Composite Load Model Decomposition: Induction Motor Contribution

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Statement of original Authorship

The work contained in this thesis has not been previously submitted for a degree or diploma at any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another except where due reference is made.

Signed: ____________________________
Date: ______________________________
Acknowledgement

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Dedication

This work is dedicated to:

- My principal Supervisor
- My Parents and family
Abstract

When a disturbance occurs in a power system, the value of load power will change in response to the voltage and frequency changes. Step changes in power generation, for example, will induce speed changes in the system frequency and this induces speed changes in any induction motors present in the load, and these speed changes will be seen as power changes to the load. Motors consume 60-70% of the energy from the power system. It is important to have good knowledge about induction motor loads, because stalling motors draw large reactive currents that can slow voltage recovery after a fault. In this thesis the effect of frequency changes ($\Delta f$) on an induction motor real ($\Delta P$) and reactive ($\Delta Q$) power changes have been modelled. Among the parameters of an induction motor, the dynamics are largely characterized by Inertia (H) and the torque-damping factor (B). The model of induction motor in these frequency and power relations that has been developed can be used to estimate B and H and these are as shown in this thesis.

Most of the works on load modelling to date have been on post disturbance analysis, not on continuous on-line models for loads. The post disturbance methods are unsuitable for load modelling unless a major external disturbance has already occurred for prediction of response to system disturbances. The bibliography in load modelling considers the variation of the power system supply parameters, but the general case for the load model is that the power system affects the load and the load affects the power system measurement. So, in this thesis a new technique has been developed for measuring the load model of an induction motor to include these aspects.

Also, the composition of loads needs to be characterized because the time constants of composite loads affects the damping contributions of the loads to power system oscillations, and their effects vary with the time of the day, depending on the mix of motors loads. Another characteristic is that there is not only a single time constant but also a mix of motor loads with a range of inertias present at any one time which gives rise to multiple time constants. Hence in this research 10 induction motors with different power ratings, inertia and torque damping constants are modelled, and their composite models are developed with different percentage contributions for each
motor. After that this thesis also shows how measurements of a composite load respond to normal power system variations and this information can be used to continuously decompose the load continuously and to extract information regarding the load into different amounts of motor loads.

To validate the simulated induction motor model, an experimental setup, which is run in the QUT laboratory, is reported. Also, to validate the proposed decomposition technique, Brisbane and Sydney West data are collected from the feeder. This data is analysed and explained in detail with regards to what types of motor load are present in a composite load.

**Key Words**

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List of symbols

P(f)  Power spectra density
X_1  Sequence of row vector
X_k  Row vector of input
L    Length of the segments
D    Overlapping segment length
K    No of segment
K    Periodogram length
N    Total data length
n    Discrete data index
A_1 (m), A_k (m) Fourier Coefficient
W (n) Window length
f_n  Normalized frequency
f_s  sampling frequency
I    Modified periodogram
U    Average periodogram
ε(t) Prediction error
θ    Parameter values
ϕ(t) Regression Matrix
\hat{θ} Estimated parameter
E    Expectation
y    Output
x    Input
X  System input
Y  System output
$s_m$  Motor slip
$\omega_s$  Synchronous speed of the motor in rad/s
$\omega_r$  Rotor speed in rad/s
$I$  The input current of induction motor
$V$  The input voltage of the induction motor
$r_r$  Rotor resistance
$R_s$  Stator resistance
$X$  sum of stator and rotor reactance
$P_e$  Real power input
$Q_e$  Reactive Power
$H$  Machine inertia
$B$  Torque-damping factor
$\omega_s$  Steady state value of supply frequency
$v_{q0}$  Q-axis steady state voltage
$\omega_b$  Base speed
$S$  Laplace constant
$k_{pf}$  High frequency gain of f-p
$k_{pv}$  High frequency gain of v-p
$s_{10}$  Steady state motor slip
$k_{qv}$  High frequency gain f-q
\( \mathbf{X}_{ls} \) Stator leakage flux

\( \mathbf{X}_{lr} \) Rotor leakage flux

\( \mathbf{\Psi}_{ad} \) Mutual flux linkage equation of direct

\( \mathbf{\Psi}_{mq} \) Mutual flux linkage equation of q-axis

\( T_e \) Electromagnetic torque

\( T_L \) Load torque

\( \mathbf{\Psi}_{qs} \) Q-axis stator flux

\( \mathbf{\Psi}_{ds} \) Direct axis stator flux

\( \mathbf{\Psi}_{qr} \) Q-axis rotor flux

\( \mathbf{\Psi}_{dr} \) Direct axis rotor flux

\( X_{ad} \) Flux

\( i_{qs} \) Q-axis stator current

\( i_{ds} \) Direct axis stator current

\( i_{qr} \) q-axis rotor current

\( i_{dr} \) Direct axis rotor current

\( V_{ds} \) Direct axis stator Voltage

\( V_{qs} \) q-axis stator Voltage

PMU Phasor measurement unit

\( X' \) Transpose input vector

\( W_1 (n) \) Input noise

\( W_2 (n) \) Output Noise
Additive Input of the correlator

$R_{w_p}(\tau)$ Cross correlation between input noise and output signal

$h(s)$ the impulse response

$\zeta$ Time Delay

$R_{w_w}(\zeta)$ Cross correlation between Input and Output signal

$R_{w_f}(\zeta)$ Cross correlation between Input noise and input signal

$G_i(\omega)$ Transfer function in frequency domain

$K_p$ The gain of the governor

$\tau$ Time constant of the governor

$Z$ Zero of transfer function

$P$ Pole of transfer function

$a_i$ Real portion of magnitude

$log_{10} \Delta \omega$ Frequency difference of log10 in Bode plot

$\omega_s$ Starting frequency in rad/s

$\omega_f$ Final frequency in rad/s

$A$ Area

$a_s$ Correction Term

$\Delta \omega$ Frequency difference in one FFT point to another FFT point in Hz

$\Delta Q_s$ Change of reactive power

$Q_{e0}$ Steady state reactive power

$s_{f0}$ Steady state slip
\( \Delta s_i \)  
Change of slip

\( \omega_{r0} \)  
Steady state rotor speed

\( \dot{\psi}_{qr} \)  
Derivative of rotor q-axis flux

\( \theta_1 \)  
Angular position1 of springy shaft load

\( \theta_2 \)  
Angular position2 of springy shaft load

\( k \)  
Spring constant

\( H_2 \)  
Inertia

\( \Delta X \)  
State vector of dimension n

\( \Delta u \)  
Input vector of dimension r

\( A \)  
State matrix of dimension n*n

\( B \)  
Input matrix of dimension n*r

\( \text{OE} \)  
Output Error

\( \text{LS} \)  
Least Square

\( \text{GA} \)  
Genetic Algorithm

\( \text{ARMA} \)  
Autoregressive Moving Average

\( \text{ANN} \)  
Artificial Neural Network
Chapter 1

1.1 Introduction

Load modelling is important for power system dynamic analyses, including; voltage stability, angle stability and also for grid operation and planning purposes. Accurate load modelling enables engineers to perform a realistic assessment of grid response to the stability concerns. It is also required to avoid overly conservative assumptions for modelling loads that may lead to unnecessary transmission investment. But accurate load modelling is a daunting task because the proportion of motor load to total load is changing with time of day and week, seasons, weather etc. The safe region of operation could be estimated much more accurately if the overall load composition at that time were known with greater certainty.

Because of the importance of load modelling there is a long history of research on this topic. One direction is the survey of equipment owned by consumers and a subsequent
prediction of how much is connected at any one time [1] [2]. Another path is to produce simplified aggregate models of a group of loads for off line studies [3, 4]. The approach that probably comes closest to reflecting reality is where measurements are made of the actual load change that occurs when test disturbances are applied [5]. There are two types of field tests. One is a staged test and the other is continuous monitoring. A Continuous monitoring phasor measurement device is used in this thesis. An experiment has been set up in the Queensland University of Technology laboratory for the purpose of this research. A phasor measurement device has been used for measuring and recording transient data. Brisbane and Sydney West feeder data are collected from a similar PMU (phasor measurement device).

Induction motors undergo transients when voltage, current and speed vary. Therefore, it is important to understand the dynamic characteristic of the motors for their influence on power system dynamics. Load modelling of induction motors using off-line methods and on-line methods can be found in Refs [6, 7]. The off line method considered the no-load and locked rotor test. In the on-line test, the motor is already connected to the industrial load bus and a 5th order model is used. In Ref [7] the motor is loaded with position-dependant loads and using a time varying frequency generalized averaging method to determine the model of a two-phase induction motor. In this thesis an online method is used to predict the response from 10 different rating induction motor models.

Refs [6, 8-25] estimated induction motor models and parameters without considering the major effect of the load changes influencing the power system as well as the power system changes influencing the load. The authors of those papers have used a feed forward model (power system affects load) but in this thesis the feedback loop (where load is also affecting the power system) in addition to the normal feed forward element is considered. Ref [4] has considered the feedback loop and estimated the transfer function of the power system rather than the transfer function of the load. Most of the papers considered the changes of the power system as measurement noise but in this thesis the changes of power system and changes of load are considered as major
information to develop the model of the induction motor; the theory for identification of a system under feedback with multiple noises has described in chapter 4.

For estimating the parameters of the system, the system model is important. Ref [22] described some models of induction motors and also described the way that the 1st order quasi stationary model is good for determining response to disturbance up to 2 Hz and for disturbances with some higher perturbation frequency the motor can be treated as a third order model which neglects the stator transient. Aiming to model an induction motor in those frequency bands, which affect the generator oscillation, this thesis simulates time domain data. For obtaining time domain data, simulation has been started from 5th order and been reduced to a 3rd order induction motor model. After that, the 3rd order model has transferred to a frequency domain model. Also, the third order model has been reduced to a 1st order model by ignoring the rotor transient. Subsequently, the 1st order model’s location of pole, zero and high frequency gain are mapped to steady state transfer function’s algebraic equation of pole, zero and high frequency gain to determine the inertia and torque damping factor. The procedure is described in chapter 2.

1.2 Motivation

This work is motivated by online measurement of natural variation of power system feeder data from a phasor measurement device and extracts the dynamic information from that data.

This work is focused on extracting an induction motor model from a composite load model. Load modelling is important for power system stability analysis such as small signal stability assessment as well as voltage stability assessment. To simulate the power system disturbance, the load model must be accurate to capture the dynamics. Dynamic load modelling is important for capturing the dynamic phenomena.

Electromechanical oscillation is an inevitable phenomenon in heavily loaded power systems. Loads with a time constant around 0.3-0.5sec have the largest influence on system damping [4]. Induction motor load time constants fall in that range. There is not
only a single time constant but also a mix of motors loads with a range of inertias present at any one time. Therefore, it is important to characterize the composite motor loads.

Voltage stability is significantly dependent on dynamic behaviours of the connected load, especially loads that are large in size in proportion to the induction motor. Load composition changes with season and time of day. A high proportion of induction motor loads are used in the summer time, and especially during the hotter parts of summer time. Irrigation and dairy farming use induction motors during certain times of day as well. Induction motors undergo transient when frequency and voltage of the power system changes due to contingencies or due to load variation. This thesis is interested in frequency/Voltage variations due to load variation only.

When motor voltage is decreased, the torque of an induction motor is decreased as well. If the load torque of the induction motor is constant then continuous decreasing of voltage decreases the induction motor torque less than the load torque. Even after recovery of disturbances, the load torque can be so low that it can’t return to a stable situation and this phenomenon is called motor stalling. Motor stalling draws significant reactive current from the system and deteriorates voltage stability and as a result voltage is collapsed in the whole system. This is also called a blackout. Some motor tripping at the time of low voltage by power electronic device is beneficial, but excessive amounts of motor tripping can cause a voltage excess in the system. This excess voltage may cause problems in others devices.

Hence induction motor characteristics are important for voltage stability and consequently are the main interest of this research.

1.3 Objective and Contribution

The main objective of the thesis is to separate different motor load components from composite load response. We consider the load to consist of motors, voltage dependent passive impedance loads and constant power loads. For a pure L or C there is a well
defined characteristic with low gain for small frequency changes for frequency
dependent passive impedance load that’s why it is not considered as a composite load
component. For these loads, it is important that if the system frequency were varied but
all voltage magnitudes were held constant, it would only be the induction motors which
would show transient and steady state changes in P (real power) and Q (reactive power).
The process of identification starts with measurements of the observed frequency
perturbations at the supplying bus and the variations of P and Q to the load. From a
correlation of these measurements a transfer function from frequency to load power can
be inferred. This process is similar to one of the traditional approaches to dynamic loads,
which makes a step in voltage and then observes the changes in load power.

The main contribution of the thesis are presented following the process steps below,

1. Derive algebraic transfer function of an induction motor and identify the model
2. Identify the motor model and parameter by using system identification.
3. Extract dynamic motor components by using area calculation.
4. Extract dynamic components by using least square identification.

1.4 Thesis Structure

This thesis has been organized into seven chapters.

Chapter.1 Introduction

In this chapter the thesis is summarised and different objectives are determined to reach
the overall goal. The publications of all researched works are also listed at the end of the
chapter.

Chapter.2 Literature review

The importance of load modelling is discussed in chapter.2. This chapter also outlines
induction motor behaviours, the importance of system identification in the frequency
domain and different model structures in the time domain. One aspect of nonparametric
identification spectra estimation is described, and the history and uses of PMU data are shown. Least square estimation and ‘one step ahead’ predictions are described in this chapter.

Chapter 3 Derive algebraic transfer functions of an induction motor and identify the model

The simplest model of an induction motor comes from its steady state equivalent circuit. Hence an algebraic equation of the transfer function of an induction motor with a linear load is developed from the steady state equations and is shown in this chapter. This chapter also shows the process of reducing from the 5th order model to the 3rd order model, and from the 3rd order model to the 1st order model’s location of pole, zero and high frequency gain are mapped to steady state transfer function’s algebraic equation of pole, zero and high frequency gain to determine the inertia and torque damping factor.

Chapter 4 Identify the motor model and parameter by using system identification

An induction motor model, which is predicted based on system identification in the frequency domain, is proposed in this chapter. The model is developed considering the closed loop system. One form of closed loop system identification using input and output additive noise is proposed in this chapter. When input signal (Frequency or Voltage) and output signal (real or reactive power) is available, one can form models to predict the next sample based on the history of input and output. When the input is a low pass filtered signal, most of the measurements can be well predicted from previous measurements. If we remove all the predictable portions of the input the remaining unpredictable portion becomes a white noise signal with a flat spectrum. This is the process to extract the component causing the measurement variations. This means that we can relate the unpredicted changes in frequency with a white noise term called input noise.

Similarly, the unpredicted changes in output (real or reactive signals) are declared to be associated with the term ‘output noise,’ which represents load changes. Having
separated input noise and output noise, the transfer functions from input noise to input signal and input noise to output, and the ratio of these terms, identifies the feed forward portion. Similarly the transfer function from output noise to input signal and output can be found and the ratio used to identify the feedback term.

The proposed model is validated by a real motor model, which is run in the laboratory.

**Chapter 5 Extracting the dynamic motor component by using area calculation**

One possibility of extracting the motor component from a dynamic composite load is that of area calculation. The area is calculated under the real component of the transfer function (frequency change to real power change) on a log of frequency plot. The proposed idea is implemented to calculate the area of transfer functions for 10 simulated induction motors with different ratings. To validate the proposed idea, Brisbane and Sydney West data are collected from the PMU and the transfer function is estimated according to a closed loop system identification process. The area is calculated and presented using a 24hr load curve for Brisbane and Sydney West data. After that the load curve is mapped to the original load curve of Brisbane and Sydney West.

**Chapter 6 Extracting the dynamic motor component by using least square identification**

Another method of extracting the motor component from a dynamic composite load follows these steps; first the real component of the transfer function of frequency change to real power change is estimated by using a system identification technique. After that it is necessary to fit the real component of the composite transfer function to three groups of ten induction motors (small, medium and large groups) as represented by their real components of the transfer function. A method to determine the percentage contribution of each group of motors by using the least square identification technique is proposed. 10 induction motors with different ratings are simulated to form a composite model and the parameters are extracted by using the proposed approach. After that, real data is collected from Sydney West PMU is analysed using this approach.
Chapter 7 Future work and conclusion

The ideas and theories that are implemented in this thesis are described. The limitations of this thesis are described in this chapter with suggestions for future work.

1.5 Publication

The ideas and results behind the thesis have been presented in the publications below,


Chapter 2: Literature Review

2.1 Literature review on dynamic Load Modelling

2.1.1 Load Modelling

A load model is a mathematical representation of real and reactive power changes to power system voltage and angle (frequency) changes [26].

Figure 2.1 Load representation in busbar
For power system analysis, load modelling is very important. It is also important for grid planning and operation [26]. Therefore accurate load modelling is important. Otherwise using overly optimistic models the grid operator will operate the system beyond its capacity which will increase the chances of widespread outages and a pessimistic model will increase the risk of power shortages in an energy deficient region. Ref [6] demonstrates the need for accurate modelling of loads. But accurate load modelling continuously is a difficult task due to several factors such as those described in Ref [1];

a. The large number of diverse load components
b. Ownership and location of load devices in customer facilities not being directly accessible to the electrical utilities
c. The changing of load composition with time of day and week, seasons, weather.
d. Lack of precise information on the composition of the load
e. Uncertainties regarding the characteristics of many load components, particularly for large frequency or voltage variations.

Traditionally there are two types of load modelling, static and dynamic [27].

**Static:** A static model expresses the active and reactive powers at any instant in time as functions of the bus voltage magnitude and frequency at the same instant. Static load model is used both for essentially static load components (e.g., resistive and lighting loads), and as an approximation for dynamic load components [28].

The exponential function of voltage can be expressed in terms of nominal operating point designed by the subscripts o.

\[ P_d = P_0 \left( \frac{V}{V_0} \right)^\alpha \]  \hspace{1cm} (2.1)

\[ Q_d = Q_0 \left( \frac{V}{V_0} \right)^\beta \]  \hspace{1cm} (2.2)

There are three types of static load modelling depending on the values of \( \alpha \) and \( \beta \);
**Constant current model:** When $\alpha$ and $\beta$ are 1, the static model power varies directly with voltage variation.

**Constant impedance model:** When $\alpha$ and $\beta$ are 2, the static load model power varies directly with the square of voltage magnitude.

**Constant power model:** When $\alpha$ and $\beta$ are zero the static model power is constant in spite of voltage magnitude variations. It’s also called a constant MVA model.

**ZIP Model:** Any combination of constant current, constant power and constant impedance model is called the polynomial ZIP model.

There are many other types of static load models which have been developed. But the focus on static load is not the main aim of this thesis.

Conventional static load models can sometimes adequately represent the characteristics of a residential/commercial feeder load [29].

Ref [4] shows that the static representation of a load which exhibits dynamic behaviours can give quite a misleading result. As a result of this, dynamic load modelling is important for predicting the characteristics of load which have their own dynamic responses to voltage and frequency.

**Dynamic:** Difference or differential equations can be used to represent dynamic loads.

\[
T_p \dot{P}_d + P_d = P_i(V) + K_p(V)\dot{V} \tag{2.3}
\]

\[
T_p \dot{Q}_d + Q_d = Q_i(V) + K_q(V)\dot{V} \tag{2.4}
\]

Ref [30] shows that static and dynamic load modelling can give a similar result for studies on security limits but for large excursions of frequency and voltage fluctuation the difference between two models can be significant. Hence, the type of load modelling depends on what types of system analysis are being studied.
Additionally, rotor angle stability, voltage stability, induction motor stability, cold-load pickup and dynamic over-supply of voltage call for unique load modelling requirements. All these terms are described in Ref [1]. For this thesis, modelling has been done for general purpose studies, which can be used in all types of stability problems.

2.2 Literature review on how Load modelling affects the power system damping angle stability and voltage stability

2.2.1 First swing

In this issue voltage is changed rapidly at the time of fault, and slowly during the first power-angle swing. The load response to this type of voltage change is important. There is also a brief frequency change at the power angle swing. The frequency characteristic of loads, which are electrically close to the acceleration or decelerating of a generator are also important.

Ref [31] examines the accuracy of modelling real power behaviour by static load models and the adequacy of using static load models for transient stability analysis. Five static load models, the PSS/E static model, ZIP model, exponential model, EPRI model and the composite Zip-Exponential model are considered in this paper. The exponential model offers a reliable and consistent result. Compared with static load models, the dynamic load models under study give a slightly improved result in modelling real power behaviour at the expense of increasing the system dimension.

2.2.2 Small signal stability damping

Small–disturbance rotor angle stability problems are of two types; local and global. Local problems involve a small part of the power system and are usually associated with rotor angle oscillations of a single power plant against the rest of the power system. Such oscillation is called a local plant oscillation mode.
Global problems involve oscillation of a group of generators in one area swinging against a group of generators in another area. Such oscillation is called inter-area oscillation.

The load characteristics have a major effect, particularly on the stability of inter-area modes [32]. One paper shows that the influence of load representation specially on the frequency of oscillation of the local modes can be neglected [33].

It is desirable that frequency and damping of power systems oscillation are accurately predicted by load modelling. In Ref [4] it was shown that dynamic load models could not only effect the damping of electromechanical modes, but also could influence which generator participated in the mode. This paper also considered the significance of load model uncertainty by varying load parameters randomly. The damping level provided by each set of parameters determined is quite widespread.

The stability of an inter-area mode depends on the operating conditions and on the locations of the loads [34, 35]. Loads with time constants around 0.3-0.5 sec have the largest influence on system damping [4] in context of this thesis. It is possible to know what type of load has significant influence on damping [4] by using residue and eigenvalue sensitivities [4, 36]. Ref [37] also used eigenvalue analysis to study an unstable low frequency oscillation incident due to double trunk line outage. The incident was experienced by the Taiwan power system. The Taiwan power system used different load models and concluded that static and dynamic load model composition provides the most accurate oscillation damping. This paper also shows that the load model can have significant effects on power system stabilizer parameters and gains value design and mentions that it is necessary to do further research on the effects of dynamic load models on designing power system stabilizer parameters.

Two basic methods are used for analysing this inter area oscillation, one is Eigen analysis and another is Prony analysis. Prony analysis and Eigen analysis are complimentary methods [38]. J.W.Pierre proposed another analysis to estimate the electromechanical modal frequencies and damping from the spectra content of the ambient noise [39]. In his paper he mentioned that his analysis is quite similar to
Prony analysis with regards to the ring down data but clearly different in the types of signals processed. In Prony analysis the system input is assumed to be a known deterministic signal. In J.W.Pierre analysis, the system input is assumed to be random white noise.

2.2.3 Voltage stability

Voltage stability is usually a long-term problem. Voltage stability refers to which maintains the steady voltages at all buses in the system after being subjected to a disturbance from a given initial operating condition [40]. The driving force for voltage instability is usually the variation of load power in response to disturbances [7]. Thus load modelling is important for voltage stability.

Ref [41] shows that parameters and models of loads have important effects on the study of voltage stability. This paper shows that the limit of voltage stability has been calculated for five types of static load models. The calculation results of voltage stability with constant resistance type models shows more conservative results or exceeds the results obtained with load parameters from real field tests. Therefore it is important to consider field tests rather than conventional load models.

The main purpose of a voltage stability study is to identify the maximum demand (load limit), which can be supplied, that shows satisfactory system performance during a steady state and in the transient period following disturbances. Ref [42] has described a set of performance criteria and planning guidelines for the purpose of dynamic analysis of voltage stability in New Zealand. In this paper, the real power of a static load is represented by a constant current model and reactive power is represented by constant impedance, which remains connected during the transient period. As a dynamic load, the induction motors are considered and grouped into three categories according to their behaviour following a three-phase fault. This paper also shows that voltage performance during the transient period is very sensitive to the amount of the motor load, which would be tripped by a connector or would drop out before fault clearance. Given the uncertainties of the load modelling, conclusively assessing the risk of voltage performance is difficult [10].
2.3 Literature review of the Induction motor model

2.3.1 Load response

Measurements in the laboratory and on the power system buses show a typical load response to a step in voltage is of the form shown in [43].

![Figure 2.2 Typical Load responses [45]](image)

Many of the real loads are very variable in nature, so it is necessary to consider dynamic behaviours of loads [44]. Motors consume 60-70\% of the available energy from the power system; therefore, the dynamic characteristics of motors are critical for dynamic load modelling [4]. Ref [45] shows that induction motors in most cases reduce system stability.

2.3.2 Induction motor

Small signal stability damping and voltage stability studies show that choice of load, including induction motor models, affects the dynamic behaviours of the system. Induction motors undergo transients, causing voltage, current and speed changes. The real and reactive power and voltage response of a large and small motor are shown in figure 2.3. The larger motor response is less damped than that of a smaller motor [46]. Regardless of motor size, the transient disappears very quickly almost in one second followed by an exponential return to a steady state, as shown in figure 2.3 taken from Ref [46]. Note that the larger motor reaches a steady state slower than the smaller motor.
The Park model of an induction motor consists of five nonlinear differential equations describing the dynamics of the motor [22]. In this paper the author shows that the Park 5th order model captures the frequency perturbation dynamics quite well. The author also mentions that iron loss needs to be considered for voltage perturbations as induction motors behave differently when saturated. This can be seen in the Park model for voltage perturbation modelling. Ref [47] shows that first order and reduced third order models have similar steady state real power characteristics, but reactive power and dynamic responses can be significantly different. It shows that a first order model is good for long term voltage stability problems whereas a third order model is good for transient voltage stability problems. In this paper the author considered the real and reactive power coupling.

Many research papers have been published on load modelling of induction motors [11-12]. The following types of methods of testing have been performed on motors [6]:

1. **Off Site Methods:** The motor is tested away from its application and the tests are the no load test and the locked rotor test. This is a very simple method and often represents the real system poorly.

