Chebyshev polynomials has been described and tested with three orbits. The results demonstrate that the orbit interpolation based on 60 second data rates results in no loss of orbit accuracy at all. The optimal choice for the degree of the Chebyshev polynomial functions depends on the number of data points. In the FedSat case, where the maximum data arc of 25 minutes per pass is used, the optimal degree is about 21 or so for each coordinate component. In general, the Chebyshev polynomial function is very suitable for Fed Sat orbit interpolations.

The described software has been provided to the UTS Ka-band group for daily operational use. This is one of the contributions of the research made to the FedSat mission.
Chapter 5

Improvement of FedSat onboard navigation solutions

5.1 The problem of FedSat onboard navigation solutions

Normally a GPS receiver onboard a LEO uses the up-looking antenna for onboard navigation, timing and precise orbit determination purposes. FedSat collects GPS data with its only aft-looking antenna, for both orbit determination and scientific applications, including atmosphere occultation and ionosphere studies. As a result, the view field of the antenna is only two-third of the hemisphere, as shown in Figure 5.1, with about half the measurements observed at negative elevation. Another problem is that FedSat is in a duty-cycle operation mode, That is, the GPS onboard FedSat only operates 10 to 20 minutes per orbit to minimise power consumption. The availability of GPS measurements is therefore greatly reduced compared to other LEO missions that utilize GPS.

5.2 FedSat in-orbit performance

The GPS antenna on FedSat is looking toward the anti-velocity direction. According to the altitude of the FedSat and the radius of the Earth, the negative elevation can reach -7.5 degree. Therefore, the view field of the aft-looking antenna is approximately two-third of the hemisphere. The mask angles for orbit determination or positioning should be 90 and -25 degrees respectively. The GPS satellite singles with the elevations between -25 and -27.5 degrees are occulted / delayed, and these measurements may be used for atmospheric occultation studies [40].

FedSat GPS measurement quality

The quality of actual flight GPS data may vary from mission to mission, although the same type of receiver may be used. In the discussion below, analysis of code measurement noise level is based on the following equations [41]:

\[ P1M (t) = [P1 (t+1)-P1 (t)] - ([L1 (t+1)-L1 (t)] \]
\[ PCM (t) = [PC (t+1)-PC (t)] - ([LC (t+1)-LC (t)] t=1,2,3 \]  

(5.1)
where PC is ionosphere-corrected code measurements; $\lambda$ is the wavelength of L1 frequency (1575.42MHz); P1M and PCM mainly contain receiver noise and multipath errors. The RMS values of the observations P1 and PC are given as:

$$
\sigma_{P1} = \sqrt{\frac{\sigma_{P1M}^2}{2}}, \quad \sigma_{PC} = \sqrt{\frac{\sigma_{PCM}^2}{2}}
$$

(5.2)

Figure 5.1. FedSat aft-looking antenna viewing the two-third of the hemisphere.

Figure 5.2. Scattered points for P1 ranging noises plotted against the elevation for Day 364, 2002. The overall P1 ranging RMS value is 1.03m for the error rejection threshold of $\pm 15$m and 0.89m for the threshold of $\pm 5$m.
Due to variations of the atmospheric conditions between epochs, $\sigma_{P1}$ and $\sigma_{PC}$ are conservative estimates of the standard deviation for the measurements $P1$ and $PC$. Figure 5.2 illustrates $P1$ ranging noises against elevation for Day 364/02. The overall $P1$ range RMS value for the data set is 1.03m. The RMS was estimated with rejection thresholds of $\pm 15$m (that is, the $P1$ ranging noises outside $\pm 15$m were excluded).

