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Prediction of minor stream delays at a limited priority freeway merge

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Abstract

This paper presents the development and application of a limited priority gap acceptance model to freeway merging. In the limited priority model, drivers in the major stream at a merge area may incur delay in restoring small headways to a larger, sustainable minimum headway between them and the vehicle in front. This allows minor stream drivers to accept smaller gaps. The headway distributions are assumed to be distributed according to Cowan's M3 model, whose terms were calibrated for this system. Minor stream minimum follow-on time was calibrated, and a realistic range of the critical gap identified.

An equation was developed for minimum average minor stream delay. A function was identified to model the relationship between minor stream average delay and degree of saturation. The shape parameter of this function was calibrated using simulated traffic flow data, under three different minor stream arrival pattern regimes.

The model provides a useful means of comparing performance, through average minor stream delay, for varying minor and major stream flow rates and minor stream critical gap, under arrival patterns that differ due to traffic control upstream of the on-ramp. Minor stream delay is a particularly useful measure of effectiveness for uncongested freeway merging as it relates directly to the distance required to merge. Observations from the model developed provide physical evidence that minor stream drivers incur lesser delay, or have a better chance of merging quickly, when they arrive at constant intervals as is the case under constant departure ramp metering, than when they arrive in bunches downstream of a signalised intersection, or even a semi-bunched state downstream of an unsignalised intersection.

1. Freeway Merge Area Driving Strategies

Freeway merge area performance is dependent upon the ability of the freeway stream to absorb vehicles entering via the on-ramp. Previous research (Makigami and Matsuo, 1990; Szwed and Smith, 1974) has applied gap acceptance theory to model this process. This theory has been adopted widely for the analysis of minor stream entries into, or across, a major stream of vehicles that exercises absolute priority. Under this system drivers in the major stream experience no delay, whilst minor stream drivers suffer all of the delay. Situations to which this behaviour pertains include roundabout and unsignalised intersection entries.

The previous research on gap acceptance in freeway merging has assumed that merging drivers seek gaps in the freeway kerb lane (adjacent to the on-ramp) and that drivers in that stream had been unaffected by the movements of the merging drivers, thereby exercising absolute priority.

The movement of vehicles on a number of merge areas in Brisbane, Australia under *uncongested* conditions was analysed by Bunker (1995) to determine whether absolute priority gap acceptance was representative of the actual merging process. Uncongested conditions were prescribed as those in which traffic is flowing in a stable state at an average speed higher than the critical speed as defined by the Highway Capacity Manual (2000), over the level of service range A through E. Upon close examination, it was revealed that two strategies were often performed by drivers in the kerb lane: (1) kerb lane drivers often slowed down sufficiently that merging drivers could merge in to the kerb lane ahead of them; and (2) kerb lane drivers often moved into gaps in the median lane to provide larger gaps in the kerb lane for merging drivers to accept. Both of these strategies represent forms of priority sharing, rather than absolute priority to the kerb lane drivers.

A theoretical basis was sought to explain Strategy 1 where kerb lane drivers slowed down thereby modifying the gaps, or headways, immediately ahead of and behind them. Cowan (1979) had previously identified four priority sharing merging models for arrivals in two streams. Cowan's four priority sharing models are: (a) first come, first served; (b) alternating merging, when a queue exists in both streams vehicles depart from each stream alternately; (c) limited priority to the major stream, where a major stream vehicle may be delayed up to a minimum headway, Δ, to accommodate a merging vehicle that accepts a gap greater than Δ in front of that vehicle; and (d) queue clearance, where queues of vehicles in each stream are exhausted alternately. The four models all provide for the same total delay; however, the proportion of delay between minor and major stream differs.

Bunker (1995) constructed time-space diagrams of vehicle trajectory data during freeway merging captured at in Brisbane, Australia. Analysis of the data identified the most likely priority sharing mechanism taking place was (c) limited priority. This behaviour was validated in further research on freeway merging by Troutbeck and Kako (1999).

Strategy 2 could be explained by a secondary gap acceptance process where some kerb lane drivers seek an acceptable gap in the adjacent freeway lane (median lane on a four lane freeway section) to either aid, or avoid being slowed by, a merging driver.

2. Limited Priority Gap Acceptance Theory

Limited priority gap acceptance theory includes headway distributions in the traffic streams and gap acceptance parameters of merging drivers.

The theory can be applied to develop the following useful measures of effectiveness of the merging process including minor stream capacity, minor stream degree of saturation, minimum average minor stream delay, and average minor stream delay. These are now discussed.

Headway Distributions

Cowan's (1975) M3 model was used to model the distribution of headways in each of the minor and major streams. Cowan's M3 model is a dichotomised distribution distributed in cumulative form by:

$$F(t) = 1 - \alpha\, e^{-\lambda(t-\Delta)} \qquad t \geq \Delta$$
$$= 0 \qquad t < \Delta \tag{1}$$

where:

t is the sample headway (s);
Δ is the minimum headway within bunches (s);
α is the proportion of non-bunched vehicles; and

the shape parameter λ is defined as follows:

$$\lambda = \frac{\alpha q}{1 - \Delta q} \tag{2}$$

where q is the inverse of the mean headway, being equal to the stream flow rate, having units of veh/s.

Gap Acceptance Parameters

Gap acceptance theory stipulates that minor stream drivers will not enter a gap in the major stream less than their critical gap, denoted T.

Further, gap acceptance theory stipulates that minor stream drivers will not follow-on behind a leading vehicle, be it a major stream vehicle or just-merged minor stream vehicle, at a headway less than their minimum follow-on time, t_f.

Although variable within and across drivers, both parameters are assumed herewith as constants. Research by Troutbeck (1991) revealed that this assumption does not lead to substantial variations in the application of gap acceptance theory to the merging process.