2. **On-site and off line methods:** In this test a motor is connected in the industrial setup and supplied by its power converter.
An induction motor model is developed using an on site and off-line method in Ref [6]. In this paper the author uses a fifth order induction motor model. Ref [48] uses an off-line standstill frequency response test to evaluate the induction motor equivalent circuits. In Ref [12] the motor is examined loaded with position-dependant loads. This paper used a time varying frequency generalized averaging method to find out the model of a 2-phase induction motor. In this paper the author also uses a position dependant load and rotor speed dependant loads.

2.4 Literature review on aggregate Load Modelling

The load at a given bus may include many types of induction motors, each having different dynamic characteristics and each operating at a different steady state condition [49] and often includes static loads as well. One single motor is not the right choice for simulating the bus bar dynamic load [50]. Many papers have been published about aggregate or composite load modelling [49, 51-57]. These papers considered the composite load as a combination of static and dynamic load models. Ref [56]] considered the static load as a combination of constant impedance load and constant power load and for a dynamic load an induction motor is included. Ref [55] used a ZIP model for static load and an induction motor for dynamic load. Ref [57] proposed an interim composite model which consists of 80% static load and 20% dynamic load.

From the results of refs [32,34} it is appropriate to model a divergent group of induction motors by one or two aggregate motors [49]. In Ref [49] two aggregation methods are considered;

**Aggregation method A:**

Each parameter of the aggregate motor is calculated as the weighted average of the respective parameter of the individual motors in the group. The relevant equation is,

\[ P_{agg} = \sum_{j=1}^{n} \sigma_j P_j \]  

(2. 5)
In the above equation $p_j$ is substituted by each parameter of the induction motor model.

The weighted coefficient $\sigma_j$ is defined as the relative KVA rating of the individual motor $j$ with respect to the KVA rating of the aggregate motor.

$$\sigma_j = \frac{KVA_j}{\sum_{j=1}^{n} KVA_j}$$  \hspace{2cm} (2.6)

**Aggregation method B:**

Aggregation method B is in some aspects similar to method A. It uses equation (2.6) to calculate inertia and loading of the aggregated motor. However to calculate the electrical parameters of the aggregate motor, method B uses a weighted average of admittance, whereas method A uses the weighted average of impedance. The relevant equation for calculating electrical parameters of the aggregate motor is,

$$\frac{1}{Z_{agg}} = \sum_{j=1}^{n} \frac{\sigma_j}{Z_j}$$  \hspace{2cm} (2.7)

**2.5 Literature review on system identification**

**2.5.1 System identification**

The term “system identification “was first defined by Lotif Zadeh: “Identification is the determination, on the basis of input and output, of a system within a specified class of systems, to which the system under test is equivalent” [58]. With this definition in mind, system identification in practice involves the following steps [59];

1. Selection of a model structure
2. Given a model structure, design of the input sequence, $u (k)$
3. Given \( u(k) \), generation of the system response \( y(k) \)
4. from the input-output dataset, estimation of the model parameters
5. Assessment of identified model quality based on the estimated model parameters
6. Iteration and model refinement as necessary

![Figure 2.4 Schematic of the system identification problem [60]](image)

The task of load modelling is in fact a system identification procedure [60]. System identification methods can be grouped into frequency domain methods and time domain methods.

### 2.5.2 Time domain identification

Two approaches for dynamic load modelling in time domain are as follows;

1. The component based approach models the load on the basis of knowledge of static and dynamic behaviour of all the individual load components of a particular load bus.

2. The field Measurement based approach uses system identification to estimate a proper model and its parameters.

   The advantage of the measurement-based technique is that it is able to obtain data directly from the actual system. Many papers [53, 55, 60, 61] model the load according to measurement based approach and there are few papers published about component based load modelling.
This thesis is considered to be a measurement-based technique. There are two types of field measurement,

1. **Staged test**: Tap changing transformer and the switching of the reactive sources are used to impose voltage perturbation artificially on system loads. In Queensland regulations the permissible voltage variation is 5% of the operating voltage, which is a limit of the extent of this staged test.

2. **Continuous monitoring**: In this test the records of load behaviour under large disturbances to small disturbances is possible by using continuous monitoring devices and installing them on the residential/commercial/industrial feeder.

Ref [62] used a staged test to calculate the coefficient of a power system model. In 1965 after the catastrophic failure of the North Eastern power grid in North America, a great deal of research was conducted on techniques for determining the state of a power system in real time based upon real time measurements. Many papers have developed and considered the continuous monitoring unit to measure the voltage and current of a power system [35, 54]. Refs [63, 64] considered the phasor measurement unit to measure the power system voltage and current phasor.

A phasor measurement unit uses the GPS signal, synchronized with a sampling clock so that the calculated phasor would have a common reference. This was first developed in the power system research laboratory of Virginia Tech. This early version is shown in figure(2.5)[65]
The early GPS system was expensive but today’s satellite system is fully deployed and a GPS receiver can be obtained for a few hundred dollars. This makes the phasor measurement unit available for use for power system voltage and frequency phasor calculation, state estimation, instability prediction, adaptive relaying and improved control [65, 66]. Figure (2.6) shows the functional block diagram of a typical PMU taken from Ref [65].
The microprocessor determines the positive sequence phasor using recursive DFT filtering [65]. The analogue voltage and current signals are derived from the transformer secondaries with appropriate anti-aliasing and surge filtering and A/D converter. Queensland University of Technology’s (QUT) phase Monitoring System has been installed many sites in Australia including Blackwall to record the real data. The QUT measurement system generates a 50Hz binary data file in order to save storage space. The unit used in this project originally used a 16-bit integer for the binary file for the processed data from a 12 bit A/D converter. It has found that a quantization error could contribute to error in data processing. To increase the resolution of data quantization, a 32-Bit data file and 16-bit A/D converter have been examined for the measurement system and the comparison has been taken with a 16-Bit data file and a 12-bit A/D converter [67]. QUT’s updated phasor measurement unit is being used in this project.

2.5.3 Frequency domain identification

The usefulness of frequency domain system identification is well known [68-70]. One approach is to consider the primary observation in the time domain and then convert the time domain data to frequency domain. Frequency domain system identification can be implemented using a cross power spectrum estimation divided by an auto power spectrum [71].

There are two types of spectra estimation;

1. Parametric spectrum estimation

2. Nonparametric spectrum estimation

Non-parametric estimation does not require prior knowledge of the signal that is under consideration and for this reason in this thesis nonparametric spectrum estimation is considered and literature review is confined to nonparametric spectra estimation. There are many techniques to estimate nonparametric spectra, as described in Refs [72, 73]. Ref [73] described the suitability of different algorithms in different situations. In this paper the author mentioned that if the dynamic range of
the spectrums being estimated is small then a rectangular averaging process is enough because problems due to spectra leakage are not a concern in the short-term. If the dynamic range is not short, spectra leakage is prominent, and in this case the Welch method or modified periodogram can be used with different types of windows, as mentioned in Ref [74]. Fast Fourier transforms are most famous and short processing time for power spectra estimation are mentioned in Ref [75], based on Welch periodograms.

The method is described below;

Let \( X(n), n=0\ldots N-1 \) be a sample from a stationary, stochastic sequence. Assume for simplicity \( \text{E}[x(n)]=0 \) and \( P(f) \) is the power spectra density of \( X(n) \). Take the segment, possible overlapping of length \( L \) with the starting points of theses segment \( D \) units apart [75].

The first such segment is,

\[
X_1(n) = X(n) \quad n = 0\ldots L-1
\]

Similarly

\[
X_2(n) = X(n+D) \quad n = 0\ldots L-1
\]

And Finally

\[
X_k(n) = X(n+(K-1)D) \quad n = 0\ldots L-1
\]

Let \( k \) such segments cover the entire record \( N= (K-1) D+L \). Here \( X_1 \) to \( X_k \) are row vectors.

Now select column vectors of data windows. The equation for computing the coefficients of a Tukey window is,

\[
W(n) = \begin{cases} 
\frac{1}{2}(1 + \cos \frac{2\pi}{r(N-1)} - \pi) & k \frac{r}{2}(N-1) + 1 \\
1 & \frac{r}{2}(N-1) + 1 \leq k \leq N - \frac{r}{2}(N-1) \\
\frac{1}{2}(1 + \cos \frac{2\pi}{r} - \frac{2\pi}{r} \frac{(k-1)}{(N-1)} - \pi) & N - \frac{r}{2}(N-1) \leq k 
\end{cases}
\]

(2.9)
n=0…L-1 for calculating modified periodogram and r is the ratio of taper to constant sections and is between 0 and 1. The sequence is \( X_i(n) W(n) \) … \( X_k(n) W(n) \) and then take the finite Fourier transformation \( A_1(m) \)….\( A_k(m) \) of these sequences. Here

\[
A_k(m) = \frac{1}{L} \sum_{n=0}^{L-1} X_k(n) W(n) e^{-2\pi j m n / L} \tag{2.10}
\]

Where \( j = \sqrt{-1} \). And finally the K modified periodograms,

\[
I_k(f_n) = \frac{L}{U} |A_k(n)|^2 \tag{2.11}
\]

Where \( k=1,2,\ldots,K \) and \( f_n = \frac{m}{L} \quad m=0,\ldots,L/2 \)

And

\[
U = \frac{1}{L} \sum_{n=0}^{L-1} W^2(n) \tag{2.12}
\]

The spectral estimates is the average of these periodograms,

\[
\hat{P}(f_n) = \frac{1}{K} \sum_{k=1}^{K} I_k(f_n) \tag{2.13}
\]

The type of windows used can have a large effect on the characteristic of a spectral estimation [76]. A rectangular window produces many negative lobes in the frequency domain, hence many types of windows are proposed to reduce this phenomena which is mentioned in Ref [77]. In this thesis a Turkey (tapered cosine) window is used.

Resolution refers to the ability to discriminate spectral features, and is a key concept in the analysis of spectral estimator performance.

In order to resolve two sinusoids that are relatively close together in frequency, it is necessary for the difference between the two frequencies to be greater than the width
of the main lobe of the leaked spectra for either one of these sinusoids. The main lobe width is defined to be the width of the main lobe at the point where the power is half the peak main lobe power (i.e., 3 dB width). This width is approximately equal to $f_s/L$.

In other words, for two sinusoids of frequencies $f_1$ and $f_2$, the resolvability condition requires that:

$$\Delta f = (f_1 - f_2) \frac{f_s}{L}$$  \hspace{1cm} (2.14)

There is a trade-off between resolution and variance when selecting different types and lengths of windows. Ake suggested those windows that give a low side lobe level [77]. If the total number of data points $N$ is quite large then the average over $K$ segments reduces the variance by $/K$ but if the total number of data points $N$ is limited, then they overlap a segment by one half of their length ($D=L/2$) to reduce the variance. It is shown by Ake that 50-75% overlap is good enough for all window lengths.

### 2.5.4 Parameter Identification

Parametric Identification Methods are techniques used to estimate parameters with given model structures. Basically it is a matter of finding (by numerical search) those numerical values of the parameters that give the best agreement between the model's (simulated or predicted) output and the measured one[78].

Most often the choices are confined to one of the following linear special structure cases.

**ARX:** $A(q)y(t) = B(q)u(t - nk) + e(t)$  \hspace{1cm} (2.15)

**ARMAX:** $A(q)y(t) = B(q)u(t - nk) + C(q)e(t)$  \hspace{1cm} (2.16)

**OE:** $y(t) = \frac{B(q)}{F(q)}u(t - nk) + e(t)$  \hspace{1cm} (2.17)
The BJ model is given by:

\[ y(t) = \frac{B(q)}{F(q)} u(t - nk) + e(t) \]  

(2.18)

With

\[ A(q) = a_0 q^n + a_1 q^{n-1} + \ldots + a_m \]

(2.19)

\[ B(q) = b_0 q^n + b_1 q^{n-1} + \ldots + b_n \]

(2.20)

\[ C(q) = c_0 q^r + c_1 q^{r-1} + \ldots + c_r \]

(2.21)

\[ F(q) = f_0 q^n + f_1 q^{n-1} + \ldots + f_p \]

(2.22)

Using the prediction error minimization approach, a suitable error function is given by the OE model:

\[ \varepsilon(t) = y(t) - \frac{B(q)}{F(q)} u(t - nk) \]

(2.23)

And the associated estimate model is:

\[ \hat{y}(t) = \phi(t) \theta + \varepsilon(t) \]

(2.24)

Where:

\[ \phi(t) = \begin{bmatrix} -y(t-nk) \ldots y(t) \ldots -u(t-nk) \ldots u(t) \end{bmatrix} \]

(2.25)

\[ \theta = \begin{bmatrix} b_0 \ldots \ldots b_n, f_0 \ldots \ldots f_p \end{bmatrix} \]

(2.26)

The above formulation requires the prior knowledge of the noise model and the noise assumes white noise properties [79].

After selecting the type of approach, a suitable field measurement unit and the structure of the model, the next step is to identify the parameter \( \theta \). Many papers have
been published about estimating the parameter using many different techniques [8-12, 14-21, 23-25, 80-82].

There are three techniques used to estimate the parameter; 1. An optimization based approach, 2. An analytical approach 3. A stochastic approach

Refs [12] described different stochastic approaches for induction motor parameter estimation. In Ref [12], the author compares eight stochastic algorithms, which represent four main groups of stochastic optimization algorithms used today; local search, generational EAs, Evolution strategies and a particle swarm optimizer. The simple population-based approach showed good performances while the advanced algorithms had the best performance.

Ref [82] estimates parameters using analytical approach-adaptive genetic algorithms. This approach can be used in a special test such as a step test but it is sensitive to measurement error. This method is used to search large, nonlinear search spaces where traditional optimization approaches fall short.

Ref [9, 25] estimated the parameter by means of a least square technique and genetic algorithms. Neither an LS algorithm nor a GA can be used on-line hence the use of off-line identification is required. LS techniques cannot be used because they require smooth data, which cannot be generated by inverter fed induction motors. The author of this paper has shown in Ref [9] that data filtered by an anti-casual filter allows use of the LS technique to avoid more complicated techniques. GA is not suitable for real-time implementation because of its long execution time. The author has shown that combinatorial optimization is capable of identifying the non-linear models and in this process the derivatives of variable cannot be numerically computed with an acceptable level of noise.

Ref [15] estimates the parameter by using recursive, least square identification which is fast and simple and may be easily implemented in real time. In Ref [81], the author mentioned that recursive least square puts too many restrictions on the noise signal. A less restrictive method, total least square method was used to estimate the parameter of an induction motor. Another online optimization algorithm is proposed in Ref [14] which is a dynamic encoding algorithm for searching, quite similar to GA
but the basic philosophies are quite different. This algorithm is effective both in accuracy of identifying the parameter, and in execution time. Ref [23] estimates the parameter of different load models by using the output error method.

Least square parameter identification is the most popular technique in an optimization-based approach. The reason for its popularity is that the method is easier to comprehend than the others and doesn’t require a knowledge of mathematical statistics [83].

From the above equation, the estimate of $\theta$ on the basis of least-error-square is;

$$\text{Residue, } \varepsilon = y(t) - \theta \phi(t) \quad (2.27)$$

Criterion $J$ is;

$$J = \sum_{i=1}^{n} \varepsilon_i^2 = \varepsilon^T \varepsilon \quad (2.28)$$

Minimized the criterion;

$$J = (y - \theta \phi)^T (y - \theta \phi) \quad (2.29)$$

$$= y^T y - \theta^T \phi^T y - y^T \phi \theta + \theta^T \phi^T \phi \theta$$

Differentiate $J$ with respect to $\theta$ and equate the result to zero to determine the condition on the estimate $\hat{\theta}$ that minimizes $J$. Thus;

$$\hat{\theta} = (\phi^T \phi)^{-1} \phi^T y \quad (2.30)$$

The estimator accuracy can be conveniently measured by a number of statistical properties such as bias, error covariance, efficiency and consistency.
Estimator unbiased means $E(\hat{\theta}) = \theta$.

Substitute equation (2.27) to equation (2.30),

$$\hat{\theta} = \theta + (\phi^T \phi)^{-1} \phi^T \varepsilon$$  \hspace{1cm} (2.31)

Taking expectations of both sides

$$E[\hat{\theta}] = E[\theta] + E[(\phi^T \phi)^{-1} \phi^T] E[\varepsilon]$$ \hspace{1cm} (2.32)

If assumed white noise, then $E[\varepsilon] = 0$

Hence;

$$E[\hat{\theta}] = E[\theta]$$ \hspace{1cm} (2.33)

In the presence of white noise, the least square estimation is unbiased. Thus the least square technique does indeed have many advantages.

Because of these advantages the least square technique is used in one-step-ahead prediction for removing the dynamic component [84].

The one-step-ahead prediction method calculates the response of the system one step in the future to an input sequence while the process outputs are known up to some instant in time [71].

Consider the ARMA process,

$$y(k) = -a_1 y(k-1) - \ldots - a_n y(k-n) + b_1 u(k-1) + \ldots + b_m u(k-n) = \phi^T(k) \theta(k)$$ \hspace{1cm} (2.34)

Where $y$ is output and $x$ is input of the system, and
\[
\varphi^T(k) = \left[-y(k-1) - ... - y(k-n); u(k-1) + ... + u(k-m)\right]
\]  

(2.35)

And

\[
\theta = [a_1 \ldots a_n; b_1 \ldots b_m]
\]  

(2.36)

To identify the output \(y(k)\), first the unknown parameter value \(\theta\) is identified by least square algorithms, which is represented in equation (2.27).

Where \(y = [y(m+1) \ y(m+2) \ldots y(N)]^T\) for \(N\) available data samples and \(k=m+1 \ldots N\) and the regression matrix \(\varphi\) is,

\[
\varphi =\begin{bmatrix}
-y(m) & -y(1) & u(m) & u(1) \\
-y(m+1) & -y(2) & u(m+2) & u(2) \\
\vdots & \vdots & \vdots & \vdots \\
-y(N-1) & -y(N-m) & u(N-1) & u(N-m)
\end{bmatrix}
\]  

(2.37)

The random component of a signal is extracted by subtracting the estimated signal from the original signal, which is called the residue. If the residue is not white (uncorrelated) then the least square estimate is biased, in which case the instrumental variable method is used to estimate the parameters instead of the least square.

### 2.5.5 Artificial neural network (ANN) Method

The artificial neural network is an alternate method to undertake system identification [85]. Recently some classes of optimization problems have been solved and programmed in the neural network [86]. ANN is good for some tasks. ANN is especially good for complex and ill-defined nonlinear systems where a decision is normally made on a human intuition basis. The system, which requires high accuracy and precision, ANN cannot be applicable in the system which requires high accuracy and precision. Commonly neural networks are adjusted, or trained, so that a
particular input leads to a specific target output [78]. Such a situation is shown below.

Figure 2.7 Neural Networks [80]

For linear relationships between input and output the single layer neuron is adequate [87]. A linear neuron with R inputs, using linear transfer functions is shown below,

Figure 2.8 Single layer linear neuron and linear transfer function [80]

For nonlinear system identification problem, multiple layers of neurons with nonlinear transfer functions allow the network to learn nonlinear and linear relationships between input and output vectors. Back propagation was created by generalizing the Widrow-Hoff learning rule to multiple-layer networks and nonlinear differentiable transfer functions.
Networks are also sensitive to the number of neurons in their hidden layers. Too few neurons can lead to underfitting. Too many neurons can contribute to overfitting, in which all training points are well fit, but the fitting curve takes wild oscillations between these points.

The architecture of a multilayer network is not completely constrained by the problem to be solved. The number of inputs to the network is constrained by the problem. The number of neurons in the output layer is constrained by the number of outputs required by the problem. However, the number of layers between network inputs and the output layer and the sizes of the layers are up to the designer.

A generalized regression neural network (GRNN) is often used for function approximation [87]. It has been shown that, given a sufficient number of hidden neurons, GRNNs can approximate a continuous function to an arbitrary accuracy.

Probabilistic neural networks (PNN) can be used for classification problems [87]. Their design is straightforward and does not depend on training. A PNN is guaranteed to converge to a Bayesian classifier, providing it is given enough training data. These networks generalize well.

The GRNN and PNN have many advantages, but they both suffer from one major disadvantage. They are slower to operate because they use more computation than other kinds of networks to do their function approximation or classification.
The larger a network you use, the more complex the functions the network can create.

2.6 Literature review on closed loop system identification

2.6.1 Traditional Load modelling

A traditional load model containing only feed forward components is shown in figure (2.10).

![Diagram of Traditional Load Modelling](image)

Where, X and Y could be variations of reactive power, active power, voltage magnitude and frequency.

Almost all previous research has performed load modelling based on traditional models which are power system variations affecting the load real and reactive power. A few papers [4, 62, and 88] considered that the load real and reactive power changes also affect power system voltage and current changes. There are three approaches of closed loop identification (power system variations affect the load and load variation affects the power system) mentioned in Ref [89]. These are,

1. **Direct approach**: This approach ignores the feedback loop and identifies the feed forward loop exactly like open loop identification

2. **The indirect approach**: This approach identifies the closed loop transfer function and determines the open loop transfer function using the knowledge of the linear feedback controller
3. The Joint input-output approach: In this approach input and output are considered jointly, even though the output of a system is driven by some extra input or set point signal and noise. Exact knowledge of the regulator parameter is not required.

In the direct approach, knowledge of feedback is not required but for the indirect method it is required [90]. Also parameterization of the noise model is important to the indirect method for accurate estimation of the parameter of the plant and controller [89]. The indirect and joint input-output methods need prior knowledge of the closed loop system. Another drawback of the indirect and joint input-output methods are that they generally give suboptimal accuracy [89]. Recently the two stages and related projection methods use an advanced method based on prediction error. A two-stage method consistently estimates the open loop system, regardless of the noise model used in case of linear feedback [89]. It is quite robust and simple to use. However it fails if the controller is nonlinear and contains some unknown disturbances. With the projection method these problems can be circumvented and can be applied to a system with arbitrary feedback mechanisms. Projection method considers a non-casual FIR model [91]. This method accuracy is also suboptimal. With undermodelling, the model can fit to the data with the arbitrary frequency weighting which is a clear advantage compared to the direct method [91].

Many of the closed-loop identifications have been focused on identification for control and the assumption is made based on the control context rather than for a load modelling task [62, 89-92]. Direct Identification using the prediction error method is known to fail in the case of low variation of reference. The residue method for the load modelling problem which is mentioned in paper [63] is still able to give good estimates of the noise when output noise continues to drive the system. The residue method is quite similar to the projection method but in the projection method one must calculate the open loop transfer function based on output and estimated input. Whereas in the residue method the open loop transfer function is calculated based on input residue and, similarly, output residue is estimated and used for calculating the feedback transfer function. The residue method can be used for a low signal to noise ratio.
Chapter 3: Derive algebraic transfer function of an induction motor and identify the model

The knowledge of induction motor response is essential for dynamic load modelling. Hence, this chapter starts from the 1\textsuperscript{st} order induction motor model with a simple linear type load, which is called a fan load. The 1\textsuperscript{st} order motor model ignores the rotor and stator transient, which is important for dynamic load modelling. Upgrades of the 1\textsuperscript{st} order fan load model to a 5\textsuperscript{th} order model of induction motor is done for dynamic load modelling purposes and for the estimation of dynamic parameters. The order is reduced from the 5\textsuperscript{th} order induction motor to 3\textsuperscript{rd} order induction motor as well as mapping the 3\textsuperscript{rd} order model to the 1\textsuperscript{st} order steady state induction motor model. The algebraic equations of an induction motor transfer function are developed from simple steady state equations and derivations of the equations are appended in appendix A. The eigenvectors are calculated from the algebraic equation of a transfer function of a 5\textsuperscript{th} order model with a fan type linear load and springy shaft type load. The reason behind calculating the eigenvalue is to examine the relationships between parameter values of the induction motor with eigenvectors.
3.1 1st order induction motor with fan load

The two types of shaft loads of an induction motor are considered in this thesis. One is an induction motor with a linear fan type load and the other is a springy shaft load. Both types of load are shown in figure (3.1)

![Figure 3.1 Induction motor with springy shaft load and fan load](image)

Figure (3.2) shows the 1st order induction motor with a linear load represented by a steady state equivalent circuit

![Figure 3.2 Steady state equivalent circuit](image)

Ignoring the mutual inductance, the equation of slip, current and power are as follows;
\[ s_m = \frac{\omega_r - \omega_s}{\omega_s} \]  
\( (3.1) \)

\[ I = \frac{V}{r_r s_m + R_l + jX} \]  
\( (3.2) \)

\[ P_e = \text{real}(VI^*) \]
\[ Q_e = \text{Imag}(VI^*) \]  
\( (3.3) \)

where \( s_m \) is motor slip, \( \omega_s \) is synchronous speed of the motor in rad/s, \( \omega_r \) is rotor speed in rad/s, \( I \) and \( V \) are the input current and voltage of the induction motor, \( r_r \) is rotor resistance, \( R_l \) is stator resistance and \( X \) is the sum of stator and rotor reactance. \( P_e, Q_e \) are real and reactive power input.