Figure 5.3 is the histogram of the $P1$ ranging errors for the period Day 083 to Day 086, 2003, showing characteristics of normal distribution. The threshold of $\pm 5$m was set for both RMS estimation and follow-up orbit estimation. Table 5.1 lists the RMS values for different data sets against elevation. It is observed that the GPS data with elevation angles below 10 degrees are normally noisier than those with higher elevation angles for the CHAMP and SAC-C missions, where the flight data for orbit determination were collected with an up-looking antenna. However, in the FedSat case, the following was observed:

- The overall RMS of the FedSat ranging errors is comparatively high compared to CHAMP and SAC-C data. For instance, $RMS_{FEDSAT}$ is nearly twice $RMS_{CHAMP}$, and three times $RMS_{SAC-C}$ if the error rejection threshold is $\pm 15$m.
- Applying the threshold of $\pm 5$m as the error rejection criteria to the data sets over the period Day 083 to Day 086, the overall RMS is below 0.60m see Figure (5.3), which is close to the RMS of 0.52 m for the CHAMP mission.
The data with elevation angle below –27 degrees are much noisier than those above –27 degrees. Other than this, the ranging measurements with negative elevation are not necessarily/significantly noisier than those with positive elevations.

Table 5.1. Summary of the RMS values from different satellite missions against elevation angles

<table>
<thead>
<tr>
<th>Mission</th>
<th>Receiver/antenna</th>
<th>Orbit period</th>
<th>All data</th>
<th>0&lt;Elev&lt;10</th>
<th>10&lt;Elev&lt;20</th>
<th>Elev&gt;25</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHAMP</td>
<td>Blackjack</td>
<td>4 days</td>
<td>0.53m</td>
<td>0.93m</td>
<td>0.64m</td>
<td>0.42m</td>
</tr>
<tr>
<td>SAC-C</td>
<td>TurboRogue</td>
<td>4 days</td>
<td>0.32m</td>
<td>0.73m</td>
<td>0.46m</td>
<td>0.17m</td>
</tr>
<tr>
<td>Topex Poseidon</td>
<td>Motorola Monarch</td>
<td>4 days</td>
<td>0.342m</td>
<td>0.38m</td>
<td>0.385m</td>
<td>0.329m</td>
</tr>
<tr>
<td>FedSat</td>
<td>Blackjack</td>
<td>364/02/02 (±15m)</td>
<td>1.03m</td>
<td>4.00m</td>
<td>0.90m</td>
<td>1.10m</td>
</tr>
<tr>
<td>FedSat</td>
<td>Blackjack</td>
<td>083/03/03 (±5m)</td>
<td>0.75m</td>
<td>1.49m</td>
<td>0.84m</td>
<td>0.67m</td>
</tr>
</tbody>
</table>

Quality of FedSat onboard navigation solutions

The FedSat Orbit Determination and Tracking (FODT) software developed at CRCSS/QUT using batch least squares estimation technique was used to process the above data sets to produce an orbit filtering solution for each day. To assess the performance of the FedSat onboard navigation solutions, we conduct the comparisons were made between the FODT and filtering solution and the onboard navigation solutions (ONS). Figure 5.4 illustrates the x, y and z errors of ONS against the FODT filtering solutions for the data points sampled over four days. Figure 5.5 is the histogram of all coordinate errors, indicating the normal distributions with long tails. For these results, we have the following comments can be made:

- There are many gross errors in the FedSat onboard navigation solution. Although the 1-sigma accuracy of each component is about 20m, there are over 11% of positioning error fall outside +/-50m, and 5% of errors outside the 100m threshold. The 3D RMS values are 35m, 87m, and 173m for the three cases respectively.
• The FedSat ONS uncertainties are approximately three times greater than those from other satellite missions.

Figure 5.4. Illustration of FedSat onboard navigation solution errors, differences between FedSat daily FODT filtering and FedSat ONS solutions over the period Day 084 to 087, 2003.

Figure 5.5 is the histogram of all the coordinate errors, showing the normal distribution characteristics, with long tails due to many gross ONS positional errors.
There are a number of error sources that explain the poor quality of Fed Sat onboard navigation solutions. Poor code measurement quality as demonstrated early, and poor satellite geometry due to the aft-looking antenna is two obvious reasons. Due to the high percentage of outlier solutions, it would be dangerous to use these data without application of detection and exclusion procedures. In the following sections, methods for improving the navigation are suggested.