Minor Stream Capacity

The capacity of a minor stream merging into a major stream is dependent upon the traffic arrival conditions in the major stream, and the gap acceptance parameters of minor stream drivers. Troutbeck (2002) quantified the minor stream capacity of a limited priority merge, $q_{2,\max}$, as follows:

$$q_{2,\max} = \frac{q_1 C \alpha\, e^{-\lambda(T-\Delta)}}{1-e^{-\lambda t_f}} \tag{3}$$

where:

q_1 is the major stream flow rate (veh/s);
α, Δ, and λ are major stream headway distribution parameters as defined in (1) above;
T and t_f are the minor stream critical gap and minimum follow-on time respectively; and the limited priority term, C, is defined as follows:

$$C = \frac{e^{\lambda t_f} - 1}{e^{\lambda t_f} - e^{-\lambda(T-t_f-\Delta)} - \lambda(T-t_f-\Delta)\, e^{-\lambda(T-t_f-\Delta)}} \tag{4}$$

It is noted that this theoretical capacity is restricted to the following range of the critical gap:

$$t_f \le T \le t_f + \Delta$$

Degree of Saturation

The minor stream degree of saturation, X, defines the demand/supply characteristic for the minor stream and is given by the ratio of minor stream demand flow rate to capacity. A degree of saturation equal to 1.0 corresponds to a capacity condition, which in the steady state yields infinite minor stream delays. A degree of saturation in excess of 1.0 indicates that the minor stream demand flow cannot be satisfied.

Minimum Average Minor Stream Delay

The minimum average minor stream delay, $D_{2,0}$, is that experienced by an isolated, random merging driver. (Subscript 2 denotes the minor stream and 0 the degree of saturation.) This delay was quantified by considering four independent components.

Figure 1 illustrates components d_1, d_2, and d_3 that are experienced by an isolated merging driver who rejects their first observed gap. The driver rejects the first observed gap between major stream vehicles 'a' and 'b', because it is less than the critical gap, T. The delay component is d_1. The successive gaps between vehicles 'b' and 'c', and then 'c' and 'd', are also less than T, so are rejected. This delay component is d_2. Finally, the driver is delayed when entering the gap they accept, in this case between 'd' and 'e', by an amount of time equal to the minimum follow-on time, t_f. This delay component is denoted d_3.

Figure 1 also illustrates the independent delay component, d_4, that is experienced by an isolated merging driver who accepts their first observed gap. When the isolated merging driver accepts their first observed gap, delay components d_1 to d_3 are not experienced. The only component is d_4. Such a delay occurs if and when the merging driver arrives subsequent to the leading major stream vehicle 'a', but within a time interval lesser in value than the minimum follow-on time, t_f.

All of the delay components, d_1 to d_4, are independent and can be summed to give the total minimum average delay to an isolated random merging driver, $D_{2,0}$. The following offers the theoretical development of this quantity.

The probability that a driver will arrive in a particular major stream gap is proportional to the length of that gap. Assuming the minor stream driver is able to note the time when the preceding major stream vehicle passed, the distribution of these first observed gaps, h(t), is given by:

$$\begin{aligned} h(t) &= q\,t\,f(t) \qquad && t \geq \Delta \\ &= 0 && t < \Delta \end{aligned} \tag{5}$$

If the first observed gap has a duration of 't' seconds, then the average delay to minor stream drivers is equal to t/2 seconds. The average delay component due to the first observed gap being less than the critical gap, T, is then given by:

$$d_1 = \int_0^T \frac{t\,h(t)}{2}\,dt \tag{6}$$

Substituting for Eq (5):

$$d_1 = \alpha\, q \left(\left(\frac{\Delta^2}{2\alpha} + \frac{\Delta}{\lambda} + \frac{1}{\lambda^2} \right) - \left(\frac{T^2}{2} + \frac{T}{\lambda} + \frac{1}{\lambda^2} \right) e^{-\lambda (T-\Delta)} \right) \tag{7}$$

The proportion of drivers delayed due to the first observed gap being less than the critical gap, H(T), is given by:

$$H(T) = \int_0^T h(t)\,dt \tag{8}$$

These drivers must review successive gaps until one is greater than the critical gap, T. The probability that a gap is greater than T is given by (1 – F(T)). If gap 'i' is accepted, then the delay is given by (i – 1)* $\overline{f}(T)$ where $\overline{f}(T)$ is the mean gap size less than T, given by:

$$\overline{f}(T) = \frac{\int_0^T t\, f(t)\, dt}{F(T)} \tag{9}$$

The average delay component for all drivers rejecting gaps after the first observed gap is then given by:

$$d_2 = H(T) \sum_{i=1}^{\infty} (i-1)\, \overline{f}(T)\, F(T)^{i-1}\, (1 - F(T))$$

$$= \frac{H(T)\, \overline{f}(T)\, F(T)}{(1 - F(T))}$$

or

$$d_2 = \alpha\, q \left(\left(\frac{\Delta^2}{\alpha^2} + \frac{2\Delta}{\alpha\lambda} + \frac{1}{\lambda^2} \right) e^{\lambda(T-\Delta)} + \left(T^2 + \frac{2T}{\lambda} + \frac{1}{\lambda^2} \right) e^{-\lambda(T-\Delta)} - \left(\frac{2T\Delta}{\alpha} + \frac{2\Delta}{\alpha\lambda} + \frac{2T}{\lambda} + \frac{2}{\lambda^2} \right) \right) \tag{10}$$

The H(T) drivers who reject the first observed gap are further delayed in the gap they accept by a time equivalent to the minimum follow-on time, t_f. This delay component, d_3, is given by:

$$d_3 = t_f\, H(T)$$

or

$$d_3 = \alpha\, q\, t_f \left(\left(\frac{\Delta}{\alpha} + \frac{1}{\lambda} \right) - \left(T + \frac{1}{\lambda} \right) e^{-\lambda(T-\Delta)} \right) \tag{11}$$

If a driver does accept their first observed gap they will still be delayed if they arrive within a time equivalent to the minimum follow-on time, after the preceding major stream vehicle.

The complete distribution of first observed gaps which are acceptable, that is, greater than the critical gap, is given by:

$$h(t|t > T) = \frac{h(t)}{(1 - H(T))} \qquad t > T$$
$$= 0 \qquad t \le T \tag{12}$$

The probability that a minor stream vehicle arrives within a small time interval of duration δt at any position within that acceptable first observed gap of duration t seconds is equal to δt/t, irrespective of when it arrived in the gap. The joint probability of a lead-time of γ seconds after the start of an acceptable first observed gap of duration t seconds is then equal to δt/t * h(t | t>T).

