MODELLING DELAY RISKS ASSOCIATED WITH TRAIN SCHEDULES

A. Higgins

L. Ferreira

E. Kozan

1 School of Mathematics, Queensland University of Technology, PO Box 2434, QLD 4001, Australia

2 School of Civil Engineering, Queensland University of Technology, PO Box 2434, QLD 4001, Australia

Address for Correspondence: L. Ferreira (as above)
ABSTRACT

The overall timetable reliability is a measure of the likely performance of the timetable as a whole, in terms of schedule adherence. The reliability of arrivals is a critical performance measure for all rail markets. This paper presents analytically based models designed to quantify the amount of delay risk associated with each track segment, train and the schedule as a whole. Three main types of delays are modelled, namely: terminal/station delays; track related delays; and rolling stock related delays. The models can be used to prioritise investment projects designed to improve timetable reliability. For example, a comparison can be made between track, terminal and rolling stock projects, in terms of their likely impact on timetable reliability. The effect of timetable changes on likely reliability can also be modelled. Using the risk models developed here, it is possible to assess the likely effect of removing/adding sidings for passing and crossing purposes, under single line train operations. The paper illustrates the use of the models using a series of tests with a timetable consisting of nine trains and six stations. The effect of changing assumptions regarding delays due to terminal congestion; track related problems; and rolling stock, are modelled and the results are summarised. Also modelled is the impact of changes to the timetable.

KEY WORDS: Rail Operations, Reliability, Risk Analysis, Modelling
INTRODUCTION

When freight and passenger trains are scheduled on a rail corridor the objective is to achieve a given level of customer service whilst minimising overall operating costs. Customer service in this context is made up of several attributes which include overall journey time of trains and the degree to which those journey times are achieved on a regular basis. Transit time reliability can be defined as the probability that the planned arrival time will be achieved for each train. In the context of freight movements, the benefits of improved reliability need to be estimated on a train by train basis. Each train is usually loaded with freight from a range of customers and origin-destination flows. The elasticity of demand with respect to transit time reliability will differ for each customer, commodity, and origin-destination combination.

The overall timetable reliability is a measure of the likely performance of the timetable as a whole, in terms of schedule adherence. The concept is important for both urban and non-urban rail passenger services, as well as rail freight transportation. The reliability of arrivals is a critical performance measure for all rail markets. The ability of rail systems to compete effectively relies to a large extent on consistent transit time reliability1,2,3,4.

The next section of the paper discusses the importance of transit time reliability when considering both a fixed level of infrastructure, and when analysing options for long term investment in terminals, rolling stock and track upgrading. The third section puts forward the general variables used in the model, the assumptions for timetable risk analysis and the classification of the types of delay; whilst in the fourth section risk delay equations are put forward for three distinct categories of delays. This is followed by the testing of the models using an example comprising of nine trains and six sidings. Finally, in section 6 the main conclusions are drawn and areas for further research are identified.

For the purposes of this paper 'siding' is a section of track which can be used for the crossing and passing of trains under single track operations. The terms 'crossing loop' or 'passing loop' are also used in some countries to describe such track sections. Figure 1 shows the concept of a siding used in this paper.

TRANSIT TIME RELIABILITY

General

A schedule which is planned to minimise overall operating cost is not necessarily the optimum schedule from the viewpoint of overall net benefit. This is particularly the case when the schedule is easily disrupted through small delays to individual trains. If a schedule is unstable, trains can be at risk of suffering heavy delay if any unexpected event occurs. Rail systems have a major interest to determining the risk associated with a given schedule. This risk may be analysed further to determine which trains are at most risk from a delay, or which track segment(s) cause the most instability to the schedule.

An example of instability with single line track is the case when the number of sidings where trains can pass or cross each other, is just enough to meet planned demand. When a train is delayed (due to breakdown or other sources such as signalling, weather or track
maintenance), considerable delay may occur on the system. There are two reasons for this, namely: the schedule is very tight and a delay to one train will cause delays to other trains; secondly, since there are few sidings, they may be far apart. In this case it may take longer to move a broken down or slowed down train to the next siding. Larger delays will occur to more trains due to the larger spaces between sidings. Similarly, when several sidings are used but are more clustered in one part of the track, there will be a larger delay risk due to some parts of the track having long distances between sidings. The risk is at a minimum when there are many sidings equally spaced (given the conflicts are evenly spread out). Therefore this minimum risk is achieved at a cost of constructing and maintaining sidings and associated way-side signalling.

