

# Damping Estimation in Highly Interconnected Power Systems

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**Abstract**—In a large interconnected power system, oscillating modes typically arise after disturbances. Estimation of the rate of rise or decay of these disturbances (i.e. of the damping factor) is important so that poorly damped modes which put the system at risk can be identified. In the past both Fourier based and parametric techniques have been used to estimate the damping factors of the oscillating modes. Parametric methods tend to perform poorly in noise and existing sliding window methods do not optimally account for the kind of noise present in real operating environments. This paper presents an improvement to existing Fourier based methods for damping factor estimation. Simulations and real data results are provided to support the claims made in the paper. A comparison with parametric methods is also made.

**Index Terms**—damping factor, white Gaussian noise, colored noise, Prony analysis.

## I. INTRODUCTION

Power system oscillations are present in the generator angle vibration measurements for any large power system in normal operation [1]. As explained in [1] the modal oscillation signal,  $y(n)$ , can be modeled as the output of an infinite impulse response (IIR) filter driven by the integral of white Gaussian noise (WGN) [1, 2]. As also explained in [1], if  $y(n)$  is differentiated the result,  $x(n)$ , can be considered to be the output of the IIR filter driven by white noise. This is illustrated diagrammatically in Figure 1.

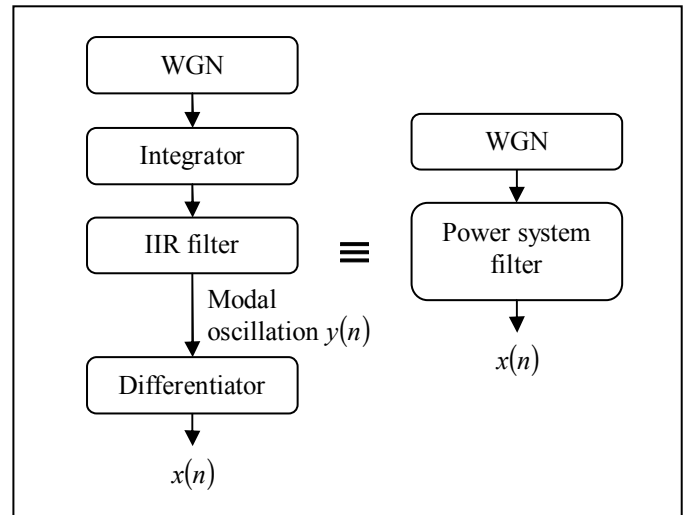


Figure 1. Power system modal oscillation model.

Assuming the model described in the previous paragraph, the IIR filter impulse response,  $h(n)$ , and the transfer function,  $H(z)$ , can be estimated from  $x(n)$  using Prony's method or some other parametric technique. From a stability monitoring prospective the key information to be extracted from  $H(z)$  is the set of damping factors corresponding to the roots of the transfer function denominator. These damping factors can alternatively be found by applying sliding window Fourier based methods to the autocorrelation function of  $x(n)$  [3, 4].

Parametric methods such as Prony's method do not perform well in noise and are thus prone to error. Additionally, existing Fourier methods fail to properly account for the type of noise actually present in real power system scenarios. The new Fourier based method presented in this paper overcomes these short-comings by effectively taking account of the true character of the noise.

Section II formally presents the signal model and the problem of interest. Section III presents the new Fourier based technique and shows how the new technique overcomes the short comings of existing methods. Section IV provides simulations to support the claims made about the new method. Section V is devoted to analyzing a real data example. Section VII presents the conclusion.

## II. THEORY

### A. Power Signal

The impulse response of the power system filter depicted in Figure 1 will initially be assumed to have the form:

$$h(n) = \sum_{m=1}^M A_m \exp((- \alpha_m + j \omega_{0m}) n T_s + j \phi_m) \quad (1)$$

where  $A_m$  = Amplitude,  $\alpha_m$  = Damping factor,  $\omega_{0m}$  = Frequency (rad/s),  $\phi_m$  = Phase (rad),  $n$  = Discrete time and  $T_s = 1/f_s$  = Sampling period.

A common approach for estimating the parameters of  $h(n)$  in (1) is to use Prony's method [5-9]. This method is described below.

In Prony analysis the  $h(n)$  is determined by setting up linear prediction equations.  $H(z)$  is then found from  $h(n)$  and the roots of the denominator of  $H(z)$  are used to determine the frequencies and the damping factors of the modes. The damping factors are determined by taking the real values of the logarithms of the system poles. The frequencies are determined by taking the imaginary values of the logarithms of the system poles.