If the stator resistance and the approximately normal operational region of low slip are ignored, the real and reactive powers are reduced to;

\[ P_e \approx \frac{s_m |V|^2}{r_r} \]  
\( (3.4) \)

\[ Q_e \approx \frac{s_m^2 X|V|}{r_r^2} \]  
\( (3.5) \)

The dynamic motion equation of a linear fan load, where \( H \) is machine inertia and \( B \) is torque-damping factor, is;

\[ \frac{d\omega_r}{dt} = \frac{1}{H} (P_e - B\omega_r) \]  
\( (3.6) \)

After manipulating equations (3.1) and linearizing the equation around the steady state value of supply frequency \( \omega_s \), q-axis steady state supply voltage \( v_{q=0} \), the
algebraic equation of the transfer function of supply frequency change to the real power change is shown below and derivation of the equation is attached in appendix A, where \( \omega_b \) is base speed in rad/s and S is Laplace constant.

\[
P_{e_\omega} = \frac{v_{qr0}^2 (S + \frac{B}{2H})}{r_\omega \omega_b (S + \frac{B}{2H} + \frac{v_{qs}^2}{2Hr_\omega \omega_b})}
\]  
(3.7)

In most cases the transfer function zero is far from the pole thus we can assume that \( \frac{B}{2H} \) is close to 0 and equation (3.7) is reduced to;

\[
P_{e_\omega} = \frac{v_{qr0}^2 S}{r_\omega \omega_b (S + \frac{v_{qs}^2}{2Hr_\omega \omega_b})}
\]  
(3.8)

And the high frequency gain of the transfer function of frequency change to real power change is;

\[
k_{pf} = \frac{v_{qr0}}{r_\omega \omega_b}
\]  
(3.9)

3.1.1 Reactive power with changing supply frequency

Considering equation (3.5) and linearizing the equation around the steady state value (slip \( s_{l0} \)), algebraic equation of the transfer function of supply frequency change to the reactive power change is shown below and derivation of the equation is attached in appendix A

\[
\frac{Q_{e_\omega}}{\omega_\omega} = \frac{2s_{l0}v_{qr0}^2 X}{r_\omega^2 \omega_b} \left( \frac{S + \frac{B}{2H}}{(S + \frac{B}{2H} + \frac{v_{qs}^2}{2Hr_\omega \omega_b})} \right)
\]  
(3.10)
Therefore, the transfer function of frequency change affecting reactive power change is:

\[
\frac{\Delta Q}{\omega_v} = \frac{2s_{10}V_{q10}^2X}{r_r^2\omega_b} \frac{S}{(S + \frac{V_{qr}^2}{2Hr_0\omega_b})}
\] (3.11)

The high frequency gain of the transfer function of frequency change affecting reactive power is:

\[
k_q = \frac{2s_{10}V_{q10}^2X}{r_r^2\omega_b}
\] (3.12)

### 3.1.2 Real power with changing voltage

Similarly using equations (3.1) and linearizing the equation around the steady state value, the algebraic equation of the transfer function of supply voltage change to the real power change is given in equation (3.13) and a derivation of the equation is attached in appendix A,

\[
\frac{\Delta P}{\Delta V_{qr}} = \frac{2V_{qr}s_{10}(S + \frac{B}{2H})}{r_r(S + \frac{B}{2H} + \frac{V_{qr}^2}{2Hr_0\omega_b})}
\] (3.13)

Again assuming that zero is far from the pole, hence \(\frac{B}{2H}\) is close to 0, the equation (3.13) reduces to,

\[
\frac{\Delta P}{\Delta V_{qr}} = \frac{2V_{qr}s_{10}(S)}{r_r(S + \frac{V_{qr}^2}{2Hr_0\omega_b})}
\] (3.14)
The high frequency gain of the transfer function of voltage change affecting real power is

\[ k_{pv} = \frac{2V_{q0}s_{l0}}{r_r} \]  

(3.15)

### 3.1.3 Reactive power changing with voltage

The transfer function of the reactive power changes to supply voltage change is given below and the derivation of the equation is attached in appendix A.

\[ \Delta Q_e \quad \Delta V_{q0} \quad \frac{X}{r_r} \quad 2V_{q0}s_{l0}^2 \quad \left( S + \frac{B}{2H} - \frac{V_{q0}^2}{2Hr_{b0}} \right) \quad \left( S + \frac{B}{2H} + \frac{V_{q0}^2}{2Hr_{b0}} \right) \]  

(3.16)

Again assuming that zero is far from the pole, so that \( \frac{B}{2H} \) is close to 0, the equation (3.16) reduces to;

\[ \Delta Q_e \quad \Delta V_{q0} \quad \frac{X}{r_r} \quad 2V_{q0}s_{l0}^2 \quad \left( S - \frac{V_{q0}^2}{2Hr_{b0}} \right) \quad \left( S + \frac{V_{q0}^2}{2Hr_{b0}} \right) \]  

(3.17)

And high frequency gain of the transfer function of the voltage changes effecting reactive power changes is,

\[ k_{qv} = \frac{X}{r_r} \quad 2V_{q0}s_{l0}^2 \]  

(3.18)
3.2 5th order induction motor formulas

In practice an induction motor response is not similar to the 1st order steady state model response. Because of this, to identify real motor response from the feeder data the order of the induction motor needs to be increased to a 5th order induction motor. A 5th order induction motor response is quite similar to a real motor. There are numerous ways of formulating the equations of an induction machine for the purpose of computer simulation [93]. If voltages of the direct and quadratic axis are taken as the independent variables and flux is taken as dependent variables, the five state equations of the induction motor in arbitrary reference frame are [94],

\[
\frac{d\Psi_{qs}}{dt} = \omega_b \left[ v_{qs} - \frac{\omega}{\omega_b} \Psi_{ds} + \frac{r_x}{x_{js}} (\Psi_{mq} - \Psi_{qs}) \right]
\]

\[
\frac{d\Psi_{ds}}{dt} = \omega_b \left[ v_{ds} - \frac{\omega}{\omega_b} \Psi_{qs} + \frac{r_x}{x_{js}} (\Psi_{md} - \Psi_{ds}) \right]
\]

\[
\frac{d\Psi_{qr}}{dt} = \omega_b \left[ v_{qr} - \left( \frac{\omega - \omega_f}{\omega_b} \right) \Psi_{dr} + \frac{r_x}{x_{lr}} (\Psi_{mq} - \Psi_{qr}) \right]
\]

\[
\frac{d\Psi_{qr}}{dt} = \omega_b \left[ v_{qr} - \left( \frac{\omega - \omega_f}{\omega_b} \right) \Psi_{dr} + \frac{r_x}{x_{lr}} (\Psi_{mq} - \Psi_{qr}) \right]
\]

\[
\frac{d\omega_f}{dt} = \frac{\omega_b}{2H} (T_e - T_L)
\]

For balanced operation of a symmetrical induction motor, the most widely used reference frame for simulation is a synchronously rotating reference frame [94]. To
change the above equations to a synchronous reference frame, the system speed $\omega$ is replaced by $\omega_s$. Here $\omega_b$ is base speed of the system.

For simulating the above five equations in MATLAB, knowledge of the stator leakage reactance ($X_{ls}$), rotor leakage reactance ($X_{lr}$), mutual flux linkage equation of direct ($\Psi_{md}$) and q-axis ($\Psi_{mq}$), electromagnetic torque ($T_e$) and load torque ($T_L$) are required. These equations are:

\[
\Psi_{mq} = X_{aq} \left( \frac{\Psi_{qs}}{X_{ls}} + \frac{\Psi_{qr}}{X_{lr}} \right) \tag{3.24}
\]

\[
\Psi_{md} = X_{aq} \left( \frac{\Psi_{ds}}{X_{ls}} + \frac{\Psi_{dr}}{X_{lr}} \right) \tag{3.25}
\]

Where, $\Psi_{qs}$ and $\Psi_{qr}$ are q-axis stator and rotor flux and $X_{ad}$ is,

\[
X_{ad} = X_{aq} \left( \frac{1}{X_M} + \frac{1}{X_{ls}} + \frac{1}{X_{lr}} \right)^{-1} \tag{3.26}
\]

Per unit electromagnetic torque equation is,

\[
T_e = \Psi_{dl}i_{qs} - \Psi_{qs}i_{ds} \tag{3.27}
\]

The torque depends on current and the current equations of direct and q-axis of stator and rotor are as follows,
\[ i_{qs} = \frac{1}{X_{is}}(\Psi_{qs} - \Psi_{mq}) \]  
(3.28)

\[ i_{ds} = \frac{1}{X_{is}}(\Psi_{ds} - \Psi_{md}) \]  
(3.29)

\[ i_{qr} = \frac{1}{X_{ir}}(\Psi_{qr} - \Psi_{mq}) \]  
(3.30)

\[ i_{dr} = \frac{1}{X_{ir}}(\Psi_{dr} - \Psi_{md}) \]  
(3.31)

And linear load torque which is considered in this thesis is [27],

\[ T_L = B\omega_r \]  
(3.32)

The MATLAB code of the 5th order induction motor is attached in CD.

3.3 Reduce order from 5th to 3rd and then to 1st

Inertia and torque damping factors are most influential parameters for characterizing the dynamic characteristic of an induction motor. To identify the transient parameter of inertia and the damping factor of an induction motor, it is good to start from a 5th order model, then;

1. Infer the relevant parameter for a 3rd order model
2. Map it to a 1st order steady state model.
3. Compare the 1st order steady state model to an algebraic equation and calculate the parameter value from the algebraic equation.

In general, the speed transients are considered slow, especially with inertia or high load, while stator flux and rotor flux transients are much faster for voltage source supply [94].
This thesis is interested in modelling induction motors specifically with respect to a slow transient and considers that an induction motor is running at a steady state. Consequently, to reduce the model order from 5th to 3rd the stator flux derivative is considered zero in a synchronous coordinate. To reduce a 3rd order to a 1st order the rotor flux derivative is considered zero. Thus, while the real measurements derive from a 5th order model, the quality of the modelling is retained for generator dynamic stability purposes, if the motor first order model is accurate. The order reduction from a 5th order to a 3rd order is elaborated in appendix B. Where \( V_{ds} \) and \( V_{qs} \) are direct and q-axis voltage and As, Bs, Ar, Br and Cr values are defined in appendix B.

\[
\begin{bmatrix}
\Psi_{ds} \\
\Psi_{qs}
\end{bmatrix} = \begin{bmatrix} A_s & B_s \end{bmatrix} \begin{bmatrix}
\Psi_{ds} \\
\Psi_{qs}
\end{bmatrix}
\]  

(3.33)

\[
\begin{bmatrix}
\frac{d\Psi_{dr}}{dt} \\
\frac{d\Psi_{qr}}{dt}
\end{bmatrix} = \begin{bmatrix} A_r & B_r \end{bmatrix} \begin{bmatrix} V_{dr} \\
V_{qr}
\end{bmatrix} + \begin{bmatrix} C_r \end{bmatrix} \begin{bmatrix}
\Psi_{dr} \\
\Psi_{qr}
\end{bmatrix}
\]

(3.34)

The simulation of equation (3.33) and equation (3.24) are executed in MATLAB. The MATLAB codes are provided in the CD.

To reduce the model order from the 3rd to 1st, set rotor direct (\( \Psi_{dr} \)) and q-axis (\( \Psi_{qr} \)) flux zero in equation (3.34). The 1st order simplified equation executed in MATLAB is as follows,

\[
\begin{bmatrix} 0 \\
0
\end{bmatrix} = \begin{bmatrix} A_r & B_r \end{bmatrix} \begin{bmatrix} V_{dr} \\
V_{qr}
\end{bmatrix} + \begin{bmatrix} C_r \end{bmatrix} \begin{bmatrix}
\Psi_{dr} \\
\Psi_{qr}
\end{bmatrix}
\]

(3.35)
3.4 How to simulate an induction machine in MATLAB

For simulating an induction motor in MATLAB, the solver ode45 is used in this thesis’s main program, where the subroutine is called by the main program. In the subroutine all the states are coded. The initial state is the steady state value of the five states which are $\omega$, $\Psi_{dr}$, $\Psi_{qr}$, $\Psi_{q^*}$, and the chosen time span is 20sec for simulating a long-term dynamic response. Simulation options are set by odeset and ode45 solver’s output function calls another subroutine to save the output values of ode45. There is no hard and fast rule of using a time span of 20sec. For predictions of system performance through simulation extending over a time range of tens of seconds to several minutes [93] A time span of 50sec can be used, but in that case a longer time is needed to run the code. In this thesis perspective, 20sec is a good choice for averaging the noise as well as estimating the model. The following figures (3.3-3.4) show an induction motor model estimated by system identification and integration times of 50sec and 20sec are used. The system identification process is described in Chapter 4. There is no substantial difference between the two models except distortion. In the 20sec simulation the data length is small, for that reason averaging couldn’t eliminate the noise totally.

For the 3rd order induction motor model, the stator transient is neglected, thereafter three steady state values $\omega$, $\Psi_{qr}$, $\Psi_{dr}$ pass through the solver and everything else is
similar to a 5\textsuperscript{th} order model. Similarly for a 1\textsuperscript{st} order, only one state, that is $\omega_r$ passes through the solver and everything else is similar to a 5\textsuperscript{th} order model.

Steps for simulating an induction motor in MATLAB are as follows

1. First write up the main code and specify the electrical parameters of an induction motor and calculate the parameters value from equations (3.24). MATLAB has several routines for numerical integration, in the main code it has to be called by the statement;

   
   
   \begin{align*}
   [t,y]=\text{solver} ('Xprime' tspan, X_0)
   \end{align*}

   Where ‘Xprime’ is an m-file to evaluate the state variable derivatives

2. Secondly write the function “xprime” with 5 state equations (3.19) and the statement is;

   
   
   \begin{align*}
   \text{Function xdot=xprime(t,x)}
   \end{align*}

   Where xdot is the vector of time derivatives of the state and t is the time and x is the vector of the initial state. This routine takes x as input and completes the specific values.

   Here xdot is the value of 5 state equations (3.19) and t is 20 sec time with time steps 0.0167sec (one-half cycle), where fast phenomena are significant and for longer term effect, time step should be 0.1s to 0.2 s. To subside the low dynamics and predominate longer term dynamic effects the higher time steps can be used [93]. \(X_0\) is the steady state value of the 5 states is those obtained from y the output vector of the solver ode45.

   After running the integration from 0 to 20 sec with 0.0167 time steps and zero initial state the motor will approach a steady state shortly. The program needs to save the steady state value for simulating the motor around a steady state.
In this function, equation (3.24) is calculated first, and then calculates the state of equations (3.19), after that, equation (3.28) is evaluated to calculate the electromagnetic torque of equation (3.27). The output of the solver contains 5 state derivatives in y column vectors after 20sec of integration.

In this thesis the “ode45” solver is used, which uses fourth order Runge-Kutta integration. It has additional options that can be set by the user. Additional options such as the real tolerance, the absolute tolerance, and an increase in the number of output points and any installable output functions are called by the solver with “odeset” after every successful step. The statements for setting options are as follows;

Options=odeset (‘name1’, value1,’name2’, value2…)

The user defined option value is chosen for this thesis as follows,

Options=odeset('RelTol',2.22045e-018,'AbsTol',1e-15,'Refine',1,'MaxStep',0.00028,'InitialStep',0.000001,'NormControl',1,'Stats','on','OutputFcn',@odeout5thorder).

For calculating the frequency domain response of the time domain integration, MATLAB code “linmod” is used for linearization purposes. Steps are as follows,

1. MATLAB code “Linmod” extracts the linear state –space model of a system around a stable operating point. The syntax is,

[A, B, C, D]=linmod (‘sys’, x, u)

Where sys is Simulink s-function from which a linear model is to be extracted, x is the initial steady state vector and u is the input vector.

2. Simulink passes the current time t, state vector x, input vector u and flag to the s-function. The flag has an integer value that indicates the task to be performed by the s-function.
This thesis uses 0, 1 and 3 flags respectively to define the s-function block characteristics of sample times and the initial condition of continuous states, calculates the derivatives of the continuous states and calculates the output of the state derivatives.

3. “Linmod” returns the linear model in state space form A, B, C, and D, which describes the linear input – output relationship. Therefore, using the state space form to calculate the magnitude and phase plot in the frequency domain by using Bode command. The syntax is:

\[
\text{Bode} (A, B, C, D)
\]

A flow-chart of the simulation of an induction motor is described below. The function name “Omegaprime” which is called by “ode45” and “diffsfunc5thorder” which is called by “linmod” are used in the flowchart.

![Flow chart of simulating an induction motor](image-url)
3.4.1 Simulation result

A 3-phase, 630kW motor with the following parameter values is used to illustrate the simulation in MATLAB.

Table 3.1. Parameter value of 630kW motor

<table>
<thead>
<tr>
<th>B</th>
<th>H</th>
<th>R_r</th>
<th>R_1</th>
<th>X_2</th>
<th>X_1</th>
<th>X_m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.0222</td>
<td>0.0453</td>
<td>0.0322</td>
<td>0.074</td>
<td>2.042</td>
</tr>
</tbody>
</table>

At first, the machine is simulated from a no load to a full load and the steady state value is determined. In the figure 3.6 the rotor speed is 0 and in the figure 3.7 slip is 1. When it runs up to near synchronous speed at steady state the rotor speed is close to 314rad/s and the slip is close to zero. Because of rotor inertia transients, rotor reaches to synchronous speed with little overshoot and slip falls down to zero with small overshoot.

![Figure 3.6. Rotor speed](image)

![Figure 3.7. Slip](image)

The d & q axis rotor and stator fluxes $\Psi_{ds}$, $\Psi_{qs}$, $\Psi_{qr}$, $\Psi_{dr}$ reaches to steady state are shown in figure 3.8 note that initial high frequency starting transients.
When the electromagnetic torque is equal to the load torque, a steady state condition is reached and shown in figure 3.12. In the figure it is also shown that at the beginning a high frequency starting transient is present and reached a steady state with an overshoot caused by the rotor inertia transient.
The direct and q-axis current in the stator are shown in figures (3.14-3.15). The high frequency starting stator transient is present in both axes. The current of q-axis reached the steady state with overshoot because of rotor and stator transient interaction, whereas the direct axis current decreased to reach a steady state without any overshoot.

After determining the steady state value of the 5 states, the machine is simulated around the steady state point with the system frequency perturbed by random noise using “rand” MATLAB command. The noise perturbed electromagnetic torque real power and rotor speed are shown in figures (3.16-3.18). As small change in torque is
seen to change the real power input. Because of the low pass filter effects, the input power does not change substantially and for that reason rotor speed changes slowly due to rotor inertia as shown in figures (3.16-3.18).

![Figure 3.16. Electromechanical Torque](image1)

![Figure 3.17. Input Real power](image2)

![Figure 3.18. Rotor speed](image3)

The 3rd order induction motor is simulated in MATLAB as mentioned in section 3.3 by ignoring the stator transient. The rotor speed reaches a steady state with a less significant high frequency stator starting transient. All the other outputs are the same as a 5th order model except that there is a lower high frequency starting transient. The rotor speed is reached at a steady state with damped overshoot shown in figure 3.19 below.
A 1st order induction motor is simulated in MATLAB by ignoring the rotor and stator transient in a similar way as mentioned in section 3.4. All the system performance values are almost the same as a 5th order model except the starting, and before reaching the steady state, the transient is absent. The 1st order rotor speed is shown in figure 3.20 as it reaches the steady state point.
The bode plot of frequency change, as it affects the real power of the 5th, 3rd and 1st order induction motor models by using “linmod” are shown in figure 3.21. From figure 3.21 it can seen that from the frequency of 0.1 rad/s to a frequency 25 rad/s, the 1st order, 3rd order and 5th order model behave similarly. Power rolls off affect both the 5th and 3rd order models because of the rotor and stator transients. There is a second resonance is visible in only 5th order model. The first resonance may be attributed to rotor transient and the second resonance to stator transient.

Figure 3.21. Bode plot of frequency change affects real power changes

In a similar way, the Bode plot of frequency change as it affects the reactive power, is calculated by using “Linmod” and shown in figure 3.22 below. In this case the frequency change affects reactive power change, the high frequency gain of the 5th order model and the 1st order model are almost identical but high frequency gain depends on slip. In the similar way, the analysis of the frequency change affecting the real power is discussed above. The figure3.22 can describe the transfer function between frequency and reactive power. In the range from 1 rad/s to 10 rad/s for the 1st, 3rd and 5th order model behave similarly. Dips are only present for the 3rd and 5th order models so the reason of occurrence of the dips is the rotor transient. The 2nd resonance does not exist for the 3rd order model. Therefore the reason for occurrence of this resonance and power roll off is the stator transient.
Figure 3.22. Bode plot of frequency change affects reactive power changes

The Bode plot of voltage change affects the real power change and is calculated using “Linmod” and shown in figure 3.23. Consequently the algebraic equation is developed and explained by equation (3.16). The high frequency gain is calculated and represented by equation (3.18). From the equation (3.18) it is shown that the high frequency gain depends on slip of the induction motor. From figure 3.23 it can be said that from frequency 0.01 rad/s to 1 rad/s, both the 1st and 3rd order models behave same and after that the difference is because of the rotor transient, which is not present in the 1st order model.
The remainder of this thesis considers how frequency change affects the real power change transfer function of an induction motor. The high frequency gain transfer function of frequency change to real power change is independent of slip and equal to steady state high frequency gain. For another task to infer the 5th order model to the algebraic equation, the transfer function of frequency change to real power change is a good choice for representation in the model.

The same motor with a springy shaft load is shown in figure 3.1a and simulated in MATLAB. The 5th, 3rd and 1st order induction motor models of frequency changes affecting on the real power changes is obtained by using a similar procedure as that followed by a linear load induction motor and the transfer function is shown in figure 3.24.

The reason for considering this type of load is that the real data collected from Brisbane and simulated in MATLAB showed some resonance that could have been related to springy shaft resonance. The estimated model consists of many sharp resonance peaks and the hypothesis is that the reasons for these sharp peaks are because of the springy shaft type load. To establish the hypothesis that the response is due to an induction motor with springy shaft type load, the system is simulated in MATLAB. From the model it is shown that the resonance peak of the 5th order model is as sharp as the fan type linear load model. The 5th order and 3rd order models are
agree with fan type induction motors. Although 1\textsuperscript{st} order model’s high frequency gain is not equal to 5\textsuperscript{th} or 3\textsuperscript{rd} order resonance peak height. We are thinking area under the shape of the curve will infer the power rating of the springy shaft induction motor. Additionally, analytical calculation of this type of load is complex. Hence to avoid the complexity, this type of load representation has been ignored in the rest of the thesis.

![Bode Plot](image)

Figure 3.24. Bode Plot of frequency changes affects the real power changes

### 3.5 Eigenvalues of Induction Motor

For a 5\textsuperscript{th} order induction motor model, the states are $X' = [\Psi_d, \Psi_{qr}, \Psi_{ds}, \Psi_{qr}, \omega_t]$

And direct and q- axis voltages of rotor and stator are inputs, $u' = [V_d, V_{qr}, V_{ds}, V_{qs}]$

The linear differential equation around a steady state operating point can be written,

$$\Delta \dot{X} = A \Delta X + B \Delta u$$  \hspace{1cm} (3.36)

Where A matrix is,
Derivation of matrix A is summarized in appendix C.

The dynamic response of an induction motor model can be obtained by calculating the eigenvectors of matrix A. The Eigen values of a 630kW motor with linear fan load and springy shaft load are tabulated below. The damping ratio of both types of load is approximated at -0.15.

Table 3.2. Calculated Eigenvalue

<table>
<thead>
<tr>
<th>Linear Fan Load</th>
<th>Sprinhy shaft load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0e+002 *</td>
<td>1.0e+002 *</td>
</tr>
<tr>
<td>-0.1702 + 3.1355i</td>
<td>-0.1362 + 3.1378i</td>
</tr>
<tr>
<td>-0.1702 - 3.1355i</td>
<td>-0.1362 - 3.1378i</td>
</tr>
<tr>
<td>-0.0622 + 0.3889i</td>
<td>-0.0501 + 0.3448i</td>
</tr>
<tr>
<td>-0.0622 - 0.3889i</td>
<td>-0.0501 - 0.3448i</td>
</tr>
<tr>
<td>-0.1172</td>
<td>-0.0925</td>
</tr>
<tr>
<td>Damping ratio:-0. 1579</td>
<td>Damping ratio=-0.1437</td>
</tr>
</tbody>
</table>

3.6 Outcomes

The real world measurements will contain all aspects of induction motor performance as reflected in the 5th otrder model. The difficulty is to interpret the measurements of real machines into simple parameters such as inertia. From the plot of a fifth order machine we can infer the plot of the corresponding first order model. From the first order model we can infer the basic parameters such as inertia. The process overall has been to know the frequency to power relationship of a real
machine, to infer the corresponding simple first order frequency to power relationship and thus maps the results to inertia and torque damping factor

This chapter successfully shows how to simulate an induction motor of 5\textsuperscript{th} order model with two types of shaft load for dynamic load modelling purposes and also shows how to reduce a 5\textsuperscript{th} order model to a 3\textsuperscript{rd} order, and then to a 1\textsuperscript{st} order model to get the dynamic parameters, e.g. the torque damping factor and inertia, which are the main influences of the transient characteristic of the induction motor. Algebraic equations of the transfer function of real and reactive power change to supply frequency and voltage change are derived in this chapter as well. It is not possible to derive an algebraic equation of eigenvalues. Hence, it is not possible to relate the eigenvector to the parameters values of an induction motor as shown in appendix C.
Chapter 4: Identify the motor model and parameters by using System Identification

A power system includes composite loads with different characteristics. Aggregating these composite loads and representing them in equivalent models are important to be able to predict system responses to disturbances.

In the power system context, for obtaining a model of the load, one form of traditional testing is to use a tap change and observing the response of the plant [3]. There is a greater range of systems that can be identified if we can make use of the variations that occur naturally in the power system to learn more of the performance.

The load response includes both the aspect of the power system affecting the load and the load changes affecting any power measurement. Hence, in this chapter a closed loop system identification technique is described. The model of an induction motor with shaft load, and the dynamic parameters of the induction motor are identified using this
technique. To validate the system identification technique and the model of the load, an induction motor was run in the laboratory and a phasor measurement unit was used to get the voltage and current phasor, from which a model was derived.

The advantage of this measurement-based technique is that it is able to obtain data directly from the actual system continuously. Because of its advantages, a measurement based dynamic load modelling technique is the topic of this thesis.

4.1 Theory for Cross-Correlation identification [95]

Estimating an impulse response from input-output measurements is a component of system identification [95].

One path is to compute the impulse response of a transfer function from the cross-correlation of its input and output signals.

This system of cross-correlation based identification is presented in figure 4.1

![Figure 4.1. Cross-correlation based identification](image)
The measured system variable could be voltage or frequency. Due to other loads causing system variations, the variations in the measurement may not be fully predictable from past measurements or load effects. This non-predictable portion is represented as a white noise input \( W_1 \). Similarly there is a component of load variation that represents the unpredictable changes in load power by customers turning switches ON and OFF. This white noise component is represented by \( W_2 \).

