Advanced Analysis of Steel Frame Structures Subjected to Lateral Torsional Buckling Effects

By

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Lateral torsional buckling, Steel I-section, Rigid frame, Advanced analysis, Nonlinear analysis, Steel frame design, Ultimate Capacity, Structural Stability, load-deflection response, and Finite element analysis
Abstract

The current design procedure for steel frame structures is a two-step process including an elastic analysis to determine design actions and a separate member capacity check. This design procedure is unable to trace the full range of load-deflection response and hence the failure modes of the frame structures can not be accurately predicted. In recent years, the development of advanced analysis methods has aimed at solving this problem by combining the analysis and design tasks into one step. Application of the new advanced analysis methods permits a comprehensive assessment of the actual failure modes and ultimate strengths of structural steel systems in practical design situations. One of the advanced analysis methods, the refined plastic hinge method, has shown great potential to become a practical design tool. However, at present, it is only suitable for a special class of steel frame structures that is not subject to lateral torsional buckling effects. The refined plastic hinge analysis can directly account for three types of frame failures, gradual formation of plastic hinges, column buckling and local buckling. However, this precludes most of the steel frame structures whose behaviour is governed by lateral torsional buckling. Therefore, the aim of this research is to develop a practical advanced analysis method suitable for general steel frame structures including the effects of lateral-torsional buckling.

Lateral torsional buckling is a complex three dimensional instability phenomenon. Unlike the in-plane buckling of beam-columns, a closed form analytical solution is not available for lateral torsional buckling. The member capacity equations used in design specifications are derived mainly from testing of simply supported beams. Further, there has been very limited research into the behaviour and design of steel frame structures subject to lateral torsional buckling failures. Therefore in order to incorporate lateral torsional buckling effects into an advanced analysis method, a detailed study must be carried out including inelastic beam buckling failures.

This thesis contains a detailed description of research on extending the scope of advanced analysis by developing methods that include the effects of lateral torsional buckling in a nonlinear analysis formulation. It has two components. Firstly, distributed plasticity models were developed using the state-of-the-art finite element analysis programs for a range of simply supported beams and rigid frame structures to
investigate and fully understand their lateral torsional buckling behavioural characteristics. Nonlinear analyses were conducted to study the load-deflection response of these structures under lateral torsional buckling influences. It was found that the behaviour of simply supported beams and members in rigid frame structures is significantly different. In real frame structures, the connection details are a decisive factor in terms of ultimate frame capacities. Accounting for the connection rigidities in a simplified advanced analysis method is very difficult, but is most critical. Generally, the finite element analysis results of simply supported beams agree very well with the predictions of the current Australian steel structures design code AS4100, but the capacities of rigid frame structures can be significantly higher compared with Australian code predictions.

The second part of the thesis concerns the development of a two dimensional refined plastic hinge analysis which is capable of considering lateral torsional buckling effects. The formulation of the new method is based on the observations from the distributed plasticity analyses of both simply supported beams and rigid frame structures. The lateral torsional buckling effects are taken into account implicitly using a flexural stiffness reduction factor in the stiffness matrix formulation based on the member capacities specified by AS4100. Due to the lack of suitable alternatives, concepts of moment modification and effective length factors are still used for determining the member capacities. The effects of connection rigidities and restraints from adjacent members are handled by using appropriate effective length factors in the analysis. Compared with the benchmark solutions for simply supported beams, the new refined plastic hinge analysis is very accurate. For rigid frame structures, the new method is generally more conservative than the finite element models. The accuracy of the new method relies on the user’s judgement of beam segment restraints. Overall, the design capacities in the new method are superior to those in the current design procedure, especially for frame structures with less slender members.

The new refined plastic hinge analysis is now able to capture four types of failure modes, plastic hinge formation, column buckling, local buckling and lateral torsional buckling. With the inclusion of lateral torsional buckling mode as proposed in this thesis, advanced analysis is one step closer to being used for general design practice.
Publications


Papers to be submitted to the ASCE Journal of Structural Engineering are in preparation, they include:

Yuan, Z. and Mahendran, M., “Modelling of Idealized Simply Supported Beams using Shell Finite Element”.


Yuan, Z. and Mahendran, M., “Refined Plastic Hinge Analysis of Steel Frame Structures Subjected to Lateral Torsional Buckling Effects”.

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Statement of Original Authorship

The work contained in this thesis has not been previously submitted for a degree or diploma at any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

Zeng Yuan

Signature:  ____________________________________________

Date:  ____________________________________________
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Notation

Abbreviations
AISC  American Institute of Steel Construction
AISI  American Iron and Steel Institute
AS4100 Australian Standard for the Design of Steel Structures
CRC  Column Research Council
FEA  finite element analysis
LRFD  load and resistance factor design
R3D4  rigid quadrilateral element with four nodes and three degrees of freedom per node
S4  quadrilateral general purpose shell element with four nodes and six degrees of freedom per node
S4R5  quadrilateral thin shell element with four nodes, reduced integration, and five degrees of freedom per node
UB  universal beam

Symbols
\( c \)  \( \cos \theta \)
\( C_b \)  moment gradient factor
\( d \)  total depth of section
\( \mathbf{d} \)  element displacement vector
\( d_1 \)  web clear depth
\( \mathbf{d}_g \)  global element displacement vector
\( d_{gi} \)  components of the global displacement vector \( \mathbf{d}_g \)
\( \mathbf{d}_l \)  local element displacement vector
\( E \)  elastic modulus
\( E_t \)  tangent modulus
\( e_o \)  member out-of-straightness imperfection
\( e_t \)  non-dimensional tangent modulus = \( E/E_t \)
\( F_{cr} \)  critical stress
\( F_y \)  yield stress
\( \mathbf{f} \)  element force vector
\( \mathbf{f}' \)  component of element force vector = \( \mathbf{f}_r + \mathbf{f}_p \)
\( \mathbf{f}_r \)  element fixed-end force vector
\( \mathbf{f}_g \)  global element force vector
\( f_i \)  local element force vector
\( f_p \)  element pseudo-force vector
\( H \)  applied horizontal load
\( H_u \)  ultimate horizontal load
\( h \)  frame height
\( I \)  second moment of area with respect to the axis of in-plane bending
\( K \)  structure stiffness matrix
\( k \)  axial force parameter = \( \sqrt{P/EL} \)
\( k \)  element stiffness matrix, or effective length factor
\( k_e \)  effective length factor
\( k_f \)  form factor for axial compression member = \( L/A_e \)
\( k_g \)  global element stiffness matrix
\( k_l \)  load height factor
\( L \)  member length or length of element chord
\( L_e \)  member effective length
\( M \)  bending moment
\( M_A \)  bending moment at element end A
\( M_B \)  bending moment at element end B
\( M_i \)  AS4100 nominal in-plane moment capacity
\( M_u \)  AS4100 elastic buckling moment under uniform moment
\( M_{ocr} \)  AS4100 reference moment
\( M_p \)  plastic moment capacity = \( \sigma S \)
\( M_s \)  AS4100 nominal section moment capacity = \( \sigma Z_e = (Z_e/S)M_p \)
\( M_{sc} \)  bending moment defining the section capacity
\( M_u \)  ultimate buckling moment
\( M_y \)  yield moment = \( \sigma Z \)
\( m \)  non-dimensional bending moment = \( M/M_p \)
\( m_{iy} \)  non-dimensional bending moment defining the initial yield = \( M_y/M_p \)
\( m_{sc} \)  non-dimensional bending moment defining the section capacity = \( M_{sc}/M_p \)
\( N_{cy} \)  AS4100 minor axis axial compression member capacity
\( N_s \)  AS4100 nominal axial compression section capacity = \( \sigma A_e = k_p P_y \)
\( P \)  axial force or applied vertical load
\( P_e \)  Euler buckling load = \( \pi^2 EI/L^2 \)
\( P_u \)  required ultimate strength of compression member, or ultimate applied vertical load
\( P_y \)  squash load = \( \sigma A_e \)
\( p \)  non-dimensional axial force = \( P/P_y \)
\( p_e \) non-dimensional Euler buckling load = \( P_e/P_y \)
\( p_{iy} \) non-dimensional axial force defining the initial yield = \( P_{iy}/P_y \)
\( p_{sc} \) non-dimensional axial force defining the section capacity = \( P_{sc}/P_y \)
\( r \) radius of gyration with respect to the axis of in-plane bending
\( s \) \( \sin \theta \)
\( s_1, s_2 \) elastic stability functions
\( T_g \) local to global transformation matrix
\( T_i \) initial force transformation matrix
\( t \) plate thickness, or variable used to define the plastic strength and section capacity
\( t_f \) flange thickness
\( t_w \) web thickness
\( u \) axial displacement
\( V \) applied vertical load
\( V_u \) ultimate vertical load
\( w \) applied beam distributed load
\( x \) distance along member from end A
\( y \) in-plane transverse displacement at location \( x \)
\( Z \) elastic section modulus with respect to the axis of in-plane bending
\( Z_e \) effective section modulus with respect to the axis of in-plane bending
\( Z_{ex}, Z_{ey} \) major axis and minor axis effective section moduli
\( \alpha \) force state parameter of section
\( \alpha' \) effective force state parameter
\( \alpha_c \) compression member factor
\( \alpha_b \) member section constant
\( \alpha_t \) member slenderness reduction factor
\( \alpha_{iy} \) force state parameter corresponding to initial yield
\( \alpha_{mn} \) moment modification factor
\( \alpha_{mu} \) force state parameter of unbraced member
\( \alpha_{sc} \) force state parameter corresponding to section capacity
\( \beta \) end moment ratio
\( \Delta \) relative lateral deflection between member ends due to member chord rotation
\( \delta \) deflection associated with member curvature measured from the member chord
\( \Phi \) curvature
\( \Phi_{sc} \) curvature corresponding to formation of a plastic hinge (i.e., section capacity)
\( \phi \) capacity reduction factor, flexural stiffness reduction factor, or non-dimensional curvature
\(\phi_a\)  flexural stiffness reduction factor for element end A
\(\phi_b\)  flexural stiffness reduction factor for element end B
\(\lambda_n\)  compression member slenderness ratio
\(\nu\)  Poisson’s ratio
\(\theta\)  rotation of deformed element chord
\(\theta_A\)  rotation at element end A
\(\theta_B\)  rotation at element end B
\(\sigma\)  stress
\(\sigma_r\)  maximum residual stress
\(\sigma_y\)  yield stress
\(\psi_o\)  member out-of-plumbness imperfection
\(\omega\)  distributed load magnitude
Chapter 1. Introduction

1.1 General

The Australian steel structures design standard AS4100 (SA, 1998) explicitly gives permission to waive member capacity checks for fully laterally restrained frames consisting of compact sections, provided the designers use an advanced analysis. For these frames, the advanced analysis has the ability to accurately estimate the maximum load-carrying capacity and to trace the full range load-deflection response (Clarke et al., 1991). Recent studies have demonstrated that advanced analysis is also suitable for two dimensional frames made of non-compact sections and three dimensional space frames made of closed sections (Liew, 1998; Teh, 1998; Avery, 1998; Kim, 2001). However, due to the presence of lateral torsional buckling effects, separate member capacity checks are still required for the majority of steel frame structures as they are not fully laterally restrained. This would be the case whether advanced analyses were used or not. Therefore elastic analysis combined with separate ultimate member capacity checks is still the most commonly used method in the steel design practice. A design process that uses a second order inelastic analysis but still requires separate member capacity checks is inefficient.

There are many disadvantages with the conventional design approach. Although the strength and stability of a structural system and its members are related, the current practice is not able to include their interdependency adequately. This problem is more important for complex redundant frame structures. The present design methods consider separately the strength and stability of individual members and the stability of the entire structure, which leads to a lower bound design solution. Since the load-deflection responses are not traced, the present design approach cannot predict the failure modes of a structural system accurately.

It is widely recognised that steel frame structures may exhibit a significantly non-linear behaviour prior to achieving their maximum load capacity. Thus, a direct, non-linear analysis is the most rational means for assessment of overall system performance. Advanced analysis has been defined as “any method of analysis which
sufficiently represents the behavioural effects associated with member primary limit states, such that the corresponding specification member capacity checks are superseded” (White and Chen, 1993). The refined plastic hinge method is a state-of-the-art advanced analysis method. Currently, it is capable of analysing two-dimensional, fully laterally restrained frames subjected to local buckling effects and three dimensional space frames consisting of closed sections.

Large numbers of steel frame structures are built with relatively slender open sections (eg., I-beams). Lateral torsional buckling failure (or out-of-plane instability as shown in Figure 1.1) often governs the limit strength design criteria, and the currently available refined plastic hinge analysis methods are not capable of taking these effects into consideration. At this point of time, the prediction of lateral torsional buckling failure is mostly based on a simplified elastic analysis and associated approximate semi-empirical equations. The elastic analysis and member capacity checks can not be integrated to obtain the load-deflection response of the members. Therefore, research must be carried out to develop suitable methods to incorporate the out-of-plane instability directly into advanced analysis procedures. This research project is aimed at extending the refined plastic hinge method to include the lateral torsional buckling effects.

Figure 1.1 Lateral Torsional Buckling of steel beams and frames

For steel frame structures, there are two types of advanced analysis methods. They are the distributed plasticity (plastic zone) method and the concentrated plasticity method. Nonlinear finite element analysis (FEA) is one of the most well known distributed
plasticity methods. Recent developments in computing hardware and commercial FEA programs have enabled the development of full scale numerical structural models. These types of computer models are able to predict the ultimate loads and trace the load-deflection characteristics to give a very good correlation with corresponding experiments. However, the FEA is too complex and computationally intensive for general design use. It is not feasible particularly for complex steel frames with multiple load cases due to the advanced engineering skills, time, and computing resources required. FEA modelling is a very effective research tool and is often used for developing benchmark solutions (Avery, 1998; Kim, 2002).

In contrast, the concentrated plasticity methods are more suitable for general design situations due to their computational efficiency. The refined plastic hinge method is one of them. When properly formulated and executed, they hold the promise of rigorous assessment of the interdependencies between the strength of structural systems and the performance of their components. With the use of these methods, comprehensive assessment of the actual failure modes and maximum strengths of steel frame structures will be possible without resort to simplified methods of analysis and semi-empirical specification equations. A plastic hinge based analysis method has the potential to extend the creativity of the structural engineer and simplify the design process.

A number of concentrated plasticity analysis methods have been developed in the past. These include:

- Quasi-plastic hinge method (Attala et al, 1994)
- Notional-load plastic-hinge method (EC3, 1993; Liew et al, 1994)
- Hardening plastic hinge method (King and Chen, 1994)
- Springs in series model (Yau and Chen, 1994; Chen and Chan, 1995)
- Refined plastic hinge method (Liew, 1992; Kim, 1996; Avery, 1998)

Among all these methods, none is capable of accounting for the lateral torsional buckling effects that are present in the majority of steel frame structures. It will be
very beneficial if the lateral torsional buckling behaviour can be captured with sufficient accuracy, thus separate member capacity checks can be eliminated.

The first publication on lateral torsional buckling attributed to Michell and Prandtl. Their work was extended by Timoshenko to include the effects of warping torsion in I-section beams. With the advent of the modern digital computer in the 60s, there was an explosion in the amount of research published on the subject (e.g., Lee, 1960; CRC Japan, 1971; Galambos, 1988; Trahair and Bradford, 1988; Bradford, 1992; Trahair, 1993). However, most of these researchers focused on the development of simplified and semi-empirical equations for ultimate member capacity calculations. The load-deflection response of the members due to lateral torsional buckling is not the main objective of these studies. Since the knowledge of load-deflection response is crucial for the development of a practical advanced analysis method, detailed investigations in this area will be a major part of this research project.

1.2 Objectives

The overall objective of the research project described in this thesis is to develop and validate a practical advanced analysis method suitable for the design of steel frame structures including the effects of lateral torsional buckling.

Specific objectives of the research project include:

1. Develop and verify shell finite element models for simply supported beams using finite element analyses. These models will include the effects of geometric imperfections, residual stresses, different load arrangements, connection details, and most importantly, lateral torsional buckling.

2. Develop shell finite element models for steel frame structures subjected to lateral torsional buckling effects. These models will also include the effects of geometric imperfections, residual stresses, different load arrangements and connection details.

3. Use the developed shell finite element models to investigate and fully understand the lateral buckling behavioural characteristics of simply supported
beams and frame structures. Both the ultimate capacities and load-deflection responses of these beams and frames can be used as benchmark solutions.

4. Based on the inplane load-deflection response and stress distribution studies from finite element models, develop suitable techniques to incorporate the effects of lateral torsional buckling into two dimensional refined plastic hinge analysis methods.

5. Develop a computer program using the refined plastic hinge analysis to include the effects of lateral-torsional buckling.

6. Calibrate and validate the new method using the finite element benchmark solutions.

7. Develop a user friendly graphical user interface (GUI) for the advanced analysis program.

1.3 Research Methodology

A thorough understanding of structural stability, advanced analysis methods and lateral torsional buckling behaviour is crucial for the completion of this research project. Therefore a comprehensive literature review was undertaken first. Computer programming skills are also essential for the development of an advanced analysis program. The knowledge and skills of C++ programming language were gained in order to compile the advanced analysis design software.

![Figure 1.2 Experimental and Numerical Analyses of Steel Frame Structures undertaken at QUT](image-url)
Due to the complexity of lateral torsional buckling, the analytical method is unable to investigate this problem. Two methods can be used to investigate the nonlinear load-deflection response of frame structures subject to lateral torsional buckling effects. They are experimental analyses and nonlinear finite element analyses. Examples of experimental and finite element models of a rectangular hollow section frame used in one of the recent research projects at the Queensland University of Technology (QUT) are shown in Figure 1.2. Large numbers of tests have been conducted on simply supported beams world wide for the development of beam curves (Fukumoto and Itoh, 1981). The behaviour of simply supported beams is well documented. In comparison, experiments on heavy frame structures with hot-rolled and welded sections are not as common mainly due to their high cost and the lack of technical support required in the academic institutions.

Shell finite element analyses have become the main research tool for steel structures. Recent QUT research has demonstrated that these analyses are capable of predicting the ultimate load and trace the load-deflection response of various full scale experimental frames even when subjected to complex local buckling and flexural buckling effects (Alsaket, 1999, Avery, 1998). Initial member and local imperfections, membrane and flexural residual stresses, gradual section yielding, spread of plasticity and second-order instability can all be explicitly modelled using the shell finite element analyses (Avery, 1998).

In this research project, a considerable number of frame analyses is required to thoroughly investigate and understand the general behaviour of steel beams and frame structures subject to lateral torsional buckling effects. It is not feasible to use the experimental method. Hence, with the confidence gained from recent QUT research projects, it was decided to use shell finite element analyses to investigate both the load-deflection response and stress distribution of steel beams and frame structures that undergo out-of-plane instability.

1.4 Organisation of the Thesis

A summary of current literature relevant to the advanced analysis and design of steel frames subject to lateral torsional buckling effects is provided in Chapter 2. It includes
the following topics: common practice of plastic frame analyses, advanced analysis methods, instability analyses of beams, and discussions on the important factors associated with lateral torsional buckling. Three design specifications concerning steel member capacities have also been reviewed including the Australian code AS4100, the US code AISC LRFD and Eurocode 3.

Chapter 3 is about the distributed plasticity analysis of simply supported beams. Four load cases have been investigated including a uniform bending moment, a midspan concentrated point load, two concentrated loads at quarter points and a uniformly distributed load. The effects of load height have also been investigated for the transverse load cases. Considerable efforts have been made on the development of suitable simply supported beam boundary conditions for three dimensional shell finite element models.

Finite element analyses of typical steel frames are presented in Chapter 4. Nine series of frame structures are included in the study. Results from these frames were used as benchmark solutions to validate the proposed refined plastic hinge method. Concentrated point loads were used in the frame models. The frame supports were modelled as either pinned or fixed. The effects of four types of rigid beam column connections and residual stresses have also been investigated using these frame models. In total, over 400 nonlinear frame analyses have been carried out in the study.

A new refined plastic hinge analysis method is proposed in Chapter 5. The effects of lateral torsional buckling are accounted for implicitly in the analysis. The formulations of the new method and the validations against the benchmark solutions are presented in this chapter. Comparisons have also been made between the new advanced analysis method and the current design method for steel frames subjected to lateral torsional buckling effects.

Chapter 6 summarizes the research work reported in this thesis. Directions for further study are also recommended.
Chapter 2. Literature Review

This chapter contains a review of current literature and design specifications relevant to the advanced analysis of steel frame structures subjected to lateral torsional buckling effects.

2.1 Common Plastic Frame Analysis Practice

Most of the current steel frame design methods are based on elastic analyses including elastic buckling analysis, linear and second-order elastic analysis. The design specifications are based on simplified elastic methods, and rely on semi-empirical equations to approximately account for non-linear behavioural effects. Occasionally, plastic analysis methods have also been used, for example, plastic collapse load calculation, first-order and second-order inelastic analyses. Plastic analyses often predict higher a load capacity and are more suitable for redundant structures. Comparison of elastic and plastic methods of analysis is shown in Figure 2.1.

This image is not available online. Please consult the hardcopy thesis available from the QUT Library.

Figure 2.1 Elastic and Plastic Analyses (From White and Chen, 1993)

Frame structures are often divided into two major types: rigid frames and semi-rigid frames. Inclusion of semi-rigid connections adds an extra dimension into frame analysis and there is no research into the relationship between connections and lateral
torsional buckling. Connections are often assumed to be rigid in the analyses dealing with lateral torsional buckling problems. For rigid frames, three types of ultimate failure modes may occur; formation of a collapse mechanism, local and global buckling of members, and lateral instability of the whole frame. These failure modes might also interact with each other. In practice, the formation of plastic hinge mechanism is the prefer failure mode since the structure will exhibit maximum ductility. The following is a brief summary of common plastic analysis methods.

2.1.1 Calculation of Plastic Collapse Loads

Depending on their geometrical configurations, there are two extreme cases for steel frame structures. At one end, when frames consist of very slender members, their ultimate capacities are governed by the elastic critical load $P_{cr}$. At the other end, the failures of the frames are controlled by the formation of a plastic collapse mechanism. Commonly two methods are used to calculate the plastic collapse loads ($P_p$), hinge by hinge method and the mechanism method. The procedure of hinge by hinge method is essentially a sequence of elastic analysis when additional plastic hinges formation as the load increases. This method is suitable for computer programming. Also, the plastic formation sequence is important in plastic design. The mechanism method involves two steps. The first step is to identify all possible failure mechanisms. Then, the virtual work method is used to determine the plastic collapse load for each mechanism. The collapse load is derived from the mechanism that gives the lowest value. In comparison, the hinge by hinge method with the aid of a computer is more suitable for design purposes.

Plastic collapse load is obtained by assuming that there are no instability effects. This is often not true for steel frame structures. In reality, steel frame failure is a result of both instability and plasticity effects. The interaction of these two effects is very complex, but some approximate interaction equations were proposed. One of the well-known equations is the Merchant-Rankine interaction equation

$$\frac{P_L}{P_{cr}} + \frac{P_f}{P_p} = 1$$

(2.1)
It has been demonstrated by Horne and Merchant (1965) that the failure load $P_f$ obtained from this equation is usually conservative and reasonably accurate for design purposes.

### 2.1.2 First Order Elastic Plastic Analysis

First-order elastic plastic analysis is the most basic type of inelastic analysis using perfect plastic constitutive material model. This method models the effects of section yielding under incremental loading. But as the name implies, it does not consider second-order stability effects. The formulation of first-order elastic-plastic analysis utilizes an elastic plastic hinge idealisation of the cross-section behaviour. The inelastic behaviour is approximated by inserting a perfectly plastic hinge in the member where the full plastic strength is reached. Members in a structure are assumed to be fully elastic prior to the formation of the plastic hinges.

The appropriate element matrix is adjusted to account for the effects of the plastic hinge. In one approach, the plastic hinge deformation is only produced by the plastic rotation. But, using more advanced techniques, the axial and rotational plastic deformations are allowed using an associated flow rule. The effects of biaxial bending, shear and bimoment can be included in the modelling of the cross-section plastic strength, but generally only biaxial bending effects are considered (Duan and Chen, 1990; ECCS, 1984; Orbison, 1982). First order elastic plastic analysis essentially predicts the same load as the conventional collapse load calculation.

### 2.1.3 Second Order Elastic Plastic Analysis

Second-order elastic plastic analysis models the decrease in stiffness due to both section yielding and large deflections. The inelastic stability limit load obtained by a second-order inelastic analysis is the most accurate representation of the true strength of the frame. However, the second-order elastic plastic analysis is the most basic type of such method (concentrated plasticity model). It employs the same principles of perfect plastic-hinge theory as the first-order elastic-plastic hinge method. The method also includes the use of an equilibrium formulation based on the deformed
structural geometry, therefore taking into account the member instability (Goto and Chen, 1986).

Due to the simplified assumptions associated with the second-order elastic-plastic hinge method, this method has several drawbacks. White and Chen (1993) summarized them as: “due to the idealisation of the members as elastic elements with zero-length elastic-plastic hinges, a second-order elastic-plastic hinge analysis may in some cases over-predict the actual inelastic stiffness and strength of the structure”. Partial yielding, distributed plasticity and associated instability behaviour can not be accurately represented. Nevertheless, this method lays the foundation for the more rigorous analysis method – the refined plastic hinge method.

### 2.2 Advanced Analysis of Steel Frame Structures

The current design procedures based on member capacity checks are “limited in their ability to provide true assessment of the maximum strength behaviour of redundant structural systems” (Liew et al, 1993). First, the current design can not provide the structural failure mode or the failure factor. Second, the elastic analysis is used to determine the forces acting on each member, whereas the inelastic analysis is used to determine the strength of each member in the system. The effects of member instability are ignored in the analysis. As a result, the strength limit state that is predicted by the design codes might be too conservative compared with the true strength of a redundant frame. To qualify as an advanced analysis method, the analysis must take into account all aspects influencing the behaviour of the steel frame, which include:

- Material properties,
- Residual stresses,
- Geometric imperfections,
- Second-order effects,
- Loading history,
- Large deflections
- Post-buckling strength and behaviour,
- Load eccentricities,
• Connection response,
• End restraints,
• Erection procedures, and
• Interaction with the foundations

With the use of advanced analysis, it is possible to achieve a comprehensive assessment of the actual failure modes and maximum strength of steel frame systems. According to Maleck et al (1995), “the primary benefit in directly assessing the capacity of a structure with the analysis is that it allows for a simplified design methodology that eliminates the need for checking of certain member interaction equations.” Currently, the Australian Standard AS4100 (SA, 1998) explicitly gives the engineer permission to disregard member capacity checks if an advanced analysis of the structural system is performed, but it only allows this for compact and fully restrained frames. Lateral torsional buckling occurs when the frames are partially restrained and therefore it is not covered by this clause.

Advanced analyses can be categorized as plastic zone (distributed plasticity) analysis or concentrated plasticity analysis, commonly referred to as plastic hinge analysis. The advantages and limitations of these methods and analyses are discussed in Sections 2.21 and 2.22.

2.2.1 Plastic Zone Analysis

Plastic zone analysis involves explicit modelling of the gradual spread of plasticity throughout the volume of the structures. Compared with plastic hinge analysis, it is capable of accommodating wider ranges of the physical attributes and behaviours of the steel structures. For example, the actual residual stresses and initial geometric imperfections can be directly modelled in the analysis.

For frame structures, two types of finite elements are often used in the plastic zone analysis. The first one is fibre elements. The analysis involves subdivision of each member of the frame into a number of beam-column elements, and each element is divided into a number of fibres. The other one is the 3-D shell element. Shell element
has been widely used in aeronaut and automobile industries and in the past decade, it has been increasingly used in thin-wall structural researches.

Plastic zone analysis is able to accommodate most of the important factors related to steel frames and can accurately predict the structural ultimate capacity. A number of literatures even considered plastic zone analysis to be capable of achieving the “exact” solution. (King et al., 1991; Liew et al., 1993). However, in order to accurately model the spread of plasticity, a relatively fine discretization is required for frame structures. Even using the latest computer technology, the intensity of computation prohibited the use of these methods for common design purposes. Therefore, the plastic zone methods are often reserved for specialised design applications and development of design charts.

Currently, the plastic zone analysis is also widely used in the development of benchmark solutions. It has often been used to replace the expensive large-scale experiments in steel structure research (Toma and Chen, 1992; Avery, 1998; Kim, 2002). Compared with experimental analyses, plastic zone methods are less time consuming, less expensive, better cope with different load cases, more repeatable, and have less operational errors.

2.2.2 Plastic Hinge Analysis

For general design purposes, the focus of advanced analysis research is on developing a simpler second-order plastic analysis method that can capture the nonlinear behaviour of steel frame structures. Substantial progresses have been made using concentrated plasticity analysis (plastic hinge type analyses). These methods use beam-column elements to model all members for the frame structures. They assume that each element remains fully elastic except at its ends, where zero-length plastic hinges may occur. When the member plastic capacity is reached, a plastic hinge is inserted at the element’s end to represent the inelastic behaviour of the members.

Plastic hinge based analysis can maintain the computational efficiency compared with the plastic-zone analysis, while providing comparable analysis accuracy for fully restrained frames. However, in their present forms, they are still not accepted as a
practical tool for general design/analysis and a great deal of research is needed. The SSRC Task Force report (White et al, 1993) lists the ten desirable attributes for plastic hinge based elements suitable for practical advanced analysis of plane frames:

1. The model should be accurate using only one element per member. The element should not be more than 5% unconservative when compared with “exact” solutions for in-plane beam-column strength.

2. The element relationship should be derived analytically and implemented in explicit form for analysis. Numerically integrated elements do not provide the degree of computational efficiency required for analysis of moderate to large size structural systems.

3. The model should be extensible to 3-D analysis.

4. The effects of inelasticity on axial member deformations should be represented because the column axial stiffness provides a significant portion of the structure’s side sway resistance in many types of frames.

5. As the axial load approaches zero, the element behaviour should approach that of the elastic-plastic hinge mode because this mode provides a good representation of the performance for beam members. The possible benefits of strain hardening should not be relied upon, due to the precise effects of yielding, strain hardening, and local and lateral torsional buckling on the full moment rotation characteristics which are not quantified adequately.

6. In the case of a member loaded by pure axial load, the element inelastic flexural stiffness should be close to that associated with the inelastic flexural rigidity $E_tI$ implied by the column strength equations of the particular design specification being used.

7. Member out of straightness effect, when important, should be accommodated implicitly within the element model. This would parallel the philosophy behind the development of most modern column strength expressions (include the effects of residual stresses, out of straightness, and out of plumbness).

8. For intermediate to high axial loads, the moment gradient along the member length should have a significant effect of the element inelastic stiffness. The reduction in stiffness due to yielding should be largest for single-curvature bending and smallest for full reversed-curvature bending.

9. Once the full plastic strength at a cross-section is reached by the effect of member second-order forces, the cross-section forces may vary with continued
loading, but these forces should never violate the strength conditions of the fully plastified section, that is, strain-hardening effects should be neglected. Thus, if a plastic hinge forms in a beam-column member, the member axial force may still be increased, but this must result in a corresponding decrease in the moment at the hinge.

10. The formation of plastic hinges within the span of a member should be accommodated using one-element per member. This is particularly important for transversely loaded members such as the beams and girders of a frame. Large saving in solution effort may be realized if these members do not need to be discretized into multiple elements to capture internal plastic hinges.

There are a number of approaches to plastic hinge based advanced analysis. They include: Notional-load plastic hinge method, Hardening plastic hinge method, Quasi-plastic hinge method and Refined-plastic hinge method.

2.2.2.1 Notional-Load Plastic-Hinge Method

One approach to improve the use of second-order elastic-plastic hinge analysis for frame design is to specify artificially large values of frame imperfections. This method uses an equivalent lateral load to generate a larger than standard erection tolerance geometric deformation, intended to cover the effects of residual stresses, gradual yielding, local buckling and member imperfections that are not accounted for in the second-order elastic-plastic hinge analysis, as shown in Figure 2.2.

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Figure 2.2 Example of Using Equivalent Notional Load (From EC3)
In the ECCS design method (Toma and Chen, 1992), 0.005 times the gravity load is used as the exaggerated notional load to avoid over-prediction of strength by traditional elastic-plastic hinge methods. In comparison with the benchmark solutions from the plastic zone analyses, this method is ideal for strong axis bending in both sway and non-sway moment resisting frames. Eurocode 3 (1990) adopts this approach for advanced analysis of frames. Both the Canadian Standard (CSA, 1989) and Australia Standard (AS4100, 1998) allow the use of this method. The biggest advantage of this technique is the simplicity; one notional load factor covers all. However, it is not a method suitable for every situation. For example, this method over-predicts the strength by up to 10% in the isolated beam-columns subject to axial forces and bending moments (Liew, 1996).

2.2.2.2 Hardening Plastic Hinge Method:

This method is also referred to as the modified plastic hinge method. It was proposed by King et al (1991). However, it includes the assumption that once a plastic hinge has formed at the end of an element, the moment at the plastic hinge will remain unchanged as the axial force is increased. This may violate the cross-section plastic strength, and does not allow for unloading of the hinge. The method was therefore found to give unconservative errors in excess of the tolerable limit (5%), and did not reduce to the behaviour of an inelastic column for members loaded by axial force alone. It was therefore considered unacceptable as an advanced analysis design tool. An alternative technique was also presented by King et al (1991), termed the beam-column strength method, to avoid these problems. However, this method tended to underestimate the column capacity in some situations and did not reduce to the behaviour of a beam member in the limit that the member is subjected only to bending moment.

A refined and extended version of the modified plastic hinge method was presented by King and Chen (1994). This method, referred to as the hardening plastic hinge method, gave load-deflection responses “almost identical to the test results” for a small number of published example analyses. Although it does not appear to have been tested as extensively as the refined plastic hinge model, the hardening plastic hinge method contains several aspects worthy of consideration. It should be noted
that the phrase “hardening” bears no reference to material strain hardening, which is in fact neglected in the proposed method. The authors use the phrase “work hardening” to indicate the concept of a degradation of tangent stiffness of a cross-section, which is calibrated against the exact moment-curvature-axial force relationship. The emphasis of this research is on the behaviour of semi-rigid steel frames with minor axis bending.

2.2.2.3 Quasi-Plastic Hinge Method

The quasi-plastic hinge method presented by Attala et al. (1994) is an intermediate approach between the plastic-zone and the elastic-plastic hinge methods. A 2-D element that accounts for gradual plastification under combined bending and axial force is developed from equilibrium, kinematic and constitutive relationships. Gradual plastification is handled by fitting nonlinear equations to moment-curvature-axial force data obtained from a plastic zone analysis. Flexibility coefficients for the full member are obtained by successive integration along its length. Thus, an inelastic-element stiffness matrix is obtained using of the incremental flexibility relationships. Initial yield and full plastification surfaces are used to represent gradual yielding effects of the cross-section.

This method predicts strengths with an error less than 5% compared with a number of plastic zone analysis models (Kanchanalai, 1977; El-Zanaty et al., 1980; Ziemian, 1990). The use of more precise plastic strength surfaces may contribute to the superior accuracy of this analysis method. Because the formulation of this method is based on flexibility relationships, it would be difficult to extend the framework of this model to three-dimensional analysis. Moreover, although this model eliminates the necessity of the refined model (eg. fibre elements) through the cross-section, it still requires many elements along the member.

2.2.2.4 Refined Plastic Hinge Method

The refined plastic hinge method is based on simple modifications of the second-order elastic-plastic hinge method, and was developed by Liew (1993). The modified hinge represents the effect of distributed plasticity through the cross-section along the whole
length of members (see Figure 2.3), assuming that the hinge stiffness degradation is smooth. Thus, the steel frames’ distinct behaviour such as residual stresses, geometric imperfections, connection flexibility, and gradual yielding can be formulated implicitly into the analysis. Large amounts of research have been conducted following this analysis approach and a 3D formulation has been developed by Liew (1998) and Kim (2002). Avery (1998) has extended this method to include the effects of local buckling. The major advantage of this approach is that it is as simple and efficient as the elastic-plastic hinge method, and it provides a more accurate assessment of strength and stability of frame structures.

Figure 2.3 Spread of Plasticity (From Chen, 1997)

The second order effects (P-Δ, and P-δ) are solved by the use of stability functions in the refined plastic analysis. Gradual yielding, distributed plasticity and the associated instability effects are accounted for by the use of two functions: the tangent modulus ($E_t$) and the flexural stiffness reduction factor ($\phi$). These functions represent the distributed plasticity along the length of the member due to axial force effects and the distributed plasticity effects associated with flexure, respectively.
Stability Functions

Statically indeterminate structures can be solved using both force and displacement methods. The solution of displacement methods is achieved by firstly deriving force-displacement relations. Then, the resulting system of equations is solved for the unknown displacement, which is then substituted into the fundamental relationships to determine the remaining response characteristics (internal forces and moments) of the structure.

The slope-deflection method is one of the displacement methods. It was first introduced by Maney in 1915. The basis of the slope-deflection method lies in the slope-deflection equations, which express the end moments of each member in terms of the end distortions of that member.

In order to establish a relationship between the end moments \( M_A, M_B \) and the end rotations \( \theta_A, \theta_B \), a non-sway beam-column subject only to end moments is considered (Figure 2.29).

![Figure 2.4 Rotation of beam-column with end moments](image)

In this case, \( \theta_A = y'_{(x=0)} \), and \( \theta_B = y'_{(x=L)} \). From the closed form solution of beam-column for this load case:

\[
y = -\left(\frac{M_A \cos kL + M_B}{EIk^2 \sin kL}\right) \sin kx + \frac{M_A}{EI k^3} \cos kx + \frac{M_A + M_B}{LEIk^2} x - \frac{M_A}{EI k^2} \quad (2.2)
\]

\[
y' = -\frac{1}{EIk} \left[ \cos kL \cos kx + \sin kx - \frac{1}{kL} \right] M_A - \frac{1}{EIk} \left[ \frac{\cos kx}{\sin kL} - \frac{1}{kL} \right] M_B \quad (2.3)
\]

therefore, \( \theta_A = y'_{(x=0)} = -\frac{1}{EI} \left[ \frac{\sin kL - kL \cos kL}{(kL)^2 \sin kL} \right] M_A + \frac{L}{EI} \left[ \frac{\sin kL - kL \cos kL}{(kL)^3 \sin kL} \right] M_B \quad (2.4)\)

\[
\theta_B = y'_{(x=L)} = -\frac{1}{EI} \left[ \frac{\sin kL - kL \cos kL}{(kL)^2 \sin kL} \right] M_A + \frac{L}{EI} \left[ \frac{\sin kL - kL \cos kL}{(kL)^3 \sin kL} \right] M_B \quad (2.5)
\]
From these equations, the slope-deflection equations can be obtained:

\[
M_A = \frac{EI}{L} \left( s_{ii} \theta_A + s_{ij} \theta_B \right) 
\]  
\[
M_B = \frac{EI}{L} \left( s_{ij} \theta_A + s_{ii} \theta_B \right) 
\]  

where:

\[
s_{ii} = \frac{kL \sin kL - (kL)^2 \cos kL}{2 - 2\cos kL - kL\sin kL} 
\]  
\[
s_{ij} = \frac{(kL)^2 - kL \sin kL}{2 - 2\cos kL - kL\sin kL} 
\]  
\[
k = \sqrt{\frac{P}{EI}} 
\]

The coefficient \(s_{ii}\) and \(s_{ij}\) are referred to as stability functions. Note that these slope-deflection equations are valid only if the following conditions are satisfied:

- The beam-column is prismatic
- Member does not sway
- There is no in-span transverse loading on the member
- The beam-column is subjected to axial compression force.

If these conditions are not satisfied, suitable modifications to the slope-deflection equations are necessary. Chen and Lui (1987) provided a number of modified equations, which include:

- Member with sway
- Member with one hinge at one end
- Member with elastically restrained ends
- Member with transverse loadings
- Member with axial tensile force

For example, the slope-deflection equations for sway members can be expressed as:

\[
M_A = \frac{EI}{L} \left( s_{ij} \theta_A + s_{ii} \theta_B - (s_{ii} + s_{ij}) \frac{\Delta}{L} \right) 
\]  
\[
M_B = \frac{EI}{L} \left( s_{ij} \theta_A + s_{ii} \theta_B - (s_{ii} + s_{ij}) \frac{\Delta}{L} \right) 
\]
It can be observed from the plot of stability functions (Figure 2.5) that when the axial force approaches zero, $s_{ii}$ equals to 4 and $s_{ij}$ is 2. It matches the regular stiffness method, where the member stiffness is $4EI/L$, and the carry over factor is $1/2$.

![Stability functions](image)

Figure 2.5 Stability functions

Tangent modulus

In the refined plastic hinge analysis, the elastic modulus in the stiffness matrix is replaced with a tangent modulus ($E_t$) to represent the distributed plasticity along the length of the member due to axial force effects. The member inelastic stiffness, represented by the axial rigidity ($E_tA$) and the bending rigidity ($E_tI$), is assumed to be a function of the axial force only. The values $E_tA$ and $E_tI$ represent the properties of an effective core of the section (see Figure 2.3). The tangent modulus can be evaluated from column member capacity curve specification equations and therefore implicitly includes the effects of residual stresses, initial geometric imperfections, and inelastic second-order effects.

Figure 2.6 illustrates the procedure used to evaluate the tangent modulus using a member capacity column curve (Avery, 1998). This procedure relies on the assumption that the tangent modulus of a compression member with a particular non-dimensional axial load ($p$) and slenderness ($\lambda_n$) can be approximated by the tangent modulus of an “equivalent stiffness” compression member with a member capacity equal to the axial load ($p$). This assumption is only strictly valid for the limiting case.
given by $\lambda_n = \lambda'_{n}$. The procedure used to evaluate the tangent modulus is described below:

1. The “equivalent stiffness” capacity of a member with a particular slenderness ($\lambda_n$) and applied non-dimensional axial force ($p$) is obtained by extrapolating line EB to the intersection with the column curve at C.
2. The slenderness corresponding to point C ($\lambda'_{n}$) and the non-dimensional Euler’s buckling load corresponding to this slenderness ($p'_{e}$) are given by the axis intercepts D and F.
3. The non-dimensional tangent modulus ($e_t = E_t/E$) is defined as the ratio $p/p'_{e}$ and is conveniently independent of the actual member slenderness ($\lambda_n$).

![Diagram showing tangent modulus calculation using column curve](image)

**Figure 2.6 Tangent modulus calculation using column curve**

The original refined plastic hinge formulation (Liew, 1992) offered a choice of two tangent modulus functions derived from the AISC LRFD and CRC column curves for members with compact cross-sections. The tangent modulus is intended to implicitly account for the effects of initial geometric imperfections, gradual yielding associated with residual stresses, and the associated instability. The tangent modulus functions recommended by Liew (1992) are appropriate only for compact sections. However, Avery (1998) has developed an improved tangent modulus function suitable for non-compact sections subject to local buckling effects.

The tangent modulus ($e_t$) derived from AISC LRFD compression member capacity is:
\[ e_i = 0.877 \quad \text{for } p \leq 0.39 \]
\[ e_i = -2.389 \ln(p) \quad \text{for } p > 0.39 \]  
\[ (2.13) \]

And the tangent modulus derived from CRC column formula is:
\[ e_i = 1.0E \quad p \leq 0.5 \]
\[ e_i = 4Ep(1-p) \quad p > 0.5 \]  
\[ (2.14) \]

where \( p \) is the non-dimensionalized axial load

**Flexural stiffness reduction factor**

Distributed plasticity effects associated with flexure are represented by introducing a gradual degradation in stiffness as yielding progresses and the section capacity at one or both ends is approached. The member stiffness gradually degrades according to a prescribed function after the element end forces exceed a predefined initial yield function from the elastic stiffness to the stiffness associated with the formation of plastic hinges at one or both ends.

For the second-order elastic-plastic hinge model, perfect plastic hinges are formed at the member ends; the elastic stiffness at the hinges will be reduced to zero when hinges reach yield capacity. The incremental force-displacement relationship is expressed as Equation 2.15. To represent this gradual transition for the formation of a plastic hinge at each end of an initially elastic beam-column element, Liew (1992) described the refined plastic hinge element incremental force-displacement relationship during the transition using a stiffness reduction factor (\( \phi \)):

\[
\begin{bmatrix}
\dot{M}_A \\
\dot{M}_B \\
\dot{P}
\end{bmatrix} = \frac{EJ}{L} \begin{bmatrix}
\phi_A \left[ s_j - \frac{s_j^2}{s_j} (1-\phi_B) \right] & \phi_A \phi_B s_2 & 0 \\
\phi_B \left[ s_j - \frac{s_j^2}{s_j} (1-\phi_A) \right] & 0 & 0 \\
0 & A/I & \dot{u}
\end{bmatrix}
\]  
\[ (2.15) \]

Liew (1992) considered several alternative functions to calculate the stiffness reduction factor for the original refined plastic hinge formulation. The most appropriate function (Equation 2.15) was established by comparison with plastic zone
analytical benchmarks provided by Kanchanalai (1977). The flexural stiffness reduction parameter ($\phi$) was assumed to be a function of the combined axial force and bending moment (represented by the force state parameter $\alpha$), and that declines according to a prescribed parabolic function following the initial yield. The stiffness reduction parameter ($\phi$) is equal to unity when the element end is elastic, and zero when a plastic hinge is formed. Intermediate values, representing partial yield, are determined using a parabolic decay function.

$$\phi = 1 \quad \text{for } \alpha \leq 0.5$$

$$\phi = 4\alpha(1-\alpha) \quad \text{for } \alpha > 0.5$$

(2.16)

The force state parameter ($\alpha$) represents the magnitude of the axial force and bending moment at the element end. The initial yield is denoted by $\alpha = 0.5$, and the plastic strength by $\alpha = 1.0$. The determination of the force state parameter ($\alpha$) depends on the functions used to define the plastic strength. Liew et al. (1993) employed the AISC LRFD bi-linear interaction equations (Figure 2.7) for a member of compact cross-section and zero length to define the plastic strength. The expressions for $\alpha$ corresponding to this definition are:

$$\alpha = p + \frac{8}{9}m \quad \text{for } p/m \geq \frac{2}{9}$$

$$\alpha = \frac{1}{2} p + m \quad \text{for } p/m < \frac{2}{9}$$

(2.17)

Figure 2.7 Bi-linear Interaction Equations (From AISC 1999)

The flexural stiffness reduction function (Equation 2.15) used in the original refined plastic hinge model (Liew, 1992) was only intended for compact sections. It was
necessary to modify this function to account for the effects of local buckling on the flexural stiffness reduction of non-compact sections. Avery (1998) generalised the function as:

\[ \phi = 1 \quad \text{for } \alpha \leq \alpha_{iy} \]

\[ \phi = \frac{\alpha (\alpha_{sc} - \alpha)}{\alpha_{iy} (\alpha_{sc} - \alpha_{iy})} \quad \text{for } \alpha_{sc} \geq \alpha > \alpha_{iy} \]

(2.18)

The symbol \( \alpha \) represents a force-state parameter that measures the magnitude of the axial force and bending moment at the element end, normalised with respect to the plastic strength. Note that the initial yield surface is denoted by \( \alpha = \alpha_{iy} \), the section capacity by \( \alpha = \alpha_{sc} \), and the plastic strength surface by \( \alpha = 1 \). The section capacity and plastic strength are identical for compact sections (i.e., \( \alpha_{sc} = 1 \)). For non-compact sections, \( \alpha_{sc} \) represents the reduction in section capacity due to local buckling. The force state parameter corresponding to initial yield (denoted by \( \alpha_{iy} \)) can simply be taken as 0.5 (as for the original refined plastic hinge model) and assumed to be independent of the section slenderness.