The linear prediction equation described in [9] is given by:

$$Ab = v \quad (2)$$

where  $A$ ,  $b$  and  $v$  are equal to:

$$A = \begin{bmatrix} z_r(L-1) & z_r(L-2) & \dots & z_r(0) \\ z_r(L) & z_r(L-1) & \dots & z_r(1) \\ \vdots & \vdots & \ddots & \vdots \\ z_r(N-2) & z_r(N-3) & \dots & z_r(N-L-1) \end{bmatrix}$$

$$b = \begin{bmatrix} b(1) \\ b(2) \\ \vdots \\ b(L) \end{bmatrix} \quad \text{and} \quad v = \begin{bmatrix} z_r(L) \\ z_r(L+1) \\ \vdots \\ z_r(N-1) \end{bmatrix}$$

where  $L$  = Prediction filter order. For backward prediction the linear prediction equation described in [10] is given by:

$$Ab = -v \quad (3)$$

where  $A$ ,  $b$  and  $v$  are equal to:

$$A = \begin{bmatrix} z_r^*(1) & z_r^*(2) & \dots & z_r^*(L) \\ z_r^*(2) & z_r^*(3) & \dots & z_r^*(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ z_r^*(N-L) & z_r^*(N-L+1) & \dots & z_r^*(N-1) \end{bmatrix}$$

$$b = \begin{bmatrix} b(1) \\ b(2) \\ \vdots \\ b(L) \end{bmatrix} \quad \text{and} \quad v = \begin{bmatrix} z_r^*(0) \\ z_r^*(1) \\ \vdots \\ z_r^*(N-L-1) \end{bmatrix}$$

Once  $b$  is found the denominator of  $H(z)$  has effectively been found [11]. The denominator can then be factorized to yield the roots. Frequencies and damping factors can then be determined as explained previously.

Solution of the linear prediction equation in (3) is ill-conditioned. Kumaresan and Tufts showed that one could improve the conditioning by using pseudo inverse techniques [10]. Accordingly, the singular values of  $A$  are found and the inverse of  $A$  is determined by selecting only the "significant" singular values and singular vectors in the inversion process [10, 12].

A disadvantage with Prony's method (with or without pseudo inverse methods) is that it does not perform well in noise. For this reason Fourier methods can sometimes be better alternatives.

## III. THE NEW FOURIER BASED METHOD

### A. Sliding Window Methods

The sliding window algorithms described in [3, 4] & [13, 14] use the fact that as a window slides along a decaying sinusoid the spectrum of the signal within that window will also decay in amplitude. The rate at which the spectrum decays (at a certain frequency) can be used to determine the damping factor of a given mode. The following formula can be used to estimate the damping factor:

$$\hat{\alpha}_{lm} = \frac{1}{N_g T_s} \ln \left[ \text{Re} \left\{ \frac{\sum_{n=n_1}^{n_1+N_g-1} z_r(n) \exp(-j \omega_{0m} n T_s)}{\sum_{n=n_2}^{n_2+N_g-1} z_r(n) \exp(-j \omega_{0m} n T_s)} \right\} \right] \quad (4)$$

where  $z_r(n)$  = Power signal,  $n$  = Discrete time,  $N_g$  = Number of samples between adjacent windows and  $N_w$  = Window length,  $z_c = \exp(-j \omega_{0m} N_g)$ .

In existing sliding window methods the damping factor is calculated at only one point in frequency. However, for the power system model depicted in Figure 1 the signal to noise ratio (SNR) will be constant at all frequency positions. Therefore one can use any or all samples in the frequency spectrum to obtain damping factor information and then use averaging to improve the accuracy of the estimates.

For a single mode, for example, one can use all the samples in the frequency domain and simply average according to:

$$\hat{\alpha}_2 = \frac{1}{N_w} \sum_{k=0}^{N_w-1} \frac{1}{N_g T_s} \ln \left[ \operatorname{Re} \left\{ \frac{Z_{r1}(k)}{Z_{r2}(k)Z_c(k)} \right\} \right] \quad (5)$$

where  $Z_c(k) = \exp(-j\omega_k N_g)$  and where  $\omega_k$  is the  $k^{\text{th}}$  frequency sample in the  $N_w$  - point discrete Fourier transform (DFT).

For multiple modes which were well separated one could use all frequency samples in the vicinity of the mode of interest and then average the resulting estimate. For multiple modes which are closely spaced, one could use the approach described in [15]. That is, one could use several sliding windows and feed the output from these sliding windows into a parametric technique such as the Kumaresan and Tufts algorithm. Moreover, one could do this for several frequency samples and then average the resulting damping factor estimates. Note that the sliding window output sequence is at a much higher SNR than the original signal and there are relatively few samples involved. The approach is therefore robust to noise and computationally efficient.

