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Service Bid Comparisons by Fuzzy Ranking in Open Railway Market Timetabling

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Abstract: In an open railway access market, the Infrastructure Provider (IP), upon the receipts of service bids from the Train Service Providers (TSPs), assigns track access rights according to its own business objectives and the merits of the bids; and produces the train service timetable through negotiations. In practice, IP chooses to negotiate with the TSPs one by one in such a sequence that IP optimizes its objectives. The TSP bids are usually very complicated, containing a large number of parameters in different natures. It is a difficult task even for an expert to give a priority sequence for negotiations from the contents of the bids. This study proposes the application of fuzzy ranking method to compare and prioritize the TSP bids in order to produce a negotiation sequence. The results of this study allow investigations on the behaviors of the stakeholders in bid preparation and negotiation, as well as evaluation of service quality in the open railway market.

Keywords: Fuzzy ranking; policies comparison; open railway access market; intelligent transportation systems

1. Introduction

Open railway markets consist of a group of independent train service providers (TSPs) attempting to gain access to a common rail network supplied by a single infrastructure provider (IP) (Jensen, 1998). In some cases, the ancillary services and maintenance services are also provided by other independent parties (Shaw, 2001). This contrasts to the conventional railways...
where the infrastructure facilities and train services are managed by a single corporation. An open railway market therefore involves multiple stakeholders arranged as a supply-chain through which railway resources, such as track capacity and rolling stock, are supplied to the TSPs to allow the ultimate train service provisions to the end-consumers. Improvements on track capacity utilization and quality of train services are anticipated with this intra-modal competition (BTRE, 2003).

The competing TSPs are often classified by their types of service provisions. Train services are first categorized into freight and passenger services. Freight services are further grouped by the nature of commodities being bulk (e.g. coal, petrochemicals) or non-bulk (e.g. foodstuffs, postal, parcels). On the other hand, passenger services are usually classified as regional or intercity according to the distances traveled. In open access markets, these railway services are operated by different stakeholders who not only compete for track access and revenues but also occasionally cooperate to provide services for mutual benefits.

To acquire track access rights for service operations, TSPs need to negotiate with the IP with their service bids which contain their preferred train timetables, as well as a payment known as track access charge (TAC). Conflicts of rights-of-way may arise when more than one TSP demands for the same track resources in the same timeslots. IP is responsible for resolving these conflicts, with the objective of maximizing its revenue and the utilization of track capacity, subject to the constraints given by the schedule times requested by the TSPs, the TSP’s willingness-to-pay for the TAC, and the availability of track resources. Rounds of negotiations are required to derive a timetable which satisfies the requirements of all stakeholders. In an open market, the business interests and objectives, as well as the technical constraints, of the stakeholders, inevitably complicate the negotiations.

During timetable development, the IP may conduct the track capacity allocation in different ways. The straightforward approach is by combinatorial generation (Tsang, 2007), in which the IP may collect all offers from the TSPs and determine the optimal allocations for all TSPs.
simultaneously. If the TSPs decide to reject the offers produced by the IP, they can revise their bids and submit them in the next round of negotiation. The process repeats until either the track access agreements are reached or the TSPs withdraw from the negotiation. However, train timetabling is a complex and time-consuming process (Watson, 2001). Combinatorial generation may be infeasible when considering the deadline for the final timetable and the scale of the solution-searching process with increasing number of TSPs. Alternatively, the IP conducts the individual negotiations with TSPs in a sequential manner and hence each negotiation only involves the IP and one TSP (Tsang, 2007). This significantly reduces the complexity of the negotiation and the overall timetable scheduling becomes manageable. Nevertheless, the IP is required to determine the order in which the individual negotiations are to be conducted.

In order to facilitate the negotiation between a single IP and a single TSP (IP-TSP transaction), the problem of IP to generate train service timetable from the negotiation has been modeled as a multi-dimensional constrained optimization problem, which was solved initially by exhaustive enumeration (Tsang & Ho, 2006), and later by a branch-and-bound algorithm (Tsang & Ho, 2008).

While it is possible for the IP to attain the optimal solution within a single IP-TSP negotiation, the cumulative revenue of the IP, the track capacity utilization and the resulting timetable may not necessarily be optimal when a number of these IP-TSP negotiations are performed (IP-TSP negotiation) in a specific sequence. It was shown in a previous study (Tsang, 2007) that the sequence in which the IP conducts the negotiations with TSPs plays a significant role on the overall revenue, capacity allocation and ultimately the timetable. The study examined two sequence generating methods, First-Come-First-Serve (FCFS) and Highest-Willingness-to-Pay-First (HW2PF). FCFS arranges the negotiation sequence according to the order that the IP receives the TSP bids while HW2PF gives priority to the TSP who is willing to pay a higher TAC. Although such rule-based methods are simple for implementation, they only utilize one of
the parameters in the TSP bids to determine the negotiation sequence, but not considering the contents in the bids as a whole.

There are a vast number of parameters within the TSP bids and the number may increase with the types or classes of the services to be offered. The characteristics of the parameters also vary as some are given by crisp values (e.g. schedule timings) and others are in linguistic terms (e.g. willingness of the TSP to relax its requirements). It is difficult even for an experienced timetable planner to produce an objective sequence by taking all parameters in the bids into account. This study thus proposes a fuzzy ranking approach to compare TSP bids with multiple parameters and produce a negotiation sequence accordingly so that a train service timetable can be attained with the flexibility for the IP to match its business and operation objectives.