### 2.2.3 Semi-rigid Frames

Very few real connections are fully rigid. In most cases, the rigidity of the connection falls between the two extreme ends (rigid or pinned). The plastic hinge based methods are particularly suitable for semi-rigid frame structures. The semi-rigid connections are often modelled as rotational springs with a moment-rotation curve based on the connection type and dimensions. However, in the view of practising engineers, the adoption of semi-rigid framing is unlikely to produce savings of sufficient magnitude to cause a switch to steel from what otherwise would have been a concrete building (Springfield, 1987).

#### 2.2.3.1 Semi-Rigid Connections

A connection is a joint through which forces and moment are transmitted from one member to another. A general set of loads that may be transmitted includes axial forces, shearing forces, bending moment, and torsion. The effect of torsions is often neglected in 2D frames and the deformations of axial and shearing forces have minor
effects on the structural behaviour. Consequently, only the rotation deformation is studied, and the characteristics of connections are expressed using the moment-rotation (M-$\theta_r$) curves. The angle $\theta_r$ is a measure of the relative rotation of the beam to the column (see Figure 2.8)

![Figure 2.8 Beam to column connection](image)

A large amount of research has been conducted on the behaviour of connections of different types based mainly on experimental analyses. Lui and Chen (1987) summarised the behaviour of real connections as follows:

- All types of connection exhibit a (M-$\theta_r$) behaviour that falls between the extreme cases of ideally pinned (the horizontal axis) and fully rigid (the vertical axis) conditions.
- For the same moment, the more flexible the connection is, the larger the value of $\theta_r$. Conversely, for a specific value of $\theta_r$, a more flexible connection will transmit less moment between the adjoining members.
- The maximum moment that a connection can transmit decreases with more flexible connections
- The (M–$\theta_r$) relationships for the semi-rigid connections are typically non-linear over virtually the entire range of loadings but are almost linear for unloading.

The nonlinearity of connection behaviour is due to a number of factors:

- Material discontinuity of the connection. The connection is composed of various combinations and arrangements of bolts and structural shapes like angles and T-stubs. This formation allows for irregular slip and movement of components relative to one another at different stages of loading.
• Local yielding of some component parts of a connection. This is the primary factor related to the nonlinear behaviour of a connection.
• Stress and strain concentrations caused by holes, fasteners and bearing contacts of elements used in a connection.
• Local buckling of flanges or web of the beam and column in the vicinity of a connection.
• Overall geometric changes under the influence of applied loads.

2.2.3.2 Connection Modelling

Finite element analyses on connection behaviour have been carried out by many researchers (Krishnamurthy et al, 1979; Patel and Chen, 1984). However, the time and cost involved as well as the uncertainty inherent in the analysis make these types of techniques unacceptable for practical use. The more reliable data of \( M-\theta_r \) curves are obtained almost exclusively from experiments. Thus the most commonly used approach to describe the \( (M-\theta_r) \) relationship is to curve-fit the experimental data with simple expressions. Many models exist describing the \( (M-\theta_r) \) relationship, they are:
• Linear models (Rathbun, 1936; Monforton and Wu, 1963; Lightfoot and LeMessurier, 1974; Tarpy and Cardinal, 1981; Lui and Chen, 1983)
• Polynomial model (Frye and Morris, 1976)
• B-spline model (Jones, Kirby, and Nethercot, 1982)
• Power models (Colson and Louveau, 1983; Ang and Morris, 1984; Kishi and Chen, 1990)
• Exponential models (Lui and Chen, 1986; Yee and Melchers, 1986; Wu and Chen, 1990)

For practical use, the Fry-Morris polynomial model, the Ang-Morris power model, and the Wu-Chen exponential model are recommended.

2.2.3.3 Analysis of Semi-Rigid Frames

The use of rigid or pinned connection models in frame analysis serves the purpose of simplifying the analysis and design. If the connections possess moment capacities in excess of their adjoining members, the ultimate load capacity of the flexibly-jointed frame are assumed to be not much different from that of a rigidly-jointed frame
(Poggi and Zandonini, 1985; Chen and Zhou, 1987). For semi-rigid frames, the modified stability coefficients can be derived using the $M-\theta_r$ curves (Liew et al, 1993). A seven step procedure for the design of semi-rigid steel frames based on advanced inelastic analysis is proposed by Liew et al (1993):

1. Perform a second-order elastic analysis based on rigid frame action.
2. Select the type of semi-rigid connection to be used. Estimate the dimensions of the connections and check for bolt tension, shear etc based on the forces obtained from step 1.
3. Determine the ultimate moment capacity and the initial stiffness of the connection according to a known moment-rotation curve.
4. Check connections for ductility.
5. Perform advanced analysis incorporating the effects of the semi-rigid connections.
6. Check the strength and serviceability limit states.
7. Modify the connection design by repeating steps 2-6 until the most economical structure is achieved.

Semi-rigid connections are beyond the scope of this project. However, lateral torsional buckling has a close association with the connection rigidity. The current advanced analyses of semi-rigid frames do not consider the interactions between connections and member lateral torsional buckling. Nevertheless, the inclusion of a semi-rigid connection in the analysis will not solve the lateral torsional buckling problem.

### 2.3 Lateral Torsional Buckling

The stability designs of steel frame structures deal with three major fields, local buckling, global buckling and structural instability due the plastic hinge forming. Lateral torsional buckling is a form of global buckling. Currently, all of the plastic hinge based methods preclude the effects of lateral torsional buckling. As a result, the potential of the advanced analysis is not fulfilled in the common design practice. A large amount of lateral torsional buckling research has been conducted, but little attempt has been made to incorporate the effects of lateral torsional buckling into
advanced frame analysis. This section reviews a range of issues associated with beam stability problems aimed at answering the following questions.

1. How does lateral torsional buckling occur?
2. How can the effects of lateral torsional buckling be quantified?
3. What effect does lateral torsional buckling have on the plastic redistribution along the beams?

2.3.1 Methods of Stability Analysis

The lack of perceptive understanding of structural stability by engineers has caused many failures, in which stability requirements have been seriously under-estimated. There are three methods in stability analysis of an elastic structural system, namely, bifurcation approach, energy approach, and dynamic approach. The bifurcation approach is an idealized mathematical approach to determine the critical conditions of a geometrically perfect system. In the bifurcation approach, the state at which two or more different but adjacent equilibrium configurations can exist is sought by an eigenvalue analysis. The lowest load that corresponds to this state is the critical load of the system. The system’s critical conditions can be identified by setting the determinant of the system’s tangent-stiffness matrix equal to zero, because the tangent stiffness of the system vanishes at the critical load. The critical conditions are represented by the eigenvalues of the system’s stiffness matrix and the displaced configurations are represented by the eigenvectors. The eigenvalue approach can not be applied to the load-deflection problem.

The system’s critical condition can also be determined by the energy approach. For an elastic system, the total potential energy of the system can be expressed as a set of functions of generalized displacements and the external applied forces. If the system is in equilibrium, its total potential energy must be stationary. Thus, the equilibrium condition can be identified by setting the first derivative of the total potential energy functions with respect to each generalized displacement equal to zero. The critical load can then be calculated from the equilibrium equations. By investigating the higher order derivatives of the total potential energy functions, the stability of the equilibrium can then be determined.
The third way to obtain the critical load is the dynamic approach. The equations describing system free vibration is written as a function of the generalized displacements and the external force. The critical load is obtained as the level of external force when the motion ceases to be bounded. The equilibrium is stable if a slight disturbance causes only a slight deviation of the system from its original equilibrium position and if the magnitude of the deviation decreases when the magnitude of the disturbance decreases. The equilibrium is unstable if the magnitude of motion increases without bound when subjected to a slight disturbance.

When lateral torsional buckling occurs, twisting and lateral deformations are coupled. Compared with column flexural buckling, this type of problem is more complex and difficult to deal with. All three analysis approaches can be used to solve the lateral torsional buckling problem, but only the bifurcation analysis can provide generalized formula for buckling moment. However, due to the complexity of lateral torsional buckling, closed form solutions exist only for limited load cases. The derivations of buckling moment for uniform moment and midspan concentrated load cases are given in Sections 2.3.2 and 2.3.3.

### 2.3.2 Beams Subjected to Uniform Bending Moment

Open cross-sections such as I-beam are particularly susceptible to lateral torsional buckling. The main reason is that the torsional rigidities of such sections are very low and their resistance to torsional instability is very limited. There are two types of torsional rigidity that might exist in a member with thin plate cross section. They are uniform torsion and non-uniform torsion (also known as warping restraint torsion) restraints.

**Uniform torsion**

Uniform torsion is a unique load case when the open cross section has no warping resistance under equal and opposite torques. In this case, the warping deformation in the beam is the same for all cross sections without inducing any axial strain on the longitudinal fibres. So the applied torque is resisted solely by shear stresses developed in the cross section. It is usually assumed that the shear stress at any point acts parallel
to the tangent to the midline of the cross section (see Figure 2.9 (a)). The magnitude of these shear stresses will be proportional to the distance from the midline of the component plate. As a result, uniform torsion ($T_{sv}$) can be expressed by the following formula:

$$T_{sv} = GJ \frac{d\gamma}{dz}$$  \hspace{1cm} (2.19)

$$\frac{d\gamma}{dz} = \text{the rate of twist}$$  \hspace{1cm} (2.20)

$G$ is the shear modulus and $J$ is the torsional constant of the cross section.

If the end of a member is fixed in the longitudinal direction, in addition to uniform torsion rigidity, there exists another type of torsion called warping restraint torsion. When the cross section is prevented from warping, axial strains and hence axial stresses will be induced in the cross section in addition to the shear stresses (see Figure 2.9 (b)). For an I-section, these axial stresses in the two flanges create a pair of equal and opposite moments, $M_f$. The magnitude of these moments can be expressed with the usual beam moment-curvature relationship:

$$M_f = EI \frac{d^2u_f}{dz^2}$$  \hspace{1cm} (2.21)

**Figure 2.9 Shear Stress Distributions due to Uniform and Non-uniform Torsions**

**Non-uniform torsion**

The magnitude of these shear stresses will be proportional to the distance from the midline of the component plate. As a result, uniform torsion ($T_{sv}$) can be expressed by the following formula:

$$T_{sv} = GJ \frac{d\gamma}{dz}$$  \hspace{1cm} (2.19)

$$\frac{d\gamma}{dz} = \text{the rate of twist}$$  \hspace{1cm} (2.20)

$G$ is the shear modulus and $J$ is the torsional constant of the cross section.
I_f = the moment of inertia of one flange about the minor axis
u_f = the lateral displacement of the flange

The pair of shear forces associated with the moment M_f provides the warping restraint torsion (T_w).

\[ V_f = -\frac{dM_f}{dz} = -EI_f \frac{\gamma}{dz^3} = -EI_f \frac{hd^3 \gamma}{2dz^3} \quad (2.22) \]

\[ T_w = V_f h = -EI_f \frac{h^2 d^3 \gamma}{2dz^3} = -EI_w \frac{d^3 \gamma}{dz^3} \quad (2.23) \]

h = the distance between the lines of action of the shear forces

\[ I_w = \frac{I_f h^3}{2} \] = warping constant of I-section

The warping restraint torsion is dependent on the end restraint and its magnitude reduces at points away from the member ends. Hence it is also called non-uniform torsion.

For general case, when a member is twisted, the internal twisting moment (T) equals to: \( T = T_{sv} + T_w \) \( (2.24) \)

The uniform torsion is always present when a member is subjected to twisting and rotates. On the other hand, warping torsion resistance will develop when a cross section is prevented from warping. Not all cross sections will have warping resistant torsion. Some cross sections have zero warping constant such as Angle sections, Tee sections and Cruciform sections.

![Figure 2.10 Lateral Torsional Buckling of a Beam subjected to Uniform Moment](image-url)
When a simply supported I-beam is subjected to uniform bending moment (see Figure 2.10), lateral torsional buckling is one of the most likely failure modes. Similar to a column under axial compression force, the critical load of a beam can be worked out using a bifurcation analysis. Idealized simply supported condition means that the twist rotation of beam end is restrained but does not include any warping constraints.

In order to simplify the derivation of internal and external force relationships, an additional set of coordinates is used. The \( x'\)-\( y'\)-\( z'\) axes are the local coordinates of the cross section plane away from the supports. The relationships of these three coordinates are shown in Figure 2.11. The external bending moment on the local coordinates can be calculated based on these relationships (Chen, 1991).

The external moments at any cross section are:

\[
\begin{align*}
M_{x'(\text{ext})} & = -\gamma M_0 \\
M_{y'(\text{ext})} & = -\gamma M_{x(\text{ext})} = -\gamma M_0 \\
M_{z'(\text{ext})} & = \frac{d \gamma}{dz} M_{x(\text{ext})} = \frac{d \gamma}{dz} M_0
\end{align*}
\]  

The corresponding internal resisting moments are:

\[
\begin{align*}
M_{x'(\text{int})} & = M_{x(\text{int})} = M_0 \\
M_{y'(\text{int})} & = 0 \\
M_{z'(\text{int})} & = 0
\end{align*}
\]
And the governing differential equations for an I-beam subjected to uniform bending are:

\[ M_{x}'(\text{int}) = -EI_x \frac{d^2v}{dz^2} \]  \hspace{1cm} (2.29)

\[ M_{y}'(\text{int}) = -EI_y \frac{d^2u}{dz^2} \]  \hspace{1cm} (2.28)

\[ M_{z}'(\text{int}) = GJ \frac{d\gamma}{dz} - EI_w \frac{d\gamma^3}{dz^3} \]  \hspace{1cm} (2.30)

The first equation is independent of the other two equations, which means it describes the in-plane behaviour of the beam before lateral buckling. The lateral-torsional buckling behaviour can be obtained from the combination of the last two equations:

\[ EI_x \frac{d^2v}{dz^2} + M_o = 0 \]  \hspace{1cm} (2.31)

\[ EI_y \frac{d^2u}{dz^2} + \gamma M_o = 0 \]  \hspace{1cm} (2.32)

\[ GJ \frac{d\gamma}{dz} - EI_w \frac{d^3\gamma}{dz^3} - \frac{du}{dz}M_o = 0 \]  \hspace{1cm} (2.33)

By solving this equation with simply supported conditions, the critical moment can be obtained:

\[ M_{ocr} = \frac{\pi}{L} \sqrt{EI_y GJ} \sqrt{1 + \frac{\pi^2 EI_w}{L^2 GJ}} \]  \hspace{1cm} (2.35)

It is interesting that the above equation assumes that the in-plane deflection has no effect on the lateral torsional buckling behaviour of the beam. This assumption is well justified if the major axis rigidity is greater than that of minor axis (such as UB section). However, if rigidities of minor and major axis are of the same of order of magnitude, the solution becomes more complicated. An approximate solution was given by Kirby and Nethercot (1985) as:

\[ M_{ocr} = \frac{\pi}{L} \sqrt{EI_y GJ} \sqrt{1 - \frac{I_y}{I_x}} \left[1 + \frac{\pi^2 EI_w}{L^2 GJ}\right] \]  \hspace{1cm} (2.36)

This formula also indicates that lateral torsional buckling will not occur when the load is applied on the plane of strong axis.
2.3.3 Transverse Loads

When a beam is subjected to transverse loads, the bending moment varies along the length of the beam. Subsequently, the governing differential equations will have variable coefficients. Furthermore, the vertical position of the applied load (load height factor) also has a significant effect on the elastic buckling moment. The following example illustrated by Chen (1991) is a simply supported beam subjected to a midspan concentrated load (Figure 2.12).

Considering that the concentrated load is applied to the shear centre of the I-beam, the external moments for any cross section (z distance from support) on the local coordinates x’-y’-z’ are:

\[ M_{x'(\text{ext})} = M_{x(\text{ext})} = P/2(L/2 - z) \]  \hspace{1cm} (2.37)
\[ M_{y'(\text{ext})} = -\gamma M_{x(\text{ext})} = -\gamma P/2(L/2 - z) \]  \hspace{1cm} (2.38)
\[ M_{z'(\text{ext})} = M_{z(\text{ext})} = -P/2(u_{\text{max}} - u) \]  \hspace{1cm} (2.39)

On the other hand, the internal moment can be expressed with respect to the x’-y’-z’ coordinates as:

\[ M_{x'(\text{int})} = -EI_x \frac{d^2y}{dz^2} \]  \hspace{1cm} (2.40)
\[ M_{y'(\text{int})} = -EI_y \frac{d^2u}{dz^2} \]  \hspace{1cm} (2.41)
So, the governing differential equations are:

\[ EI_s \frac{d^2 v}{dz^2} + \frac{P}{2} \left( \frac{L}{2} - z \right) = 0 \]  \hspace{1cm} (2.43)

\[ EI_y \frac{d^2 u}{dz^2} + \frac{1}{2} \left( \frac{L}{2} - z \right) = 0 \]  \hspace{1cm} (2.44)

\[ GJ \frac{d\gamma}{dz} - EI_w \frac{d^3 \gamma}{dz^3} + \frac{P}{2} (u_{\text{max}} - u) - \frac{du}{dz} \frac{L}{2} \left( \frac{L}{2} - z \right) = 0 \]  \hspace{1cm} (2.45)

The critical moment can be solved from the combination of second and third equations in a form of:

\[ EI_w \frac{d^4 \gamma}{dz^4} - GJ \frac{d^2 \gamma}{dz^2} - \frac{\gamma}{EI_y} \left[ \frac{P}{2} \left( \frac{L}{2} - z \right) \right] = 0 \]  \hspace{1cm} (2.46)

Timoshenko and Gere (1961) have solved this equation using the method of infinite series. However, the result is not a closed form solution, and also this solution is only valid for the case of load applied at the shear centre. In practice, it would be difficult to determine what the exact load height is. Therefore, the load is generally assumed to be acting at either the shear centre, top or bottom flanges. For each load height, separate differential equations have to be solved.

The example of a simply supported beam subjected to midspan point load demonstrates that it is not possible to use the analytical method to derive a closed form solution for member lateral torsional buckling under various load conditions. Since there are infinite combinations of such load cases, suitable stability functions can not be derived for lateral torsional buckling based on solving the differential equations of equilibrium. It seems that using empirical equations is more suitable for beam instability problems.

### 2.3.4 Moment Gradient

For simply supported beams subjected to a uniform moment, the governing equation has constant coefficients in terms of z-axis. However, in most situations, the bending moment changes along the length of the beam. This means the governing differential equation will have variable coefficients. In these cases, the closed-form solutions can
not be obtained. Therefore, a numerical procedure is necessary to obtain the solutions involving the use of series or special functions, and these methods are not feasible for general design practice. Commonly, an approximate method that uses an equivalent moment factor ($\alpha_m$) method is used to obtain the beam buckling moment. Salvadori (1955) has demonstrated that the effects of moment gradient can be accounted for by an empirical factor. For example, the critical moment for beams with unequal end bending moment can be calculated by:

$$M_{cr} = \alpha_m M_{ocr}$$

where: $\alpha_m = 1.75 + 1.05\beta + 0.3\beta^2 \leq 2.3$

$$\beta = \frac{M_A}{M_B}$$

$\beta$ is positive when the beam bends in double curvature and is negative when the beam bends in single curvature.

There are a number of equations of equivalent moment factor used in design practice for common moment distribution cases. Appropriate formulae for general load cases have also been derived. The general formula needs to deal with a wide range of load cases. Its accuracy may vary from case to case. Using a different equation for different bending moment distribution significantly complicates the computer coding of beam capacity calculations.

The equivalent moment factors were first used in the case of elastic buckling moment, and have been adapted to calculate the inelastic beam strength (SA, 1998) even though it is difficult to fully verify the precision of equivalent moment method for all the load cases and the inaccuracies of this method are often acknowledged. It is also the only available practical method to deal with the effects of moment gradient.

### 2.3.5 Effects of Restraints

When the effects of various restraints are considered, instability analyses of beams become even more complex. First, the application of loads and restraints are often inseparable. The applied transverse load always induces a lateral restraint to the member. Second, there are many degrees of freedom needed to be considered when dealing with lateral torsional buckling. Even for a simply supported beam, restraints
on 14 degrees of freedom in terms of constraints (three displacements, three rotations, and warping restraint at each end of segment) must be accounted for. For a continuous beam, the restraint conditions are more complicated. Other factors such as top or bottom flange supported, bracing rigidities, etc also have profound effects on the beam’s capacity. In the design practice, the effective length concept is used to cope with various restraint conditions. The effective length factor of a beam is conceptually different from that of a column. It is significantly more complex and involves more issues such as warping restraints, partial restraints, and load height factor.

Commonly, the beam segments are classified into four groups: simply supported beams, restrained beams, continuous beams and cantilever beams.

### 2.3.5.1 Restrained Simply Supported Beams

An idealized simply supported beam is the fundamental case of restrained beams. The closed form solution for beams subjected to a uniform bending moment is based on the restraint conditions of idealized simply supported beams. The boundary conditions for idealized simply supported beam (Trahair, 1993) are defined as:

- Both ends fixed against in-plane vertical deflection but unrestrained against in-plane rotation, and one end fixed against longitudinal horizontal displacement.
- Both ends fixed against out-of-plane horizontal deflection and twist rotation but unrestrained against minor axis rotation and warping displacement.

When the boundary conditions of a member are different from those of the idealized simply supported beam and the beam segment is subjected to a uniform bending moment, the elastic beam capacity can be expressed as:

\[
M_{ocr} = \frac{\pi}{K_b L} \sqrt{\frac{EI_y}{GJ}} \sqrt{\left(1 + \frac{\pi^2 EI_w}{(K_c L)^2 GJ}\right)}
\]

(2.48)

The effective length factor \(K_b\) corresponds to the lateral bending effect while \(K_t\) corresponds to the twisting restraint. Typical \(K\) values are given in Table 2.1 (Vlasov, 1959).
In a study of beam segments subject to a uniformly distributed load or a concentrated load at mid-span, Nethercot (1979) has taken a different approach. The critical moment is expressed as:

\[ M_{cr} = C_{bs} M_{ocr} \]

where \( C_{bs} \) is the moment gradient factor but it also accounts for different end conditions of the beams.

The effective length factor \( K \) used in \( M_{ocr} \) equals to:

\[
K = \frac{\pi \sqrt{E I_y GJ}}{\sqrt{2L(C_{bs} M_{ocr})}} \left\{ 1 + \left[ 1 + \frac{4(C_{bs} M_{ocr})^2}{E I_y GJ GJ} \right]^{0.5} \right\}^{0.5}
\]

(2.49)

\( C_{bs} \) equals to:

\[
C_{bs} = \begin{cases} 
AB & \text{for load at bottom flange} \\
A & \text{for load at shear centre} \\
A/B & \text{for load at top flange} 
\end{cases}
\]

The reduction factors \( A \) and \( B \) for three common restraint cases (Figures 2.31 to 2.33) are from Table 2.2 (where \( W = (\pi/L)\sqrt{E I_w / GJ} \)): 

---

This image is not available online. Please consult the hardcopy thesis available from the QUT Library.
Table 2.2 Moment reduction factor for different load cases

<table>
<thead>
<tr>
<th>Loading</th>
<th>Case</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>q</td>
<td>1</td>
<td>$1.643 + 1.771W - 0.405W^2$</td>
<td>$1 + 0.625W - 0.339W^2$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$1.2 + 0.402W + 0.416W^2$</td>
<td>$1 + 0.571W - 0.225W^2$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$1.9 + 0.006W - 0.12W^2$</td>
<td>$1 + 0.806W - 0.1W^2$</td>
</tr>
<tr>
<td>P</td>
<td>1</td>
<td>$1.916 + 1.851 - 0.424W^2$</td>
<td>$1 + 0.923W - 0.466W^2$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>$1.43 + 0.463 + 0.485W^2$</td>
<td>$1 + 0.6913W - 0.317W^2$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$2.0 + 0.304 - 0.074W^2$</td>
<td>$1 + 1.047W - 0.207W^2$</td>
</tr>
</tbody>
</table>

For design purposes, it is often difficult to judge the exact restraint conditions for a beam segment. The effective length factors are often taken as:

- 1.0 if both ends are simply supported
- 0.7 if one end is simply supported and the other end is fixed
- 0.5 if both ends are fixed

A conservative measure is taken if the restraint conditions are doubtful and in which case the effective length factor is often taken as one.
2.3.5.2 Continuous Beams

A continuous beam refers to the member with lateral restraint(s) between its end supports. These lateral bracings can significantly increase the buckling moment of the beams and they change the beam’s buckle shape. The simplest case is a simply supported beam with additional lateral bracing at midspan. When this beam is subjected to a uniform bending moment, the lateral buckling mode will be a complete sine wave (see Figure 2.16). Therefore, the effective length equals to half the length of the beam, and the corresponding critical moment can be calculated accordingly.

When a continuous beam has more than two lateral bracings, partial end restraints will develop between adjacent spans. Theoretically, the lateral buckling load depends on the relative stiffness of the segments, the type of bracing or constraint used for the intermediate support, and the type and relative magnitude of the loads on the beam. In practice, most of these parameters are impossible to quantify. Commonly, the critical loads for each segment of a continuous beam are evaluated separately. The selection of a K value for each beam segment is often based on engineering intuition with the assumption that each segment is simply supported. The lowest value of the critical loads is taken as the buckling of the continuous beam. The method neglects the interactions between each beam segments but the results are generally conservative.

2.3.5.3 Cantilever Beams:

The elastic buckling moment of a cantilever subjected to a uniform moment $M_o$ is given by:

$$M_{ocr} = \frac{\pi}{L/2} \sqrt{E I_y G J} \sqrt{1 + \frac{\pi^2 E I_w}{(L/2)^2 G J}}$$

(2.50)

where:
L is the length of the cantilever beam
\( I_w \) is the warping constant, \( J \) is the St. Venant torsional constant
E is the modulus of elasticity, \( G \) is the shear modulus
\( I_y \) is the minor axis second moment of area

For other loading conditions, no analytical procedure can be applied to obtain the solutions. Kirby and Nethercot (1985) have shown that using the effective length factor similar to the one used in columns gives a conservative estimate of \( M_{cr} \) for most applications. The equation for elastic critical loads is:

\[
M_{cr} = \frac{\pi}{KL} \sqrt{\frac{EI}{GJ}} \sqrt{\left(1 + \frac{\pi^2 EI_w}{(KL)^2 GJ}\right)}
\]  
(2.51)

Typical effective length factors (\( K \)) are given in Table 2.5.

Table 2.3 Effective length factors for cantilevers (Kirby and Nethercot, 1985)

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2.3.6 Inelastic Beams

The critical moment derived from an elastic buckling analysis is only valid for the cases where material yielding does not take place in the member. When a beam has a high slenderness ratio (\( L/r_y \)), instability occurs before yielding. For beams of
intermediate slenderness ratios, the member capacities are related to both yielding and buckling. Since some portions of the member are yielded when buckling commences and only the elastic portion of the cross section will remain effective in providing resistance to lateral buckling, the inelastic beam buckling behaviour may differ significantly from that of an elastic beam.

In the inelastic column analysis, tangent modulus concept has been successfully applied. This concept has been used for solving the inelastic beams in a similar fashion. For example, Attempt has been made for a simply supported I beam with a uniform bending moment and no residual stresses (Chen, 1987). The inelastic critical moment can be obtained from a simple modification of elastic critical formula \( M_{ocr} \) as:

\[
M_{ocr} = \frac{\pi}{L} \sqrt{(EI_y)_e (GJ)_e} \sqrt{1 + \frac{\pi^2 (EI_w)_e}{L^2 (GJ)_e}}
\]  

(2.52)

where \((EI_y)_e\), \((GJ)_e\), \((EI_w)_e\) are the effective bending rigidity, torsional rigidity, and warping rigidity.

However, when using the tangent modulus concept for lateral torsional buckling cases the method is unable to account for the residual stresses present in the steel beams. The residual stresses can greatly reduce the lateral torsional buckling strength in the inelastic range (Chen, 1987).

The moment gradient is another problem in the analysis of inelastic beams. When beam segments are subjected to a uniform bending moment, the distributions of yielding are fairly constant along the member. For general load cases, this is not the case. For example, when a beam is subjected to equal end moments, bending the beam into a double curvature, yielding is usually confined to sections at the supports of the beam. Furthermore, many load and restraint conditions will produce local yielding. Their effects on the inelastic beam buckling behaviour can be very significant.
Since inelastic buckling is such a complex phenomenon, analytical methods are not adequate and only two methods can be used to solve this problem: the numerical solution and the experimental solution. Trahair (1993) has summarized the numerical methods used for inelastic flexural torsional buckling. They include finite difference, finite integral, transfer matrix and finite element methods. Among all the numerical methods, 3D finite element analyses provide the most comprehensive solutions. On the other hand, large amounts of experiments have also been conducted on simply supported beams. It is the results of beam testing that lay the foundations for design codes in dealing with lateral torsional buckling issues. Figure 2.17 shows the beam testing results that led to the development of inelastic beam strength curves in the Australian design specification AS4100 (SA 4100).

2.4 Design of Members Subjected to Lateral Torsional Buckling Effects

The design of steel beams is based on semi-empirical equations for the ultimate section and member capacities. The member capacities are compared with the member internal loads corresponding to the ultimate applied forces, typically obtained from an elastic analysis. This section presents the relevant design rules from three
design specifications (AS4100, AISC LRFD, and Eurocode3) regarding beam and column capacities and the member out-of-plane capacity.

2.4.1 Australian Standard - AS4100

Member capacity for bending
The nominal member moment capacity \( M_b \) is defined in Clause 5.6.1.1 (SA, 1998) as:

\[
M_b = \alpha_m \alpha_s M_s \leq M_s
\]

(2.53)

where: \( \alpha_m \) = a moment modification factor
\( \alpha_s \) = a slenderness reduction factor
\( M_s \) = nominal section capacity

The moment modification factor can be obtained from AS 4100 Table 5.6.1, or by using the following equation:

\[
\alpha_m = \frac{1.7 M_m^*}{\sqrt{M_2^* + M_3^* + M_4^*}} \leq 2.5
\]

(2.54)

where: \( M_m^* \) = maximum design bending moment in the segment
\( M_2^*, M_3^*, M_4^* \) = design bending moments at the quarter points of the segment
\( M_3^* \) = design bending moment at the midpoint of the segment

The slenderness reduction factor is given as:

\[
\alpha_s = 0.6 \left[ \sqrt{\left( \frac{M_s}{M_o} \right)^2 + 3} - \left( \frac{M_s}{M_o} \right) \right]
\]

(2.55)

\( M_o \) is called the reference buckling moment which is obtained from elastic analysis of simply supported beams under a uniform bending moment.

\[
M_o = \sqrt{\left( \frac{\pi^2 EI_y}{l_e^2} \right) \left( GJ + \left( \frac{\pi^2 EI_w}{l_e^2} \right) \right)}
\]

(2.56)

where: \( E, G \) = the elastic moduli
\( I_y, J \) and \( I_w \) = section constants
\( l_e \) = the effective length
The effective length is given by:

\[ l_e = k_t k_l k_r l \]

where: \( k_t \) is a twist restraint factor, \( k_l \) is a load height factor and \( k_r \) is a lateral rotation restraint factor.

\( l \) is taken as

a) the segment length, for segments without intermediate restraints, or for segment unstrained at one end, with or without intermediate lateral restraints.

b) the sub-segment length, for sub-segments formed by intermediate lateral restraints in a segment which is fully or partially restrained at both ends.

**Member capacity for axial compression**

The nominal member capacity \( (N_c) \) is defined in Clause 6.3.3 (SA, 1998) as

\[ N_c = \alpha_c N_s \leq N_s \]  \hspace{1cm} (2.57)

where \( N_s \) is the nominal section capacity

\[ \alpha_c = \frac{N_c}{N_s} = \xi \left[ 1 - \sqrt{1 - \left( \frac{90}{\xi \lambda} \right)^2} \right] \]  \hspace{1cm} (2.58)

\[ \xi = \frac{(\lambda/90)^2 + 1 + \eta}{2(\lambda/90)^2}; \quad \lambda = \lambda_n + \alpha_n \alpha_b \]

\[ \eta = 0.00326(\lambda - 13.5) \geq 0 \]

\[ \lambda_n = \left( \frac{L_e}{r} \right) k_f \sqrt{\frac{f_y}{250}} \]

\[ \alpha_n = \frac{2100(\lambda_n - 13.5)}{\lambda_n^2 - 15.3\lambda_n + 2050} \]

The reduction factor \( \alpha_c \) is based on experimental testing of a range of compact and non-compact sections and is appropriate for either major or minor axis column buckling. The equations can be used for a wide variety of common section types (hot-rolled I-sections, welded I-sections, cold-formed rectangular hollow sections, etc.) by the selection of appropriate member section constant \( (\alpha_b) \), as shown in AS4100 Tables 5-1 and 5-2.
Member capacity for combined action of axial force and bending moment

Three aspects of member capacities must be checked for members subjected to a combined action of axial force and bending moment. They are section capacity, in-plane capacity, and out-of-plane capacity. The out-of-plane member capacity ($M_{ox}$) is related to lateral-torsional buckling effect and is defined as follows (Clause 8.4.4.1).

$$M_{ox} = M_{bx} \left(1 - \frac{N^*}{\phi N_{cy}}\right)$$ (2.59)

where: $M_{bx}$ is the nominal member moment capacity about major axis.

$N_{cy}$ is the nominal member capacity in axial compression for buckling about minor axis (out-of-plane buckling).

$\phi$ is the capacity reduction factor.

For members subject to axial tension, the out-of-plane member capacity (Clause 8.4.4.2) is:

$$M_{ox} = M_{bx} \left(1 - \frac{N^*}{\phi N_{tx}}\right) \leq M_{rx}$$ (2.60)

where: $N_{tx}$ is the nominal section capacity in axial tension.

2.4.2 AISC (American) Design Specification – LRFD

Member capacity for bending - LRFD beam curve

For beams of compact sections, there are two possible types of failure: (1) plastic yielding, (2) lateral torsional buckling. The design curve is shown in Figure 2.18.

The beam design curve is divided into three sections. If $L_b \leq L_p$, the beam is considered to have adequate lateral support. If $L_p < L_b \leq L_r$, the beam is considered to be laterally unsupported and inelastic lateral-torsional buckling may occur. When $L_b \geq L_r$, the nominal moment ($M_n$) is equal to elastic buckling solution. For double symmetric I-sections, the nominal moment capacity is defined as in Chapter F2 (AISC, 1998) for various ranges of $L_b$. 
The limiting laterally unbraced lengths for full plastic bending \( (L_p) \) and inelastic lateral-torsional buckling \( (L_r) \) are obtained from experimental data. They are defined as follows:

\[
L_p = \frac{300r_y}{\sqrt{F_y}} \quad (2.61)
\]

\[
L_r = \frac{r_y X_1}{(F_y - F_r)} \left( 1 + \sqrt{1 + X_2 (F_y - F_r)^2} \right) \quad (2.62)
\]

where: 
- \( r_y \) = radius of gyration about the weak axis in inches
- \( F_y \) = yield stress in ksi
- \( L_p \) = limiting laterally unbraced length for full plastic bending
- \( L_r \) = limiting laterally unbraced length for lateral-torsional buckling

\[
X_1 = \frac{\pi}{8} \sqrt{\frac{EAGJ}{2}} \quad (2.63)
\]

\[
X_2 = \frac{4C_w}{I_y} \left( \frac{S_x}{GJ} \right)^2 \quad (2.64)
\]

\( S_x \) = section modulus about strong axis \( (\text{in}^3) \)

\( E, G, A, J, C_w, I_y \) = section constants \( (\text{ksi, ksi, in}^2, \text{in}^4, \text{in}^4, \text{in}^6) \)

\( F_r \) = compressive residual stress in the flange

when \( L_b \leq L_p \):

\[ M_n = M_p \]
where: $M_p$ is the plastic section capacity

when $L_p < L_b \leq L_r$;

$$M_n = C_b \left[ M_p \left( M_p - M_r \frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p$$  \hspace{1cm} (2.65)

where: $C_b = 1.75 + 1.05 \left( \frac{M_1}{M_2} \right) + 0.3 \left( \frac{M_1}{M_2} \right)^2 \leq 2.3$  \hspace{1cm} (2.66)

$M_1, M_2$ = numerically smaller to larger end moments,

$$M_r = S_x (F_y - F_i)$$

where: $S_x$ = section modulus about the strong axis (in$^3$)

$F_y$ = yield stress (ksi)

when $L_b > L_r$;

$$M_n = M_{cr} \leq M_p$$

where: $M_{cr}$ = the critical elastic moment

Member capacity for axial compression forces

The design compression strength for flexural buckling (compact sections) is defined in Chapter E2 (AISC, 1998) as follows:

$$P_n = A_g F_{cr}$$  \hspace{1cm} (2.67)

for $\lambda_c \leq 1.5$:

$$F_{cr} = \left( 0.658 \frac{\lambda_c^2}{\lambda_c^2} \right) F_y$$  \hspace{1cm} (2.68)

for $\lambda_c > 1.5$:

$$F_{cr} = \frac{0.877}{\lambda_c^2} F_y$$  \hspace{1cm} (2.69)

where: $A_g$ = gross area of the member

$F_{cr}$ = nominal critical stress

$$\lambda_c = \frac{K l}{r \pi} \sqrt{\frac{F_y}{E}}$$  \hspace{1cm} (2.70)

$K$ = effective length factor

$l$ = lateral unbraced length of member

$r$ = governing radius of gyration about the axis of buckling
Member capacity for combined action of axial force and bending

The AISC LRFD format is based on the exact inelastic solutions of 82 beam-columns. The bending moment and axial force interaction is in a form of bi-axial bending capacity equation. Using this format, the in-plane and out-of-plane member capacity checks are combined into one.

The interaction equations are defined in Chapter H1 (AISC, 1995) as follows:

For \( \frac{P_u}{\phi P_n} \geq 0.2 \)

\[
\frac{P_u}{\phi P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0
\]

(2.71)

For \( \frac{P_u}{\phi P_n} < 0.2 \)

\[
\frac{P_u}{2\phi P_n} + \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0
\]

(2.72)

where: Reduction factor for compression \( \phi = 0.85 \)

Reduction factor for Flexure \( \phi_b = 0.90 \)

\( P_u, M_{ux}, M_{uy} \) = member design loads

2.4.3 European Standard - EC 3 (ENV1993) Part 1.10

Member capacity for bending (lateral-torsional buckling)

The design buckling resistance moment of laterally unrestrained beam is defined in Clause 5.5.2 (EC3, 1993)

\[
M_{b, Rd} = \chi_{LT} W_{pl,y} f_y / \gamma_M
\]

(2.73)

where: \( W_{pl,y} = \) plastic section modulus

\( \gamma_M = \) partial safety factor

\( \chi_{LT} = \) lateral torsional buckling reduction factor
\[ \chi_{LT} = \frac{1}{\phi_{LT} + \left[ \phi_{LT}^2 - \lambda_{LT}^2 \right]^{0.5}} \]  
\hspace{1cm} (2.75)

in which
\[ \phi_{LT} = 0.5 \left[ 1 + \alpha_{LT} \left( \chi_{LT} - 0.2 \right) + \lambda_{LT}^2 \right] \]  
\hspace{1cm} (2.76)

\[ \lambda_{LT} = \sqrt{\frac{\left[ \frac{k}{k_w} \right]^2 I_w + 0.039 I_{LT}^2 I_w}{I_z}} \]  
\hspace{1cm} (2.77)

\[ M_{cr} = C_1 \frac{\pi^2 EI_z}{l_{LT}^2} \sqrt{\left( \frac{k}{k_w} \right)^2 I_w + 0.039 I_{LT}^2 I_w} \]  
\hspace{1cm} (2.78)

\( \alpha_{LT} \) = imperfection factor 0.21 for hot-rolled sections

\( M_{cr} \) = elastic critical moment

\( L_{LT} \) = kL effective length for out-of-plane bending

k = effective length factor for out-of-plane bending

\( k_w \) = effective length factor for warping

\( I_z \) = minor axis second moment of area,

\( I_w \) = warping constant \( I_t \) = torsional constant

**Member capacity for axial compression**

The ultimate capacity for flexural buckling is carried out using the formula:

\[ \frac{N}{\chi N_{pl}} \leq 1 \]  
\hspace{1cm} (2.79)

where: \( N_{pl} = A f_y \), ultimate axial capacity

N = design axial force

\( f_y \) = specified yield stress

\( \chi \) = reduction coefficient, depending on the reference slenderness ratio \( \lambda \) and the buckling curves

\[ \chi = \frac{1 + \alpha (\chi - 0.2) + \lambda^2}{2 \lambda^2} - \frac{1}{2 \lambda^2} \sqrt{\left[ 1 + \alpha (\chi - 0.2) + \lambda^2 \right]^2 - 4 \lambda^2} \]  
\hspace{1cm} (2.80)

\[ \lambda_{pl} = \pi \frac{E}{f_y} \]  
\hspace{1cm} (2.81)
\[
\lambda = \frac{\lambda_{pl}}{N_{ki}} = \sqrt[\lambda_{pl}]{N_{ki}}
\]

(2.82)

\(\alpha\) = column curve coefficient, depending on the member cross-section properties

\(N_{ki}\) = elastic buckling load

**Member capacity for combined action of axial force and bending moment**

**a) Bending and axial tension**

The net calculated stress \(\sigma_{\text{com,Ed}}\) in the extreme compression fibre due to the vectorial effects should be determined from

\[
\sigma_{\text{com,Ed}} = \frac{M_{Sd}}{W_{\text{com}}} - \psi_{\text{vec}}\frac{N_{t,Sd}}{A}
\]

(2.83)

where:

\(\psi_{\text{vec}} = 0.8\)

\(W_{\text{com}}\) = the elastic section modulus

\(N_{t,Sd}, M_{Sd}\) = the design loads

The design moment obtained from

\[
M_{\text{eff, Sd}} = \psi_{\text{com}}\sigma_{\text{com,Ed}} \leq M_{b,Rd}
\]

(2.84)

where:

\(M_{b,Rd}\) = design buckling resistance moment

**b) Bending and axial compression**

For members that the lateral-torsional buckling is a potential failure mode, the member capacities should satisfy

\[
\frac{N_{Sd}}{\chi_z A f_y / Y_{M1}} + \frac{k_{LT} M_{Sd}}{\chi_L T W_{pl, y} f_y / Y_{M1}} + \frac{k_M M_{Sd}}{W_{pl, z} f_y / Y_{M1}} \leq 1
\]

(2.85)

in which

\(k_{LT} = 1 - \frac{\mu_{LT} N_{Sd}}{\chi_z A f_y} \) but \(k_{LT} \leq 1\)

(2.86)

\(\mu_{LT} = 0.15 \chi_z \beta_{M,LT} - 0.15 \) but \(\mu_{LT} \leq 0.90\)

(2.87)

\(\beta_{M,LT}\) = equivalent uniform moment factor for lateral-torsional buckling
2.4.4 Comparison of Design Specifications

All of the design codes are based on the well established theory of elastic lateral torsional buckling, which is the upper bound for all design criteria. The plastic moments are taken as the ultimate compact beam capacities at short unbraced length for all three design specifications. For intermediate and long members, different forms of empirical transition are used in these design codes to obtain the ultimate capacities from the elastic buckling curve.

![Comparison of Beam Curves (uniform bending moment case)](image_url)

Figure 2.19 Comparison of Beam Curves (uniform bending moment case)

Figures 2.19 and 2.20 are comparisons of the beam curves for two load cases. The first load case is uniform bending moment and the second one is midspan concentrated point load. The moment modification factors (or inverse of equivalent factor in EuroCode 3) are taken as 1.35 in the second load case. The differences of ultimate capacities from these design specifications are very significant. When the moment modification factor is involved, the variations of ultimate capacities become more profound especially for beam segments with intermediate length. The predictions from the American code are substantially higher than the others. The
reasons for the discrepancies of design code include: “1] because of simplifying approximations or 2] because of deliberate introduction of conservative criteria, 3] because some specifications aim to represent mean predictions while others aim for a lower bound of the tests and the predictions” (Fukumoto, 1991).

![Figure 2.20 Comparison of Beam Curves (midspan point load case)](image)

In comparison, the beam curve used in the Australian design specification (AS4100) adapts a lower bound approach. Various effects including residual stresses, initial geometrical imperfections, etc are taken into considerations. In the beam capacity formula, the separation of moment modification factor and slenderness reduction factor with AS4100 is significantly different to other codes.