5.3 Improvement of FedSat navigation solutions with geometric methods

There are basically two categories of methods to improve the LEO onboard solutions. The first are the geometric or kinematic methods, which make use of delta-position solutions derived from carrier phase differences between two epochs to smooth the code-based navigation solutions. The second are dynamic methods, which use orbital dynamics information for orbit improvements. In this section, a method from the first group developed and tested with real flight data. The method for estimation of Two Line Elements from the position solutions is actually a simplified dynamic method for improvement of LEO GPS navigation solutions, which will be discussed in Chapter 6.

5.3.1 Smoothing methods with carrier phase measurements

When pseudoranges and continuous carrier phase are brought together, point-positioning solutions based on code measurements can be improved by exploiting the precise delta-position solutions obtained by phase difference between two consecutive phase measurements. The concept is illustrated in Figs 5.7a-c. A sequence of $N$
independent navigation solutions \( \hat{x}_k \) is shown in Figure 5.7a. The true time-varying orbit is represented by the dashed line.

![Figure 5.7. Concept of Smoothing Methods with carrier phase measurements](image)

(a) Successive code based positional measurements

(b) Successive measurements of continuous (but biased) delta carrier phase differences between from epochs

(c) Absolute phase derived by adjusting mean of b to mean of a

Figure 5.7. Concept of Smoothing Methods with carrier phase measurements

If \( x_k \) is the true position at time \( t_k \), one can write the following:

\[
\hat{x}_k = x_k + n_k
\]

(5.3)

where, \( \hat{x}_k \) are the navigation solutions derived by point positioning based on code measurements.

\[
\hat{x}_k = (B_k^T B_k)B_k^T p_k
\]

\( B_k \) is the design matrix and \( p_k \) is the difference between computed and observed code ranges.

For simplicity, one assumes \( n_k \) is a white noise process with standard deviation \( \delta_n \).

With carrier phase differences between epochs:

\[
\Delta\hat{x}_{k,k-1} = (B_k^T B_k) B_k^T (L_k - L_{k-1})
\]

Figure 5.6b shows the record of positional change obtained by tracking carrier phase over the same arc. This can be regarded as a series of positions, \( \bar{x}_k \), having a much smaller random error and a common bias. Thus, one can write:
\[ \bar{x}_k = \hat{x}_1 + \Delta \hat{x}_{2,1} + \Delta \hat{x}_{3,2} + \ldots + \Delta \hat{x}_{k-1,k-1} = x_k + b + e_k \quad (5.4) \]

where \( b \) is the bias and \( e_k \) is a white noise process with standard deviation \( \sigma_e \).

Estimating the bias \( b \) by averaging the difference between the \( \bar{x}_k \) and \( \hat{x}_k \)

\[ \hat{b} = \frac{1}{N} \sum_{k=1}^{N} \bar{x}_k - \hat{x}_k \quad (5.5) \]

or

\[ \hat{b} = \frac{1}{N} \sum_{k=1}^{N} e_k - n_k \quad (5.6) \]

Because \( \sigma_e \) is typically much smaller than \( \sigma_n \), the approximate component error on the bias estimate is

\[ \sigma_b = \frac{\sigma_n}{\sqrt{N}} \quad (5.7) \]

Thus, 20 metre-level random noise on 1-s pseudorange-derived coordinates can give a metre level bias estimate within a few minutes. Subtracting Equation (5.6) from Equation (5.4) eliminates the bias in Equation (5.4) to give a precise record of absolute positions. As shown in Figure 5.7c, the corrected navigation solutions sit close to, and have nearly the exact shape of, the true coordinate sequence. The corrected navigation solutions will have an approximate error:

\[ \sigma_x = (\sigma_b^2 + \sigma_e^2)^{\frac{1}{2}} \quad (5.8) \]

where \( \sigma_b \) represents the residual bias common to all data points and \( \sigma_e \), is the point-to-point random error. A sequence of position solutions derived from the corrected initial position at epoch 1 will have the precision of a pure carrier-based positional solution, with a bias that is a fraction of the typical onboard navigation position error.