The paper is organized as follows. Section 2 discusses the characteristics of the TSP bids. Section 3 presents the concepts of fuzzy ranking methodologies and the applications of fuzzy ranking algorithms on TSP bid comparisons. Simulation setup to evaluate the fuzzy ranking approach, and the results and discussions are given in Section 4. Conclusions are then made in Section 5.

2. Track access rights negotiation

Each IP-TSP negotiation is modeled as an iterative process to settle on a mutually agreed price, if possible, for a product (track access rights) between a buyer (TSP) and a seller (IP). The negotiation may either lead to a deal on the product or end with the buyer withdrawing from the process.

2.1 Track access rights

A track access rights specifies the conditions for track usage by the TSP. It consists of a service schedule describing the train movements in space and time and the type of rolling stocks
to be operated on rails. In addition, a parameter ‘flex’ is adopted to denote the flexibility with which the TSP is willing to adjust the schedule time after each round of the negotiation has completed (Gibson et al., 2002). Flex is defined as a set of levels where the lowest and highest levels refer to the minimum and maximum flexibilities to shift the time schedule respectively. The levels may be given by linguistic descriptions, implicitly indicating the progressive willingness of the TSP to make concessions during negotiation (e.g. ‘strongly willing’, ‘willing’, ‘neutral’ and ‘not willing’). Different flex levels on different parameters, such as service timings, in the track access rights are allowed. The TSP also has to agree on a payment of track access charge (TAC) in order to obtain the permission for train operation.

A track access rights $P$, in its simplest form, is defined in Eq. (1), where $c \in \{1, 2, \ldots, \infty\}$ is the TAC (in $\text{\$}$ or appropriate currencies); $\Psi$ is the train service schedule as defined in Eq. (2); $\omega \in \{\omega_i\,|\,i = 1, \ldots, n_\omega\}$ is the rolling stock selected for operation ($n_\omega$ is the total number of types of rolling stock); and $\phi \in \{\phi_i\,|\,i = 1, \ldots, n_\phi\}$ is the chosen flex level ($n_\phi$ is the total number of available flex levels).

$$P = \{c, \Psi, \omega, \phi\} \quad (1)$$

A train service schedule $\Psi$ consists of a set of IDs $S = \{s_i\,|\,i = 1, \ldots, n_s\}$ identifying the sequence of stations to be visited ($n_s$ is the total number of train stations). The movement of train in time is described by the service commencement time (i.e. the arrival time at the first station) $\zeta$ (in hh:mm), the dwell times at each station $T_D = \{t_{di}\,|\,i = 1, \ldots, n_s\}$ (in min), and the inter-station runtimes $T_R = \{t_{ri}\,|\,i = 1, \ldots, n_s - 1\}$ (in min) between adjacent stations. Hence, $\Psi$ is formally defined as a 4-duple as follows.

$$\Psi = \{S, \zeta, T_D, T_R\} \quad (2)$$

Other parameters, such as safety and service quality records, and even the credibility of the TSPs, are also commonly adopted in the track access rights bids for the purpose of comparisons.
More parameters inevitably complicate the comparison process, but also further urging the need of a proper methodology for comparison.

2.2 Comparisons

In order to establish the negotiation sequence with TSPs according to their track access rights bids, the IP is required to conduct direct and objective comparisons among their bids. The comparisons should allow relative evaluations on individual parameters within the bids and the combination of these evaluations to form an overall ranking among the bids.

As the parameters in the bids are in either crisp values or linguistic descriptions, the comparisons of both quantitative and qualitative variables are to be realized and the variations of the parameters should be confined within the same scales across the bids (i.e. normalization of comparison indices). A mechanism to combine the comparison results of the individual parameters is also required. From the viewpoint of the IP, it is also desirable to incorporate the flexibility of being able to weigh selected parameters in the combination process according to certain business and operational requirements. To summarize, the generation of negotiation sequence for IP can be regarded as a problem of ranking multiple-aspect alternatives or policies where precise information may not be available.

3. Fuzzy Ranking

Fuzzy analysis has found many applications of comparing policies, processes and solutions, in the areas of transportation, engineering, manufacturing and even finance (Heung & Ho, 2005; Huang et al., 2008; Kang & Lee, 2007; Lai, 2008;). Fuzzy logic allows imprecision, ambiguity or vagueness in the available information to be captured. Fuzzy numbers are employed to assess the preference of one imprecise value over another. It is particularly useful for decision-makers
who do not always have the crisp-value data on hands because of the intrinsic natures of the data and the fuzziness within the sources of the data.

In this application of track access rights bids comparison, the representations of the parameters within the bids are first aligned by appropriate fuzzy membership functions. The corresponding parameters are then compared across the bids. Subsequently, the comparison results are merged according to the assigned importance of the parameters in order to attain the ranking of the bids. In other words, it consists of three processes: a) parameters fuzzification; b) comparisons of fuzzified parameters; and c) aggregation of comparison and ranking.

3.1 Fuzzification

Fuzzification is the process to obtain a set of fuzzy numbers to represent the bid parameters. Two types of fuzzification processes are required to handle the two groups of bid parameters respectively. The usual fuzzy analysis approach with predefined membership functions is adopted for qualitative (or intangible) parameters, while the concept of fuzzy line segment (Carnahan, 1994) is applied to deal with quantitative (or tangible) parameters.