The complete distribution of lead-times, γ, amongst all acceptable first observed gaps is given by:

$$g(\gamma|t>T)=\int_T^\infty \frac{h(t|t>T)}{t}\,dt$$

or

$$g(\gamma|t>T)=\frac{\alpha\, q\, e^{-\lambda(T-\Delta)}}{\left(1-H(T)\right)} \tag{13}$$

Considering that (1 – H(T)) drivers accept their first observed gap, the average delay component is then given by:

$$d_4=(1-H(T))\int_0^{t_f} g(\gamma|t>T)(t_f-\gamma)\,d\gamma$$

or

$$d_4=\alpha\, q\left(\frac{t_f^{\ 2}}{2}e^{-\lambda(T-\Delta)}\right) \tag{14}$$

The total average delay to an isolated, random merging driver, $D_{2,0}$, is equal to the sum of the independent delays, d_1 to d_4. Summing Eqs (7), (10), (11) and (14):

$$D_{2,0}=\frac{\alpha q}{2}\left(T-t_f\right)\left(T-t_f+\frac{2}{\lambda}\right)e^{-\lambda(T-\Delta)}+\frac{e^{\lambda(T-\Delta)}}{\alpha q}-\left(2T-t_f\right)-\frac{1}{\lambda}+\frac{\left(\lambda\,\Delta^2-2\Delta+2\Delta\alpha\right)}{2\left(\lambda\Delta+\alpha\right)} \tag{15}$$

This minimum average minor stream delay occurs when the minor stream flow rate approaches zero. This delay can also be used to describe the average delay experienced by a pedestrian in a group that accepts gaps in a limited priority fashion, that is, with the traffic being prepared to slow to allow the pedestrians to cross.

Average Minor Stream Delay

Eq (15) enables the minimum average minor stream delay to be estimated for the isolated, random merging driver. Under general traffic flow conditions, however, the arrival flow in the minor stream is higher. This results in larger delays, due to the queuing component of delay, where the driver must wait for minor stream vehicles ahead of them to merge. Under steady state conditions, the average minor stream delay increases more rapidly as the minor stream flow rate approaches capacity, where it tends to infinity.

The form of the Pollarcezc-Klintchine function (Newell, 1982) was chosen to represent this relationship based upon data acquired from simulation of limited priority merging behaviour. The function is similar to that used to represent an M/G/1 queuing process and is listed below.

$$D_{2,X} = D_{2,0}\left(1 + \frac{\varepsilon X}{1 - X}\right) \tag{16}$$

where ε is a shape parameter that determines the rate of rise of the delay/saturation function.

A larger value provides a greater rate of rise in the delay/saturation function, thus higher average minor stream delays for all degrees of saturation between 0 and 1, and a poorer performance of the merge. It is noted that a value of ε equal to 1 corresponds to random service times in traditional queuing theory.

The term ε was found to be a function of the headway distribution parameters in the major and minor streams, and the minor stream gap acceptance parameters, or:

$$\varepsilon = fn\left(q_1, \alpha_1, \Delta_1, q_2, \alpha_2, \Delta_{2,} T, t_f\right) \tag{17}$$

A theoretical relationship for ε is complicated by the extensive number of influencing parameters. Instead, an empirical function for ε was developed from regression of simulation model data under an appreciable range of traffic flow conditions. The following sections discuss the calibration of parameters and development of a general function to estimate ε.

3. Calibration of Parameters

Headway Distributions

Headway data was captured at four freeway merge sites in Brisbane, Australia to calibrate major stream (kerb lane net of vehicles that moved to the adjacent lane) and minor stream (on-ramp) headway distributions. The objective was to develop indicative relationships to show the extent of the effect of limited priority.

For each site the data consisted of 15 minute samples collected under a range of uncongested flow conditions. The Cowan's M3 model (Eq 1) was fitted to each stream using the Maximum Likelihood Technique (Luttinen, 1999). The minimum headway, Δ, was set constant to 1s in all cases.

Figure 2 is a plot of (α, q) data pairs from the major stream for all four sites. Assuming that the data all belong to the same population, the following dichotomised function was developed empirically to estimate α as a function of flow rate:

$$\begin{aligned} \alpha_1 &= e^{-0.55(q_1-0.025)} & q_1 &\geq 0.025 veh/s \\ &= 1 & q_1 &< 0.025 veh/s \end{aligned} \tag{18}$$

This function was developed using a limited major stream flow range between 300veh/h and 1,000veh/h. The reduction in α with increased flow rate is explained by increased bunching between vehicles.

The characteristics of headway distributions on-ramps located downstream of unsignalised intersections are shown in Figure 3. The following indicative function was developed empirically for α:

$$\alpha_2 = e^{-1.5 q_2} \tag{19}$$

This function was developed with a minor stream flow range between 150veh/h and 900veh/h. The reduction in α with flow rate is higher than that in the major stream, and is explained by a greater level of bunching between vehicles that depart the unsignalised intersections upstream of the on-ramp.

Similar data for traffic on an on-ramp downstream of a signalised intersection is shown in Figure 4. The following indicative function was developed to estimate α as a function of flow rate:

$$\alpha_2 = e^{-1.7(q_2+0.35)} \tag{20}$$

This function was developed with a limited minor stream flow range between 650veh/h and 1,050veh/h. The proportion of bunching will depend on the signal settings and traffic flows. Eq (20) should not be considered to apply universally.

All three functions demonstrate that α reduces with increasing flow suggesting bunching increases as flow rate does. In comparison with the minor streams downstream of unsignalised intersections, the α values are significantly lower, meaning bunching is significantly higher, which is explained by the signalised intersection generating pulsed traffic flow. Similarly the level of bunching on either on-ramp is higher than the freeway kerb lane.

Table 1 lists the analysis of variance on the regression performed according to the constants in each of Eqs (18), (19) and (20) respectively. The table states the significance of the F ratios, which indicate that Eqs (18) and (19) are adequate models for estimating α given q for the data set used in each regression. The F ratio for Eq (20) exceeds the critical value at a five percent significance, which is attributed to the small sample size and narrow band of available data.