This technique can be readily generalised to provide real-time recursive estimation of the position of an unpredictably moving space vehicle. Consider a receiver that produces an instantaneous point position solution \( \hat{x}_k \) at time \( t_k \), and a position change solution \( \Delta \hat{x}_k \), obtained by continuously tracking carrier phase from \( t_{k-1}, t_k \). An estimate \( \hat{X}_{n+1} \) of the position at time \( t_{n+1} \), is given by:

\[ \hat{X}_{n+1} = \frac{n}{n+1} (\hat{X}_n + \Delta \hat{x}_{n+1,n}) + \frac{1}{n+1} \hat{x}_{n+1} \quad (5.9) \]

Note that this is a variation on the recursive formula for a simple average:
The position change information $\Delta \hat{x}_{n+1}$, maps the current position estimate $\hat{X}_n$ forward to the next time point for averaging with the point position $\hat{x}_{n+1}$, computed at that time. Carrier phase, in effect, aids the sequential averaging of point position solutions to refine the phase bias estimate. The procedure can be tuned by weighting each $\hat{x}_k$ by its inverse covariance.

A principal virtue of this technique is its simplicity. A filter to track unpredictable motion (or the relative positions of multiple vehicles) can be realised in a few lines of software code. It is, however, sub-optimal. Correlations between the $\Delta \hat{x}_{k,k-1}$ are not properly accounted for, and it does not fully exploit the information in the carrier phase. Another drawback is its exclusion of external information about platform dynamics. The solution becomes vulnerable to outages that might easily be bridged with simple dynamic models. These weaknesses are remedied in a more robust technique that employs the Kalman filter formalism [23].

### 5.3.2 Experiment results with SAC-C flight data

The algorithms were tested using a 24-h SAC-C GPS data set collected for 12 February, 2002, with an up-looking antenna [42]. Figure 5.8 illustrates the 3D orbit errors resulting from JPL precise GPS orbits with smoothing techniques. The 3D orbit errors were computed against JPL’s POD SAC-C orbit solutions. It is obvious that the improvement of 3D orbit accuracy has been achieved via a smoothing process over different smoothing periods. The results also demonstrate the limitations of the methods, which cannot eliminate these outlying errors in some cases, where both the onboard navigation solutions and delta-positional solutions suffer from the effects of poor satellite geometry. Another observation is that the optimal smoothing period for SAC-C data is 5 to 10 minutes. Smoothing over longer periods does not necessarily produce better results.
Figure 5.8. illustrates the 3D orbit errors resulted from precise GPS orbits with smoothing techniques.
Chapter 6

Orbit representation with Two Line Elements

6.1 Overview of Two Line Elements

The concept of representing an orbit with Two Line Elements (TLE) has been mentioned in Chapter 1. TLE is the NORAD/NASA modification of Keplerian Elements that describe a mean orbit (including orbital perturbations and their long and short-term periodic variations.) Most tracking programs use the Space Command Simplified General Perturbation (SGP4) orbit propagation algorithms in order to maintain compatibility with the Keplerian elements. FedSat ground operations depend on the daily NORAD TLE sets for TT&C communication with the satellite.

In order to determine the position of an Earth-orbiting object using the standard NORAD TLE sets, it is necessary that the proper orbital model be used. Since the observations taken by NORAD are reduced to orbital elements using the SGP4 model, the SGP4 model must be used to then generate accurate determination of an object's position and velocity. The primary reason for this is that different orbital model handle perturbations in a different manner.

The SGP4 orbital model takes into account the following perturbations:

- atmospheric drag (based on a static, non-rotating, spherically-symmetric atmosphere whose density can be described by a power law),
- fourth-order zonal geopotential harmonics (J2, J3, and J4),
- spin-orbit resonance effects for synchronous and semi-synchronous orbits, and
- solar and lunar gravitational effects to the first order

The latter two terms are less important for low-Earth orbit, for period less than 225 minutes. Therefore, there are two classes of SGPS 4 models: SPG4 for objects in orbits with periods less than 225 minutes, and SDP4 for objects in orbits greater than or equal to 225 minutes. Details of SPG4 and SDP4 are found in to [43].