3.1.1 Intangible parameters

A finite set of fuzzy numbers: \( U = (u_1, u_2, ..., u_n) \) is first established in the universe of discourse \( U \rightarrow [0, 1] \) to denote the imprecise or qualitative descriptors, such as “Good”, “Medium” and “Poor”. A linguistic scale is then used to enable these intangible parameters to be represented by fuzzy numbers. Each scale is described by a membership function \( \mu_i \). Triangular membership function is a typical and commonly adopted example while others are equally valid. For a fuzzy number, the triangular membership function is expressed by Eq. (3) and illustrated in Fig. 1, where a, b and c correspond to the vertices of the triangle and they are known as the triplet points of the membership function.
The number of possible “values” or descriptions that an intangible parameter can take should match with that of the fuzzy numbers available. The higher number of possible values implies better resolution of the fuzzy representation of the parameter. However, the computational demand in the subsequent processes becomes higher. At the end of this stage, the intangible parameters are represented by the membership functions of the respective fuzzy numbers.

3.1.2 Tangible parameters

The concept of fuzzy line segment allows fuzzification to be performed on quantitative parameters (i.e. tangible variables) or crisp values. The transformation of a tangible parameter $x$ to a fuzzy variable $z$ follows the steps described below.

The relative value $r_x$ of a variable $x \in [x_{\min}, x_{\max}]$ is the normalized value in the closed interval $[0, 1]$, and it can be computed by Eq. (4).

$$r_x = (x - x_{\min}) / (x_{\max} - x_{\min})$$  

Let $n$ be the number of triangular membership functions distributed over the normalized interval $[0, 1]$. If $\{a_i, b_i, c_i\}$ denotes the triplet points for the triangular membership function $i$, for $1 \leq i \leq n$, and $a_i = b_i, a_n = b_n, b_i = a_{i+1}, c_i = b_{i+1}$, for $1 \leq i \leq n-1$, then the alpha value $\alpha_x$ of variable $x$ is defined in Eq. (5).

$$\alpha_x = (r_x - b_k) / (b_{k+1} - b_k) \quad \text{for } k = \arg\{b_i \leq r_x \leq b_{i+1}\}$$

Given the relative and alpha values, $r_x$ and $\alpha_x$, the triplet points and membership function for the fuzzified variable $z$ are determined. The triplet points $\{a_z, b_z, c_z\}$ for fuzzy variable $z$ are then computed by Eqs. (6)-(8).
\[
\begin{align*}
a_z &= (1-\alpha)a_k + \alpha a_{k+1} \quad (6) \\
b_z &= (1-\alpha)b_k + \alpha b_{k+1} \quad (7) \\
c_z &= (1-\alpha)c_k + \alpha c_{k+1} \quad (8)
\end{align*}
\]

The increasing portion \( f : z \to x \) for the membership function of \( z \) can be found from the inverse function \( f^{-1} : x \to z \) given in Eq. (9).

\[
z = (1-\alpha)(x-a_i)/(b_i-a_i) + \alpha(x-a_{i+1})/(b_{i+1}-a_{i+1}) \quad (9)
\]

Similarly, the decreasing portion \( g : z \to x \) for the membership function of \( z \) is attained from the inverse function \( g^{-1} : x \to z \) given in Eq. (10).

\[
z = (1-\alpha)(c_i-x)/(c_i-b_i) + \alpha(c_{i+1}-x)/(c_{i+1}-b_{i+1}) \quad (10)
\]

The tangible parameters are then represented by the corresponding fuzzy membership functions.

3.2 Parameter comparison

The fuzzy preference relation (Zimmermann, 1987) of each corresponding parameter in the bids is first determined by directly comparing their fuzzy membership functions. The fuzzy preference relation \( P(u_1, u_2) \) between two fuzzy numbers \( u_1 \) and \( u_2 \) (represented by the fuzzy membership functions) indicates the degree of preference of \( u_1 \) over \( u_2 \). If \( P(u_1, u_2) > 0.5 \), \( u_1 \) is regarded to be more preferable to \( u_2 \) and vice versa. On the other hand, if \( P(u_1, u_2) = 0.5 \), \( u_1 \) is said to be indifferent (i.e. equally preferable) to \( u_2 \). By definition, \( P(u_1, u_2) \) and \( P(u_2, u_1) \) are complementary to each other so that \( P(u_1, u_2) + P(u_2, u_1) = 1 \) (Thurson & Carnahan, 1992).

One of the commonly adopted means to determine the fuzzy preference relation is to utilize the maximum and minimum set derived from the membership functions of the two fuzzy variables to be compared (Chen, 1985). Formulation of the fuzzy preference relation involves
tedious graphical manipulation of membership functions. As the number of bids to be compared increases, the computation time escalates drastically, which makes this method inefficient for the negotiation-sequencing problem. An alternative is to employ the Hamming distance approach (Tseng & Klein, 1989) which comes from the concept of dominance or indifference, defined by the extent or lack of overlapping areas of the membership functions of two fuzzy variables. The Hamming distance approach is therefore a more suitable approach because of shorter computation time required. The preference relation value is determined by Eq. (11), where $S(u_1, 0)$ and $S(u_2, 0)$ are the total areas of the membership functions of $u_1$ and $u_2$ respectively. $S(u_1, u_2)$, as shown in Fig. 2, is the area in the membership functions where $u_1$ dominates $u_2$ (assuming $u_1$ is in the right-hand side of $u_2$; otherwise, $S(u_1, u_2) = 0$), and $S(u_1 \cap u_2, 0)$ is the overlapping area of $u_1$ and $u_2$, which is illustrated in Fig. 3.

$$P(u_1, u_2) = \frac{S(u_1, u_2) + S(u_1 \cap u_2, 0)}{S(u_1, 0) + S(u_2, 0)}$$  (11)

If there are $m$ bids, there are a set of $m$ fuzzy numbers $U = \{u_i \mid i = 0, 1, \ldots, m\}$ which are to be compared as one of the parameters in the bids, the fuzzy preference relation values between each pair of numbers can be denoted by a matrix $P = [P(u_i, u_j)]_{m \times m}$, in which all the diagonal elements $P(u_i, u_i) = 0.5$. The matrix $P$ thus contains the information of comparison results of one particular parameter in the $m$ bids.

