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An Analysis of Trust Transitivity Taking Base Rate into Account

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Abstract

Trust transitivity, as trust itself, is a human mental phenomenon, so there is no such thing as objective transitivity, and trust transitivity therefore lends itself to different interpretations. Trust transitivity and trust fusion both are important elements in computational trust. This paper analyses the parameter dependence problem in trust transitivity and proposes some definitions considering the effects of base rate. In addition, it also proposes belief functions based on subjective logic to analyse trust transitivity of three specified cases with sensitive and insensitive based rate. Then it presents a quantitative analysis of the issue of exaggerated beliefs in Mass Hysteria based on subjective logic.

1. Introduction

Trust transitivity is the most explicit form of computational trust, meaning for example that if Alice trusts Bob, and Bob trusts Claire, then by transitivity, it can be computed that Alice will also trust Claire [3]. This assumes that Bob recommends Claire to Alice. This simple principle, which is essential for human interaction in business and everyday life, manifest it in many different forms. This paper investigates the parameter dependence problem in trust transitivity and proposes possible formal computational models that can be implemented using belief reasoning based on subjective logic. With adequate computational trust models, the principles of trust propagation can be ported to online communities of people, organizations and software agents, with the purpose of enhancing the quality of those communities.

In uncertain probability theory [14], the metric which express belief is called opinion. Under a defined scope, trust relationships can support transitivity [18,20]. The use of trust in transitive chains requires the existence of a common purpose which needs somehow to be derived from or given by a specific transitive chain [17,21]. Subjective logic takes both the uncertainty and individuality of beliefs into account while still being compatible with standard logic and probability calculus. The migration from the assumed towards the perceived world is achieved by adding an uncertainty dimension to the single valued probability measure, and by taking the individuality of beliefs into account. Trust can be interpreted as a belief about the reliability of an object, and as a decision to depend on an object [10]. In this paper, trust is interpreted as a belief about reliability. As a calculus of beliefs, subjective logic can therefore be used for trust reasoning. Although this model can never be perfect, and able to reflect all the nuances of trust, it can be shown to respect the main intuitive properties of trust and trust propagation.

As soon as one attempts to perform computations with input parameters in the form of subjective trust measures, parameter dependence becomes a major issue. If Alice for example wants to know whether tomorrow will be sunny, she can ask her friends, and if they all say it will be sunny she will start believing the same. However, her friends might all have based their opinions on the same weather-forecast, so their opinions are dependent, and in that case, asking only one of them would be sufficient. It would in fact be wrong of Alice to take all her friends' opinions into account as being independent, because it would strengthen her opinion without any good reason. Being able to identify cases of dependent opinions is therefore important, though it is difficult.

In section 2, we have introduced the trust computational model with subjective logic. We have identified two problems of trust transitivity in section 3. In section 4, we have shown the effect of not being aware of dependence between opinions. By giving an appropriate example, we have shown that it is possible for recommended opinions to return to their originator through feedback loops, resulting in even more exaggerated beliefs and with repeated loops, it may create a mass hysteria. In section 5, we have concluded the outcome of this research.

2. Computing Trust

Trust has become important topic of research in many fields including sociology, psychology, philosophy, economics, business, law and IT. It is not a new topic to discuss. In fact, it has been the topic of hundreds books and scholarly articles over a long period of time. Trust is a complex word with multiple dimensions. A vast literature on trust has grown in several area of research but it is relatively confusing and sometimes contradictory, because the term is being used with a variety of meaning [19]. The most cited definition of trust is given by Dasgupta where he defines trust as "the expectation of one person about the actions of others that affects the first person's choice, when an action must be taken before the actions of others are known" [11]. This definition captures both the purpose of trust and its nature in a form that can be reasoned about.