### 2.5 Summary

Over the past several decades, a large body of knowledge has been accumulated on the behaviour, analysis and design of steel frame structures. The information relevant to this research project includes a wide range of methods on frame analysis and a large amount of detailed studies on lateral-torsional buckling of simply supported beams. In current practice, advanced analysis is not used in steel frame design. Simple
elastic analyses are the most popular methods. On some occasions, plastic hinge analysis has been performed. However, the plastic hinge analysis can only account for one type of frame instability, the hinge collapse mechanism. Hence, it is not sufficient for comprehensive structural assessments. Advanced analysis can be categorized into two kinds: distributed plasticity analysis and concentrated plasticity analysis. This chapter has reviewed the limitation and capability of the currently available advanced frame analysis methods. It was found that the refined plastic hinge analysis is the most promising method. The available literature also shows that no existing practical advanced analysis method is capable of dealing with member lateral torsional buckling problems.

Lateral-torsional buckling effects are one of the most critical elements in the behaviour and design of steel frame structures. The characteristics of out-of-plane instability due to bending are extremely complicated. The analytical solutions cannot be used to obtain the capacity of beams subjected to lateral torsional buckling effects. The current beam capacity curves are derived mainly from testing data of simply supported beams. Since the focus of previous studies was to obtain the beam’s buckling capacity, very limited research has been conducted on the beam’s load-deflection behaviour. Also, the research into the effects of lateral torsional buckling of frame structures is virtually non-existent. It is therefore necessary to conduct research in these areas, preferably using the latest distributed plasticity analysis tools. In the design practice, the member out-of-plane capacity is checked by using semi-empirical equations. It is found that the principles behind the design specifications in different countries are the same, but the formulations can be quite different. Lateral-torsional buckling is related to a number of factors. The important ones are section properties, slenderness ratio, member end constraints, load conditions, member imperfections, residual stresses, and moment gradient along the member. These factors have to be included in the proposed advanced analysis.

Currently, the refined plastic hinge method is capable of dealing with three instability problems: plastic hinge formation, inelastic column buckling and local buckling. The formulation of this method has been extended to cover semi-rigid connections and 3D space frames issues. The most significant achievement of refined plastic hinge analysis is that the calculations for the column effective length factors are eliminated.
Instead, the stability functions and the tangent modulus concept are used to directly account for member buckling issues due to axial compression force. It has the potential to become a practical design tool due to the computational efficiency. However, because of the lack of closed form solution for lateral torsional buckling, solving the beam’s instability problem directly is not likely. It seems that the effects of lateral torsional buckling should be accounted for implicitly by using the plastic hinge based method. Further a thorough understanding of beams and frame structures including the ultimate strength and load-deflection response is required to include these effects in the practical advanced analysis method. The latest distributed plasticity analysis will be used for these studies.
Chapter 3. Distributed Plasticity Analyses of Simply Supported Beams

Lateral torsional buckling of a simply supported beam is a fundamental failure case in steel frame structures. Although it has been intensively studied, there are still many issues that have to be addressed, including the load-deflection response. Therefore a comprehensive study using finite element analysis was conducted to investigate the lateral torsional buckling behaviour of simply supported beams. Details of this study are presented in this chapter.

In general, there are three methods to determine the ultimate capacity of a member in steel frame structures subject to such buckling effects, viz. Analytical, Experimental, and Numerical. Among the analytical methods, Ritter (1888) conducted the earliest work on the theory of inelastic buckling for compression members. He developed a graphical method of constructing load–deflection curves of columns corresponding to the numerical procedures still implicitly used in computer programs. However, no adequate analytical solution has been developed since Ritter developed his graphical procedure for compression members and beam-columns. In 1987, Schwartz made a new attempt to derive the column load-deflection curves through double integration of the curvature (Thurlimann, 1990). In the rapidly advancing computer age, the advantage of an analytical solution over a numerical solution has lost its significance. Any further attempt on the more complicated lateral buckling problem using the analytical method seems unlikely.

Experimental method generally offers the most persuasive results. Many tests were conducted on lateral buckling of I-section beams by Fukumoto (1977, 1980 and 1981) and Nasahiro (1988). However, the experimental method also has many technical problems. Lateral torsional buckling capacity is sensitive to the initial geometric imperfections and residual stresses of the member. The imperfections of the test specimen are very difficult to quantify. The relationship between the initial imperfections and the ultimate strength is not well understood (Fukumoto, 1980). Residual stress measurements are often found to vary significantly. Most importantly, the experimental boundary conditions of supports and loading have always had
unknown side effects on the results. For example, the load application often introduces undesirable lateral restraints in the test specimen. Other major difficulties in carrying out experimental investigations on full-scale steel frame structures are due to reduced financial and technical support in research institutions. In the previous studies, most lateral buckling experiments associated with lateral buckling problems were on simply supported beams. This is because the purpose of these experiments was to verify the nominal design strength given by the design specifications. Full scale frame tests require significant amount of resources and hence only a limited number of tests had been conducted (Vacharajittiphan, 1973; Hancock, 1978; Bradford, 1986; Shanmugam, 2001). Due to the problems associated with the experimental tests, the non-linear behaviour of individual members and their failures (lateral buckling modes) in steel frame structures have not been investigated adequately.

Among the numerical methods, the robust general-purpose finite element analysis (FEA) programs and ever-increasing computer power have opened a new era in the field of non-linear analysis of steel structures. Shell elements have been used in many stability problems with great success, for example, ultimate capacity of non-compact steel frame structures and lateral-torsional behaviour of hollow flange beams (Avery and Mahendran, 2000b). Experimental studies were also conducted in these researches. The comparison of results from experimental and numerical analyses shows that the finite element analysis is a preferred method for steel frame structural research. Therefore in this project, distributed plasticity shell finite element models with different end restraints were used to investigate the buckling and non-linear ultimate strength behaviour of doubly symmetric steel I-beams subject to various combinations of bending moment and axial force.

The objective of conducting distributed plasticity analyses in this research is to gain a better understanding of lateral torsional buckling phenomenon in steel frame structures, and, eventually, to develop a series of benchmark solutions for steel frame structures. Bridge et al (1991) has suggested that test problems for benchmarking and verification should be graded in complexity, so the initial analyses exhibit the simplest structural behaviour while later analyses incorporate more complex phenomena. Using this strategy, it is reasonable to commence the modelling with
simply supported beams and then increase the complexity by considering models of full scale frame structures. In this way, the source of any inconsistencies occurring during the verification process can be isolated and the accuracy of the model can be better understood.

3.1 Model Description

The deformations of beams in lateral torsional buckling mode are quite complex, especially when approaching their ultimate failure state in plastic ranges. The “stick” type distributed plasticity analysis models do not provide enough degree of freedoms to describe them. Coarse assumptions are often made when modelling the boundary conditions using these beam elements. However, both load and boundary conditions have very significant affects on the inelastic lateral torsional buckling failure mode. In order to develop benchmark solutions of steel frame structure subject to lateral torsional buckling failures, three dimensional finite elements must be used because such models would capture the real structural behaviour including not only global effects of the member but also various local effects.

In this research, simply supported I-beam models were developed using shell elements. These models are capable of taking both geometrical and material non-linearities into account. Using shell elements, both geometrical imperfections and residual stresses can be explicitly modelled. Effects of loading and support conditions can also be studied in greater detail.

3.1.1 Elements

To study the non-linear behaviour associated with lateral buckling failures, shell elements are required because they can provide sufficient degrees of freedom to explicitly model buckling deformations and spread of plasticity effects. Many commercial finite element analysis packages are capable of performing second-order inelastic analysis. Among them, ABAQUS (HKS, 1998) was found to be the most appropriate due to the ease of residual stress input. The geometry of I-section beams is very easy to model and 4-noded shell elements are suitable for this type of
geometry. Three types of 4-noded shell elements are available in ABAQUS Standard version 5.8 (S4, S4R, and S4R5). Both S4 and S4R elements are doubly curved general-purpose, finite membrane strain shell elements. “R” stands for reduced integration with hourglass control. These two elements are often used for shell structures with thicknesses larger than 1/15 of the element length for which transverse shear deformation is important and Kirchhoff constraint is satisfied analytically. The S4R5 element is a thin, shear flexible, isoparametric quadrilateral shell with four nodes and five degrees of freedom per node, utilizing reduced integration and bilinear interpolation schemes. This element imposes the Kirchhoff constraint numerically. In comparison, S4 and S4R elements have six degrees of freedom per node and have multiple integration locations for each element. They will be more accurate than the S4R5 element for thick shell structures, but are significantly more computationally expensive. For the simply supported beam models, the preliminary analyses indicated that the difference between the ultimate load results using S4 and S4R5 elements was negligible (less than one percent). Therefore, S4R5 element was selected for the analysis of all the models.

Choice of mesh density depends on the geometrical characteristics of the structures and CPU speed. In order to achieve an optimal aspect ratio, and minimise the localised stress concentrations at the beam supports while keeping the computer run time manageable, it was necessary to have 16 elements for the web and 10 elements for the flanges. An element length of 50 (mm) (aspect ratio = 3.4) gave element numbers in the range from 1000 to 7000 depending on the model length. Modelling support boundary conditions require the use of special type rigid elements. For idealised pin supported models, four types of multi-point constraints (MPC) elements were used to achieve the support boundary conditions. They include a Rigid Beam, Tie, Pin, and two explicit type MPCs for linking nodes with only one specific degree of freedom. For the two warping restrained models, Rigid Surface element R3D4 was utilized. This element is a rigid quadrilateral with four nodes and three translational degrees of freedom at each node. As a rigid surface attached to the end of the I-beam, all nodes in the cross section will remain in a plane and warping deformations are effectively controlled. The CPU run time on the Silicon Graphics Power Challenge with R1000 processors was between half to one hour for non-linear analysis with more than one hundred steps.
3.1.2 Material Properties

The ABAQUS classical metal plasticity model was used in all the analyses. This model implements the von Mises yield surface to define isotropic yielding and associated plastic flow theory. i.e. as the material yields, the inelastic deformation rate is in the direction of the normal to the yield surface. This assumption is generally acceptable for most calculations with metals. Perfect plasticity behaviour is assumed in all the models. The reasons for this assumption are: strain-hardening data is not readily available and the aim of this research is not to verify experimental results, but to establish benchmark solutions. Strain hardening may affect the capacity of members that are not simply supported. However, unless full scale frame experiments are carried out, it would be difficult to quantify its effect on lateral buckling failures. Therefore the strain-hardening behaviour was not implemented into the models. The yield stress $\sigma_y$ of both the web and flanges was taken as 300 MPa. The elastic modulus and the Poisson’s ratio were taken as 200000 MPa and 0.3, respectively.

3.1.3 Load and Boundary Conditions

The most fundamental load case in lateral-torsional buckling research is the simply supported beam in uniform bending (Figure 3.1(a)). The theoretical elastic buckling moment is well established (Trahair, 1993). However, an analytical solution is still not available for inelastic beams. The real beam capacity formula was derived from large amount of experimental data. Since it is difficult to achieve the same boundary conditions assumed by analytical elastic buckling solutions in experiments, it is important to develop a numerical model capable of achieving these idealised simply supported conditions. This allows the results from elastic buckling analysis to be verified against the theory and then to use this model for the study of inelastic beams. Since moment gradients have significant effects on lateral-torsional buckling failures, three other different load cases are also used in the distributed plasticity models (see Figures 3.1 (b) to (d)). They are: 1) a concentrated load at mid span, 2) two concentrated loads at quarter points, and 3) a uniformly distributed load. The empirical equations in current design specifications are in general based on testing of simply supported beams under these load cases.
Because of the symmetric loading and geometrical conditions, half length beam models were used for all these load cases with symmetric boundary conditions applied to the mid-span of the beams.

Effects of warping restraints are a difficult issue in dealing with the lateral-torsional buckling in steel frame structures. Therefore, two fully warping-restrained models were also studied. They are: warping restrained with free out-of-plane rotation support, and warping restrained with fixed lateral rotation support. Details of establishing these boundary conditions in shell element models are given as follows.

3.1.3.1 Idealised Simply Supported Boundary Condition

These boundary conditions were used to replicate idealised simply supported beams with uniform bending moment conditions assumed in theoretical elastic buckling analyses. The models with these boundary and load conditions are suitable for the development of beam design curves and for the study of lateral buckling behaviour under combined actions of bending moment and axial force. According to Trahair (1993), the idealised pin support boundary conditions (Figure 3.2) are required to satisfy the following requirements.

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**Figure 3.1 Loading Configurations of Simply Supported Beams**

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- Simply supported in plane: that is, both ends fixed against in-plane vertical deflection but unrestrained against in-plane rotation, and one end fixed against longitudinal horizontal displacement.

- Simply supported out-of-plane: that is, both ends fixed against out-of-plane horizontal deflection and twist rotation but unrestrained against minor axis rotation and warping displacement.

![Figure 3.2 Idealised Simple Support Boundary Conditions of the Models](image_url)

The idealised load and boundary conditions do not exist in real beams. Since the theoretical buckling analyses are based on the case of idealised load and boundary conditions, it is important to achieve these conditions in numerical models for comparison purposes. Using 2D elements, the implementation of the idealised boundary conditions is straightforward. However, the reproduction of these boundary conditions is very difficult when using three dimensional models (both in experiments and FEA). Ideally, the applied forces (axial force and/or bending moment) need to be spread evenly/smoothly across the end of members, while allowing all nodes to move in a fashion that satisfies pin support requirements. A range of boundary conditions has been studied so that modelling of idealised pin supports can be finalised.

1. The first trial of simple support condition is shown in Figure 3.3. The analysis results show that tying all the nodes (all translation degrees of freedom) to the centre of cross-section would not produce the desired effects of the idealised simple support (pin) conditions. This type of boundary conditions use (a) “rigid beam” type MPC or (b) rigid surface to spread the load applied at the centroid.
across the support, but the rigid ends have the undesirable warping restrained effects.

Figure 3.3 First Trial of Simple Support Boundary Conditions

Figure 3.4 Second Trial of Simple Support Boundary Conditions

2. Since warping deformation must be allowed, the models with loads spread along the web only were used in the second trial (Figure 3.4). There are three types of elements that can be used to achieve this: a) MPC, b) RB3D4 – a type of rigid beam element in ABAQUS, and c) stiff beam element with large artificial elastic
The problem for these types is that the bending moment or axial force cannot be spread evenly across the support ends. Thus it causes stress concentrations during non-linear analysis and leads to premature failure.

Figure 3.5 Third Trial of Simple Support Boundary and Load Conditions

3. The third trial of support conditions is shown in Figure 3.5. In this trial, bending moment is applied using a vertical load at the end of a cantilever. There are two problems when using this arrangement. Firstly, if the end of the cantilever segment of the beam is allowed to deform out-of-plane, a second-order torsional moment will be introduced into the system that causes additional stress at the supports. Further, unless the cantilever segment length equals half the beam length (where cantilever segment and main beam segment have equal and opposite warping deformation), warping restraint will remain to certain extent at the support. On the other hand, when the cantilever segment of the beam is restrained in the out-of-plane direction, at the support, warping deformation is nearly non-existent and the main segment of the beam becomes warping restrained. These boundary conditions are commonly adapted for experimental study of beam subjected to uniform bending moment. The same shortcoming will be experienced in the test. Compared with Trahair’s definition (1998), this configuration can only
achieve “approximate” simply supported beam conditions and is not suitable for beams with a large warping constant.

Figure 3.6 Fourth Trial of Simple Support Boundary and Load Conditions

4. Figure 3.6 shows the fourth trial of simple support boundary conditions. It is assumed that this boundary condition would spread loading more evenly into the supports. However, the use of rigid MPC elements prevented any warp or skew in the web section at the supports, thus adding undesirable warping effects into the supports. The third and fourth trials indicate that the correct way to achieve the idealised simple support conditions is to have all the boundary conditions in a plane perpendicular to the beam.

5. The fifth trial (Figure 3.7) was an improved version of the second trial. Stiffer beams were attached to the flanges to reduce the stress concentration. However, the elastic buckling loads were sensitive to the stiffness of these artificial beams. This means that such models may be either over-constrained or cause local stress concentration problems. One other problem is that the rigid beam attached to the web does not allow twisting deformations. In certain cases, this affects the warping deformation at the end of the beam.
Table 3.1 shows the examples of a six metre long simply supported beam (250UB37.5) using these five trial boundary conditions. Both elastic bifurcation buckling and non-linear analyses were carried for these models. The theoretical elastic buckling moment \( M_o \) of this beam is 71 kNm and the ultimate moment capacity \( M_{bu} \) is 56.1 kNm according to the Australian Steel Design Standard AS 4100 (SA, 1998). Comparing with \( M_o \) and \( M_{bu} \) from AS 4100, none of the trial support boundary conditions are fully satisfactory. These results also demonstrate that lateral-torsional buckling of beams is very sensitive to their end boundary conditions. For example, the difference between the rigid Beam MPC and the rigid shell element R3D4 used for modelling end plate (Trial 1) is very small, yet the buckling moments show a 6% difference.

In Trial 2 type boundary conditions, the elastic buckling moments are lower than that predicted by the elastic theory. This indicates that the supported end is under-restrained. In the non-linear analysis, very significant local stress concentrations were found at the support which caused a premature failure (see Trial 2b in Table 3.1). To overcome this problem, elastic material properties were assigned to a strip of element adjacent to the rigid beam. However, the ultimate moment capacity was increased depending on the width of this strip. Using such procedures is tedious and inconsistent in a parametric study.
Table 3.1 Elastic Buckling and Ultimate Moments for different Boundary Conditions

<table>
<thead>
<tr>
<th>Trial</th>
<th>Support conditions</th>
<th>Elastic moment</th>
<th>Ultimate moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 a. Rigid Surface MPC</td>
<td><img src="image1.png" alt="Image" /></td>
<td>106.3 kNm</td>
<td>84.1 kNm</td>
</tr>
<tr>
<td>1 b. Rigid Surface R3D4</td>
<td><img src="image2.png" alt="Image" /></td>
<td>99.6 kNm</td>
<td>82.0 kNm</td>
</tr>
<tr>
<td>2 a. Rigid Beam MPC</td>
<td><img src="image3.png" alt="Image" /></td>
<td>66.5 kNm</td>
<td>54.5/59.3 kNm (with elastic strips – yes/no)</td>
</tr>
<tr>
<td>2 b. Rigid Beam RB2D2</td>
<td><img src="image4.png" alt="Image" /></td>
<td>66.5 kNm</td>
<td>54.5/59.3 kNm</td>
</tr>
<tr>
<td>2 c. Stiff Beam</td>
<td><img src="image5.png" alt="Image" /></td>
<td>66.5 kNm</td>
<td>54.5/59.3 kNm</td>
</tr>
<tr>
<td>3.</td>
<td><img src="image6.png" alt="Image" /></td>
<td>82.9 kNm (loading end free)</td>
<td>78.9 kNm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>159 kNm (end laterally retrained)</td>
<td>115 kNm</td>
</tr>
<tr>
<td>4.</td>
<td><img src="image7.png" alt="Image" /></td>
<td>108 kNm</td>
<td>85.3 kNm</td>
</tr>
<tr>
<td>5. Combined type</td>
<td><img src="image8.png" alt="Image" /></td>
<td>67.0 kNm</td>
<td>60.6 kNm</td>
</tr>
<tr>
<td>6.</td>
<td>AS 4100</td>
<td>70.9 kNm</td>
<td>56.1 kNm</td>
</tr>
</tbody>
</table>
Third and fourth trials are obviously over-constrained. The elastic buckling and non-linear analysis results are much higher than those predicted by AS 4100. Third trial boundary conditions are commonly used in experiments. However, the numerical model shows that beam testing with this configuration will not achieve the idealised simply supported beam conditions.

The elastic buckling analysis result from the fifth trial indicates that this type of boundary condition would be the most suitable model. A correct idealised pin support boundary condition must be able to overcome the restraint problems seen in Trials 1, 3, and 4. Furthermore, the bending moment and axial force applied must not cause local stress concentrations which may lead to premature failure during inelastic non-linear analyses.

Figure 3.8 Final Version of Idealised Simple Support Conditions
The finalized boundary conditions are the improved version from Trial 5, and are shown in Figure 3.8. The main improvement is by allowing the rigid beams attached to the flanges and the web to move independently. These load and boundary conditions are achieved by using a system of Multiple Point Constraints (MPC). This system includes the following three components:

1. Three “rigid beam” type Multiple Point Constraint (MPC) elements, “ab”, “cd”, “ef”. The independent nodes are the centres of the “rigid beams”. These elements were used to spread the point load at the centroid of the cross-section evenly to the web and flanges.
2. Two “Pin” type MPCs connecting web and flange MPC elements, “ag” and “bh”. These links allow the flanges to rotate independently about the minor axis (ie, warping restraint was eliminated).
3. Explicit type MPC elements linking “rigid beams” (“ab”, “cd” “ef”) to the corresponding nodes on the edge of the section. For the nodes on the web, only x and z translational degrees of freedom are linked whereas for the flanges, only z translation was linked. This will allow the web or flanges to expand without distortion at the support, thus eliminating the possible stress concentrations.

A strip of elastic elements was also included in the model adjacent to its pinned end to further eliminate any effect of undesirable stress concentrations (the effects are small). An elastic strip width of 25 mm was used for all models. All nodes on the “rigid beam” are dummy nodes located in the same plane as the beam’s end. The centroid of the section was connected to the independent node at the web rigid beam with a “Tie” MPC. A concentrated axial compression load and a bending moment about its major axis were applied at point “A”. Thus, the entire member will have the required uniform bending moment and axial force. A single point constraint of “126” was applied to the centroidal node “A”, representing an ideal pinned support. For all the nodes on the other end of the model representing the plane of symmetry, the boundary condition used was “345”. The degrees of freedom notation “123” corresponds to translations in x, y and z axes whereas “456” relate to rotations about x, y and z axes, respectively.
3.1.3.2 Warping Restrained Simply Supported Boundary Conditions

The results from trial models of a simply supported beam show that a small increase in warping restraint may cause large differences in buckling capacity. Warping restraint issues are generally not being addressed in design specifications. They are often ignored to achieve conservative designs. Nevertheless, understanding the difference between idealised and warping restrained pins is important.

A warping restrained pin model must be able to distribute the axial force and bending moment to the member and also restrain end warping of the flanges. Trial 1 type supports (with rigid end plate) are suitable for this purpose. However, the Trial 1a type support restrains all in-plane deformation in the end cross-section – not even the cross-sectional distortions are allowed (effectively Poisson’s ratio equals zero at the end cross-section). This was considered over-restrained. Therefore, Trial 1b type support was considered to be the most appropriate boundary condition. It simulates a rigid loading plate attached to the ends of beam. The boundary conditions and loads are applied only to the centroid of plate, thus achieving the simply supported beam boundary conditions. This loading plate also prevents warping at the support.

A simply supported beam with warping end restrained model is shown in Figure 3.9. ABAQUS R3D4 element was adopted to model the rigid end plate. A concentrated
nodal force $P$ and a bending moment $M$ are applied to the centroid of this rigid plate. Effects of stress concentrations caused by the rigid elements were eliminated by including an elastic strip (width equals 25 mm). At the support end, a point constraint of “126” was applied to the centroidal node, representing the simply supported conditions. As for the idealised pin model, the boundary condition for symmetric plane was “345” for all nodes on the cross-section.

### 3.1.3.3 Laterally Fixed Simply Supported Boundary Conditions

The models for laterally fixed simply support are the same as that for the warping restrained pin support except the support boundary condition is 1256 instead of 126 (see Figure 3.9). Using this boundary condition, the out-of-plane rotation is restrained.

In a steel frame structure with rigid connections, the degrees of constraints of the beams are often between this case and the previous one (Case 2). In the elastic theory, the effective length of a beam segment with such support conditions equals half of that with idealised pin support conditions. Compared with Case 1 boundary conditions, the finite element models should show the same correlations.

### 3.1.4 Initial Geometric Imperfections

Very little information is known about the geometric imperfections along the length of structural steel members. Experimental data for geometric imperfections are limited. The existing data do not give a direct indication of how lateral buckling is triggered by real imperfections. In general, two parameters are considered important; the magnitude and the shape of imperfection. They are considered to have a direct link on the ultimate capacity. However, in an average steel member, the imperfections appear randomly. Therefore it is possible that these random imperfections only initiate the buckling deformation, but the ultimate member failure capacity is mainly determined by the primary buckling mode.

In the finite element analysis, the first (lowest) eigenmode is used for the imperfection distribution. A maximum imperfection value is used in a simple way to trigger the
occurrence of buckling failure. The maximum imperfection value with the “worst possible imperfection shape” is used to provide the conservative lower bound results.

Maleck et al (1995) recommends the use of AISC erection and fabrication tolerances as the maximum imperfection value (AISC, 1995). The same tolerances are specified in Sections 14.4 and 15.3.3 of the Australian Standard AS4100 (SA, 1998) for compression members. They are:

- Out-of-straightness: L/1000, but not less than 3 (mm).
- Out-of-plumbness: h/500, but not more than 25 (mm) for h<60 metres.

These values have been used by other researchers in their finite element analysis to establish steel frame benchmark solutions (Avery, 1998).

For simply supported beam cases, only the out-of-straightness imperfection is considered. A nominal maximum magnitude of L/1000 is used as the maximum lateral-twisting imperfection. A study was carried out to examine the effects of imperfection magnitudes and different eigenmode shapes. Figure 3.10 plots the non-dimensionalized ultimate moment capacity against the imperfection magnitudes. The simply supported beam is six metres long. The beam (250UB37) used in the study is a
compact section. The boundary conditions used are the idealised simply supported conditions.

The graphs in Figure 3.10 show that the maximum value of imperfections and the initial imperfection shapes have significant effects on the ultimate capacity of the six metre beam. When the imperfection value was chosen as L/1000 recommended by Maleck et al (1995), the results from the non-linear analysis using the first eigenmode were very close to what the design code (AS4100) predicted (3.5% more conservative).

One interesting finding was that even though the non-linear analyses started with different imperfection shapes/modes, at ultimate states, the member always failed with similar deformations (see Figure 3.12). This numerical result confirms the lateral buckling experimental results which showed that the beams failed in the same manner, but the varying degree of imperfections affected the ultimate beam capacities (Avery and Mahendran, 2000b). In the non-linear analysis, it appears that a very small initial imperfection may trigger the out-of-plane buckling, and also that the final buckling shape is different from the initial mode unless the first buckling mode is used as the initial input. Because the magnitude and shape of the initial imperfection chosen have such a big effect on the non-linear analysis results, the choice of accurate imperfection input is as important as choosing the accurate boundary conditions.

![Figure 3.11 Initial Geometric Imperfections of the Model](image)

In this study, the worst possible deformation mode (from the 1st eigenmode) was used in the non-linear analysis to determine the ultimate member capacity. The initial
geometric imperfection of models is shown in Figure 3.11. Using ABAQUS 5.8, the initial shape is introduced by modifying the nodal coordinates of the model using a vector field created by scaling the lateral torsional buckling eigenvector obtained from an elastic buckling analysis. The node that has the maximum out-of-plane deformation from elastic buckling analysis will have the largest imperfection magnitude of L/1000.

<table>
<thead>
<tr>
<th>Eigenmode</th>
<th>Initial Imperfection Shape</th>
<th>Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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*Figure 3.12 Initial Imperfection Shape and the Ultimate Failure Mode*

### 3.1.5 Residual Stresses

The residual stress of steel frame member affects the member stiffness. It causes premature initial yielding, and may even reduce the ultimate strength. For hot-rolled or welded structural members, the residual stress is associated with the cooling effect
during manufacturing processes. Since many variations exist in the process, the exact residual stress can be very different from one beam to the other. For example, it was often found that upper flanges have higher residual compressive and tensile stresses than those in lower flanges due to differences in cooling conditions. Measuring residual stresses is also a difficult task. For instance, the relatively low mean value of the compressive residual stresses at the flange end causes a large value (COV= 0.44) of coefficient of variation (Fukumoto, 1980). The compressive residual stresses at the flange ends are often considered as having the undesirable effect of causing lateral-torsional buckling. These kinds of uncertainty may lead to unreliable numerical models. Figure 3.13 illustrates the variation of residual stresses obtained from experiments (Fukumoto, 1980). In hot-rolled and welded sections, the membrane component (constant through the thickness) of the residual stresses is considered to be significant and is in the longitudinal direction.

This image is not available online. Please consult the hardcopy thesis available from the QUT Library.

Figure 3.13 Variation of Residual Stress Patterns (Fukumoto, 1980)

In advanced analyses, idealised residual stress patterns must be used to simplify the problems. For hot-rolled I-sections, the longitudinal membrane residual stress distributions are recommended by the ECCS Technical Committee 8 (1984) and have been adopted by numerous other researches (Avery, 1998). These residual stress
distribution patterns were also used in the simply supported beam models described in the thesis (see Figure 3.14).

Figure 3.14 Residual Stress Contours for a Typical I-section

The residual stress subroutine was added to each input file using the ABAQUS 5.8 *INITIAL CONDITIONS option. This subroutine defined the local components of the initial stress as a function of the global coordinates. The initial stresses were applied in a *STATIC step with no loading and standard model boundary conditions to allow equilibration of the initial stress field before starting the response history.

As global coordinates were used to define the local stress components, the residual stresses have to be determined by the deformed shape. Two polynomial functions with the length of member as a variable were used to represent the imperfection of top and bottom flanges approximately. In real steel frame members, the membrane residual stresses are self-balanced. In numerical models, because the residual stresses are defined on a “curved” geometry, they cause undesirable additional initial displacement for the following non-linear analysis step because of the unbalanced stress distributions. To overcome this, an additional analysis was conducted with the residual stresses subroutine and a boundary condition of all nodes fixed in the x, y, and z translation degrees of freedom. A reaction force field was obtained from this analysis. This force field was then used to reverse the extra initial deflection effects in the non-linear analysis stage.
3.2 Analysis Methods

Two methods of analysis were used: elastic buckling and non-linear static. Elastic buckling analysis was used to obtain the eigenvectors for the input of geometric imperfections. Non-linear static analyses were used twice: firstly, to obtain the reaction force field used to eliminate the undesirable initial deformation caused by the residual stress input. The second non-linear analysis was used to obtain the ultimate capacity of the member.

The Newton-Raphson solution technique and convergence tolerances of $10^{-8}$ were used in all the non-linear analyses. Failure occurred when equilibrium could not be achieved without reducing the increment size to less than this minimum. In general, the post-buckling path can be observed using the RIKS method (Riks, 1979) during non-linear analyses. However, because the load on the models with Case 1 boundary condition is applied by motion of a rigid surface and the structure buckles away from the constraints, the RIKS algorithm no longer has any beneficial effect (HKS, 1998). As a result, the RIKS method (using ABAQUS 5.8) was not used for the idealised simply supported boundary condition.

The parameters used for non-linear analyses are:

- maximum number of load increments = 100,
- initial increment size = 0.05,
- maximum increment size = 0.05,
- minimum increment size = 0.000001,
- automatic increment reduction enabled, and large displacements enabled.

The applied loads were approximately 1.1 times the design capacity, ensuring that the applied loads would exceed the ultimate capacity and provide a consistent and reasonable initial load increment size.

The following is a summary of the procedure used to prepare a non-linear analysis model including initial local and member imperfections, using MSC/Patran:
1. Define geometric surfaces for web and flanges.
3. Define loads, simply supported and symmetric boundary conditions, elastic material properties, element properties, and bifurcation buckling analysis parameters.
4. Run bifurcation buckling analysis using ABAUQS to obtain the first buckling eingenmode.
5. Import buckling analysis results into the Patran database.
6. Plot out-of-plane top and bottom flange displacements against the length of the member using MSC/Patran. The displacement data are then imported into an EXCEL spreadsheet. Using the curve-fitting function from the spreadsheet, two polynomial functions are found to represent the top and bottom flange deformation shape. These two functions are multiplied by a scale factor by which these maximum displacements should be multiplied to give the required member imperfection magnitudes. After that, the functions are used in residual stress input subroutine.
7. Using the Patran “advanced results processing vector plot option”, generate spatial FEM continuous vector field for the first eigenmode.
8. Using the “nodes modify by field” form located in the utilities FEM menu, offset the nodes of the model using the FEM vector fields (Step 6) and scale factors (Step 7).
9. Define the non-linear material properties and non-linear static analysis parameters.
10. Apply additional restraints to all nodes except both end edges in x, y, z direction.
11. Run the first non-linear analysis with single “load step” using initial stress input subroutine. The subroutine utilizes the polynomial functions obtained in bucking analysis that representing the top and bottom flange imperfection magnitude (Step 6). In this analysis, the axial force and bending moment are not applied.
12. Import first non-linear analysis result into Patran database. Using the vector plot option, generate spatial reaction force vector field as shown in Figure 3.15.
13. Run the second non-linear analysis. This run include two “load steps”:
a. Residual stress input “load step” - with simply supported and symmetric boundary conditions. The reaction force field is acting on every node except the ones at both ends of the member
b. Ultimate capacity load factor step - with concentrated axial force and bending moment applied.

\[\text{Figure 3.15 Force Vector Field}\]

### 3.3 Results and Discussions

Research on local buckling and flexural buckling problems of steel frame structures demonstrated that finite element analysis using the shell elements can achieve very accurate results compared with experimental data (Avery and Mahendran, 2000a; Alsaket, 1999). Therefore, it was expected that similar finite element analyses could be used in lateral torsional buckling problems in this project. The results obtained from finite element analysis can be verified using other methods: 1) compare the finite element analysis results of elastic bifurcation buckling with the results from the well established elastic theory, 2) compare non-linear static analysis results with design specifications.
The “beam curves” used in the design standard such as AS 4100 (SA, 1998) were developed based on rigorous testing of simply supported beams. The load cases shown in Figure 3.1 were used in the experiments where lateral torsional buckling occurred. If the non-linear analysis results such as the ultimate moment capacity agree well with the design curves, then these finite element models can be considered as reliable. Therefore the lateral torsional buckling behaviour including the moment-rotation relationship up to the ultimate failure state as obtained from the non-linear analyses can be used as benchmark solutions in the development of a simplified analysis method.

The results of finite element analyses are divided into four load cases.

The first case is the simply supported beam under uniform bending moment as shown in Figure 3.1(a). Different aspects of this classical load case have been studied; this includes four series of analyses.

- The first series uses the idealized simply supported boundary conditions shown in Figure 3.8. The finite element models of this series are subjected to only uniform bending or axial compression load.
- The second and third series focus on the lateral-torsional buckling behaviour of beams with supports that have warping restraints (boundary conditions 2 and 3 as discussed in Section 3.1.3).
- The fourth series of analyses involved combined loading of axial compression force and bending moment.

In these analyses, Load case 2 investigates the beam behaviour under one concentrated load at mid-span. Load case 3 is for two concentrated loads applied at quarter points. Load case 4 is for beam under uniformly distributed loads.

Load cases 2 to 4 involve beams with changing bending moment along the member. In order to reduce the degree of complication, the effects of axial force are not considered in these three load cases. Idealized simply supported boundary conditions are applied to these models.
In Australia, the standard hot-rolled Universal beam sections are listed in the BHP product brochures (BHP, 1994). These beam products have an $I_x$ to $I_y$ ratio ranging from 7 to 27 and are used primarily for carrying transverse loads. The cross section selected for finite element modelling is a 250UB37.3, a compact hot-rolled section. Local buckling will not affect the strength of this member. The properties of the section are shown in Figure 3.16. The effect of different slenderness was investigated in all load cases. The length of the models ranged from 0.5 m to 12 m.

![Figure 3.16 Section Properties of 250UB37.3](image)

**250UB37.3**
- $b_f = 146$ mm
- $t_f = 10.9$ mm
- $d_w = 245.1$ mm
- $t_w = 6.4$ mm
- $f_y = 320$ MPa

### 3.3.1 Simply supported beams subjected to a uniform bending moment and an axial compression force

#### 3.3.1.1 Series 1: Idealized Simply Support Boundary Conditions

a) Under axial compression load

Column flexural buckling due to axial compression force is one of the most critical aspects in steel frame structural design. This phenomenon has been extensively studied in the past. For inelastic flexural buckling, two-dimensional representation of cross-section partial yielding theory is able to match the observed behaviour of individual column buckling (eg. using tangent modular concept). The “column curve” documented in various design specifications is based on this research supported by a large amount of testing.
The fundamental case for column flexural buckling is a column with a pin support at one end and a roller support at the other end where axial load is applied (effective length $L_e$ equals physical length $L$). The release of warping restraint used in idealized simply supports is not necessary in the study of column buckling. Even though the boundary conditions developed for an idealized simply supported beam are more complicated, there should not be any adverse effect by using these boundary conditions in flexural buckling studies. For this reason, analyses were conducted to examine the effectiveness of the boundary conditions under axial loading. The analysis processes are the same as described in Section 3.1.6. A concentrated axial compression load is used instead of a concentrated bending moment in the models. The maximum initial imperfection is also taken as $L/1000$.

Finite element analysis results are plotted in Figure 3.17. The member capacities are non-dimensionalized using the section yield capacity ($P_y$). They are plotted against the member slenderness $\lambda_c$ which is defined as:

$$\lambda_c = \frac{L_e}{\pi} \sqrt{\frac{f_y E}{P_y}}$$

(3.1)

The results show that elastic bifurcation buckling analysis produces the same load as predicted by elastic theory:

$$P_e = \frac{\pi^2 EI}{L_e^2}$$

(3.2)

Also, the non-linear static analysis results agree very well with that predicted by Australian Standard AS 4100 (SA, 1998). In the design standard, the member capacities are further reduced by a factor $\phi (\phi = 0.9)$. This factor is taken as one in Figure 3.17 for comparison purposes. These results show that generally, the idealized pin supported finite element model can accurately predict the capacities of members subjected to axial compression load. Only for very short members, the finite element analysis predictions are slightly higher than the “column curve”. This is because the general ABAQUS perfect plastic material properties are used in the analysis. For stocky members, material failure criteria are the more critical factors in determining the member capacities. However, this research focuses on global out-of-plane buckling, and the members of interest will not be stocky. Using perfect plastic material properties is therefore adequate.
b) Under uniform bending moment

Figure 3.18 illustrates the moment capacities of an idealized pin supported beam model from elastic bifurcation and static non-linear analyses. The moment capacities are presented in a non-dimensionlized form \( \frac{M_u}{M_p} \), \( \frac{M_e}{M_p} \) against member slenderness \( \frac{L_{cr}}{r_y} \). The \( M_e \) and \( M_u \) are the elastic buckling and maximum inelastic buckling moments from finite element analysis where as \( M_p \) is the plastic moment capacity of the section.

The elastic buckling analysis results agree well with the elastic beam theory, where the elastic buckling moment is:

\[
M_{cr} = \frac{\pi^2 EI_y GJ}{L} \sqrt{1 + \frac{\pi^2 EI_y}{L GJ}}.
\]  

(3.3)
Figure 3.18 Moment Capacities of Idealized Simply Supported Beams

The design beam curve according to AS 4100 (SA, 1998) is also plotted in Figure 3.18. The capacity reduction factor $\phi$ used in the design specification is taken as one. The non-linear static results are within 3% of the results predicted by AS 4100. Figure 3.19 shows the comparison of beam curves from AS 4100 (SA, 1998), LRFD-AISC (AISC, 1998), and Eurocode3 (EC3, 1993). It shows that the difference in elastic-plastic range from the three approaches is significant. According to Fukumoto (1991), the reasons for these differences are due to 1) simplifying approximations, 2) deliberate introduction of conservative criteria, 3) some specifications aim to represent mean predictions while others aim for a lower bound of the tests. The good agreement between the finite element analysis results and AS 4100 beam curve may not imply that one design specification is better than the other, because the beam curve had to cover a wide range of design issues relating to the differences between rolled and welded sections, methods to treat moment gradient, incorporation of initial out-of-straightness, etc. In fact, the Australian beam curve was derived from the test results of 159 hot-rolled beams, which might have led to the good agreement with FEA results. Nevertheless, this good agreement demonstrates that the idealized pin simply supported beam FEA model is capable of capturing the real beam behaviour.
According to Trahair (2000), the beams of compact cross-section can be divided into four categories, 1) short beam – fully plastic, 2) intermediate – inelastic buckling, 3) long – elastic buckling, and 4) very long – failure about minor axis (see Figure 3.20). Figures 3.21 (a) to (c) show the longitudinal stress contours of the idealized pin supported models. These stress distributions demonstrate that the beam has significant difference in its failure modes when its length varies.
Figure 3.21 (a) shows the results of a short beam. Initial imperfection was introduced based on its first eigenmode (not lateral torsional buckling mode). Due to the short length of the beam, maximum value of initial imperfection is taken as d/150, where d is the depth of the cross-section. The result shows that both top and bottom flanges are fully yielded. The initial imperfection input does not alter the member bending capacity compared with the full plastic section capacity.

The longitudinal stress contour for a member with intermediate length (inelastic buckling range) is shown in Figure 3.21 (b). When the member reaches the ultimate state (buckling occurs), yielding occurs in only one leg of the top and bottom flanges. The yielding spreads from the mid span of the member, and as it moves towards the end supports, the stresses are reducing.

Figure 3.21 (c) shows the longitudinal stress contour for a long beam. Long beams will suffer only elastic buckling. When long beams reach the ultimate state, a large part of the beam remains elastic with the occurrence of yielding only in one leg of the compression flange and no spreading towards the end supports. The coupling of twisting and out-of-plane bending contributes to the unequal stress distribution in the flanges. Since this is a long beam, the twist restraints at the member ends have less effect on the beam’s twisting deformation. Therefore, at the mid-span of the beam, the stress distribution of flanges is similar to that when the beam is subjected to minor axis bending. However, the stress contours at the support show high level of stress concentration when the beam is in its post-buckling state.

This means that the long beam is very likely to have a connection failure other than failure by forming a plastic hinge in the minor axis direction. In other words, without extra reinforcement at the supports, the ultimate failure load of a long simply supported beam would be smaller than the moment capacity about its minor axis.
a) Short beam with $L_e/r_y = 5.78$

b) Beam with intermediate length $L_e/r_y = 86.7$

c) Long beam with $L_e/r_y = 260$

Figure 3.21 Longitudinal Stress Distributions at Failure
From the finite element analysis, the graphs for beam’s end rotation versus applied bending moment can be obtained easily. Figure 3.22 plots the typical moment versus end in-plane rotation curves for the beams with initial geometrical imperfection input. For comparison purposes, the curves for fully laterally restrained beams are also plotted.

Figure 3.22 Moment versus Inplane End Rotation Curves for Idealized Simply Supported Beams

When the non-linear static analysis does not include the input of initial geometrical imperfection, lateral torsional buckling will not occur and full plastic section capacity (\(M_p\)) can be achieved. The members are able to undergo large plastic deformations. Effectively, the perfect straight member models behave the same as fully laterally restrained beams. The moment-rotation curves of fully laterally restrained beams can be divided into three distinguishable stages: 1) linear elastic, 2) gradual yielding, and 3) plastic hinge stage. Figure 3.22 shows that the three beams with different member slenderness have similar behavioural characteristics.
On the other hand, when the initial geometrical imperfection is considered, the moment-rotation curves have a very different characteristic. In terms of in-plane rotation, as the member reaches the buckling moment, rotation decreases abruptly. The buckling moment is also the maximum member strength. For members classified as plastic or inelastic (such as 2, 3 m length for 250UB37.3), after lateral torsional buckling occurs, the simply supported beam does not have a large in-plane rotational capacity. The out-of-plane bending and twisting are the dominant deformations. Because of the lack of post-buckling strength, the lateral torsional buckling for simply supported beam is not a desirable failure mode and should be avoided in design practice.

The other interesting behaviour these curves reveal is that before lateral-torsional buckling occurs, the in-plane bending stiffness of the member is the same as for the member with full lateral restraints. The implication of this behaviour is that the initial out-of-plane geometrical imperfection of the member does not affect the in-plane stiffness in the pre-buckling load path.

In the post-buckling stage, the in-plane moment capacity declines. The secondary internal forces (such as minor axis bending moment and torsion) take over and lead to the instability of the member. The minor axis bending moment limit $M_{p, \text{minor axis}}$ is also plotted in Figure 3.22. Generally, the simply supported beam will completely fail before it reaches this value due to excessive yielding/distortion or support failure. Figure 3.22 also shows that the slender members ($L/2 = 4$ m) are able to undergo larger end rotation. For shorter members, the ultimate failure occurs not long after the maximum buckling load is reached. This observation implies that the lateral torsional buckling failure mode is very different from the failure mechanism for plastic hinge formation. As loading exceeds the maximum buckling resistance, the stiffness and stability of the laterally unrestrained beams become unpredictable. Therefore the post-buckling capacities of the idealized simply supported beams are not reliable.

### 3.3.1.2 Series 2: Warping restrained simply supported boundary conditions

The moment capacities from elastic bifurcation and non-linear static analyses are plotted in Figure 3.23. For comparison, the theoretical elastic buckling curve and
AS4100 beam design curve for idealized simply supported beams are also plotted in the same graph.

Figure 3.23 Moment Capacities of Simply Supported Beams with Warping Restrained Ends – $k_e = 1$

Figure 3.24 Moment Capacity of Simply Supported Beams with Warping Restrained Ends – $k_e = 0.94$. 
The slenderness ratio in Figure 3.23 was calculated by assuming the effective length factor $k_e$ to be one ($L = L_e$). The results show that the elastic buckling capacity of simply supported beam increases by approximately 9% due to the warping restraints. Using the trial and error method, it can be shown that the finite element elastic bifurcation analysis results agree reasonably well with the elastic theory when the effective length factor of the beams equaling $k_e$ is taken as 0.94. This agreement is demonstrated in Figure 3.24 using the same analysis data.

End warping restraints are usually ignored in design practice. For a short I-beam, the nonlinear analysis results show that the difference in moment capacity caused by warping restraints is very small (see Figure 3.23). As the member slenderness increases, the difference between the two boundary conditions is increased by up to 20%. This improvement is the result of stress redistribution at the supports.

Figure 3.25 is a sequence of stress distribution snapshots for a beam with warping restraints. With the warping restraints, significant yielding can be observed at the end supports. Yielding of the four flanges at the supports is not even and is related to the deformed shape of the beam. The pre-buckling out-of-plane deflections transform the flanges into two different out-of-plane arches. The warping restraint forces the ends of these two arches in the same plane and more strain energy can be stored under loading. As a result, the buckling resistance increases. Also, yielding of flanges is more significant along the member after the beam reaches its maximum load.