It should be noted that other means of comparing and ranking fuzzy numbers are possible. Many methods have been proposed (Adamo, 1980; Chen & Cheng, 2005; Klir & Yuan, 1995; Lee & Li, 1988; Mabuchi, 1988) and they have their merits and limitations. With certain modifications, they are also applicable in this particular study.

3.3 Aggregation and Ranking

The aggregation process combines the fuzzy preference relation matrices of all the
parameters in \( m \) bids (i.e. \( n \times m \times m \) matrices when there are \( n \) parameters in the bids) into a single matrix, the global preference relation matrix \( Q \), which indicates the overall comparisons among the \( m \) bids. The process presented in this study is based on the modifications of Tanino’s aggregation rule (Wang, 1997), which is described below.

Let \( Z = \{z_k\} \) be a set of fuzzified parameters in the bids, for \( 1 \leq k \leq n \). Further, let \( P_k(u_i, u_j) \) be the fuzzy preference relation for \( z_k \), for \( 1 \leq i, j \leq m \). The elements in the global preference relation matrix \( Q = [q(t_i, t_j)]_{m \times m} \) can then be obtained from Eq. (12), where \( a \lor b \) refers to the maximum of \( a \) and \( b \), \( u_i \) is a parameter in the bid \( t_i \) and \( u_j \) is the corresponding parameter in \( t_j \).

\[
q(t_i, t_j) = \begin{cases} 
\sum_{k=1}^{n} w_k [(P_k(u_i, u_j) - 0.5) \lor 0] & \text{for } i \neq j \\
\sum_{k=1}^{n} w_k |P_k(u_i, u_j) - 0.5| & \text{for } i = j \\
0.5 & \text{for } i = j
\end{cases}
\]

\( w_k \) is the weighting of \( z_k \), subject to \( \sum_{k=1}^{n} w_k = 1 \). Weighting plays an important role to denote the importance of the parameters. There are different approaches to assign or compute the weightings of the parameters. An experienced IP may be able to provide a set of weightings to capture the knowledge of the relative importance of the parameters. With such subjective judgment on the weightings, Analytic Hierarchy Process (AHP) (Saaty, 1980) can be applied. Conventional AHP employs exact values to score the weightings but the direct association of a number may not fully reflect the expert judgment or perception. When linguistic descriptors are more appropriate for assigning importance of the parameters, fuzzy AHP allows such imprecise and vague information to be processed (Cheng, 1996 & 1999; Duran & Aguilo, 2008; Huang et al., 2008).
With the global preference relation matrix \( Q \), the Degree of Dominance (DOD) among the bids is obtained from Eq. (13). \( t_i \) is preferred over \( t_j \) if \( DOD(t_i) > DOD(t_j) \), while \( t_i \) is indifferent to \( t_j \) if \( DOD(t_i) = DOD(t_j) \). Hence, the resulting descending order of the DOD values lists explicitly the sequence from the highest to the lowest rank of the \( m \) bids.

\[
DOD(t_i) = \sum_{\substack{j \leq m \\text{if} \ i \neq j}} q(t_i, t_j) \quad (13)
\]

The overall process of the fuzzy ranking analysis for the TSP bids is summarized in the flowchart as shown in Fig. 4.

4. Results and Discussions

In order to demonstrate the effectiveness and flexibility of the fuzzy ranking approach on negotiation sequence generation, case studies on an IP collecting bids from a number of TSPs and generating negotiation sequence accordingly are conducted. Comparisons with other negotiation sequencing methods with respect to the impact on the quality of the resulting timetables are given and the evaluations are made through statistical analysis.

4.1 Open Market

In this open market test-bed, there are 1 IP and 5 TSPs. The TSPs offer 3 different services, freight, intercity and regional services over a section of track with 5 stations (A to E) and spanning over 85 km. The inter-station track lengths are shown in Table 1. One freight service and one intercity service are operated on the line while there are 3 regional services running on the same line and serving all stations.

From Eq. (1), the track access rights from the TSP service bids contain 4 components. There is one parameter each for the track access charge \( c \), rolling stock \( \omega \) and flex level \( \phi \). From the
train schedule in Eq. (2), as 5 stations are on the line, there are 5 station dwell times $T_d$ and 4 inter-station runtimes $T_R$, as well as 1 service commencement time. The total number of parameters in the service bids is 13, which are to be compared one by one across the bids in the fuzzy ranking analysis.

4.2 Negotiation Sequence to Timetable

With the negotiation sequence available, the impact of the sequence on the quality of service can only be examined by the subsequent negotiations and the resulting timetable. In order to realize the negotiations between IP and TSP, a multi-agent system for open railway access market (MAS-ORAM) developed in a previous study (Tsang, 2007) has been employed. The MAS-ORAM is built on the popular middleware called JADE (Java Agent DEvelopment Framework) which provides the essential software components for agent development. In this model, each stakeholder in the railway open market is considered as a self-interested entity, or agent, that is capable of interacting with other agents in the system through an iterative process of bid-offer submission. A fuzzy-constraint based model (Tsang & Ho, 2004) has been devised for the TSPs to enable their agents to submit the desired track access rights bids and relax the constraints according to the IP reactions during the negotiations. By employing the Buyer-and-Seller-Behavior Protocol (Luo et al., 2003), the negotiation between a single IP and a single TSP agent (IP-TSP transaction) is guaranteed to settle at the Pareto-optimal solution if it exists. MAS-ORAM is thus the tool to convert the negotiation sequence into a train service timetable according to the contents of the TSP service bids.