Eqs (18), (19) and (20) provide a means of predicting orders of magnitude for values of the proportion of non-bunched vehicles, α, given stream flow rate, q. Ideally, a much greater amount of data is desirable, from a much greater number of sites displaying a greater range of characteristics, and where possible across a greater range of flows.

Critical Gap and Minimum Follow-on Time

Only very infrequently do vehicles follow eachother into the merging opportunities in the kerb lane traffic. Accordingly, it is difficult to find robust estimates of the follow-on time. Reasonable estimates were obtained from the characteristics of the headways between on-ramp vehicles, which were measured at the four Brisbane freeway merge sites by recording arrival times of their vehicles at the end of the merge tapers. From the measured data of each site, the modes of the distributions were identified, from which the means of the minimum follow-on times were estimated. Figure 5 illustrates the probability density functions of all follow-on times from all 15 minute data sets combined for each site, in the range 0s to 3s.

Table 2 lists the minor stream minimum follow-on times that were subjectively estimated from this limited data. It is estimated that the general value for freeway merging is equal to 1s, with a larger value of 1.2s on more restricted alignments.

Calibration of minor stream population critical gap for freeway merging was also found to be difficult. The Siegloch technique (Kyte *et al*, 1994) could not be used because it requires saturated conditions on the merge, whereas freeway merges are undersaturated during uncongested operation, to which this study was limited. Brilon *et al* (1997) examined several methods of estimating critical gaps, of which the maximum likelihood technique (Troutbeck, 1990) and Hewitt's method were shown to have given the best results. The former requires for each minor stream driver their largest rejected gap and their accepted gap, between which their critical gap is estimated to lie. An absence of rejected gaps and lags precluded use of this technique. The latter requires all observed gaps and lags to be recorded along with the number of rejected gaps and lags. This basis of this technique was used to calibrate population critical gap in this study.

The lags rejected and accepted were measured at each Brisbane area freeway merge site studied were converted into gaps, by adding the site minimum follow-on time. This conversion was made because gaps were defined between front bumpers of vehicles. The distributions of gaps offered were then categorised into 0.5s intervals. Similar to Hewitt's method, the probability of acceptance was calculated from the data for each interval, providing an estimate of the acceptability of a gap of a given size.

Based upon the calculated probabilities of acceptance of each site, the population critical gap is estimated to be larger than the minimum follow-on time, below which the probability of acceptance was equal to zero, and less than the sum of the minimum follow-on time and the major stream minimum headway, beyond which the probability of acceptance was equal to one. In the absence of a means of discretely estimating critical gap, this study considers critical gap to be a variable within this range.

4. Estimation of Minor Stream Delays

A simulation model was constructed to produce average minor stream delays, given the major stream and minor stream headway distribution parameters, and minor stream gap acceptance parameters calibrated above. Arrivals in each stream were simulated independently.

For a particular value of major stream flow rate and minor stream critical gap, the minor stream flow rate was varied to produce nine data pairs of minor stream average delay and degree of saturation ($D_{2,X}$, X). Figure 6 illustrates an example data set where the critical gap was set to 1.75s and the major stream flow rate 0.2veh/s (720veh/h).

For each data set between degree of saturation 0.1 and 0.8 an estimate of shape parameter, ε, was calculated by least squares regression between Eq (16) and the simulation data. Figure 6 illustrates the regression function for the example data set. The point corresponding to degree of saturation 0.9 was excluded from regression because of the variability in delay at high degrees of saturation, and the effect of a finite simulation time on recorded delay values.

It is useful to estimate ε across a range of major stream flow rate and critical gap. This was achieved by regressing the estimates of ε, using the technique described above, across a range of flows and critical gap values.

Critical gap was varied between 1s and 2s in 0.25s increments, and major stream flow rate between 0.1 and 0.9veh/s, in 0.1s increments, to yield 45 estimates of ε. A range of functions was trialled to determine those best describing the influence of major stream flow rate, and of critical gap, on the logarithm of ε. A cubic polynomial was found to best reflect the influence of the former while a quadratic the latter; this resulted in the following general function:

$$\ln(\varepsilon) = \left(aT^2 + bT + c\right)q_1^{\,3} + \left(dT^2 + eT + f\right)q_1^{\,2} + \left(gT^2 + hT + i\right)q_1 + \left(jT^2 + kT + l\right) \tag{21}$$

where terms *a…l* are regression constants.

These constants were calibrated for three separate merge area conditions; with minor stream headway distributions reflecting traffic departing an unsignalised intersection, a signalised intersection, and a constant departure ramp meter. Table 3 lists the set of regression constants for each merge area condition. The marked differences in values of each of the constants between conditions are attributed to the different level of minor stream queue interaction associated with bunched versus semi-bunched and non-bunched arrivals.

Table 4 lists the analysis of variance on the regression used to determine the constants *a…l* for each of the three minor stream arrival conditions. It is apparent from the F ratio for each of the analyses that Eq (21) is an adequate model for estimating ε given critical gap and major stream flow rate.

The constants listed in Table 3 correspond to a minimum follow-on time of 1s. An alternative set of constants may be developed for the higher minor stream minimum follow-on time of 1.2s corresponding to tighter alignments. However, the relative effect would be expected to be similar. Eq (21) and the associated constants are intended to quantify the relative influence of the ramp traffic conditions. They are not seen to be universally applicable and are dependent on the follow-on times and critical gaps.

5. Effect of Minor Stream Arrival Pattern

The boundary conditions of the steady state minor stream average delay/degree of saturation functions are identical between all three minor stream arrival patterns resulting from upstream intersection control systems, viz. unsignalised, signalised, and constant departure ramp metered. These boundary conditions being minimum average minor stream delay, $D_{2,0}$, given by Eq (15), and maximum average minor stream delay, $D_{2,1}$, equal to infinity for steady state conditions. Between these boundaries, the rate of increase in average minor stream delay with degree of saturation is reflected by the value of shape parameter, ε.