\( W_1 \) is added to the normal steady-state operation signal \( f(n) \). \( f(n) \) is either steady-state voltage or steady-state supply frequency and the sum of normal operational variations around a stable operating condition \( v(n) = W_1(n) + f(n) \) forms the input to the identified system (induction motor with different kinds of shaft load). We can consider an induction motor as an ideal linear system which doesn’t generate any internal noise at the output \( y(n) \) of which a noise signal \( W_2(n) \) is added. Assuming the noise is largely independent of both \( W_1(n) \) and \( y(n) \), we can model the noise that is internally generated by any real system. In this thesis any internal noise source such as friction of the induction motor shaft is ignored. Switching on or off of the load is considered as the output noise \( W_2(n) \) which is inside the loop. The load bus switching the load on/off changes the network voltage/frequency and that is considered as input noise \( W_1(n) \).

\( W_1(n) \) is assumed to be connected to the first input of the correlator, the other input of which is supplied by the additive signal, \( p(n) = y(n) + W_2(n) \) as measured at the output of the system and input of the correlator thus produces an estimate of the correlation function \( R_{n,p}(\tau) \).

The internal linear block is described by the input–output convolution relationship

\[
y(n) = \alpha \int_{0}^{\alpha} h(s)v(n-s)ds
\]

(4.1)

Where, \( h(s) \) is the impulse response of the induction motor with different kinds of shaft load.

If realized in a discrete time domain, the cross-correlation function depends on the delay \( \xi \) as
\[ R_{w_p} = \mathbb{E} \{ W_1(n) \, p \, (n+\xi) \} \quad (4.2) \]

After substitution of the value of \( p \),

\[ R_{w_p} = \mathbb{E} \{ W_1(n) \, (y(n+\xi) + W_2(n)(n+\xi)) \} \quad (4.3) \]

\[ R_{w_1p}(\xi) = \alpha \int_0^\infty h(s)v(n+\xi-s)ds + W_2(n+\xi)) \quad (4.4) \]

Expanding the product and using the linearity of the mean-value operator that enables us to interchange it with the integrator operator, we can obtain;

\[ R_{w_1p}(\xi) = \mathbb{E} \{ W_1(n)W_2(n+\xi) \} + \int_0^\infty h(s)E \{ W_1(n+W_1(n+\xi-s)) \} \, ds \quad (4.5) \]

\[ = R_{w_1w_2}(\xi) + \int_0^\infty h(s)(R_{w_1w_2}(\xi-s) + R_{w_1w_2}(\xi-s)) \, ds \]

Based on assumption of independence of both the noise \( W_2(n) \) and the production signal \( f(n) \) on the auxiliary measurement signal \( w_1(n) \), the cross-correlation function \( R_{w_1w_2}(\xi) \) and \( R_{w_1f}(\xi) \) are equal to zero and we finally obtain
If the equation (4. 6) is transformed into a frequency domain, the frequency response of the system can be determined as [95]

\[
G_1(\omega) = \frac{S_{w_1p}(\omega)}{S_{w_1w_1}(\omega)}
\]  \tag{4. 7}

Which is given by the ratio of the cross and power spectrum and can be obtained by Fourier transformation of the measured correlation function.

Following the same procedure to get the transfer function between \( W_1 \) and \( v \),

\[
G_2(\omega) = \frac{S_{w_1v}(\omega)}{S_{w_1w_1}(\omega)}
\]  \tag{4. 8}

We can assume that, \( v(n) = f(n) + W_1(n) \) is the voltage/frequency change of the induction motor input. Hence the real power model of the induction motor by changing voltage/ supply frequency is:

\[
G = \frac{\Delta p}{\Delta v} = \frac{G_1}{G_2} = \frac{S_{w_1p}(\omega)}{S_{w_1v}(\omega)}
\]  \tag{4. 9}
Similarly, the reactive power model of the induction motor by changing voltage/supply frequency is,

\[
G = \frac{\Delta q}{\Delta v} = \frac{G_1}{G_2} = \frac{S_{wq}(\omega)}{S_{wv}(\omega)}
\]

(4.10)

4.2 Theory for identification of a system under feedback with multiple noises

The block diagram of the interaction between power system and load is shown in figure 4.2. \(W_1\) is the white noise of the power system and \(W_2\) is the white noise of load. These two indicate a disturbance of the power system and load. The symbol \(X\) is the voltage or frequency/angle changes in a power system and \(Y\) is the real or reactive power of the load.

![Block diagram of the interaction between power system and load](image)

Figure 4.2. Identification when there are multiple disturbances and feedback

The more general case as in figure 4.2 can refer to cases where the power system affects the load and the load affects the power measurement. When the load is significant compared with the power system strength then the feedback structures becomes important.

In figure 4.2 if \(W_2=0\) then the transfer function between \(X\) and \(Y\) would identify the feed forward system. If \(W_1=0\) then the transfer function between \(Y\) and \(X\) would identify the
feedback system. When both terms are present, there is no clear separation between the effects.

The idea behind this processing is to find the best predictor for X and Y. The white noise component as the residuals for X and Y will thus be the white noise inputs \( W_1 \) and \( W_2 \). Thus the process is able to find the transfer function from \( W_1 \) to both X and Y and the ratio will give the feed forward system \( A(s)/B(s) \), provided that \( W_1 \) and \( W_2 \) are uncorrelated. Similarly we can find the transfer function from \( W_2 \) to Y and to X and the ratio will give the feedback system \( C(s)/D(s) \).

### 4.3 Process of estimating a model in MATLAB by using the system identification toolbox

In chapter one “Linmod” is used to find out the system response in a frequency domain when states of the system are measurable. In this chapter, the system identification code “tfdemlate” is used to find out the system response in a frequency domain when no information about the states of the system is available. “Linmod” computes a linear state-space model by perturbing the model inputs and model states. But, for analysing real data, especially for a black box case, there is no prior knowledge about the system. Since “Linmod” is not applicable in this case, the following steps are taken to find out the model when only input and output data are available:

1. Run an induction motor in MATLAB as explained in chapter.3 using ode45.
2. Get the input value and output value from the solver ode45 output file.
3. These input and output values are used to find out the system as described in figure 4.1.
4. The system described in figure 4.1 is for an open loop system. If the system is corrupted with input and output noise and the system is determined to be a closed loop. Then we follow the theory explained in section 2.2 which would give a good system model rather than calculating a direct system using input and output values as in figure 4.1.
5. As part of this process, the input and output residue needs to be calculated by using one–step-ahead prediction for removing the dynamic component by calling MATLAB code “lscd” least square with a common denominator function. The random component of the signal is calculated by subtracting the estimated signal from the original signal.

\[ W_1 = X(n) - E(X(n)) \] and \[ W_2 = Y(n) - E(Y(n)) \]

The \( W_1 \) and \( W_2 \) power spectra are to be checked to determine whether the spectrum is flat or not. If the spectrum is flat, it means that the residue noise is white. Otherwise, the cross correlation sequence order length is to be changed until the noise becomes white. The “ttestimate” estimates the transfer function from input noise and input \( X \) and the algorithms are based on cross correlation identification. The syntax of the “ttestimate” is,

\[
[T_{W1X}, F] = \text{tfestimate}(W_1, X, \text{window}, \text{noverlap}, \text{nfft}, \text{fs})
\]

\( W_1 \) and \( X \) are divided into overlapping sections of the specified window length. This window can be a hanning, hamming, and Kaiser or Tukey window. Window length depends on how much the input and output data are corrupted with noise. Power spectra density (PSD) and cross power spectra density (CPSD) estimate the transfer function using the specified FFT length \( nfft \). \( F_s \) is the sampling frequency in Hz. \( F \) is the same size as \( T_{W1X} \),

Similarly, the transfer function \( T_{W1Y} \) between input noise and output real power changes is estimated by “ttestimate.” Dividing \( T_{W1Y} \) by \( T_{W1X} \) estimates the feed forward transfer function \( T_{XY} \). In the similar way, \( T_{W2X} \) and \( T_{W2Y} \) are calculated and the feedback transfer function \( T_{YX} \) is estimated. The estimated transfer function versus properly scaled frequency can be plotted by using command semilogx (f, Txy). The sequence is described in the flowchart,
For simulating the real data, the steps 4 to 6 are to be followed and the flow chart is as below:
4.4 Simulated result of the estimated model of an induction motor

4.4.1 Open loop (without feedback):

A 630kW Induction motor with the parameters and the simulation processes mentioned in chapter 3 is used here to get the input and output data of the induction motor. The model of the induction motor is estimated by using system identification code “tfestimate” by following steps_1 to 6, as mentioned in section 4.3. The system frequency change is the input and the real power change is the output. Using the 1-step ahead prediction method of cross-correlating data sequence length 100, the white noise of input and output, which are $W_1$ and $W_2$ respectively, are estimated.
MATLAB code XCORR is used to find out the cross correlation between input frequency change and input noise $W_1$ and the figure 4.5 shows the cross correlation between them. There is no strong cross correlation at negative lags and certain cross correlation exhibits at positive lags. At lags 0 to 65 the cross correlation is totally zero because those cross correlation sequences are deducted when calculating the input noise $W_1$. In the similar way, cross correlation is calculated between input frequency change $df$ and output noise $W_2$ and no correlation is exhibited between them which is shown in figure 4.6.

Figure 4.5. Correlation between input noise and input signal

Figure 4.6. Correlation between input signal and output noise

Figure 4.7 shows that there is no strong cross correlation between input and output noise. In another way, it can be said that it is an open loop because there is no correlation exhibited in negative lags. No correlation has been found between input noise and output power change also, which is shown in figure 4.8. No strong correlation has been found between output power change and output noise in figure 4.9.
The frequency content of input frequency change and output real power change are shown in figures (4.10-4.11). In a Natural Variation of input frequency, most of the output energy is concentrated between 2 to 10 Hz, as seen in figure (4.10). In figure (4.11) at 5 Hz the energy is highest. But in the input, substantial energy exists between the ranges of 15 Hz to 20Hz as seen in figure (4.11).
The energy content of input noise and output noise are quite flat and according to white noise definition, both are like white noise shown in figure (4.12) and figure (4.14). The autocorrelation of the noise in figure (4.13) and figure (4.15) is quite substantial at lag=0 which also demonstrates whiteness.
Consequently the transfer function of real magnitude and phase between input frequency change and output real power change are shown in figures (4.16-4.17). When transfer function is calculated dividing the cross power by auto power the output highest energy will be at 5 Hz and most of the energy will be in between 2 to 15 Hz.
4.4.2 Closed loop (Feedback):

In the traditional approach to modelling the induction motor, it is assumed that a power system will affect the load only. In many real cases the model of an induction motor is estimated when the power system affects the load and load affects the power system governor mode as well.

\[
\begin{align*}
\omega_s &= \frac{KP}{Pe} \left( S + \tau \right) \\
\end{align*}
\]  

(4.11)

Figure 4.18. Closed loop system identification

Here a simulation is carried out in MATLAB in exactly the same way as in an open loop, which is described in chapter 3, except another state equation is added with the simulation that output power is changing the input frequency. Governor response is considered as low pass filter and the additional state is calculated using the equation (4.11)
Here $\omega_i$ is input frequency in rad/s and $P_e$ is the real induction motor input power and $K_p$ and $\tau$ are the gain of the governor and time constant respectively. In this case $K_p=0.01$ and $\tau=0.5$. It is good to select the value of parameters $K_p$ and $\tau$ in such a way that those will not change the input system frequency above the stable limit to cause a system instability situation. Therefore, the input and output value of the model is calculated according to the simulation described in chapter 3. After that, the residue of the input and output are calculated using a one-step ahead technique which is described in chapter 2 and the transfer function is estimated using the steps 1 to 6 in section 4.3.

The cross correlation between input, output, input noise and output noise are calculated and shown in figures (4.19-4.24). From the cross correlation between input and output, it is evident that the system is anti-casual which means a closed loop system. Certain cross correlation exists between input noise with input frequency change and output noise with output real power change. There is no correlation exists between input and output noise. In figures (4.25-4.26) the input and output noise power spectrum is flat indicating the whiteness of the residue. Here, the cross-correlation sequence length is 100, for this reason no correlation exist in these 100 sequence length lagged data.
Crosscorrelation between input frequency change and output noise

Figure 4.21. Correlation between input signal and output noise

Crosscorrelation between input noise and output real power

Figure 4.22. Correlation between input noise and output signal

Crosscorrelation between output real power change and output noise

Figure 4.23. Correlation between output signal and output noise

Crosscorrelation between input and output noise

Figure 4.24. Correlation between input and output noise

Frequency content of \( W_1 \)

Figure 4.25. Input noise frequency content

Frequency content of \( W_2 \)

Figure 4.26. Output noise frequency content
Frequency contents of input and output are shown in figures (4.27-4.28). In inputs all energy is concentrated between 5 Hz to 25Hz in the same manner as output energy.

Using the simulation procedure that is described in section 4.3.2 and the estimated feed forward and feedback transfer function magnitude values are shown in figures (4.27-4.28), the feed forward transfer function magnitude shape is quite similar to figure (3.21) except that most of the energy is exhibited between 2 to 6.36 Hz instead of 2 to 15 Hz. The shape is also similar to the bode plot of 630kW motor real magnitude plot, which is shown in chapter 3. The feedback transfer function in figure 4.30 has a local mode that is indicative of the governor mode.
The phase plot of the feedback system is shown in figure 4.31, the shape of which is quite similar to a phase plot of the 630kW motor in chapter 3.
4.5 Parameter Identification

By using an induction motor continuous state space model and assuming that the slip of the induction motor is working in a linear region of torque –speed curve and also input bus frequency is perturbing with white noise, we can find out that the linear time invariant transfer function is;

\[
\frac{\Delta P_e}{\Delta f} = K_f \frac{S + Z}{S + P}
\]  \hspace{1cm} (4.12)

Where, the gain constant is,

\[
K_f = \frac{V_{m}^2}{r \omega}
\]  \hspace{1cm} (4.13)

Zero of the transfer function,

\[
Z = \frac{B}{2H}
\]  \hspace{1cm} (4.14)

Pole of the transfer function

\[
P = \frac{B}{2H} + \frac{V_{s0}^2}{2Hr \omega}
\]  \hspace{1cm} (4.15)

4.5.1 Simulation Result of parameter estimation:

A 3-phase, 10HP motor with the following parameter values is used for simulating in MATLAB.

Table 4.1.10 HP Induction motor parameter value

<table>
<thead>
<tr>
<th>B</th>
<th>H</th>
<th>r</th>
<th>R1</th>
<th>X2(stator)</th>
<th>X1(rotor)</th>
<th>Xm(magnetizing)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.0222</td>
<td>0.0453</td>
<td>0.0322</td>
<td>0.074</td>
<td>2.042</td>
</tr>
</tbody>
</table>
A frequency domain plot of the 3rd order and 1st order real power transfer function of the induction motor between $f$ and $P_e$ is shown in figure 4.32.

![Bode plot of 3rd and decompose 1st order from 3rd order using invfreqs](image)

Figure 4.32. Bode plot of 1st and 3rd order model simulated in Matlab

The way to find out pole and zero locations from 1st order transfer functions is indicated in figure 4.33.

![Bode plot of 3rd and decompose 1st order from 3rd order using invfreqs](image)

Figure 4.33. Bode Plot of 1st and 3rd order model simulated in MATLAB

All the values are tabulated in table 4.2 below from the plot.
Table 4.2. Gain, Location of pole and Location of zero from simulated Bode plot of an Induction motor

<table>
<thead>
<tr>
<th>The location of zero</th>
<th>The location of pole</th>
<th>The gain 20logA=-11</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4842</td>
<td>78.37</td>
<td>0.2724</td>
</tr>
</tbody>
</table>

From equation (4.12), parameter values are calculated and tabulated in table 4.3 below,

Table 4.3. Calculated Parameter value of an induction motor

<table>
<thead>
<tr>
<th>( r_r ) (pu)</th>
<th>( H(pu) )</th>
<th>( B(pu) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0292</td>
<td>0.441</td>
<td>0.423</td>
</tr>
</tbody>
</table>

The estimated value is almost equal to the original value with a 10-12% error.

4.5.2 Experimental result

Nameplate Reading:
A.C motor: 1.5 kW
D.C motor: H.P-1, 180/200V
415V-Y, 3.5A, 50 Hz 5A, RPM-1750

The experimental setup of running an induction motor with a dc load is as below,
In this case the value of the variable resistance is zero.

![Figure 4.34. Real Induction motor Experiment](image)
A 1.5kW, 3-phase motor with a shaft load of 1HP dc machine is running in the laboratory and a phasor measurement unit with 16bit data acquisition card is used to store the input voltage, current and phase. Real and reactive powers are calculated by using these data. The measured voltage magnitude, voltage phase, current magnitude and current phase are shown below in figure 4.35.

Real and reactive powers are calculated by using these voltage magnitude, current magnitude and difference between voltage and current phase. Before calculating the transfer function, the real and reactive power initial transient and any dc components are ignored by using 1st-order low pass butterworth filter with cutoff frequency 0.0012 and zero phase digital filters called filtfilt in Matlab. From the plot of frequency content of frequency and power it’s clear that most of the energy is in the low frequency range. To
predict the transfer function for that range, decrease the sampling rate by factor of 4. Before this change the data sampling rate was 0.02 now it’s 0.005.

The transfer function \( G \left( \frac{\Delta P_e}{\Delta \omega}, \frac{\Delta Q_e}{\Delta \omega} \right) \) is estimated by using the theory developed in this chapter. After the transfer function is identified using steps 4 to 6 in section 4.3, fit it to a 3rd order model and decompose a 3rd order model to a 1st order model according to the description illustrated in chapter 3. Therefore, compare the 1st order model pole and zero location to the algebraic equation and estimate the value of B and H. The real motor transfer function \( \frac{\Delta P_e}{\Delta \omega} \) is shown in figure 4.36.

From the location of pole, zero and gain are calculated and tabulated in table 4.4 below,

Table 4.4. Gain, Location of poles and Location of zeros from experimental Bode Plot of an Induction motor

<table>
<thead>
<tr>
<th>Location of pole</th>
<th>Location of zero</th>
<th>Kf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>0.002397</td>
<td>0.8843</td>
</tr>
</tbody>
</table>

The values are substituted in equations (4.13),

The value of H=7.840 p.u And B=0.0375 p.u.

Direct measurement of real power from the experiment setup is 0.035pu. If we assume rotor angular frequency is similar to system angular frequency, then the value of B is equal to the value of real power.

Experimental value of B and estimated value of B are close, with a 5.7% error.
Per unit inertia estimated from bode plot is 7.850s. In kg m\(^2\) the value is 0.954 kg m\(^2\). The size of rotor used in the experiment lab is almost 1kg of 1m radius. The estimation error of calculating rotor weight is 4.6% which is not much.

4.6 Outcomes

This chapter has shown that if input and output data are available then using those data one can generate a transfer function by using the closed loop identification technique. This closed loop identification technique is based on input and output signals an contains an unpredicted portion which is called input and output noise. After that the two main mechanical transient parameters, the inertia and torque damping factors are estimated from the transfer function which has been shown in this chapter also. Whether the closed loop theory is working or not has been proved by an experiment run in QUT’s lab. After that the parameter value has been estimated from the transfer function of a real motor. The estimated and original values are matched quite accurately with each other.
Chapter 5: Extracting the Dynamic Component Using Area Calculation

The peak gain of a simulated 5\textsuperscript{th} order motor transfer function of frequency change’s affect on the real power change is close to the high frequency gain of an electrical steady state model of the induction motor model with a fan load. This high frequency gain is able to yield to the power of the motor if the slip is known. Voltage and current phasors are the only available data at substation bus. To calculate machine power from the high frequency gain, the slip requirement should be avoided. Consequently, it is necessary to determine an alternate way to calculate the power without specific information about the slip of individual motors. Here ten induction motors are simulated in MATLAB to examine rated slip for different size motors. In all these cases examined, we have found that the zeros of the system are sufficiently far from the peak. So the shape of the curve is dominated by the poles and thus the magnitude of the real portion of the peak closely determines the area under the curve. Calculation of area under the transfer function curve can be one way of calculating the power of the induction motor, as shown in this
chapter. The ten-induction motors simulated results are also presented in this chapter. Area calculation theory is presented with the simulation results. The real data from the Brisbane phasor measurement unit and the Sydney phasor measurement unit are used to validate the theory. There are two-methods developed in this thesis to infer motor parameters. The first one is an area calculation under the curve and the other one is the calculated percentage contribution of each type of motor. Both are contributing to the composite measurements. The first method is elaborated in this chapter.

5.1 Identify the motor loads

The simplest model of an induction motor comes from its steady state equivalent circuit. When driving a fan type load the transfer function of frequency change affects the real power change, as described in chapter 3 in equation (3.7).

If motor slip was known the high frequency gain of the model would be proportional to the motor power by using equation (3.4).

In practice there are more dynamics from rotor and stator transients but as we see in figure (3.28) the magnitude of the peak of the f-P transfer function is the same for the 5th and 3rd order models and can be used to infer the high frequency gain of the steady state transfer function.

5.2 Identify group of induction motors

Sets of induction motor parameters ranging from 4kW to 630kW are given in appendix D. The transfer function from frequency changes to real power changes is evaluated on a per unit basis for each machine. From linearization at rated load, the real component plot becomes similar to that shown in figure (5.1). For each of these wide ranges of machines, we see a very similar plot of the real portion of the transfer function from the frequency changes to the real power changes for these wide ranges of machines. Note that this requires the plot to be against the log of frequency. For each of these machines
the peak of the plot is proportional to \( \frac{V^2}{R, \omega_i} \). The lower rated machines have a higher rated slip and a higher frequency of the peak.

5.3 Area Calculation

5.3.1 Area calculation from Bode plot

The area of a real power to frequency change plot depends on the rated slip and the damping factor. When plotted against the log of frequency, the shape of the individual machine frequency change to real power change is almost the same, except the location of the resonance and height of the resonance are different for each machine. Frequency to power change transfer function (f-p) curves of ten machines are observed and shown in figure (5.1). In each case the zeros are quite far from the poles of each machine thus the shape is dominated by the poles and the values of the peak. Hence the total area under the f-p curve is indicative of the power of the motor. The terms pole and zero of the machine are explained in chapter 4.

The step by step procedure to calculate the area of an induction motor is as follows:
1. Linearize the transfer functions of frequency to real power change at normal operating point.
2. Find the magnitude and phase of the transfer function in the frequency domain.
3. Use the real part of the magnitude only to calculate area. The reason behind using the real part is explained in chapter 3.

Equation of the area is

\[
\sum_{i=1}^{f} \omega_s a_i \log_{10} \Delta \omega
\]

(5.1)

Where \(a_i\) is the real portion of magnitude, \(\log_{10} \Delta \omega\) is the log of frequency difference of \(\log_{10}\) in the bode plot, and \(\omega_s\) is the starting frequency in rad/s and \(\omega_f\) is final frequency in rad/s.

Because of the change in rated slip, the area calculation is not correctly showing rated power, as is shown in simulation section 5.4 later. Thus we need to develop a procedure to correct the area. Calculating the scaling factor is a way of correcting the calculated area, which is shown in section 5.3.2 below.

### 5.3.2 Area correction (scaling factor)

Each machine is operating at 1p.u power its own base. If it is possible to make all the machine area close to 1, then from equation (5.2) it is possible to find out the scaling factor or correction term. The rated power of the individual machine is,

\[
\frac{A}{P * a_s} = 1
\]

(5.2)

Where, A is area, \(P\) is rated power and \(a_s\) is correction term
To calculate the scaling factor the following steps are taken:
4. Take the area from step 3 in section 5.3.1 and the frequency point of each machine is normalized by dividing the rated power of individual machines.
5. Use MATLAB function “POLYFIT” to fit the area and calculate the area correction at each frequency point.
6. Extrapolate the “POLYFIT” output manually to those frequency points desired to fit the area.
8. Divide the magnitude by the scaling factor.
9. Follow the area calculation procedure at step 3 in section 5.3.1 to calculate the corrected area.
10. If area is not equal to 1, use the manual correction factor to correct the area.

5.3.3 Area calculation from system identification model

The above mentioned procedure is applicable to calculate the area for a simulated motor model, not for the model, which is estimated from real data. This is done here to get ideas to solve the real data. How to estimate the model from real data using a system identification tool is explained in the previous chapter, chapter 4. The following session describes the area calculation from the estimated model.

Step1.
Note the transfer function magnitude and frequency by using the system identification theory, as mentioned in previous chapter in section 4.3.

Step2:
Calculate the area in those frequency ranges, which gives the accurate model of the real system by using that real portion of the magnitude and the difference between those frequency points which give the area of the model. In this case instead of logarithm scale of frequency, linear scale of frequency has been used. For the reason, the magnitude of Bode plot models is different from estimated models using by using ‘tfestimate’

The equation of the area is,
\[ \sum_{\omega_s}^{\omega_f} a_i \Delta \omega \]  

(5.3)

Where, \( a_i \) is the real part of the magnitude in each FFT point, \( \Delta \omega \) is frequency difference between one FFT point and another FFT point in Hz. \( \omega_s \) is the starting frequency point in Hz and \( \omega_f \) is final frequency point in Hz.

### 5.4 Simulation result

#### 5.4.1 Area calculation from Bode plot

Ten induction machines having power ratings of 4kW, 7.5kW, 11kW, 15kW, 18.5kW, 22kW, 30kW, 45kW, 180kW and 630kW are simulated in MATLAB. The Bode plots of these machines are shown in figure (5.1). The area is calculated following steps 1-4 in section 5.3.1-5.3.2. The ten machines calculated power and original power are shown in table 5.1 using power units as kW (kW) and per unit (p.u).