The resulting timetables are to be compared against a set of indices with those attained from the two commonly used negotiation-sequence generating strategies, first-come-first-served (FCFS) and highest-willingness-to-pay-first (HW2PF) through statistical analysis, in terms of IP revenue and service quality. The first comparison index is the total IP utility ($IPU_T$) which is the sum of revenue to be collected by the IP through the succession of negotiations with the
TSPs. \( IPU_T \) is given in Eq. (14) where \( n_k \) is the total number of successful negotiations with the TSPs.

\[
IPU_T = \sum_{i=1}^{n_k} U_i \tag{14}
\]

From the perspective of passengers and freight customers, a better quality of service is perceived when the overall journey time can be reduced. Thus, the second comparison index, \( EJT_\theta \), measures the average deviation in journey time of a train service operated by a TSP of type \( \theta \) (i.e. freight, regional, intercity, etc.) from its desired schedule. For a TSP operating a set of \( n_\theta \) services, \( EJT_\theta \) is defined by Eq. (15), where \( \tilde{t}_{ij} \) and \( \hat{t}_{ij} \) (in min) are the actual and expected inter-station runtime of train \( i \) between stopping station \( j \) and \( j+1 \) respectively.

\[
EJT_\theta = \frac{1}{n_\theta} \sum_{i=1}^{n_\theta} \sum_{j=2}^{n_j} \max(\tilde{t}_{ij} - \hat{t}_{ij}, 0) \tag{15}
\]

When \( EJT_\theta = 0 \), the timetable is described as ‘without extension’ because all trains arrive at the stations no later than the time requested in the service bid. When \( EJT_\theta \) takes a value other than zero, the timetable is said to be ‘extended’, in which one or more of the services suffers from extension in journey time.

Trains are preferred to arrive at a station at equally spaced time intervals. Any deviations, either earlier or later, may lead to discontentment arising from overcrowding at platforms and trains. The third comparison index, \( DFR_\theta \), is the mean deviation from regularity of TSP \( \theta \) at all stopping station \( j \) and defined by Eq. (16). \( \hat{n}_\theta \) is the expected number of trains in an one-hour operation, \( n_\theta \) is the actual number of trains in service, \( t_{ij} \) (in min) is the arrival time of the \( i \)-th train at station \( j \), and \( t_{ij,1} = t_{ij} + 60 \), which assumes the timetable repeats in the subsequent hour.

\[
DFR_\theta = \frac{1}{n_\theta} \sum_{j=1}^{n_j} \sum_{i=1}^{n_i} |t_{ij,1} - t_{ij} - 60 / \hat{n}_\theta| \tag{16}
\]
When $DFR_\theta = 0$, the timetable is referred as ‘periodic’ because the TSP operates trains with equally spaced time intervals at all stations. If $DFR_\theta$ takes a value other than zero, the timetable is said to be ‘non-periodic’.

4.3 Statistical Analysis

As the TSP service bids contain many parameters and there are a huge number of combinations of values for these parameters, case-based comparison does not offer the most pragmatic approach because conclusions drawn from the results are only valid to the specific set of input values, which may hardly be representative in practice. In order to obtain generalized findings associated with the negotiation based on the sequence given by fuzzy ranking, a statistical analysis is more appropriate.

In a statistical analysis with an appropriate simulation tool, the set of input variables $\Theta_y = \{y_i | i = 1, 2, ..., v\}$ are modeled by a set of known probability functions $P_i : y_i \rightarrow [0,1]$. A random instance $\hat{y}_i$ is generated for each variable and they are delivered to a simulator, which produces a set of output instances $\hat{x}_i$ for the variable set $\Theta_x = \{x_i | i = 1, 2, ..., u\}$. If the process is repeated for $m$ times, it is possible to construct the sample distribution $X_i$ and compute the sample mean $\bar{x}_i$ for each output variable. Although the population distributions are unknown, the distributions of their sample means $\bar{X}_i$ are approximately normal if the sample size $m$ is sufficiently large (Walpole et al., 1998). As a result, by selecting a suitable test-statistics (e.g. $z$-test or $t$-test statistics) to analyze the output data, the population means can be estimated. The process is summarized in Fig. 5.

In this study, the parameters in the services bids are made random variables with known probability density functions (pdfs) and instances are drawn from the respective pdfs. The MAS-ORAM provides the simulator tool to facilitate the negotiations and produce the resulting
timetables and hence the comparison indices, $IPU_{T}$, $EJT_{\theta}$, and $DFR_{\theta}$. To enable comparisons among the negotiation-sequence generating methods over a substantial number of simulation runs, a set of two-sample hypothesis tests on the mean of the indices obtained from different negotiation-sequence generating methods are performed. The hypothesis tests are based on the $t$-test statistics because the population variances are unknown. A 95% confidence interval (i.e. $\alpha = 0.05$) is employed in the hypothesis tests.

4.4 Case Studies

A few case studies are given here to illustrate the operation and performance of the fuzzy ranking approach to generate negotiation sequences. In the case studies, the IP agent issues a Request-For-Bid (RFB) message to the 5 TSP agents. The service commencement time spans from 07:00 to 07:59. Interested TSP agents are allowed to submit their bids after the issue of RFB.