For a given degree of saturation, X, a higher value of ε yields a higher average minor stream delay, $D_{2,X}$, than the minimum average minor stream delay, $D_{2,0}$. The value of the shape parameter indicates the degree of interference experienced by minor stream drivers in merging, due to minor stream vehicles merging ahead of them, which is a consequence of the arrival pattern.

Figure 7 illustrates relationships between shape parameter, ε, and major stream flow rate, q_1, for the three minor stream arrival patterns under consideration, with critical gap equal to the upper bound of 2s and given the indicative headway distribution characteristics. It can be seen that the shape parameter of unsignalised departures is approximately one-half that of signalised departures, across all major stream flow rates. The shape parameter of constant metered departures is approximately one-third that of unsignalised departures, across all major stream flow rates. These conditions are more prominent at the lower major stream flow rates. These results indicate that, for a given degree of saturation, the average delay incurred by minor stream drivers relative to the minimum average minor stream delay, is not as great when they arrive at constant intervals, as when they arrive in a semi-bunched state downstream of an unsignalised intersection, and in turn as when they arrive in bunches downstream of a signalised intersection. Similar and more pronounced effects have been found for the critical gap equal to the mid range value of 1.5s.

6. Relationship between Shape Parameter and Flows

Figure 7 shows that shape parameter, ε, reduces as major stream flow rate increases, suggesting that for a given minor stream degree of saturation, there is more interference in the minor stream merging process under lower major stream flow rates. This was investigated.

It is important to note first that a given degree of saturation does not represent the same minor stream conditions between two different major stream flow rates, because the different major stream flow rates yield different boundary conditions of capacity and minimum average minor stream delay.

Figure 8 illustrates this idea more specifically, for the case of a signalised intersection upstream of the merge, and a critical gap of 2s. It shows the relationships between average minor stream delay and arrival flow rate as major stream flow rate, and accordingly ε, vary. It is clear that a degree of saturation of 0.5 corresponds to markedly different average minor stream delay and arrival flow rate, across the eight major stream flow rates depicted. From Figure 8 it is apparent that ε has little influence on average minor stream delay in comparison with flow rate in either stream. This is evident from the apparent distortion in the curves closer to the vertical axis, which corresponds to lower minor stream capacities, than those extending further across the horizontal axis, corresponding to higher capacities. Even so, it is useful to understand why ε reduces as major stream flow rate increases.

Consider the leftmost and rightmost curves shown of Figure 8. Examination of Figure 4 for the minor stream reveals a higher proportion of non-bunched vehicles, or less bunching, under the narrow minor stream flow range evident in the leftmost curve of Figure 8, than under the wider minor stream flow range evident in the rightmost curve. Conversely, extrapolation from Figure 2 for the major stream reveals a lower proportion of non-bunched vehicles, or more bunching, under the higher major stream flow rate corresponding to the leftmost curve of Figure 8, than under the low major stream flow rate corresponding to the rightmost curve.

In the case of the leftmost curve, for a particular degree of saturation, minor stream mergers are more likely to be separated from others, hence average delay closer to the minimum average minor stream delay, and a smaller value of ε. In the case of the rightmost curve, for a particular degree of saturation, minor stream mergers are more likely to encounter interference due to minor stream vehicles ahead of them attempting to merge into a more dispersed major stream.

It is not particularly useful to compare the value of ε as major stream flow rate varies, because the major stream flow rate itself has a significantly greater influence on minor stream conditions. This shape parameter is a more useful quantity in comparing the influence of minor stream arrival patterns on average delay when the boundary conditions are the same, as is the case under a constant major stream flow rate.

7. Practical Example in Comparison of Minor Stream Arrival Patterns

An example illustrates the effects of the three minor stream arrival patterns. Consider a freeway merge with good geometry, a minor stream (on-ramp) flow rate of 700veh/h and a major stream (kerbside freeway lane) flow rate of 840veh/h., giving a combined downstream flow rate of 1,540veh/h in the kerbside lane, reflecting uncongested conditions.

With critical gap equal to the upper bound value of 2s, using the data presented in previous sections the minimum average minor stream delay, $D_{2,0}$, is estimated to be 0.31s and minor stream degree of saturation, X, is estimated to be equal to 0.29. From Eq (21) and Table 3 the shape parameter, ε, is estimated as follows: (i) 2.67 under unsignalised intersection control upstream of the on-ramp; (ii) 5.16 under signalised intersection control upstream of the on-ramp; and (iii) 0.84 under constant metered departures upstream of the on-ramp. The corresponding average minor stream delays, $D_{2,0.29}$, are estimated from Eq (16) to be: (i) 0.65s under unsignalised intersection control upstream of the on-ramp; (ii) 0.96s under signalised intersection control upstream of the on-ramp; and (iii) 0.42s under constant metered departures upstream of the on-ramp.

In this particular case, the average delay incurred when minor stream traffic departs an unsignalised intersection upstream of the on-ramp is two-thirds that incurred when it departs a signalised intersection. In turn, the average delay incurred when minor stream traffic is metered constantly is approximately 40% that incurred when it departs a signalised intersection. The average delay is related to the distance travelled by the on-ramp driver before they can merge.

This example illustrates that limited priority gap acceptance provides a useful means of quantifying freeway merge area operation, particularly as minor stream average delay is a direct measure of the ease of merging. Hence its use in "level of service" assignment for other unsignalised intersection types (TRB, 2000).

When quantifying the probability of minor stream drivers being delayed, drivers have a brief amount of time to merge freely at speed on a freeway merge, so excessive delay cannot be tolerated. This makes minor stream average delay a critical parameter in quantifying the merge area performance. In this context, the previous example highlights the significance of the effect of minor stream arrival condition on minor stream delay, and therefore on merge area performance.

8. Conclusions

The limited priority gap acceptance model presented in this paper provides a useful means of comparing performance, through average minor stream delay, for varying minor and major stream flow rates and minor stream critical gap, under arrival patterns that differ due to traffic control upstream of the on-ramp. Minor stream delay is a direct measure of the ease of merging, hence its use as a primary measure of effectiveness for other intersection types. For uncongested freeway merging, to which the model is limited, it is a more critical measure as delay relates directly to the distance required to merge. Drivers only have a limited amount of time to merge freely at speed so excessive delay cannot be tolerated.

