GEOMETRICAL MODEL FOR MOBILE RADIO CHANNEL WITH HYPERBOLICALLY DISTRIBUTED SCATTERERS

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Abstract– A geometrical channel model with hyperbolically distributed scatterers for a macrocell mobile environment is presented. This model provides the statistics of Direction-of-Arrival (DOA) information, as well as amplitude, time delay and phase data for wireless communication channel. The model is useful for designing a forward link beamformer. Simulation results of the channel impulse response and the probability density function for some channel parameters are presented. The data produced from the model is consistent with real world wireless channel characteristics.

Keywords- Multipath; channel model; hyperbolic; fading; macrocell.

I. INTRODUCTION

Over the past few decades, radio communication systems have undergone extensive developments. The demands that a radio system must fulfil are greater by the day. The propagation of radio signals on both forward (base station to mobile) and reverse (mobile to base station) links is affected by the physical channel in several ways. These physical objects and structures are: buildings, hills, streets, and trees. The collection of objects in any given physical region describes the propagation environment [1].

A signal propagating through a wireless channel usually arrives at the destination along a number of different paths, referred to as multipaths. These paths arise from scattering, reflection, refraction or diffraction of the radiated energy off objects that lie in the environment. The received signal is much weaker than the transmitted signal due to phenomena such as mean propagation loss, slow fading, and fast fading. For analysing the performance of wireless communication systems, a statistical channel model (which provides the Direction-of-arrival (DOA) and time-of-arrival (TOA) of the multipath components) is required.

A common channel modelling strategy is the statistical description of time variant fading effects of the physical channel due to moving terminals, moving obstacles and the transmission environment [2]. However, those scalar stochastic channel models do not provide any directional information. Therefore, they are not directly applicable for systems with multiple antennas [3]. A possible extension of these models is the Directional Gaussian Scattering (DGS) model [4], which assigns directional information to uncorrelated scatterers. This information has to be obtained from practical measurements.

Liberti and Rappaport [5] developed a Geometrical Based Single Bounce model (GBSB) for microcells. The GBSB model assumes that scatterers are uniformly distributed in space and have equal scattering cross sections. This model provides a structure in which short delay multipath components are more likely to arrive with direction-of-arrival near the direct path, while multipath components with longer delays are more uniformly distributed in DOA.

In this paper, we present a statistical channel model for a macrocell environment. We assume that the scatterers are arranged circularly around the mobile station (MS), whereby the distances, R_k, between scatterers and mobile stations are subject statistically to a hyperbolic distribution. This assumption is more realistic and flexible than other possible probability density functions (pdf's), like an exponential-decaying pdf [6]. For simplicity, this model assumes that each multipath component is created by a specular reflection of the wave fronts at the remote object. This model provides spatial information. Once the co-ordinates of the scatterers are drawn from a reciprocal cosh distribution of the distance between the mobile and the scatterer, all necessary channel characteristics, including direction of arrival, can be derived.

II. GEOMETRICAL CHANNEL PARAMETERS

The multipath geometrical channel parameters are illustrated in Fig.1. The position of the reflectors relative to mobile and base station is determined by a reciprocal cosh distributed scatterers. A signal from the mobile travels through number of paths, each with its own delay [7]. These paths arrive at the receive antenna array with different angles of arrival. The composite multipaths induce a different multipath at each channel because of differences in relative phasing of the paths.

A. Channel model with hyperbolically distributed scatterers

In this section we introduce a geometrically based statistical macrocell channel model and derive the probability density function of the time delay and the angle of arrival of the

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multipath components in a macrocell at the base station. Signals received at the base station are assumed to be plane waves arriving from the horizon and hence the angle of arrival calculation includes only the azimuthal coordinate [8].

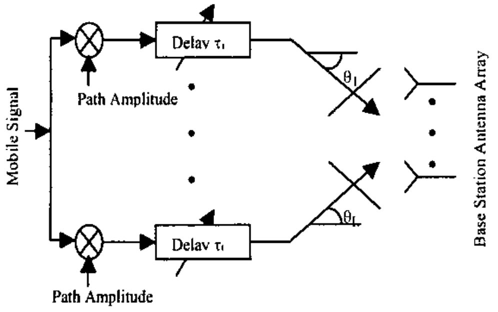

Fig. 1. The Geometrical model parameters

The complex baseband discrete impulse response is given by

$$h_b(t,\theta_k,\tau_k)=\sum_{k=0}^{L-1}\alpha_k e^{j\phi_k}p(t-\tau_k) \tag{1}$$

Where θ_k is the angle-of-arrival of the multipath components, α_k is the complex amplitude of the k^{th} multipath component, τ_k is the path delay for that component, $p(t-\tau_k)$ is a very narrow pulse, and ϕ_k is the random phase of the received signal. The parameter L is the number of multipath components; it is application dependent.