Table 2 lists the probability density functions and the range of values of the bid parameters. $U(a_{1}, a_{2}, ..., a_{n})$ denotes a uniform distribution among feasible discrete values of $a_{1}, a_{2}, ..., a_{n}$. $U(a_{1}:a_{n})$ specifies a similar distribution with values $a_{1}, a_{1}+1, ..., a_{n}$. $N(\mu, \sigma^{2})$ denotes a normal distribution with population mean $\mu$ and variance $\sigma^{2}$. $P(a, \lambda, t)$ represents a right-shifted Poisson distribution by $a$ units with decay constant $\lambda$ and time interval $t$. The flex level is set the same for all TSPs.

As given in Table 2, the intercity service is prepared to pay higher track access charge and it only stops at selected stations. The regional services need to stop at all stations but the station dwell times are short. When there are 3 regional services, regularity becomes an important service quality measurement. On the other hand, the freight service tends to pay lower track access charge. The train is slower and spends more time at stations (i.e. it occupies the track over longer time span).
For the fuzzification process, a linguistic scale of seven membership functions is adopted for the intangible parameters, which is a trade-off between reasonable resolution and computational demand. To ensure consistency for the tangible parameters, seven triangular membership functions over the normalized interval are also employed to derive the corresponding fuzzy variables.

To keep Type I and Type II errors within acceptable level in the hypothesis tests (Watkins et al., 2004), sample instances have to be drawn from the parameter pdfs and passed through the simulation to attain the negotiation sequence for a sufficient number of times in order to ensure adequate sample size for the statistical analysis. In the case studies, a total of 155 negotiation sequences are generated from random samples of the parameters. Three test cases are discussed here, with a) equal weightings on the parameters; b) 90% weighting on track access charge only; and c) 90% weighting on station dwell times only (i.e. the remaining 10% weighting shared among other parameters). The fuzzy ranking method (FRM) is compared against and the two negotiation-sequence generating methods, FCFS and HW2PF. The results are summarized in Tables 3-8.

4.5 Discussions

Table 3 lists the number of successful negotiations of each service from the 155 negotiation sequence generations. The unsuccessful negotiation implies that the requested service cannot be fitted into the timetable and the corresponding TSP is excluded from the negotiation sequence. In such cases, the TSP bids are not included in the statistical analysis. As the successful rates are always over 90%, there are sufficient cases to derive valid conclusions in the statistical analysis.

Table 4 indicates the IP revenue under different negotiation-sequence generating methods while Tables 5-7 give the service extension times on the intercity, freight and regional services respectively. As there are only one intercity and one freight service, service regularity only applies to the regional services, which is illustrated in Table 8.
In the test case (a) where the weightings on the bid parameters are the same, the resulting timetables do not give better IP revenue or service timings than FCFS and HW2PF do. It is however not unexpected as FRM is not intended to improve the timetable, but to provide a flexible means to establish a negotiation sequence, from which the timetable is derived. Indeed, certain bid parameters are in conflict with others. If no preference is imposed on any bid parameter, the negotiation sequence and hence the timetable do not favor the specific evaluation indices in IP revenue or service timings. In the extreme cases, the bid parameters are in such a conflict that the negotiation between the IP and the TSP does not lead to any mutually agreed service in the timetable. On the other hand, when the parameter preferences are indicated as in the test cases (b) and (c), the desired effects on service timetabling can be reflected on the corresponding service evaluation indices.

From Table 2, the intercity service TSP tends to pay higher TAC. As a result, it is more favorable in the test case (b), which means the intercity service bid is given the first priority in the negotiation sequence in most cases. As the track access is allocated to the intercity service first, IP revenue is inevitably higher and no service extension time is needed. As shown in Table 5, extension time comparison on intercity service is not available (or actually not necessary). In fact, test case (b) is very close to HW2PF in which the willingness to pay higher TAC is given the priority. Thus, HW2PF and FRM case (b) produce very similar results on timetable evaluation. However, the priority of the intercity service is at the expense of the quality of the other services. In particular, the regional services suffer worse as they have a slightly lower successful negotiation rate and the service regularity deteriorates substantially.

Station dwell time is given a dominating weighting in the test case (c). While the regional services require much shorter dwell times, they are given the priority in the negotiation sequence and subsequently in track access allocation. On the other hand, the IP revenue is lower when compared with the test case (b). Both service extension time and regularity are reduced significantly. The freight service carries the longest dwell time and hence its track access
allocation is often given last. The successful negotiation rate drops and the service extension time is lengthened.

The test cases described above only represent a few typical IP-TSP negotiation scenarios while others are also possible. They serve the purposes of demonstrating the flexibility of the fuzzy ranking approach and highlighting the effects of the service bid contents on the negotiation sequence and then the timetable.

5 Conclusions

Negotiations among the stakeholders to establish a service timetable in an open railway market requires appropriate negotiation sequence. It involves consideration of the service bid contents and comparisons of a large number of individual bid parameters which are of qualitative and quantitative characteristics. Prioritizing the service bids for negotiation and then attaining certain objectives on resource allocation, cost and service optimization thus requires multi-criteria comparison and combination among the service bids. This study proposes a fuzzy ranking approach to support this decision-making process.