Deutsch [12] states that "trusting behaviour occurs when a person encounters a situation where she perceives an ambiguous path. The result of following the path can be good or bad and the occurrence of the good or bad result is contingent on the action of another person" [16]. Another definition for trust by Gambetta is also often quoted in the literature: "trust (or, symmetrically, distrust) is a particular level of the subjective probability with which an agent assesses that another agent or group of agents will perform a particular action, both before he can monitor such action (or independently of his capacity ever to be able to monitor it) and in a context in which it affects his own action" [14]. But trust can be more complex than these definitions. Golbeck et al [15] has proposed a definition of trust suitable for use in web-based social networks with a discussion of the properties that influenced its use in computation. They have also presented two algorithms for inferring trust relationships between individuals that are not directly connected in the network.

Subjective logic is a belief calculus specifically developed for modeling trust relationships [3,9]. In subjective logic, beliefs are represented on binary state spaces, where each of the two possible states can consist of sub-states. Belief functions on binary state spaces are called *subjective opinions* and are formally expressed in the form of an ordered tuple $\omega_x^A = (b, d, u, a)$, where b, d, and u represent belief, disbelief and uncertainty respectively where $b, d, u \in [0, 1]$ and $b+d+u = 1$. The base rate parameter $a \in [0, 1]$ represents the base rate probability in the absence of evidence, and is used for computing an opinion's probability expectation value $E(\omega_x^A) = b + au$, meaning that a determines how uncertainty shall contribute to $E(\omega_x^A)$. A subjective opinion is interpreted as an agent A's belief in the truth of statement x. Ownership of an opinion is represented as a superscript so that for example A's opinion about x is denoted as ω_x^A.

The fact that subjective logic is compatible with binary logic and probability calculus means that whenever corresponding operators exist in probability calculus, the probability expectation value $E(\omega)$ of an opinion ω that has been derived with subjective logic, is always equal to the probability value that would have been derived had simple probability calculus been applied. Similarly, whenever corresponding binary logic operators exist, an absolute opinion (i.e. equivalent to binary logic TRUE or FALSE) derived with subjective logic, is always equal to the truth value that can be derived with binary logic.

Subjective logic has a sound mathematical basis and is compatible with binary logic and traditional Bayesian analysis. Subjective logic defines a rich set of operators for combining subjective opinions in various ways [1–9]. Some operators represent generalizations of binary logic and probability calculus, whereas others are unique to belief calculus because they depend on belief ownership. With belief ownership it is possible to explicitly express that different agents have different opinions about the same issue. The advantage of subjective logic over probability calculus and binary logic is its ability to explicitly express and take advantage of ignorance and belief ownership [3]. Subjective logic can be applied to all situations where probability calculus can be applied, and to many situations where probability calculus fails precisely because it can not capture degrees of ignorance. Subjective opinions can be interpreted as probability density functions, making subjective logic a simple and efficient calculus for probability density functions.

3. Analyzing Trust Transitivity

Assume two agents *A* and *B* where *A* trusts *B*, and *B* believes that proposition *x* is true. Then by transitivity, agent *A* will also believe that proposition *x* is true. This assumes that *B* recommends *x* to *A*. In our approach, trust and belief are formally expressed as opinions. The transitive linking of these two opinions consists of discounting *B*'s opinion about *x* by *A*'s opinion about *B*, in order to derive *A*'s opinion about *x*. This principle is illustrated in Fig.1 below. The solid arrows represent initial direct trust, and the dotted arrow represents derived indirect trust.

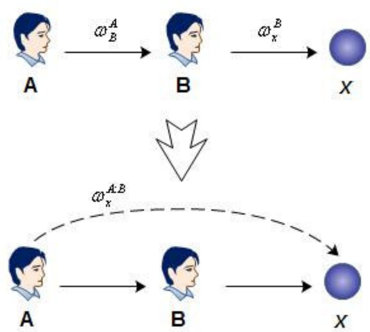

Fig. 1. Principle of Trust Transitivity

Trust transitivity, as trust itself, is a human mental phenomenon, so there is no such thing as objective transitivity, and trust transitivity therefore lends itself to different interpretations. We have identified two main difficulties. The first is related to the effect of *A* disbelieving that *B* will give a good advice. What does this exactly mean? We will give two different interpretations and definitions. The second difficulty relates to the effect of base rate trust in a transitive path. We will briefly examine this, and provide the definition of a base rate sensitive discounting operator as an alternative to the two previous which are base rate insensitive.