In the post-buckling range, the end supports are experiencing even more yielding and stress concentrations. The member becomes unstable when the substantial local deformations occur at mid span. The failure of the compression flange is similar to that from minor axis bending (see final state in Figure 3.25).
The in-plane end rotation versus nondimensionalized bending moment curves are plotted in Figure 3.26. The simply supported beams with end warping restraints have a very similar loading path to idealized simply supported beams. Before reaching the
maximum load, the in-plane stiffness is the same as that of fully laterally restrained members. With warping restraints at the supports, the simply supported beams become more “ductile” in the post-buckling range. The in-plane end rotations are able to reach up to 8 degrees as the moment resistance of the beam approaches minor axis plastic moment capacity. Using these boundary conditions with finite element beam models, it is possible to derive the post-buckling stiffness for slender members using a straight line. For shorter members, the reduction of member stiffness exhibits a more non-linear behaviour.

![Figure 3.26 Moment versus End Rotation Curves for Simply Supported Beams with Warping Restrained Ends](image)

**Figure 3.26 Moment versus End Rotation Curves for Simply Supported Beams with Warping Restrained Ends**

### 3.3.1.3 Series 3: Laterally fixed simply supported boundary conditions

If the end rotation degrees of freedom are fixed for a column under axial compression force, the effective length factor $k_e$ of this member equals 0.5. Similarly, if a beam has out-of-plane rotation degree of freedom fixed, the effective length for bending is half its member length.

The results from elastic bifurcation and non-linear analyses are plotted in Figure 3.27. For each data point on the graph, the corresponding member slenderness was
calculated by assuming an effective length factor of 0.5. As shown in the graph, the finite element analysis results agree very well with the elastic buckling theory and design specification.

Figure 3.27 Moment Capacities of Simply Supported Beams with Laterally Fixed Ends (k_e = 0.5)

Figure 3.28 Longitudinal Stress Distribution at Failure for Beam with Laterally Fixed Ends (L_e/r_y = 86.7)
Figure 3.28 shows the stress contour plot for the member at failure load. The mid-span and supporting ends are the regions with excessive yielding. The mid-span stress distributions in the flanges are very similar to those arising from minor axis bending. The transformation from major axis bending to dominant minor axis bending at mid-span occurs quickly after the member exceeds its maximum/buckling load. At the supports, yielding patterns are very complicated. Complex effects due to the actions of combination of warping and uniform torsion and bending moments are difficult to separate.

![Figure 3.29 Moments versus End Rotation Curves for Simply Supported Beams with Laterally Fixed Ends](image)

The applied moment versus in-plane end rotation curves are shown in Figure 3.29. In the pre-buckling range, these curves follow the same load path as members with full lateral restraints. During the post-buckling state, large end rotations are still possible, but the change in member stiffness appears to be highly non-linear.
Figure 3.30 Moments versus End Rotation Curves for a 4 m Simply Supported Beam with Different End Boundary Conditions

The bending moment versus in-plane end rotation curves for a member with three different end boundary conditions are plotted in Figure 3.30. One common feature of these curves is the declining moment resistances towards minor axis plastic moment capacity. The non-linearity of the transition is related to the end restraint condition. The restraint conditions for these three cases are arbitrary. The real simply supported beam end supports are only approximate compared with these boundary conditions. For design purposes, it is very difficult to quantify the degree of warping or out-of-plane restraints. In general, simplifying the member end conditions is sufficient to calculate the buckling resistance. However, considering the complex yielding patterns along the member and especially at the supports during post-buckling state, it is very hard to derive a simple rule for post-buckling in-plane stiffness reduction.

3.3.1.4 Series 4: Combined loading of axial compression force and uniform bending moment

Lateral torsional buckling of simply supported beams subject to combined loading of axial compression force and uniform bending moment were also studied. Idealised
simply supported end conditions were chosen for the analyses. A range of analyses with different combinations of axial compression load to bending moment (p/m ratio) was conducted with three different member slenderness ratios (L_y/r_y). The p/m ratio is the nondimensionalised compression force and bending moment ratio, i.e. \( p = \frac{P}{P_y} \) and \( m = \frac{M}{M_p} \) where \( P_y \) and \( M_p \) are the axial yield and plastic bending capacities. The p/m values considered were 0, 0.05, 0.11, 0.18, 0.25, 0.43, 1, 2.33, 4, 5.68, and infinity. The member slenderness ratio values were 86.7, 116 and 173, corresponding to member lengths of 3, 4 and 6m, respectively.

![Figure 3.31 Interaction Diagram for Beam columns with L_y/r_y = 86.7, 116 and 173](image)

Figure 3.31 is the compression force and bending moment interaction diagram obtained from these analyses. Assuming the capacity reduction factor \( \phi \) as 1.0, the relationship for \( p \) and \( m \) is a straight line using AS 4100 (SA, 1998). These lines for members with different slenderness ratios are also plotted in Figure 3.31.

\[
p + m \leq 1
\]

They can be expressed as:
\[
m = \frac{M_{bx}}{M_p}
\]
\[
p = \frac{N_{cy}}{N_y}
\]

\( M_{bx} \) is the nominal member moment capacity about the major axis.

\( N_{cy} \) is the nominal member capacity in axial compression for buckling about minor axis (out-of-plane buckling).
The results from these analyses agree well with the design specification AS 4100 (SA, 1998) for members subject to combined loading of axial compression force and uniform bending moment. It is worth noting that the end support conditions, residual stress distributions and material properties of these models are assumed to be the same. The initial geometrical imperfection input was based on the “worst shape” and the maximum value was taken as L/1000. The good correlation between the analysis results and design code predictions shows that the chosen parameters are “reasonable”.

In summary, the elastic buckling moments and beam curves (ultimate strength) for the uniform bending moment case can be accurately predicted using finite element analyses. The good correlations between the finite element analysis results and the design code predictions indicate that the member stiffness transformation behaviour from the numerical analyses is reliable.

3.3.2 Simply supported beams subjected to transverse loads

When transverse concentrated loads are applied to a member, the bending moment along its length will not be uniform. There are no closed-form solutions for these load cases. Often, an equivalent moment factor method is used to estimate the buckling moments. Transverse load cases are more complicated than those subjected to end moments. The lateral torsional buckling moments are not only dependent on the moment gradient variation, but also on the location of applied load from the shear centre. Three loading situations were studied in this research: 1) central concentrated loads, 2) two equal concentrated loads applied at quarter points of the beam, and 3) uniformly distributed load applied at the shear centre of member. Idealized simply supported boundary conditions are used in the finite element modelling.

3.3.2.1 Series 1: Central concentrated loads

According to AS 4100 (SA, 1998), the moment capacities of beams subjected to a central concentrated load at the shear centre can be approximated by $M_u = 1.35M_{uu}$ (shown in Figure 3.32), where $M_{uu}$ is the member capacity under uniform bending
moment and $M_u$ is the maximum ultimate moment of the beam. It was also found that the applied load height has significant effects on the capacity of beams. The concentrated loads in this series are applied independently at the middle of web (shear centre), top flange and bottom flange.

![Figure 3.32 Bending Moment Diagram for Midspan Concentrated Load](image)

$\alpha_m = 1.35$

$$M_u = \frac{PL}{4}$$

**Case 1: Load applied to the shear centre at the mid-span of the beam**

Details of the finite element model including the applied loads and boundary conditions are shown in Figure 3.33. In order to avoid high stress concentrations and associated premature failures, loads were applied to all the nodes in the web at the symmetry plane. Because the loads were applied to the symmetry plane, these applied loads equal half of $P$ as shown in Figure 3.33 and the peak moment calculated from the finite element model is $PL/8$.

![Figure 3.33 Finite Element Model of a Simply Supported Beam with a Central Point Load at the Shear Centre](image)
Figure 3.34 plots the graph of nondimensionalised moment capacity versus slenderness of the beams, where the beam slenderness is expressed as $\sqrt{M_p/M_o}$, where $M_o$ is the elastic buckling moment for beams subject to a uniform bending moment. This beam slenderness expression is a more universal format and is suitable for various buckling modes. For example, the column slenderness can be expressed as $\sqrt{N_p/N_e}$. For simply supported beam with transverse load cases, it is more logical to use this format of slenderness to accommodate the effects of various moment gradients.

![Graph showing non-linear analysis results compared to AS4100 beam curve and buckling analysis](image)

**Figure 3.34 Maximum Ultimate Moments of Simply Supported Beams with a Central Point Load at the Shear Centre**

The results show that the non-linear static analyses agree well with the AS4100 design code predictions. For a short beam, the moment capacity is slightly lower than the design beam curve. This might have been caused by the local stress concentrations in the models. In real life, this reduction may also occur if the applied load is concentrated in a very small area. The elastic buckling analysis results and the theoretical elastic buckling curve also agree well. The moment modification factor ($\alpha_o$) is used in deriving the theoretical elastic buckling curve.
Figure 3.35 shows the typical longitudinal stress contours at the ultimate load. The stress distribution in this transverse load case is very similar to that of a simply supported beam subjected to a uniform bending moment (see Figure 3.21).

**Figure 3.35 Longitudinal Stress Distribution at Failure for a Beam with a Central Point Load at the Shear Centre**

![Stress Contours](image)

**Figure 3.36 Moment versus Rotation for Simply Supported Beams with a Central Point Load at the Shear Centre**

![Moment-Rotation Graph](image)
The results for the largest bending moment along the beam versus in-plane end rotation are plotted in Figure 3.36. Moment versus rotation curves for fully laterally restrained beams are also plotted in the same graph. The lateral torsional buckling curves reveal two distinguishable characteristics, 1) the beams follow the same load path as if they are fully restrained before buckling occurs, 2) the end rotation capacities are relatively small for this load case after the occurrence of buckling. The post-buckling stress distribution contours show that excessive yielding occurs at the top flange of the member.

Case 2: Load applied to the top flange at the mid-span of the beam

When a concentrated load was applied to the top flange, a slight modification was made to the finite element model. Instead of applying point loads to the nodes at the top flange of the symmetry plane, a rigid beam type MPC element was adapted to link all nodes together (see Figure 3.37). Even though this arrangement does not alter elastic buckling results (less than 5%), it reduces local yielding of flange significantly during non-linear static analyses and prevents the premature member failure.
Treatments to account for the load height effects are not uniform among the various design specifications. In AS 4100, the effective length for this load case is multiplied by a factor of 1.4. The AS 4100 design beam curve and finite element results are plotted in Figure 3.38, which shows a reasonably good agreement. For more slender beams, the nonlinear analysis results are slightly higher than the elastic bifurcation analysis results. This is caused by significant second order effects in the post-buckling stage. The applied load causes certain degrees of restored moment as the beam rotates out-of-plane.

![Figure 3.38 Maximum Ultimate Moments of Simply Supported Beams with a Central Point Load on the Top Flange](image)

In general, the moment-rotation curves for this load case (Figure 3.39) are similar to those when the concentrated load is applied at the shear centre. Short and intermediate beams change in-plane stiffness rapidly after the maximum/ultimate load is reached. For very slender members, lateral torsional buckling takes place in a gradual manner. The significantly lower moment capacity is due to the second-order effects when concentrated load is applied to the top flange.
Case 3: Load applied to the bottom flange at the mid-span of the beam

A modification similar to that in Case 2 was made to the finite element models for this load case. A rigid type MPC was used to apply the concentrated load to the nodes at the bottom flange in the symmetry plane (see Figure 3.40). In AS 4100, there is no special treatment for this load case, implying that the effects due to loading below the shear centre can be ignored. The finite element analysis results are plotted with the design beam curve and the theoretical elastic buckling curve in Figure 3.41.

In Figure 3.41, the effective length factors for the beam curve and the theoretical elastic buckling curve are taken as unity. This assumption underestimates the member capacities. When the load is applied to the bottom flange, the restoring moment/torsion due to second-order effects improves the member capacity significantly. The beam and elastic buckling curves in Figure 3.42 assume an effective length factor of 0.75. With this assumption, the finite element analysis results agree very well with the design formula.
Figure 3.40 Finite Element Model of a Simply Supported Beam with a Central Point Load on the Bottom Flange

Figure 3.41 Maximum Ultimate Moments of Simply Supported Beams with a Central Point Load on the Bottom Flange – Assume $k_e = 1.0$
Figure 3.42 Maximum Ultimate Moments of Simply Supported Beams with a Central Point Load on the Bottom Flange – Assume $k_e = 0.75$

Figure 3.43 Moment versus Rotation for Simply Supported Beams with a Central Point Load on the Bottom Flange
The load height factor not only affects the member capacities, but also has a greater influence on the members’ post-buckling behaviour. Figure 3.43 demonstrates that the members are more “ductile” with the bottom flange loading configuration even though the pre-buckling loading paths are similar to other load cases.

The gradual yielding behaviour of slender beams is related to the magnitude of initial geometrical imperfection. Smaller imperfection input in the finite element models leads to higher member strength with a more abrupt change in member stiffness (see Figure 3.44). This behaviour is more significant for slender members.

![Figure 3.44 Effect of Initial Imperfection on the Behaviour of Slender Beams](image)

**3.3.2.2 Series 2: Equal concentrated loads applied at quarter points of the beam**

When two equal concentrated loads are applied to quarter points of the simply supported beam (Figure 3.46), the effect of bending moment variation along the member is considered as not very significant. The moment modification factor $\alpha_m$ is often taken as 1.09. Because of the use of symmetry boundary conditions and only
half the beam is modelled, the applied loads in the finite element models are
equivalent to a concentrated force at the shear centre as shown in Figure 3.46.

$$\alpha_m = 1.09 \quad M = \frac{PL}{4}$$

Figure 3.45 Bending Moment Diagram for the Load Case of Two Concentrated Loads at
Quarter Points of the Beam

Figure 3.46 Finite Element Model of a Simply Supported Beam with Two Concentrated
Loads at Quarter Points

The finite element analysis results are plotted in Figures 3.47 and 3.48. Assuming the
effective length factor to be one, the moment capacities of the member agree well
with the AS 4100 predictions. The bending moment versus in-plane end rotation
curves show a very similar behaviour to that of a single concentrated load at mid-span
case (shear centre). In general, the end rotation capacity is small in the post-buckling
stage.
Figure 3.47 Maximum Ultimate Moments of Simply Supported Beam with Two Concentrated Loads at Quarter Points

Figure 3.48 Moment versus Rotations for Simply Supported Beam with Concentrated Loads at Quarter Points
3.3.2.3 Series 3: Uniformly distributed load at shear centre

The effect of a uniformly distributed load (UDL) can also be studied using the idealized simply supported beam model. The moment modification factor for this load case is often taken as $\alpha_m = 1.13$ (Figure 3.49). The loads are applied to all nodes at the centre of web of the finite element model (see Figure 3.50).

\[ \alpha_m = 1.13 \quad M = \frac{wL^2}{8} \]

Figure 3.49 Bending Moment Diagram for the Load Case of UDL

Figure 3.50 Finite Element Model of a Simply Supported Beam with A Uniformly Distributed Load at the Shear Centre

Figure 3.51 shows that the non-linear finite element analysis results agree well with AS4100 predictions. Also, the elastic buckling analyses agree with the theoretical elastic buckling curve based on $M_o$. The moment versus end rotation curves (Figure 3.52) follow the same trend seen in the cases of concentrated load at shear centre. Essentially, the simply supported beam behaviour is similar to that of fully laterally restrained beams prior to buckling. During the post-buckling stage, load carrying
capacities decrease except for very slender beams, and overall end rotation capacities are less than those for beams with full lateral restraints.

Figure 3.51 Maximum Ultimate Moments of Simply Supported Beams with a Uniform Distributed Load at the Shear Centre

Figure 3.52 Moment versus Rotation for Simply Supported Beams with a Uniformly Distributed Load
3.4 Summary

Due to the complexity of lateral torsional buckling, the design specifications around the world are based on empirical equations to predict the member bending capacities. The design equations are derived from various experiments on simply supported beams. With the advances in computing power, distributed plasticity analyses are able to reproduce the results from these experimental studies.

Finite element analyses of simply supported beam models with different load cases, support boundary conditions and length have been presented in this chapter. These results show that the shell finite element models are suitable for the study of lateral torsional buckling. Compared with experimental tests, finite element analysis requires many fewer resources. Further, this method allows identification of all the factors that may affect the beam behaviour. For example, idealized load and boundary conditions can be achieved using numerical models. This allows the isolation of any particular factor relating to beam stability behaviour without the unforeseeable problems often occurring in experiments.

In this research, considerable time and effort was spent on establishing an idealized simply supported shell finite element model. Effects of boundary conditions on lateral torsional buckling have been studied in depth. It was found that the proposed MPC system is the most suitable boundary condition for simply supported I-beams. This end boundary condition is comparable with the beam elastic buckling theory. Even though this end support is an artificial one and does not exist in real beams, with appropriate initial geometrical imperfection and residual stress input, the finite element nonlinear analysis produces results that agree very well with the lower bound test results used in the development of AS 4100 beam curve.

Four types of common load cases applied to a simply supported beam were modelled and a number of sub-load cases were also included in each load case. Nonlinear analyses of these finite element models not only predicted the ultimate lateral torsional buckling capacities but also provided a full picture of the pre- and post-buckling behaviour. The major findings are given next.
Figure 3.53 Typical Moment versus Rotation Curve of a Simply Supported Beam

Finite element analyses demonstrated that lateral torsional buckling of simply supported beam is an unfavourable type of failure. A typical moment versus in-plane end rotation curve can be divided into two stages, pre- and post-buckling as shown in Figure 3.53. In the pre-buckling stage, the in-plane stiffness of a beam is no different from that of a fully laterally restrained member. Once the load path goes beyond the ultimate load, the tangent stiffness of the member becomes negative. The rate of stiffness reduction varies depending on the many parameters such as member slenderness, end restraints, and load conditions, and it is often not linear. Therefore, the post-buckling load paths of simply supported beam are very unpredictable. Further, the laterally unrestrained beam has much less end rotation capacity than the fully laterally restrained beams.

The stress contour plots show that the ultimate load is reached when one side of the top flange has fully yielded due to the compression forces in the flange. The spread of yielding has by then extended significantly along the member length. After the ultimate capacity is reached, the second order moment/torsion and plastic deformation produce very complex stress contours along the member including local stress concentration at mid-span of the member.
In general, the nonlinear analysis was terminated well before the member deformed sufficiently to become a case of minor axis bending. This implies that final beam strength is much higher than the minor axis plastic moment capacity. Short and intermediate length members will fully yield along the member at this stage. Only the very slender members behave similar to the minor axis bending case after initial yielding. However, beams with such large slenderness are not often used in real design due to severability criteria. Thus, the use of minor axis bending moment capacity as the limit for post-buckling strength is not correct.

The finite element analysis results conclude that the post-buckling strength of simply supported beams is not reliable. Beams that are susceptible to lateral torsional buckling should not be loaded beyond their member ultimate capacities of the code predictions. These criteria will be used in the development of advanced analysis methods for steel frame structural design presented in Chapter 5.

Since the shell finite element models can accurately capture the nonlinear behaviour of simply supported beams, the same method will also be used for steel frame structures to investigate the lateral torsional buckling behaviour of their members. The next chapter presents the details of this research into the behaviour of frame structures.
Chapter 4. Distributed Plasticity Analyses of Frame Structures

Two types of element are often used in the distributed plasticity analysis of steel frame structures. They are the fibre and shell elements. When using fibre elements, the structural members are divided into line segments, and the cross section of each segment is subdivided into fibre elements. This type of analysis considers only the normal stress of the members and has limited capacity in dealing with the localised deformations caused by local buckling, web shear failure, etc. Shell elements are the more suitable choice for steel frame structures when accuracy is a priority. The finite element analyses (FEA) using shell elements consider the combined effects of both normal and shear stresses. Since the shell elements have higher degrees of freedom, most parameters (initial imperfections, residual stresses etc) of a real structure can be explicitly incorporated into the analysis. Therefore, analyses using shell elements are widely used in structural engineering researches in recent years and have been used to develop benchmark solutions for planar frame structures (Avery, 1998; Kim, 2002). Some of these results have been successfully validated using experimental results and other types of analytical solutions (Avery, 1998). In the case of simply supported beams subject to lateral torsional buckling effects, the shell finite element model using HKS/ABAQUS 5.8 (HKS, 1998) is able to reproduce the AS 4100 “beam curve” that was developed based on experimental tests (see Chapter 3). These beam analysis results demonstrate that shell element models are suitable for the study of lateral torsional buckling effects in steel frame structures.

The design specifications (AS4100, AISC-LRFD, Eurocode3) do not discuss about how to deal with lateral torsional buckling in plastic hinge type structural analysis. In general, only plastic-hinge collapse mechanism is considered in the analysis for plastic frame design. The plastic design rules are to both prevent the occurrence of lateral torsional buckling at moments less than the plastic moment capacity (\(M_p\)) and to ensure adequate in-plane rotation capacity even if the final failure (leading to unloading) is triggered by buckling (Nethercot, 1992). Since the prediction of lateral torsional buckling is based on semi-empirical equations and required results from elastic analysis, a two stage procedure is still necessary for plastic design calculations.
Advanced steel frame analysis is intended to combine these two procedures into one step. If lateral torsional buckling failures are to be checked in the plastic hinge type analysis, relevant failure criteria must be specified. However, there is a lack of research into the behaviour of rigid steel frames subject to lateral torsional buckling.

In order to develop reliable and practical advanced analysis methods, the researchers must have a good understanding of the true structural behaviour of frame structures. Various benchmark solutions have been provided for steel frame structures (Kanchanalai, 1977; White, 1985; Vogel, 1985; Clarke, 1992; Avery, 1998; Kim, 2002), but none of these frames are subjected to member lateral torsional buckling failures. In a recent publication, Trahair (2003) states that “There appears to have been few corresponding analyses made of the out-of-plane behaviour of beam-column, and none of frame. Such analyses need to be developed so that they can be used to provide benchmark solutions for plastic hinge advanced analyses”. The objective of the distributed plasticity steel frame analyses described in this section was to develop a set of analytical benchmark solutions for steel frame structures subjected to lateral torsional buckling effects that can be used in the development of practical advanced analysis methods.

For simply supported beams, the lateral torsional buckling failure is related to five major factors: 1) geometry (cross-section and member length), 2) loading conditions (eg. moment gradient, load height), 3) end restraint conditions, 4) material properties, 5) initial geometrical imperfections and residual stresses. As for members in the frame structures (consist of more than one member), they are still the dominant factors. However, the end restraint conditions of individual beam segments in frame structures are not straightforward. Also, the bending moment distribution of the beam segments may change during the plastic stages. Two additional major factors must be addressed for frame structures. They are: 1) the interaction behaviour between members, and 2) how does a frame structure respond after one of its member fails in lateral torsional buckling mode?

Since there are unlimited combinations of load cases, member end restraints, and frame configurations, it is impossible to include all the complex cases in the
investigation and then to attempt to isolate the problem related to lateral torsional buckling. Therefore, a series of simple steel frame structures with idealised load cases was studied in this chapter. These frames have partial lateral restraints at the ends of the members to prevent the out-of-plane buckling failure of the entire frame. Since the effects of semi-rigid connections are not considered in this research, all models are assumed to be rigid frame structures. The load cases for these finite element models were chosen so that the structure would primarily fail in the lateral torsional buckling mode. Ultimate strength capacities and load-deflection curves were obtained from these models for each load case and analysed to fully understand the steel frame behaviour. The FEA results were also compared with the predictions based on AS4100 design rules (SA, 1998). This chapter presents the details of the distributed plasticity analyses of these steel frame structures and the results.

4.1 Steel Frame Model Description

Benchmark frame solutions have been developed for fully laterally restrained frames using distributed plasticity advanced analysis methods (Vogel, 1985; Ziemian, 1990; Avery, 1998). These benchmark frames can be categorised into three different types: 1) single bay, single storey portal frame, 2) multi-storey or multi-bay frames, 3) single bay single storey gable frame. These frames include both sway and non-sway problems. The column bases can be both pinned and fixed, and the frames are all rigid.

The frames chosen in this project include all these categories (see Figures 4.1, 4.3, and 4.4). Since cantilever beams have been treated as a special case in lateral torsional buckling research, the finite element models also include frame structures consisting of cantilever members (Figure 4.2). The nine frame models are:

1) Non-sway portal frame with fixed bases,
2) Non-sway portal frame with pinned bases,
3) Sway portal frame with fixed bases,
4) Sway portal frame with pinned bases,
5) Γ shape frame, consisting of cantilever beam attached to a fixed based column,
6) Single bay single storey frame with a cantilever beam,
7) Two bay single storey frames,
8) Single bay two storey frames,
9) Single bay single storey gable frame.

Figure 4.1 Single Bay Single Storey Frames

Figure 4.2 Frames with a Cantilever Segment
The descriptions of these frame models include five aspects: the elements used, material models, loads and boundary conditions, residual stresses, and initial imperfections. The effects of local buckling and interactions between local and lateral buckling can also be investigated using these finite element models. Since this project was limited to lateral torsional buckling effects, only compact I-sections were used in these frames. The compact sections are capable of achieving full section capacities without being affected by local buckling effects.
4.1.1 Elements

The ABAQUS S4R5 element was used in the analysis of all the frame models. Details of this element are described in Section 3.1.1. Due to the large models, the element density of the frame models was reduced compared with the simply supported beam models. The cross-section of the frame members had 10 elements in the web and 6 elements in the flanges. The element length was approximately 50-mm. The size of frame models range from 3500 to 15000 elements. The analyses were conducted on Silicon Graphic Origin 3000 with 60 processors (28 R12000 chips + 32 R14000 chips). The computer’s theoretical processing capability is 52 Giga Flops. Since ABAQUS was configured to support only up to a maximum of four processors, about two hours were needed to run the largest model for non-linear analysis with more than 100 steps. A steel frame structure may have more than half a million elements, i.e., thirty two \(2^5\) times the size of the largest frame in this project. If 20 load cases are considered, the computer run time may be 1200 hours using the current supercomputer. Assuming the computing power of PC (personal computer) to be five years behind the current supercomputer, and that, computer power doubles every 18 months, it will take more than 20 years before shell element models can be used in practical designs (run time on a PC less than one hour for 500,000 shell element model with 20 load cases). Rigid beam type MPC elements have also been used to model the rigid connections between beams and columns and to simulate the pinned supports at the column bases.

4.1.2 Material model and properties

The ABAQUS classical metal plasticity model was used in all the analyses. This model implements the von Mises yield surface to define isotropic yielding and associated plastic flow theory. The material steel is assumed elastic-perfect plastic. Strain hardening was neglected for all of the frame models, even though strain hardening could have a significant effect on the capacity of frames, especially for those with a higher degree of redundancy. The increase of frame capacities was only about 5% in the previous study of fully laterally restrained frames (Avery, 1998). Also, the finite element analysis (FEA) results for simply supported beam show that perfect plastic material properties can achieve very accurate results unless the models
are deep beams. The yield stress $\sigma_y$ of both web and flange was taken as 320 MPa. The elastic modulus and the Poisson’s ratio were taken as 200 GPa and 0.3, respectively.

### 4.1.3 Loads and boundary conditions

Soil, foundation elements and foundation to structure connections can be modelled in structural analysis. When a frame structure is isolated from its surrounding, the boundary conditions have significant effects on the analysis results. Generally, the boundary conditions include three areas: 1) frame supports, 2) beam-to-column connections, and 3) supports along the frame members. Supports along the members are not considered in this research in order to reduce the complexity of the problems. For the same reason, only concentrated loads were used in the load cases. A range of base and connection supports, boundary conditions and loading configuration has been studied. It included three types of column base supports and four types of beam-to-column connections. For non-sway frame structures, symmetry boundary conditions were also used to reduce the element numbers.

### 4.1.4 Frame base support boundary conditions

When lateral torsional buckling failure is of concern, the segment end boundary conditions of a three dimensional I-section include at least nine degrees of freedoms. They are:

1. Lateral displacement of top flange,
2. Lateral displacement of bottom flange,
3. Lateral rotation of top flange,
4. Lateral rotation of bottom flange,
5. Warping deformations,
6. Axial displacement,
7. In plane transverse displacement,
8. In plane rotation,
A realistic column base is a combination of restraints in these degrees of freedoms. Attending to all these boundary conditions in finite element modelling are both troublesome and unnecessary. In practical design situations, most column supports are considered to have either fixed bases or pinned bases. Fully fixed bases can be easily achieved for steel frame models using shell elements, i.e. fix all nodes of the cross-section at the column end in all directions (see Figure 4.5). Pinned supports require a closer examination when lateral torsional buckling failure is involved. Generally, there are two types of pinned restraints: Pinned support without end plate and Pinned support with warping restraint (Figures 4.6 (a) and (b)).

Figure 4.5 Fully Fixed Support

Figure 4.6 Pinned Supports
Pinned support without end plate is similar to the idealized pinned support discussed in Chapter 3. The support provides no warping restraint. However, the column supports shown in Figure 4.6(a) are not commonly used. Its axial compression capacity is much less than full section capacity. These types of column support will cause premature failure in rigid frame structures. Therefore, it is irrelevant to use the idealized pinned supports for rigid frame structures.

When an end plate is attached to the end of the column, the column base boundary conditions become warping restrained pinned support. There can be three scenarios in this constraint: a) column base is allowed to rotate about the minor axis, and b) no minor axis rotation is allowed, and c) some minor axis rotation (most common as shown in Figure 4.7 (b)). These types of supports can be simulated using the rigid beam MPC as shown in Figure 4.7 (a). Since the lateral torsional buckling of columns is not the dominant failure mode in the frames involved in this project, and the preliminary studies showed that the frame analysis results are similar with scenarios (a) and (b) mentioned above, the column base out-of-plane rotation restraints are ignored in the FEA.

Figure 4.7 General Type Pinned Supports
4.1.5 Beam-column connection

In order to prevent the overall frame out-of-plane buckling, all the structures that have been studied are laterally restrained at the beam-column connections. There can be many types of beam column connections. Based on their moment-rotation behaviour, connections can be classified as rigid and semi-rigid types. This research only considered rigid type connections. In the design practice, little attention is paid to the connection rigidities in structural analysis when the joint is considered as rigid. Following details are for some commonly used (Figure 4.8) fully restrained rigid connections (Trahair, 1993). Besides the difference in the connecting method (welded or bolted), the internal structures of a joint can also be different (e.g. with/without web stiffeners), but they are indistinguishable in member capacity calculations.

A) Fully welded connection, B) Angle seat and cleat to web
C) Bolted connection, D) Mitre joint connection

Figure 4.8 Commonly Used Rigid Beam-Column Connections
When considering member lateral torsional buckling, the rigidity difference in a connection has a large influence on the capacity of members. Assuming that the member connection is infinitely rigid might lead to unrealistic results in the frame structure modelling. In order to understand the frame behaviour relating to connection stiffness, four connection details were studied in the finite element analysis. These models simulate the welded connections with various degrees of web stiffening. The effects of bolted connection and mitre joint connections are complicated for member lateral torsional buckling, and hence are not considered in this research. The connections considered and their boundary conditions are shown in Figure 4.9:

![Figure 4.9 Beam-column Connection Models](image)

**Figure 4.9 Beam-column Connection Models**
4.1.6 The use of symmetry boundary conditions

When a structure is symmetrical under applied loads, symmetry boundary conditions can be used to reduce the finite element model size. For single bay single storey non-sway frames (Figures 4.1 (a) & (b)), only half of the structure needs to be modelled due to the symmetry of both structure and loading conditions (see Figure 4.10). The translational degrees of freedom along the member ($T_x$) and in-plane and out-of-plane rotational degrees of freedoms ($R_y$, $R_z$) are fixed along the symmetry plane.
4.1.7 Loading conditions

Only point loads were considered in the finite element analyses, i.e., either as vertical loads at the mid-span of beams or horizontal loads at the top of the columns. Loads are assumed to be applied at the shear centre unless specified, i.e., the centre of web of I-sections. In order to reduce the stress concentration problems, MPC elements were used at locations where the loads were applied in order to spread the point loads evenly into the web or flanges.

4.1.8 Initial geometric imperfections

Initial geometrical imperfections were used to trigger the development of buckling in the nonlinear analysis. Without the imperfection input, the simply supported beam models behave as if they were fully laterally restrained, thus initial imperfections have significant effects on the structural ultimate capacity. Real structural imperfections appear randomly. In general, the worst possible imperfection shapes were used to predict the ultimate behaviour of structures in static nonlinear analyses. This method produces satisfactory outcomes in simply supported beam analyses when compared with design beam curves which are based on test results.
Two types of initial geometrical imperfections have been included in the analysis as shown in Figure 4.11. Firstly, a nominal maximum magnitude of $L/1000$ was used for out-of-plane imperfection, based on the AS4100 (SA, 1998) fabrication tolerance for compression members. This type of imperfection is often called the out-of-straightness. For sway frames, an in-plane imperfection with a magnitude of $h/500$ is used (called out-of-plumbness). This type of imperfection causes $P-\Delta$ effect to the columns.

The out-of-straightness imperfection was introduced by modifying the nodal coordinates $(x, y, z)$ of the model by scaling the lateral torsional buckling eigenvector obtained from an elastic buckling analysis. The modification could occur both in the beams and columns depending on where lateral torsional buckling is more likely to take place. In all the frames studied, lateral torsional buckling occurs in the beam. The in-plane out-of-plumbness imperfection was modelled during the creation of the model (columns were modelled as inclined members). No out-of-plumbness was included in the non-sway frames.

4.1.9 Residual stresses

A commonly used residual stress model for hot-rolled I-sections (ECCS, 1984) was used for the frame models as shown in Figure 4.12. The residual stress subroutine was
added to each input file using the ABAQUS6.3 *INITIAL CONDITIONS, option with TYPE=STRESS, USER. This subroutine defined the local components of the initial stress as a function of the global coordinates. The initial stresses were applied before the initial geometrical imperfection input and the nonlinear analysis step.

In order to reduce the complexity of the residual stress input, and since this research is only dealing with rigid frame structures, no initial stress is included in the beam-column connection. The study of simply supported beam finite element models (see Chapter 3) shows that residual stress input is more important for shorter beam members. It is not expected that they will have decisive effects with the frame models where none of them consist of short members.

This image is not available online. Please consult the hardcopy thesis available from the QUT Library.

![Figure 4.12 Residual Stress Contours for a Typical I-section (ECCS, 1984)](image)

4.2 Use of Patran Command Language (PCL)

Shell finite element analyses are not often used in steel frame design practices. This is due to two major reasons: a) complicated structures may require computer power that does not exist, b) conducting an analysis requires special skills and the development of finite element model is time consuming. The first obstacle was discussed in Section 4.1.1. The second problem is more solvable with the current technology. The use of Patran Command Language (PCL) is one of the examples.
In this research MSC/Patran was used for pre-processing, the ABAQUS input file was generated by the running of Patran commands, and all these commands are carried out by Patran Command Language. PCL is a full functional computer programming language delivered as part of the MSC/Patran product. It is a high level language that the syntaxes are interpreted by a C program to produce a more efficient binary code. PCL is able to integrate programs developed externally into the Patran environment and it aims to eliminate tedious, repetitive procedures.

In this research project, PCL subroutines were written to create a “Frame Wizard” program (see ). The user of the “Frame Wizard” is able to define the overall and section dimensions of the frame structures. When a particular PCL function is called, the program executes the following steps to generate the ABAQUS input file:

1. Create frame geometry; points, lines, surfaces, and coordinates,
2. Generate mesh including create mesh seeds, mesh, remove duplicate nodes,
3. Create MPCs if needed
4. Create load sets,
5. Create material sets,
6. Create properties sets,
7. Create analysis steps.

In the “Frame Wizard” program, parameters such as frame dimensions and constraints are defined using dialog boxes. Other parameters are predefined. This program is not compiled for general frame design used, but to demonstrate that the use of PCL can greatly reduce the finite element modelling time.

Common frame analysis programs (such as Microstran, Space Gass) use only beam column elements. One advantage of these programs is the ease of usage. Because only “stick” diagram is needed to draw, the simple frame modelling process is significantly simplified compared with the analysis using shell elements. However, by using PCL, it is possible to develop a program that “reads” stick diagram and turns it into shell element model (see Figure 4.14).
Figure 4.13 Screenshot of Frame Wizard using PCL
4.3 Methods of Analysis

Two analysis methods were used in the frame analyses: elastic buckling and non-linear static. Elastic buckling analysis was used to obtain the eigenvectors for the initial geometrical imperfections, and nonlinear analysis to obtain the ultimate capacities of frames. The new version of program (ABAQUS 6.3) was also used in the analysis of these steel frame structures. It includes a minor change in the input method of residual stresses and initial geometrical imperfections. The new version of ABAQUS is able to directly read buckling analysis results and impose suitable buckling shapes into the model before the nonlinear analysis step. This step was carried out by Patran as one of the pre-processes in the simply supported beam models while using ABAQUS 5.8. The new version eliminated the disadvantage of entering imperfections before residual stresses, and significantly simplified the finite element modelling process. Non-linear static analyses were used three times for each frame structure:

1. Non-linear static analysis of the frame structure with members fully laterally restrained including residual stress input. The results represent the frame behaviour without lateral torsional buckling effects,
2. Non-linear static analysis of the frame structure with out-of-straightness imperfection,
Newton-Raphson solution technique was used for all the non-linear analyses of fully laterally restrained frames. Using this method, the failure occurred when equilibrium could not be achieved without reducing the increment size to less than this minimum. For frames with initial geometrical imperfection, RIKS method was used to study the unloading characteristics of the structures. The maximum load factors obtained from the RIKS method are the ultimate capacities of the steel frame structures.

The use of PCL “frame wizard” program simplified the procedure of frame modelling. Frame analyses include four steps:
1. Run “Frame Wizard” in Patran environment,
2. Run elastic buckling analysis command,
3. Run nonlinear static analysis command,
4. Use Patran to read ABAQUS analysis results.

### 4.4 Distributed Plasticity Analysis Results and Discussion

Chapter 3 demonstrates that the distributed plasticity analyses are able to predict the simply supported beam capacities under various loading conditions compared with empirical equations. Since the empirical equations are based on experimental tests of simply supported beams, the good agreement between analytical results and design specifications can be regarded as satisfactory correlation between FEA and physical testing. For full-scale frame FEA models, direct comparisons are more difficult because of the lack of frame testing data. Using the current design method, frame capacities can only be obtained indirectly by using both elastic frame analysis and separate plastic member capacity calculations. The indirect method is not intended to predict the failure mode of frame structures but is used as a design check. Generally, the frame capacities predicted are conservative. Comparing the FEA results with design code predictions for frame structures may not be as meaningful as the simply supported beam case. Nevertheless, the frame capacities from the design code can be treated as lower bound solutions.

The FEA program ABAQUS is suitable for the nonlinear analysis of simply supported beams subjected to lateral torsional buckling effects and it has also been used
successfully in verifying frame structures subjected to local buckling failures (Avery, 1998). Alsaket (1999) also demonstrated the adequacy of ABAQUS finite element models is simulating the flexural buckling and collapse behaviour of steel portal frames by comparing with full scale experimental results. Therefore, it is reasonable to deduct that ABAQUS has the capacity to deal with the frames that are susceptible to out-of-plane failures, providing the structures are correctly modelled. The analytical models described in Section 4.1 were therefore used to develop a comprehensive range of analytical benchmark solutions for laterally unsupported frames that can be used for the verification of simplified methods of analysis. These analytical benchmark models included:

- Idealised fixed or pinned column supports and various degrees of rigid beam-column connections.
- Concentrated loads applied at mid-span of beam members and horizontal load applied at intersection of the beam and column centre lines.
- Global geometric imperfections based on the construction and fabrication tolerances.
- Residual stress distributions. Bi-linear stress-strain material properties with no strain hardening.

The effects of various parameters will be discussed in detail with each series of frames as follows.

4.4.1 Single bay single storey non-sway portal frames (Series 1 and 2)

Only half of the frame structures needs to be modelled for non-sway frames with pinned and fixed bases. These frame structures are made from a standard I-section 250UB37.3 (compact section). The dimensions of the frames are shown in Figure 4.15. The portal frames are identified using a simple convention. For example, p24 means pin bases, column height equals 2 metres, and half beam length is 4 metres. For each type of structure, four different types of connections have been modelled (see Figure 4.9). The two series of frames include 64 finite element models. Each model includes four analyses; one elastic buckling analysis and three nonlinear analyses with various parameters, in total 256 analysis runs.
4.4.1.1 Elastic buckling analysis

Lateral restraints are applied to the beam-column connections for Series 1 and 2 frames thus the out-of-plane buckling can occur only in the individual members. The buckling shape of the beam in the portal frame is similar to the primary buckling mode of a simply supported beam (see Figure 4.16).
In a frame structure, higher order buckling modes often include a combination of buckling deformations in different members. For examples, modes 2, 3, and 4 include both lateral torsional buckling of beam and local/global column buckling.

When the moment gradient is not unity in the beam segment, the maximum moment must be converted to an equivalent moment for comparison purposes using the moment modification factor $\alpha_m$, or vice versa. In order to obtain $\alpha_m$, a linear elastic analysis must be conducted first (see Figure 4.17 where the results are shown). For Series 1 and 2 frames, the moment modification factor was calculated based on the formula:

$$\alpha_m = 1.35 + 0.36 \beta_m$$

where: $\beta_m = M_1 / M_2$

Table 4.1 shows that the average value of $\alpha_m$ for beam segments is approximately 1.6. Therefore, a maximum theoretical elastic buckling curve for Series 1 and 2 frames can be developed based on $M_e = \alpha_m M_o = 1.6 M_o$ for a given slenderness ratio.

<table>
<thead>
<tr>
<th>Frame ID</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$\beta_m$</th>
<th>$\alpha_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>f22</td>
<td>87.54</td>
<td>132.46</td>
<td>0.661</td>
<td>1.588</td>
</tr>
<tr>
<td>f23</td>
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<td>0.743</td>
<td>1.617</td>
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<td>f24</td>
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<td>245.56</td>
<td>0.792</td>
<td>1.635</td>
</tr>
<tr>
<td>f25</td>
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<td>301.46</td>
<td>0.824</td>
<td>1.647</td>
</tr>
<tr>
<td>f42</td>
<td>73.25</td>
<td>146.75</td>
<td>0.499</td>
<td>1.530</td>
</tr>
<tr>
<td>f43</td>
<td>123.6</td>
<td>206.40</td>
<td>0.599</td>
<td>1.566</td>
</tr>
<tr>
<td>f44</td>
<td>175.77</td>
<td>264.23</td>
<td>0.665</td>
<td>1.589</td>
</tr>
<tr>
<td>f45</td>
<td>228.85</td>
<td>321.15</td>
<td>0.713</td>
<td>1.607</td>
</tr>
<tr>
<td>p22</td>
<td>82.32</td>
<td>137.68</td>
<td>0.598</td>
<td>1.565</td>
</tr>
<tr>
<td>p23</td>
<td>134.68</td>
<td>195.32</td>
<td>0.690</td>
<td>1.598</td>
</tr>
<tr>
<td>p24</td>
<td>188.1</td>
<td>251.90</td>
<td>0.747</td>
<td>1.619</td>
</tr>
<tr>
<td>p25</td>
<td>242.02</td>
<td>307.98</td>
<td>0.786</td>
<td>1.633</td>
</tr>
<tr>
<td>p42</td>
<td>65.97</td>
<td>154.03</td>
<td>0.428</td>
<td>1.504</td>
</tr>
<tr>
<td>p43</td>
<td>114.17</td>
<td>215.09</td>
<td>0.531</td>
<td>1.541</td>
</tr>
<tr>
<td>p44</td>
<td>164.91</td>
<td>275.09</td>
<td>0.599</td>
<td>1.566</td>
</tr>
<tr>
<td>p45</td>
<td>216.98</td>
<td>333.02</td>
<td>0.652</td>
<td>1.585</td>
</tr>
</tbody>
</table>
On the other hand, the calculations of FEA buckling moment curves are not as straightforward. The bending moment distributions (eg., $\beta_m$) must be obtained first before converting an elastic buckling load ($P$) into an elastic buckling moment ($M_e$). For shell finite element model, it would be impractical to obtain the bending moment diagram directly. As a substitute, moment distributions from linear elastic analyses (see Figure 4.17) were used to convert FEA buckling loads into buckling moments.

$$M_e = \frac{PL}{4} \left( 1 - \frac{\beta_m}{2} \right)$$

The beam slenderness ratio for FEA model can be expressed as: $\lambda = \sqrt{\frac{M_p}{M_o}}$. The effective length factor $k_e$ is assumed to be one in the moment capacity chart. $M_p$ is the plastic moment of the I-section, and $M_o$ is the reference elastic buckling moment (uniform moment case):

$$M_p = \sqrt{\left( \frac{\pi^2 EI_y}{L_e^2} \right) \left( GJ + \left( \frac{\pi^2 EI_w}{L_e^2} \right)^2 \right)}$$

where: $L_e =$ effective length = effective length factor $k_e \times$ Length (L)

Figure 4.17 Screen Shot of Linear Elastic Analysis
Figures 4.18 to 4.21 show the elastic buckling capacities from FEA models and theoretical predictions. In these figures, the elastic buckling capacities for frames with Type 1 beam-column connection are plotted in Figure 4.18. Type 1 connection is the weakest type of connections. The beam segment ends offer little out-of-plane rotation restraints and the connection itself undergoes some degree of distortion when lateral torsional buckling occurs in the beam. Subsequently, the frame elastic buckling capacities are lower than those predicted by the theoretical buckling curve.

The results also show that the column’s in-plane stiffness has very little effect on the adjacent beam’s lateral torsional buckling capacity. When the height of columns doubles or the supports alter from a pin to a fixed base, the in-plane column stiffness is increased significantly. However, they do not affect the out-of-plane rotation and twisting restraints at the beam ends. The increase of column in-plane stiffness may change the moment gradient along the beam slightly, but the variation of $\alpha_{m}$ is less than 4%. The increase of moment gradients coincides with the minor differences of elastic buckling capacities of frames with the same bay width. These trends are also observed in frames with different types of connections.
Elastic buckling results of frames with Type 2 connections are plotted at Figure 4.19. Due to presence of the web stiffeners, the warping deformation of beam ends was restrained by the beam-column connections. The beam-column connections are still undergoing small distortions during buckling. As a result, the effective length of the beam is slightly reduced compared with the idealized simply supported beams, and the elastic buckling capacity increases accordingly.