#### IV. SIMULATIONS

The new sliding window algorithm was applied to damping factor estimation of a simulated power system according to the model in Figure 1. The IIR filter contained a single pole. The impulse response corresponded to the autocorrelation function of the differentiated modal oscillation signal,  $x(n)$ . Figure 2 shows the mean square error (MSE) of the resultant damping factor estimate for various SNRs on the autocorrelation function. The power system filter single mode had the following parameters:  $f_s = 4$  Hz,  $\alpha = 0.02$ ,  $\omega_b = 6.2832$  rad/s. The window length used for the sliding window algorithm was  $N_w = 256$  and the separation between windows was  $N_g = 256$ . 100 simulations were used for each SNR value. MSEs for the new algorithm are also compared with MSEs obtained from Prony's method, Kumaresan-Tufts method and the existing sliding window method. For the Kumaresan-Tufts method the prediction filter order was set to  $L = 100$ . The new method is seen to clearly outperform the three alternatives.

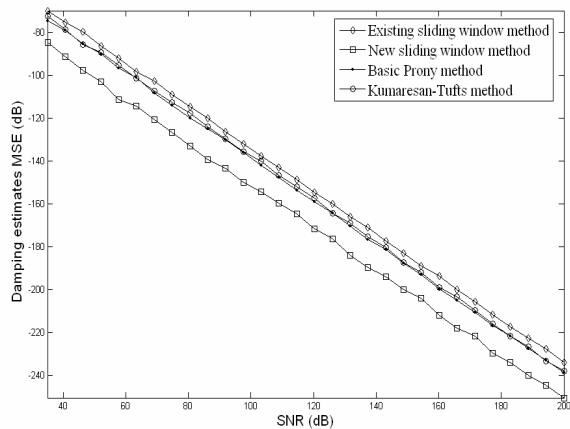


Figure 2. Damping factor mean square error.

#### V. REAL DATA ANALYSIS

Vibration measurement data sampled at 10 Hz was acquired from the Tasmanian power system grid. The autocorrelation function was formed to yield the decaying oscillating modal signal shown in Figure 3 below.

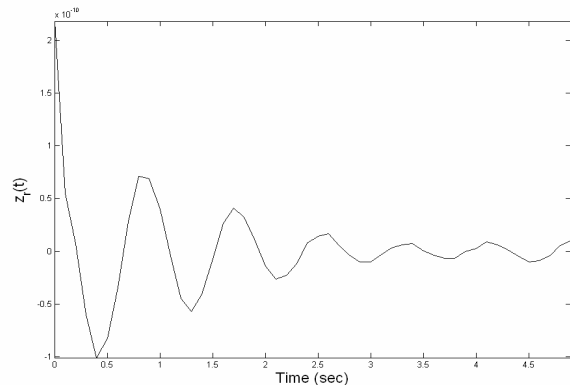


Figure 3. Autocorrelation function of disturbance signal from Tasmanian grid.

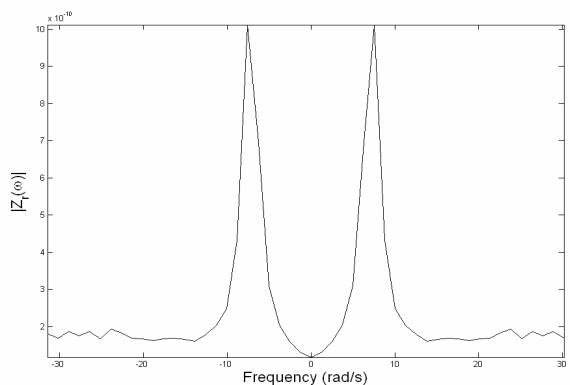


Figure 4. FFT of the autocorrelation function in Figure 3.

The window length,  $N_w$ , was set to  $N_w = 16$  samples and the window separation,  $N_g$ , was set to  $N_g = 16$  samples. The estimate of damping factor obtained with the new method was 0.45378. This appears consistent with the decay apparent in Figure 3. 4 of the 16 FFT samples were used to create damping factor estimates and averaging was used to obtain the overall estimate. Only 4 FFT samples were used because there appeared to be a small amount of spectral energy away from the main mode.

## VI. DISCUSSION

Simulations have shown that the new method can outperform both Prony's method and existing Fourier based methods, at least for the case of a single mode. The new method should in principle be extendable to multiple modes but this still needs to be verified through simulation. This will be the subject of future work. The new method also has been effective on some real data obtained from the Tasmanian power system grid.

## VII. CONCLUSION

A new Fourier sliding window method has been introduced which outperforms existing Fourier methods for estimating damping factor in power system disturbance monitoring. The method has also been used on real data and has given very plausible results.

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