3.1 Uncertainty Favoring Trust Transitivity

A's disbelief in the recommending agent *B* means that *A* thinks that *B* ignores the truth value of *x*. As a result *A* also ignores the truth value of *x*.

Definition 1 (Uncertainty Favoring Discounting).

Let A, B and be two agents where A's opinion about B's recommendations is expressed as $\omega_B^A = \{b_B^A, d_B^A, u_B^A, a_B^A\}$, *and let x be a proposition where B's opinion about x is recommended to A with the opinion* $\omega_x^B = \{b_x^B, d_x^B, u_x^B, a_x^B\}$ *Let* $\omega_x^{A:B} = \{b_x^{A:B}, d_x^{A:B}, u_x^{A:B}, a_x^{A:B}\}$ *be the opinion such that:*

$$\begin{cases} b_x^{A:B} = b_B^A b_x^B \\ d_x^{A:B} = d_B^A d_x^B \\ u_x^{A:B} = d_B^A + u_B^A + b_B^A u_x^B \\ a_x^{A:B} = a_x^B \end{cases}$$

then $\omega_x^{A:B}$ *is called the uncertainty favoring discounted opinion of A. By using the symbol* $\otimes$ *to designate this operation, we get* $\omega_x^{A:B} = \omega_B^A \otimes \omega_x^B$.

It is easy to prove that this operator is associative but not commutative. This means that the combination of opinions can start in either end of the path, and that the order in which opinions are combined is significant. In a path with more than one recommending entity, opinion independence must be assumed, which for example translates into not allowing the same entity to appear more than once in a transitive path. Fig.2 illustrates an example of applying the discounting operator for independent opinions, where ω_B^A = {0.1, 0.8, 0.1} discounts ω_x^B = {0.8, 0.1, 0.1} to produce $\omega_x^{A:B}$ = {0.08, 0.01, 0.91}.

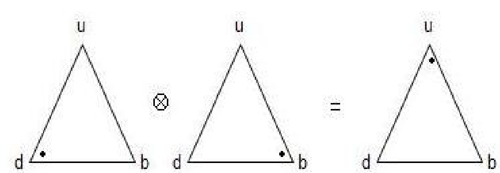

Fig. 2. Example of applying the discounting operator for independent opinions

3.2 Opposite Belief Favoring

A's disbelief in the recommending agent B means that A thinks that B consistently recommends the opposite of his real opinion about the truth value of x. As a result, A not only disbelieves in x to the degree that B recommends belief, but she also believes in x to the degree that B recommends disbelief in x, because the combination of two disbeliefs results in belief in this case.

Definition 2 (Opposite Belief Favoring Discounting).

Let A, B and be two agents where A's opinion about B's recommendations is expressed as $\omega_B^A=\{b_B^A, d_B^A, u_B^A, a_B^A\}$, *and let x be a proposition where B's opinion about x is recommended to A as the opinion* $\omega_x^B = \{b_x^B, d_x^B, u_x^B, a_x^B\}$. *Let* $\omega_x^{A:B}=\{b_x^{A:B}, d_x^{A:B}, u_x^{A:B}, a_x^{A:B}\}$ *be the opinion such that:*

$$\begin{cases} b_x^{A:B} = b_B^A b_x^B + d_B^A d_x^B \\ d_x^{A:B} = b_B^A d_x^B + b_B^A d_x^B \\ u_x^{A:B} = u_B^A + (b_B^A + d_B^A) u_x^B \\ a_x^{A:B} = a_x^B \end{cases}$$

then $\omega_x^{A:B}$ *is called the opposite belief favoring discounted recommendation from B to A. By using the symbol* $\otimes$ *to designate this operation, we get* $\omega_x^{A:B} = \omega_B^A \otimes \omega_x^B$.