Observations from the model developed provide physical evidence that minor stream drivers incur lesser delay, or have a better chance of merging quickly, when they arrive at constant intervals as is the case under constant departure ramp metering, than when they arrive in bunches downstream of a signalised intersection, or even a semi-bunched state downstream of an unsignalised intersection. The more even separation resulting from metering reduces interference between merging drivers and those merging ahead of them.

The priority sharing model as a basis may be extendable to congested conditions; however, the mechanism may differ from limited priority. Consequently the gap acceptance theory developed may not be directly applicable, nor its parameters as calibrated here. Further research would be useful to enhance the understanding of congested merging operation.

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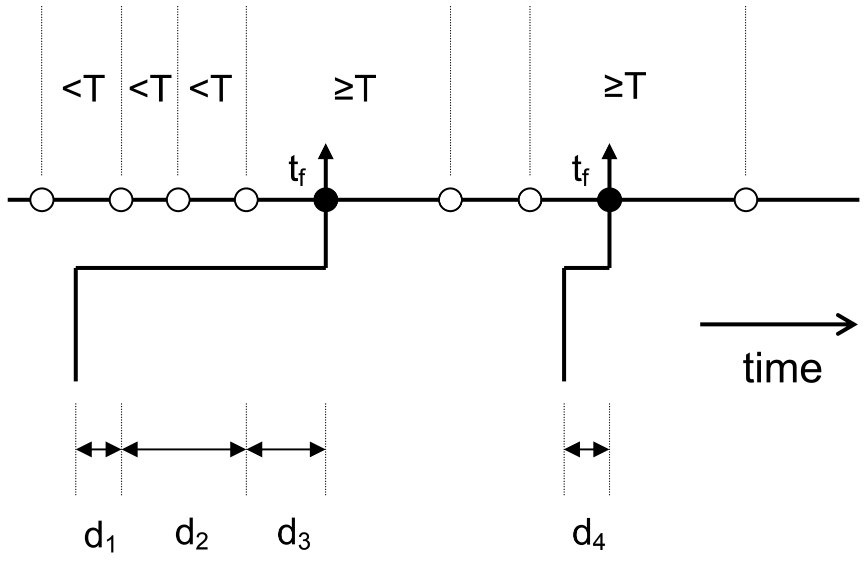

Fig. 1. Independent delays, d_1, d_2, d_3, and d_4 to isolated merging drivers

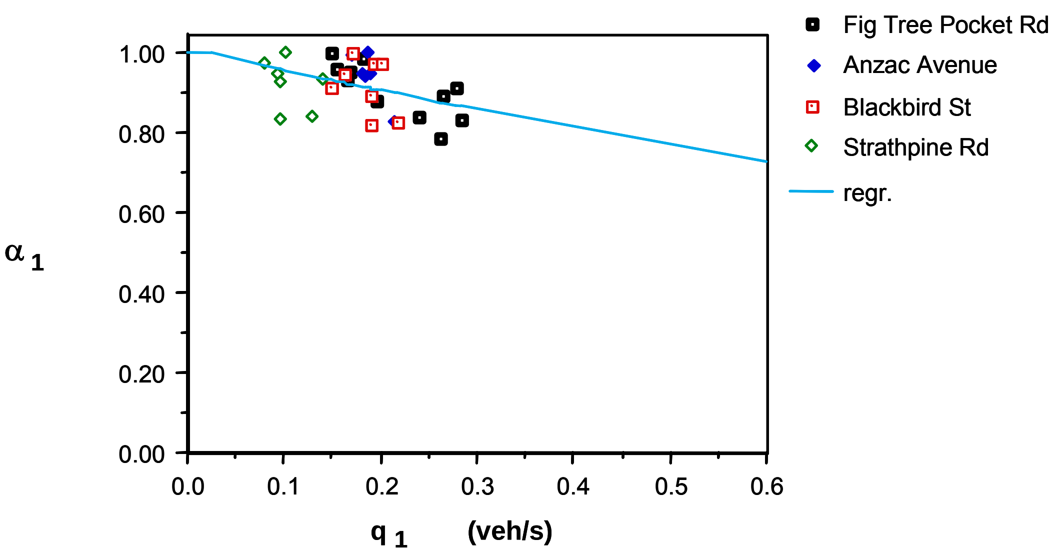

Fig. 2. Regression of major stream 15 minute (α, q) data over four freeway merge sites

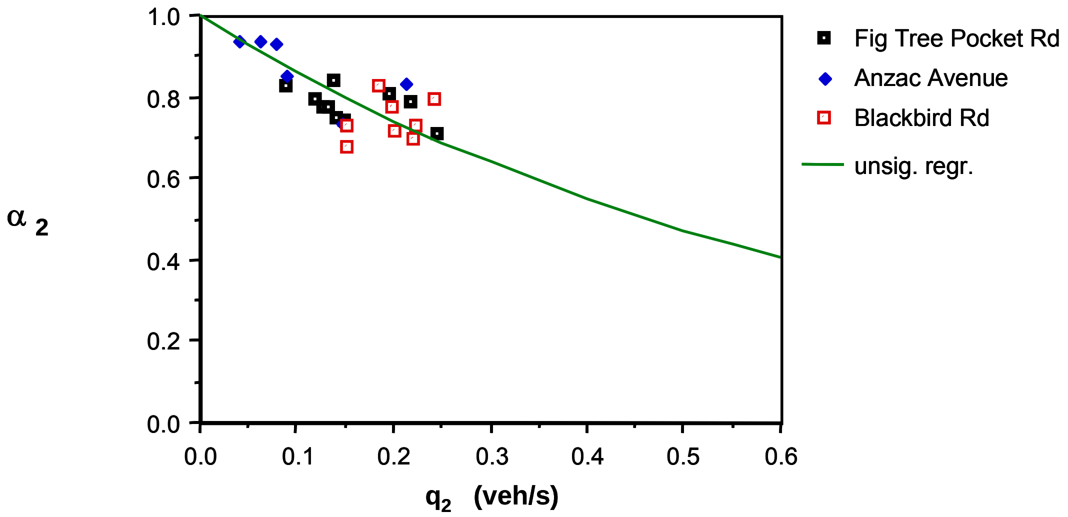

Fig. 3. Regression of minor stream 15 minute (α, q) data over three freeway merge sites downstream of unsignalised intersections