Infrastructure and Rolling-stock Upgrading

A risk model can be used to analyse the timetable risks associated with given levels of investment in track, terminals and rolling-stock. This is important for determining which categories of investment are likely to yield the greatest benefit in terms of increased transit time reliability. For example, if a track segment causes high unexpected delay due to its low speed restriction, then the effects of increasing the speed limit can be analysed for possible upgrading. Therefore the outputs of a risk model can be used to undertake a cost benefit analysis of rail projects.

There are several ways in which investment in track related infrastructure can reduce delays and hence improve transit time reliability. Four main investment strategies may have significant impact on the probabilities associated with train delays, namely:

(a) Investment in major track strengthening to increase maximum allowable speeds. The higher speeds have the potential to reduce conflict related delays and improve train recoverability;

(b) Investment designed to alter track alignments, both vertical and horizontal, thus increasing average train speeds;

(c) Investment in additions to the number and length of sidings where trains can cross and pass each other on single line track. Conflict related delays are directly affected by the number, length and location of sidings; and

(d) Investment in advanced train control and communication systems to allow trains to proceed at shorter headways, and with less stops required for safe train operations.

Reduced delay probabilities will also result from investments in non-track related areas, such as: terminal infrastructure and information systems designed to improve freight handling operations, thereby improving on-time departure performance; and new locomotives capable of higher maximum speeds and improved self-diagnostics capability to reduce breakdown incidents.

PROBLEM CHARACTERISTICS AND DEFINITIONS
Background

There has been considerable research into the optimisation of train operations under a single line regime\(^5,6,7,8\). All of these authors resolve the crossing/passing of trains so as to minimise some objective function. The calculated optimal schedule may only be optimal if no other delays occur to the train operations. There has been very little research which considers what happens to the schedule if unexpected delays occur. The effects of such delay has been considered\(^9\) for the purpose of analysing jam capacity. The latter measures the maximum flow of trains along a track corridor. When the flow of trains is near the capacity, the system is unstable due to the unexpected delay. A rail line with low risk of unexpected delay will be able to operate near capacity with very little instability. The risk delay to a train when it is travelling in the same direction as another train which is delayed, is analysed (Carey et al 1994). The first train is subject to random delay and the risk to the second train is dependent on the headway between the two trains. Estimates of the expected trip time of the train due to the delay are calculated by non-linear regression and heuristic methods assuming this stochastic dependence. The model is simple but useful for determining ideal headways between trains travelling in the same direction.

Chen and Harker (1991) are the first to develop a line delay model to determine the train delay of a given unresolved schedule. The authors model the probability of a train controller delaying a particular train due in a conflict. This probability model is based on a historical dispatching behaviour. The actual conflict delay between two trains is based on this probability and the probability of the two trains interfering with each other. The latter probability is dependent on the outcome of prior conflicts in the schedule and unforeseen events. The unforeseen events is treated as a departure time distribution from the origin station. The resulting model (system of equations) is solved using iterative methods. The model was enhanced by Harker and Hong (1990) to include a partially double track corridor. Hallowell (1993) improves upon several of the deficiencies of these two models by: allowing the trains to enter/exit at any point of the track corridor; accounting for an optimal meet/pass process; and allowing train priorities to be a function of expected delays. All models are a bit unrealistic for the following reasons:

- The sidings are assumed to be equally spaced. This means the model would not be very accurate when applied to a track corridor with many short and long track segments.
- Unforeseen events is only treated as a departure time distribution and does not consider over the track delays explicitly.
- More importantly, the model does not consider the nature train movements after an unforeseen event. After an unforeseen event a severe bottleneck can occur. Trains will clear in bunches so as to keep the overall delay as small as possible. The model does not restrict one train at a siding at one time so the effect from a long unforeseen event cannot be modelled effectively.
- The total conflict delay for a particular train is the sum of delays for each conflict. In a congested schedule when the trains are bunched, the situation often occurs where a train must wait at a siding for two opposing trains to cross. The total conflict delay is less than the sum of the individual conflict delays.
- Lastly, the model is too complicated to be used in an objective function for schedule optimisation.
The only models developed for determining the risk delay for an entire schedule are by Chen and Harker (1991), Harker and Hong (1990) and Hallowell (1993).