This operator models the principle that *"your enemy's enemy is your friend"*. That might be the case in some situations, and the operator should only be applied when the situation makes it plausible. It is doubtful whether it is meaningful to model more than two arcs in a transitive path with this principle. In other words, it is doubtful whether the enemy of your enemy's enemy necessarily is your enemy too.

3.3 Base Rate Sensitive Transitivity

In the transitivity operators defined in Sec.4.1 and Sec.4.2 above, a_B^A had no influence on the discounting of the recommended (b_x^B, d_x^B, u_x^B) parameters. This can seem counterintuitive in many cases such as in the example described next.

Imagine a stranger coming to a town which is know for its citizens being honest. The stranger is looking for a car mechanic, and asks the first person he meets to direct him to a good car mechanic. The stranger receives the reply that there are two car mechanics in town, David and Eric, where David is cheap but does not always do quality work, and Eric might be a bit more expensive, but he always does a perfect job. Translated into the formalism of subjective logic, the stranger has no other info about the person he asks than the base rate that the citizens in the town are honest. The stranger is thus ignorant, but the expectation value of a good advice is still very high. Without taking a_B^A into account, the result of the definitions above would be that the stranger is completely ignorant about which if the mechanics is the best.

An intuitive approach would then be to let the expectation value of the stranger's trust in the recommender be the discounting factor for the recommended (b_x^B, d_x^B) parameters.

Definition 3 (Base Rate Sensitive Discounting).

The base rate sensitive discounting of a belief $\omega_x^B = \{b_x^B, d_x^B, u_x^B, a_x^B\}$ *by a belief* $\omega_B^A=\{b_B^A, d_B^A, u_B^A, a_B^A\}$ *produces the transitive belief* $\omega_x^{A:B}=\{b_x^{A:B}, d_x^{A:B}, u_x^{A:B}, a_x^{A:B}\}$ *where*

$$\begin{cases} b_x^{A:B} = E(\omega_B^A) b_x^B \\ d_x^{A:B} = E(\omega_B^A) d_x^B \\ u_x^{A:B} = 1 - E(\omega_B^A)(b_x^B + d_x^B) \\ a_x^{A:B} = a_x^B \end{cases}$$

where the probability expectation value $E(\omega_B^A) = b_B^A + a_B^A u_B^A$.

However this operator must be applied with care. Assume again the town of honest citizens, and let the stranger A have the opinion ω_B^A = (0, 0, 1, 0.99) about the first person B she meets, i.e. the opinion has no basis in evidence other than a very high base rate defined by a_B^A = 0.99. If the person B now recommends to A the opinion ω_x^B = (1, 0, 0, a), then, according to the base rate sensitive discounting operator of Def.4, A will have the belief $\omega_x^{A:B}$ = (0.99, 0, 0.01, a) in x. In other words, the highly certain belief $\omega_x^{A:B}$ is derived on the basis of the highly uncertain belief ω_B^A, which can seem counterintuitive. This potential problem could be amplified as the trust path gets longer. A safety principle could therefore be to only apply the base rate sensitive discounting to the last transitive link.

There might be other principles that better reflect human intuition for trust transitivity, but we will leave this question to future research. It would be fair to say that the base rate insensitive discounting operator of Def.2 is safe and conservative, and that the base rate sensitive discounting operator of Def.4 can be more intuitive in some situations, but must be applied with care.

4. The Effects of Unknown Dependence

One of the strengths of this work is in its analytical capabilities. As an example, consider how mass hysteria can be caused by people not being aware of dependence between opinions. Let's take for example; person A recommend an opinion about a particular statement x to a group of other persons. Without being aware of the fact that the opinion came from the same origin, these persons can recommend their opinions to each other as illustrated in Fig.3.