For practical reasons, the lateral restraint at the beam-column connection has to be at more than one point to eliminate the stress concentration during nonlinear analysis. When the web stiffeners are added, these lateral restraints will also prevent some degrees of beam out-of-plane rotation. However, the degrees of out-of-plane rotation restraints are difficult to quantify by using simple methods. These rotational restraints are due to the bending of web plate in the connection and therefore may not be very significant. The results show that the beam-column connection stiffness has significant effects on the beam end restraint conditions and subsequently the additional restraints lead to the increasing frame buckling capacities.

The elastic buckling capacities of frames with Type 3 connections are the highest among the four connection series. With the diagonal stiffeners, the beam end warping...
deformations are effectively eliminated. However, the out-of-plane “patch” restraint given by the diagonal stiffeners are also preventing beam out-of-plane rotation. The out-of-plane rotation restraints are much higher than that in frames with Type 2 connection, and this is the major cause for increased capacities. In real frames, these out-of-plane rotation restraints may also occur when a strong bracing member runs perpendicular into the portal frame corners.

![Image](image.png)

**Figure 4.20 Maximum Elastic Buckling Moments of Beams with Type 3 Connection**

Type 4 connection is an idealized case. With the use of MPC, the connection can be considered as fully rigid. There would be no warping deformation allowed at the beam’s ends. On the other hand, only a point lateral restraint was used at the centre of the connection. No additional or undesirable beam end out-of-plane restraint was introduced into the frame system. Frame elastic buckling capacities of such models are between that of connection Type 2 and Type 3 frames. This means that MPC type connection is more rigid than Type 2 connection that allows no internal deformation but compared with Type 3 connection, it allows more out-of-plane beam end rotations.
Elastic buckling analyses of non-sway frames with pin and fixed bases demonstrate that the elastic buckling moment for the beam as part of a frame structure is not different from the beam as an isolated member. Given the same moment gradient and end restraint conditions, the beams in these two cases should have the same elastic buckling capacities. For example, if the beams in frames with different connections are given the following effective length factors in Table 1.2, the nondimensionalised elastic buckling load fit well into simply supported beam elastic buckling curve that assumes $\alpha_m = 1.6$ as shown in Figure 4.22.

Table 4.2 Effective Length of the Beams in Various Frame Structures

<table>
<thead>
<tr>
<th>Connections Type</th>
<th>$k_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.15</td>
</tr>
<tr>
<td>2</td>
<td>0.98</td>
</tr>
<tr>
<td>3</td>
<td>0.80</td>
</tr>
<tr>
<td>4</td>
<td>0.70</td>
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</tbody>
</table>

In summary, elastic buckling capacities of Series 1 and 2 frames are governed by individual members' lateral torsional buckling capacities. The connection types for the frame have very significant effects on the frame capacities. Using the current
design procedure, the calculations of individual member’s elastic buckling capacities require linear analysis to obtain the moment gradient factors. The minimum value of members’ capacities would be the frame’s elastic buckling capacity if lateral restraints are provided at member connections. For Series 1 and 2 frames, the design code prediction for frame capacities are nearly the same as finite element analyses.

4.4.1.2 Nonlinear static analysis

Nonlinear FEA results of simply supported beams indicated that inelastic lateral torsional buckling problems are different from the in-plane bending problems where the members will undergo large plastic deformations while still holding the maximum applied loads. For simply supported beams, lateral torsional buckling often occurs abruptly with limited post-buckling capacity. Also, the in-plane stiffness of the beam in the elastic range would be the same with and without the lateral torsional buckling effects. Frame structures that are not fully laterally restrained are also expected to exhibit similar behaviour. However, unlike simply supported beams, Series 1 and 2 frames have higher degrees of redundancy. Plastic redistribution and interactions between members play an important role in achieving the frame’s ultimate strength. Therefore, the comparisons of frame member capacity with that of simply supported beam are complicated unlike in the case of elastic buckling.
Tables 4.3 and 4.4 give the ultimate load capacities of Series 1 and 2 frames. When residual stresses are included in the analyses, the ultimate load capacities are reduced. The reductions range from 0.03% to 2% with an average of 0.5%. In other words, the effect of residual stresses on the ultimate frame capacities is insignificant.

Table 4.3 Ultimate Loads of Series 1 and 2 Frames

<table>
<thead>
<tr>
<th></th>
<th>Type 1 (kN)</th>
<th>Type 2 (kN)</th>
<th>Type 3 (kN)</th>
<th>Type 4 (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>f22</td>
<td>160.6</td>
<td>165.9</td>
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</tr>
<tr>
<td>f23</td>
<td>104.4</td>
<td>104.5</td>
<td>119.0</td>
<td>119.4</td>
</tr>
<tr>
<td>f24</td>
<td>75.1</td>
<td>75.5</td>
<td>82.7</td>
<td>82.8</td>
</tr>
<tr>
<td>f25</td>
<td>56.2</td>
<td>56.3</td>
<td>61.3</td>
<td>61.3</td>
</tr>
<tr>
<td>f42</td>
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<td>159.0</td>
<td>176.7</td>
<td>178.0</td>
</tr>
<tr>
<td>f43</td>
<td>102.5</td>
<td>103.3</td>
<td>115.9</td>
<td>116.8</td>
</tr>
<tr>
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<td>74.8</td>
<td>81.3</td>
<td>81.7</td>
</tr>
<tr>
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<td>60.6</td>
</tr>
<tr>
<td>p22</td>
<td>160.8</td>
<td>165.1</td>
<td>185.0</td>
<td>188.5</td>
</tr>
<tr>
<td>p23</td>
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<td>104.6</td>
<td>119.1</td>
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<tr>
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<td>154.4</td>
<td>167.5</td>
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<tr>
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<td>113.5</td>
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<td>74.3</td>
<td>79.8</td>
<td>80.7</td>
</tr>
<tr>
<td>p45</td>
<td>56.2</td>
<td>56.7</td>
<td>60.0</td>
<td>60.4</td>
</tr>
</tbody>
</table>

Note: Unshaded columns give the results for no residual stress input.

Table 4.4 Ultimate Loads of Series 1 and 2 Frames with full lateral restraints

<table>
<thead>
<tr>
<th></th>
<th>Type 1 (kN)</th>
<th>Type 2 (kN)</th>
<th>Type 3 (kN)</th>
<th>Type 4 (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>f22</td>
<td>267.2</td>
<td>301.0</td>
<td>302.3</td>
<td>307.0</td>
</tr>
<tr>
<td>f23</td>
<td>179.4</td>
<td>202.8</td>
<td>204.5</td>
<td>207.1</td>
</tr>
<tr>
<td>f24</td>
<td>133.9</td>
<td>150.1</td>
<td>151.6</td>
<td>153.2</td>
</tr>
<tr>
<td>f25</td>
<td>106.3</td>
<td>118.3</td>
<td>119.5</td>
<td>120.8</td>
</tr>
<tr>
<td>f42</td>
<td>268.9</td>
<td>295.0</td>
<td>297.1</td>
<td>299.6</td>
</tr>
<tr>
<td>f43</td>
<td>181.4</td>
<td>198.0</td>
<td>202.4</td>
<td>204.0</td>
</tr>
<tr>
<td>f44</td>
<td>135.6</td>
<td>151.0</td>
<td>152.2</td>
<td>153.4</td>
</tr>
<tr>
<td>f45</td>
<td>108.0</td>
<td>120.3</td>
<td>121.0</td>
<td>122.2</td>
</tr>
<tr>
<td>p22</td>
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<td>298.8</td>
<td>300.8</td>
<td>303.6</td>
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<tr>
<td>p23</td>
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<td>202.7</td>
<td>204.4</td>
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<tr>
<td>p24</td>
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<td>152.8</td>
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<td>120.8</td>
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<tr>
<td>p42</td>
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<td>p43</td>
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<td>108.5</td>
<td>120.1</td>
<td>120.8</td>
<td>121.8</td>
</tr>
</tbody>
</table>
From the nonlinear static analysis, a maximum or ultimate load can be obtained \( (P_{\text{max}}) \). In order to compare the beam capacity predicted by the design code, this ultimate load was converted to the peak bending moment \( M_u \) that may exist in the beam using the formula: \[ M_u = \frac{P_{\text{max}} L}{4} \left( 1 - \frac{P_{\text{max}}}{2} \right) \]. The ultimate moment \( M_u \) is plotted against the beam slenderness \( \sqrt{M_p / M_u} \). The calculations of beam slenderness utilise the results from frame elastic buckling analyses, i.e., beam’s effective length factor for Type 1 connection is taken as 1.15, and so on. On the other hand, the design beam curve (AS4100) that assumes \( \alpha_m = 1.6 \) is also plotted in the same chart.

The ultimate moments for the frames with Type 1 connection are plotted in Figure 4.23. The results are very surprising. In essence, they do not agree with the design code predictions. It was suspected that Type 1 connection may be too “flexible” to be classified as a rigid connection. If the bay width is short, severe yielding may occur at the connections and cause premature failures of frames. Even though it did not occur in the frames studied here, but from the data projection it is very likely to happen for the short span frames.

\[ \text{Figure 4.23 Maximum Ultimate Moments of Beams with Type 1 Connection} \]
In general, the FEA results are much higher than the design code predictions, with the worst case (p45) being 78% higher. Further, unlike in the elastic buckling analysis in which the column stiffness does not affect the frame buckling load significantly, the column stiffness has a more important role in frame ultimate capacities. An interesting aspect is that the frame ultimate capacities are reduced with increasing column stiffness compared with the predictions. For example, the pin base frames and frames with longer column perform better than expected.

The stress contour plots (see Figure 4.24) may offer some explanation about this behaviour. Firstly, for structures with longer beam and less stiff column, the displacements of beams are bigger than otherwise. Therefore, the bending moment redistributions in the plastic stage for these frames move away further from their elastic stages. More importantly, the plastic deformation of the beam changes quite differently from the initial geometrical imperfection input, i.e., the elastic buckling shape. In the ultimate stages, the beam’s contraflexure points move further away from the connection therefore the effective length of the beam may become shorter. Consequently, the predictions based on elastic moment distribution are in disarray.

![Figure 4.24 Deformations of Frame f22 and p44](image-url)

The results from Type 2 connection are similar to that of previous type (see Figure 4.25). The FEA results are plotted against the beam slenderness that assumes an
effective length factor $k_e$ of 0.98. In the elastic buckling analysis, the results from these frames agree very well with the beam curve which implies that the beam in the frame behave very similar to an idealized simply supported beam. However, the nonlinear static analyses results give a fairly different picture. The ultimate beam capacities are generally higher than the beam curve predictions when connection distortion is disregarded (frames with short bays).

Figure 4.26 shows the results of case of Type 3 connection. The effective length factor is taken as $k_e = 0.7$ based on the elastic buckling frame analyses. The results for these frames are the closest to the design code predictions. Type 3 connection with associated lateral restraints is the most “rigid” among the four types of connection. Using this connection, the interactions between column and beam should be the smallest due to the additional out-of-plane rotational restraints by the connection boundary conditions. Therefore the beam deformation shapes at the ultimate load are relatively similar to elastic buckling of isolated beams. In other words, the elastic effective lengths of the beams are similar to real effective lengths after frames undergo plastic deformation.

![Figure 4.25 Maximum Ultimate Moments of Beams with Type 2 Connection](image-url)
Figure 4.26 Maximum Ultimate Moments of Beams with Type 3 Connection

Figure 4.27 Maximum Ultimate Moments of Beams with Type 4 Connection

Type 4 connection results are similar to Type 3 connection as shown in Figure 4.27. Compared with elastic buckling analysis results, the nonlinear analysis results for
ultimate frame capacities are scattered. The elastic buckling analysis does not seem to allow for beam-column interaction. This is not the case for nonlinear analyses. Frames f23, f43, p23, and p43 have the same bay width (the beam length), for which the maximum difference in elastic buckling moments is 3.6% but is 13.9% for ultimate moments. This means that the beam-column interactions had significant effects on the beam ultimate capacities.

Figures 4.28 and 4.29 illustrate the mid-span deflection behaviour of frames whose out-of-plane deformations are restrained. $P_u$ is the load that causes the beam to form three hinges (collapse mechanism). Frames with Type 1 connection achieve only 90% of the plastic collapse load. Therefore, these frames should not be called rigid frames. Type 2 connection has improved the frame’s ultimate capacities. However, the unloading behaviour shown in the graphs indicate that these frames are also not capable of reaching theoretical collapse mechanism.

![Figure 4.28 Load-Deflection Curves for Fully Laterally Restrained Frames with Fixed Bases](image-url)
The results from fixed and pinned base frames are very similar. The connection Type 4 is the most rigid type of connection even though the elastic buckling analyses indicate that Type 3 connection had the highest buckling capacities.

Figures 4.30 and 4.31 are the typical load-deflection curves for frames with lateral restraints only at the beam-column connections. They demonstrate that Types 1 and 2 connections are more flexible than the real “rigid” connections once yielding occurs. Even though these two connections are not fully rigid, the maximum loads these frames achieved are still higher than design code predictions.

Before initial yielding commences, the behaviour of frames with different connections is nearly the same. This means that the connection rigidities introduce changes only during the plastic stages. The plastic rigidity differences of connections are not easy to account for in the design practice because there are many parameters involving connection rigidities. A lower bound approach must be used in dealing with this issue. From this point of view, it makes sense to treat Types 1 to 4 connections as the same in common design calculations.

Figure 4.29 Load-Deflection Curves for Fully Laterally Restrained Frames with Pinned Bases

The mid-span deflection (mm) is plotted against the load ratio \( \frac{P}{P_u} \), where \( P \) is the applied load and \( P_u \) is the ultimate load. The graph shows the behavior of frames with different connection types under lateral restraints.
The frames shown in Figures 4.30 and 4.31 consist of a beam with intermediate length (L = 6 m). All the frames exhibit significant ductility. Since only the basic material properties are considered in the analyses (properties such as metal fracture, strain...
limits, and weld effects are not considered), ductility issues should be investigated further. The stress contours show that stress concentrations are very significant at beam-column connections.

Table 4.3 shows that the ultimate capacity differences are insignificant whether or not residual stresses are considered. In terms of load-displacement relationships, the initial residual stress input has important effects on frames. As shown in Figure 4.32, the load-deflection curves change more abruptly without residual stress input during the nonlinear analyses. This behaviour must be taken into account in the simplified advanced analysis methods.

Load-deflection curves for frames with a 6 m bay width are plotted in Figure 4.33. Type 3 connection was used in these frames. The column stiffness of these frames is altered by changing the support conditions or the column lengths. The results show that higher column stiffness leads to a higher frame capacity. This does not contradict what is shown in Figure 4.26 which shows that decreasing column stiffness gives higher frame capacities. This is because the design code predictions and real capacities are two different issues.

![Figure 4.32 Effects of Initial Residual Stress on the Ultimate Capacity of Frames](image)
When the load-deflection curves for the frames with the same column stiffness are plotted together, another picture emerges (Figure 4.34). The figure is for frames with 4 m fixed base columns. It shows that the ultimate capacities of frames are significantly reduced as a result of lateral torsional buckling. Compared with frames that are fully restrained against out-of-plane displacement, the elastic load paths are the same for frames that undergo lateral torsional buckling. The frames with short beams require less plastic deformation to reach their ultimate capacities. The load paths for these frames can be clearly divided into three stages. They are: elastic stage, plastic stage and post-buckling/unloading stage. Frame f42 shows an apparent unloading stage after reaching its maximum strength.

Frames with longer beam such as f45 shown in Figure 4.34 behave quite differently. Plastic deformation may start during the very early stages of the load path. The frame structures have a highly nonlinear behaviour. The unloading path for these frames can not be observed before the analyses encountered converging problems.
In summary, the observations from the load-deflection curves of Series 1 and 2 frames are:

- The commonly used Types 1 and 2 connections exhibit considerable amount of plastic redistribution within the connections. It is questionable whether the frames with these types of connections should be considered as rigid frames. On the other hand, in general the frames consisting of these two types of connections still achieve higher capacities than design code predictions.

- The ultimate capacities of the frames with Types 2, 3 and 4 connections are reasonably close, but the load-deflection behaviours are dissimilar. If the frames are assumed to be truly rigid, connection Types 3 and 4 should be used in future modelling.

- The interactions between the column and beam members have significant effects on the frame capacities. The capacities that are unaccounted for in the design code are partly due to these interactions. Other reasons are: a) real effective length of beam in the frames is shorter than that calculated from elastic buckling analysis, b) bending moment gradients after yielding are different from those prior to yielding.
• Residual stresses are not very important for the analyses aimed at determining the ultimate frame strengths. Their effects are seen mainly on the nonlinear load-deflection characteristics.

4.4.2 Single bay single storey sway portal frames (Series 3 and 4)

Series 3 and 4 frames are two sway rigid frames subjected to various ratios of horizontal and vertical loads (H/V). For each frame, a horizontal load is applied to the beam-column connection, while a vertical load is applied at the midspan of the beam. The dimensions of these two series of frame are shown in Figure 4.35.

Different types of beam-column connections were studied in the non-sway single bay single storey frame series. It was found that the stiffness of the connections has very significant effects on the structural behaviour. However, in order to keep the computer run time manageable, Series 3 and 4 frames considered only idealized rigid connections using MPC. For the same reason, the nonlinear analyses without residual stress input or with full lateral restraints were not undertaken for these series of frame.

![Figure 4.35 Dimensions of Series 3 and 4 Frames](image)

4.4.2.1 Elastic buckling analysis

Each FEA model consists of seven load cases. The horizontal to vertical load ratio (H/V) of each load case are shown in Table 4.5. The bending moment distributions are significantly different for different load cases. After conducting linear analyses,
the moment modification factor $\alpha_m$ values can then be obtained and these values are also shown in Table 4.5.

Table 4.5 Horizontal to Vertical Load Ratio (H/V) and $\alpha_m$

<table>
<thead>
<tr>
<th>H/V</th>
<th>$\alpha_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>f46</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.091</td>
</tr>
<tr>
<td>3</td>
<td>0.303</td>
</tr>
<tr>
<td>4</td>
<td>0.727</td>
</tr>
<tr>
<td>5</td>
<td>1.818</td>
</tr>
<tr>
<td>6</td>
<td>4.545</td>
</tr>
<tr>
<td>7</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

Figure 4.36 shows the first buckling modes of f46 frame under only horizontal or vertical load. The elastic buckling for these two extreme load ratios are similar. In fact, the analysis results show that instabilities always take place in the beams for all the load cases. The cases with larger horizontal load exhibit small twisting rotation. This is due to the higher axial loads in the beam segments.

![Figure 4.36 Elastic Buckling of Beam under Horizontal and Vertical Loads](image)

The theoretical elastic buckling loads of the frames are calculated based on the beam buckling moments. Elastic buckling analyses of Series 1 and 2 demonstrate that the beam’s effective length factor $k_e$ is approximately equal to 0.7 with Type 4 connections. Therefore, $k_e = 0.7$ is used to derive the buckling load curves for beams.
with various slenderness ratios. The theoretical buckling loads for Series 3 and 4 frames are obtained using the following steps:

1. Conduct a linear elastic analysis for each load case to obtain the bending moment diagram for each frame.

2. For each load case, the buckling moment is calculated using the empirical method with a moment modification factor:

\[ \alpha_m = \frac{1.7M^*_m}{\sqrt{\left(M^*_2\right)^2 + \left(M^*_3\right)^2 + \left(M^*_4\right)^2}} \leq 2.5 \]

where \( M^*_m \) is the maximum bending moment in the beam, \( M^*_2, M^*_3 \) and \( M^*_4 \) are the bending moment at quarter points.

The maximum buckling moment of the beam is \( M_e = \alpha_m M_o \).

3. The bending moment diagram is scaled up or down in such a way that the maximum bending moment in the beam equals \( M_e \). The original input loads (\( H \) or \( V \)) are multiplied by the scale factor to obtain the frame’s buckling loads (ultimate horizontal load \( H_e \) or ultimate vertical load \( V_e \)).

The results from the elastic buckling analyses are plotted in Figure 4.37. Compared with the theoretical frame buckling loads based on the effective length factor \( k_e \) of 0.7 for beams and the empirical moment modification factor \( \alpha_m \), the FEA results can either be higher or lower. The general trends from both methods are similar.

When the frame structures are subjected to only the vertical loads, the FEA results are within 5% of the theoretical buckling loads. This implies that the assumption of \( k_e = 0.7 \) is capable of representing the boundary conditions used in the finite element models. However, the discrepancies are larger for other load cases. There are two explanations for this problem. First, axial forces in the beam are not considered during the derivation of the theoretical frame buckling loads. The elastic buckling loads of a beam segment subjected to combined actions of axial compression force and bending moment are not well studied. Second, the interactions between adjacent members are taken into account by the FEA. However, considering the member interaction is difficult and not practical for the simplified method. The second factor is believed to have more significant effects.
Figure 4.37 Elastic Buckling Loads of Sway Portal Frames
4.4.2.2 Nonlinear static analysis

Nonlinear analysis is sometimes called “Mechanical simulation” or even “Destructive testing” in some FEA packages. These labels are obviously a marketing tactic and maybe an overstatement. Nevertheless, they recognised the potential of inelastic nonlinear analysis. These analyses are able to provide a better understanding of the real structural behaviour.

The FEA results and the ultimate loads calculated using the design code AS4100 are shown in Figure 4.39. The ultimate horizontal and vertical loads ($V$ and $H$) are non-dimensionalized by the loads that cause the first plastic hinge ($V_u$ or $H_u$). According to the simple plastic hinge method, a frame structure is able to form more than one hinge. However, the hinge that causes a collapse mechanism could be difficult to determine in some cases e.g., two hinges may form at the same time. In order to overcome this problem, the loads which cause the first plastic hinge to form are used as a reference point (see Figure 4.38).

![Figure 4.38 Bending Moment Diagram of Frames subject to $V_u$ and $H_u$](image)
Figure 4.39 Ultimate Loads of Sway Portal Frames
For frames f46 and p46, the FEA results and the design code predictions agree reasonably well. These frames have a relatively short bay width. When the frames are subjected to only horizontal loads, the nondimensionalized factors are nearly equal to one. It implies that the failure modes of the frames are the same as shown in Figure 4.38. The FEA results show that the frame’s ultimate capacities are lower than the loads that cause the collapse mechanism but much higher than the loads at the first hinge formation. When the structures are subjected to predominantly vertical loads, the FEA results are slightly higher than the design code predictions. This is because the design code method does not consider the stress redistributions while the frames are undergoing lateral torsional buckling.

For frames f48, p48, f410, and p410, the FEA results are generally much higher than the design code predictions when vertical loads are predominant. Neglecting the stress redistribution in the current design code obviously leads to an over-conservative solution. For fixed base frames (f48 and f410) with more flexible beams, the column stress redistributions are more beneficial. The frame failure modes are closer to that of collapse mechanism than that shown in Figure 4.38.

In Figure 4.37, the elastic buckling loads from FEA are only slightly lower than the theoretical predictions. In the nonlinear analysis cases, the results seem to be opposite with FEA producing higher predictions. This implies that even though the elastic buckling loads of the frame can be accurately obtained with advanced frame analysis techniques, correct ultimate loads are not warranted.

Trahair (1998) also observed that the capacity of members in portal frame structures is significantly higher than the design code predictions. He mentioned two factors:

1. According to AS4100 design rules, the member capacity is calculated using two factors. One of these factors, the moment modification factor - $\alpha_m$ depends on the bending moment gradient. However, when the frame reaches its ultimate capacity, plasticity redistribution occurs and the moment gradient changes in favour of carrying greater loads.
2. The design code assumes no warping restraint at the end of the members for conservative reasons. However, in rigid frames, the warping restrained condition exists. Therefore, the member capacities from FEA are greater.

The FEA results demonstrate that there is an important third factor. That is, the design code does not consider that interaction between adjacent members. When considering the plastic redistribution in a frame structure, it should not be limited to the members undergoing lateral torsional buckling but also to the members connected to them. The moment redistributions due to yielding in any members increase the overall frame capacity.

Figure 4.40 (a) is a screen shot of the frame p46 subjected to a predominated horizontal load case. This is a typical failure mode due to the formation of a collapse mechanism. A plastic hinge is clearly formed in the right hand side column. The failure modes predicted by FEA for such frame/load cases generally agree well with the theoretical predictions.

Figure 4.40 (b) shows a typical lateral torsional buckling failure mode. Excessive yielding can be observed at midspan of the beam. The deformation shape at the maximum load stage is similar to the elastic buckling shape. For this fixed base frame, instead of forming three plastic hinges (collapse mechanism), the frame capacity is limited by the member lateral torsional buckling.

Figure 4.40 Frames subject to Different Load Cases

Figure 4.40 (b) shows a typical lateral torsional buckling failure mode. Excessive yielding can be observed at midspan of the beam. The deformation shape at the maximum load stage is similar to the elastic buckling shape. For this fixed base frame, instead of forming three plastic hinges (collapse mechanism), the frame capacity is limited by the member lateral torsional buckling.
The failure mode for frame p410 subjected to a predominant horizontal load is another type of lateral torsional buckling (Figure 4.41). One main difference from the previous case is that the beam has a double curvature buckling shape. The elastic buckling analysis of this load case provides a single curvature geometrical imperfection input but the nonlinear analysis leads to a different result. This behaviour will be difficult to predict using the simplified analysis method.

![Figure 4.41 Frame p410 with a H/V Load Ratio of 1.82](image)

Observations of frame deformation from the nonlinear analysis confirm that there are three types of failure modes for Series 3 and 4 frames: 1) formation of plastic hinges, 2) lateral torsional buckling of member, and 3) combination of the two modes. Simple plastic hinge method considering only a collapse mechanism is not applicable to all of the steel frame structures. A specific failure under a certain load case will be difficult to predict unless using very sophisticated methods such as the shell finite element analysis.

The beam midspan load-deflection curves for frame p46 are shown in Figure 4.42. According to the current design code, the member capacities of beam and columns are approximately equal to the plastic section capacities. In other words, the structure should behave similar to that of a fully laterally restrained frame. The load-deflection curves agree well with this prediction. Even though when the frame is subjected to a predominant vertical load, the structure exhibits some degrees of snap through behaviour. Large plastic deformation can be observed, and the ultimate capacities are only slightly higher than AS4100 design code predictions.
Figure 4.42 Load versus Beam Midspan Deflection Curves for Frame p46

Figure 4.43 Load versus Knee Drift Curves for Frame p46
The knee drift versus load curves (Figure 4.43) show the same behaviour. When the structure is subjected to a predominant horizontal load, the frame is able to withstand large deformations. Loads are decreasing after the maximum capacities for most load cases. This is due to the use of perfect plastic material properties in the FEA. When strain hardening is considered, the frame structure should be able to withstand the maximum loads for larger deformations. In general, plastic deformation stages are short compared with elastic stages. It means that stress redistribution does not play an important role in increasing the frame capacities. After all, the pinned base frames do not have high degrees of redundancy.

Figure 4.44 shows the beam midspan load-deflection curves for frame f48. This is a typical graph for fixed base frames. Even though the beam undergoes lateral torsional buckling, the beam is still able to endure large plastic deformation without capacity reduction. This behaviour is significantly different from that of simply supported beams. Comparison of these curves with those from pin base frames shows that the plastic deformations have a greater contribution to load capacities.
The loads versus knee drift curves for frame f48 are shown in Figure 4.45. The behaviour of these curves is not much different from that where lateral torsional buckling is not an issue (as for frame p46). Fixed base frame has higher degrees of redundancy. When the member in the rigid frame undergoes lateral torsional buckling, the buckling beam does not seem to stop redistributing the stress to other members. The results show that moment redistributions are possible and the frame’s redundancy has clearly increased the ultimate capacities of frames.

In summary, the failure behaviour due to lateral torsional buckling in rigid frame structures is not fundamentally different from that based on hinge formation collapse mechanism. Instead of forming hinge/hinges in a member, the beam simply reaches its member capacity, and if the connection failures are disregarded, this member is able to undergo large plastic deformation without capacity reductions.

When a member in frame structures undergoes lateral torsional buckling, its behaviour is more complicated compared with isolated members with known boundary conditions. The current design methods are not capable of capturing the plastic behaviour of steel frame structures, especially when lateral torsional buckling
failure is involved. Nevertheless, the FEA shows that the current design process is a conservative design approach.

4.4.3 The \( \Gamma \) shape frame (Series 5)

The dimensions of Series 5 frames are shown in Figure 4.46. For each frame, four different beam-column connections have been modelled (as shown in Figure 4.9). There are 48 finite element models in this series. Four analyses were conducted for each model to study the behaviours of elastic buckling, the effects of residual stress, and the fully laterally restrained frames.

![Figure 4.46 Dimension of “Γ” Shape Frames](image)

4.4.3.1 Elastic buckling analysis

Clark and Hill (1960) gave the effective length factor of 1.0 for cantilever beam subjected to concentrated load at the end. However, the factor \( k_e \) is considered as two times the overhang length according to the design code (AS4100). It seems that very conservative measures are taken in the design practice. When a concentrated point load is applied to the tip of the cantilever, the moment modification factor is \( \alpha_m = 1.25 \). When a cantilever is attached to a column, the lateral torsional buckling of overhang segment may behave quit differently. The frame c2-3 shown in Figure 4.47 (a) is 2 metres high, and has a 3 metres long overhang beam. The buckling shape of this beam is significantly different from the cantilever with the same properties. Very little twisting deformation is presented in this type of \( \Gamma \) shape structure. In the other case (c2-4), the overhang is 4 m long (Figure 4.47 (b)). The twisting deformation is more substantial. The buckling shapes of the overhang are closely
related to the stiffness ratio between the adjacent members. When the twisting components are not presented in the buckling modes, the elastic buckling loads are much smaller. For comparison purposes, analyses for cantilever have also been conducted (Figure 4.47 (c)).

![Figure 4.47 Lateral Torsional Buckling of Γ shape frames and Cantilever](image)

The elastic buckling moments ($M_e$) for frames with Type 1 connection are plotted at Figure 4.48. $M_e$ is the applied load (P) times the overhang length (L). The FEA results show that the short overhang segments are much weaker than they are supposed to be. Obviously, the restraints that the columns provided are far from fully fixed. On the other hand, when the overhangs are slender, they agree better with the theoretical predictions (assumed $k_c = 1.0$). The abilities to provide out-of-plane rotation restraints are crucial in the capacity predictions of overhang segments.

Figure 4.49 shows the elastic buckling moment graphs for frames with Type 2 connections. The stiffeners in the beam-column connections make the “patch” lateral restraints (see Figure 4.9) more efficient. As a result, the elastic buckling capacities are closer to those of fixed end cantilever beam. For the 2 metres height frames that have Type 2 connections, the combined effects of column twisting restraints and “patch” lateral restraints produce a result that is nearly the same as fixed end cantilever. However, rotational restraints from the 4 metres tall columns must be
weaker. For shorter beam, the columns are relatively more slender compared with the cantilever segments. The combined effects of column twisting resistance and “patch” lateral restraints at the beam-column connection are not sufficient to produce fully fixed conditions.

The results for Type 3 frames are shown in Figure 4.50. They are slightly higher than the theoretical predictions. Type 3 connection has two diagonal stiffeners. These stiffeners are able to make the lateral restraints “patch” more effective for the overhang segment. Using this connection, the interactions between adjacent members are significantly reduced. The results show that the columns length has insignificant effect on the elastic buckling capacities. The overhang segments with this connection yield similar results to those of normal cantilever beams.

The elastic buckling moments for frames with Type 4 connection are plotted in Figure 4.51. This connection is an idealized case. The boundary conditions at the connections provide warping and lateral displacement restraints but no additional beam out-of-plane rotational restraints. The out-of-plane rotational restraints were supplied only by the column. Therefore, the stiffness of column becomes a decisive factor affecting the elastic buckling capacities. When the frames have 2 metre columns, the FEA results follow the same trend as theoretical lateral torsional buckling curve. When the columns are 4 metres, the primary buckling loads are much lower than the theoretical predictions. The overhang segments undergo lateral buckling without twisting which is similar to that shown in Figure 4.47 (a). The columns are obviously not strong enough to provide sufficient twisting stiffness for the beam segment to undergo classical lateral torsional buckling.

In summary, the elastic buckling capacities of the overhang segments in the Γ shape frames do not agree well with those of cantilever beams. The main reason is that the column may not be able to provide sufficient restraints to the overhang segments. The overhang capacities are very sensitive to the column stiffness and the connection details. The beam – column interactions are also very important with Γ shape frames. When the out-of-plane rotational restraints are insufficient, the overhang segment may experience a different buckling mode. The twisting rotation may not be present and
the elastic buckling capacity of the overhang can be much lower. The beam-column connection details are a critical element for the elastic buckling analyses. Without attending to all the details, the elastic buckling analyses are not likely to provide consistent solutions. Since the effects of beam-column interactions and the connections details are difficult to account for, choosing a suitable effective length factor to calculate the overhang segment elastic buckling capacities accurately would be difficult for this type of structure.

![Graph](image)

**Figure 4.48 Maximum Elastic Buckling Moments of Beams with Type 1 Connection**

![Graph](image)

**Figure 4.49 Maximum Elastic Buckling Moments of Beams with Type 2 Connection**
4.4.3.2 Nonlinear static analysis

Elastic buckling analyses demonstrate that the overhang segment capacities depend on some factors not often considered by the design specifications, eg., the details of beam-column connections and adjacent member interactions. There are some
discrepancies between the capacities of overhang segment and normal cantilever beam. Nonlinear analyses indicate that these differences are even more significant.

When the frames are fully laterally restrained, the ultimate applied load \( (P_u) \) multiplied by the corresponding overhang length \( (L) \) is roughly the same as the beam plastic moment \( (M_p) \). This implies that the frame structures fail via the formation of a plastic hinge. One interesting aspect (see Figure 4.52) is that the ultimate loads are nearly the same for the same frame with different connections. This may explain that connections Type 1 to 4 can be treated as the same (Rigid) in the current design practice.

Figure 4.53 shows the ultimate loads for normal Series 5 frames. Some nonlinear analyses did not include residual stress (RS). In general, the results from these analyses are slightly higher than those which included residual stresses (on average 2.75% higher). It appears that the inclusions of residual stress in the FEA are not very critical for frame analysis.

![Figure 4.52 Ultimate Capacities of Fully Laterally Restrained Series 5 Frames](image-url)
Figure 4.53 Ultimate Capacities of Series 5 Frames

Figure 4.54 Plastic Deformations of Overhang Segments and Cantilever

Observations of the final deformations indicate that the overhang segments include two failure modes, which is similar to those from elastic out-of-plane buckling.
Depending on the overhang end constraints, some frames at the ultimate state consist of very little twisting component (Figure 4.54 (b)), while others act resembling cantilever beam (Figure 4.54 (c)). As with the elastic buckling analysis, the buckling shape has significant effects on nonlinear analysis. When twisting components are absent in the failure shapes, the overhang segment capacities are significantly lower than the expected values.

Plotting the bending moment and slenderness data pairs with the design beam curve can be very beneficial in understanding the lateral torsional buckling behaviour of the overhang segments. For each data pair, the ultimate moment capacity \( M_u \) is calculated by the cantilever length \( L \) times applied load \( P \). Corresponding beam slenderness is taken as \( \sqrt{\frac{M_p}{M_o}} \). \( M_o \) is the elastic buckling moment capacity of cantilever beam subject to a uniform bending moment. The effective length factor is assumed to be 1.0 in the \( M_o \) calculation \( (L_e = L) \) as predicted by Clark and Hill (1960). The bending moment capacities for the overhang are shown in Figures 4.55 to 4.58 as follows.

![Figure 4.55 Maximum Ultimate Moments of Overhang with Type 1 Connection](image)

\[
\sqrt{\frac{M_p}{M_o}}
\]
Figure 4.56 Maximum Ultimate Moments of Overhang with Type 2 Connection

Figure 4.57 Maximum Ultimate Moments of Overhang with Type 3 Connection
The results for overhangs with Type 1 connection are shown in Figure 4.55. These overhang segments behave quite differently from both the design code prediction and FEA results for cantilever beams. The ultimate bending moments in the segments are nearly the same despite the changing of slenderness. The results demonstrate that the overhang segment capacities are dictated by the connection details and they can be affected by the interaction between members and connections significantly. When the frames are fully laterally restrained, Type 1 connection is able to achieve plastic moment ($M_p$). However, in lateral torsional buckling case, twisting and out-of-plane bending actions are also presented in the connections. The combined actions will lead to significant reduction on connections moment resistances. Also, due to the twisting and warping restraints against the end of the overhang are relatively weak, the deformed shapes of the overhang segments are unlike those of the cantilever beams; they lack the twisting components (same as Figure 4.54 (b)) even when twisting initial geometrical imperfections are presented. With fewer restraints, naturally, the overhang segments will have lower moment capacities.

With flanges extend into the column, Type 2 connection is substantially stronger. For frames with 2 m columns, the overhang segments behave similar to the cantilever
beams and follow the trend predicted by the design code (Figure 4.56). The moment capacities of overhang decrease as slenderness increased. However, when the columns are 4 m high, the frames still lack out-of-plane rotation restraints. The overhang segments have very small twisting rotations. With a different buckling mode, the overhang segment capacities are much lower than the FEA results of cantilevers.

The ultimate capacities for overhangs with Type 3 and 4 connections are shown in Figures 4.57 and 4.58. The results for these two types of connections are similar. Both of them are scattering and do not fully agree with the cantilever beam results. One aspect is certain, the theoretical effective length factor of 1.0 is not suitable to use for the common overhang segments in frame structures. If the connections or the rest of the structure can not provide fully fixed constraints to the overhang segments, the frame capacities will be significantly reduced.

Initial geometrical imperfections are considered as a trigger that initiates the buckling failure of the structures. Generally, the buckling shapes from nonlinear analyses would be the same as the initial input and the ultimate capacities obtained from the analyses represent the inelastic buckling loads. For the Γ shape frames, the relationship between the initial geometrical imperfection and the ultimate load is not as straightforward. Firstly, the overhang segments may behave differently when using nonlinear analyses. The twisting component is often not present when the frames reach the ultimate loads. Secondly, the ultimate capacities in some frames are even higher than the primary elastic buckling loads (frame c4-2 to c4-5 with Type 4 connections) due to the post-buckling strength of the structures.

The load-deflection curves reveal the load path of the structures which provide further information on lateral torsional buckling behaviour. Figure 4.59 shows the load versus beam end deflection behaviours for Series c2-2 frames. These frames are fully laterally restrained and their results agree well with the established plastic hinge method. The applied loads \( P \) in nonlinear analyses are non-dimensionlized by the loads that may cause forming of plastic hinge \( P_u \). \( P_u \) can be calculated as \( P_u = M_u / L \). \( M_u \) is the section plastic moment, and \( L \) is the cantilever length. On average, all the frames achieve 97% of the load that predicted by the plastic hinge.
method. The structures with Type 2 to 4 connections have very similar results and are able to maintain large displacements. Type 1 connection is more flexible but the frame is also able to reach the near plastic capacity.

Figure 4.59 Load-Deflection Curves for Fully Laterally Restrained Frame c2-2

Figure 4.60 Load-Deflection Curves for Frame c4-2
Figure 4.60 shows the results for the same frames (c2-2) without fully laterally restraints. All of these curves have typical behaviour of beams subjected to lateral torsional buckling. The overhang segments behave linearly until they reach the ultimate capacities. For Type 1 frame, the post-buckling part of the curve is substantially different from others. The sharp reduction of frame capacity is due to the failure of the beam-column connection.

The effects of residual stress input can be clearly seen in the load-deflection graphs shown in Figure 4.61. Without the residual stress, the structures will reach the loads sometimes up to 9% higher. Also, the lateral torsional buckling failure will be more abrupt without considering the residual stress. These curves demonstrate that residual stress input is quite important and should be considered in steel frame analysis when it is possible.

The load-deflection curves for frames c2-3 and c4-3 are plotted in Figure 4.62. When both frames are fully laterally restrained, the ultimate capacity for the taller frame is slightly lower (both frames have 3 m long overhang). The differences between the two are not very significant. In the lateral torsional buckling case, the twisting stiffness of the columns plays a more important role. Frame with short column
achieves much higher load. This implies that the interaction between adjacent members has significant effects on lateral torsional buckling.

![Figure 4.62 Effects of Column Stiffness (Frames with Connection Type 2)](image)

In summary, the predictions for Γ shape frame capacities are very difficult using the simplified method. Firstly, the restraints for overhang segments are not easy to quantify. The end conditions for overhangs are very different from those of cantilevers. Secondly, the interactions between adjacent members are usually ignored in the simplified method. This often produces over-estimated frame capacities. The finite element analyses reveal that beam-column connection details have very significant effects on the lateral torsional buckling behaviour. The finite element analyses also indicate that the inelastic lateral torsional buckling modes could be different from those of elastic buckling. Subsequently, the capacities of long overhang segments can be much higher than the design code predictions. Due to all these uncertainties about the overhang end restraints and the effects of adjacent members, the use of effective length factor as 2.0 in the design code (AS4100) is well justified.
4.4.4 Portal frames with an overhang member (Series 6)

Series 1 to 5 frames consist of only one member that is susceptible to lateral torsional buckling. In general, two or more members subjected to certain load cases may undergo lateral torsional buckling in a frame structure. However, research in this area has been limited. Trahair (1993) has presented some studies on elastic buckling capacities of continuous beams. Some simplified methods for elastic buckling load calculations are given, but the studies have not been extended to include material yielding. Series 6 frames are portal frames with overhang members. The geometrical configurations of these frames are shown in Figure 4.63. Depending on the length of the overhang, the primary elastic buckling mode of the frame structure may occur either in the beam or the overhang. Figures 4.64 and 4.65 show the primary buckling shapes for these two cases.

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<td>2.5 m</td>
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<tr>
<td>3.0 m</td>
<td>F2-3.0</td>
<td>F4-3.0</td>
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Figure 4.63 Dimensions of Series 6 Frames

Figure 4.64 Elastic Buckling of the Overhang Segment
Figure 4.65 Elastic Buckling of the Beam Segment

The shell finite element analyses are able to consider the interactions between adjacent members automatically. On the other hand, it is very difficult for simple elastic methods to take member interactions into account accurately. A zero interaction method is the simplest solution and will provide a lower bound elastic buckling load. The example presented in Trahair (1993) shows that the zero interaction method gives 20% lower buckling load than the FEA for a two span continuous beam subject to an end bending moment. Trahair has summarized the zero interaction method as follows:

a) Determine the properties $EI_y$, $GJ$, $EI_w$ and $L$ of each of the $n$ segments

b) Analyse the in-plane bending moment distribution throughout the beam for an initial load set, and determine the initial maximum moment $M_{mi}$ for each segment.

c) Assume that the effective length factor $k$ of each segment is unity.

d) Estimate the maximum moment $M_m$ at elastic buckling for each segment, and corresponding buckling load factor $\lambda_n = (M_m/M_{mi})_n$.

e) Determine a lower bound estimate of the beam buckling load as the lowest value of $\lambda_n$.

Two improved methods are also provided in the same paper (Trahair, 1993). Essentially, the effective length factor of individual critical member using these methods is modified by the stiffness ratio of the adjacent members. The process is quite tedious and may involve an iterative procedure. During the simplified theoretical calculations for Series 6 frames, the effective length factors for the beam segments are
taken as 0.7 and the effective length factors for the overhanging segment are assumed to be 2.0.

The elastic buckling loads from FEA and the simplified theoretical calculations using Trahair’s method are presented in Figure 4.66. The FEA results from frames with 2 m and 4 m columns are very similar while subjected to the same loading conditions. Because there are only vertical loads applied to the frames, the interaction between column and beam segments does not have significant effects on the buckling loads. However, Figure 4.66 shows that the discrepancy between the FEA results and the hand calculations are enormous. The elastic buckling loads are up to 280% higher for finite element analyses (F4-2.5). Further, the buckling mode from the two methods could also be different. According to FEA, the lateral torsional buckling of overhang segment occurs only in the frames with 3 m overhang. First mode buckling takes place in the beam segments for all other frames. However, the theoretical calculations that assumed $k_e$ of 2.0 for overhang reveal that the beam segments are critical only for the frames with 1.5 m overhanging. The simplified methods have a significant shortcoming in estimating the elastic buckling loads of overhang segments.

The ultimate loads of the frame structures can also be derived using similar procedures as for elastic buckling loads. The effective length factors for the beam segments are taken as 0.7 and 2.0 for the overhang segments. Appropriate $\alpha_m$ factors
based on the moment distribution in the segments also used. The ultimate loads calculated based on the design code AS4100 are shown in Figure 4.67. The calculations show that the overhang segments are the critical members in all frames thus governing their capacities.

Figure 4.67 Ultimate Loads (P) for Series 6 Frames

Figure 4.68 The use of Microstran Design Software
The results from the design package Microstran are also plotted in Figure 4.67. The computer program assumes zero interaction between adjacent segments therefore the effective length factors \((k_e)\) of all the beam segments are taken as unity. The program also takes the \(k_e\) of overhang segments to be 1.0. These assumptions are not following the design specification (AS4100). As a result, the beam segments become the critical members in all the design checks. A screen shot of design check using Microstran is given in Figure 4.68.

Since both the beam and overhang segments of the frame structure may undergo lateral torsional buckling, initial geometrical imperfections for both the buckling shapes were applied to the models before the nonlinear static analyses. For Series 6 frames, the first and second buckling modes correspond to the lateral torsional buckling of beam and overhang segments. The maximum imperfection magnitudes for the beam segments are assumed to be \(L_b/1000\), where as the maximum values for the cantilever segments are \(L_c/500\). \(L_b\) and \(L_c\) are the length of beam and overhang segments respectively.

The nonlinear analyses indicate that the overhang segments are the critical members in most of the frame structures except the ones with the shortest overhang (1.5 m). A typical failure shape is shown in Figure 4.69. Both in-plane and out-of-plane deflections at the tip of the overhang segment are significantly larger than that at the midspan of the beam segment.