**Classifying the Delay**

Modelling train delays is made more complex as the causes of those delays increase. For modelling purposes, it is important to deal with each of the main delay sources separately. In this way, investment strategies designed to reduce delays from individual sources can be assessed. Three main delay categories are identified here, namely:

(a) Track related delays. These can be sub-divided into: those delays which are caused by a train having to slowdown due to track problems (eg. temporary speed restrictions); and delays caused by a complete stoppage for a period of time. Signal failure and obstacles on the track are examples of this type of delay.

(b) Train dependent delays. These are delays caused by a train which breaks down or is forced to slowdown in a line section for reasons other than track problems ( eg. locomotive failure).

(c) Terminal/schedule stop delay. These are delays which occur at scheduled stops and at the origin station. This may involve delays associated with loading/unloading, train connections, fuelling and crew changes.

**Definition of Variables**

The sidings are represented by the set $Q = \{1, 2, \ldots, NS\}$ where $NS$ is the total number of sidings in the track system. Let $I = \{1, \ldots, M, M+1, \ldots, N\}$ represent the set of train trips where the inbound train trips are numbered from $1$ to $M$ and the outbound train trips are numbered $M+1$ to $N$. The general variables used in this paper are defined as follows:

- $X^{'aq}_{i I q} = \text{actual arrival time of train } i \in I \text{ at station } q \in Q \text{ if no risk delays occur}$
- $X^{'dq}_{i I} = \text{actual departure time of train } i \in I \text{ from station } q \in Q \text{ if no risk delays occur}$
- $X^{'Di}_{i} = \text{arrival time of train } i \in I \text{ at its destination station if no risk delays occur}$
- $X^{'Oi}_{i} = \text{actual departure time of train } i \in I \text{ from its origin station } O_i$
- $d^p_p = \text{length of segment } p \in P$
- $Y^{'Oi}_{i} = \text{planned departure time of train } i \in I \text{ from its origin station}$
- $Y^{'Di}_{i} = \text{planned arrival time of train } i \in I \text{ at its destination station}$
- $v^p_p = \text{velocity of train } i \in I \text{ on segment } p \in P$
- $\bar{v}_p = \text{maximum achievable average velocity of train } i \in I \text{ on segment } p \in P$
- $ED_i^q = \text{expected remaining conflict delay for train } i \in I \text{ from siding } q \in Q$
- $W^i = \text{the initial priority of train } i \in I$
- $P^i_q = \text{number of remaining track segments for train } i \in I \text{ to travel, from siding } q \in Q$
- $h = \text{the minimum time distance between two trains which follow each other (ie. headway)}$. 
Assumptions

The two main assumptions are:

- An actual resolved train schedule is made available, indicating the arrival time of trains at all sidings if there is no unexpected delay.
- The schedule repeats itself after a period of time.

The following information is to be provided by the user:

- The initial priority of each train. This may be a function of train type, (e.g., passenger vs. freight train), type of freight and customer considerations.
- The upper velocities of each train on each siding of the trains path. This is a major factor for determining if the train can recover from a delay.
- The distribution of length of delays, caused by unexpected events on tracks. A different set of distributions for each type of delay will have to be obtained.
- The probability of a train being delayed per kilometre of track for the train caused delay. This will also be estimated from past data.
- The probability distribution for the track related delays. Such a distribution is dependent on the track segment.
- The probability distribution for terminal/siding related delays.

FORMULATION OF RISK DELAY

Line segment risk involves the delay placed on the system when a train deviates from its schedule for any reason such as those described earlier. In this section each of the three types of delay mentioned earlier will be modelled. For example, if a train breaks down on a track segment, it may have to slow down to get to the next siding. If the segment is long with a large amount of congestion the system will suffer considerable delay. The risk delay is determined by the probability of slowing down for each unit of distance; the lengths of the individual track segments; and the congestion on these segments. In the case of a breakdown due to locomotive failure, the distance from the nearest depot also plays a major role as a replacement locomotive will have less distance to travel to reach the broken down train.