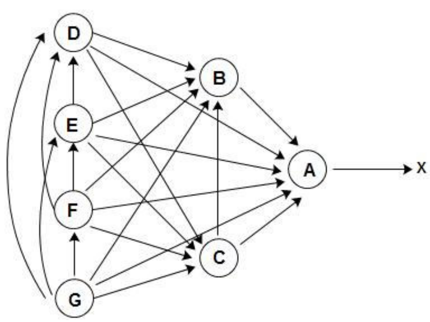

Fig. 3. The effects of unknown dependence

The arrows represent trust so that for example $B \longrightarrow A$ can be interpreted as saying that B trusts A to recommend an opinion about statement x. The actual recommendation goes, of course, in the opposite direction to the arrows in Fig.3. It can be seen that A recommends an opinion about x to 6 other agents, and that G receives 6 recommendations in all. If G assumes the recommended opinions to be independent and takes the consensus between them, his opinion can become abnormally strong and in fact even stronger than A's opinion.

As a numerical example, let A's opinion ω_x^A about x as well as the agents' opinions about each other ($\omega_A^B, \omega_A^C, \omega_B^C, \omega_A^D, \omega_B^D, \omega_C^D, \omega_A^E, \omega_B^E, \omega_C^E, \omega_D^E,$ $\omega_A^F, \omega_B^F, \omega_C^F, \omega_D^F, \omega_E^F, \omega_A^G, \omega_B^G, \omega_C^G, \omega_D^G, \omega_E^G, \omega_F^G$) all have the same value given by (0.7, 0.1, 0.2, a).

In this example, we will apply the consensus operator for *independent* beliefs to illustrate the effect of unknown dependence. We also apply the uncertainty favoring discounting operator which does not take base rates into account. Taking all the possible recommendations of Fig.6 into account creates a relatively complex trust graph, and a rather long notation. In order to reduce the size of the notation, the transitivity symbol ":" will simply be omitted, and the cumulative fusion symbol _ will simply be written as ",". Analyzing the whole graph of dependent paths, as if they were independent, will then produce:

$$\omega_x^{\left(\begin{array}{l}\text{GA, GBA, GCA, GCBA, GDA, GDBA, GDCA, GDCBA, GEA, GEBA, GECA,}\\ \text{GECBA, GEDA, GEDBA, GEDCA, GEDCBA, GFA, GFBA, GFCA, GFCBA,}\\ \text{GFDA, GFDBA, GFDCA, GFDCBA, GFEA, GFEBA, GFECA, GFECBA,}\\ \text{GFEDA, GFEDBA, GFEDCA, GFEDCBA}\end{array}\right)}$$
= (0.76, 0.11, 0.13, a)

For comparison, if G only took the recommendation from A into account (as he should), his derived opinion would be $\omega_x^{G:A}$ = {0.49, 0.07, 0.44, a}.

In real situations it is possible for recommended opinions to return to their originator through feedback loops, resulting in even more exaggerated beliefs. When this process continues, an environment of self amplifying opinions, and thereby hysteria, is created.

5. Conclusion

Trust propagation does not manifest itself as a physical phenomenon in nature, but only exists on the mental and cognitive level. It is therefore difficult to assess whether computational models for trust propagation are adequate and reflect the way people reason about trust. A number of principles have been described to model the propagation of trust. We have discussed the shortcomings of computational trust based on subjective logic by the risk of over counting of trust evidence when opinions are not independent and have described a set of computational trust principles that reflect intuitive trust propagation constructs. Different situations require different trust models. With appropriate computational trust models, the principles of trust propagation can be ported to online communities of people, organizations and software agents, with the purpose of enhancing the quality of those communities. The specific computational trust operators must therefore be selected as a function of the situation to be modelled.

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