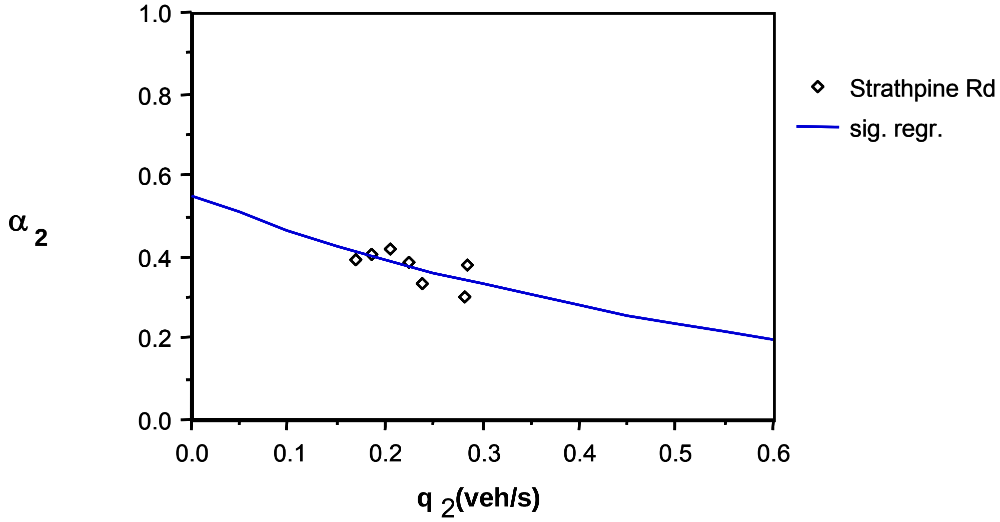

Fig. 4. Regression of minor stream 15 minute (α, q) data at a freeway merge site downstream of a signalised intersection

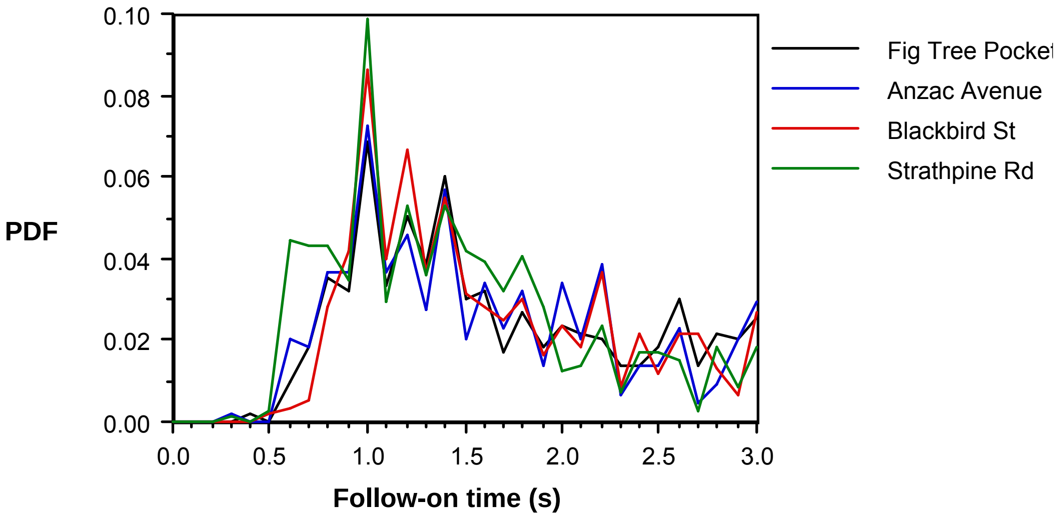

Fig. 5. Probability density functions of all merging follow-on times in the range 0s to 3s over four Brisbane freeway merge sites

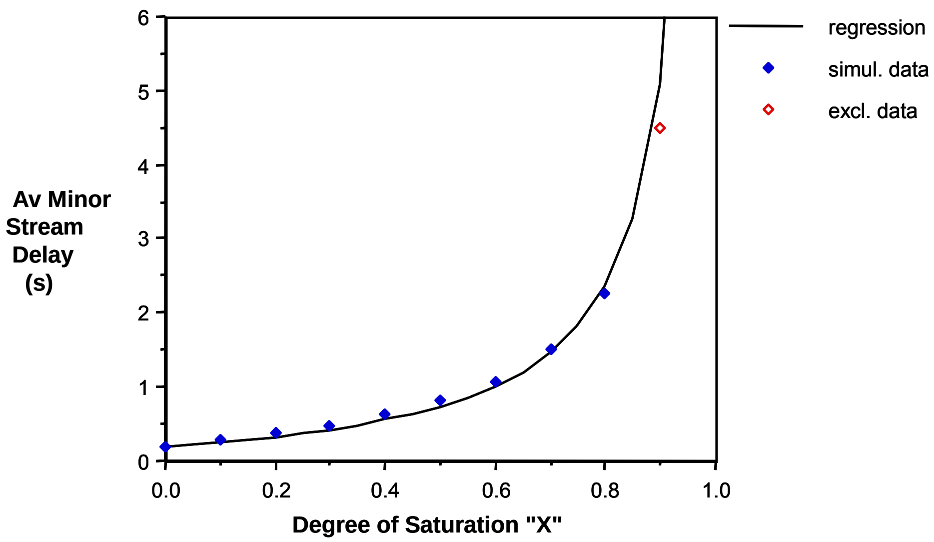

Fig. 6. Example of regression function representing minor stream average delay vs degree of saturation against simulated data