For frames with a 1.5 m overhang segment, the failure deformation shape is shown in Figure 4.70. In this case, since the ultimate load for both the beam and cantilever segments are reasonably close, lateral torsional buckling occurs in both segments. One interesting aspect of finite element modelling is that the failure modes from elastic buckling and nonlinear analyses could be completely different. Material yielding obviously plays an important role for inelastic buckling. Yielding patterns in various locations of the frames are very complex. There may not be any simple solutions to obtain these patterns using the simplified methods.
Compared with the design code predictions using the zero interaction method, the FEA ultimate loads are significantly higher (up to 170%). It indicates that the design code is significantly underestimating the frame capacities. However, while using the design code, flexibilities of frame connections are not considered during the structural analysis stage. The design code must also cope with the semi-rigid type frames. Therefore, the over-conservativeness may be present only when dealing with the rigid frame structures.

Load-deflection curves for two extreme cases are plotted in Figure 4.71. One frame has the shortest overhang segment while the other has the longest. Frame F215 suffers
from large plastic displacement at midspan of the beam segment. Slight unloading can also be observed at the tip of overhang which indicates that lateral torsional buckling takes place in the segment. On the other hand, the beam segment of frame F43 remains elastic. Plastic deformation occurs only in the overhang segment. The load-deflection curves demonstrate that when one of the members in the frame structures undergoes lateral torsional buckling, the frame structures reach their ultimate capacities.

![Figure 4.71 Load – deflection Curves for Frame F215 and F43](image)

In summary, Series 6 frames have significantly higher elastic buckling and ultimate loads than the design code (AS4100) predictions. The current design procedure is not able to predict either the elastic buckling or the ultimate failure modes accurately. Also, the finite element analyses demonstrate that the elastic buckling and ultimate failure modes are not necessarily the same. This implies that the current frame design method has a considerable deficiency.

### 4.4.5 Two bay single storey frames (Series 7)

Series 7 frames are single storey two bay structures subjected to eight (8) different load cases. The dimensions of this frame are shown in Figure 4.72. In the previous
frame series (Series 6), the overhang portions of the structures often govern the load capacities due to their lack of moment redistribution abilities. For this series, the two beam segments in the structures have the same restraint conditions, and both of them have three degrees of redundancy. If the frame is fully laterally restrained, it is able to form plastic hinges one after the other until the structure reaches a collapse mechanism. In lateral torsional buckling cases, an important question for redundant frame structures is: “should the structure be allowed to carry more loads after one of its segments exceed its member capacity?”

![Figure 4.72 Dimension of Series 7 Frames](image)

Figure 4.73 illustrates the load configurations and designations for Series 7 frames. The first three load cases are vertical load cases, and the axial compression forces are not very significant in the beam segments. The horizontal loads are presented in other load cases. The magnitudes of the horizontal forces are four times the size of the larger vertical load. Because of the variations in the bending moment diagrams for these load cases, the moment modification factors ($\alpha_m$) of beam segments range from 1.66 to 2.50.

Results from the finite element elastic buckling analysis and the simple elastic buckling load calculations are plotted in Figure 4.74. In the hand calculations, the effective length factors of the beam segments are assumed to be 0.7 and 1.0 to account for the beam end warping and out-of-plane rotation restraints. The assumption of 1.0 significantly under-predicted the elastic buckling load especially for the load cases with smaller axial compression force (1g1-1, 1g1-1, and 1g1-2). When the effective length factor is taken as 0.7, the hand calculated loads are larger.
than FEA results. The hand calculations have not considered the effects of axial compression forces. If the internal axial forces are taken into account, the agreement with FEA results is likely to improve.

Figure 4.73 Load Cases of Series 7 Frames

Figure 4.74 Elastic Buckling Loads (p) for Series 7 Frames

Some first-mode elastic buckling shapes are shown in Figures 4.75 and 4.76. In general, lateral torsional buckling deformations are present only in one beam segment (see Figure 4.76). The out-of-plane displacements are much small in the other segment. The deformation shown in Figure 4.75 is a special case where both applied
vertical loads are equal. Therefore, the out-of-plane deflections of both beam segments are equal and in opposite directions.

Figure 4.75 Primary Buckling Shape of Frame 1g1-1

Figure 4.76 Primary Buckling Shape of Frame 2g1-0

In the nonlinear static analyses, the first buckling mode deflection shapes are used to introduce the initial imperfections. The maximum value of geometrical imperfections is assumed to be $L/1000$, where $L$ is the length of the beam segment. The results from the nonlinear analyses are shown in Figure 4.77. For comparison, the results from the hand calculations based on beam lateral torsional buckling capacities (design code AS 4100) are also plotted in the same graph. According to this graph, the design code predictions are very conservative if the effective length factors ($k_e$) are assumed to be one. If $k_e$ is assumed to be 0.7 in the hand calculations, the ultimate loads for the
horizontal load cases exceed the FEA results for load cases 2g1-1, 2g0-1, 2g1-2, 2g2-1 and 2g10. The axial compression loads in the failed beam are significantly higher than the first three load cases. The combined actions of bending and compression load lead to a lower FEA results. Nevertheless, the FEA demonstrates that the frame capacities are directly related to the out-of-plane buckling of one beam segment.

![Figure 4.77 Ultimate Loads (p) of Series 7 Frames](image1)

![Figure 4.78 Deformation of Frame 1g1-1 at the Ultimate Load](image2)
Figure 4.78 illustrates the deflections and longitudinal stress contours of Frame 1g1-1. It is interesting that only one beam segment is undergoing lateral torsional buckling even though both beam segments are subjected to the same vertical loads. Similar behaviour can be observed in Frame 2g1-0 (see Figure 4.79). In this case, one beam segment is subjected to the worst in-plane bending moment, hence this member will fail first in lateral torsional buckling mode. Nevertheless, the failure can be found only in one beam segment at the ultimate state as if a local collapse mechanism has developed.

![Figure 4.79 Deformation of Frame 2g1-0 at the Ultimate Load](image)

The in-plane bending moment diagrams of Frames 1g1-1 and 2g1-0 are quite different, but the frame deformations and even the stress contours of the failed beams are very similar. A plastic hinge is formed at the midspan of the beam segment. Also significant yielding can be observed at both ends of the beam resembling the formation of three hinges in the failed segment.

Selected cases of load-deflection curves for the beam segment are shown in Figure 4.80. They are for Frames 1g1-1 and 2g2-1. For each load case, the midspan deflections of left (a) and right (b) beam segments are presented. These load-deflection graphs reveal some interesting behaviour: 1) All the failed beam segments are undergoing much more significant plastic deformations. 2) The unloaded beam in 2g2-1 is hogging up and remains elastic while the structure reaches its ultimate state.
3) Strictly speaking, the finite element frame model is not symmetrical with the columns’ out-of-plumbness modelled. This subtle difference causes the behaviour variations in the left and right beams for load case 1g1-1. As a result, only one beam undergoes lateral torsional buckling at the ultimate state.

In summary, Series 7 frames indicate that only one beam segment may fail in a steel frame structure. The failure of the beam segment forms a local collapse mechanism. In general, the ultimate loads of a rigid frame are much higher than those predicted by the current design code (AS 4100).

4.4.6 Single bay two storey frame (Series 8)

Series 8 frames are single bay two storey fixed-base frames subjected to eight different load cases. The bay width is 8 meters, and the height of each storey is 4 meters. The various load cases are shown in Figure 4.81. The first three load cases (1h1-1, 1h1-0, and 1h0-1) do not include horizontal loads. Out-of-plane restraints are applied only to the beam column connections. For the other five load cases, equal vertical loads are applied to both top and bottom beams. A horizontal force is applied to either the first or second storey.
The finite element elastic buckling analysis results are shown in Figure 4.82. The buckling loads calculated using simple elastic theory are also plotted in the same graph. The hand calculations assume the effective length factor to be 0.7, and the results are slightly higher than those from finite element analyses (within 10%).

![Figure 4.81 Load Cases of Series 8 Frames](image)

![Figure 4.82 Elastic Buckling Loads (vertical load p) for Series 8 Frames](image)

The elastic buckling modes of the structure are shown in Figures 4.83 and 4.84. For the load cases with only the vertical loads, lateral torsional buckling may occur either in the top or bottom beams. When the applied loads are the same for both beams, the buckling occurs in the top beam. For the load cases including horizontal loads, only
the top beam will be subjected to lateral torsional buckling effects because of the additional lateral restraints at the bottom beam.

![Figure 4.83 Elastic Buckling Modes of the Structure subject to Vertical Loads Only](image)

Figure 4.83 Elastic Buckling Modes of the Structure subject to Vertical Loads Only

![Figure 4.84 Elastic Buckling Mode of the Structure subject to Horizontal and Vertical Loads](image)

Figure 4.84 Elastic Buckling Mode of the Structure subject to Horizontal and Vertical Loads

Nonlinear static analysis results are plotted in Figure 4.85. They are generally higher than the AS4100 predictions assuming that one of the members undergoes lateral torsional buckling (up to 39%). For load cases 2h2, 2h3, 3h1, and 3h2, the first plastic hinge may form before the beam segment reaches its member capacity. The loads that cause the first plastic hinge are also plotted in the same chart.
The deformation shape and longitudinal stress contour of frames 1h1-1 and 1h0-1 are shown in Figure 4.86. Lateral torsional buckling occurs in either the top or bottom beam segments at the ultimate load. The frame structures can not carry further loads once they form a local collapse mechanism. Prior to the beam segment failure, there are not any significant yielding anywhere in the structures.

On the other hand, when the structure is subjected to horizontal loading, the frame may behaviour quite differently. The stress contours for Frame 2h1, 2h2, and 2h3 are shown in Figure 4.87. The vertical loading conditions are the same in these three load cases. The horizontal loads are one, one and half and two times that of the vertical load. Frame 2h1 behaves similar to that of vertical only load cases. No plastic hinge
formed after the buckling of beam segment. On the other hand, there is no significant lateral torsional buckling shown in Frame 2h3. The structure forms a plastic hinge collapse mechanism at the ultimate load. Frame 2h2 is an intermediate case, for which the plastic hinges are formed before the lateral torsional buckling in the top beam. It is interesting that yielding occurs only at midspan and one end of the beam segment for Frame 2h2. Significant yielding can be found in five locations of the structure. Frame 3h1 behaves similar to Frame 2h1, and Frame 3h2 is similar to Frame 2h3. The frame deformations for Frames 3h1 and 3h2 at the ultimate load are shown in Figure 4.88.

![Figure 4.87 Deformations of Frames 2h1, 2h2, and 2h3 at the Ultimate Load](image1)

![Figure 4.88 Deformations of Frames 3h1 and 3h2 at the Ultimate Load](image2)
Figure 4.89 Vertical Load – Midspan Deflection Curves for the Beams in Series 8 Frames

Figure 4.90 Horizontal Load - Deflection Curves for Series 8 Frames
Typical load-deflection curves for Series 8 frame are plotted in Figures 4.89 and 4.90. Curves (1), (2) and (3) shown in Figure 4.89 are the midspan deflection curves for the top beam segments. Large plastic deformations are presented with Curves 1 and 2. Both structures fail due to lateral torsional buckling of the top beam. On the other hand, there is minimum plastic deformations taking place in Curve (3), and the frame is experiencing significant unloading once it reaches the maximum capacity. The plastic deformations occur elsewhere in the structure.

The three horizontal load – deflection curves shown in Figure 4.90 illustrate the effects of plastic hinge formation. Curve 2h1 shows limited plastic deformations and indicates that plastic hinges did not form in the structure. Both Curves 2h2 and 2h3 have large plastic deformations. There is no significant unloading behaviour in Curve 2h2. This implies that plastic hinge formation may not be the only cause for the structural failure. Conversely, the large unloading portion of Curve 2h3 suggests that a hinge collapse mechanism is achieved. Frame 2h3 does not fail due to lateral torsional buckling.

In summary, Series 8 frames consist of two unconnected beam segments. The interaction of these two segments is not clearly evident while one of them undergoes lateral torsional buckling. The finite element analyses of Series 8 frames indicate that once a member undergoes lateral torsional buckling the structure may not carry further loads nor form additional plastic hinges.

4.4.7 Single bay gable frames (Series 9)

Single bay gable frames are commonly used in commercial and industrial buildings. Usually, purlins are placed on the top flanges of the frames to support roof sheeting. The uplift wind load cases often produce lateral torsional buckling failure of beams when insufficient fly braces are used. Series 9 frames are idealized frames with lateral restraints only applied to the knees and apex of the frame structures. The columns are 4 m high and the bay widths are 16 m. The apex is 1 m higher than the knee. The six applied load cases for Series 9 frames are shown in Figure 4.91. The first three frames are subjected to downward load cases, and the other three frames are subjected to uplift loads.
The results from finite element elastic buckling analyses are shown in Figure 4.92. The buckling loads calculated from simple elastic theory are also plotted in the same graph. The hand calculations assume the effective length factor to be 0.7. The simplified method yields nearly the same results as the finite element analyses for downward load cases. Their differences are within 7%. However, the hand calculations are significantly lower for the uplift load cases (up to 51%). In the simplified method, the elastic buckling loads of beams are the same irrespective of the direction of the vertical loads (upward or downward). It means that the bending moment gradients for the two cases are similar. One aspect that the hand calculations did not considered is the effect of axial loads. The beams are in compression when the structures are subjected to downward loads and in tension when subjected to uplift loads. It seems that the tensile forces in the beams are able to increase the elastic buckling loads for portal frames significantly.
The elastic buckling modes of Series 9 frames are shown in Figure 4.93. When the structures are subjected to only a pair of vertical loads, lateral torsional buckling occurs in both beam segments (see I1 and I4 in Figure 4.93). On the other hand, when horizontal loads are applied, only one segment buckles in its primary mode due to the unsymmetrical bending moment distributions. The unusual aspect of the buckling shapes for these load cases (I2, and I5) is that the first buckling modes do not occur in the other segment that has the maximum bending moment based on linear elastic analyses.

The nonlinear static analysis results are plotted in Figure 4.94 together with the ultimate loads calculated using the design code AS4100 (SA, 1998). The ultimate loads derived from the finite element analyses are 23% to 53% higher than the design code predictions. The effective length factors are assumed to be 0.7 in the code method. The ultimate load graph shows that the differences between the downward and uplift loads are not very significant. The ultimate loads from finite element analyses exhibit a similar pattern as predicted by the design code.
The longitudinal stress contour and deformations at the ultimate load are shown in Figure 4.95. From observations, there are significant differences in the frame displacements from one load case to another. Also, the locations of plastic yielding seem difficult to predict. In some cases, plastic hinges are clearly formed. In others, there is extensive yielding along the beam segments. The common pattern is that only one beam segment displays the lateral torsional buckling characteristic at the ultimate failure state for all the load cases.
Figure 4.96 Horizontal Load-sway Curves for Series 9 Frames

Figure 4.97 Vertical Load – Deflection Curves for Series 9 Frames
Figures 4.96 and 4.97 show the load – deflection curves obtained from the finite element analyses. In Figure 4.96, the knee drift characteristics for the same horizontal load are significantly different when the structures are subjected to upward or downward load. Compared with curves I2 and I5 (or I3 and I6), the second order effects of vertical forces are very substantial. Figure 4.97 captures the midspan deflections of left or right beam segments. The curves I1 left and I1 right are nearly identical until they approach the unloading state. The results show that even with the same vertical loads applied to two identical beam segments in a symmetry frame, only one beam segment is undergoing full lateral torsional buckling. Simultaneous lateral torsional buckling does not seem to happen in a frame structure with proportional loading. The beam midspan deflection graphs also illustrate that there are subtle differences for downward and upward loads. In downward load cases (I3 left and I3 right), the lateral torsional buckling of left beam segment delays the deflection in the right beam segment. For uplift cases, the left and right beam segments have similar plastic deformation characters.

In summary, the behaviour of portal gable frame structures is more difficult to predict compared with frames with only orthogonal members. The elastic buckling loads for uplift load cases are significantly higher than those from the current design code method. On the other hand, this behaviour is not shown by nonlinear analyses. The finite element analyses of gable frame also indicated that only one member undergoes lateral torsional buckling and leads to the failure of the structures. When defining a failure criterion for frame structures, the lateral torsional buckling of one beam segment in the frame can be treated as another mechanism similar to the hinge forming collapse mechanism.

4.5 Summary

The objective of this research described in this chapter was to develop a comprehensive set of analytical benchmark solutions for steel frame structures subjected to lateral torsional buckling effects. The shell finite element models described in this chapter were developed for this purpose. The current commercial finite element programs (such as ABAQSU/implicit) are capable of dealing with
nonlinear static analysis with basic steel material properties. With the use of initial geometrical imperfection as a triggering mechanism, these shell finite element models can effectively capture the nonlinear behaviour of lateral torsional buckling failure mode, and are also able to explicitly account for the effects of gradual cross-sectional yielding, longitudinal spread of plasticity, and even residual stresses. Compared with the use of fiber element, shell elements are more suitable for developing benchmark solutions of steel frame structures subject to lateral torsional buckling.

A frame structure consists of more than one member and connection and may be subject to many possibilities of load combinations. Simple relationships between various parameters of simply supported beam and their load capacities can be derived using finite element models, but it is impossible for full scale structural frame systems. Therefore, research into the behaviour of frame systems can only be based on case by case studies. In this chapter, the behaviour of nine series of frame structures was investigated in detail. The investigations include all the structural cases that have been studied in the previous research of fully laterally restrained frames (Vogel, 1985; Ziemian, 1993; Avery, 1998).

Two types of finite element analyses were conducted for the nine series of frame models: they are elastic bifurcation analysis and nonlinear static analysis. Comparisons were made between the results from FEA and a simplified method based on linear elastic analysis and backward deductions according to individual member elastic or plastic capacities. In general, the elastic buckling loads agree reasonably well. However, the ultimate frame capacities from the simple method may be significantly lower than those from finite element analyses.

For each series of frame models, elastic buckling and ultimate failure deformation shapes are also presented. It appears that there are many subtle differences between the idealised elastic buckling and the real modes. With elastic buckling analysis, the buckling segment appears to be a half wave sine curve. On the other hand, the inelastic buckling deformations are much more irregular.

Typical load-deflection curves are provided for each frame series. With the exception of frames with overhang members, the results demonstrate that rigid frames are
reasonably ductile even though their members fail due to lateral torsional buckling. This behaviour seems to contradict the lateral torsional buckling of simply supported beams. Detailed studies on the effects of beam-column connections were also conducted for Series 1, 2 and 5 frames. They show that the connection rigidities play a very important role in reaching the frame capacities. The details of the beam-column connections have a significant influence on the beam buckling behaviour. To obtain accurate ultimate frame capacities due to lateral torsional buckling failure, it is not possible to avoid modelling the connection details for more general type frame structure (as opposed to rigid frame structure). However, the amount of details that needs to be attended explicitly is far exceeding what is allowed for in normal design circumstances. Engineering intuitions / judgements are needed for dealing with the connection problems inevitably. Due to all these uncertainties, lateral torsional buckling failures should be treated as an unfavourable failure mode even for rigid frame structures.

In the current design code, the effective length factor concept is introduced to deal with the various end boundary conditions of beam segments. The adoption of this concept has many drawbacks. It is an empirical method, and issues such as interaction between adjacent members and details of beam segment end restraints are difficult to quantify while using this design concept. The finite element analyses demonstrate that the current design approach yields substantially lower frame capacities. This is mainly due to the separation of analysis and capacity check process in the design approach. However, it is assuring to note that the effective length concept is a conservative method. Compared with detailed connection modelling and using effective length factor with good engineering judgements, the latter is easier to achieve. The simplicity of the use of this concept may offset its drawbacks while alternative tedious modelling may require the use of FEA method.

The hand calculations of beam segment capacities are only partially related to frame analysis (bending moment diagrams are required for $\alpha_m$). The capacities of steel frame structure are determined implicitly when using the design code method. On the other hand, advanced analyses are capable of establishing the actual failure behaviour of the steel frame structures directly. But comparing the results from design code
predictions and finite element analyses, the use of moment modification factor $\alpha_m$ seems to be able to capture the general trend of frame capacities when it is related to moment variation along the structural members.

The shell finite element analyses are able to capture the hinge by hinge forming process of the frame structures. The results show that it is possible that hinges are formed before the members eventually failed in lateral torsional buckling. The analyses also indicate that, of all the frames studied, there is no formation of plastic hinges or additional member buckling after lateral torsional buckling occurred in a structure. This means that the ultimate capacity of a frame structure depends on the lateral torsional buckling of single members. The lateral torsional buckling failure should be avoided in the design of steel frame structures, and from a structural failure criteria point of view, no further formation of plastic hinges should be allowed after one structural member fully suffers from lateral torsional buckling.
Chapter 5. Development of a New Advanced Analysis Method for Frame Structures Subjected to Lateral Torsional Buckling Effects

Ultimate strength requirement is one of the most important aspects of structural design. The structure and its component members and connections are designed for the strength limit state so that the capacities of all members and connections are greater than the design actions. In the current design approach, the member design actions are obtained from elastic analysis of the structural systems whereas the member capacities are determined by inelastic analysis. The main disadvantage of this design method is that it does not give an indication of the nonlinear structural behaviour.

Advanced analysis methods are an alternative direct design process. These methods are aimed at capturing the behavioural effects associated with member primary limit states, such that the corresponding specification member capacity checks are superseded. The analyses are capable of taking into account all aspects influencing the behaviour of the frame, which may include: material inelasticity, residual stresses, geometric imperfections, second-order (instability) effects, connection response, local buckling and global out-of-plane buckling. Using these methods, a comprehensive assessment of the actual failure modes and maximum strength of overall framing system is possible. Since the structure capacity is directly assessed within the analysis, separate member and section capacity checks are not required. It should simplify the design process considerably while achieving a higher degree of accuracy.

The distributed plasticity analysis method used in Chapters 3 and 4 is an advanced analysis method. Since most of the factors affecting the structural capacity are explicitly modelled, such analyses are capable of capturing the inelastic behaviour including the out-of-plane buckling effects. However, this type of analysis is not practical for general design use due to the additional computational resources required. A primary research objective was therefore to develop a practical advanced analysis method that adequately captures the inelastic behaviour of frame structures subjected to out-of-plane buckling effects.
Advanced analysis methods are based on distributed plasticity formulation or the simplified concentrated plasticity formulation. Refined plastic hinge analysis is one of the common concentrated plasticity methods. This method uses stability functions for second-order effects. The non-linear material behaviour is accounted for by using tangent modulus and flexural stiffness reduction concepts. Because of its simplicity, the refined plastic hinge analysis is becoming the popular choice for the next generation design methods. Recent research has extended this method to include local buckling (Avery, 1998) and 3-D frames effect (Liew, 1998; Kim, 2001). In spite of this, in its current state, the refined plastic hinge method is still not able to cope with lateral torsional buckling effects. These methods yield little advantage over the design of many structures using open sections (eg. I sections) since separate out-of-plane member capacity checks are still required. Because of this, an advanced analysis approach is yet to be used for practical design purposes.

Out-of-plane buckling includes three important modes; beam lateral torsional buckling, column flexural buckling and distortional buckling. The main focus of this research project is on the beam lateral-torsional buckling. There are many factors in the structure affecting the lateral-torsional buckling failure. They include: geometrical configuration of the members, connection details, load conditions, moment gradient along the members and material properties. This failure mode is extremely complex even for isolated members, and closed-form elastic buckling solutions exist only for a limited number of load cases. It is impossible to solve all the differential equations of equilibrium for different load cases and derive a beam-column element by the use of stability functions. Currently, the member capacities of this failure type are obtained from curve fitting of experimental results. Since the direct modelling of lateral-torsional buckling involves too many unknown parameters and many of them are difficult to account for, standard frame analysis software is incapable of modelling these types of instabilities, and it is primarily for this reason that SSRC Technical Memorandum No.5 recommends that separate checks are made for frame and member instabilities (Galambos, 1998). It seems that an implicit approach has to be taken in dealing with this failure mechanism, otherwise lateral torsional buckling may never be incorporated into the refined plastic hinge method.
The advanced analysis method proposed in this chapter is based on the refined plastic hinge approach. Structural member out-of-plane buckling checks (column buckling and lateral-torsional buckling) are incorporated in the analysis while still retaining the original capabilities. The basic principles of the refined plastic hinge method are first reviewed followed by a discussion of out-of-plane buckling characteristics of steel frame members. The formulation of out-of-plane buckling capacity check is based on the current Australian design standard - AS4100 (SA, 1998). The implementations using C++ programming language are presented in Section 5.4. The comparison between the new method and the finite element analyses from Chapters 3 and 4 are presented in Section 5.5. Section 5.6 discusses the graphic interface issues related to the new practical advanced analysis method.

5.1 Refined plastic hinge method

The refined plastic hinge method was developed based on second-order elastic plastic hinge analysis. Compared with the current design method using non-linear elastic analysis, the second-order elastic plastic hinge method uses stability functions to account for the geometric non-linearity effects, and allows the formation of perfect plastic hinges and associated plastic redistribution. The advantage of this approach is that it captures the most common frame structural failure mode, the formation of a collapse mechanism. The shortcoming is that the simplified assumption associated with the second-order elastic plastic hinge method often over-predicts the actual inelastic stiffness and strength of the structures.

The main improvement of the refined plastic hinge method is that it solves the problems associated with partial yielding and distributed plasticity of the structural members. Two novel concepts are introduced in the method: the tangent modulus ($E_t$) and the flexural stiffness reduction factor ($\phi$). The tangent modulus function represents the distributed plasticity along the length of the member due to axial force effects and the flexural stiffness reduction factor simulates the distributed plasticity effects associated with flexure. For steel frame structures that are fully laterally restrained, this method achieves a very high degree of accuracy compared with plastic zone analysis, or the so called “exact solution”. There are two types of numerical
implementation; simple incremental and the incremental-iteration methods. Since the first one easily achieves convergence and is more straight-forward in accounting for formation of hinges, it becomes the more accepted approach even though it requires more steps. The three main components associated with the refined plastic hinge method are described as follows, they are: element force-displacement relationship, tangent modulus and flexural stiffness reduction factor.

5.1.1 Frame element force-displacement relationship

Structural analysis is a process of determining the unknown forces and displacements. Using the equations of equilibrium and laws of compatibility, a force-displacement relationship of a structural element can be developed.

![Figure 5.1 Beam Column Element](image)

The force-displacement relationship for a frame structural element subject to static loading is of the form:

\[ f = kd + f' \] (5.1)

It is convenient to derive the force-displacement relationship of a beam-column element using a local co-ordinate system (see Figure 5.1). The vector \( \mathbf{d} \) contains the displacements corresponding to the independent degrees of freedom of the element. A typical two dimensional beam-column element has three independent degrees of freedom in the local co-rotational co-ordinate system, rotation at end A (\( \phi_A \)), rotation at end B (\( \phi_B \)), and axial displacement (u). The local displacement vector (\( \mathbf{d}_l \)) is therefore given by:

\[ \mathbf{d}_l = \begin{bmatrix} \theta_A \\ \theta_B \\ u \end{bmatrix} \] (5.2)

The vector \( \mathbf{f} \) represents the element forces corresponding to the degrees of freedom of the element. The element force vector of a typical beam-column element comprises
the bending moment at end A ($M_A$), the bending moment at end B ($M_B$), and the axial force ($P$). The local element force vector ($f_i$) is therefore given by:

$$f_i = \begin{bmatrix} M_A \\ M_B \\ P \end{bmatrix}$$  

(5.3)

The matrix $k$ is known as the stiffness matrix. The $kd$ term represents the forces induced by the displacements corresponding to the element degrees of freedom. The element stiffness matrix will always be square. Korn and Galambos (1967) showed that the effects of curvature shortening are negligible in a second-order inelastic analysis. The flexural and axial force-displacement relationships are therefore independent, hence the zero terms in the stiffness matrix. The stiffness matrix of a beam-column element in the local co-rotational co-ordinate system ($k_l$) will always be of the form:

$$k_l = \begin{bmatrix} k_{11} & k_{12} & 0 \\ k_{21} & k_{22} & 0 \\ 0 & 0 & k_{33} \end{bmatrix}$$  

(5.4)

The vector $f'$ represents the element forces that are not due to the displacements corresponding to the element degrees of freedom. This vector may contain fixed-end forces ($f_f$) required to balance the distributed loads acting on the element and the concentrated loads not acting at the element nodes, or pseudo-forces ($f_p$) occurring as a result of plastic hinge formation.

For computational purposes, it is necessary to express the force-displacement relationship in an incremental form. In a local co-ordinate system, the incremental force-displacement relationship for an element can be written symbolically as:

$$\dot{f}_e = k_e \dot{d}_e$$  

(5.5)

where $\dot{f}_e$ and $\dot{d}_e$ are the incremental element end forces and displacements, respectively, and $k_e = \frac{\partial f_e}{\partial d_e}$ is the element basic tangent stiffness matrix after omitting the higher order terms. In order to solve the unknown force and displacement of the structural system, the element force-displacement relationship must be converted into a global Cartesian co-ordinate system. Considering rigid-body motion of the member, element displacements in the local and global systems have the following relationship:
\[ \theta_A = \theta_0 + d_{g3} + \tan^{-1} \left( \frac{y_0 + d_{g5} - d_{g2}}{x_0 + d_{g4} - d_{g1}} \right) \]  
(5.7)

\[ \theta_B = \theta_0 + d_{g6} + \tan^{-1} \left( \frac{y_0 + d_{g5} - d_{g2}}{x_0 + d_{g4} - d_{g1}} \right) \]  
(5.8)

\[ u = L_f - L_0 = \frac{(2x_0 + d_{g4} - d_{g1})(d_{g4} - d_{g1}) + (2y_0 + d_{g5} - d_{g2})(d_{g5} - d_{g2})}{L_f + L_0} \]  
(5.6)

where \( d_{g1}, d_{g2}, \ldots \) and \( d_{g3} \) are the global translational and rotational degrees of freedom.

By differentiation of these equations with respect to pseudo-time, the incremental kinematics relationship relating to two sets of displacement vectors may be written as:

\[ \dot{d}_c = \frac{\partial d_c}{\partial d_g} \dot{d}_g = T_g \dot{d}_g \]  
(5.9)

where:

\[ \dot{d}_c = \begin{bmatrix} \dot{\theta}_A & \dot{\theta}_B & \dot{\theta}_T \end{bmatrix} \]

\[ \dot{d}_g = \begin{bmatrix} \dot{d}_{g1} & \dot{d}_{g2} & \dot{d}_{g3} & \dot{d}_{g4} & \dot{d}_{g5} & \dot{d}_{g6} \end{bmatrix} \]

\[ T_g = \begin{bmatrix} -s/L_0 & c/L_0 & 1 & s/L_0 & -c/L_0 & 0 \\ -s/L_0 & c/L_0 & 0 & s/L_0 & c/L_0 & 1 \\ -c & -s & 0 & c & s & 0 \end{bmatrix} \]

in which \( c = \cos\phi \), \( s = \sin\phi \), and \( \phi \) is the angle of inclination of the chord of the deformed member.

Similarly, element forces in the two co-ordinate systems are related by:

\[ f_g = T_g^T f_c \]  
(5.10)

Taking derivatives on both sides gives

\[ \dot{f}_g = T_g^T \dot{f}_c + \ddot{T}_g^T f_c. \]  
(5.11)

Therefore, incremental element force-displacement relationship expressed with respect to the global coordinate system can be derived.

\[ \dot{f}_g = k_g \dot{d}_g + f_{sp} \]  
(5.12)

where:

\[ k_g = T_g^T k_c T_g + M_A T_1 + M_B T_2 + P T_3 \]
\( \mathbf{f}_{gp} \) is the pseudo-force vector.

\[
T_1 = T_2 = \frac{1}{L_0} \begin{bmatrix}
    -2sc & c^2 - s^2 & 0 & 2sc & s^2 - c^2 & 0 \\
    2cs & 0 & s^2 - c^2 & -2sc & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 \\
    -2sc & c^2 - s^2 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]  \hspace{1cm} (5.13)

\[
T_3 = \frac{1}{L_0} \begin{bmatrix}
    s^2 - sc & 0 - s^2 & sc & 0 \\
    c^2 & 0 & sc & -c^2 & 0 \\
    0 & 0 & 0 & 0 & 0 \\
    s^2 - sc & 0 & sc & -c^2 & 0 \\
    0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]  \hspace{1cm} (5.14)

For non-linear analysis, the stiffness matrix is a function of the total forces and/or displacements. The forces must therefore be applied incrementally, with the appropriate tangent stiffness matrix calculated for each increment.

5.1.2 Tangent modulus

In an elastic analysis method, structural stiffness is assumed to be unchanged when the structure approaches its ultimate failure state. This is one of the main shortcomings of the current design procedure. With the refined plastic hinge method, the elastic modulus is replaced by a tangent modulus \( (E_t) \) to represent the distributed plasticity along the length of the member due to member yielding caused by axial force. The inelastic member stiffness is represented by axial rigidity \( (E_tA) \) and bending rigidity \( (E_tI) \). The value of tangent modulus is a function of the member’s axial loading state, and is often evaluated from column member capacity specification equations. The original refined plastic hinge formulation (Liew, 1992) gave a choice of two tangent moduli functions derived from the CRC and AISC LRFD column curves. Since the column curves in the design codes around the world are not identical, the tangent moduli they derive will be different. The tangent modulus for hot-rolled I sections based on the Australian steel design standard AS4100 can be
calculated using the following procedure (Avery, 1998):

1. Obtain the “equivalent stiffness” member slenderness reduction factor \( (\alpha'_c) \) for a given non-dimensional axial force \( (p) \). \( k_f \) is the form factor for members subject to axial compression. By including this factor, the tangent modulus includes the stiffness reduction due to local buckling of non-compact sections.

\[
\alpha'_c = \frac{p}{k_f}
\]  

(5.15)

2. Calculate the corresponding slenderness ratio \( (\lambda') \).

\[
\lambda' = 90 \left( \frac{0.1467\sqrt{\alpha'_c} - \sqrt{-1.934\alpha'_c + 0.956\alpha'_c^2 + 1}}{(\alpha'_c - 1)\sqrt{\alpha'_c}} \right)
\]  

(5.16)

3. Calculate the non-dimensional Euler buckling load \( (p'_e) \).

\[
p'_e = \frac{k_f \pi^2 E}{250\lambda'_c^2}
\]  

(5.17)

4. Determine the tangent modulus for the given non-dimensional axial force.

\[
e_t = \frac{p}{p'_e}
\]  

(5.18)

5. Check the limiting values of the tangent modulus.

\[
e_t = 0 \quad \text{for } \alpha'_c = 1, \lambda' < 13.5
\]  

(5.19)

Tangent modulus is based on empirical specification equations. A comparison of tangent modulus from various design specifications such as CRC, AISC LRFD, and AS4100 for compact I-sections indicates that the AS4100 prediction is the most conservative (Avery, 1998).

5.1.3 Second-order effects and flexural stiffness reduction factor

To represent the second-order effects and gradual transition for the formation of a plastic hinge at each end of an initially elastic beam-column element, Liew (1992) described the refined plastic hinge element incremental force-displacement relationship using the following equation:
\[
\begin{bmatrix}
\dot{M}_A \\
\dot{M}_B \\
\dot{p}
\end{bmatrix} = \frac{EIL}{L} \begin{bmatrix}
\phi_A s_1 - \frac{s_2^2}{s_j} (1 - \phi_B) & \phi_A \phi_B s_2 & 0 \\
\phi_B s_1 - \frac{s_2^2}{s_j} (1 - \phi_A) & 0 & 0 \\
0 & 0 & A/I
\end{bmatrix} \begin{bmatrix}
\dot{\theta}_A \\
\dot{\theta}_B \\
\dot{u}
\end{bmatrix} + \dot{f}_{ip} \tag{5.20}
\]

\(s_1\), and \(s_2\) are stability functions based on the closed form solution of beam-column element differential equations. The stability functions are used to account for both member chord rotation \((P-\Delta)\) and member curvature \((P-\delta)\) second-order effects (moment magnification effects). Since the exact solution inherits a numerical problem when axial force equals to zero, a simplified version is often used for these stability functions:

\[
s_1 = 4 + \frac{2\pi^2 \rho}{15} - \frac{(0.01 \rho + 0.543) \rho^2}{4 + \rho} + \frac{(0.004 \rho + 0.285) \rho^2}{8.183 + \rho} \tag{5.22}
\]

\[
s_2 = 2 - \frac{\pi^2 \rho}{30} + \frac{(0.01 \rho + 0.543) \rho^2}{4 + \rho} - \frac{(0.004 \rho + 0.285) \rho^2}{8.183 + \rho} \tag{5.21}
\]

where: \(\rho = \frac{P}{P_e} = \frac{PL^2}{\pi^2 EI}\)

\(\phi_A\) and \(\phi_B\) are the flexural stiffness reduction factors. These factors describe the member stiffness gradual degradation associated with the formation of plastic hinges at one of the two ends. The member stiffness gradually degrades according to a prescribed function after the element end forces exceed a predefined initial yield function from the elastic stiffness to the stiffness associated with the formation of plastic hinges at one or both ends. For fully laterally restrained frames, flexural stiffness reduction factor is a function of the section capacity surface. The section capacity surfaces are also based on empirical equations and hence depend on the design standard used.

\(\dot{f}_{ip}\) is the incremental pseudo-force vector. This factor accounts for the change in moment corresponding to a change in axial force at plastic hinge locations. It is given by Liew (1992) as:
\[
\hat{\mathbf{f}}_{|A} = \begin{cases} 
\frac{\dot{M}_{scA}}{s_j - (1 - \phi_A) s_j^2 / s_j} & \text{when } \phi_A = 0 \text{ and } \phi_B > 0 \\
0 & \text{when } \phi_B = 0 \text{ and } \phi_A > 0 
\end{cases} \quad (5.23)
\]

\[
\hat{\mathbf{f}}_{|B} = \begin{cases} 
\frac{\phi_A s_j M_{scB}}{s_j - (1 - \phi_A) s_j^2 / s_j} & \text{when } \phi_B = 0 \text{ and } \phi_A > 0 \\
M_{scB} & \text{when } \phi_A = \phi_B = 1 \\
0 & \text{when } \phi_A = \phi_B = 1 
\end{cases} \quad (5.25)
\]

where \( M_{sc} \) is the change on plastic surface at element end A as axial force changes.

In its current state, the refined plastic hinge method is straight-forward in principle and implementation. It is able to deal with structural failures due to column inelastic buckling and formation of plastic hinges (or reaching section capacity surface) while considering second-order effects. However, structures built with open sections (eg. I-sections) are often subject to lateral torsional buckling failures. The refined plastic hinge method is not able to handle this type of failure mode. This is the main reason why advanced analysis has not been widely used as a practical design tool.

### 5.2 Characteristics of out-of-plane buckling

Structural members such as hot-rolled I-sections often have a high ratio of major/minor axis bending capacity. Even if the members are subjected to only a major axis bending moment and an axial load, they often fail in out-of-plane buckling mode when they do not have sufficient lateral restraints. Out-of-plane buckling includes
three failure modes: column buckling, beam buckling, and distortional buckling. Distortional buckling and column out-of-plane buckling are beyond the scope of this research project. This study concentrates on solving the beam lateral torsional buckling problem using a two dimensional refined plastic hinge method. Therefore the characteristics of lateral torsional buckling are the main focus of this research. When lateral torsional buckling occurs, twisting and lateral deformations are coupled. This type of problem is more complex than flexural buckling of column. Generally speaking, the strength of beams subject to lateral-torsional buckling relates to five major factors:

1) geometry (cross-section and member length),
2) loading conditions (eg. moment gradient, load height),
3) end restraint conditions,
4) material properties,
5) initial geometrical imperfections and residual stresses.

Among these factors, the loading conditions are unique to beam lateral torsional buckling compared with column flexural buckling. From the elastic analysis point of view, the differential equations that represent deformation and force relationship are significantly different if the moment gradient changed in the lateral torsional buckling case. The closed-form elastic solutions for this failure mode are available only in limited load cases. It is therefore not possible to derive stability functions to capture the elastic buckling behaviour as in the column buckling circumstances. The shortfall of elastic theory has significant effects on the understanding of lateral torsional buckling behaviour in frame structures. It is difficult, if not impossible, to formulate a precise element force-displacement relationship that includes this buckling mode. The lack of stability functions to capture beam buckling failure is the major drawback of the refined plastic hinge method.

The second obstacle in dealing with lateral torsional buckling is that numerous member restraint parameters must be dealt with in the frame analysis. The restraint details of beams have significant effects on their buckling loads. In general, members with continuous restraints on the compression flange are considered as fully restrained. Otherwise, the members in steel frames are susceptible to lateral torsional
buckling. In order to accurately capture the beam buckling behaviour in a frame structure, the following details of segment end need to be considered:
1. Out-of-plane displacement restraint (top and bottom flange)
2. Out-of-plane rotation restraint (top and bottom flange)
3. Warping restraint
4. Axial twist restraint
5. In-plane displacement restraint (whether cantilever or not)

Regardless of whether the distributed plasticity or concentrated plasticity advanced analysis method is used, accounting for all these connection details is a tedious task if lateral torsional buckling is involved. The direct approach must specify the connections’ out-of-plane moment-rotation relationships, torsion-twist (uniform/non-uniform torsion) relationships, and the moment-torsion relationships. The finite element analyses presented in Chapter 4 show that the connection details have a very significant impact on the ultimate frame capacities. For example, different types of “rigid” connection could lead to 50% differences in the ultimate loads for the same frame. Precise solutions for frame capacities are very difficult to obtain without attending to excessive modelling details. However, in practice, the design cost in the foreseeable future would not permit this kind of in-depth structural modelling. As a result, when dealing with lateral torsional buckling, the deterministic approach is not suitable for general structural design.

In determining the member out-of-plane capacities, the current design specifications use an empirical approach. The effects of member restraint conditions are considered implicitly using the well known effective length concept. To simplify the design process, the nonlinear behaviour of connections is not considered in the analysis steps. Therefore, there is no special treatment for end restraints even though their plastic behaviour may have significant effects on the ultimate capacities. Nevertheless, the current approach is straight-forward and is generally conservative.

Material properties such as yield strength also have a significant effect on the lateral torsional buckling behaviour. Normally, the material parameters have more effects on shorter structural members. For stocky beams, lateral instability does not usually occur. The structural failure mode becomes the collapse mechanism when sufficient
plastic hinges have formed. In general the member capacities relate to two issues, yielding and stability. For slender members (higher slenderness ratio $L/r_y$), instability is the predominant factor. For beams of intermediate slenderness ratios, the member failures are the result of the interaction between yielding and stability. In common cases, since some portions of the members are yielding when buckling commences, and as only the elastic portions of the cross section will remain effective in providing resistance to lateral torsional buckling, the inelastic beam buckling behaviour may significantly differ from the elastic beam behaviour. The results presented in Chapters 3 and 4 indicate that yielding patterns along the members can be very complicated as a result of lateral torsional buckling. The material yielding may be spread unevenly along the member and across the section. The yielding patterns even change while plastic deformations progress. It is difficult to predict the stress distributions for lateral torsional buckling in simple beam column element analysis as in the formation of plastic hinges.

There are other factors associated with material nonlinearity which cause a reduction in member bending capacity. Residual stress is one that always exists in steel beams. The magnitude of the residual stress at the flange tips can be very important. High compressive residual stresses lead to early yielding at the compression flange tips and create an unfavourable mono-symmetry effect. However, the true effects of residual stress on lateral torsional buckling are difficult to gauge in a real structure. First of all, it is difficult to measure the residual stress in the structure accurately. Approximations of residual stress are often used in the distributed plasticity analyses. The results from the analyses show that the inclusion of residual stress will lower the ultimate capacity, but by not more than 10%. The most significant effect of residual stress in the nonlinear analysis is the occurrence of premature yielding in the structures.

Initial geometrical imperfection is the other factor that has significant effects on member capacities. A real beam always has geometrical imperfections. These geometrical imperfections induce second order effects and trigger the occurrences of member instability. Shapes of these imperfections are random in nature. It is not practical to obtain the true geometrical imperfection data of the frames and use them for design purposes. In order to directly account for the effects of imperfections, the first eigen mode shapes from the elastic buckling analyses are often used in the
nonlinear analysis. Design code specified tolerances are often used as the maximum magnitude of imperfection. For a single member, the implementation of this technique is straight-forward. However, for a frame structure, when multiple load cases are considered, the input of initial geometrical imperfection for different load cases becomes a troublesome process because the critical shapes vary under different load cases. Furthermore, more than one mode of geometrical imperfection may have to be introduced into a complex frame analysis. The interactions due to imperfections of different members are difficult to judge. Therefore, for design purposes, the effects of geometrical imperfections are best accounted for by implicit means in the frame analysis.

Currently, the lateral-torsional buckling capacity predictions are mainly based on experimental tests. The majority of these experiments are conducted using simply supported beams. Due to the lack of testing on frame structures, the effects of lateral torsional buckling are not fully understood. With the advances of computing technology, higher order finite element analysis (FEA) can be used to study the behaviour of lateral torsional buckling in frame structures. In this project, finite element analyses were undertaken on a large range of simply supported beams and typical simple frame structures. It is interesting to note that the simply supported beam models agree very well with the current Australian design code predictions (AS 4100) as shown in Chapter 3. On the other hand, there is quite a discrepancy in some of the rigid frame cases.

Simply supported beam models show that lateral torsional buckling is an unfavourable type of failure. After the members reach the maximum loads, the post-buckling strength declines as the beam deflection increases. Generally, the members’ in-plane load-displacement relationships in the pre-buckling stage are the same as if the beams were fully laterally restrained. For simply supported beam models, the elastic buckling loads match well with the method based on analytical solution of uniform bending beam and moment modification factors. The ultimate loads agree well with the code predictions. It is therefore fair to say that the shell finite element analyses are able to capture the experimental results accurately.
Lateral torsional buckling behaviour in the frame structures is much more complicated than that in the simply supported beams. Frame structures are not only made of members but also various types of connection. Many of these connections can be classified as rigid connections. However, the subtle differences between these connections have significant effects on the lateral torsional buckling behaviour. The current design specifications do not adequately allow for the interaction between members and their connections. For example, the beam warping restraints that come from the connections are often ignored.