Calculation of Train Related Risk

This sub-section models the delay to the delaying train, as well as any other trains that may be delayed by it. The following assumptions will be made in conjunction with the risk delay model in this sub-section:

- A delaying train is a train that breaks down or slows down due to train dependent problems and causes delay to other trains.
- A siding will usually have enough room to accommodate only one train.
- When a delaying train clears the track segment, all trains waiting which are to travel in the same direction will be given right of way, since they can follow the delaying train on the same track segment. The delay will be lower if all trains travelling in the same direction as the delaying train go first.
Figure 2a shows the regions which trains would be delayed due to a delaying train where $T_q$ is the length of time of delay. The same situation will also apply to the case when an inbound train is the delaying train.

Referring to Figure 2a, if an inbound train is to arrive at station $q+1 \in Q$ at the time indicted by region a, it will experience a delay of at least the time indicated by b plus the time for the trains in the other direction to clear. If more than one train is to arrive at station $q+1 \in Q$ in periods a and b, they will have to wait at stations $q+2, q+3 \in Q$ and so on (if there are any outbound trains in region c) since the trains travelling in the same direction as the delaying train will clear first (in bulk). The lower bound on the delay that will be suffered by the $z^{th}$ train (train $j$) is

$$
\sum_{s=1}^{z-1} \frac{d_{p+s}}{v_{p+s}} + \frac{d_{p+z}}{v_{p+z}}
$$

(1)

where $c_f$ is the slowest delayed train travelling in the same direction as the delaying. This delay is shown in Figure 2b. Referring to Figure 2a, region c represents the trains directly affected by the delaying train (travelling in the same direction). If a second outbound train has already departed station $q \in Q$ when the delaying train stops or slows down, it will wait on that track segment until the first train starts moving again. Region d represents inbound trains which are not directly affected by the delaying train but are delayed by the clearing outbound trains. This is because the clearing outbound trains will be bunched and running late, so they will almost certainly have a higher priority. Region e is the set of trains travelling in the same direction as the delaying train in period e; and is the set of trains affected by the clearing of trains (in bulk) in the opposite direction to the delaying train. Region e will only apply if there are no trains in region c as the delaying train will wait at siding $q+1 \in Q$ for the other trains to cross. Region d will apply when region e does not.

Both Figure 2a and Figure 2b refer to the case when an outbound train is the delaying train. To construct the model, the following variables are defined using Figure 2a (for when the delaying train $i \in I$ is outbound). The variables for when train $i \in I$ is inbound are defined in identical fashion.
Let $\text{Downset}_{i,q}^{Tq}$ be the set of trains travelling in the same direction as the delaying train $i \in I$ and arrive at siding $q \in Q$ in the time marked by region c and let $NDS_{i,q}^{Tq}$ be the number of trains in this set. Let $\text{Oppdownset}_{i,q}^{Tq}$ be the set of inbound trains that arrive at station $q+1 \in Q$ in the periods indicated by a and b, $\text{NODS}_{i,q}^{Tq}$ be the number of trains contained in this set (when delaying train $i \in I$ is outbound). Let $\text{Rgiond}_{i,q}^{Tq}$ be the set containing any train(s) $j \in I$ travelling in the opposite direction to the delaying train that passes through siding $q+1 \in Q$ in the period indicated by d and the corresponding track segment $w \in P$ of conflict with the delaying train given the delaying train's new delayed path (ie. $(j,w) \in \text{Rgiond}_{i,q}^{Tq}$). These are the trains which are affected by the clearing of trains from region c after the delaying train has cleared. $\text{Rgione}_{i,q}^{Tq}$ is the set containing any train(s) $j \in I$ travelling in the same direction as the delaying train that arrive at siding $q \in Q$ in period e and the corresponding track segment $w \in P$ of conflict with the delaying train given the delaying train's new delayed path (ie. $(j,w) \in \text{Rgione}_{i,q}^{Tq}$).