Table 6.1 describes the NORAD TLE set format with the following ‘mean’ elements:

\[ n_0 = \text{the SGP type “mean” mean motion at epoch} \]
e₀ = the “mean” eccentricity at epoch
i₀ = the “mean” inclination at epoch
M₀ = the “mean” mean anomaly at epoch
ω₀ = the “mean” argument of perigee at epoch
Ω₀ = the “mean” longitude of ascending node at epoch
\( \dot{n}_0 \) = the time rate of change of “mean” mean motion at epoch
\( \ddot{n}_0 \) = the second time rate of change of “mean” mean motion at epoch
B = the SGP4 type drag coefficient

The original ‘mean’ mean motion and ‘mean’ semi-major axis are recovered from these elements, including: \( n_0 \), \( e_0 \), \( i_0 \), gravitational constant GM, the equatorial radius of the Earth, and the second gravitational zonal harmonic of the Earth. Based on these ‘mean’ elements, time since epoch and other gravitational parameters, the orbit elements at any other time can be computed.

In general, NORAD TLE method can be summarised as follows:

- Firstly, it can be considered a simplified orbit determination method: estimation of six mean orbit elements and one drag coefficient with reduced orbit dynamic force models over certain observation arcs. NORAD uses measurements such as directions and distances from their ground-tracking network. However, other information, such as GPS code measurements or onboard navigation solutions, can be used for the estimation of these elements.

Data for each satellite consists of three lines in the following format:

```
AAAAAAAAAAAAAAAAAAAAAAAAAAAA
1 NNNNNN NNNNNAAA NNNNN NNNNNNNN +NNNNNNNN +NNNNN-N +NNNNN-N N NNNNN
2 NNNNN NNN.NNNN NNN.NNNN NNNNNNN NNN.NNNN NNN.NNNN NNN.NNNN NNN.NNNN NNN.NNNN NNN.NNNN NNN.NNNN
```

Example for Fed Sat:
```
FEDSAT
1 27598U 02056B 03346.12531011 .00000038 00000-0 33287-4 0 2759
2 27598 98.6307 56.8337 0009821 29.2062 330.9678 14.27804023 51811
```

Line 0 is a twenty-four character name (to be consistent with the name length in the NORAD SATCAT). Lines 1 and 2 are the standard Two-Line Orbital Element Set Format identical to that used by NORAD and NASA. The format description is given as the follows
Secondly, it uses the six ‘mean’ elements and one drag coefficient as parameters and SGP4 specified force models to compute the position and velocity at any time over an orbit. Therefore, propagation of orbit with SGP4 models from the TLE set is an orbit representation method.

Thirdly, the same models must be used in both TLE estimation and orbit propagation with TLE data sets to maintain orbit accuracy. In other words, use of precise force models in TLE estimation, despite it being feasible, would lead to poor
orbit accuracy if the propagation models are not the same as accurate as those used in the TLE estimation.

In the following section, firstly, the technical description for orbit determination of LEO mean orbit elements with positional measurements is presented. The numerical treatment of the estimation of TLE elements is also outlined. Secondly, the FedSat orbit accuracy achieved with different modelling schemes is experimentally tested. This orbit accuracy is referred to as orbit representation accuracy with the proposed method. The findings of this chapter are summarised in the final section.

### 6.2 TLE estimation with positional measurements

As mentioned previously, the FedSat operation depends on NORAD TLE data sets updated daily. To be able to autonomously track the FedSat, it would be necessary to create the TLE data instead. A straightforward solution is to create TLE from the FedSat navigation solutions, or predicted x, y, z solutions over a longer orbit data arc, for instance, 24 or 12 hours. In either case, the problem now is to determine the TLE data sets from positional measurements. The basic description for the numerical orbit estimation method was given in Chapter 2, in particular, Section (2.3) and Section (2.4). The orbit propagation using Equation (2.9) and the orbit estimation with the method as described in Section (2.4). However, algorithms have to be developed in order to address the problem properly. In the following paragraphs the TLE estimation procedures with SGP4 models is first described, followed by the numerical strategies for TLE estimation.