<table>
<thead>
<tr>
<th>Unit</th>
<th>Mac1</th>
<th>Mac2</th>
<th>Mac3</th>
<th>Mac4</th>
<th>Mac5</th>
<th>Mac6</th>
<th>Mac7</th>
<th>Mac8</th>
<th>Mac9</th>
<th>Mac10</th>
</tr>
</thead>
<tbody>
<tr>
<td>KW (original)</td>
<td>4</td>
<td>7.5</td>
<td>11</td>
<td>15</td>
<td>18.5</td>
<td>22</td>
<td>30</td>
<td>45</td>
<td>180</td>
<td>630</td>
</tr>
<tr>
<td>KW (calculated)</td>
<td>0.3072</td>
<td>0.6623</td>
<td>1.1329</td>
<td>1.3636</td>
<td>2.0128</td>
<td>2.3851</td>
<td>3.8736</td>
<td>4.9274</td>
<td>23.346</td>
<td>91.909</td>
</tr>
<tr>
<td>Per unit (calculated)</td>
<td>0.0768</td>
<td>0.0883</td>
<td>0.1030</td>
<td>0.0909</td>
<td>0.1088</td>
<td>0.1084</td>
<td>0.1291</td>
<td>0.1095</td>
<td>0.1297</td>
<td>0.1459</td>
</tr>
</tbody>
</table>

There is a difference between the original area and calculated area. Hence to calculate the area correctly the MATLAB command “POLYFIT” is used. Ten machines original area and the area calculated by using the MATLAB command “POLYFIT” of each frequency point are shown in figure (5.2).
From the figure (5.2) above it is shown that the ten machines areas are calculated for frequency ranges from 40 rad/s to 90 rad/s and the original ten machines transfer function frequency range, is seen in figure (5.1), and is extended up to 160 rad/s. Thus it is necessary to extrapolate the correction factor from 0 to 160 rad/s. Extrapolation has been performed manually. After extrapolation the real magnitude of the transfer function of ten machines and the corrected area of ten machines are plotted in figure (5.3).
Now each frequency point’s scaling factor or correction term is already known. Hence we divide the original magnitude by the scaling factor and then compute the area. The corrected and original areas are shown in figure (5.4). Note that the original magnitude was not expected to be near 1 since there was no correction for the frequency scales.

![Figure 5.4. Phase of induction motor frequency to Power transfer function](image)

The ten machines areas are around 1 in the above curve.

Now, the scaling factor $a_s$ and area $A$ are already known. Hence these value are substituted in equation (5.3). The rated power is,

$$\frac{A}{a_s} = P \quad (5.4)$$

The ten machines original and corrected power levels are shown in the table 5.2 below in kW and also in per unit (p.u)

Table 5.2 shows Ten Induction motors calculated rated power in kW and in per unit
5.4.2 Area calculation from system identification

5.4.2.1 Area calculation from open loop system

A 15kW induction machine is simulated in MATLAB as an open loop system where the power system affects the load only. After that the value of real power change and the frequency change data are collected from the estimated transfer function. The theory to estimate the 15 kW machines transfer function of frequency changes effect on real power change is explained in chapter 4 in section 4.3 and is used here to estimate the transfer function. The estimated transfer function is shown in figure (5.5) below. To compare this model to one of the standard induction motor models, the relevant data is in the frequency range from 10 rad/s to 100 rad/s. This frequency range consists of around 5731 frequency points. Hence to calculate the area, each frequency point and the magnitude of each frequency point is required to be considered.

Therefore the equation of the area is,

$$\sum_{\omega_s}^{\omega_f} a_i \Delta \omega$$

(5.5)

In this case \( \omega_s = 1.6 \) Hz and \( \omega_f = 15.9 \) Hz and there are 5731 FFT points.

The area is= \((\text{real (a (638:6369)'))* (\text{diff ((\omega (638:6370)))) })) = 13.12\)

This is almost similar to the original power of the machines (15kW).

<table>
<thead>
<tr>
<th>Unit</th>
<th>Mac1</th>
<th>Mac2</th>
<th>Mac3</th>
<th>Mac4</th>
<th>Mac5</th>
<th>Mac6</th>
<th>Mac7</th>
<th>Mac8</th>
<th>Mac9</th>
<th>Mac10</th>
</tr>
</thead>
<tbody>
<tr>
<td>KW (Original)</td>
<td>4</td>
<td>7.5</td>
<td>11</td>
<td>15</td>
<td>18.5</td>
<td>22</td>
<td>30</td>
<td>45</td>
<td>180</td>
<td>630</td>
</tr>
<tr>
<td>KW (Calculated)</td>
<td>3.7385</td>
<td>7.7347</td>
<td>11.9918</td>
<td>13.4044</td>
<td>16.3026</td>
<td>21.0484</td>
<td>35.3374</td>
<td>41.2413</td>
<td>211.1042</td>
<td>882.8554</td>
</tr>
<tr>
<td>Pu (Calculated)</td>
<td>0.9347</td>
<td>1.0313</td>
<td>1.0902</td>
<td>0.8936</td>
<td>0.9893</td>
<td>0.9567</td>
<td>1.1779</td>
<td>0.9165</td>
<td>1.1728</td>
<td>1.0839</td>
</tr>
</tbody>
</table>
5.4.2.2. Area calculation from direct feed through and feedback

A 15kW induction motor is simulated with a 1\textsuperscript{st} order feedback system as explained in chapter 4 in section 4.4.2. The estimated transfer function between the frequency change and the real power change is given below in figure (5.6).
To calculate the area of the transfer function of an induction motor within a feedback system, we follow equation (5.3). The transfer function is a bit distorted because of the feedback loop. The output is filtered through a feedback filter and is cumulative with the input and consequently increases the output. In this case, the area is calculated between the 400\textsuperscript{th} frequency points to 5370\textsuperscript{th} frequency point and this frequency range transfer function looks like an induction motor transfer function. The specific range is indicated in figure (5.6) with black arrows.

\[ \text{Area} = (\text{real } (a(400:5369))) \times (\text{diff } ((w(400:5370)))) = 13.4636 \text{ kW} \]

The actual power is 15.834kW

Error is = 15.834 - 13.4636 = 2.37 = 15%

5.4.2.3. Area calculation of feedback power system with constant impedance and constant current load

The ten induction motors from appendix D are simulated in MATLAB and the power combined with that constant impedance (10%) and constant power (2420kW) static load as well as feedback system. The simulation included representation of the feedback system of the composite load affecting the measured bus frequency. From the composite load model (10 induction motor models, constant impedance model and constant power model), the composite 10-induction motor models of transfer function of frequency change to real power change is extracted by using the procedure described in chapter 4 section 4.4.2. This procedure is applicable in spite of the constant impedance and constant power loads because frequency changes only affect the induction motors real power.

Follow equation (5.3) to calculate the area of the composite induction motor. Because of the feedback system the measurement of frequency combines with the original input signal to apparently change the transfer function. The frequency change to real power change transfers function is shown figure (5.7) below,
Area = (real (a (400:5369)))*(diff ((w (400:5370)))) = 5305 kW

Actual composite power is around 8916 kW and constant power is 2420 kW, hence the actual composite motor power is 6496 kW. Estimation error is 18%.

5.5 REAL DATA

5.5.1 SYDNEY WEST REAL DATA

Sydney West data was collected from the substation using the QUT phasor measurement device. By using a Tukey window length of 128, the samples are processed to reduce noise [78]. To eliminate the mean and low frequency components a high pass filter with a cut off frequency 0.012 rad/s for the measured current phasor is used. Note that the strength of the current signal is not substantial below 0.0012 rad/s and the strength of the voltage signal is substantially low for the voltage phasor up to 0.012 rad/s frequency range. Hence a cut off of 0.012 rad/s for current and 0.0012 rad/s for voltage phasor are used in the high pass filter for filtering current/voltage phasor. After removing means and low frequency, the input frequency and output real power for 30 minutes of data are shown in the time domain in figure (5.8) below,
24 hrs worth of Sydney West data is processed in MATLAB. After that we divide the 24hrs data into 30 minute window lengths and calculate the area of each 30 minutes data length by using steps 1 and 2 in section 5.3.3. In a similar way a 90 minute window length and a 120 minute window length are used to calculate the area. The area plot of 90 minutes, 120 minutes and 30 minutes data lengths are shown in figures (5.9-5.11).

To find out the optimum window length both 55 minutes window length and 100 minutes window length are also used to calculate the area. Those window lengths couldn’t estimate a better result so the optimum window length search is confined to three possible window lengths 30 min, 90 min and 120 min.

Sydney west data had been collected on the 24th January 2002 during the hot summertime. By midnight, most residential customers have turned off the majority of electrical equipment and industrial electrical equipment has also been turned off at that time. Hence demand starts decreasing at that time. The 30, 90 and 120 minutes data length area calculation follows this decreasing trend from 12am to 4 pm in figures (5.9-5.11).
One contributing factor to the daily variation is weather, and especially hot weather, during which residential customers use the most air conditioning. AC induction motors
have been widely used as the fan motors of the air conditioner indoor unit[96]. In the morning, residential customers go to the office/school and therefore residential customers shut off the air conditioning which is shown in a decreasing trend of 120 minutes data length and also 90 minutes data length but in 30 minutes data length the decreasing trend is not clear.

30 minutes data length is quite variable in nature. In practice the load is not varying substantially as it is in the 30 min data length. 90 minutes data follows the decreasing load trends from 12 to 4am better than 120 minutes data length. From 4am to 12 mid day the load is increasing, which can be predicted from the 120 & 90 minute data length area calculation curves in figures (5.9-5.11). In this time gap of 90 minutes, the data follows the trend better than in the 120 minutes data length. The 120 minutes data length does not show the load variation exactly and it follows the increase trend rather than showing the variation trend.

From 12 mid day to 4pm the load is increasing in the 90 min window length but not in the 120 min window length. If it is a residential type load then it is obvious to increase the load at that time because school-going children and also people who start work early finish work and come home at this time. Therefore it is obvious that at this time the load

Figure 5.11.Area calculations of the Sydney west data 30 minute’s lengths
would be increased. If it is an industrial type load then it’s obvious that at that time industrial work is going at full pace. From 4pm to 8pm the load is increasing for a few hours then starts decreasing in the 90 and 120 minutes data lengths. From 8pm to midnight the load trend is varying and about at 10pm it is increasing suddenly for half an hour. This trend is visible in the 90 minute data but not in the 120 or 30 minute data.

Frequency change affects the induction type load. In this case, the real power is changing by changing the frequency which is calculated and plotted in the semi log plot and after that the area is calculated using the area calculation theory of system identification and is plotted in a graph. The area is telling about the motor type load, power measured per unit. It might be from industrial type motors or might be a residential type motor load or might be a commercial type motor load or it might be combination of all three types of motor load. From observing the three types of window length load profiles we can say that a 90 min data length area calculation follows the real load changing phenomena more clearly than other data length area calculations.

The load profile of the industrial load and residential load of Sydney West data are collected from an integral energy survey and shown in figure (5.12) and figure (5.13). Consequently we must try to map the trend of the area plot with the residential type load profile and industrial type load profile. After following the trend of the 120, 90 & 30 minute data length area calculations with the residential and industrial load profiles, 90 minutes data length area calculation is seen to follow the load profile trend, although bring the shortest one, more clearly shows the trends.
5.5.2 BRISBANE REAL DATA

On 17th October data was collected from the Brisbane phasor measurement unit. The input frequency deviation and output real power after removing trends and means of Brisbane data are shown below in figure (5.14),
The deviation of input frequency and output real power after removing trends and means (remove dc component) of Brisbane data are shown in figure (5.15). In Brisbane data the frequency signal is smaller and thus probably subject to more relative noise. Hence for analysing Brisbane data the difference of frequency and difference of real power are considered because the difference of data pushes the high frequency component and supersedes the low frequency component and also reduces the noise that is shown in figure (5.15) below. The characteristic of the load is such that the change of load is approximately equal to the white noise or the integral effect of white noise. To eliminate this integral effect, differentiations of measurement have been done.
In a similar way, the way that Sydney West data area is calculated, three different data lengths are used to calculate area. The area plot of 90 minutes, 120 minutes and 30 minutes data lengths of Brisbane data are shown in figures (5.16-5.18).

From 4am to 6am, the load is decreasing, from 6am to 7am the load profile is constant and from 8 am to 12 mid day the load is increasing. It is decreasing until 4pm in the calculated area of the 90 minute window length but in the 30 min and 120 min window lengths the increasing and decreasing of the load are quite sharp. The 90 min load profile may be of the residential type because from 12 mid day it starts decreasing until 7 pm and again it starts increasing from 7 pm.

Figure 5.15. Output real power change and input frequency change
Figure 5.16. Area calculation of 120 min length

Figure 5.17. Area calculation of 30 min length
The Queensland summer and winter load profiles have been collected from the Energex website. The winter peak of the load profile is quite similar to the 90 min area calculation of the Brisbane data.
5.6 Outcomes

This chapter shows a significant method to calculate the power of an induction motor by calculating the area under the transfer function of the frequency change as it affects the real power change and also this process can quite accurately calculate ten simulated induction motors’ area, which is presented in this chapter. 24 hrs of Sydney West and Brisbane data is used to calculate area with different data window lengths. After that the load profile is generated. The 90 min data window length load profile is quite well matched with the integral survey load profile. The calculated load profile of Brisbane data and the surveyed load profile from Energex didn’t match quite as well because Brisbane data has been collected when the system frequency was reasonably stable, which further makes the load changes due to frequency changes difficult to estimate.
Chapter 6: Extracting the Dynamic Component Using Least Squares Identification

The aim of this chapter is to show how to decompose a measurement of a composite motor load. In the power system transmission buses, load can be represented by static and dynamic load. The Induction motor is considered as a dynamic load and in the practice for major transmission buses there will be many and various induction motors contributing. Particularly in an industrial bus most of the loads are of the dynamic type. Rather than trying to extract models of many machines this thesis seeks to identify three groups of induction motors to represent the dynamic load. In this case, as a chosen compromise between resolution and accuracy we choose to simulate three groups of induction motors in MATLAB. One is the small group (less than 15kW), another one is the medium group (15kW to 180kW) and the other one is the large group (above 630kW). We evaluate composition based on these groups with different percentage contributions for each of the machine groups. The composite model is simulated in MATLAB and each group of motor percentage contributions is estimated by using least square algorithms, which is the main aim of this chapter. In commercial and residential buses, the percentage of the static load is higher than the dynamic load. To apply this
theory to other types of buses such as residential and commercials, it is good practice to represent the total load as a combination of the three composite motor loads, constant impedance load and constant power load. Additionally, to validate the theory, the 24hrs Sydney West data is decomposed into the three groups of motor models and the simulation result is shown in detail.

6.1 Theory of Dynamic load Aggregation/Composition

Many papers have been published about the aggregation/composition of induction motors and the representation of a group of motors as a single motor to facilitate the computational process. In Ref [49] aggregation is not being done for representing groups of induction machines as a single machine, rather it is done to represent explicitly the real power system load as accurately as we can, consistent with the quality of the identification. In this chapter, two methods of aggregation are described. One is method A, which uses the weighted average of the respective parameter and method B is similar to method A but for calculating the electric parameter it uses the weighted average of admittance rather than impedance. Both have been described in the literature review section in chapter 1. The aggregation outcomes of all the techniques are approximate in nature [49]. In this work, method A is being used to aggregate 10 induction motors shown in figure (6.1).

![Figure 6.1. Composite Load Model](image-url)
The total power of 10 aggregated motors is,

\[ P_{agg} = \sum_{i=1}^{10} a_i P_i \]

(6.38)

Where \( P_i \) = individual motor power and \( a_i \) = contribution of each machine, where \( n_i = \) no of motors = 10.

6.2 Theory of Dynamic Loads Decomposition

Least square identification is being used to decompose the measured composite motor response. The least square equation is,

\[ X \theta = Y \]

(6.39)

Here \( Y \) is the real magnitude of aggregated real power to frequency change transfer function. \( \theta \) is percentage contribution of each motor, and \( X \) is real magnitude of individual motor’s real power to frequency change transfer function.

Therefore if \( Y \) and \( X \) data are available it is easy to calculate the percentage contribution of each motor. However power system variation usually affects the load and load variation affects the power system. Therefore, to compare the real system with a simulated system the four types of representations of power systems are considered in this thesis,

1. Open loop power system with composite induction motor.
2. Feedback power system with composite induction motor.
3. Feedback power system with composite induction motor, constant impedance load and constant power load.

4. Feedback power system with composite induction motor, constant impedance load, constant power load and variable frequency/voltage.

The four types of representations of power systems are described below.

### 6.2.1 Open loop power system with composite induction motor

In this case the power system affects the load. To determine the time domain, the real and reactive input power of the induction motor is simulated in MATLAB code “ode45” by using a 5th order induction motor model over a time duration of 20sec which is described step by step in chapter 3. The step by step procedure to perform composition and decomposition using least square method in MATLAB is as follows;

1. Firstly identify the different percentage motor from composite induction motor response by choosing the percentage contribution of each motor. Then use equation (6.1) to determine composite power of the 10 motors in time domain by adding up the time domain real power of each motor.

2. Secondly, calculate the frequency domain transfer function of each individual motor and aggregated motor of small change of system frequency. MATLAB function TFESTIMATE is used to find out the transfer function that is described also in chapter 4.

3. Finally substitute the real magnitude value of each individual motor transfer function and composite motor in equation (6.2) and calculate the percentage contribution of each motor.
The flow chart of the process described above is shown in figure (6.2)

6.2.2 Feedback systems with composite induction motor

Load dynamics can be thought of as feedback mechanisms, which influence system behaviour. Therefore to understand how load is affecting the system, a closed loop
model is developed and implemented to decompose the load. In section 6.2.1 steps 1-3 for the open loop process consider the natural variation of system frequency. To calculate the closed loop system, the closed loop transfer function is incorporated with the open loop transfer function as illustrated in chapter 4. To do aggregation and decomposition of a closed loop system the theory, as explained in chapter 4, is used and one follows steps 1-3 in section 6.2.1.

6.2.3 Feedback Power system with composite induction motor, constant impedance load and constant power load.

One example of total aggregated power when constant impedance and constant power load are included is as follows:

$$P_{agg} = \sum_{i}^{10} \alpha_i P_i + \frac{V^2}{20} + 2100$$  \hspace{1cm} (6.40)

To represent the load at any bus, constant impedance load and constant power load are incorporated with aggregated motor load. For aggregation and decomposition steps 1-3 in section 6.2.1 are followed. Constant impedance load and constant power load will not affect the f-p output because constant impedance load is dependant on voltage and constant power load is not dependant on either frequency or voltage. The voltage is assumed constant at this stage. Therefore, the aggregated transfer function of frequency change to real power change will be the same and also the percentage contribution will be the same. This can be understood from simulation results in section 6.5 below.

6.2.4 Feedback systems with composite induction motor, constant impedance load, constant power load and variable frequency/voltage

Change in frequency causes change in slip of the induction motor, that in turn changes the voltage at the system bus [84]. Therefore, if a small change of frequency is considered in the system, a small change of voltage should be considered as well. The
closed loop system is simulated again with a composite motor and constant impedance and constant power. In this case both system frequency and voltage are changing simultaneously. The composite motor transfer function of frequency to real power change will be affected by the voltage perturbation and that will also change the percentage contribution of each motor.

The simulation section shows the step-by-step process sequentially.

6.3 Decomposition of the real data

In chapter 5, the transfer function of frequency change affecting the real power is already described. The method of processing the real data and developing the transfer function from real data is also described in that chapter. The real data is collected from substation Phasor measurement unit. In that substation, the load consists of constant impedance load, constant power load and also dynamics load. It’s already been shown that only a frequency change affects the induction type load. Therefore, frequency change as it affects the real power transfer function is the representation of the composite induction motor load.

There is no information available to understand whether the data represents residential/commercial induction motor loads or industrial/agricultural induction motor loads. Usually large motors are used in industry and small power rating motors are used in residential premises. Consequently if it is possible to decompose the composite induction motor load then by using the power rating the nature of the load type can be determined.

Section 6.2 has already explained how to decompose the composite motor if the individual motor transfer function’s real magnitude data is available since, knowledge about the individual motor is necessary. Hence the following steps are adopted to decompose the real data:
1. Select one motor from each group and determine its transfer function using equation (6.1) and use it as a template composite motor model.

2. Use the value of the real transfer function of the small, medium and large template motors as X in equation (6.2) and the value of real data as Y. Calculate the residue. If the residue is not around zero then check the frequency content of the residue. If the residue energy is substantially higher in the low frequency band than in the high frequency band, then the large motor template is not accurate. Thus we need to change the large motor parameter to fit it well. Decrease or increase the large/small/medium motor power in the composite template motor model and again follow the same procedure mentioned above to fit it with real data and calculate the residue. If the residue is around zero then follow the next steps, otherwise if there is room for another motor by plotting the real transfer function, template composite transfer function and individual template transfer function then include another motor in the template composite motor and follow the same procedure to fit it with the real data. The same motors if shifted a bit to the right or left will fit perfectly with the real data. By changing inertia the motor can be shifted in the frequency domain.

3. The motor power ratings that exist in real data are already known. Therefore, using equation (6.2) it is possible to calculate the percentage contribution of each motor group.

Extracting the dynamic motor component from real data is explained clearly in simulation section 6.5 below.
6.5 Simulation Result

6.5.1 Open Loop systems

Ten induction motors are simulated in this regards in MATLAB with different types of system. Such as,

1. Open Loop Systems

2. Closed Loop or Feedback Systems

3. Feedback system with composite induction motor, constant impedance and constant power load

4. Feedback system with composite induction motor, constant impedance, constant power load and variable voltage/frequency

Simulation is mainly done to see how well least square is able to estimate the percentage contribution of each motor with different types of systems.

Ten induction motors’ power ratings and parameters are already mentioned in chapter 4 and are simulated in MATLAB as an open loop-power system which affects the load only. Natural frequency variation is considered and voltage is fixed. To make all the motor powers equal to 630kW, the contributions of the motors in the composite motor are shown in tabular form below.

Table 6.1.10 Induction motor power rating in kW and their kW contribution in composite motor

<table>
<thead>
<tr>
<th>Power rating (kW)</th>
<th>11</th>
<th>15</th>
<th>4</th>
<th>7.5</th>
<th>18.5</th>
<th>22</th>
<th>30</th>
<th>45</th>
<th>180</th>
<th>630</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contribution (kW)</td>
<td>57</td>
<td>42</td>
<td>155</td>
<td>84</td>
<td>34</td>
<td>29</td>
<td>21</td>
<td>14</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>
The time domain power is converted to frequency domain power. The real value of magnitude of transfer function is plotted in figure (6.3). The process of generating transfer function is described in chapter 4. The composite real magnitude and individual motor real magnitude are shown in figure (6.3) using different line styles.

![Figure 6.3. Transfer function of 10 motors and aggregated motors](image)

The real magnitude value of composite one and individual motor one are substituted in equation (6.1) to estimate the percentage contribution of each motor. In this case, only 10 rad/s to 100 rad/s frequency range of real magnitude value are considered. In this range, the magnitude value looks like a simulated 5th order motor model, which is described in chapter 4.

The input are, 
\[ x_i = \text{Real (tfab (637:6370)) real (tfab2 (637:6370)) real (tfab3 (637:6370)) real (tfab4 (637:6370)) real (tfab5 (637:6370)) real (tfab6 (637:6370)) real (tfab7 (637:6370)) real (tfab8 (637:6370)) real (tfab9 (637:6370)) real (tfab10 (637:6370))}; \]

Here, tfab is the real magnitude of the transfer function of motor 1 to 10 and the output is, 
\[ y_i = \text{Real (tfabagg (637:6370))}; \]

Where, tfabagg is the composite real magnitude of the transfer function. The input and output values are substituted in equation (6.1) to estimate the contribution.
The original and calculated kW contribution of machine 1 to 10 are shown in figure (6.4).

![Figure 6.4](image)

Figure 6.4. Original and estimated percentage contribution of 10 motors

15kW, 4kW, 7.5kW, 45kW, 180kW and 630kW percentage contribution of real and estimated one is exact but for 11kW, 4kW, 18.5kW, 22kW, 30kW percentage contribution the estimation is not exact and error is around 20%.

### 6.5.2 Feedback Systems

In this case, ten induction motors are simulated in MATLAB with a feed-through power system, which is affecting the load and feedback that cause load changes, which is affecting the supply frequency. Governor response is considered as a low pass filter. Therefore, a low pass filter transfer function is inserted as a feedback transfer function with feed-through. Then the whole system is simulated as a closed loop system. Individually, ten motors and composite motor feed-through transfer function’s real magnitude values are shown in the figure (6.5) below.
In this case, the small motor, medium motor and large motor groups are fitted as closely as possible into the composite motor and also we tried to peel the percentage contribution of each group motor.

Hence, the value of input is three motors transfer function real magnitude value \( x_1 = \text{real (tfab4 (637:6370)) real (tfab22 (637:6370)) real (tfab630 (637:6370))}; \)

and the value for output is 10 composite motors transfer function real magnitude value, \( y_2 = \text{Real (tfbagg (637:6370))}; \)

This value is substituted in equation (6.2), and the kW contribution of the groups of motors are tabulated below,

Table 6.2. Actual and estimated kW contribution of small, medium and large group of motor

<table>
<thead>
<tr>
<th>Contribution(kW)</th>
<th>Small group motor</th>
<th>Medium group motor</th>
<th>Large group motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>301.0313</td>
<td>172.7778</td>
<td>0.7684</td>
</tr>
<tr>
<td>Actual</td>
<td>(57+155+84) =296</td>
<td>(42+34+29+21+14+3) =143</td>
<td>1</td>
</tr>
</tbody>
</table>

Estimation is quite similar to actual data with 1.6% error for low group, 20% error with medium group and 23% error with large group.
6.5.3 Feedback system with composite induction motor, constant impedance and constant power load

Ten induction motors with a closed loop system are simulated and combined with a constant impedance type load and constant power type load. Constant impedance load power is proportional to square of voltage and voltage is constant here. Therefore, constant impedance power is constant in this case. The transfer function of the frequency change affects the real power of ten machines and composite machines are shown in figure (6.6) below.