A train $j \in I$ is classed as being in region a or b if

\[
X_{aq}^j > X_{dq}^i + Tq, \quad X_{aq+1}^j < X_{dq+1}^i + Tq \quad \text{and train } i \in I \text{ is outbound}
\]

\[
X_{aq+1}^j > X_{dq+1}^i, \quad X_{dq}^j < X_{aq}^i + Tq \quad \text{and train } i \in I \text{ is inbound}
\]

A train $j \in I$ is classed as being in region c if

\[
X_{dq}^j > X_{dq}^i, \quad X_{dq}^j < Tq + X_{dq}^i \quad \text{and train } i \in I \text{ is outbound}
\]

\[
X_{dq+1}^j > X_{dq+1}^i, \quad X_{dq}^j < Tq + X_{dq+1}^i \quad \text{and train } i \in I \text{ is inbound}
\]

A train $j \in I$ is classed as being in region d if

\[
X_{dq+1}^j > X_{aq+1}^i + Tq, \quad X_{aq}^j < X_{aq+1}^i + Tq + \sum_{k \in P^{aq+1}_c} d_k / \psi_{ij}^k \quad \text{and train } i \in I \text{ is outbound}
\]

\[
X_{dq}^j > X_{aq}^i + Tq, \quad X_{aq}^j < X_{aq}^i + Tq + \sum_{k \in P^{aq}_c} d_k / \psi_{ij}^k \quad \text{and train } i \in I \text{ is inbound}
\]

This represents any train which crosses the path of the clearing of the trains travelling in the same direction as the delaying train. The path of the clearing of trains commences from when the trains travelling in the same direction as the delaying train begin to clear until either the termination station is reached or a major rail yard is reached for these clearing trains.

A train $j \in I$ is classed as being in region e if

\[
X_{dq}^j > X_{dq}^i + Tq, \quad X_{dq}^j < X_{aq}^i + Tq + \sum_{k \in P^{aq}_c} d_k / \psi_{ij}^k \quad \text{and train } i \in I \text{ is outbound}
\]

\[
X_{dq+1}^j > X_{aq+1}^i + Tq, \quad X_{aq}^j < X_{aq}^i + Tq + \sum_{k \in P^{aq+1}_c} d_k / \psi_{ij}^k \quad \text{and train } i \in I \text{ is inbound}
\]

The prioritised delay that would be suffered due to a delaying train taking time $Tq$ to clear to a siding is in the form of priority*(delay due to delaying train - recoverability). The recoverability component is required to make an estimate of the delay at the destination. The actual recoverability of a train would not be known unless the schedule is resolved or optimised for each delay $Tq$. The time to do this would be prohibitive, so an unbiased estimate of the recoverability is made. The analytical equation for the delay suffered by outbound trains due to a delaying outbound train $i \in I$ is
\[
\sum_{j \in \text{Downset}_{q}^{\text{Downset}}} W^{j} \cdot ((X^{i}_{aq+1} + Tq - X^{i}_{aq+1}) - (X^{i}_{dj} - (X^{i}_{aq+1} + Tq))) - ED^{q+1}_{j} - \sum_{k \neq \text{Downset}_{j}} \frac{d_{k}}{V^{j}_{k}})
\]

and the delay suffered by the inbound trains is

\[
\sum_{j \in \text{Oppdownset}_{q}^{\text{Oppdownset}}} W^{j} \cdot (X^{i}_{aq+1} + Tq + NDS^{Tq}_{i,q} \cdot h \cdot \sum_{z=1}^{z=1} \frac{d_{p+s}}{V^{j}_{p+s}} - X^{j}_{dq+z} - (X^{j}_{dj} - X^{i}_{aq+1} - Tq)) - NDS^{Tq}_{i,q} \cdot h - \sum_{z=1}^{z=1} \frac{d_{p+s}}{V^{j}_{p+s}} - ED^{q+1}_{j} - \sum_{k \neq \text{Downset}_{j}} \frac{d_{k}}{V^{j}_{k}})
\]

where

\[
z = \begin{cases} 
\text{position of train } j \text{ in } \text{Oppdownset}_{i,q}^{Tq} & \text{if } NDS^{i}_{q} \neq 0 \\
0 & \text{if } NDS^{i}_{q} = 0
\end{cases}
\]

The delay is similar for inbound delaying trains and is given by equations 2.3 and 2.4.