#### 6.2.1 The algorithm of TLE estimation with positional measurements

The SGP4 model, which is denoted by the symbol $S$, relates the spacecraft state vector:

$$Y(t) = \begin{pmatrix} r(t) \\ v(t) \end{pmatrix} = S(\vec{a}_0, B, t)$$

(6.1)

at time $t$ to a set of (SGP4 specific) mean elements $\vec{a}_0 = (\vec{a}_0, \vec{e}_0, \vec{i}_0, \vec{O}_0, \vec{\omega}_0, \vec{M}_0)$ at epoch $t_0$ and a ballistic coefficient $B$ describing the effective area-to-mass ratio. Here, we have chosen a Keplerian set of orbital elements, which is identical to the NORAD
elements set, except for the semi-major axis $a$. The latter parameter is identical to the "original mean motion $a$" of the SGP4 theory that results from the "mean mean motion" $n$ by removing the secular $J$ perturbations from the associated Keplerian semi-major axis. SGP4 is considered as a 6-dimensional, continuous and differentiable function of time, depending on seven dynamic parameters. Because of the well-known singularity of Keplerian orbital elements for orbits that are either circular or equatorial, a different parameterisation of the SGP4 model is, however, required for the adjustment of orbital parameters from observations. Therefore, the concept of a "mean SGP4 state vector" is introduced, which is free from singularities, and in the ideal sense free from orbital perturbations. Making use of the well known mapping between osculating Keplerian elements $a = (a,e,i,\Omega,\omega,M)$ and the six-dimensional (osculating) state vector $y = k(a)$, one can define the mean state vector at epoch $t_0$ by the expression:

$$\bar{y}_0 = k(\bar{a}_0) \quad (6.2)$$

where, again, $\bar{a}_0$ denotes the SGP4 mean elements at the same epoch. The inverse function of Equation (6.2) is [44]:

$$\bar{a}_0 = k^{-1}(\bar{y}_0) \quad (6.3)$$

Equations (6.1) and (6.2) may be combined into the resulting expression:

$$y(t) = s(k^{-1}(\bar{y}_0),B,t) = s(x,t) \quad (6.4)$$

which relates the osculating state vector for a given time $t$ to the combined parameter vector $x = (\bar{y}_0,B)$ via the composite functions. Compared to the original formulation, $s$ is non-singular even for circular or equatorial orbits, and the partial derivatives of $s$ with respect to the orbital parameter $x$ can be well defined.

### 6.2.2 Osculating to mean state vector conversion of SGP4 model

For epoch $t_0$ and given ballistic coefficient $B$ (e.g. $B=0$), Equation (6.4) may be inverted using a fixed point iteration of the form:

$$\bar{y}_0^{(i)} = y_0, \quad \bar{y}_0^{(i+1)} = \bar{y}_0^{(i)} + (y_0 - s((\bar{y}_0^{(i)}),B,t_0)) \quad (6.5)$$
to find the mean state $\bar{y}_0$ at this epoch from the corresponding osculating state vector $y_0 = y(t_0)$. While Equation (6.5) provides a useful point-to-point conversion from osculating to mean state vectors, the result is only approximate due to inevitable modelling deficiencies in the SGP4 theory. Considering the neglected higher order perturbations as well as sectorial and tesseral gravity field components, Equation (6.4) should properly be expressed as:

$$y(t) = s((\bar{y}_0, B), t) + e(t),$$

(6.6)

where $e$ denotes the time-dependent model errors. As a rule of thumb, the neglected terms in the SGP4 orbit model give rise to position errors of 2km. Assuming that these errors have a zero mean value over multiple revolutions [45], an iterative least-squares fit may be used for a rigorous determination of $\bar{y}_0$ from a given $y_0$, which is expressed as:

$$y_i = y(t_i, y_0)$$

(6.7)

giving a reference trajectory at discrete time steps $t_i$ computed from the osculating state at epoch $t_0$ using a reliable force model and numerical integration.