![Graph](image)

Figure 6.6. Transfer function of aggregated motor and ten individual motors

Similarly, the small motor, the medium motor and the large motor groups are fitted as closely as possible in the composite motor. After that we compute the kW contribution of each group motor.

Here, the value of input is three motors transfer function real magnitude value

\[
X_1 = \text{real } \left( \text{tfab4 (637:6370)} \right) \text{ real } \left( \text{tfab22 (637:6370)} \right) \text{ real } \left( \text{tfab630 (637:6370)} \right)
\]

and the value for output 10 composite motors transfer function real magnitude value,

\[
Y_2 = \text{Real } \left( \text{tfabagg (637:6370)} \right)
\]
These values are substituted into equation (6.2), the kW contribution of the groups of motor are tabulated below,

Table 6.3. Actual and estimated kW contribution of small, medium and large group of motor

<table>
<thead>
<tr>
<th>Contribution(kW)</th>
<th>Small group motor</th>
<th>Medium group motor</th>
<th>Large group motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>303.5253</td>
<td>171.3685</td>
<td>0.8385</td>
</tr>
<tr>
<td>Actual</td>
<td>(57+155+84) =296</td>
<td>(42+34+29+21+14+3) =143</td>
<td>1</td>
</tr>
</tbody>
</table>

Thus, estimation is quite similar to actual data with 2.3% error with small group motor, 19% error with medium group motor and 16% error with large group motor.

6.5.4 Feedback system with composite induction motor, constant impedance, constant power load and variable voltage/frequency

Ten induction motors are simulated with a closed loop system in MATLAB. After that ten induction motors are composite with constant impedance type load and constant power type load. Constant impedance load power is proportional to the square of voltage and voltage is not constant here. Therefore constant impedance power is variable in this case. The transfer function of the frequency change affects the real power change of all ten individual machines and the composite machine as shown in figure (6.7)
In the similar way, the small motor, medium motor and large motor group are fitted as closely as possible into the composite motor and we also tried to decompose the kW contribution of each groups of motors.

The value of input is three motors transfer function real magnitude value
\[ x_1 = \text{real}(\text{tfab4 (637:6370)}) \text{ real}(\text{tfab22 (637:6370)}) \text{ real}(\text{tfab630 (637:6370)})]; \]
and the value for output 10 composite motors transfer function real magnitude value,
\[ y_2 = \text{Real}(\text{tfabagg (637:6370)}); \]
Put this value in equation (6.2), the kW contribution of the groups of motors are tabulated below,

Table 6.4. Actual and estimated kW contribution of the small, the medium and the large groups of motor

<table>
<thead>
<tr>
<th>Contribution(kW)</th>
<th>Small group motor</th>
<th>Medium group motor</th>
<th>Large group motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>324.5159</td>
<td>162.8380</td>
<td>0.8834</td>
</tr>
<tr>
<td>Actual</td>
<td>(57+155+84)</td>
<td>(42+34+29+21+14+3)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>=296</td>
<td>=143</td>
<td></td>
</tr>
</tbody>
</table>
Since the estimation is quite similar to actual data with a 9% error of small group motors, 13% error with medium group motors and 11% error with the large group of motors.

### 6.6 Real Data

How to process real data and develop the transfer function of real data has been described in chapter 5. This chapter starts with the transfer function of frequency change to the real power change of real data and tries to decompose it based on the ideas already proposed in this chapter in section 6.3.

At first, we tried to fit two motors from small and medium groups into this real data in figure (6.8). But it seems there is still some room to fit another motor. Therefore we tried to fit a third motor from the large groups of motors into the real data, which is shown in figure (6.9). After that, the real value of the transfer function of real data is compared with the real value of the transfer function of the composite model of 15kW motors, 180kW motors and 630kW motors. To fit the composite one to the real one, the power of each motor is decreased in figure (6.10). In figure (6.11) the inertia is changed of the 630kW and 15kW machines is changed to match the composite one with the real one.

It is clearly shown in figure (6.12) that if inertia is increasing, motor real magnitude of transfer function is moving from right to left and vice versa in figure (6.13). Therefore in figures (6.14-6.15) the process continues to fit the composite model to real data by changing the inertia and power of each motor. Whenever the composite one is exactly matched with the real data, the three motors’ real magnitude values are considered as a template motor value and these three motor values can be used as the input of the least square input X and the value of the transfer function of real data as an output Y. The values of X and Y are substituted in equation (6.2) and the percentage contribution of each machine (180kW, 15kW and 630kW) is estimated from real data.

The same three template motors modelled data is used to estimate a composite motor model which, exist in 24 hours of Sydney west data. In chapter 5, it is shown that
optimum window length lies between 30 to 120 minutes. Therefore the first 24hrs of data is divided into 30 minute window lengths and the transfer function is developed as described in chapter.5 for each 30 minutes window length. Then the contribution of each template motor is estimated by using the procedure that has been described above in section 6.6. In the similar way next 30 minutes window length data is used to estimate a transfer function and decomposed according to section 6.6 with same template motor and so on. To find out the optimum window length again divided the same 24hrs data is divided into 90 minute window lengths and estimate the percentage contribution of each of the template motors is estimated. In a similar way, the 24hrs data is divided again into 120 minutes window lengths and the percentage contribution is estimated. Here, different window lengths are used to estimate the percentage contribution of each template motor. The reason behind using different window lengths is to estimate the variation of the contribution of motor, which is similar to the real load variation of a feeder. The percentage contributions of the template motor in 24hrs with three different window data lengths of 30 minutes, 90 minutes and 120 minutes are shown in figures (6.16-6.18).
Figure 6.8. Transfer function

Figure 6.9. Transfer function with additional motor inserted

Figure 6.10. Transfer function after decreased power

Figure 6.11. Matching composite transfer function with real data

Figure 6.12. Transfer function after inertia changed

Figure 6.13. Transfer function of composite value and real value

Figure 6.14. Transfer function matching process

Figure 6.15. Exactly matched composite transfer function to real data
The variation of a 15kW motor is quite substantial in the 120 minutes data window length which infers that small types of appliances are switching on and off quite often in 24hrs time but the large motor 630kW is switched on from approx 8am ‘til 6pm and then it starts to be switched off. Most of the large type motors are used in industry and it is to be expected that the big motors are turned on during daytime and turned off in the evening.

![Graph showing different motor contribution in 24hrs data, 120 min window](image)

Figure 6.16. 630kW, 15kW and 180kW motors percentage contribution in 24hrs a day

The variation is quite large for all types of motors in the 30 minutes window length, which does not match with any real load variation phenomenon, and is shown in figure (6.17)

![Graph showing different motor contribution in 24hrs data, 30 min data length](image)

Figure 6.17. 630kW, 15kW and 180kW motors percentage contribution in 24hrs a day
In the 90 minutes window length the variation is quite similar to the 120 minutes window length. But the 15kW motor variation is different from the 120 min window length shown in figure (6.18)

![Figure 6.18. 630kW, 15kW and 180kW motors percentage contribution in 24hrs a day](image)

In this respect the 120 minutes data length or 90 minutes data length gives a result, which is quite similar to a real world load changing phenomenon. Instead of using the real value of the transfer function, the complex value is used to calculate the percentage contribution and the similar result is obtained. For this reason, the complex value magnitude of the transfer function is not considered here.

### 6.7 Outcomes

In this thesis, two-different approaches are taken to infer motor components from composite loads. The first one is an area calculation under the curve and another one is calculated percentage contribution of each types of motor those are aggregated in the composite motor. The first approach is described in chapter.5 and the accuracy of area calculation is based on manually fitting .There is another drawback of the area calculation process that is by using that process we can’t tell what type of motor are consists in the composite one. In the second aspect least square theory has been used to estimate the percentage contribution from a composite load model. There are open loop, closed loop power systems with composite induction motor loads, constant power loads
and constant impedance loads which are deployed in simulation section of this chapter to determine whether or not least square can estimate the percentage contribution or not. Simulation section shows the ten induction motors simulation result and composite result and using the simulated and composited magnitude to calculate the percentage contribution of individual small groups, medium groups and large groups shows in this chapter. Accordingly least square theory has been applied in real data to estimate the percentage contribution of each type of motor. It describes, how to decompose Sydney west data by using three groups of template motors. After that using those template motor, 24hrs of Sydney west data is decomposed according to three different window length. Hence this chapter has successfully shown that from real measurement it is possible to identify the composite motor model. Additionally from composite motor models it is also possible to estimate the composition of different types of induction motor models and using 90 minutes or 120 minutes window length of 24hrs Sydney West real data it is possible to identify the real load change phenomena.
Chapter 7: Conclusion

This thesis demonstrates the feasibility of the extraction of dynamic load models from the normal operational data of a power system. One critical aspect is that the loads must almost always be modelled as feedback systems. From the frequency to power relations, the load dynamics can be extracted and motor load portions inferred. A key issue that makes the separation into feed-forward and feedback components is that there is a low pass characteristic in this relation.

The process of load modelling of identification from normal system variations in this thesis offers the promise of separation of motor load components from total load and the identification of the overall motor parameters.

7.1 Summary of the results

Summary of each chapter of this thesis is given below,
1. Derive the algebraic transfer function of an induction motor and identify the model

The algebraic transfer functions of frequency changes to changes in the real reactive power as well as the transfer function of voltage changes causing real and reactive power changes in an induction motor with linear shaft load are derived using the motion equation. An induction motor is simulated using a 5th order model in MATLAB without consideration of flux saturation or losses. In the next step, the 5th order model is reduced to a 3rd order model by ignoring the stator flux changes and then to a 1st order model by ignoring rotor flux changes. The 1st order simulated model is exactly mapped to the algebraic equation of the transfer function. We can say that the way simulation is done is correct and the algebraic equation is sufficient to explain the induction motor. In this case, the model is identified by using MATLAB function “LINMOD” but this approach is not suitable for identification using real data. For this reason, cross-correlation identification has been implemented to identify the model in the next chapter.

2. Identify the motor model and parameters by using system identification

To identify the induction motor model, cross-correlation identification is used in closed loop identification. A new and simple technique has been applied to estimate the dynamic parameter values of an induction motor using the f-P Bode plot.

The cross-correlation identification, which has been applied and implemented to closed loop identification, is theoretically exact under assumptions that there is no correlation between w1, w2 and f(t). However, the correlations are not totally zero. Any correlation is assumed to be sufficiently small in comparison with the desired component so that their influence will be negligible. Also, the signal sequence length is important for precision correlation analysis. So it is important to choose a reasonably good length of signal. In this problem, the signal length is 10000 which provides a better estimate of the cross correlation Rw1p. The process divides the 10000-sample sequences into 8 windows, averaging them into 512 samples. The cross–correlation identification is
applicable for discrete modelling of continuous time systems when the upper limit of their working frequency band is lower than the Nyquist frequency of the sampling that has been used. Therefore, the sampling rate is an important parameter to achieve satisfactory results. A 0.2 sec sampling rate has been used in this thesis. It is found that the estimation of a system under feedback with multiple noise sources can give an erroneous answer if the structure of the system is not carefully observed. This report shows one method of processing the data such that the separate components of the model can be extracted. The process was shown to yield reasonable results for the two components of a system on load and the load on the system for the particular case of the induction motor load. For frequencies where the signal level is poor, there is still a fundamental limit in that the quality of estimation is reduced.

Estimating motor parameters from a bode plot is quite an accurate and easy method proved by estimated parameters of a real motor in the QUT lab. This method is successful to show that if power system operation data is available, using this data, it is easy to obtain the f-P Bode plot and from this Bode plot, it is easy to calculate the motor parameter values. Therefore, it is possible to identify the induction motor load in a power system.

3. Extracting dynamic motor component by using area calculation

One aspect of extracting the motor component from a composite measurement of a load is to use an area calculation. The proposed idea is that the real power to frequency change transfer function of ten induction motors has been observed and all of the zeros of the transfer function f-P of the motors are quite far from the poles so that the area under the transfer function is related to the power of the induction motor. For establishing the idea, ten induction motor areas are calculated and the calculated per unit area should be 1 on its own base. If it is not 1, the scaling factor is calculated to make the per unit power of each motor around 1. After that, extrapolation is done to extend the frequency point over the range 90 rad/s to 160 rad/s. Consequently, if the power is still not around 1 manual fixing is done to get the power around 1. Using Brisbane and
Sydney West real data, the proposed theory is validated. This data is collected from a feeder phasor measurement unit and a load curve is generated. The load curve of Sydney West data is quite similar to the load curve generated by an Integral Energy survey. Though this method calculates the power well enough with an error of 6-13% another aspect has been proposed to extract the dynamic component that doesn’t require manual fixing, and the calculation process is straightforward.

4. Extracting the dynamic component by using least square identification

Another aspect of extracting the dynamic component is to get the composite f-P transfer function real/complex magnitude value and use the template motors' real/complex magnitude f-P value. From a selected set of templates it is possible to extract the motor percentage contribution to the composite load by using least square identification. To validate the proposed idea, ten induction motors are combined at first with a different percentage contribution of each motor and then each motor percentage contribution is extracted by using least square identification. Estimated and actual percentage contributions are not accurate, with a 20% error. Consequently the motor component is extracted from the feedback system and from a composite motor load with a static load. The simulation result is presented in ch4. The estimation is more exact with a 9-13% error. Whenever the system is exactly like a real system, estimation is better. Real Sydney West data is collected from a feeder phasor measurement unit. Before estimating and extracting the motor component, we use a filter process to estimate the model by using the closed loop system identification theory already mentioned in chapter 4. The motor component is extracted by using least square identification. Additionally, the extracted motor component is plotted against a 24hr time axis with a different data window length. The load curve of extracted motors is closely related to the expected real world variation in motor load over a 24 hr period. But this method could not decompose Brisbane data. Because the data collected from Sydney may be mainly from an industrial area which is rich in induction type motors and Brisbane data may be collected at light load times, it may not be strongly influenced by motor loads.
7.2 Potential limitation

One potential limitation in this thesis is that a composite load is considered at a single load bus, with the frequency and voltage having minimal perturbation occurring at that bus. An actual system load is not concentrated in a load bus but rather in a few buses in a radial system, and the perturbation is happening at the measured bus. A small perturbation of frequency at the measured bus doesn’t really matter for considering the composite load in a bus or in distributed bus. With small variations at the measured bus we can form a linear transfer function and use the load modelling approach via transfer functions. If there is large voltage excursions the response may be nonlinear and linear transfer functions cannot be reliably fitted. In particular, magnetic saturation may be a concern.

In this thesis it is assumed that V and f are totally uncorrelated. So we can measure the f-P transfer function without considering the f-V relation triggering the v-P effect. Preliminary measurements of the f-V relations have shown it to be of low magnitude but the results may be able to be improved by a more detailed investigation.

Closed loop identification is applied to the load-modelling task and in this application the unpredictable input voltage/frequency changes and unpredictable output real/reactive power changes are considered as an additive white noise. So this load modelling approach is only applicable when there are mechanisms providing variations in the source V/f as well as having customer load switching changes.

The quality of identification is dependant on the degree of frequency variation visible in the system. When there is a low level of frequency variations in a particular frequency band, the quality of the load identification will be affected. When a particular frequency band is rich in electromechanical oscillation modes, the quality of motor load identification is good.

In this thesis frequency change to real power change is largely insensitive to motor impedance. This is a good characteristic when trying to extract a motor component from
a composite load, but for knowing the motor fully, the knowledge of impedance is essential because the stalling of the motor depends significantly on the value of impedance. Hence to understand induction motor characteristic completely, the value of impedance may be able to be determined from frequency change to reactive power change. This aspect has not been examined in this thesis.

7.3 Future work

The extraction of the separate motor components from a composite load measurement is done offline in this thesis. Variations in the load components can be tracked online by using recursive least square estimation. To get optimal filter estimates in this coloured noise, the least square method can be replaced by a maximum likelihood methods or instrumental variables.

The induction motor model has been identified in this thesis by the cross-correlation method. Instead of a correlation method the system can be identified by artificial neural network (ANN) if the nonlinearities were thought to be a dominant error.

The aluminium smelter load is extremely big considering the load, which already is in the local power system and can easily cause instability. This thesis has developed the idea that the composite load consists of an aggregated motor load and a static load. In the future the motor load component of an aluminium smelter may be able to be associated with an aggregated motor load and static load.

Hence the same procedure can be applied to extract the motor load and a new technique needs to be invented to extract the aluminium smelter and static loads, because in this thesis only the motor component extraction has been proposed.

Small perturbation as well as large perturbations should be considered in future to extract the composite load model for an aluminium smelter. The method that has already been developed in this thesis can be used in large perturbation if losses and flux
saturation are incorporated. Therefore, the model that is used in this thesis needs to be modified for applying to large perturbation. One of the dominant factors appears to be that in large disturbances a certain fraction of motors can stall and another fraction of the load will trip. Future works will emphasise methods to identify these fractions.

Not all induction motors used in industry are direct feed. These motors are not considered in this thesis. Motors with electronic drives are expected to be independent of small V and F changes and hence are more in the constant P category.

Distribution generation (DG) is a growing form of energy supply. So how DG influences the composite load model should be considered in future. If DG is synchronous, then machine type could be a problem, in which case further investigation is essential. However, if DG is induction generator type it is possible to explain the induction generation influence in relation to the induction motor. If DG is PV/fuel cell and inverter based then behaviour is constant P type generator and can easily explain its influence on a composite load model.
References


[22] Thiringer, Torbjorn, "Modelling of the induction machine for supply voltage disturbances."


[44] A.Hiskens, Ian, "Significance of load modeling in power system dynamic."


[87] Matlab, "Neural Network Toolbox."
Appendix A

Transfer function of frequency change affects the real power change:

Slip equation of an induction motor is,

\[ s_m = \frac{\omega_v - \omega_r}{\omega_b} \quad (A-1) \]

And power of an induction motor is,

\[ P_c = \frac{v_{qs} s_m}{r_r} \quad (A-2) \]

Dynamic motion equation is,

\[ 2H \frac{d\omega_r}{dt} = P_e - B\omega_r \quad (A-3) \]

The equation (A-1) and equation (A-2) are substituted in equation (A-3),

\[ 2HS\omega_r = \frac{2v_{qs}}{r_r} \left( \frac{\omega_v - \omega_r}{\omega_b} \right) - B\omega_r \quad (A-4) \]

After manipulating equation (A-4) the rotor speed is,

\[ \omega_r = \frac{v_{qs}^2 \omega_v}{r_r \omega_b (2HS + B + \frac{v_{qs}^2}{r_r \omega_b})} \quad (A-5) \]
Again the equation (A-1) is substituted in equation (A-2) and the real power is,

$$P_e = \frac{v_{qs}}{r_r} \frac{\omega_v - \omega_r}{\omega_b} \quad (A-6)$$

After manipulating, equation (A-6) becomes,

$$P_e = \frac{v_{qs}^2 \omega_v}{r_r \omega_b} - \frac{v_{qs}^2 \omega_r}{r_r \omega_b} \quad (A-7)$$

Now equation (A-5) is substituted in equation (A-7), and the real power is,

$$P_e = \frac{v_{qs}^2 \omega_v}{r_r \omega_b} - \frac{v_{qs}^2 \omega_v}{r_r \omega_b} - \frac{v_{qs}^2 \omega_v}{r_r \omega_b} \quad (A-8)$$

Taking the common term out from equation (A-8), the power is,

$$P_e = \frac{v_{qs}^2 \omega_v}{r_r \omega_b} (1 - \frac{v_{qs}^2}{r_r \omega_b (2HS + B + \frac{v_{qs}^2}{r_r \omega_b})}) \quad (A-9)$$

After organizing the denominator the power is,

$$P_e = \frac{v_{qs}^2 \omega_v}{r_r \omega_b} (1 - \frac{v_{qs}^2}{2HSr_r \omega_b + Br_r \omega_b + v_{qs}^2 \omega_b}) \quad (A-10)$$

After manipulating equation (A-10),
\[ P_e = \frac{v_{q_k} \omega_v}{r_r \omega_b} \left( \frac{2HS_r \omega_b + Br_r \omega_b}{2HS_r \omega_b + Br_r \omega_b + v_{q_k}^2} \right) \]  
(A-11)

\[ P_e = \frac{2 v_{q_k} \omega_v (2H + B)}{(2HS_r \omega_b + Br_r \omega_b + v_{q_k}^2)} \]  
(A-12)

\[ P_e = \frac{v_{q_k}^2 \omega_v 2H (S + \frac{B}{2H})}{2H r_r \omega_b (S + \frac{B}{2H} + \frac{v_{q_k}^2}{2H r_r \omega_b})} \]  
(A-13)

\[ P_e = \frac{v_{q_k}^2 \omega_v (S + \frac{B}{2H})}{r_r \omega_b (S + \frac{B}{2H} + \frac{v_{q_k}^2}{2H r_r \omega_b})} \]  
(A-14)

Hence, the transfer function of frequency changes affect on the real power change is,

\[ P_e = \frac{v_{q_k}^2 (S + \frac{B}{2H})}{r_r \omega_b (S + \frac{B}{2H} + \frac{v_{q_k}^2}{2H r_r \omega_b})} \]  
(A-15)

If we assume \( \frac{B}{2H} \) is close to 0 then the transfer function is,
\[
\frac{P_e}{\omega_v} = \frac{v_{qs}^2 S}{r_r \omega_b (S + \frac{v_{qs}^2}{2Hr_r \omega_b})}
\]  \quad (A-16)

And high frequency gain is,

\[
k_{pf} = \frac{v_{qs}}{r_r \omega_b}
\]  \quad (A-17)

**Transfer function of frequency change affects the reactive power change:**

An induction motors’ reactive power equation is,

\[
Q = \frac{v_{qs}^2 s m X}{r_r^2}
\]  \quad (A-18)

Small reactive power change from steady state value is,

\[
Q_{e0} + \Delta Q_e = \frac{V_{qs0}^2 (s_{l0} + \Delta s_f) X}{r_r^2}
\]  \quad (A-19)

Expanding equation (A-19),
\[ Q_{e0} + \Delta Q_e = \frac{V^2 q_{s0} s^2_{l0} + 2s_{l0} \Delta s_l + \Delta s_l^2}{r_r^2} X \]  \hspace{1cm} (A-20)

Ignoring the highest variable term,

\[ Q_{e0} + \Delta Q_e = \frac{V^2 q_{s0} s^2_{l0} + 2s_{l0} \Delta s_l}{r_r^2} X \]  \hspace{1cm} (A-21)

Consider only the variable term,

\[ \Delta Q_e = \frac{V^2 q_{s0} 2s_{l0} \Delta s_l}{r_r^2} X \]  \hspace{1cm} (A-22)

Equation (A-1) is substituted in Equation (A-22),

\[ \Delta Q_e = \frac{V^2 q_{s0} 2s_{l0} \frac{\omega_v - \Delta \omega_r}{\omega_b}}{r_r^2} X \]  \hspace{1cm} (A-23)

Expanding equation (A-23),

\[ \Delta Q_e = \frac{V^2 q_{s0} 2s_{l0} \frac{\omega_v}{\omega_b} - X}{r_r^2} - \frac{V^2 q_{s0} 2s_{l0} \frac{\Delta \omega_r}{\omega_b} X}{r_r^2} \]  \hspace{1cm} (A-24)

Equation (A-16) is substituted in equation (A-24),
Expanding equation (A-26),

$$\Delta Q_e = \frac{2s l_0 v_{qs0}^2 X}{r_r^2 \omega_b} \left( \frac{2HSr_r \omega_b + r_r \omega_b B + v_{qs}^2 - v_{qs}^2}{r_r^2 \omega_b \left( 2HS + B + v_{qs}^2 \frac{r_r \omega_b}{r_r \omega_b} \right)} \right) \omega_v$$  \hspace{1cm} (A-27)

Assume $\frac{B}{2H}$ is close to 0,
\[
\Delta Q_e = \frac{2s_{10}V_{qs0}^2 X}{r_f \omega_b} \left( \frac{S}{r_f^2 \omega_b} \right) \omega_r 
\]

(A-30)

Therefore,

\[
\frac{\Delta Q_e}{\omega_r} = \frac{2s_{10}V_{qs0}^2 X}{r_f^2 \omega_b} \frac{S}{r_f^2 \omega_b} \left( S + \frac{v_{qs}^2}{2Hr_f \omega_b} \right) 
\]

(A-31)

High frequency gain is,

\[
k_{qf} = \frac{2s_{10}V_{qs0}^2}{r_f \omega_b} 
\]

(A-32)

At high frequency, the ratio of real power and reactive power gain is,

\[
\frac{k_p}{k_q} = \frac{r_f}{2s_{10}X} 
\]

(A-33)

**Transfer function of voltage change affects the real power change:**

For varying voltage,

Slip is varying,

\[
s_m = s_{l0} + \Delta s_l 
\]

(A-34)

Rotor speed is varying,
\[ \omega_r = \omega_{r0} + \Delta \omega_r \]  

(A-35)

Real power is varying,

\[ P_e = P_{e0} + \Delta P_e \]  

(A-36)

And voltage is varying,

\[ V_{qs} = V_{qs0} + \Delta V_{qs} \]  

(A-37)

All these values are substituted in equation (A-3),

\[ 2HS(\omega_{r0} + \Delta \omega_r) = \frac{(V_{qs0} + \Delta V_{qs})^2}{r_r} (s_{l0} + \Delta s_l) - B(\omega_{r0} + \Delta \omega_r) \]  

(A-38)

Expanding equation (A-38),

\[ (2HS + B)(\omega_{r0} + \Delta \omega_r) = \frac{(V_{qs0}^2 + 2V_{qs0}\Delta V_{qs} + \Delta V_{qs}^2)(s_{l0} + \Delta s_l)}{r_r} \]  

(A-39)