\[
\sum_{j \in \text{Oppdownset}_{q}^{\text{Oppdownset}}} W^{j} \cdot ((X^{i}_{aq} + Tq - X^{i}_{aq}) - (X^{i}_{dj} - (X^{i}_{aq} + Tq))) - ED^{q+1}_{j} - \sum_{k \neq \text{Downset}_{j}} \frac{d_{k}}{V^{j}_{k}})
\]

and the delay suffered by the outbound trains is

\[
\sum_{j \in \text{Oppdownset}_{q}^{\text{Oppdownset}}} W^{j} \cdot (2 \cdot X^{i}_{aq} + 2 \cdot Tq + 2 \cdot NDS^{Tq}_{i,q+1} \cdot h \cdot 2 \cdot \sum_{s=1}^{s=1} \frac{d_{s}}{V^{j}_{s}} + 2 \cdot \sum_{s=1}^{s=1} \frac{d_{s}}{V^{j}_{s}} - X^{j}_{dr+z+1}) - X^{j}_{dr+1-z} - (X^{j}_{dj} - ED^{q+1+z-1}_{j} - \sum_{k \neq \text{Downset}_{j}} \frac{d_{k}}{V^{j}_{k}})
\]

where

\[
z = \begin{cases} 
\text{position of train } j \text{ in } \text{Oppdownset}_{i,q+1}^{Tq} & \text{if } NDS^{i}_{q} \neq 0 \\
0 & \text{if } NDS^{i}_{q} = 0
\end{cases}
\]

The delay to trains in region d given an outbound delaying train is

\[
\sum_{(j,w) \in \text{Region}_{d}^{\text{Region}_{d}}} W^{j} \cdot (2 \cdot X^{i}_{aq} + 2 \cdot Tq + 2 \cdot NDS^{Tq}_{i,q} \cdot h + 2 \cdot \sum_{s=1}^{s=1} \frac{d_{s}}{V^{j}_{s}} + 2 \cdot \sum_{s=1}^{s=1} \frac{d_{s}}{V^{j}_{s}} - X^{j}_{dw+z+1}) - (X^{j}_{dj} - ED^{q+1+z-1}_{j} - \sum_{k \neq \text{Downset}_{j}} \frac{d_{k}}{V^{j}_{k}})
\]

where

\[
z = \begin{cases} 
\text{position of train } j \text{ in } \text{Region}_{i,q+1}^{\text{Region}_{i,q}} & \text{if } NDS^{i}_{q} \neq 0 \\
0 & \text{if } NDS^{i}_{q} = 0
\end{cases}
\]

The delay to trains in region d given an inbound delaying train is

\[
\sum_{(j,w) \in \text{Region}_{d}^{\text{Region}_{d}}} W^{j} \cdot (2 \cdot X^{i}_{aq} + 2 \cdot Tq + 2 \cdot NDS^{Tq}_{i,q} \cdot h + 2 \cdot \sum_{s=1}^{s=1} \frac{d_{s}}{V^{j}_{s}} + 2 \cdot \sum_{s=1}^{s=1} \frac{d_{s}}{V^{j}_{s}} - X^{j}_{dw+1}) - (X^{j}_{dj} - ED^{w-z}_{j} - \sum_{k \neq \text{Downset}_{j}} \frac{d_{k}}{V^{j}_{k}})
\]

where

\[
z = \begin{cases} 
\text{position of train } j \text{ in } \text{Region}_{i,q+1}^{\text{Region}_{i,q}} & \text{if } NDS^{i}_{q} \neq 0 \\
0 & \text{if } NDS^{i}_{q} = 0
\end{cases}
\]

The delay to trains in region e given an outbound delaying train is:
The risk delay to the delaying train itself is:

\[ W T q X X Tq ED d \]

\[ W^i (Tq + (X^i_{aq} - (X^i_{aq} + Tq) - ED^i_q - \sum_{k \in P^i_{aq+1}} \frac{d_k}{V_k})) \]

if train \( i \in I \) is outbound \hspace{1cm} (2.9)

\[ W^i (Tq + (X^i_{aq} - (X^i_{aq} + Tq) - ED^i_q - \sum_{k \in P^i_{aq+1}} \frac{d_k}{V_k})) \]

if train \( i \in I \) is inbound \hspace{1cm} (2.10)