The service bid parameters are represented by fuzzy numbers and compared across the bids. They are then combined according to their relative importance in order to determine the ranking of the bids. The negotiation sequence is evaluated by the resulting negotiated timetable, in terms of cost return and service quality. The evaluation is carried out by statistical analysis and the negotiation is facilitated by a multi-agent open railway market simulation system in which the stakeholders are acted by self-interested agents. The results show that the fuzzy ranking method provides an objective and systematic means to integrate the bid parameters of different natures and compare bids on a well-defined and open basis, as well as the flexibility for the railway operator and regulator alike to carry out various ‘what-if’ studies on open railway market.
The introduction of the open railway market is gathering momentum worldwide. Prior to the legislation and implementation, it is essential to predict the possible effects of the business objectives, service provisions, negotiation behaviors of the stakeholders; market regulations; and market demands on the viability of the market, customer affordability and service quality. Numerous studies on a wide range of issues have been underway and this research work enables further investigations on service bid contents and negotiations between IP and TSPs (or other types of stakeholders) when the number of stakeholders increase.

Acknowledgements

The authors gratefully acknowledge the generous financial support provided by the Research Grant Council of Hong Kong (PolyU 5174/06E) and the Hong Kong Polytechnic University.

References


Fig. 1. Triangular fuzzy membership function

Fig. 2. Area of $u_1$ dominating over $u_2$, $S(u_1, u_2)$

Fig. 3. Overlapping area between $u_1$ and $u_2$, $S(u_1 \cap u_2, 0)$
Fig. 4. Fuzzy ranking analysis on TSP service bids
Fig. 5. Statistical analysis for comparing timetables attained from different negotiation sequences.
Table 1. Track configuration

<table>
<thead>
<tr>
<th>Origin</th>
<th>Destination</th>
<th>Track Length (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>20</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>30</td>
</tr>
<tr>
<td>C</td>
<td>D</td>
<td>15</td>
</tr>
<tr>
<td>D</td>
<td>E</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 2. Service bid parameter pdfs

<table>
<thead>
<tr>
<th></th>
<th>Intercity</th>
<th>Regional</th>
<th>Freight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of service</td>
<td>U(1)</td>
<td>U(3)</td>
<td>U(1)</td>
</tr>
<tr>
<td>TAC</td>
<td>N(1600, 625)</td>
<td>N(1500, 625)</td>
<td>N(1375, 100)</td>
</tr>
<tr>
<td>Commencement time</td>
<td>U(0:59)</td>
<td>U(0:19)</td>
<td>U(0:59)</td>
</tr>
<tr>
<td>Dwell Time at A</td>
<td>N(5, 0.25)</td>
<td>P(1, 0.2, 1)</td>
<td>N(15, 1)</td>
</tr>
<tr>
<td>Dwell Time at B</td>
<td>-</td>
<td>P(1, 0.2, 1)</td>
<td>N(15, 1)</td>
</tr>
<tr>
<td>Dwell Time at C</td>
<td>-</td>
<td>P(1, 0.2, 1)</td>
<td>-</td>
</tr>
<tr>
<td>Dwell Time at D</td>
<td>-</td>
<td>P(1, 0.2, 1)</td>
<td>N(15, 1)</td>
</tr>
<tr>
<td>Dwell Time at E</td>
<td>N(5, 0.25)</td>
<td>P(1, 0.2, 1)</td>
<td>N(15, 1)</td>
</tr>
<tr>
<td>Runtime at AB</td>
<td>P(11, 0.3, 1)</td>
<td>P(15, 0.5, 1)</td>
<td>P(24, 0.7, 1)</td>
</tr>
<tr>
<td>Runtime at BC</td>
<td>P(16, 0.3, 1)</td>
<td>P(24, 0.5, 1)</td>
<td>P(35, 0.7, 1)</td>
</tr>
<tr>
<td>Runtime at CD</td>
<td>P(9, 0.3, 1)</td>
<td>P(14, 0.5, 1)</td>
<td>P(23, 0.7, 1)</td>
</tr>
<tr>
<td>Runtime at DE</td>
<td>P(11, 0.3, 1)</td>
<td>P(15, 0.5, 1)</td>
<td>P(24, 0.7, 1)</td>
</tr>
</tbody>
</table>

Table 3. Successful negotiations

<table>
<thead>
<tr>
<th>Services</th>
<th>FCFS</th>
<th>HW2PF</th>
<th>FRM Case (a)</th>
<th>FRM Case (b)</th>
<th>FRM Case (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercity</td>
<td>155</td>
<td>142</td>
<td>155</td>
<td>83</td>
<td>155</td>
</tr>
<tr>
<td>Freight</td>
<td>155</td>
<td>150</td>
<td>151</td>
<td>155</td>
<td>150</td>
</tr>
<tr>
<td>Regional</td>
<td>465</td>
<td>461</td>
<td>460</td>
<td>462</td>
<td>458</td>
</tr>
<tr>
<td>Total:</td>
<td>775</td>
<td>753</td>
<td>766</td>
<td>700</td>
<td>763</td>
</tr>
<tr>
<td>Successful rate (%)</td>
<td>97.16</td>
<td>98.84</td>
<td>90.9</td>
<td>98.45</td>
<td>98.06</td>
</tr>
</tbody>
</table>
### Table 4. IP Revenue $\text{IP}_U$

<table>
<thead>
<tr>
<th>Sequence policies</th>
<th>FCFS</th>
<th>HW2PF</th>
<th>FRM Case (a)</th>
<th>FRM Case (b)</th>
<th>FRM Case (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean ($/service$)</td>
<td>6791</td>
<td>6958</td>
<td>6268</td>
<td>6918.3</td>
<td>6846.68</td>
</tr>
<tr>
<td>Standard deviation ($/service$)</td>
<td>528</td>
<td>335.4</td>
<td>753.1</td>
<td>378.2</td>
<td>429.33</td>
</tr>
</tbody>
</table>