Both experimental tests and FEA show that lateral torsional buckling is an abrupt failure mode for simply supported beams. The results presented in Chapter 4 indicate that this may not be the case for rigid frame structures. In some cases, where very slender members are present in the frames, lateral torsional buckling can also be ductile and exhibit similar behaviour to that of plastic hinge formation. For these cases, the nonlinear analyses generally yield higher results than the design code predictions.

In flexural buckling cases, the frame buckling shapes obtained from elastic and nonlinear analysis are generally the same. For lateral torsional buckling, the elastic and inelastic buckling shapes can be quite different. An example is shown in Figure 5.2. The portal frame structure is subjected to a loading dominated by a horizontal load (H). The elastic buckling shape of the beam resembles a half sine curve. However, due to the yielding of the far end corner, the inelastic buckling shape is
irregular. Judging by the distance between contraflexure points, the one from nonlinear analysis is shorter. This implies that the effective length concept may be slightly flawed in the current design approach.

It is possible that the structure may carry more loads with stage loading schemes. However, the stage loading effects will significantly increase the difficulty levels of the lateral torsional buckling problem. Currently, the proportional loading is the most commonly used analysis procedure. It would not be practical to use the stage load method in practical advanced analysis. All the finite element analyses presented in Chapter 4 utilise the proportional loading scheme. With the use of this analysis method, the frame structures appear to carry lower loads after one of their members undergoes lateral torsional buckling. This characteristic of the frame structures is very useful for developing a simplified advanced analysis method. It means that lateral torsional buckling failures are significantly different from plastic hinge mechanism where more than one hinge may form in the frame structures. When lateral torsional buckling occurs, the frames are considered to have reached their ultimate state.

Some of the factors associated with lateral torsional buckling have not been well investigated, such as the interactions between adjacent members and connections, the relationships between moment gradients and yielding zone in the members and various warping restraint effects of the joints. In dealing with all these uncertainties, the current design specifications generally take a conservative approach.

In summary, since inelastic buckling is such a complex phenomenon, only two methods can solve this problem: the higher order plastic zone analysis solution and the experimental solution. For design purposes, both methods are impractical. In the current design practice, empirical equations are used to check the lateral torsional buckling capacity while separate elastic/inelastic analysis must be carried out first. No practical software is available incorporating the lateral torsional buckling check directly. Therefore the design of steel frame structures remains a two-step process. The goal of this research project is to develop a practical advanced analysis method that can account for both the plastic behaviour and the lateral torsional buckling effects.
5.3 Consideration of lateral torsional buckling in refined plastic hinge analysis

The current refined plastic hinge method is able to capture the two most common modes of frame failure, gradual formation of plastic hinges and member buckling under axial forces. This method has been extended to account for connection flexibility (Liew et al, 1993b) and local buckling effects (Avery, 1998). The method was originally developed for 2D frames but now it also includes a 3D formulation. One of the main assumptions for all practical advanced analysis methods is that the structural members will not undergo lateral torsional buckling failures.

Column inelastic buckling is effectively solved by using appropriate stability function and tangent modulus concepts in the refined plastic hinge method. This approach successfully eliminates the use of effective length factor in dealing with member instabilities due to axial compression forces. For lateral torsional buckling, due to the non-uniform bending moment along the member, there is no closed-form solution or stability function that can be derived. Thus, a similar approach can not be used for this failure mode. Furthermore, due to the non-uniform nature of bending moment, using a constant tangent modulus to represent partial yielding caused by bending moment along the member is also not applicable. The tangent modulus concept has been used for solving inelastic beams in a similar fashion in the finite element method using one-dimensional elements. However in some cases, the theoretical predictions are significantly higher than the experimental results with no satisfactory explanation (Trahair, 1993). It seems that it is extremely difficult to account for lateral-torsional buckling directly in any practical advanced analysis method. To cope with the lateral torsional buckling problems using the refined plastic hinge method, an implicit approach using the effective length factor method appears to be the most appropriate option.

The distributed plastic analysis presented in Chapters 3 and 4 demonstrates that the lateral torsional buckling behaviour and the plastic hinge formation have some similarities. First, prior to the formation of first plastic hinge or beam buckling, the in-plane stiffness generally follows the same trend for both cases. Second, when the hinge formed or members reach maximum loading due to beam buckling, excessive yielding often occurs in small localised zones in the members. The excessive yielding
in lateral torsional buckling case can be treated as the formation of a “virtual” hinge where the maximum bending moment is located. The comparison is illustrated in Figure 5.3

![Diagram showing plastic hinge formation and lateral torsional buckling](image)

**Figure 5.3 Comparison of Plastic Hinge Formation and Lateral Torsional Buckling**

The difference between the two failures is that more plastic hinges may form in the plastic hinge collapse mechanism whereas additional “virtual” hinges are not allowed in the lateral torsional buckling case.

The steel frame behaviour observed in the finite element analyses is consistent with the current design philosophy in dealing with lateral torsional buckling. Nethercot (1992) pointed out that the design rules are to both prevent the occurrence of lateral torsional buckling at moments less than the plastic moment capacity \( M_p \) and to ensure adequate in-plane rotation capacity even if the final failure (leading to unloading) is triggered by buckling. In the Australian Standard AS4100, advanced analysis method can only be performed on two-dimensional frames comprising members of compact section with lateral restraints which prevent lateral-torsional buckling. This restriction is to ensure that full inplane frame strength can be achieved with adequate inelastic rotation capacity. In other words, when one of its members in the structures fails in lateral torsional buckling mode, plastic analysis (includes advanced analysis) should not be continued any further.

Using this principle as a guideline, it is possible to develop a practical 2D advanced analysis to deal with lateral torsional buckling. It is true that the out-of-plane buckling
deformations might have significant effects on the unloading behaviour of steel frames. 3D element formulations must be used to capture this biaxial bending and torsional effects in the post-buckling stage. However, the post-buckling performances are not important for general design purposes. The in-plane deflections are the dominant activity prior to the occurrence of lateral torsional buckling. Even though the 2D analysis method can only solve the in-plane load-deflection problems, it is adequate to capture the pre-buckling behaviour. The new refined plastic hinge method will perform member out-of-plane capacity checks during the analysis. When one of the members in the structure reaches its member capacity, it will assume that the ultimate load of the structure has been reached. The member capacity calculations are based on the existing design rules, therefore the new method is consistent with the current design specifications.

5.3.1 Stiffness reductions due to out-of-plane buckling

In the refined plastic hinge method, the stiffness reductions due to yielding are accommodated by using the concepts of tangent modulus \( E_t \) and flexural stiffness reduction factor \( \phi \). The tangent modulus was introduced to cope with the gradual yielding along the member for column buckling. When a column is subjected to axial force, the tangent modulus is constant along the member. It is solely dependent on the magnitude of axial force and is independent of the actual member slenderness. For lateral torsional buckling, due to the complicated yielding configuration in the member, it would be difficult to find simple relationships among \( E_t \), \( G_t \), \( EI_w \) and the internal forces in various cross-sections. While using the refined plastic hinge method, it seems that tangent modulus concept is not the most suitable method for lateral torsional buckling.

The flexural stiffness reduction factor is used to simulate the gradual degradation of member stiffness at the plastic hinge location. If lateral torsional buckling is not an issue, the stiffness reduction factor depends on the ratio of internal forces and cross-section capacities. This factor is also independent of member slenderness. The original flexural stiffness reduction factor is proposed by Liew (1992). Avery (1998) modified it to suit the local buckling problem. The generalized parabolic function proposed by Avery (1998) is of the form:
\( \phi = 1 \) for \( \alpha \leq \alpha_{iy} \)

\[
\phi = \frac{\alpha (\alpha_{sc} - \alpha)}{\alpha_{iy} (\alpha_{sc} - \alpha_{iy})} \quad \text{for} \quad \alpha_{sc} \geq \alpha > \alpha_{iy}
\]

(5.26)

The symbol \( \alpha \) represents a force-state parameter that measures the magnitude of the axial force and bending moment at the element ends, normalised with respect to the plastic strength, whereas \( \alpha_{iy} \) represents the initial yielding surface and \( \alpha_{sc} \) the section capacity surface. Since the calculation of section capacity includes the effects of local buckling, the refined plastic hinge method using this formulation can also be used for non-compact sections.

In the lateral-torsional buckling cases, the in-plane moment-rotation relationship at the maximum bending moment location is similar to that of the plastic hinge formation. As the maximum bending moment in the member reaches its lateral torsional buckling capacity, the rotational stiffness at the corresponding location is reduced to zero. Hence, a one-off “virtual” hinge is formed. The relationship between the in-plane rotation and the internal forces can be calculated according to current design standards. With limiting modification to the existing refined plastic hinge method, the stiffness reduction factor can be adapted to account for lateral torsional buckling failure. Gradual yielding along the member is lumped to the “virtual” hinge when lateral torsional buckling occurs.

Using the virtual hinge approach, the problems of lateral torsional buckling are solved by defining the in-plane stiffness reduction at the locations of maximum bending moment. The stiffness reduction factor is calculated by comparing the in-plane internal forces’ load state parameter and the member out-of-plane capacities. The process of obtaining stiffness reduction factor for lateral torsional buckling is similar to that in the plastic hinge case.

In the proposed method, the stiffness reduction factors due to both in-plane formation of plastic hinge and out-of-plane buckling will be calculated separately. Avery’s (1998) formulations of stiffness reduction factor are used for in-plane plastic hinge forming effects. The generalized equation (5.26) can be used for both AS4100 and
AISC LRFD design codes. The force state parameters ($\alpha$ and $\alpha_{sc}$) can also be calculated as follows:

$$\alpha = p + \frac{1}{c'_i} \text{ for } p/m \geq c'_3$$

$$\alpha = c'_2 p + m \text{ for } p/m < c'_3$$

where $p = P/P_y$ and $m = M/M_p$ are the non-dimensionized axial compression force and bending moment and:

AS4100: $c'_1 = 1.18; \quad c'_2 = 0; \quad c'_3 = 0.153$

AISC LRFD: $c'_1 = \frac{9}{8}; \quad c'_2 = 0.5; \quad c'_3 = \frac{2}{9}$

$$\alpha_{sc} = \frac{k_f c_1 (c'_i p/m + 1)}{c'_i (c'_1 p/m + k_f S/Z_e)} \text{ for } p/m \geq c'_3$$

$$\alpha_{sc} = \frac{k_f c_1 (c'_2 p/m + 1)}{c_1 p/m + k_f S/Z_e} \text{ for } c'_3 > p/m \geq c_3$$

$$\alpha_{sc} = \frac{k_f (c'_2 p/m + 1)}{c_2 p/m + k_f S/Z_e} \text{ for } p/m < c_3$$

(5.28)

The initial yield force state parameter ($\alpha_{iy}$) can be defined by considering the maximum residual stresses for a particular section. The non-dimensional axial force and bending moment corresponding to initial yield can be defined as a linear interaction equation:

$$p_{iy} + \frac{S}{Z} m_{iy} = \left( 1 - \frac{\sigma_y}{\sigma_y} \right)$$

(5.29)

The force state parameter corresponding to initial yield (denoted by $\alpha_{iy}$) can be determined from the initial yield interaction equation:

$$\alpha_{iy} = \begin{cases} \left( 1 - \frac{\sigma_y}{\sigma_y} \right) \frac{(p/m + 1/c'_i)}{(p/m + S/Z)} & \text{for } p/m \geq c'_3 \\ \left( 1 - \frac{\sigma_y}{\sigma_y} \right) \frac{(c'_2 p/m + 1)}{(p/m + S/Z)} & \text{for } p/m < c'_3 \end{cases}$$

(5.30)
For a hot-rolled universal beam I-section, initial yielding starts at approximately 70% of yielding surface \((\sigma_r/\sigma_y = 0.3, Z_e/S = 0.90)\).

The flexural stiffness reduction factor is calculated based on the section capacity. It is applicable to both compact and non-compact sections. With the introduction of a hinge softening concept, the stiffness reduction factor is also used to deal with problems associated with post local buckling.

\[
\phi = 1.5 - \sqrt{2.25 - 2e_s/e_t}
\]

where: \(e_s = \frac{E_s}{E} = \alpha_{sc} - 1 \leq 0\)

The in-plane flexural stiffness reduction at the virtual hinge location will be calculated based on the member out-of-plane capacities specified by the design code (AS4100). The out-of-plane member capacities are not only dependent on the cross-section geometry as with the section capacity calculations, they are also related to the member length between the joints, details of the connections and bending moment distribution pattern along the whole member. The effective length factor is used in the design code to deal with member slenderness calculations and to account for member restraint effects. Since the member capacity calculations are used in the new refined plastic hinge analysis, the effective length factors and moment modification factors are also adapted in the new method.

The stiffness reduction function for members susceptible to out-of-plane buckling can be expressed as:

\[
\phi = \begin{cases} 
1 & \text{for } \alpha \leq \alpha_{iyo} \\
\frac{\alpha(\alpha_{mo} - \alpha)}{\alpha_{iyo}(\alpha_{mo} - \alpha_{iyo})} & \text{for } \alpha_{mo} \geq \alpha > \alpha_{iyo}
\end{cases}
\]

where: \(\alpha_{mo}\) is the member out-of-plane capacity surface, \(\alpha_{iyo}\) is the initial yielding surface for out-of-plane buckling situation.

\[
\alpha_{mo} = \begin{cases} 
\frac{k_j \alpha_s (p/m + 1)}{p/m + k_j \alpha_s S/(\alpha_{sc}Z_e)} & \text{for } p/m \geq 1 \\
\frac{k_j \alpha_s (m/p + 1)}{1 + (m/p)k_j \alpha_s S/(\alpha_{sc}Z_e)} & \text{for } p/m < 1
\end{cases}
\]
When out-of-plane buckling occurs, the initial yielding could occur at less than 70% of section capacities. Therefore, the initial yield surface for out-of-plane buckling will be a different formation of plastic hinge case. It is assumed that when the maximum load in the member reaches 70% of the member capacity, the initial yielding will begin. The initial yielding surface for out-of-plane buckling cases can be expressed as:

\[ \alpha_{vo} = 0.7 \alpha_{mc} \]  

The relationship of plastic surface, section capacity surface, member capacity surface, and initial yielding surface are shown in the following diagrams (Figures 5.4 and 5.5). In the new refined plastic hinge analysis, each element may take an alternative load path, formation of plastic hinge or reaching out-of-plane member capacities. The member capacity surfaces for axial tension and compression cases are slightly different. For most of the cases, member capacity surface for axial tension case coincides with section capacity surface.

![Diagram](image)

Figure 5.4 Capacity Surfaces with Axial Compression Force

In the existing refined plastic hinge analysis, a member (segment between joints) is often divided into a number of elements. A sensibility study by Avery (1998) suggests that four elements per member are generally sufficient to achieve 95% accuracy.
(compared with finer discretization). Therefore, not less than four elements are used in the new refined plastic hinge analysis for each member.

![Figure 5.5 Capacity Surfaces with Axial Tension Force](image)

If out-of-plane buckling is not considered, only section capacity checks are required for each element. These existing 2D elements do not include the details of the member they belong to. In the out-of-plane buckling case, additional properties must be included for each element. They include the member slenderness ratios (for out-of-plane buckling) and bending moment gradients of the members (not element). The main achievement of existing refined plastic hinge analysis is the elimination of the use of column slenderness (\(\lambda\)). The use of effective length factor in the analysis has two major shortfalls. First, this method does not provide the most accurate solution because plastic redistributions can not be considered and second it is often regarded as tedious and not suitable for programming coding. Since there is no better method to calculate the member capacity, the use of effective length factor to account for out-of-plane buckling is inevitable. Also, the plastic redistributions after lateral torsional buckling need not be considered. Therefore using the out-of-plane effective length factor would not have as many shortcomings as the in-plane column buckling cases. On the other hand, the programming problems are solvable with the virtual hinge approach. The effective length and moment gradient factors can be treated as an
independent data block, and each element is linked to the data array. With the separation of member and element properties, the program coding is relatively straightforward.

The example shown in Figure 5.6 illustrates the relationship between member and elements. A portal frame structure has three members (AB, BC, and CD). Each member is divided into four elements (denoted elements 1 – 12). The out-of-plane buckling properties of elements 1 – 4 will be calculated based on member AB’s effective length and bending moment distribution. Since they all belong to the same member, they would have the same in-plane flexural stiffness reduction factor function. The value of stiffness reduction factor at each node (for elements 1 – 4) depends on the out-of-plane capacity of member AB and the load state parameter (\(\alpha\)) of each element end. Obviously, when the node with the maximum bending moment reaches member AB’s capacity, out-of-plane buckling occurs and the stiffness reduction factor at that node becomes zero.

![Figure 5.6 Separation of Member and Element Properties](image)

Member capacities of AB, BC and CD require moment modification factor and slenderness reduction factor \(\alpha_s\). In AS4100 the member out-of-plane bending capacity \((M_b)\) is calculated as:

\[
M_b = \alpha_m \alpha_s M_s \leq M_s
\]  

(5.35)

where: \(M_s\) is the section capacity.
The parameters required for the calculation of slenderness reduction factor $\alpha_s$ can be defined prior to the analysis.

$$\alpha_s = 0.6 \left[ \sqrt{\left( \frac{M_s}{M_o} \right)^2 + 3} - \left( \frac{M_s}{M_o} \right) \right]$$ (5.36)

where: $M_o = \sqrt{\left( \frac{\pi^2 EI_G}{L_e^2} \right) \left( GJ + \left( \frac{\pi^2 EI_w}{L_e^2} \right) \right)}$ (5.37)

The effective length $le$ is given by:

$$L_e = k_t k_l k_r L$$

where: $k_t$ is twist restraint factor, $k_l$ is load height factor, $k_r$ is lateral rotation restraint factor, and $L$ is taken as:

(a) the segment length, for segments without intermediate restraints, or for segment unstrained at one end, with or without intermediate lateral restraints.

(b) the sub-segment length, for sub-segments formed by intermediate lateral restraints in a segment which is fully or partially restrained at both ends.

Various details are given in the design code AS4100 in relation to the calculations of $k_t$, $k_l$, and $k_r$ factors. They include the classification of restraints and effective length factors for assorted restraint combinations. The classifications of member end restraints are simple. They consist of five types: fully restrained, laterally restrained, rotationally restrained, partially restrained and unrestrained. Fully restrained refers to the restraints or supports that effectively prevent both the lateral deflection of critical flange and section twist rotation. Three common cases are shown in Figure 5.7. Partially restrained refers to the restraints which prevent the lateral deflection of some point in the cross-section other than the critical flange and partially prevent twist rotation of the sections (Figure 5.8). A cross-section which is fully or partially restrained may also be considered as rotationally restrained when rotation of critical flange is prevented (Figure 5.9). On the other hand, when lateral deflection of critical flange is prevented and twist rotation is still possible, it is called laterally restrained (Figure 5.10).
Figure 5.7 Fully Restrained Cross-section as defined in Figure 5.4.2.1 of AS4100 (SA, 1998)

This image is not available online. Please consult the hardcopy thesis available from the QUT Library.

Figure 5.8 Partially Restrained Cross-section as defined in Figure 5.4.2.2 of AS4100 (SA, 1998)

This image is not available online. Please consult the hardcopy thesis available from the QUT Library.
The effective length factors $k_t$, $k_l$, and $k_r$ factors are given in Tables 5.1 to 5.3. The twist restraint factor ($k_t$) is usually close to 1.0, except for long shallow beams with very thick flanges and very thin webs. For members restrained in both ends, the load height factor ($k_l$) is either 1.4 for loads applied above shear centre or 1.0 for otherwise. For cantilever beams, the load height factor ($k_l$) equals 2.0 when loads are applied on critical flange (tension flange) otherwise it is also 1.0.

Table 5.1 Twist Restraint Factors $k_t$, as defined in Table 5.6.3(1) of AS4100 (SA, 1998)
Table 5.2 Load Height Factor $k_l$ as defined in Table 5.6.3(2) of AS4100

This table is not available online. Please consult the hardcopy thesis available from the QUT Library.

Table 5.3 Lateral Rotationally Restraint Factor $k_r$ in Table 5.6.3(3) AS4100

This table is not available online. Please consult the hardcopy thesis available from the QUT Library.

The lateral rotation restraint factors ($k_r$) of 0.85 and 0.70 are for values of intermediate stiffness compared with the theoretical values of approximately 0.7 for one pin and one rigid end case and 0.5 for both rigid end case. Due to the complexity of the restraint details, it is obvious that simplifications have been made while using the effective length method. The effects of warping restraints are not considered in general design practice. For rigid frame structures, the lateral rotational restraints often depend on the stiffness ratio of adjacent members. The simplified and conservative approach can easily underestimate the true effective length by 20%.

The other parameter required for member bending capacity calculation is the moment modification factor ($\alpha_m$). This parameter requires the results from the linear elastic
analysis for a specific load case. Commonly, this factor is taken from AS 4100 Tables 5.61 and 5.62 (SA, 1998).

Table 5.4 Moment Modification Factors for both ends restrained segments as defined in Table 5.61 of AS4100

This table is not available online. Please consult the hardcopy thesis available from the QUT Library.
Table 5.5 Moment Modification Factors for one end unrestrained Segments as defined in Table 5.62 of AS4100 (SA, 1998)

This table is not available online. Please consult the hardcopy thesis available from the QUT Library.

The moment modification factor can also be calculated using the following equation:

\[
\alpha_m = \frac{1.7M^*_m}{\sqrt{(M^*_2)^2 + (M^*_3)^2 + (M^*_4)^2}} \leq 2.5
\]

\(M^*_m\) = maximum design bending moment in the segment

\(M^*_2, M^*_4\) = design bending moments at the quarter points of the segment

\(M^*_3\) = design bending moment at the midpoint of the segment

Using an equation to calculate the moment modification factor is more suitable for computer coding. The users need not know what the bending moment pattern will be for each beam segment. It is interesting to note that the equation for moment modification factor varies depending on the design code.

In the AISC LRFD (1999), it is expressed as:

\[
C_b = \frac{12.5M_{\text{max}}}{2.5M_{\text{max}} + 3M_A + 4M_B + 3M_C}
\]

where: \(M_A, M_B\) and \(M_C\) are the absolute values at the midspan and quarter points, and \(M_{\text{max}}\) is the absolute maximum moment within the segment.
In the British Code BS5950 (BS, 2003), it is expressed as:

\[ m_{LT} = 0.2 + \frac{0.15M_A + 0.5M_B + 0.15M_C}{M_{\text{max}}} \geq 0.44 \]  

(5.40)

where: all the moments are taken as positive, \( M_A, M_B \) and \( M_C \) are the values at the midspan and quarter points, and \( M_{\text{max}} \) is the maximum moment in the segment. Note: the inverse of \( m_{LT} \) is equivalent to the moment modification factors used in AISC and AS4100.

The equation for end moments is widely used in various design codes (see first row in Table 5.4). A comparison of general equations of AS4100, AISC LRFD, BS5950 and the end moments form of moment modification factor is plotted in Figure 5.11. There are noticeable differences between all these equations. The differences between the general case and the end-moments case with AS4100 is up to 7%. In the proposed new refined plastic analysis method, options are given for the use of different equations. When using the formula other than the general form, the users need to obtain the bending moment distribution pattern before the nonlinear analysis.

![Figure 5.11 Comparison of Moment Modification Factors](image)

**Figure 5.11 Comparison of Moment Modification Factors**
In partially laterally braced frame structures, out-of-plane buckling is often caused by a combination of lateral torsional buckling and minor axis column buckling. With 2D formulation, there is no suitable method to separate the in-plane stiffness reduction due to these two buckling conditions. When a member of rigid frame fails predominantly by minor axis column buckling, plastic hinges will form and plastic redistributions can be significant, thus the plane frame structure may carry slightly higher loads. This is not the case for a predominant lateral torsional buckling case where only a minimum plastic redistribution is allowed after buckling. Since it is not possible to separate the two failure modes, both buckling cases should be treated as the same: i.e., once out-of-plane buckling occurs, the frame structure is considered to have reached its ultimate state. Using this approach, out-of-plane column buckling can also be calculated using the effective length factor method in the analysis. The member’s minor axial capacity can be predefined when the segment slenderness ratio is given. The effective length in the axial compression case can differ longer significantly from that in lateral torsional buckling. With out-of-plane axial capacity and moment capacity defined, the out-of-plane member capacity surface is derived based on a linear interaction (see Figures 5.5 and 5.6).

If a 3D line element formulation is used, the out-of-plane column buckling can be treated separately from lateral torsional buckling. The flexural stiffness reduction factors would be used solely for the beam buckling case. The column buckling issues will be solved by the use of tangent modulus and 3D stability functions thus eliminating the use of effective length factor for columns. The major axis stiffness reduction factor will relate to a 3D capacity surface as shown in Figure 5.12. The 3D element formulation is not within the scope of this project. Correctness of the use of 3D member capacity surface will not be investigated. Nevertheless, the use of empirical minor axis column buckling capacity is a significant shortfall in the new 2D refined plastic hinges analysis method. For rigid frame structures, the effective length factors of minor axis column buckling depend on the flexural and torsional stiffness ratios of adjacent members. Accurate predictions of the effective length factors will be a tedious task and the stiffness ratio approach has not been incorporated into the new method. Therefore, the analysis results from the improved refined plastic hinge analysis may be over-conservative as compared to the current design standards.
The effects of load height have not been investigated in the finite element analyses presented in Chapters 3 and 4. The other issue that has not been studied is the effects of loading/restraints at either the top or bottom flanges. In lateral torsional buckling, the compression flange is often called the critical flange (overhanging segment is the opposite). When a lateral restraint is applied to a beam on its critical flange, the member is effectively divided into two segments. However, if the load changes its direction and the bending moment diagram reverses, the existing lateral restraint will not be effective. The advanced analysis must be able to capture this structural behaviour. In the proposed refined plastic hinge method, the member properties are load case dependant. A simple gable frame example is shown in Figure 5.13. The beams are laterally restrained by the purlins on the top flanges. Under the gravity load case, the top flanges of the beam are the critical flanges and both beams can be treated as four segments. On the other hand, in the wind uplift case, the bottom flanges are generally in compression and become critical. Therefore, in calculating the member capacities for the elements, the 12 m beams can not be divided into shorter segments.

A bending moment diagram for the gable frame under uplift loading is shown in Figure 5.14. Supposedly, the 12 m beam AE is divided into four equal length elements using the proposed advanced analysis. The nodes of each element coincided with the lateral restraints from member ends and purlins. Since there is no intermediate restraint at the bottom flange, the member capacity for each element will

\[ N_s \text{ – axial section capacity} \]
\[ N_{cy} \text{ – minor axis member capacity} \]
\[ M_{sy} \text{ – minor axis section capacity} \]
\[ M_{bx} \text{ – major axis member capacity} \]
be calculated according to the length of AE (12 m). The effective length factors of the member depend on the restraint conditions of apex and knee joints (say $k_e \approx 0.85$). This illustrates that, in order to deal with varying restraint conditions on the top or bottom flanges, the member properties of the elements have to be load case dependant.

![Simple Gable Frame Structure with Purlins](image1)

**Figure 5.13 Simple Gable Frame Structure with Purlins**

![Bending Moment Diagram for Uplift Load Case](image2)

**Figure 5.14 Bending Moment Diagram for Uplift Load Case**

The moment distribution diagram indicates that the beam AE is deflected in double curvature. The modification factor for this type of bending moment distribution is approximately 2.5. It implies that the tendency of lateral torsional buckling is significantly reduced from that due to large slenderness ratio. Element AB in uplift...
load case could be governed by the capacity of beam AE, but there can be another scenario. When the top flange of segment AB is in compression and lateral torsional buckling occurs within the segment, the segment’s member capacity is calculated based on the segment’s effective length (say 3 m) and the moment gradient factor $\alpha_m$ is closer to 1.0. In some load cases and frame configurations, when the member capacity of segment AB is smaller than the member capacity of beam AE, lateral torsional buckling would occur only in segment AB. The occurrence of lateral torsional buckling within different regions of partially restrained beams can pose a problem with the definition of member capacity in the refined plastic hinge method. If the member capacity is based on a wrong segment (L), the analysis result will be meaningless. The solution for this problem is to introduce a second member property. The first member capacity is calculated based on beam AE (the length of unrestrained bottom flange). The second member capacity is based on segment AB (unrestrained top flange) where as the refined plastic hinge element consists of these two properties simultaneously. During the analysis, the lesser of the two member capacities would be used to define the in-plane stiffness degradation of the element. In the benchmark analyses, none of the frames is partially restrained at only either the top or bottom flange. The use of second member property in the analysis is therefore not verified.

Linking end node stiffness reduction to member out-of-plane capacities does not contradict the existing refined plastic hinge formulation. When the member capacity is smaller than the section capacity, lateral torsional buckling occurs. The structure dose not allow for further plastic redistribution and for the carrying of more loads. If the member capacity $M_{bx}$ equals the section capacity $M_{sx}$, the member is effectively fully laterally restrained. The member failure mode for a laterally braced frame is only due to in-plane actions. Plastic redistributions will proceed until the formation of a hinge mechanism.

Using the existing design rules to determine the out-of-plane member capacity surface has a number of limitations. First, the effective length concept has to be adapted to account for lateral-torsional buckling. The effective length factor is based on elastic theory. Yielding of connections may have significant effects on the “plastic effective length”. Second, this method does not consider the interaction between beam
segments. The member capacity may be inaccurate. Third, empirical equations for moment modification factor are used in the calculations. Choosing a specific formula in the analysis increases the programming difficulty. Also, the minor axis compression member capacities are required in two-dimensional frame analysis. The use of effective length factor for minor axial column buckling is inevitable. Furthermore, the interactions between minor axis axial forces and lateral torsional buckling are not well studied, especially for irregular moment gradient cases. The existing member capacity surfaces are generally very conservative.

In-plane stiffness reduction is a complex phenomenon due to lateral torsional buckling. Ideally, the structure should be considered as a whole in the advanced analysis to account for all the interactions between members, connections, restraints and loadings. The use of empirical member capacities calculated from isolated segments would not yield a comprehensive solution as with that from shell element analysis. However, since lateral-torsional buckling is an unfavourable failure mode, and in the absence of more explicit methods, the proposed solution can effectively retain the advantage of the existing refined plastic analysis while checking codified out-of-plane capacity requirements. It is an improvement from the current solutions which still require separate member capacity checks. The results from the proposed method will be conservative. Accounting for inelastic lateral-torsional buckling is easy to implement with the new approach. The implementations of the new method using object-orientation programming are presented in the following section.

5.3.2 Numerical implementation in refined plastic hinge analysis

The proposed refined plastic hinge analysis uses a linear incremental algorithm. The linear incremental method is the simplest and most direct non-linear solution technique, evaluating non-linear response by employing a stepwise linear procedure. The load is applied as a series of small increments, and for each of these increments the change in deformation is determined using a linear analysis. A tangent stiffness matrix based on the geometry and internal forces that exist at the beginning of the step is used to calculate the change in deformation caused by the load increment. The stiffness matrix remains constant throughout the step. The total displacements and
internal forces existing at the end of any step are determined by summing the incremental changes in displacements and internal forces up to that point.

This method is very efficient in tracing the sequence of plastic hinge formation and checking the beam segment does not exceed its member capacity surface. One disadvantage of this method is that within a load incremental step, the equilibrium between the external applied loads and internal element forces may not be achieved. However, past research found that adequate accuracy can be achieved by adapting suitable small increments in the inelastic range.

A proportional loading scheme is used to determine the capacity of the structure. For example, gravity and lateral loads will be applied to the frame structures simultaneously. This method is widely used in the current design practice. Such approach does not consider unloading, and generally under-predicts the strength of frames subjected to sequential loads. For lateral-torsional buckling, the effects of unloading are largely unknown. Since this failure mode can also be unpredictable, it is not worth while accounting for the load recovery benefits. The adapted loading scheme reduces many complications that may arise in the analysis which does not consider unloading.

The basic algorithm for each load increment of a refined plastic hinge analysis using the linear incremental solution method is outlined below:

1. Calculate the trial applied load increment equal to the total applied loads multiplied by an incremental load factor.
2. Form the local tangent stiffness matrix and pseudo-force vector for each element using the forces and displacements evaluated during the previous load increment.
3. Transform the element tangent stiffness matrices and incremental pseudo-force vectors from local to global coordinates.
4. Assemble the structure tangent stiffness matrix and incremental pseudo-force vector. If the structure tangent stiffness matrix is not positive definite (indicating that the structure is unstable), restart the increment with a reduced incremental load factor.
5. Solve the structure force-displacement relationship for the structure global incremental displacements.

6. Extract the element global incremental displacements from the structure global incremental displacements.

7. Transform the element incremental displacements from global to local coordinates.

8. Determine the element local incremental forces for each element using the appropriate element local force-displacement relationships. Calculate moment modification factor for the member, which consists of several elements. If the member capacity is less than the section capacity, out-of-plane buckling may occur (Case 1). Otherwise plastic hinges are allowed to form (Case 2).

   **Case 1** If the calculated forces reach the member capacity surface, the analysis terminates. This means one of the members in the structure undergoes out-of-plane buckling. Structure should not be permitted to carry further loads.

   **Case 2** If the calculated forces indicate that new plastic hinges have formed, ie, member capacity equals section capacity, determine whether the force state parameters corresponding to the new hinge locations are within a predefined tolerance of the corresponding section capacity force state parameters. If this tolerance is exceeded, restart the increment with a reduced incremental load factor. The increment should also be restarted with a reduced incremental load factor if the incremental element local forces at any plastic hinge location are excessive (defined by $\Delta\phi > 0.1$).

9. Update and store the total applied load factor, the total element local forces, the total structure global displacements defining the deformed geometry, and the list of active plastic hinges.

Steps 1 to 9 are repeated until satisfaction of either of the following criteria causes the analysis to terminate:

1. The maximum load steps have been achieved. If this occurs the analysis of the structure must be repeated with increased applied loads in order to obtain the ultimate capacity of the structure.
2. A solution cannot be achieved without exceeding a predefined maximum number of increments or reducing the load increment factor to less than a predefined minimum. If these failure criteria are appropriately defined, the applied loads at the final solved increment represent the ultimate capacity of the structure.

3. Hinge collapse mechanism formed. Plastic hinge is taken as perfect hinge in the calculations. Thus it prevents unrealistic large structural displacement.

The object-oriented (O-O) paradigm is used to design the computer program. In O-O design, the basic building block is an object. An object binds the data and the various functions together. This program framework is suitable for large and complex numerical problems. The refined plastic hinge analysis program was written in C++ language, and consists of four types of objects. They are:

- **Element Objects** (eleData, eleFunc, eleforceFunc): These objects include the data and functions for individual element. Each element may have two different member properties if the top and bottom flanges support conditions are different.

- **Member Objects** (memberProperties, memberFunc): These objects deal with the beam segments data. A beam member is the unbraced length between either top or bottom flanges. Each member object has a number of elements. Member objects are used to evaluate the member out-of-plane capacity surface. One of the variables - moment modification factor ($\alpha_m$) is updated in every load increment. Currently, there are three types of member objects; one calculates $\alpha_m$ from the member end moments, one from the member quarter points and maximum bending moments and one for the overhang segments. More member objects can be developed for special loading configurations.

- **Frame Object** (frmData): This object consists of the global information about the structure. A frame object is instantiated/created for a load case. It includes a member object, an element object, incremental load array, etc.

- **Incremental displacement solver Object** (dgsiSolver): It is used to solve for global incremental displacement from structural matrix ($K$) and predefined incremental load (a proportion of input loads $f_i$).
The flowchart shown in Figure 5.15 outlines the relationship between the various objects within each load incremental loop of the improved plastic hinge analysis.

![Figure 5.15 Program Flow Chart](image-url)

### 5.4 Verification of the new advanced analysis method

Two comparisons are made to verify the accuracy of the new refined plastic hinge method: 1) with design specification AS4100 and 2) with benchmark solutions from the distributed plasticity analyses. The verifications of the new method include two groups of structures: simply supported beams and typical rigid frame structures. The capacities of simply supported beams can be directly calculated from the current...
design code. For frame structures, the frame capacities are based on elastic analysis and are calculated implicitly. The benchmark solutions were established using shell finite element analyses and are presented in Chapters 3 and 4.

5.4.1 Simply supported beams

Experimental studies of lateral torsional buckling phenomenon were mainly conducted on simply supported beams. They generally included three load cases; uniform bending moment, midspan concentrated load and quarter-point concentrated loads. The existing beam design curves are in general based on curve fitting of these experimental data. Very limited testing has been undertaken on the lateral torsional buckling effects of frame structures due to their complexity and high cost. Therefore, for frame structures, the verifications of a simplified analysis method rely more on the results from comprehensive numerical methods. For simply supported beams, the accuracy of any proposed method is relatively easy to verify.

5.4.1.1 Simply supported beams subjected to a uniform bending moment

Uniform bending moment is the most critical load case for simply supported beams associated with lateral torsional buckling. A closed-form solution is available for idealised simply supported elastic beams and a large number of experiments have been conducted for this load case. For this load case, the finite element analyses taking into account factors such as geometrical imperfections, residual stresses, and material non-linearity agree very well with both the closed form solution and the beam curve. This implies that the numerical solutions are able to simulate both stability and yielding behaviours of the inelastic beams accurately. The new refined plastic hinge method is also able to predict the member capacities of this basic case. Figure 5.16 shows the member capacities obtained from the new method. They agree very well with both the design beam curve and the FEA results.

Figure 5.17 shows the moment versus end rotation curves for three simply supported beams (250UB37.5 with lengths of 2m, 3m, and 4m). For idealised simply supported beams, the effective length factors are assumed to be 1.0 in the new refined plastic hinge analyses. In the finite element analyses, the idealised pin supported boundary
conditions follow very strict rules and they are difficult to achieve using 3D modelling. A complex MPC system was used in the finite element modelling. The load paths around the peak loads from the finite element models indicate a very abrupt change of the member stiffness. The new refined plastic hinge analysis assumes a more gradual yielding behaviour, thus moment-rotation curves are significantly different in the peak load regions. A suitable modification can be made in the refined plastic hinge method to cope with this behaviour. For example, if the initial yielding is assumed to commence at 80% or 90% of the member capacities, the load paths from the new method are closer to that from the finite element analyses (see Figure 5.18). However, the abrupt failure behaviour is more associated with simply supported beams. More gradual yielding was found in the rigid frame structures subject to a lateral torsional buckling failure. Also, the use of 70% as the initial yielding point is consistent with that of the case of plastic hinge formation. Therefore, it was decided to use the 70% of member capacity as the initial yielding surface in the new method to deal with out-of-plane buckling failure.

![Figure 5.16 Moment Capacity Curves of Idealized Simply Supported Beams subject to a Uniform Moment](image)

Figure 5.16 Moment Capacity Curves of Idealized Simply Supported Beams subject to a Uniform Moment
Figure 5.17 Moment versus End Rotation Curve for Idealized Simply Supported Beams subject to a Uniform Moment

Figure 5.18 Effects of Initial Yielding on the Moment versus End Rotation Curves
The moment-end rotation curves in Figure 5.17 indicate that, for a fully laterally restrained member, the results from the FEA and the new method are virtually identical. Also, the new method does not consider unloading or the post-buckling load path.

The uniform bending moment load cases have also been considered for two other series of simply supported beams. The first series of beams have warping restrained ends. The analysis results show that the effective length factors for these models are approximately equal to 0.7. The effective length factor of $k_e = 0.7$ is used in the new refined plastic hinge analysis to represent the warping restraint effects. The member bending capacity results are plotted in Figure 5.19. Comparable results are obtained from the new method compared with the design code predictions and the finite element analyses as shown in Figure 5.19.

![Figure 5.19 Moment Capacity Curves for Beams with Warping Restrained Simply Supported Ends](image)

The dimensionless moment versus end rotation curves for warping restrained cases are plotted in Figure 5.20. The new method is very good at tracing the elastic load path and achieving the member ultimate bending moment. When the members are
assumed as fully laterally restrained along the whole length, the new method can obtain the same results as the FEA. Two approaches of treating the initial yielding are also shown in the figure. It seems that the assumption of initial yielding at 90% of the member capacities can provide slightly improved results.

![Figure 5.20 Moment versus Rotation Curves for Beams with Warping Restrained Simply Supported Ends](image)

When the ends of a simply supported beam are fixed against out-of-plane rotation, the effective length factor becomes 0.5. The finite element analyses are able to clearly demonstrate this behaviour. When the effective length factor of 0.5 is used in the new refined plastic hinge analysis, very good results are obtained. The member capacities obtained from the design code, the finite element analyses and the new method are shown in Figure 5.21. The dimensionless moment versus end rotation curves are plotted in Figure 5.22. In these out-of-plane rotation restrained models, the new method is able to achieve near identical load path as finite element analyses with the assumption that initial yielding commences when the beam reaches 90% of the member capacity. Simply supported beams are classified as determinate structures. They do not have extra degrees of redundancy. This shows that the beams are not
significantly affected by the residual stress for the lateral torsional buckling cases. The graduate formation of plastic hinge and initial yielding before lateral torsional bucking is significantly different for the simply supported beam case.

Figure 5.21 Moment Capacity Curves for Beams with Laterally Fixed Simply Supported Ends

Figure 5.22 Moment versus Rotation Curves for Beams with Laterally Fixed Simply Supported Ends
In partially laterally braced frame structures, lateral torsional buckling and minor axis column buckling often co-exist. The new refined plastic hinge analysis is using the effective length factor concept to calculate the out-of-plane column buckling capacity. Figure 5.23 shows that the column capacities for the new method are nearly identical to those derived from the finite element analyses.

Figure 5.24 shows the interaction diagrams for simply supported beams subjected to combined action of uniform bending moments and axial forces. The finite element models consist of idealized simply supported boundary conditions. The effective length factors for both lateral torsional buckling and minor axis column buckling are assumed to be 1.0 in the new refined plastic hinge analyses. The results from the new method are slightly lower than the design code predictions. The differences between the new method and the code predictions are within 5%. The linear interaction curves used in design code AS4100 are conservative. Some researchers (Rasmussen and Hasham, 1997) proposed the use of convex interaction curves. This improvement can be easily implemented in the new method if the design code allowed the change.

![Figure 5.23 Minor Axis Column Buckling Curve](image-url)
Three types of transverse load cases were studied using FEA; a concentrated load applied at midspan, two concentrated loads applied at quarter points, and a uniformly distributed load. The program coding of the first two load cases has been completed in the new refined plastic hinge method and their verifications are presented next.

When simply supported beams are subjected to transverse loads, the beams’ capacities vary depending on their bending moment gradients. The new method uses the empirical moment modification factors to account for the effects of various moment gradients during the analysis. Accuracy of the analysis will be compromised inevitably when compared with the uniform bending moment cases. For the case of simply supported beams subjected to a midspan point load, the maximum buckling moments for various slenderness ratios are plotted in Figure 5.25. The new method agrees very well with the design code AS4100 predictions that assumes $\alpha_m = 1.35$. However, the results from both the design code and the new method are slightly higher than that from finite element analyses.
The load-midspan vertical load deflection curves are shown in Figure 5.26. It appears that the finite element models are slightly softer than the refined plastic hinge models.
An elastic load-deflection line is also plotted in the same chart for a 3m span. The elastic load-deflection relationship is based on the formula:

$$\Delta = \frac{PL^3}{48EI}$$

This theoretical line agrees very well with the refined plastic hinge analysis in the elastic range. It implies that some unsolved issues associated with the distributed plasticity analysis models should not be considered as trivial. For example, due to the initial geometrical imperfections, the vertical loads used in the finite element models are applied at the shear centre. They also generate torsions. Their effects have not been quantified. Also, half length models are used in the analyses. Simply supported beams are members with one pin and one roller ends. These boundary conditions do not provide perfect symmetry. Therefore there might be some differences between the half length and the full length models. Full length models are more complicated and are difficult to use in the uniform moment load cases.

When the concentrated load is applied to the top flange in the transverse load case, the tendency of lateral torsional buckling is significantly increased. The treatments for the load height effects are not the same for different design specifications. In the Australian design specification AS4100, a load height factor is introduced to modify the effective length factor. For a transverse load at the top flange, the load height factor \( k_l \) is taken as 1.4. The results for this load case are plotted in Figure 5.27. The load height factor of 1.4 was used in the new refined plastic hinge analyses. The beam curves demonstrate that the new method agrees well with the design code predictions. As for the previous load case where the concentrated loads were applied to the centre of the cross-section at midspan, the FEA results are also slightly lower than the results of the new method for less slender members.

The top flange loads versus deflection curves at midspan are shown in Figure 5.28. In the elastic range, the new refined plastic analysis agrees well with the elastic theory and is slightly stiffer than the finite element models. For intermediate and slender beams, the ultimate capacities from the two analyses are very close. The capacity differences are within 5%.
Figure 5.27 Moment Capacity Curves for Beams subject to a Midspan Concentrated Load applied at Top Flange

Figure 5.28 Load-Deflection Curves for Beams subject to a Midspan Concentrated Load Applied to Top Flange
Comparisons are also made for quarter point concentrated load cases. For this load case, moment modification factor 1.09 was used in the new refined plastic hinge analysis. The beams’ capacities are plotted in Figure 5.29. Good correlations were
achieved for this load case in terms of the ultimate capacity. The differences between FEA and the simplified method in the load versus deflection relationships are more significant (see Figure 5.30). The initial yielding is assumed to commence at 70% of the member capacity for all transverse load cases. The load-deflection curves shown in Figures 5.26, 5.28 and 5.30 indicate that this assumption is quite reasonable. Compared with the FEA, the new method is generally more conservative and agrees well with the AS4100 design code predictions.

In summary, for simply supported beams, the results from the finite element analyses and the new refined plastic hinge analyses agree fairly well with the design code predictions for out-of-plane buckling failures. The slight differences in elastic stiffness between the simplified method and finite element analysis indicate that lateral torsional buckling is a complicated failure mode and there are still some unresolved issues with the distributed plasticity models. On the other hand, some shortfalls with the new method are inevitable due to the use of empirical effective length and moment modification factors.