The difference in the three position components between the reference trajectory is given by Equation (6.7) and the one computed by the SGP4 orbit model (equation (6.4)) can be defined as:

$$\delta z_i = \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}_i$$

(6.8)

The reference position vectors here can be used as measurements, which can be expressed by the function:

$$h_i = \begin{bmatrix} I_{3 \times 3} \\ 0_{3 \times 3} \end{bmatrix} \cdot s(x, t)$$

(6.9)

Linearisation about the reference values $x_0^{ref} = (\bar{y}_0^{ref}, B^{ref})$ of the mean state vector at epoch and the ballistic coefficient then yields the least-squares solution:

$$x = x_0^{ref} + (H^T H)^{-1} (H^T \delta z).$$

(6.10)

which may be iterated until convergence is achieved. In Equation (6.10), $\delta Z$ is the combined measurement vector and $H$ denotes the partial derivatives of the associated measurement model vector with respect to the estimated parameters.
The resulting state vector is finally converted to SGP4 mean elements using the inverse of Equation (6.3).

In the above procedures, the technical basis for the estimation of TLE from a reference trajectory defined by positional vectors at discrete epochs and the SGP4 models was outlined. In applications, a reference trajectory could be defined by a precisely predicted orbit or an “observed orbit” is defined by GPS-derived positional onboard navigation solutions. The previous is on orbit representation problem where the TLE data sets are used to represent the orbit with certain level of accuracy; while in the latter case one directly adjusts the six orbit mean elements and one drag element to a given positional data of the orbit in a least-squares procedure. Due to the processing of all measurements in a single batch as well as the use of multiple iterations, the least-squares approach to the orbit determination problem is robust enough to handle erroneous data points or bad a priori parameter values. Furthermore, the process can be implemented in a self-starting manner, since an a priori state vector can always be derived from the GPS navigation measurements. This makes it particularly attractive for automated, ground-based orbit determination of satellites carrying GPS receivers, as in the case of the FedSat mission.

6.2.3 TLE estimation with numerical strategies

As indicated previously, the SPG4 model only includes the effects of the gravitational terms J2, J3 and J4. Due to the complexity of analytical expression for perturbation force terms, it is impractical to include more terms. However, as highlighted in Table 6.1, there are many coefficients under the order 6 and degree 6 having greater values than C30 or C40. In the following, the mean elements are directly estimate using numerical integration. On the one hand, this tests the new approach to determine TLE sets, and on the other hand, provides the means assessing the effects of gravitational
Chapter 6

perturbation forces of higher terms on the TLE accuracy. The numerical integration will
compute the reference trajectory, the partials of the coordinate respect to the TLE
parameters, and drag coefficient, as well as the orbit with simplified models. The
software for this study is based on the OPass software, as shown in Figure 3.4, used
for orbit performance analysis in Chapter 3.

6.3 Numerical results of TLE estimation

The estimation strategies are summarised in Table 6.2. Figure 6.1 is a plot of the along-
track orbit errors of the TLE estimates under Schemes II, III, IV and V against the
reference orbits computed under Scheme I. Figure 6.2 compares the along-track errors
of Scheme VI with respect to Scheme IV, while Figure 6.3 compares the errors of
Scheme VII against Scheme V.

From these three figures, one can make the following observations:

Table 6.2. Example of Earth Gravity Model 96 (6x6)

<table>
<thead>
<tr>
<th>N</th>
<th>m</th>
<th>( C_{nm} )</th>
<th>( S_{nm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>(-0.484165371736E-03)</td>
<td>0.000000000000E+00</td>
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<tr>
<td>2</td>
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<td>(-0.186987635955E-09)</td>
<td>0.119528012031E-08</td>
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<td>2</td>
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<tr>
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<td>0</td>
<td>(0.957254173792E-06)</td>
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</tr>
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<tr>
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<td>3</td>
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<td>1</td>
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<tr>
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<td>2</td>
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</table>
With the EGM 4x0, which is equivalent to the SPG4 model, the along-track orbit uncertainty of the TLE estimates reaches $\pm 1000m$ within the fitting data arc of 24 hours. This error range will reduce to $\pm 200m$ when the EGM4x4 is used, and $\pm 100m$ for the EGM10x10.

- Use of EGM2x2 can achieve the same the accuracy level as the EGM4x0.
- By shortening the fitting data arc from 24hour to 12hour, there is a slight improvement for TLE estimates with the EGM4x0 model.