Equation 2.1 is made up of two parts, namely the delay due to the delaying train \((X^i_{aq+1} + Tq - X^i_{aq+1})\) and the recoverability time with respect to future delay. The trains waiting in the same direction will clear the track segment one after another with the minimum headway \( h \) between each. The recoverability is the amount of time that could be regained given the expected remaining conflict delay and the train's upper achievable velocity. This component is made up of three parts, namely; the remaining time available to reach the destination station on time (scheduled destination station arrival time minus departure time after delay from delaying train \((X^i_{aq+1} + Tq)\) minus the expected remaining conflict delay) minus the fastest time which the train can travel along the remaining track segments. This is important since the risk delay will be higher if a train is no longer able to reach the destination at its scheduled time. If a train is able to recover from the delay from the delaying train, (ie. the recoverability time is greater than delay from delaying train) there will be no risk delay. Since some trains have a higher priority than others, the risk delay is pre-multiplied by the priority. Equation 2.2 concerns the delay to the trains travelling in the inbound direction when an outbound train causes the delay. It is made up of two components, namely; the delay due to the delaying train when it departs the siding \( q+z \) where it waits for the clearing of outbound trains minus the recoverability from this siding. In Figure 2b, since train 2 is the second inbound train to be delayed, it will wait at siding \( q+2 \) \((z=2)\), and the recoverability will be measured from this siding. The actual departure time of train 2 from siding \( q+2 \) is:

\[ X^i_{aq+1} + Tq + NDS^i_{aq} * h + \sum_{s=1}^{z-1} \frac{d_{pq+1}}{V_{pq+1}} \]
where \( z = 2, cf = 4, i=4 \). This is the actual arrival time of train \( i=4 \) at siding \( q+1 \) (including risk delay \( Tq \)) plus the travel time of this train to siding \( q+2 \) plus the number of trains clearing multiplied by the headway. The larger \( z \) is, generally the longer the delay to the train in position \( z \) of \( \text{Oppdownset}_i^{Tq} \). If another train travelling in the same direction as train 1 and belonged to \( \text{Oppdownset}_i^{Tq} \) it would have to wait at siding \( q+3 \). If in Figure 2b, train 4 was not affected by the delaying train 3, then \( NDS_{i,q} \) would be empty and train 2 would not have to wait for the clearing of outbound trains. Equations 2.3 and 2.4 are analogous to equations 2.1 and 2.2 except for an inbound delaying train. Equations 2.5 and 2.6 refer to region d and are small extensions to equations 2.2 and 2.4. In equation 2.5, \( w+z+1 \) is the siding where a particular train in \( \text{Region}_{i,q} \) will be delayed and depends on the position in \( \text{Region}_{i,q} \), number of trains in region a and b and the number of available sidings between the track segment \( w \in P \) (where the train in \( \text{Region}_{i,q} \) conflicts with the clearing of outbound trains) and the track segment where the delaying train \( i \in I \) is delayed. In Figure 2b, train 3 belongs to \( \text{Region}_{i,q} \) and \( w=q+2 \). Now, \( z=0 \) which means there are fewer trains before train 3 (in regions a,b, and d) than the number of available sidings between track segment p and w. Therefore, train 3 can wait at the siding b immediately before it conflicts with the clearing of trains (ie. \( q+3 \)). If \( z>0 \), then there are more trains than available sidings and train 3 will be delayed at an earlier siding in its journey. Equations 2.7 and 2.8 refer to region e and the logic is analogous to equations 2.5 and 2.6. Equation 2.9 and 2.10 refer to the risk delay to the delaying train and is the delay \( Tq \) minus the recoverability time. If the recoverability time > \( Tq \), there will be no risk delay to the delaying train.

From equations 2.1 to 2.10, the expected delay caused by train \( i \in I \) which is of type \( Ty \) is

\[
\text{DEL}(Ty) = \sum_{i=1}^{\infty} (2.1)+\ldots+(2.10) \int f(Tq, Ty) dTq
\]

The risk factor for the train dependent delay is:

\[
P_{\text{DELAY}_i} = \sum_{i \in I} \sum_{p \in P} P_{\text{DELAY}_i} \ast d_p \ast \text{DEL}(Ty)
\]

where \( P_{\text{DELAY}_i} \) = the probability of a breakdown or delay occurring to train \( i \in I \)

\( f(Tq, Ty) \) = the probability a train of type \( Ty \) will suffer a delay of length \( Tq \).

Since \( f(.,.) \) may not be in the form of a function, the integral may have to be replaced with a summation approximation and estimates of \( f(.,.) \) will be known at discrete intervals. Usually a small number of discrete intervals is sufficient to provide a reasonable approximation to the integrals.
Figure 1: Siding Definition: Single Track Operations