5.4.2 Frame structures with rigid connections

Nine series of rigid frame structures have been studied using shell finite element analysis. They are: non-sway portal frames (Series 1 and 2), sway portal frames (Series 3 and 4), Γ shape frames (Series 5), portal frames with overhang (Series 6), two bays/storey frames (Series 7 and 8), and single bay gable frames (Series 9). In Series 1 to 4, the effects of different beam-column connections were studied. The results indicate that the connection details have very significant effects on the frame capacities. The FEA results also indicate that the adjacent members’ interactions are very important for the lateral torsional buckling failure. This issue is not addressed in the current design specifications. The main shortfall of the current design process and the new refined plastic hinge analysis is that the interactions among members and between members and connections are not accounted for directly. The accuracy of the simplified method may be compromised compared with the more detailed FEA.
5.4.2.1 *Single bay single storey non-sway portal frames (Series 1 and 2)*

The geometrical and load configuration of the first two series of frames are shown in Figure 5.31. Series 1 frames have fixed bases while series 2 frames have pin bases. Both beam and columns are made of 250UB37.5 sections. The out-of-plane restraints are only present at the beam-column connections. A point load is applied at the midspan of the beam. According to the finite element analysis, lateral-torsional buckling of beam will be the dominant failure mode. Sixteen frames have been studied. Each frame is identified by its support type, column height and half its bay width. For example, a fixed support frame that is 4 metre high and 6 meters wide is called f43.

![Series 1: Non-sway frame with fixed bases](image1)

![Series 2: Non-sway frame with pinned bases](image2)

**Figure 5.31 Configuration of Simple Non-sway Portal Frames**

The results from the shell finite element analyses indicate that the ultimate load of the frame structures has a close relationship with beam-column connection types. When plastic deformations occur at the beam-column connections, the out-of-plane and twisting rotation restraints they provide for the beam are significantly altered. For example, the effective length factors can be used successfully to describe restraint conditions of the beams for a variety of lengths if the frame structures are considered as elastic (see Figure 4.24). However, the correlations between the ultimate capacities of the beams in the frame structures with corresponding elastic restraint conditions are not very satisfactory (see Chapter 4). Slender beams in the portal frames have much higher ultimate capacities than isolated beams.
Four different types of beam-column connections were modelled and analysed. Type 2 (extended flange) connections are the most common rigid connection and Type 4 (idealized rigid) connections allow no deformation within the connections (see Figure 4.9). The ultimate loads of Series 1 and 2 with these two types of connections are shown in Figure 5.32.

In general design practice, the connections are considered as either rigid or flexible (either fully connected or pinned). Only in rare occasions are semi-rigid connections considered in the analysis. However, the effects of the details about the rigid connections on out-of-plane and twisting rotation restraints are always neglected. This leads to a significant shortfall. For example, the ultimate load discrepancies between frames with Type 2 (extended flange connection) and 4 (idealized rigid) connections are generally above 15%. For shorter span frames, the differences may reach up to 40%. The frames with Type 2 and 4 connections are both regarded as rigid frames and are treated the same in analysis. However, their capacities are significantly different. Therefore, a conservative approach must be taken in design analysis if the connection details are not considered. The stiffness equivalent to Type 2 connections should be adapted in general practice. The ultimate capacities calculated using the current design approach (AS4100) are shown in Figure 5.32. The ultimate loads were
calculated using the design software Microstran 8.0 with the design load factors for each member obtained from linear elastic analyses. The ultimate load was taken as the initial load times the minimum load factor in the structures. For comparison purposes, the capacity factor ($\phi$) is not considered in the calculations. During the design checks, the effective length factors are assumed as one for all the members (both forms of out-of-plane buckling).

The frames’ ultimate loads from the new method are also plotted in Figure 5.32. The effective length factors for all members are also assumed to be 1.0. These effectively ignore the additional out-of-plane and twisting rotational restraints of beams from the adjacent members and connections. The results are surprisingly close to those calculated from the normal design procedure (AS4100) and generally lower than those from finite element analyses which considered full beam-column connection details. For the two metre (shortest) span frames (f22, p22, f42, and p42), the ultimate loads from the new method are higher than the FEA results. For small span structures, the strains in the beam-column connections are much higher than those in the large span structures at ultimate loads. The new method does not consider the lateral torsional buckling effects on the connections. Therefore, the over-prediction of ultimate loads may occur when the connections are the critical component.

![Figure 5.33 Ultimate Loads of Series 1 and 2 Non-sway Frames ($k_e = 0.7$)](image)
Figure 5.33 shows the ultimate load results when the effective length factors are taken as 0.7 in the new refined plastic hinge analysis for both lateral torsional and column flexural buckling. Finite element analysis results for frames with Type 4 connections are also shown in the same graph. The results show reasonable correlation between the two analyses. However the additional capacities predicted by the new method for small span structures may not be acceptable (up to 15% in one case). The effective length factors of 0.85 may be more appropriate for portal frame structures with rigid connections.

A different approach can be used to study the ultimate loads of the portal frame structures, ie., comparing the isolated failed beams in the structures to simply supported beams with equivalent lateral restraint conditions. A beam curve is plotted in Figure 5.34 is based on a moment modification factor $\alpha_m$ of 1.6. The $\alpha_m$ value comes from averaging all the factors based on linear elastic analyses.

![Graph showing moment capacity of beams in portal frames with Type 2 connection](image)

**Figure 5.34 Moment Capacity of Beams in Portal Frames with Type 2 Connection**

The maximum bending moment of the beam can be calculated from FEA results based on certain assumptions ($\alpha_m$, $k_e$, etc.). The FEA predictions are much higher than the design beam curve, whereas the new refined plastic hinge analyses agree better with the design code predictions. In the new method, the effective length factor for
each member is taken as 1.0. These assumptions supposedly represent the elastic out-of-plane and twisting rotational restraints that existed in the frames with Type 2 connections. However, the discrepancy between the new method and distributed plasticity model is significant especially for larger span structures.

One logical step forward for improvement is that the new method needs to update its moment modification factor calculations after initial yielding occurs in the member, and hopefully, the ultimate frame capacities will agree better with the finite element analysis results. Unfortunately, the modification of the program does not yield satisfactory results. While the ultimate loads for larger span frames increase, the small span frames also increase proportionally. The increase of beam ultimate failure load in the frame is the result of member-to-connection interaction in the plastic state which depends on beam and adjacent member interaction. When these two issues are not resolved, the simplified method as proposed here may yield overly conservative results.

![Figure 5.35 Moment Capacity of Beams in Portal Frames with Type 4 Connection](image)

Finite element analysis results for frames with Type 4 connections and the ultimate loads from the new refined plastic hinge analyses that assume the effective length factor of each member to be 0.7 are plotted in Figure 5.35. The trends are similar to...
frames with Type 2 beam-column connections. The beams’ maximum failure loads from the new method agree reasonably with those predicted by the beam curves ($\alpha_m = 1.6$). However, they are significantly lower than the results from the distributed plasticity analyses of larger span frames. It is worth noting that the maximum moments from the new method are lower than the corresponding maximum elastic buckling moments. On the other hand, the maximum moments for large span frames are higher than those in the elastic buckling analyses if the finite element models are used. In these cases, the post-buckling strength of the frame structures is significant. If the portal frame structure has a bigger span, it has more significant post-buckling capacity. It would be difficult to simulate this behaviour using the simplified analysis technique without considering the details of connections and adjacent members’ interactions for out-of-plane buckling.

Load-deflection relationships for 4m, 6m, and 8m span frames are shown in Figures 5.36 to 5.39. Each graph consists of data from both the finite element analyses and the new refined plastic hinge analyses. In general, the finite element models are slightly more flexible than the refined plastic hinge models in the elastic range. Sometimes this can be quite significant. For example, p22 and f22 frames are considerably stiffer with the new method (see Figure 5.36).
Figure 5.37 Load-Deflection Curves of 6 m Span Frames

Figure 5.38 Load-Deflection Curves of 8 m Span Frames
When using the new analysis method, the load-deflection curves for the 6m and 8m span frames agree quite well with the finite element analyses in the elastic range. For these frames, the proportions of connection size to failed beam’s length are smaller. The effects of beam-column connections on the frame stiffness are negligible.

The ultimate loads from the distributed plasticity analyses are significantly higher for larger span frames. The additional capacities come from the more advanced treatment of plastic redistributions using FEA. The simplified analysis method considers plastic redistribution, however, the new refined plastic hinge formulations include many approximations. For example, the effective length factors and the moment modification factors are assumed to be unchanged after initial yielding occurs. When plastic deformations occur in the connections and members, the assumed load and boundary conditions of the members will change. These second order effects should be considered in the analysis, otherwise the additional plastic capacities would not be captured. Since the new method uses member out-of-plane capacity checks according to the current design specifications (AS4100), it inherits many of the limitations from the current two-step design philosophy. Unfortunately, the effects of yielding at connections and members related to frame structures’ lateral torsional buckling are
not well studied, therefore the improvement for the simplified method are difficult to carry out.

When more appropriate effective length factors are used in the new refined plastic hinge analysis, a higher ultimate capacity can be obtained and the result is closer to that from finite element analyses. Figure 5.39 shows the load-deflection curve for frame f24. The load-deflection curve from the refined plastic hinge analysis fits better with that from the finite element model with Type 4 connections. In the refined plastic hinge analysis, the effective length factors \( k_e \) are assumed to be 0.7 for all members.

**5.4.2.2 Single bay single storey sway portal frame (Series 3 and 4)**

Series 3 and 4 frames are simple portal frame structures. The geometrical configurations of these frames are the same as the first two series. Since the loads are not symmetrical, the benchmark finite element analyses use full frame models instead of half frame models. Series 3 frames have fixed bases whereas Series 4 frames have pinned bases (see Figure 5.40). The height is 4 meters for all the frames. The bay widths of the structures ranged from 6 to 10 meters. There are six frame structures in these two series. Each frame is subjected to seven load cases. The horizontal to vertical load ratios are 0, 0.91, 0.33, 0.727, 1.818, 4.545, and \( \infty \). The beam-column connections for these two series of frames are modelled as truly rigid types in the benchmark finite element analyses.

![Figure 5.40 Configurations of Series 3 and 4 Frames](image)
Frames with truly rigid beam-column connections have considerably higher capacities compared with common rigid frames. For rigid frames, the columns are able to provide some degrees of out-of-plane rotational restraints to the beam. The rigid joints prevent warping deformations in the beam member if it undergoes out-of-plane buckling. The truly rigid frames modelled for benchmarking in Chapter 4 are the most extreme cases that disregard the plastic deformation effects on frame capacities. The stiffness of connections plays a very important role in the structural failures related to stability issues. However, in the common design practice, warping and out-of-plane rotational restraints are often neglected in determining the beam’s capacities. For example, the effective length factors for all members are treated as one when a design package Microstran is used to model the benchmark frames. As a result, the rigid frame capacities are often under-estimated. Using the new refined plastic formulation, the effects of beam-column rigidities can be taken into consideration indirectly by specifying the effective length factors in the analysis. The results from the new method are expected to be closer to the finite element analyses.

Since different ratios of horizontal and vertical loads are applied to Series 3 and 4 frames, the bending moment gradients along the beam changed for each load case. In Series 1 and 2 frames, the bending moment distributions are similar because the same load case is used. The effects of moment modification factors \( \alpha_m = 1.6 \) in the analysis would not be noticed. For Series 3 and 4 frames, the \( \alpha_m \) values calculated from the elastic analysis ranged from 1.57 to 2.50. The moment modification factors are calculated using the midspan, quarter point and maximum bending moment in the members. The empirical equation is considered suitable for general cases.

Figures 5.41 to 5.46 show the frame capacities from four analyses, the nonlinear finite element analyses, the new refined plastic hinge analyses assuming effective length factors of each member to be 0.7 and 1.0, and the linear analyses using design program Microstran. The horizontal loads are normalised by the corresponding load that cause the formation of the first plastic hinge when acting on its own. The same treatment is used for the vertical loads.
Figure 5.41 Ultimate Capacities of Frame f46

Figure 5.42 Ultimate Capacities of Frame p46
Figure 5.43 Ultimate Capacities of Frame f48

Figure 5.44 Ultimate Capacities of Frame p48
Figure 5.45 Ultimate Capacities of Frame f410

Figure 5.46 Ultimate Capacities of Frame p410
Based on the results of Series 3 and 4 frames, the following observations can be made regarding the performance of the new refined plastic hinge model:

1. When the effective length factors are assumed to be 1.0 in the refined plastic hinge analysis, the frame capacities agree very well with those from the linear elastic analyses/design using Microstran. This indicates that the formulation of the new refined plastic model does not contradict the current two-step design procedure. The new method is able to reduce the one step and the results are not “worse off” using the same frame design procedure.

2. The refined plastic hinge analyses significantly under-predicted the idealized rigid frame capacities even when warping constraints and out-of-plane rotational restraints from the column are considered. For the frames with a larger span (10m), the predictions from the new method are only half those of finite element analyses. The new refined plastic hinge method does not allow plastic redistribution after one of the members reaches the corresponding isolated maximum member capacity. The segment’s member capacity is basically unchanged since the moment modification factor is established during the early stage of the analysis. Realistically, the restraint conditions of beam segments will change as yielding progresses at the segment ends. The increase of frame capacities in real structures is due to these second-order effects. In some cases, the ultimate capacities are higher than elastic buckling capacities. This implies that post-buckling capacities exist in some rigid frames which is similar to the local buckling phenomenon in thin-wall steel structures. The new method is not able to take these effects into consideration dynamically during the analysis (eg. changing the effective length factors), and hence the analyses is very conservative in general.

3. Using the current design specifications, the effective length factors are obtained through observations of frames’ geometrical configurations. The guidelines on choosing the $k_e$ value are very brief. Generally speaking, the $k_e$ value are over-simplified and over-conservative. The effective length factor of 0.7 seems suitable to describe the beams in rigid portal frames according to
elastic finite element analyses. However, it is not suitable to define the beam behaviour in both the nonlinear finite element analyses and the new refined plastic hinge models. Unlike the distributed plasticity analysis that simulates lateral torsional buckling based on the initial imperfection input, the new refined plastic hinge analysis uses the effective length concept in the analysis to capture the lateral torsional buckling and flexure buckling behaviour indirectly. The new method is expected to have some limitations. Nevertheless, the frame capacity charts show that the general trend from both numerical benchmark model and the new method are similar.

4. When the portal frame is subjected to a predominant horizontal force, the structural failure mode may switch from beam lateral torsional buckling without forming a plastic hinge in the bases to plastic hinge(s) formation followed by beam failure. In Figure 5.43, the frame capacities from both of FEA benchmark model and the new method have horizontal load factors in excess of 1.0. This implies that the frame capacities are larger than those calculated based on first plastic hinge formation. The new refined plastic hinge analysis is able to cope with the case of plastic hinge formation followed by out-of-plane buckling.

Some load deflection curves for Series 3 and 4 frames are shown in Figures 5.48 to 5.51. Observations of these curves indicate two shortcomings of the new refined plastic hinge analysis. First, the plastic hinge models are slightly more flexible than the benchmark frames in vertical deflections, especially in the cases of short span frames. The beam-column connections in the finite element models are assumed to be truly rigid. The stiffness of the frames is related to the ratio of connection’s size to span of the frame. The new method does not take this subtlety into consideration. For example, the plastic hinge may assume to form at the centre of the connections. As a result, the frame models have larger deflection in the elastic ranges. Second, the new refined plastic models are not able to fully trace the plastic load path in some load cases. This is believed to be due to the usage of moment modification factors in the proposed method. Load cases “H=20 V=66” in frame p46 and “H=100 V=55” in frame f48 produce bending moment diagrams with the maximum moment located at the member end. The moment gradient is very steep (see Figure 5.47) near the
maximum moment while a large part of the beam has opposite moment. Therefore, with slight increase of load during the analysis, the peak moment increases significantly. In both these load cases, the analyses terminate very abruptly. In finite element analyses, the plastic deformations of these two load cases are concentrated at the right end of the beam connections. The new method is not able to capture this behaviour.

Figure 5.47 Bending Moment Diagram of Frames p46 and f48

Figure 5.48 Vertical Load versus Midspan Deflection Curves of Frame f46
Figure 5.49 Horizontal Load versus Knee Deflection Curves of Frame f46

Figure 5.50 Vertical Load versus Midspan Deflection Curves of Frame f48
5.4.2.3 Frames with a overhang segment (Series 5 and 6)

Series 5 are Γ shape frames and Series 6 frames are portal frames with an overhang (see Figure 5.52). Both these frames consist of a cantilever segment that is susceptible to lateral torsional buckling failure due to the application of a concentrated load at the end of the beams. According to current design specifications, the capacities of cantilever segments can be calculated based on an effective length factor equal to 2.0 and a moment modification factor equal to 1.25. However, the finite element analyses indicate that the cantilever segment may have significantly higher capacities than the code predictions for beam segments in rigid frame structures.

Series 5 includes 12 frames. The ultimate loads from both the FEA benchmark models and the new refined plastic hinge models are shown in Figure 5.53. There are two series of refined plastic hinge analysis results. One series of models assume the effective length factor of cantilever segment to be 2.0 based on the design code, and the others assume it to be 1.5. Comparisons of the results from the new method with
the benchmark solutions indicate that the simplified method based on the current design code effective length factor is very conservative. The ultimate loads predicted by finite element analyses with ideal rigid connections are 30% to 70% higher than those predicted by the new method. When the effective length factor is taken as 1.5, the predicted capacities from the new method increase substantially but they are still significantly lower than the benchmark models.

Figure 5.52 Configurations of Series 5 and 6 Frames

Figure 5.53 Ultimate loads of Series 5 Frames
The ultimate loads shown in Figure 5.53 follow a similar trend for both finite element analyses and plastic hinge analyses. However, if the ultimate bending moments in the overhanging segments are plotted with the beam curve, a very different picture will emerge. Obviously, the maximum bending moments from the refined plastic hinge analyses match well with the beam curve that assumed a moment modification factor of 1.25, but the results from the finite element analyses are completely out-of-character. The finite element models (see Figure 5.54) consist of ideal rigid beam-column connections (modelled by MPC). Material properties of the models and the analysis techniques are the same as that of simply supported beam models. The simply supported beam models are very consistent with the design code predictions. Further investigations are essential to fully understand the behaviour of these structures.

Selected load-deflection curves for Series 5 frames are shown in Figure 5.55. The FEA results are from two models. One consists of ideal rigid beam-column connections and the other has no web stiffener. The refined plastic hinge analyses agree reasonably well with the rigid benchmark frames in the elastic range. The plastic load paths are also similar. On the other hand, the ultimate load predictions
using the simplified method are not satisfactory. Considering the differences between the rigid finite element models and the models without web stiffeners, the simplified method is unlikely to trace the exact load path of the frames without additional consideration of connection details.

![Figure 5.55 Vertical Load versus Overhang End Deflection Curves of Series 5 Frames](image)

Series 6 includes 8 frames. The frame heights are either 2 m or 4 m. The overhanging segments range from 1.5 m to 3 m. The ultimate loads from the finite element analyses and the refined plastic hinge analyses are shown in Figure 5.56. The finite element analyses use MPC to simulate the rigid beam-column connections. The results from the finite element models are significantly higher than those from the simplified analysis method. In the refined plastic hinge analyses, the effective length factors of overhanging segments are assumed to be either 2.0 or 1.5 for comparison purposes. All the other members have an effective length factor of 0.7. The refined plastic hinge analyses show that, apart from frame F4-15, the failure of these frames is due to lateral torsional buckling of the cantilever segment. The same behaviour is also observed in the FEA benchmark frames.
Typical load-deflection curves for frame F4-2 are shown in Figure 5.57. These curves indicate that the finite element models are significantly stiffer than the refined plastic hinge models. Linear load-deflection curves obtained from the elastic analysis package Microstran are also plotted in the same figure. They agree well with the refined plastic hinge analysis results. This implies that the rigid MPC connections in
the finite element models play an important role in determining the structure’s displacements. Using the simplified method, adapting different effective length factors for the beam segment is intended to simulate the effects of member end restraints. The results show that this technique has a greater influence on the frame capacities. However, this method can not successfully simulate the structure’s load deflection relationship if smaller appropriate effective length factors are selected.

5.4.2.4 Double bay/storey frame structures (Series 7 and 8)

The geometrical and load configurations of Series 7 and 8 frames are shown in Figure 5.58. Eight load cases are applied to Series 7 frames. In the first three load cases, the horizontal loads (P1) are zero. The load ratios of P2 and P3 are 1:1, 0:1, and 1:2 for these three load cases. The other five load cases include a 400kN of horizontal forces. The vertical load ratios for P2 and P3 are also varied in different load cases.

Ultimate frame capacity results from five models are plotted in Figure 5.59. The effective length factors for each frame member were taken as 0.5, 0.7, and 1.0 in the refined plastic hinge analyses. The ultimate frame capacities from FEA and the design capacities from Microstran are shown as line graphs. In the Microstran models, the effective length factors for each member are taken as 1.0. Capacity reduction factors are not used with Microstran results for comparison purpose.
When the effective length factors are assumed to be 1.0 in the refined plastic hinge analyses, the ultimate loads are nearly the same as those predicted by Microstran using the current design code method. When the effective length factors are taken as 0.5, the predictions of the new method are 10% higher than the finite element analysis for the first three load cases. In these three load cases, the dominant internal forces in the beam segments are the bending moment where the calculations of the beam’s lateral torsional buckling capacities are over-estimating the segment’s constraint and load conditions. For the other five load cases, out-of-plane buckling due to compression forces is the main issue for the beam segments. The assumptions of $k_e = 0.5$ for compression must be reasonable. As a result, the new refined plastic hinge method agrees well with the finite element analyses. When the effective length factors are taken as 0.7, the refined plastic hinge analysis results are 15% to 30% lower than the FEA results. Using the average value of 1.0 and 0.5 for effective length factor is a sensible approach for common rigid frame structural design.

![Figure 5.59 Ultimate Capacities of Series 7 Frames](image)

Typical midspan load deflection curves for Series 7 frames are plotted in Figure 5.60. When the effective length factors are assumed to be 0.5, the simplified method agrees reasonably well with the FEA results. The refined plastic hinge models exhibit some plastic deformation characteristics but can not capture the full extent of plasticity as the shell finite element models do.
When the effective length factors are taken as 0.7, the refined plastic hinge models do not show any plastic deformations. It implies that the proposed refined plastic hinge analysis does not produce “false” plastic behaviour when conservative constraint conditions are assumed. Similar behaviour is also found with the refined plastic hinge analyses of the other series of frames.

Figure 5.60 Typical Load-Deflection Curves of Series 7 Frames

Series 7 frames have six degrees of redundancy and therefore plastic redistributions would be more significant compared with determinate structures. Shell finite element analyses are able to explicitly account for these details. It is very difficult for the simplified models to simulate the same level of plastic behaviour. Series 8 frames also have six degrees of redundancy. Considerable discrepancies also exist between the finite element analyses and simplified methods including the current design method.

With Series 8 frames, eight load cases were studied using the shell finite element analyses. The first three load cases do not include horizontal forces. Vertical concentrated loads are applied at either midspan of first or second floors. In the next
five load cases, equal loads are applied at midspan of both floors. A horizontal concentrated load is presented for each load case. In load cases 4 to 6, a horizontal load is applied to the first floor (2h1 – 2h3). In load case 7 and 8, it is applied to the second floor (3h1 – 3h2).

The results from finite element analyses, design package Microstran, and the new refined plastic hinge analyses are plotted in Figure 5.61. As for Series 7 frames, the Microstran predictions are almost identical to the results from the refined plastic hinge analysis with effective length factors of 1.0. When the effective length factors are assumed to be 0.5 for every member in the structure, the ultimate load predictions from the new method are within 15% to those from finite element analyses. When \( k_e = 0.7 \) was used, the new method will yield the most appropriate results for design purpose.

![Figure 5.61 Ultimate Capacities of Series 8 Frames](image)

Typical load-deflection curves are shown in Figures 5.62 and 5.63 for load case 2h3. Figure 5.62 shows the beam midspan vertical deflections. It indicates that the refined plastic hinge model is less stiff than the finite element model. Two reasons may contribute to this problem. First, the simplified method does not consider the size of the rigid beam-column connections. Maximum bending moments are allowed to exist at the centre of connections. Second, the FEA is able to account for the combined effects of both normal and shear stresses. The vertical loads used in the finite element
model are in the form of point loads in the member’s web. The local shear deformations are accounted for in the finite element analyses. On the other hand, the refined plastic hinge method is not able to consider the effects of the local shear deformations at the location of applied load. When the refined plastic hinge analysis uses the effective length factor of 0.5, the load path pattern of the analyses is very similar to those from the FEA. The plastic deformations are more significant in the first floor than those in the second floor.

Figure 5.63 illustrates the horizontal deflections at the first floor level. In the elastic range, both finite element model and refined plastic hinge model are identical. When the effective length factors are assumed to be 0.5, the refined plastic hinge analysis is also able to demonstrate significant plastic deformations. It seems that the beam-column connection details play an important role in the frame’s plastic behaviour. Using effective length factors to represent the end connection performances is not a perfect solution but should be considered as adequate for common design purpose.

![Figure 5.62 Vertical Load versus Midspan Deflection Curves of Series 8 Frames](image-url)
5.4.2.5 Single bay gable frames (Series 9)

Series 9 frames are non-orthogonal structures (see Figure 5.64). The lateral torsional buckling behaviour of a member with skew end connections is less known. For design purposes, the effects of skew member ends are disregarded. Therefore, there is no special treatment for these types of frames in the refined plastic hinge analysis. There are six load cases with Series 9 frames. The first three load cases consist of two downward vertical loads of 110kN and the others have two upward loads of 110kN. The horizontal load in each load case is either 0, 110, or 220kN. As for all other series of frames, the refined plastic hinge models of Series 9 frames use centreline dimensions of the benchmark frames.

The ultimate capacities of the portal frame are shown in Figure 5.65. The refined plastic hinge analyses include three models with various effective length factors to simulate the member end constraint conditions. When the effective length factor is 0.5, the new method has the best agreement with the FEA results. However, the simplified method over-predicts the frame’s capacities for the downward load cases compared with the finite element analyses. When the effective length factor is
assumed to be 1.0, the new method agrees very well with the design package Microstran predictions. Compared with the finite element analyses, the assumption of effective length factor of 1.0 for rigid frame produces overly conservative results. When the refined plastic hinge models assume $k_e = 0.7$, the predicted ultimate capacities are all below the finite element analyses results. There are large margins between $k_e = 0.5$ and $k_e = 1.0$ models. For design purposes, it is best to take the middle ground, i.e., assume the effective length factors to be approximately 0.75 for member in rigid frame structures while using the simplified analysis method.

![Diagram](image)

**Figure 5.64 Configurations of Series 9 Frames**

![Graph](image)

**Figure 5.65 Ultimate Loads of Series 9 Frames**

Figures 5.66 and 5.67 are the vertical and horizontal load-deflection curves for load case 2h3. This load case includes a 220kN horizontal load applied at the knee joint.
and two vertical loads applied at the midspan of rafters. The vertical load deflection curves indicate that there are some differences between the refined plastic hinge method and the finite element analyses. The finite element model is slightly stiffer whereas the refined plastic hinge model and the linear elastic model (Microstran) have a good agreement in the elastic stage. For knee drift cases, all three analyses agree very well in the elastic range.

![Vertical Load versus Midspan Deflection Curves for Series 9 Frame](image)

**Figure 5.66 Vertical Load versus Midspan Deflection Curves for Series 9 Frame**

The observations from the FEA indicate that the left rafter (where horizontal load was applied) was the member that first undergoes lateral torsional buckling. Plastic deformation within this beam segment starts at 70% of the ultimate load. For the right hand side rafter, plastic deformation does not start until near structural failure. In this load case, the predicted ultimate loads from FEA and the refined plastic hinge model that assumes $k_e = 0.5$ are nearly identical. The load-deflection curves for the failed rafter also have a good agreement. However, the new method’s prediction of the plastic behaviour of the right rafter is slightly different from the FEA. Obviously, lateral torsional buckling failure is a very complex phenomenon. When some subtle issues are neglected such as interactions due to out-of-plane and twisting deformations, the simplified method is not expected to fully agree with the distributed
plasticity analysis. On the other hand, there are many aspects that idealized in the finite element model and the results from these analyses by no means fully represent the real structure behaviour.

![Graph showing Knee Deflections (mm) vs Applied Horizontal Loads (kN)](image)

**Figure 5.67 Horizontal Load versus Knee Deflection Curves for Series 9 Frame**

### 5.5 Graphical User Interface

The graphical user interface (GUI) is one of the most important components of engineering programs. Currently, elastic frame analysis packages are widely adapted by design engineers. User-friendliness of the graphical interface has become the main focus for frame analysis software development. These programs are able to carry first order or second order elastic analysis with integrated member capacity checks. The steel frame design has become very straightforward with this development. It is certain that without an advanced graphical interface, engineers would be reluctant to adapt the advanced analysis for its potential economical benefit. Therefore, a GUI has been developed to demonstrate that steel frame design can be further simplified and improved with the new refined plastic hinge method. This GUI is intended to illustrate
how the improved refined plastic hinge method can provide the designer with additional understanding of frame structures.

Modelling of a structure using the proposed GUI includes two steps. First step is to setup the geometry of the structure. A graphical input screenshot is shown in Figure 5.68. When a member is drawn with a mouse or a keyboard, the program automatically divides the drawn member into a designated number of elements. In other words, when an element is created, it will associate with a corresponding member property set. The behaviour of the elements will depend on the effective length and end restraint conditions of the member. The out-of-plane constraint conditions of the member are expressed in terms of effective length “k” factors following the Australian steel design code (AS4100).

![Figure 5.68 Graphical input of Structural Geometry](image)

It is quite often that lateral restraints are only present in one flange along the frame members (e.g., intermediate top flange lateral supports along the members). In these cases, sets of second member properties are assigned to the individual elements to
cope with different modes of lateral torsional buckling failure. As shown in Figure 5.14, lateral torsional buckling may occur on both sides of tension and compression flanges. The secondary member properties dialog is shown in Figure 5.69. This dialog will appear when individual elements are selected to represent the intermediate supports. The capacities of the beam segments are indeterminate prior to the first step of nonlinear analysis. As the analysis proceeds, the smaller of the two bending moments calculated from different sets of member properties will be adapted as the element’s lateral torsional buckling capacity.

The second step of modeling is to input boundary conditions and loads (see Figure 5.70). Only nodal load input is available for the refined plastic analysis in its current state. Some modifications will need to be made in the future to handle various types of distributed member loads. Other desirable features include the input of load cases. When using advanced analysis, each load case will be analyzed separately. Internal force envelope design approach is not suitable for advanced analysis method. The bending moment or axial force envelope approach is generally over-conservative and
is often not correct when the structures exhibit significant plastic characteristics. With nonlinear analyses run separately for different load cases, the ultimate load factors from each load case can be ranked thus the designers are able to have a better understanding of the structure’s behaviour.

Using the new refined plastic hinge method, the analysis time is of the same order as the current elastic analysis/design processes. The time required to run the structure shown in Figure 5.71 is approximately two seconds for one load case. The effective length factor of 1.0 is assumed in the example frame shown in Figure 5.71. The refined plastic hinge analysis shows that the steel frame is able to carry about 50% of the input loads until one of the beams failed in the out-of-plane buckling mode. For comparison, a design check was carried out using Microstran for the same structure and the result is shown in Figure 5.72. The member with the smallest of capacity factors in the Microstran design check coincides with the failed member in the refined plastic hinge method. The lowest factor from member capacity check also indicates that the structure will also fail under half of the input loads.
Since the effective length factor is assumed to be 1.0 in this example, there is not much advantage in using the advanced analysis. However, if more appropriate factors are used to account for the effects of the rigid connection (say, assuming $k_e = 0.7$ for all members), the predicted failure load from advanced analysis will be significantly higher. The ultimate load becomes 70% of the input load. Material yielding starts to play a role in reaching the ultimate load when $k_e = 0.7$. Using the current design method can not capture those plastic deformations even with the same assumptions on member capacities. Predictions for the current method (AS 4100) will be very conservative for frames with rigid connections for which the plastic redistributions are significant.
For post-processing purposes, the GUI for the new refined plastic hinge analysis is able to give the bending moment diagram and deflections for different loading stages. One significant difference from the general frame design packages (e.g., Microstran, SpaceGass, and the new GUI, and etc) is that the new GUI is also able to plot load-deflection curves for a selected nodal point. With the detailed information for load paths up to the structural failure for each load case, design engineers can produce more economical and reliable structures.

In the current state, the GUI for the new refined plastic hinge method is still very basic and the new method will not be able to compete with the mainstream elastic analysis/design packages. The three most important advantages of the new method are the input of member properties before analysis, ability of plotting load-deflection relationship and direct prediction of ultimate capacities. The new method has the potential to be widely accepted for general design practice in the near future.
5.6 Summary

It is essential for the advanced analysis method to be able to deal with member out-of-plane buckling problems. Absence of this ability will significantly reduce the usefulness of the advanced analysis method in general frame design. The lateral torsional buckling failures are complex phenomena. They are difficult to account for in the frame analysis. The state-of-the-art finite element analysis methods are able to explicitly deal with this failure mode but they are not applicable to general design practice.

A two dimensional refined plastic hinge method is developed and is capable of including out-of-plane member capacity checks in the analysis. The effects of lateral torsional buckling are handled by the use of the inplane stiffness reduction factor. Two characteristics of lateral torsional buckling are identified as the most critical factors in the new stiffness matrix formulation. The first one is bending moment variation along the beam segment. The effects of moment gradient are captured by using the moment modification factors, as for those in the Australian design code AS4100. The other critical factor is beam segment restraints. In rigid frame structures, the restraints of a member depend on both the rigidities of the connections and adjacent members. The connection details were found to be very important in the FEA models (see Chapter 4). The effects of beam segment restraints are modelled by using the effective length factors in the new refined plastic hinge analysis.

In the new refined plastic hinge analysis, each unrestrained beam segment is divided into a number of elements. The beam segment properties are shared by all of the sub-elements including the segment’s end restraints and bending moment gradient. The stiffness of each sub-element is related to not just the section capacity but also the beam segment’s out-of-plane capacities. The accuracy of the new method relies heavily on the user’s judgement on beam segment restraints (choosing of the beam effective length factors).

Compared with the results from the current design method and numerical benchmark solutions (see Chapters 3 and 4), the accuracy of the new refined plastic hinge method is adequate. Even though it achieves a very good agreement with benchmark solutions
for simply supported beams, due to the use of approximate equations for out-of-plane capacity in the formulation, the new method is generally more conservative for rigid frame structures. Using this method, separate member capacity checks can be eliminated. More importantly, load-deflection response of the frames can be obtained. Since the new formulation uses the empirical equations to deal with the lateral torsional buckling effects, the proposed refined plastic hinge method will not contradict the current design rules.
Chapter 6. Conclusions

6.1 Conclusions

A practical advanced analysis method provides a more efficient alternative to the current two-step design method. However, before the advanced analysis method can realize its potential as a tool for practical design, a number of limitations must be overcome. Currently available advanced analysis methods are capable of including the effects of plastic hinge formation, column buckling and local buckling, but not lateral torsional buckling, which is one of the most critical failure modes. Without resolving this issue, the use of advanced analysis as a practical design tool will be severely restricted.

This thesis has presented the details of an extensive investigation aimed at widening the scope of advanced analysis to include the design of steel frame structures subject to lateral torsional buckling effects. A comprehensive literature review was undertaken related to both advanced analysis and lateral torsional buckling. The investigation on the structural member’s lateral torsional buckling behaviour and load-deflection response involved a large amount of finite element analyses of simply supported beams and rigid frame structures. Over 150 analyses of simply supported beams and 400 analyses of rigid frames were conducted to study their lateral torsional buckling behaviour. Based on the numerical analyses, a new refined plastic hinge analysis has been developed. The numerical analysis results of simply supported beams and rigid frame structures were used to verify the new advanced analysis. Since the finite element analyses included the effects of material yielding, initial geometrical imperfections and residual stresses, their results can also be used as benchmark solutions in future studies. The following is a summary of the most significant findings arising from the research project.

- The analytical studies of lateral torsional buckling are still in the elastic realm due to the warping torsional restraint and moment gradient problems. Closed form solutions for this structural instability are difficult to obtain. The inelastic lateral torsional buckling is virtually unsolvable using analytical methods and
the ultimate beam capacities are derived from a large amount of testing of simply supported beams. Three design specifications AS4100, AISC LRDF and Eurocode 3 were reviewed in this research. It was found that the warping restraints are handled by the use of effective length factors in all these design codes. Moment modification factors are used to simulate the effects of moment gradient. In the Australian standard (AS4100), the use of moment modification factor is slightly different. Instead of using the maximum elastic buckling moment ($M_E = \alpha_m M_o$), the elastic buckling moment for uniform bending ($M_o$) is used in the beam slenderness reduction ($\alpha_c$) factor calculations. Hence, the effects of moment gradient are considered separately from member slenderness.

- Distributed plasticity analyses based on shell elements are able to capture the effects of lateral torsional buckling accurately for simply supported beams for various load and support conditions. The numerical analysis results agree very well with both the elastic analytical buckling moment and the design beam curve (AS4100), which is based on experiment data.

- In order to simulate the idealized simply supported conditions using three-dimensional shell finite element models, ten types of boundary conditions were used. One of them, which used a multipoint constraint (MPC) system, was found to be able to replicate the idealized simply supported conditions used in the analytical solutions. The finite element model with such idealized simply supported boundary conditions will be very useful for various beam column models in the future research.

- Two other types of simply supported boundary conditions were also investigated. They are the warping restrained and the laterally fixed simple supports. Rigid surface elements were used to achieve these two boundary conditions. The elastic and inelastic buckling loads of the beams with warping restrained supports are very similar to those of idealized simply supports. The effective length factor for these end restraints is approximately 0.94. On the other hand, laterally fixed supports prevent out-of-plane rotation and also
provide complete warping restraints at the beam ends. The effective length factor of this type of beams is 0.5. Modelling laterally fixed boundary conditions is much simpler than those with idealized simply supports. If beam design curves need to be derived, the laterally fixed supports should be used in both numerical and experimental analyses.

- Effects of initial geometrical imperfections and residual stresses were investigated in this research. The imperfection shapes and their magnitudes have a significant effect on the beam ultimate capacity. The Australian design beam curve is based on the lower bound of test data. The worst possible imperfection shape with the maximum construction tolerance was used in the numerical analyses. The good agreement between the two shows that appropriate initial geometrical imperfections have been used in the finite element models. Residual stresses, on the other hand, do not have a significant effect on the beam ultimate capacities. The residual stresses cause premature yielding and significant stiffness reduction.

- Four loading situations were investigated in the finite element analyses of simply supported beams. They are: 1) uniform bending moment, 2) midspan concentrated point load, 3) quarter point loads and 4) uniform distributed load. The load height effects were also studied. It was found that the empirical equation in AS4100 that considers the moment modification factor and the slenderness factor separately is very straightforward. The effects of moment gradient in inelastic beams can be treated independent of member slenderness.

- The load-deflection curves can be easily obtained from the shell finite element analyses. The load-deflection responses of simply supported beams indicate that lateral torsional buckling is not a desirable failure mode. The beam strength drops abruptly once it reaches its ultimate capacity. There is not much reserve of strength once buckling commences. In general, prior to the occurrence of the buckling failure, the inplane load paths of the beams are nearly the same as those with full lateral restraints. The reductions of beam strength in the post-buckling range are impossible to predict. They might
relate to a range of factors including moment gradients, end constraints and member slenderness.

- Nine series of frame structures were investigated in this research including non-sway and sway single bay single storey frames, \( \Gamma \) shape frames, portal frames with an overhang, two bay single storey frames, single bay two storey frames and single bay gable frames. Patran Command Language (PCL) was used to develop suitable computer subroutines for more efficient finite element modelling. Initial member geometrical imperfections, membrane residual stresses, gradual section yielding, spread of plasticity, second-order instability, and lateral torsional buckling deformations were all explicitly and accurately modelled in these frame analyses.

- The nonlinear finite element analysis results are significantly higher than the design code predictions (AS4100). For frames consisting of slender beams, the design code (AS4100) underestimates the frame ultimate capacities. However, this is not the case when compared with the elastic buckling and elastic analytical solutions. With the use of an appropriate effective length factor in the analysis, a very good agreement can be achieved between the FEA results and hand calculations. This leads to the useful conclusion that the effective length factors should be changed when yielding occurs in the beam members. A more stable state can be achieved with material yielding for frames with slender beams. However, this effect is very difficult to quantify. The presence of higher frame capacities can be treated as a form of post-buckling strength.

- Four types of beam-column connections were modelled in the finite element analyses of frame structures. If the frames are fully laterally restrained, all these connections can be treated as rigid connections. However, the details of beam-column connections have very significant effects on the ultimate capacities of laterally unbraced frames. Stiffeners in the connections can greatly increase the beam segment’s resistance to lateral torsional buckling. Effects of connection details are more important for the nonlinear analyses than for the elastic buckling analyses. This means that the “elastic” effective
length factor and the moment gradient factor used in the current design methods are very conservative and are not able to account for the inelastic strength contributed from the redundancy of rigid frames.

- The analyses of frames with an overhang segment show that the out-of-plane rotational restraint is most crucial for the overhang segments. If this degree of constraint is not present, the overhang segment’s ultimate capacity will be significantly reduced. In general, the use of an effective length factor of 2.0 for overhang segments according to the design code is very conservative.

- In a frame structure, the beam segment restraints relate to the connection rigidities and the stiffness of the adjacent members. These two elements are both difficult to quantify. Besides the segment end restraints problems, the load height effects of lateral torsional buckling are also difficult to handle. It seems that it is more feasible for the designers to solve the beam segment capacity using the empirical methods with sound engineering judgement.

- The full range load-deflection responses indicate that the lateral torsional buckling behaviour of rigid frames is significantly different to that of simply supported beams. Generally, the rigid frame structures are able to withstand substantial displacements while maintaining the ultimate loads during the occurrence of out-of-plane buckling.

- The finite element analyses of frame structures show that when one of the members in the frame structures fails in lateral torsional buckling mode, the frames are not able to carry further loads when the proportional loading scheme is used in the analyses. There is a similarity between plastic hinge formation type failure and lateral torsional buckling failure in the rigid frame structures. The out-of-plane failure of a beam segment in the frames is the same as the plastic hinge formation with less capacity.

- Compared with fully laterally restrained frames, the inplane load-deflection response of a rigid frame is nearly the same in the elastic range. This
behaviour is similar to the simply supported beam. Also, once a lateral torsional buckling failure has developed in one of the beam segments, the rigid frame is not able to carry additional loads in a proportional loading situation. The restriction of plastic redistribution after lateral torsional buckling is the same for both simply supported beams and frame structures. This behaviour was incorporated into the proposed refined plastic hinge analysis.

- Results from the simply supported beams and frame structures were used as benchmark solutions to validate the new refined plastic hinge method.

- Distributed plasticity analysis of steel frames subjected to lateral torsional buckling is not suitable for practical design due to its complexity and requirements of excessive computational resources. Simple concentrated plasticity methods are feasible for steel frame design use.

- A new two dimensional refined plastic hinge analysis method has been developed and presented in this thesis. Lateral torsional buckling is considered implicitly in the analysis. The formulations of the new method adopt the existing treatments of lateral torsional buckling in the Australian design standard (AS4100). The moment gradient issue is dealt with by using the appropriate moment modification factors. The effects of connections rigidities and interaction of adjacent members are handled by using beam effective length factors.

- In order to account for the moment gradient and end restraints effects, the new method introduces a member property concept. An unbraced beam segment is modeled with four or more elements. All these elements share a common member property that includes information about end restraints and moment modification factors.

- Since no stress redistribution exists in the simply supported beams, the new method agrees very well with the finite element analyses in the prediction of both the ultimate beam capacities and the load-deflection response.
For frame structures, the gradual formations of plastic hinge are still allowed prior to lateral torsional buckling, and hence the ultimate frame capacities from the new method are normally higher than those from the current design procedure.

The plastic redistributions in rigid frames can be significant during lateral torsional buckling. These effects are very complicated and have not been considered in both the design codes and the new refined plastic hinge method. Hence, compared with the numerical benchmark solutions, the new method is generally more conservative.

The load-deflection responses from the new method are comparable with the FEA results when realistic beam segment restraints are used. With the use of the new refined plastic hinge analysis, member out-of-plane capacity checks are no longer needed.

A computer program implementing the new refined plastic hinge analysis has been developed with a Microsoft Window graphical user interface. A procedure is given for the program operation. Design of steel frame structures has thus become very straightforward using the new refined plastic hinge program. The program can even take into account the restraints in either the top and bottom flanges.

The new refined plastic hinge method takes lateral torsional buckling effects into consideration during the analysis. Using the new method, the designers are able to obtain the load-deflection response of general frame structures and achieve better understanding of the structural behaviour. Even though empirical equations are used for beams subjected to lateral torsional buckling effects, plastic hinges are allowed to form for the members with sufficient lateral restraints. The new method has significant benefits for redundant frame structures compared with the current design method.
6.2 Future research

Since this research was limited to two dimensional steel frames, the spatial frame behaviour has not been considered. Three dimensional refined plastic hinge methods have been developed by Liew (1998) and Kim (2002). However, these methods are not able to take lateral torsional buckling effects into consideration. To realise the full potential, the refined plastic hinge analyses have to consider the lateral torsional buckling behaviour for space frames. Priority should be given to this research topic.

Combined actions of axial compression load and in-plane bending have been studied in this research. The combined effects of torsion, axial compression, bending, and bi-axial bending must also be investigated for various end restraint conditions. The simply supported beam models developed in this project will be suitable for this purpose.

Large numbers of benchmark solutions for rigid frames subjected to lateral torsional buckling have been presented in this thesis. However, all these frames are two dimensional structures. Development of three dimensional benchmark frame solutions will be very useful for the future development of advanced analysis methods.

There are many difficult problems (eg, 3D connections effects) that must be solved prior to the development of a comprehensive three dimensional advanced analysis method. The immediate tasks for the improvement of the new method are to include the effects of semi-rigid connections (using M-θ, curves), distributed member loading, and consideration of multiple load cases while retaining the ability to include the lateral torsional buckling effects.
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