In general, the studies show the effects of the unmodelled forces/perturbations in the SPG4 models on the Fed Sat orbit could reach $\pm 1000m$. This only includes the orbit representation errors with TLE data sets. The total FedSat orbit propagation should include both the orbit propagation and orbit representation terms.

Figure 6.1. Illustration of the along-track orbit errors of the TLE estimates under Schemes II, III, IV and V, plotted against the reference orbits computed under Scheme I
<table>
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<th>Fed Sat orbit Characteristics</th>
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<td>Numerical eccentricity</td>
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<tr>
<td>Right ascension of ascending node</td>
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<td>Argument of perigee</td>
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<tr>
<td>Mean Anomaly</td>
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<tr>
<td>Mean motion</td>
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<tr>
<td>Epoch</td>
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<table>
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<td>Atmospheric model</td>
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<tr>
<td>Solar and Lunar gravity</td>
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<tr>
<td>Data arc and sample rates</td>
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<td>Atmospheric model</td>
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<td>Solar and Lunar gravity</td>
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<th>Scheme III</th>
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<td>Atmospheric model</td>
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<tr>
<td>Solar and Lunar gravity</td>
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<tr>
<td>Data arc and sample rates</td>
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<table>
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<table>
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<th>Scheme VI</th>
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<table>
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<th>Scheme VII</th>
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<td>Solar and Lunar gravity</td>
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<td>Data arc and sample rates</td>
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Figure 6.2. Illustration of the along-track errors of Scheme VI with respect to Scheme III, showing the slight improvement if the fitting data arc is shortened from 24 hours to 12 hours.

Figure 6.3. Illustration of the along-track errors of Scheme VII with respect to Scheme V, showing the slight improvement if we shorten the fitting data arc is shortened from 24 hours to 12 hours.
Chapter 7

Conclusion

Focusing on two addressed operational needs in the FedSat mission, the research effort is directed towards the development and testing of simplified orbit determination and orbit representation techniques. The major contributions of this research:

- A covariance analysis has been performed before the launch of FedSat to assess the orbit performance under different operational modes. In summary, under the assumption of expected GPS standalone positioning performance, 3D positional RMS accuracy between 10m and 15m, the effective data set of 2-by-5 minute per orbit for 24 hours can still result in quality predicted orbits for 48 hours. Longer prediction may still be acceptable. Considering all the effects, including the atmospheric drag, the accuracy requirements of 100 metres in each component can be satisfied with 48 hours of prediction.

- A post-launch in-orbit performance was conducted with the flight data of several days, which demonstrated that there are many gross errors in the FedSat onboard navigation solution. Although the 1-sigma accuracy of each component is about 20m, over 11% of positioning error fall outside +/-50m, and 5% of the errors are greater than 100m. The 3D uncertainties would be 35m, 87m, and 173m in the three cases respectively. The FedSat ONS uncertainties are believed to be approximately three times greater than those from other satellite missions.

- Due to the high percentage of outlier solutions, it would be dangerous to use these data without implementing detections and exclusion procedures. Two simplified orbit determination methods have been proposed to improve the navigation solutions. One is the geometric method, which makes use of delta-position solutions derived from carrier phase differences between two epochs to smooth the code-based navigation solutions. The algorithms were described and tested using SAC-C GPS data, showing some improvement. Further tests will be required for the analysis of FedSat data, with an expectation of improvements. The second method is the dynamic method, which uses orbital dynamics information to improve the orbit parameters while detecting and eliminating outlier navigation solutions. This concept has been mentioned to this thesis, but not considered any further. However,
the estimation method for Two Line Elements has been discussed, and provides the technical basis for such the analyses.

- through the research efforts. FedSat onboard payload supports Ka-band tracking experiments, which require a pointing accuracy of 0.03 degree. The QUT GPS group provides the GPS precise orbit solutions on a daily basis to the Ka-band earth stations. As orbit determination and prediction software only provide satellite states at discrete time points, an orbit interpolation algorithm based on Chebyshev polynomials was developed to represent satellite orbit as continuous trajectory. This software has been by the UTS Ka-band ground station for daily operation. This is another useful contribution made
References


References


References


[42] “NORAD TWO-Line Element set format”,
http://www.celestrak.com/NORAD/elements/