Fig.2 illustrates the geometrical based reciprocal cosh-distributed scatterers model. In this model, the distance R_k between the mobile and the scatterers are distributed according to the hyperbolic pdf. The base station is marked BS and the mobile is MS. A distance D separates the base station and the mobile unit, and the angle ψ_k is uniformly distributed in the interval $[0,2\pi]$. The angle θ_k is the direction of arrival for the path delay.

R_k and ψ_k are assumed to be random variable with the following probability density function

$$f_{R,\psi}(R_k,\psi_k)=\frac{1}{2\pi}\frac{a}{\cosh^2(aR_k)} \quad where \quad 0\le\psi\le 2\pi \tag{2}$$
$$0\le R<\infty$$

Since the desired joint pdf of path delay and DOA, $f_{\tau,\theta}(\tau_k,\theta_k)$, is a rather complex expression and the calculation of an analytic expression of its probability function is difficult, a different approach is used for the channel model. Using the separability of the marginal densities $f_R(R_k)$ and $f_\psi(\psi_k)$, the values for R and Ψ can be evaluated. The value of R_k is

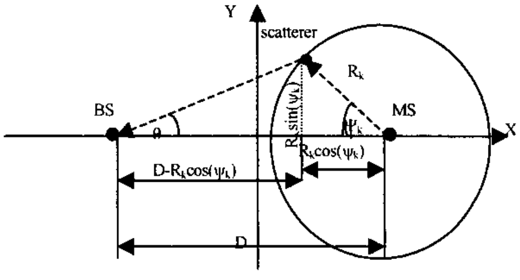

Fig. 2. Geometrical channel model with spatially hyperbolic-distributed scatterers geometry

$$R_k=\frac{1}{a}\tanh^{-1}(x_k) \tag{3}$$

Where x is a uniform random number distributed on [0,1]. The angle ψ_k is given by

$$\psi_k=2\pi y_k \tag{4}$$

Where y is the uniformly distributed on [0,1]. The path delay, τ_k, of the multipath component is given by

$$\tau_k=\frac{l_k}{c}=\frac{1}{c}\left(R_k+\sqrt{R_k^2+D^2-2R_kD\cos(\psi_k)}\right) \tag{5}$$

Where c is the speed of the light. The angle of arrival for the k^{th} path is given by

$$\theta_k=\begin{bmatrix}\tan^{-1}\left(\dfrac{R_k\sin(\psi_k)}{D-R_k\cos(\psi_k)}\right) & for\ R_k\cos(\psi_k)\le D\\ \tan^{-1}\left(\dfrac{R_k\sin(\psi_k)}{D-R_k\cos(\psi_k)}\right)+\pi & for\ R_k\cos(\psi_k)>D\end{bmatrix} \tag{6}$$

From equations (2), (5), and (6), it can be shown that the TOA and DOA probability density functions are as follows

$$f_{\tau,\psi}(\tau,\psi)=\frac{ac(\tau^2c^2-2D\tau c\cos(\psi)+D^2)}{4\pi[\tau c-D\cos(\psi)]^2\cosh^2(g(\tau,\psi))} \tag{7}$$

$$f_{\theta,\psi}(\theta,\psi)=\frac{aD\sin(\psi)\sec^2(\theta)}{2\pi[\sin(\psi)+\cos(\psi)\tan(\theta)]^2\cosh^2[g(\theta,\psi)]} \tag{8}$$

Where, $g(\tau,\psi)=\frac{a(\tau^2c^2-D^2)}{2(\tau c-D\cos(\psi))}$, and

$g(\theta,\psi)=\frac{a\,D\tan(\theta)}{\sin(\psi)+\cos(\psi)\tan(\theta)}$.

The mean power of each multipath component depends on the propagation delay τ_k and is usually defined by a characteristic power delay profile (PDP) $P(\tau_k)$ as follows [9]

$$P(\tau_k)=P_{ref}-10n\log\left(\frac{\tau_k}{\tau_{ref}}\right) \tag{9}$$

The path loss exponent, n, depends on the propagation scenario to be simulated [3]. P_{ref}, is a reference power that is measured at a distance d_{ref} from the transmitting antenna when omnidirectional antennas are used at both the transmitter and the receiver.

$$P_{ref}=P_T-20\log\left(\frac{4\pi\,d_{ref}f_c}{c}\right) \tag{10}$$

Where P_T is the transmitted power in dB and f_C is the carrier frequency [9]. The power in each multipath component, P_k, is related to the complex amplitude, α_k, of the kth multipath in equation (1) by $P_k=P_0+20\log|\alpha_k|$, hence $\alpha_k=10^{\frac{(P_k-P_0)}{20}}$.