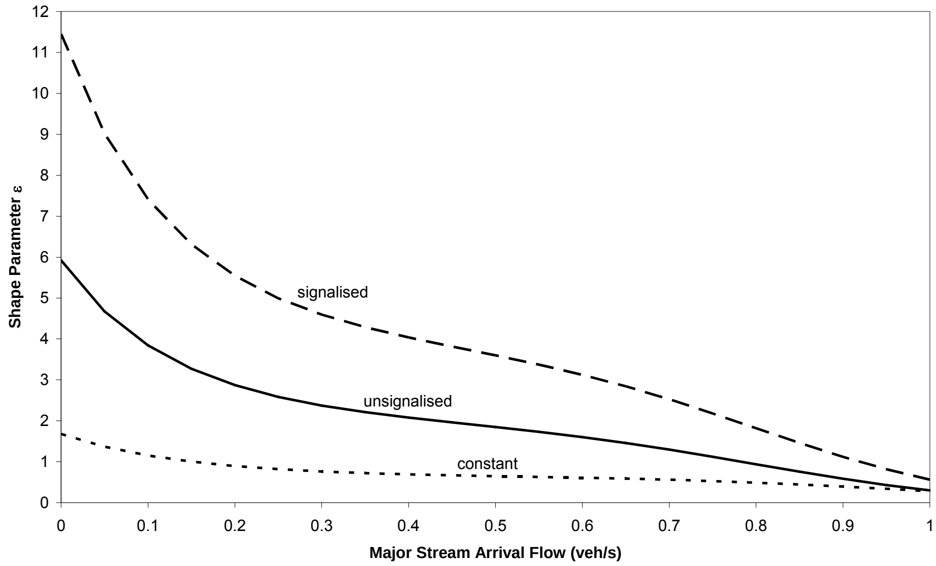

Fig. 7. Shape parameter vs major stream arrival flow for various minor stream arrival conditions, critical gap equal to 2s

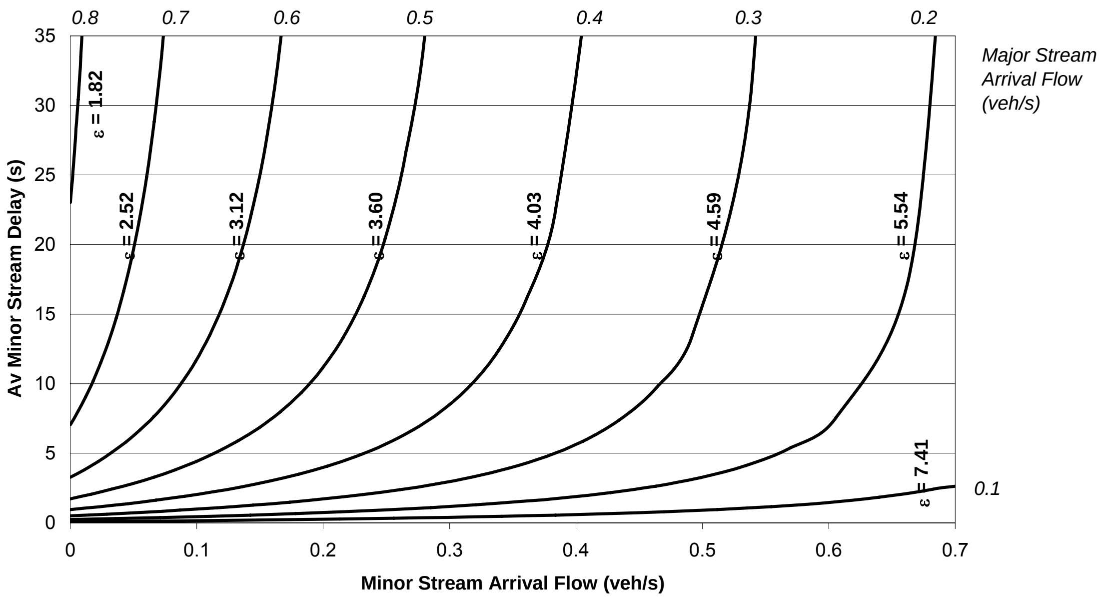

Fig. 8. Average minor stream delay vs arrival flow, downstream of a signalised intersection, critical gap equal to 2s

Table 1
Analyses of variance on merge area headway distribution parameter relationships

Source	DF	SS	MS	F ratio
Eq (18) major stream				
Regression	1	0.023	0.023	6.33
Residual	30	0.110	0.004	$p = 0.025$
Total	31	0.133		
Eq (19) minor stream (downstream of unsignalised intersections)				
Regression	1	0.102	0.102	29.01
Residual	20	0.070	0.004	$p < 0.01$
Total	21	0.172		
Eq (20) minor stream (downstream of signalised intersection)				
Regression	1	0.005	0.005	3.53
Residual	5	0.007	0.001	$p > 0.05$
Total	6	0.012		

Table 2
Preliminary calibration of freeway merge area critical gap and minimum follow-on time

Site Configuration	Minimum follow-on time	Lower bound of critical gap range	Upper bound of critical gap range
Good geometry	1.0	1.0	2.0
Tight geometry	1.2	1.2	2.2

Table 3
Regression constants required to estimate in Eq (21) shape parameter, ε

Constant	Minor stream (ramp) upstream intersection control		
	Unsignalised	Signalised	Constant metered departures
a	-1.970	1.491	-4.012
b	-6.292	-17.15	10.05
c	13.44	21.13	-8.870
d	5.887	2.197	10.00
e	-1.747	9.292	-25.47
f	-10.83	-17.95	18.40
g	-2.405	-1.430	-4.858
h	-0.4451	-3.238	10.83
i	5.326	6.975	-6.667
j	0.5594	0.3602	1.084
k	-0.7053	-0.1971	-2.386
l	0.9522	1.391	0.9541

Table 4
Analysis of variance on Eq (21)

Source	DF	SS	MS	F ratio
		Unsignalised		
Regression	11	8.239	0.749	52.01
Residual	33	0.475	0.014	p = 0.0001
Total	44	8.714		
		Signalised		
Regression	11	9.136	0.831	172.3
Residual	33	0.159	0.005	p = 0.0001
Total	44	9.295		
		Constant metered departures		
Regression	11	3.516	0.320	33.72
Residual	33	0.313	0.009	p = 0.0001
Total	44	3.829		