Expanding equation (A-39),

\[ 2HS(\omega_{r0} + \Delta \omega_r) = \frac{(V_{qs0}^2 + \Delta V_{qs})^2}{r_r} (s_{l0} + \Delta s_l) - B(\omega_{r0} + \Delta \omega_r) \]  

(A-40)

Expanding equation (A-40),
\[(2HS + B)(\omega_{r0} + \Delta \omega_r) = \frac{(V_{q0}^2 + 2V_{q0} \Delta V_{q} + \Delta V_{q}^2)(s_{r0} + \Delta s_r)}{r_r}\]

\[
= \frac{V_{q0}^2 s_{r0}}{r_r} + \frac{2V_{q0} \Delta V_{q} s_{r0}}{r_r} + \frac{\Delta V_{q}^2 s_{r0}}{r_r} + \frac{2V_{q0} \Delta V_{q} \Delta s_{r0}}{r_r} + \frac{\Delta V_{q}^2 \Delta s_{r0}}{r_r}
\]

Ignore the highest term of \(\Delta V\) and \(\Delta V \Delta s_{r0} = 0\)

\[
= \frac{V_{q0}^2 s_{r0}}{r_r} + \frac{2V_{q0} \Delta V_{q} s_{r0}}{r_r} + \frac{\Delta V_{q}^2 \Delta s_{r0}}{r_r}
\]

\[(A-41)\]

The value of equation (A-1) is substituted in equation (A-42),

\[
\frac{V_{q0}^2 s_{r0}}{r_r} + \frac{2V_{q0} \Delta V_{q} s_{r0}}{r_r} + \frac{\Delta V_{q}^2 (1 - \frac{\omega_{r0}}{\omega_{b}} - \frac{\Delta \omega_r}{s_{r0}})}{r_r}
\]

\[(A-42)\]

Expanding equation (A-43),

\[
\frac{V_{q0}^2 s_{r0}}{r_r} + \frac{2V_{q0} \Delta V_{q} s_{r0}}{r_r} + \frac{V_{q0}^2}{r_r} \frac{\omega_{r0} + \Delta \omega_r}{r_r} - \frac{V_{q0}^2}{r_r} \frac{1}{r_r} \frac{\omega_{r0}}{\omega_{b}} - \frac{V_{q0}^2 s_{r0}}{r_r}
\]

\[(A-44)\]

Therefore, equation (A-44) becomes,

\[
\frac{V_{q0}^2}{r_r} \frac{\omega_{r0} + \Delta \omega_r}{r_r} = \frac{2V_{q0} \Delta V_{q} s_{r0}}{r_r} + \frac{V_{q0}^2}{r_r}
\]

\[(A-45)\]

Consider only the changing term,
\[(2HS + B + \frac{V_{q0}^2}{r_r \omega_b})\Delta \omega_r = \frac{2V_{q0} \Delta V_{q0} \cdot S_{s0}/r_r}{r_r} \quad (A-46)\]

Hence the transfer function of voltage change, change the speed is,

\[\frac{\Delta \omega_r}{\Delta V_{q0}} = \frac{2V_{q0} \cdot S_{s0}/r_r \omega_b}{2Hr_r \omega_b S + r_r \omega_b B + V_{q0}^2} \quad (A-47)\]

We know power is,

\[P_{q0} + \Delta P_c = \frac{(V_{q0} + \Delta V_{q0})^2}{r_r} \cdot (S_{s0} + \Delta S_f) \quad (A-48)\]

Expanding the above equation,

\[= \frac{V_{q0}^2 \cdot S_{s0}/r_r}{r_r} + \frac{2V_{q0} \cdot \Delta V_{q0} \cdot S_{s0}/r_r}{r_r} + \frac{V_{q0}^2 \cdot \Delta S_f}{r_r} \quad (A-49)\]

Equation (A-1) is substituted in equation (A-46),

\[= \frac{V_{q0}^2 \cdot S_{s0}/r_r}{r_r} + \frac{2V_{q0} \cdot \Delta V_{q0} \cdot S_{s0}/r_r}{r_r} + \frac{V_{q0}^2 \cdot \Delta S_f}{r_r} \cdot (1 - \frac{\omega_{s0} + \Delta \omega_r}{\omega_b} - S_{s0}) \quad (A-50)\]

Expanding above equation,

\[= \frac{V_{q0}^2 \cdot S_{s0}/r_r}{r_r} + \frac{2V_{q0} \cdot \Delta V_{q0} \cdot S_{s0}/r_r}{r_r} + \frac{V_{q0}^2 \cdot \Delta S_f}{r_r} \cdot \frac{\omega_{s0} + \Delta \omega_r}{\omega_b} - \frac{V_{q0}^2 \cdot S_{s0}/r_r}{r_r} \quad (A-51)\]
\[
\frac{2V_{q_0} \Delta V_{q_0} S_{f_0}}{r_r} + \frac{V_{q_0}^2 \omega_{q_0}}{r_r \omega_b} - \frac{V_{q_0}^2 \Delta \omega_t}{r_r \omega_b} = \frac{V_{q_0} S_{f_0}}{r_r} \frac{2V_{q_0} \omega_b \Delta V_{q_0}}{2H r_b \omega_b S + r_r \omega_b B + V_{q_0}^2} \quad \text{(A-52)}
\]

Equation (A-44) is substituted in equation (A-49),

\[
\frac{2V_{q_0} \Delta V_{q_0} S_{f_0}}{r_r} + \frac{V_{q_0}^2 \omega_{q_0}}{r_r \omega_b} - \frac{V_{q_0}^2 \omega_b^2}{2H r_b \omega_b S + r_r \omega_b B + V_{q_0}^2} = \frac{2V_{q_0} S_{f_0} \omega_b \Delta V_{q_0}}{2H r_b \omega_b S + r_r \omega_b B + V_{q_0}^2} \quad \text{(A-53)}
\]

After expanding and manipulating equation (A-53),

\[
\frac{2V_{q_0} \Delta V_{q_0} S_{f_0}}{r_r} + \frac{V_{q_0}^2 \omega_{q_0}}{r_r \omega_b} - \frac{V_{q_0}^3 \omega_b^2}{2H r_b \omega_b S + r_r \omega_b B + V_{q_0}^2} = \frac{2s_{f_0} \Delta V_{q_0}}{2H r_b \omega_b S + r_r \omega_b B + V_{q_0}^2} \quad \text{(A-54)}
\]

Consider the variable term only if the variable power is,

\[
\Delta P_r = \frac{2V_{q_0} \Delta V_{q_0} S_{f_0}}{r_r} - \frac{V_{q_0}^3}{r_r} \frac{2s_{f_0} \Delta V_{q_0}}{2H r_b \omega_b S + r_r \omega_b B + V_{q_0}^2} \quad \text{(A-55)}
\]

\[
= \frac{2V_{q_0} S_{f_0}}{r_r} \left(1 - \frac{V_{q_0}^2}{2H r_b \omega_b S + r_r \omega_b B + V_{q_0}^2}\right) \Delta V_{q_0} \quad \text{(A-56)}
\]

\[
= \frac{2V_{q_0} S_{f_0}}{r_r} \left(\frac{2H r_b \omega_b S + r_r \omega_b B + V_{q_0}^2 - V_{q_0}^2}{2H r_b \omega_b S + r_r \omega_b B + V_{q_0}^2}\right) \Delta V_{q_0} \quad \text{(A-57)}
\]

Therefore the transfer function of voltage change’s affect on the real power change is,
\[
\frac{\Delta P_{e}}{\Delta V_{qf}} = \frac{2V_{qf0} s_{10}(S + \frac{B}{2H})}{r_e(S + \frac{B}{2H} + \frac{V_{qf}^2}{2Hr_c\omega_b})}
\]  
(A-58)

Assume \( \frac{B}{2H} \) is close to 0,

The transfer function is,

\[
\frac{\Delta P_{e}}{\Delta V_{qf}} = \frac{2V_{qf0} s_{10}(S)}{r_e(S + \frac{V_{qf}^2}{2Hr_c\omega_b})}
\]  
(A-59)

High frequency gain is,

\[ k_{pv} = \frac{2V_{qf0} S_{10}}{r_e} \]  
(A-60)

Transfer function of voltage change affects the real power change:

Now the equations (A-34-A-37) are substituted in equation (A-18),

\[
Q_{e0} + \Delta Q_e = \frac{X}{r_e} \left( V_{qf0} + \Delta V_{qf} \right)^2 (s_{10} + \Delta s_f)^2
\]  
(A-61)

Expanding the equation, it becomes,

\[
Q_{e0} + \Delta Q_e = \frac{X}{r_e} \left( V_{qf0}^2 + 2V_{qf0} \Delta V_{qf} (s_{10}^2 + 2s_{10} \Delta s_f) + \Delta V_{qf}^2 \right)
\]  
(A-62)
\[ Q_{e0} + \Delta Q_e = \]
\[ \frac{X}{r_r^2} \left( V_{q0}^2 s_{l0}^2 + 2V_{q0} \Delta V_{qs} s_{l0}^2 + 2s_{l0} \Delta s_l V_{q0}^2 + 4V_{q0} \Delta V_{qs} s_{l0} \Delta s_l \right) \]  
\[ (A-63) \]

\[ = \frac{X}{r_r^2} \left( V_{q0}^2 s_{l0}^2 + 2V_{q0} \Delta V_{qs} s_{l0}^2 + 2s_{l0} \Delta s_l V_{q0}^2 \right) \]  
\[ (A-64) \]

\[ = \frac{X}{r_r^2} V_{q0}^2 s_{l0}^2 + \frac{X}{r_r^2} 2V_{q0} \Delta V_{qs} s_{l0}^2 + \frac{X}{r_r^2} 2s_{l0} \Delta s_l V_{q0}^2 \]  
\[ (A-65) \]

Equation (A-10) is substituted in equation (A-65),

\[ = \frac{X}{r_r^2} V_{q0}^2 s_{l0}^2 + \frac{X}{r_r^2} 2V_{q0} \Delta V_{qs} s_{l0}^2 + \frac{X}{r_r^2} 2s_{l0} \Delta s_l V_{q0}^2 + \frac{X}{r_r^2} 2s_{l0} (1 - \frac{\omega_{e0} + \Delta \omega_e}{\omega_b} - s_{l0}) V_{q0}^2 \]  
\[ (A-66) \]

Expanding the above equation,

\[ Q_{e0} + \Delta Q_e = \]
\[ \frac{X}{r_r^2} V_{q0}^2 s_{l0}^2 + \frac{X}{r_r^2} 2V_{q0} \Delta V_{qs} s_{l0}^2 + \frac{X}{r_r^2} 2s_{l0} V_{q0}^2 - \frac{X}{r_r^2} 2s_{l0} V_{q0}^2 \frac{\omega_{e0} + \Delta \omega_e}{\omega_b} \]  
\[ (A-67) \]

Expanding the above equation,
\[
\frac{X}{r_r^2} V_{q_0}^2 s_{1_0}^2 + \frac{X}{r_r^2} V_{q_0}^2 \omega_{q_0}^2 + \frac{X}{r_r^2} \omega_{q_0}^2 + \frac{X}{r_r^2} \omega_{q_0}^2 \omega_{q_0}^2 + \frac{X}{r_r^2} \omega_{q_0}^2 \omega_{q_0}^2 - \frac{X}{r_r^2} \omega_{q_0}^2 \omega_{q_0}^2 \Delta \omega
\]

(A-68)

\[
= \frac{X}{r_r^2} 2 \omega_{q_0} V_{q_0}^2
\]

(A-69)

Therefore only considering the variable term,

\[
\Delta Q_r = (\frac{X}{r_r^2} 2 V_{q_0} s_{1_0}^2 - \frac{X}{r_r^2} 4 V_{q_0}^3 s_{1_0}^2) \Delta V_{q_s}
\]

(A-70)

\[
= \frac{X}{r_r^2} 2 V_{q_0} s_{1_0}^2 (1 - \frac{2 V_{q_0}^2}{(2 H \omega_{b} S + r \omega_{b} B + V_{q_0}^2)^2}) \Delta V_{q_s}
\]

(A-71)

\[
= \frac{X}{r_r^2} 2 V_{q_0} s_{1_0}^2 \left( \frac{2 H \omega_{b} S + r \omega_{b} B + V_{q_0}^2}{2 H \omega_{b} S + r \omega_{b} B + V_{q_0}^2} \right)^2 \Delta V_{q_s}
\]

(A-72)

Hence the transfer function of reactive power change by changing the supply voltage is,

\[
\frac{\Delta Q_r}{\Delta V_{q_s}} = \frac{X}{r_r^2} 2 V_{q_0} s_{1_0}^2 \left( \frac{S + \frac{r \omega_{b} B - V_{q_0}^2}{2 H \omega_{b}}}{S + \frac{r \omega_{b} B + V_{q_0}^2}{2 H \omega_{b}}} \right) = \frac{X}{r_r^2} 2 V_{q_0} s_{1_0}^2 \left( \frac{S + \frac{B}{2 H} - \frac{V_{q_0}^2}{2 H \omega_{b}}}{S + \frac{B}{2 H} + \frac{V_{q_0}^2}{2 H \omega_{b}}} \right)
\]

(A-73)
Assume $\frac{B}{2H}$ is close to 0,

\[
\frac{\Delta Q_e}{\Delta V_{qs}} = \frac{X}{r_r^2} 2V_{s0} S \left( S - \frac{V_{q0}^2}{2Hr_0\omega_b} \right) \left( S + \frac{V_{q0}^2}{2Hr_0\omega_b} \right)
\]  

And high frequency gain is,

\[
k_{qv} = \frac{X}{r_r^2} 2V_{q0}s_{s0}^2
\]

Therefore, at high frequency the ratio between the real and the reactive power is,

\[
\frac{k_{pv}}{k_{qv}} = \frac{r_r}{s_{s0}X}
\]

### Appendix B

Decompose 3rd order to 1st order

**3rd order Induction motor:**

An induction motors d & q axis mutual flux’s equations are,

\[
\Psi_{md} = X_{m0} \left( \frac{\Psi_{db}}{X_{db}} + \frac{\Psi_{dq}}{X_{dq}} \right)
\]  

(B- 1)
\[ \Psi_{mq} = X_{aq} \left( \frac{\Psi_{qa}}{X_{ls}} + \frac{\Psi_{qr}}{X_{lr}} \right) \] (B-2)

Let

\[ X_a = \frac{X_{mq}}{X_{ls}} \] (B-3)

\[ X_b = \frac{X_{mq}}{X_{lr}} \] (B-4)

From equation (B-1) and equation (B-2) the vector form of mutual flux is,

\[
\begin{bmatrix}
\Psi_{nd} \\
\Psi_{mq}
\end{bmatrix}
= \begin{bmatrix}
X_a & 0 & \Psi_{ds} \\
0 & X_a & \Psi_{qs}
\end{bmatrix} + \begin{bmatrix}
X_b & 0 & \Psi_{dr} \\
0 & X_b & \Psi_{qr}
\end{bmatrix}
\] (B-5)

An induction motors d&q axis stator flux equation are,

\[ \Psi_{ds} = \frac{\omega}{\omega_b} \left[ V_{qs} + \frac{r_s}{X_{ls}} (\Psi_{mq} - \Psi_{qs}) \right] \] (B-6)

\[ \Psi_{qs} = \frac{\omega}{\omega_b} \left[ V_{qs} - \frac{r_s}{X_{ls}} (\Psi_{nd} - \Psi_{ds}) \right] \] (B-7)

Let,

\[ \omega_{qs} = \frac{\omega}{\omega_b} \] (B-8)
From equation (B-6) and equation (B-7) the matrix form of stator fluxes are,

\[
\begin{bmatrix}
\Psi_{ds} \\
\Psi_{qs}
\end{bmatrix} = W_{qs} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} Vds \\ Vqs \end{bmatrix} + W_{qs} Xs \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \Psi_{ds} \\ \Psi_{qs} \end{bmatrix} + W_{qs} Xs \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \Psi_{md} \\ \Psi_{mq} \end{bmatrix}
\]  
(B-10)

Equation (B-5) is substituted in equation (B-10),

\[
\begin{bmatrix}
\Psi_{ds} \\
\Psi_{qs}
\end{bmatrix} = W_{qs} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} Vds \\ Vqs \end{bmatrix} + W_{qs} Xs \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \Psi_{ds} \\ \Psi_{qs} \end{bmatrix} + \begin{bmatrix} Xa & 0 \\ 0 & Xa \end{bmatrix} \begin{bmatrix} \Psi_{ds} \\ \Psi_{qs} \end{bmatrix} + \begin{bmatrix} Xb & 0 \\ 0 & Xb \end{bmatrix} \begin{bmatrix} \Psi_{dr} \\ \Psi_{qr} \end{bmatrix}
\]  
(B-11)

\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} - \begin{bmatrix} 0 & -W_{qs} Xs \\ W_{qs} Xs & 0 \end{bmatrix} - \begin{bmatrix} 0 & W_{qs} Xs \\ -W_{qs} Xs & 0 \end{bmatrix} \begin{bmatrix} Xa & 0 \\ 0 & Xa \end{bmatrix} \begin{bmatrix} \Psi_{ds} \\ \Psi_{qs} \end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} - \begin{bmatrix} 0 & -W_{qs} Xs \\ W_{qs} Xs & 0 \end{bmatrix} - \begin{bmatrix} 0 & W_{qs} Xs \\ -W_{qs} Xs & 0 \end{bmatrix} \begin{bmatrix} Xa & 0 \\ 0 & Xa \end{bmatrix} \begin{bmatrix} \Psi_{dr} \\ \Psi_{qr} \end{bmatrix}
\]  
(B-12)

Let,

\[
J = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} - \begin{bmatrix} 0 & -W_{qs} Xs \\ W_{qs} Xs & 0 \end{bmatrix} - \begin{bmatrix} 0 & W_{qs} Xs \\ -W_{qs} Xs & 0 \end{bmatrix} \begin{bmatrix} Xa & 0 \\ 0 & Xa \end{bmatrix}
\]  
(B-13)

Equation (B-13) is substituted in equation (B-12),
\[
\begin{bmatrix}
\Psi_{dr} \\
\Psi_{q_r}
\end{bmatrix} = J^{-1} \begin{bmatrix}
0 & W_{qs} \\
W_{qs} & 0
\end{bmatrix} \begin{bmatrix}
V_{ds} \\
V_{qs}
\end{bmatrix} +
J^{-1} W_{qs} X_s \begin{bmatrix}
0 & 1 \\
-1 & 0
\end{bmatrix} \begin{bmatrix}
X_a & 0 \\
0 & X_a
\end{bmatrix} \begin{bmatrix}
\Psi_{dr} \\
\Psi_{q_r}
\end{bmatrix}
\]

Hence, compact form of stator flux matrix is,

\[
\begin{bmatrix}
\Psi_{dr} \\
\Psi_{q_r}
\end{bmatrix} = A_s \begin{bmatrix}
V_{ds} \\
V_{qs}
\end{bmatrix} + B_s \begin{bmatrix}
\Psi_{dr} \\
\Psi_{q_r}
\end{bmatrix}
\]

Where,

\[
A_s = J^{-1} \begin{bmatrix}
0 & W_{qs} \\
W_{qs} & 0
\end{bmatrix}
\]

and

\[
B_s = J^{-1} W_{qs} X_s \begin{bmatrix}
0 & 1 \\
-1 & 0
\end{bmatrix} \begin{bmatrix}
X_a & 0 \\
0 & X_a
\end{bmatrix}
\]

The d&q axis rotor fluxes derivatives are,

\[
\Psi_{dr} = \omega_b \left[ V_{dr} + \left( \frac{\omega - \omega_r}{\omega_b} \right) \Psi_{q_r} + \frac{r_r}{X_{lr}} (\Psi_{md} - \Psi_{dr}) \right]
\]

(B- 16)

\[
\Psi_{q_r} = \omega_b \left[ V_{qr} - \left( \frac{\omega - \omega_r}{\omega_b} \right) \Psi_{dr} + \frac{r_r}{X_{lr}} (\Psi_{mq} - \Psi_{q_r}) \right]
\]

(B- 17)

The vector representation of equation (B-16) and equation (B-17) are,
\[
\begin{pmatrix}
\Psi_{dr}
\end{pmatrix} = \omega_b \begin{pmatrix}
1 & 0
0 & 1
\end{pmatrix}
\begin{pmatrix}
V_{dr}
V_{qr}
\end{pmatrix} + WT \ast \omega_b \begin{pmatrix}
0 & 1
-1 & 0
\end{pmatrix}
\begin{pmatrix}
\Psi_{dr}
\Psi_{qr}
\end{pmatrix} + XR \ast \omega_b \begin{pmatrix}
1 & 0
0 & 1
\end{pmatrix}
\begin{pmatrix}
\Psi_{md}
\Psi_{mq}
\end{pmatrix}
\]
\[
- XR \ast \omega_b \begin{pmatrix}
1 & 0
0 & 1
\end{pmatrix}
\begin{pmatrix}
\Psi_{dr}
\Psi_{qr}
\end{pmatrix}
\]

After manipulating equation (B-18),

\[
\begin{pmatrix}
\Psi_{dr}
\Psi_{qr}
\end{pmatrix} = \omega_b \ast \begin{pmatrix}
1 & 0
0 & 1
\end{pmatrix}
\begin{pmatrix}
V_{dr}
V_{qr}
\end{pmatrix} + \begin{pmatrix}
0 & WT \ast wb
-WT \ast wb & 0
\end{pmatrix}
\begin{pmatrix}
XR \ast wb & 0
0 & XR \ast wb
\end{pmatrix}
\]
\[
\begin{pmatrix}
\Psi_{dr}
\Psi_{qr}
\end{pmatrix} + XR \ast \omega_b \begin{pmatrix}
1 & 0
0 & 1
\end{pmatrix}
\begin{pmatrix}
\Psi_{md}
\Psi_{mq}
\end{pmatrix}
\]

Let,

\[
Br' = \begin{pmatrix}
0 & WT \ast wb
-WT \ast wb & 0
\end{pmatrix}
\begin{pmatrix}
XR \ast wb & 0
0 & XR \ast wb
\end{pmatrix}
\]

And equation (B-5) is substituted in equation (B-19),

\[
\begin{pmatrix}
\Psi_{dr}
\Psi_{qr}
\end{pmatrix} = \omega_b \ast \begin{pmatrix}
1 & 0
0 & 1
\end{pmatrix}
\begin{pmatrix}
V_{dr}
V_{qr}
\end{pmatrix} + Br' \begin{pmatrix}
\Psi_{dr}
\Psi_{qr}
\end{pmatrix} + XR \ast \omega_b \begin{pmatrix}
1 & 0
0 & 1
\end{pmatrix}
\begin{pmatrix}
\Psi_{md}
\Psi_{mq}
\end{pmatrix}
\]
\[
\begin{pmatrix}
Xa & 0
0 & Xa
\end{pmatrix}
\begin{pmatrix}
\Psi_{ds}
\Psi_{qs}
\end{pmatrix} + \begin{pmatrix}
Xb & 0
0 & Xb
\end{pmatrix}
\begin{pmatrix}
\Psi_{dr}
\Psi_{qr}
\end{pmatrix}
\]
\[
\begin{bmatrix}
\dot{\Psi}_{dr} \\
\dot{\Psi}_{qr}
\end{bmatrix} = \omega_b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_{dr} \\ V_{qr} \end{bmatrix} + B_r \begin{bmatrix}
\dot{\Psi}_{dr} \\
\dot{\Psi}_{qr}
\end{bmatrix} + X R^* \omega_b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_a \\ 0 \end{bmatrix}
\]

(B-22)

\[
\begin{bmatrix}
\dot{\Psi}_{dr} \\
\dot{\Psi}_{qr}
\end{bmatrix} + X R^* \omega_b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_b \\ 0 \end{bmatrix} \begin{bmatrix}
\dot{\Psi}_{dr} \\
\dot{\Psi}_{qr}
\end{bmatrix}
\]

Equation (B-15) is substituted in the above equation, the equation of the derivative are,

\[
\begin{bmatrix}
\dot{\Psi}_{dr} \\
\dot{\Psi}_{qr}
\end{bmatrix} = \omega_b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_{dr} \\ V_{qr} \end{bmatrix} + B_r \begin{bmatrix}
\dot{\Psi}_{dr} \\
\dot{\Psi}_{qr}
\end{bmatrix} + X R^* \omega_b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_a \\ 0 \end{bmatrix} + A_s \begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} + B_s \begin{bmatrix}
\dot{\Psi}_{dr} \\
\dot{\Psi}_{qr}
\end{bmatrix}
\]

(B-23)

\[
+ X R^* \omega_b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_b \\ 0 \end{bmatrix} \begin{bmatrix}
\dot{\Psi}_{dr} \\
\dot{\Psi}_{qr}
\end{bmatrix}
\]

After expanding equation (B-23),

\[
\begin{bmatrix}
\dot{\Psi}_{dr} \\
\dot{\Psi}_{qr}
\end{bmatrix} = \omega_b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_{dr} \\ V_{qr} \end{bmatrix} + X R^* \omega_b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_a \\ 0 \end{bmatrix} + A_s \begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} + X R^* \omega_b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_b \\ 0 \end{bmatrix} + X R^* w_b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_a \\ 0 \end{bmatrix} \begin{bmatrix}
\dot{\Psi}_{dr} \\
\dot{\Psi}_{qr}
\end{bmatrix}
\]

(B-24)

\[
\begin{bmatrix}
X_b \\ 0
\end{bmatrix} + X R^* w_b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_a \\ 0 \end{bmatrix} \begin{bmatrix}
\dot{\Psi}_{dr} \\
\dot{\Psi}_{qr}
\end{bmatrix}
\]

Let,

\[
C_r = \{ X R^* \omega_b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_b \\ 0 \end{bmatrix} + X R^* w_b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_a \\ 0 \end{bmatrix} \}
\]

(B-25)
\[ A_r = \omega_r \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]  \hspace{1cm} (B-26)

\[ B_r = X R \omega_b \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} Xa & 0 \\ 0 & Xa \end{bmatrix} = A_s \]  \hspace{1cm} (B-27)