B. Adjustment of the proposed model to the simulation environment

The maximum delay path τ_{max} for the model simulation is controlled by the value of a. This delay occurs when ψ_k=180, then from equation (5) we have

$$\tau_{max}=\frac{1}{c}(D+2R_{max}) \tag{11}$$

$$p=p_r(R_k>R_{max})=1-F_R(R_{max}) \tag{12}$$

The probability distribution of R_{max} is given by

$$F_R(R_{max})=\int_0^{R_{max}}\frac{a}{\cosh^2(aR)}dR=\tanh(aR_{max}) \tag{13}$$

By inserting (13) and (11) into (12) we get

$$a=\frac{2\tanh^{-1}(1-p)}{c(\tau_{max}-D)}=\frac{2\tanh^{-1}(1-p)}{\tau_{e\max}c} \tag{14}$$

Where p is a number in [0,1] and $\tau_{e\max}=(\tau_{max}-\tau_0)=(r_{max}-1)\tau_0$ is the maximum excess delay. Different values of a are produced by varying the maximum excess delay.

III. SIMULATION RESULTS

The vector channel model is designed to model a great variety of possible mobile channel scenarios. It produces directional information of the mobile at the environment. This information includes the path delay (τ_k), DOA (θ_k), and the power (P_k) for multipath components.

In simulating multipath component parameters by this model, it is necessary to generate samples of random variables distributed uniformly on [0,1]. The first step is to evaluate the value of a, from equation (14), which controls the maximum path delay, τ_{max}. The efficient value of a is in (0,1). When a is determined, the distance R between the scatterers and the mobile can be evaluated from equation (3). The next step is to generate the time delay of each path τ_k, and its angle of arrival θ_k by evaluating equations (5), and (6) respectively.

The probability density function, $f_{\tau,\psi}(\tau,\psi)$ for τ_k, conditioned on the angle ψ_k, is shown in Fig.3 when the distance, D, between the mobile and the base station is 1km and a=0.02. From this figure it is evident that the pdf is accumulated around the long delays.

The reciprocal cosh distributed scatterer model has been simulated for an urban area. In this simulation we considered n=4, the number of simulated multipaths component L=100 the maximum excess delay $\tau_{e\max}=(\tau_{\max}-\tau_0)$ has been limited to 400ns, and a=0.014. Fig. 4 shows the channel impulse response, h(τ,t), generated by the model. The mobile speed results in a maximum Doppler shift of $f_{D\max}=vf_c/c=30Hz$. Fig.5 shows a sample transfer function, H(f,t) of the channel impulse response, h(τ,t). A delay spread (Δτ) and an angular spread (Δθ) of the exponential model are 1.2 μs and 12^0, respectively [6]. The proposed hyperbolic model has a better angular spread (Δθ ≈ 18^0) in comparison with the exponential model.

IV. CONCLUSION

In this paper, a new statistical geometric propagation model for a macrocell mobile environment is developed. This model assumes that each multipath component is created by a specular reflection of the wave fronts of the remote object, and the distance between the scatterer and the mobile is hyperbolically distributed. The co-ordinates of the scatterers are drawn from a reciprocal cosh distribution, of the distance between the mobile and the scatterer. This model provides: the power of each path, the Time-of-Arrival (TOA), and the Direction-of Arrival (DOA) of the multipath component.

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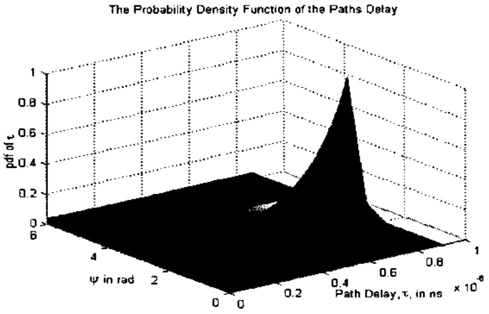

Fig.3. Probability density function, $f_{\tau,\psi}(\tau,\psi)$, for the path delay τ_k (a=0.02 and D=1km)

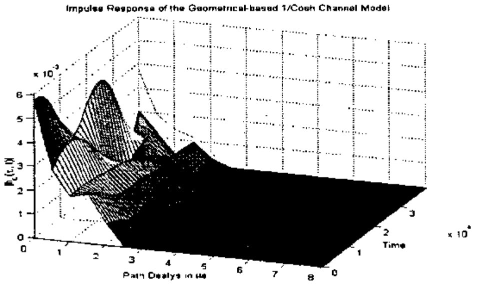

Fig.4. The channel impulse response generated by the model (maximum excess delay= 400ns, a=0.014, and D=1km)

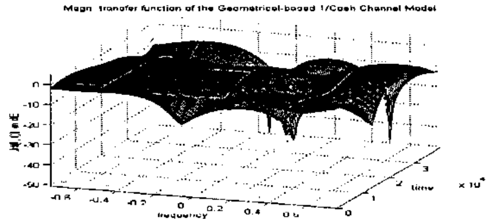

Fig. 5. The sample transfer function, H(f,t) of the channel impulse response, h(τ,t), (maximum excess delay= 400ns, a=0.014, and D=1km)

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