$H_0$ (null hypothesis): $\mu_{\text{FRM}} = \mu_{\text{FCFS}}$

$H_1$ (alternative hypothesis): $\mu_{\text{FRM}} > \mu_{\text{FCFS}}$

Critical t-score ($\alpha=0.05$)

<table>
<thead>
<tr>
<th></th>
<th>FRM Case (a)</th>
<th>FRM Case (b)</th>
<th>FRM Case (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t-score</td>
<td>-3.245</td>
<td>-10.341</td>
<td>2.143</td>
</tr>
</tbody>
</table>

Conclusions: Accept $H_0$

### Table 5. Intercity service - extension time $\text{EJT}$

<table>
<thead>
<tr>
<th>Sequence policies</th>
<th>FCFS</th>
<th>HW2PF</th>
<th>FRM Case (a)</th>
<th>FRM Case (b)</th>
<th>FRM Case (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (mins/service)</td>
<td>13.6</td>
<td>0</td>
<td>15.25</td>
<td>0</td>
<td>17.25</td>
</tr>
<tr>
<td>Standard deviation (mins/service)</td>
<td>7.77</td>
<td>--</td>
<td>6.23</td>
<td>--</td>
<td>4.57</td>
</tr>
</tbody>
</table>

$H_0$ (null hypothesis): $\mu_{\text{FRM}} = \mu_{\text{FCFS}}$

$H_1$ (alternative hypothesis): $\mu_{\text{FRM}} < \mu_{\text{FCFS}}$

Critical t-score ($\alpha=0.05$)

<table>
<thead>
<tr>
<th></th>
<th>FRM Case (a)</th>
<th>FRM Case (b)</th>
<th>FRM Case (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t-score</td>
<td>-2.002</td>
<td>-2.661</td>
<td>--</td>
</tr>
</tbody>
</table>

Conclusions: Accept $H_0$

### Table 6. Freight service - extension time $\text{EJT}$

<table>
<thead>
<tr>
<th>Sequence policies</th>
<th>FCFS</th>
<th>HW2PF</th>
<th>FRM Case (a)</th>
<th>FRM Case (b)</th>
<th>FRM Case (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (mins/service)</td>
<td>4.86</td>
<td>4.89</td>
<td>5.5</td>
<td>4.865</td>
<td>5.716</td>
</tr>
<tr>
<td>Standard deviation (mins/service)</td>
<td>1.16</td>
<td>4.23</td>
<td>4.4</td>
<td>3.924</td>
<td>4.51</td>
</tr>
</tbody>
</table>

$H_0$ (null hypothesis): $\mu_{\text{FRM}} = \mu_{\text{FCFS}}$

$H_1$ (alternative hypothesis): $\mu_{\text{FRM}} < \mu_{\text{FCFS}}$

Critical t-score ($\alpha=0.05$)

<table>
<thead>
<tr>
<th></th>
<th>FRM Case (a)</th>
<th>FRM Case (b)</th>
<th>FRM Case (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t-score</td>
<td>-0.691</td>
<td>-0.674</td>
<td>--</td>
</tr>
</tbody>
</table>

Conclusions: Accept $H_0$
### Table 7. Regional service - extension time *EJT*

<table>
<thead>
<tr>
<th>Sequence policies</th>
<th>FCFS</th>
<th>HW2PF</th>
<th>FRM Case (a)</th>
<th>FRM Case (b)</th>
<th>FRM Case (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong> (mins/service)</td>
<td>6.9</td>
<td>7.419</td>
<td>9.637</td>
<td>6.646</td>
<td>3.638</td>
</tr>
<tr>
<td><strong>Standard deviation</strong> (mins/service)</td>
<td>6.372</td>
<td>5.676</td>
<td>9.085</td>
<td>5.465</td>
<td>2.653</td>
</tr>
</tbody>
</table>

- $H_0$ (null hypothesis) $\mu_{FRM} = \mu_{FCFS}$
- $H_1$ (alternative hypothesis) $\mu_{FRM} < \mu_{FCFS}$

| t-score | -2.975 | -1.841 | 0.387 | 1.367 | 3.616 | 6.324 |
| Conclusions | Accept $H_0$ | Accept $H_0$ | Accept $H_0$ | Accept $H_0$ | Accept $H_1$ | Accept $H_1$ |

### Table 8. Regional service - deviation from regularity *DFR*

<table>
<thead>
<tr>
<th>Sequence policies</th>
<th>FCFS</th>
<th>HW2PF</th>
<th>FRM Case (a)</th>
<th>FRM Case (b)</th>
<th>FRM Case (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong> (mins/service)</td>
<td>19.614</td>
<td>19.231</td>
<td>33.98</td>
<td>27.288</td>
<td>11.19</td>
</tr>
<tr>
<td><strong>Standard deviation</strong> (mins/service)</td>
<td>21.603</td>
<td>12.92</td>
<td>59.24</td>
<td>50.12</td>
<td>7.856</td>
</tr>
</tbody>
</table>

- $H_0$ (null hypothesis) $\mu_{FRM} = \mu_{FCFS}$
- $H_1$ (alternative hypothesis) $\mu_{FRM} < \mu_{FCFS}$

| t-score | -2.512 | -2.465 | -1.077 | -0.927 | 4.005 | 6.198 |
| Conclusions | Accept $H_0$ | Accept $H_0$ | Accept $H_0$ | Accept $H_0$ | Accept $H_1$ | Accept $H_1$ |