The equation (B-24) is,

\[
\begin{bmatrix}
\mathbf{\dot{\Psi}}_{dr} \\
\mathbf{\dot{\Psi}}_{qr}
\end{bmatrix} = A_r \begin{bmatrix} V_{dr} \\ V_{qr} \end{bmatrix} + B_r \begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} + C_r \begin{bmatrix} \mathbf{\Psi}_{dr} \\ \mathbf{\Psi}_{qr} \end{bmatrix}
\]  \hspace{1cm} (B-28)

In 1st order case when rotor flux derivative are zero the equation (B-28) becomes,

\[
\begin{bmatrix} 0 \\ 0 \end{bmatrix} = A_r \begin{bmatrix} V_{dr} \\ V_{qr} \end{bmatrix} + B_r \begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} + C_r \begin{bmatrix} \mathbf{\Psi}_{dr} \\ \mathbf{\Psi}_{qr} \end{bmatrix}
\]  \hspace{1cm} (B-29)

Hence the rotor d and q axis flux’s are,

\[
C_r \begin{bmatrix} \mathbf{\Psi}_{dr} \\ \mathbf{\Psi}_{qr} \end{bmatrix} = -A_r \begin{bmatrix} V_{dr} \\ V_{qr} \end{bmatrix} - B_r \begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix}
\]  \hspace{1cm} (B-30)

**Appendix C**

**Eigenvalue calculation:**

From equation (B-13) the value of J is,
\( J^{-1} = \begin{bmatrix} \frac{1}{\%1} & \frac{W_{qs}XS(-1+Xa)}{\%1} \\ \%\frac{W_{qs}XS(-1+Xa)}{\%1} & 1 \end{bmatrix} \) \hspace{1cm} (C-1)

Where,

\( \%1 = 1 + W_{qs}^2 X_s^2 - 2 W_{qs}^2 X_s^2 X_a + W_{qs} X_a^2 X_s^2 \) \hspace{1cm} (C-2)

Equation (C-3) is substituted in equation (C-2),

\[
J^{-1} = \begin{bmatrix} 1 & \frac{W_{qs}X_s(-1+X_a)}{1+W_q\delta X_s^2 - 2 W_q\delta X_s X_a + W_q\delta X_s X_a^2} \\ W_{qs}X_s(-1+X_a) & \frac{1}{1+W_q\delta X_s^2 - 2 W_q\delta X_s X_a + W_q\delta X_s X_a^2} \end{bmatrix}
\]

We know,

\[
A_j = J^{-1} \begin{bmatrix} 0 & W_{qs} \\ W_{qs} & 0 \end{bmatrix}
\] \hspace{1cm} (C-4)

Then equation (C-4) is substituted in equation (C-5),

\[
A = \begin{bmatrix} 1 & \frac{W_{qs}X_s(-1+X_a)}{1+W_q\delta X_s^2 - 2 W_q\delta X_s X_a + W_q\delta X_s X_a^2} \\ \frac{1}{1+W_q\delta X_s^2 - 2 W_q\delta X_s X_a + W_q\delta X_s X_a^2} & 1 \end{bmatrix}
\]

After expanding equation (C-6),
A_s = \begin{bmatrix}
\frac{W_{qs}^2X_s(-1 + X_s)}{1 + W_{qs}^2X_s^2 - 2W_{qs}^2X_s^2X_e + W_{qs}^2X_s^2X_e^2}
\frac{W_{qs}}{1 + W_{qs}^2X_s^2 - 2W_{qs}^2X_s^2X_e + W_{qs}^2X_s^2X_e^2}
\end{bmatrix}
\quad \text{(C-6)}

We also know that,

\[ B_s = J^{-1}W_{qs}X_s \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} X_b & 0 \\ 0 & X_b \end{bmatrix} \quad \text{(C-7)} \]

The value of \( J \) is substituted in the equation (C-8),

\[ B_s = \begin{bmatrix}
\frac{1}{1 + W_{qs}^2X_s^2 - 2W_{qs}^2X_s^2X_e + W_{qs}^2X_s^2X_e^2}
\frac{W_{qs}X_s(-1 + X_s)}{1 + W_{qs}^2X_s^2 - 2W_{qs}^2X_s^2X_e + W_{qs}^2X_s^2X_e^2}
\end{bmatrix}
\begin{bmatrix}
W_{qs}X_s
\end{bmatrix}
\quad \text{(C-8)}

\[ \begin{bmatrix} 0 & W_{qs}X_s \\ -W_{qs}X_s & 0 \end{bmatrix} \begin{bmatrix} X_b & 0 \\ 0 & X_b \end{bmatrix} \]

After expanding equation (C-9),

\[ B_s = \begin{bmatrix}
\frac{W_{qs}^2X_s^2(-1 + X_s)}{1 + W_{qs}^2X_s^2 - 2W_{qs}^2X_s^2X_e + W_{qs}^2X_s^2X_e^2}
\frac{W_{qs}X_s}{1 + W_{qs}^2X_s^2 - 2W_{qs}^2X_s^2X_e + W_{qs}^2X_s^2X_e^2}
\end{bmatrix}
\begin{bmatrix} X_e \\ 0 \end{bmatrix} \quad \text{(C-9)}
\]

\[ \begin{bmatrix} 0 & W_{qs}X_e \\ -W_{qs}X_e & 0 \end{bmatrix} \begin{bmatrix} X_e & 0 \end{bmatrix} \]

\[ \begin{bmatrix}
\frac{W_{qs}^2X_s^2(-1 + X_s)}{1 + W_{qs}^2X_s^2 - 2W_{qs}^2X_s^2X_e + W_{qs}^2X_s^2X_e^2}
\frac{W_{qs}X_s}{1 + W_{qs}^2X_s^2 - 2W_{qs}^2X_s^2X_e + W_{qs}^2X_s^2X_e^2}
\end{bmatrix}
\begin{bmatrix} X_e \\ 0 \end{bmatrix} \quad \text{(C-10)} \]
Let,

\[ D = 1 + W_{qs}^2 X_s^2 - 2W_{qs}^2 X_s^2 X_a + W_{qs}^2 X_s^2 X_a^2 \]  

(C-11)

Then equation (C-11) is,

\[
B_s = \begin{bmatrix}
\frac{W_{qs}^2 X_s^2 (-1 + X_a) X_b}{D} & \frac{W_{qs} X_s X_b}{D} \\
\frac{-W_{qs} X_s X_b}{D} & \frac{-W_{qs}^2 X_s^2 (-1 + X_a) X_b}{D}
\end{bmatrix}
\]  

(C-12)

We know,

\[
B_r' = \begin{bmatrix}
0 & WT * wb \\
-WT * wb & 0
\end{bmatrix} - \begin{bmatrix}
XR * wb & 0 \\
0 & XR * wb
\end{bmatrix}
\]  

(C-13)

\[
C_r = \{XR * \omega_t \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X_b \} + \{XR * wb \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} X_a \} * B_s + B_r'
\]  

(C-14)

\[
= \begin{bmatrix} XR * \omega_t & 0 & Xb & 0 \\ 0 & XR * \omega_t & 0 & Xb \end{bmatrix} + \begin{bmatrix} XR * \omega_t & 0 & Xa & 0 \\ 0 & XR * \omega_t & 0 & Xa \end{bmatrix} * B_s + B_r'
\]  

(C-15)

Equation (C-14) is substituted in equation (C-15),

\[
C_r = \begin{bmatrix} XR * \omega_t & 0 & Xb & 0 \\ 0 & XR * \omega_t & 0 & Xb \end{bmatrix} + \begin{bmatrix} XR * \omega_t & 0 & Xa & 0 \\ 0 & XR * \omega_t & 0 & Xa \end{bmatrix} * B_s + B_r'
\]  

(C-16)
The value of $B_s$ is substituted in equation (C-18),

$$
\mathbf{Cr} = \begin{bmatrix} X R q X_b & 0 \\ 0 & X R q X_b \end{bmatrix} \begin{bmatrix} X R q X_b & 0 \\ 0 & X R q X_b \end{bmatrix} + \begin{bmatrix} \frac{W_c^2 X_c^2 (\pm X_c) X_b}{D} & \frac{W_c X_c}{D} \\ \frac{-W_c X_c}{D} & \frac{-W_c^2 X_c (\pm X_c) X_b}{D} \end{bmatrix}
$$

$$
\begin{bmatrix} -X R w_b & W T w_b \\ -W T w_b & -X R w_b \end{bmatrix}
$$

$$
= \begin{bmatrix} X R q X_b (-X R w_b) & W T^2 w_b \\ -W T w_b & X R q X_b (-X R w_b) \end{bmatrix} + \begin{bmatrix} \frac{-W_c X_c (\pm X_c) X_b X R q X_b}{D} & \frac{X R q X_c X_c}{D} \\ \frac{-W_c X_c (\pm X_c) X_b X R q X_b}{D} & \frac{W_c^2 X_c (\pm X_c) X_b}{D} \end{bmatrix}
$$

After manipulating equation (C-20),

$$
\mathbf{Cr} = \frac{-W_c^2 X_c (\pm X_c) X_b X R q X_b}{D} + X R q X_c X_c + \frac{X R q X_c X_c}{D} + W T^2 w_b
$$

$$
\begin{bmatrix} -X R q X_c X_c & W T w_b \\ -W T w_b & -X R q X_c X_c \end{bmatrix}
$$

$$
\frac{-W_c X_c (\pm X_c) X_b X R q X_b}{D} - W T^2 w_b + X R q X_c X_c + \frac{X R q X_c X_c}{D}
$$

We also know,

$$\mathbf{Ar} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
$$

This value is substituted in $\mathbf{Br}$,

$$\mathbf{Br} = X R \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & X a \end{bmatrix} \begin{bmatrix} 0 & X a & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathbf{As}$$
\[
\begin{bmatrix}
XR \omega_b & 0 & Xa & 0 \\
0 & XR \omega_b & 0 & Xa
\end{bmatrix} * As
\]

also value of As is substituted in Br,

\[
Br = \begin{bmatrix}
XR^r \omega_b^r X_a & 0 \\
0 & XR^r \omega_b^r X_a
\end{bmatrix} *
\[
\begin{bmatrix}
W_{w_q} X_a (-1 + X_a) \\
1 + W_q s X_a^2 - 2 W_q s X_a X_a + W_q s X_a^2
\end{bmatrix}
\]

After manipulating Br is,

\[
\begin{bmatrix}
W_{w_q} X_a (-1 + X_a) XR \omega_a X_a \\
1 + W_q s X_a^2 - 2 W_q s X_a X_a + W_q s X_a^2
\end{bmatrix}
\]

We have six state equations in a 3rd order induction motor with a springy shaft load,

\[
\Psi_{dq} = A_1 (1,1) \Psi_{dq} + B_1 (1,1) V_{dq} + B_1 (1,2) V_{qw} + \frac{W_{w_q} X_a (-1 + X_a) X_a XR \omega_a X_a}{D} + XR^r \omega_b^r X_b - XR^r w_q^r +
\]

\[
\Psi_{w_q} = A_1 (1,2) \Psi_{dq} + B_1 (1,2) V_{dq} + B_1 (2,2) V_{w_q} + \frac{XR \omega_a X_a W_a X_a}{D} - \frac{W_q X_a (-1 + X_a) X_a XR \omega_a X_a}{D} + XR^r \omega_b^r X_b - XR^r w_q^r
\]

Where,
\[ D_m = X_{rs}X_{ss} - X_m^2 \]  \hspace{1cm} (C-22)

\[ X_{rs} = X_{lr} + X_m \]  \hspace{1cm} (C-23)

\[ X_{ss} = X_{ls} + X_m \]  \hspace{1cm} (C-24)

\[ T_e = \frac{X_m}{D_m} (\Psi_{dr}\Psi_{qs} - \Psi_{qr}\Psi_{ds}) \]  \hspace{1cm} (C-25)

\[ \dot{\omega}_2 = \frac{X_m}{D_m} (\Psi_{dr}\Psi_{qs} - \Psi_{qr}\Psi_{ds}) - k(\theta_1 - \theta_2) \]  \hspace{1cm} (C-26)

\[ \dot{\omega}_1 = \frac{k(\theta_1 - \theta_2) - \frac{B\omega_1}{\omega_b}}{2H} \]  \hspace{1cm} (C-27)

\[ \dot{\theta}_1 = \frac{\omega_1}{\omega_b} \]  \hspace{1cm} (C-28)

\[ \dot{\theta}_2 = \frac{\omega_1}{\omega_b} \]  \hspace{1cm} (C-29)

So states are,

\[ X' = \begin{bmatrix} \Psi_{dr} \Psi_{qs} \omega_r \omega_1 \theta_1 \theta_2 \end{bmatrix} \]  \hspace{1cm} (C-29)

And input,
\[ u' = [V_{d1}, V_{q1}, V_{d2}, V_{q2}] \]  

\[ \Delta \dot{X} = A\Delta X + B\Delta u \]  

\[ \begin{bmatrix}  
-\frac{XR_qW_{qr}^3}{D}X_{\omega\omega}^{\psi\psi} + XR_qX_{\omega}^{\omega\omega} - XR_y \Psi \psi  \\
\frac{XR_qW_{qr}^3}{D}\Psi \omega^{\omega\omega} + \Psi \omega^{\omega\omega} + q \Omega \D & -\frac{XR_qW_{qr}^3}{D}X_{\omega\omega}^{\psi\psi} + XR_qX_{\omega}^{\omega\omega} - XR_y \Psi \psi \\
0 & \frac{XR_qW_{qr}^3}{D}\Psi \omega^{\omega\omega} + \Psi \omega^{\omega\omega} + q \Omega \D & -\frac{XR_qW_{qr}^3}{D}X_{\omega\omega}^{\psi\psi} + XR_qX_{\omega}^{\omega\omega} - XR_y \Psi \psi \\
0 & 0 & \frac{XR_qW_{qr}^3}{D}\Psi \omega^{\omega\omega} + \Psi \omega^{\omega\omega} + q \Omega \D \\
0 & 0 & 0 & \frac{1}{\Omega} \D \\
\end{bmatrix} \begin{bmatrix}  
\Delta \Psi \\
\Delta \Psi \\
\Delta \Psi \\
\Delta \Psi \\
\end{bmatrix} = \begin{bmatrix}  
\frac{X_{\omega\omega}^{\psi\psi}}{2H} \D \Psi \omega \omega \\
\frac{X_{\omega\omega}^{\psi\psi}}{2H} \Psi \omega^{\omega\omega} + \Psi \omega^{\omega\omega} + q \Omega \D \\
0 & \frac{X_{\omega\omega}^{\psi\psi}}{2H} \Psi \omega^{\omega\omega} + \Psi \omega^{\omega\omega} + q \Omega \D & -B \Omega \D \\
0 & 0 & -B \Omega \D & \frac{1}{\Omega} \D \\
\end{bmatrix} \begin{bmatrix}  
\psi_{\omega} \\
\psi_{\psi} \\
\omega_{\psi} \\
\omega_{\omega} \\
\theta \\
\end{bmatrix} \]

\[ \begin{bmatrix}  
C_{\omega}(1,1) & C_{\omega}(1,2) & 0 & 0 & 0 & 0 \\
C_{\omega}(2,1) & C_{\omega}(2,2) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{\Omega_{\omega}^2}{2H} & -\frac{\Omega_{\omega}^2 k}{2H} \\
0 & 0 & 0 & -B & -\frac{\Omega_{\omega}^2 k}{2H} & -\frac{\Omega_{\omega}^2 k}{2H} \\
0 & 0 & \frac{1}{\Omega_{\omega}^2} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{\Omega_{\omega}^2} & 0 & 0 \\
\end{bmatrix} \]

\[ A \]
For calculating the eigenvalue,

\[ \text{Det}(sI - A) = 0 \]

(C-35)
\[
\begin{bmatrix}
    s - C_r(1,1) & -C_r(1,2) & 0 & 0 & 0 & 0 \\
    -C_r(2,1) & s - C_r(2,2) & 0 & 0 & 0 & 0 \\
    0 & 0 & s & 0 & \omega_k & -\omega_k \\
    0 & 0 & 0 & \frac{-B}{2H_2} + s & -\omega_k & \frac{2H}{\omega_k} \\
    0 & 0 & -\frac{1}{\omega_b} & 0 & s & 0 \\
    0 & 0 & 0 & -\frac{1}{\omega_b} & 0 & s \\
\end{bmatrix}
\]

\[
\det = 0 \quad \text{(C-37)}
\]

Add col (5) to col (6),

\[
\begin{bmatrix}
    s - C_r(1,1) & -C_r(1,2) & 0 & 0 & 0 & 0 \\
    -C_r(2,1) & s - C_r(2,2) & 0 & 0 & 0 & 0 \\
    0 & 0 & s & 0 & \omega_k & -\omega_k \\
    0 & 0 & 0 & \frac{-B}{2H_2} + s & -\omega_k & \frac{2H}{\omega_k} \\
    0 & 0 & -\frac{1}{\omega_b} & 0 & s & 0 \\
    0 & 0 & 0 & -\frac{1}{\omega_b} & 0 & s \\
\end{bmatrix}
\]

\[
\det = 0 \quad \text{(C-38)}
\]

Sub row (6) from row (5),
\[
\begin{vmatrix}
    s - C_r, (1,1) & -C_r, (1,2) & 0 & 0 & 0 & 0 \\
    -C_r, (2,1) & s - C_r, (2,2) & 0 & 0 & 0 & 0 \\
    0 & 0 & s & 0 & \frac{\omega_b k}{2H} & 0 \\
    0 & 0 & 0 & -\frac{B}{2H^2} + s & -\frac{\omega_b k}{2H} & 0 \\
    0 & 0 & 0 & -\frac{1}{\omega_b} & s & 0 \\
    0 & 0 & 0 & -\frac{1}{\omega_b} & 0 & s \\
\end{vmatrix} = 0 \quad \text{(C-39)}
\]

Discard col (6) and row (6),

\[
\begin{vmatrix}
    s - C_r, (1,1) & -C_r, (1,2) & 0 & 0 & 0 \\
    -C_r, (2,1) & s - C_r, (2,2) & 0 & 0 & 0 \\
    0 & 0 & s & 0 & \frac{\omega_b k}{2H} \\
    0 & 0 & 0 & s - \frac{B}{2H^2} & -\frac{\omega_b k}{2H^2} \\
    0 & 0 & 0 & -\frac{1}{\omega_b} & 1 \\
\end{vmatrix} = 0 \quad \text{(C-40)}
\]

Multiply 3\textsuperscript{rd} row by \(s/H_2\), 4\textsuperscript{th} row by \(s/H\) and 5\textsuperscript{th} by \(\frac{\omega_b k}{2H_2 H}\), and then divide by

\[
\frac{s^2 \omega_b k}{2H^2 H_2^2}
\]

\[
\begin{vmatrix}
    s - C_r, (1,1) & -C_r, (1,2) & 0 & 0 & 0 \\
    -C_r, (2,1) & s - C_r, (2,2) & 0 & 0 & 0 \\
    0 & 0 & \frac{s^2}{H_2} & 0 & \frac{\omega_b k s}{2H H_2} \\
    0 & 0 & 0 & (s - \frac{B}{2H^2}) \frac{s}{H} & -\frac{\omega_b k s}{2H^2 H} \\
    0 & 0 & -\frac{\omega_b k}{\omega_b 2HH_2} & -\frac{\omega_b k}{\omega_b 2HH_2} & \frac{\omega_b k s}{2HH_2} \\
\end{vmatrix} = 0 \quad \text{(C-41)}
\]
\[
\det \frac{2H^2 k^2}{s\omega k} \omega, k s \\
\begin{bmatrix}
    s - C_r(s), 1,1 & -C_r(s), 1,2 & 0 & 0 & 0 \\
    -C_r(s), 2,1 & s - C_r(s), 2,2 & 0 & 0 & 0 \\
    0 & 0 & \frac{s^2}{H^2} & 0 & 1 \\
    0 & 0 & 0 & (s - \frac{B}{2H^2}) \frac{s}{H} -1 \\
    0 & 0 & -\frac{k}{2HH^2} & \frac{k}{2HH^2} & 1
\end{bmatrix} = 0
\] 
(C-42)

Add row (4) to row (3) and row (5) to row (4).

\[
\det HH_2 \begin{bmatrix}
    s - C_r(s), 1,1 & -C_r(s), 1,2 & 0 & 0 & 0 \\
    -C_r(s), 2,1 & s - C_r(s), 2,2 & 0 & 0 & 0 \\
    0 & 0 & \frac{s^2}{H^2} & (s - \frac{B}{2H^2}) \frac{s}{H} & 0 \\
    0 & 0 & -\frac{k}{2HH^2} & (s - \frac{B}{2H^2}) \frac{s}{H} + \frac{k}{2HH^2} & 0 \\
    0 & 0 & -\frac{k}{2HH^2} & \frac{k}{2HH^2} & 1
\end{bmatrix} = 0
\] 
(C-43)

\[
\det HH_2 \begin{bmatrix}
    s - C_r(s), 1,1 & -C_r(s), 1,2 & 0 & 0 \\
    -C_r(s), 2,1 & s - C_r(s), 2,2 & 0 & 0 \\
    0 & 0 & \frac{s^2}{H^2} & (s - \frac{B}{2H^2}) \frac{s}{H} \\
    0 & 0 & -\frac{k}{2HH^2} & (s - \frac{B}{2H^2}) \frac{s}{H} + \frac{k}{2HH^2}
\end{bmatrix} = 0
\] 
(C-44)

Interchange col (3) with col (4)
\[
\begin{vmatrix}
  s - C_r(1,1) & -C_r(1,2) & 0 & 0 \\
  -C_r(2,1) & s - C_r(2,2) & 0 & 0 \\
  0 & 0 & (s - \frac{B}{2H_s}) \frac{s}{H} & \frac{s^2}{H_s} \\
  0 & 0 & (s - \frac{B}{2H_s}) \frac{s}{H} + \frac{k}{2H_s} & -\frac{s^2k}{2H_s}
\end{vmatrix} = 0
\]  
(C-45)

Multiply row (3) by \(k/2H\) and row (4) by \(s^2\) and divide by \(\frac{s^2k}{2H}\).

\[
\begin{vmatrix}
  s - C_r(1,1) & -C_r(1,2) & 0 & 0 \\
  -C_r(2,1) & s - C_r(2,2) & 0 & 0 \\
  0 & 0 & (s - \frac{B}{2H_s}) \frac{sk}{2H^2} & \frac{s^2k}{2HH_s} \\
  0 & 0 & (s - \frac{B}{2H_s}) \frac{s^3}{H} + \frac{ks^2}{2HH_s} - \frac{s^2k}{2HH_s}
\end{vmatrix} = 0
\]  
(C-46)

\[
\begin{vmatrix}
  s - C_r(1,1) & -C_r(1,2) & 0 & 0 \\
  -C_r(2,1) & s - C_r(2,2) & 0 & 0 \\
  0 & 0 & (s - \frac{B}{2H_s}) \frac{sk}{2H^2} & 1 \\
  0 & 0 & (s - \frac{B}{2H_s}) \frac{s^3}{H} + \frac{ks^2}{2HH_s} - 1
\end{vmatrix} = 0
\]  
(C-47)

Add row (4) to row (3),
The three states equations of the 3rd order induction motor with fan load are,

\[ \Psi_{dr} = A_{(1,1)}\Psi_{dr} + B_{(1,1)}\Psi_{d} + B_{(1,2)}\Psi_{q} + \left( \frac{-W_{d}^{2}X_{a}^{2}}{D} + XR + \omega \right) \Psi_{d} + \left( \frac{-W_{d}^{3}X_{a}^{2}X_{r}(1 + X_{a})}{D} + \omega \right) \Psi_{q} + \left( \frac{W_{d}^{2}X_{a}^{2}}{D} \right) \Psi_{q} \]

\[ \Psi_{qr} = A_{(1,2)}\Psi_{dr} + B_{(2,1)}\Psi_{d} + B_{(2,2)}\Psi_{q} + \left( \frac{-W_{q}^{2}X_{a}^{2}}{D} + XR + \omega \right) \Psi_{d} + \left( \frac{-W_{q}^{3}X_{a}^{2}X_{r}(1 + X_{a})}{D} + \omega \right) \Psi_{q} + \left( \frac{W_{q}^{2}X_{a}^{2}}{D} \right) \Psi_{q} \]
\[ \dot{\omega}_r = \frac{\left[ T_e - B \frac{\omega_r}{\omega_b} \right] \omega_b}{2H} \]  

(C-54)

Where

\[ D_m = X_{rr} X_{ss} - X_m^2 \]  

(C-55)

And

\[ X_{rr} = X_{lr} + X_m \]
\[ X_{ss} = X_{ls} + X_m \]  

(C-56)

Electromagnetic torque is,

\[ T_e = \frac{X_m}{D_m} \left( \Psi_{dr} \Psi_{qs} - \Psi_{qr} \Psi_{ds} \right) \]  

(C-57)

And derivative of rotor equation is,

\[ \dot{\omega}_r = \frac{\left[ \frac{X_m}{D_m} \left( \Psi_{dr} \Psi_{qs} - \Psi_{qr} \Psi_{ds} \right) - B \frac{\omega_r}{\omega_b} \right] \omega_b}{2H} \]  

(C-58)

After manipulating the three states the state matrix of the 3rd order induction motor is,
For calculating eigenvector the value of A is substituted into equation (C-35) and we follow the same process listed above from equation (C-35) to equation (C-51).

**Appendix D**

A set of induction motor parameters from 4kW to 630kW is given below,

<table>
<thead>
<tr>
<th>Power (kW)</th>
<th>Wb</th>
<th>p(pole)</th>
<th>Rr</th>
<th>slip</th>
<th>Rr/Power</th>
<th>Slip/Power</th>
<th>J (kgm²/s²)</th>
<th>H</th>
<th>Wr</th>
<th>V</th>
<th>B</th>
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<td>0.2</td>
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<td>0.073</td>
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