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Freeway Traffic Estimation in Beijing based on Particle Filter

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Abstract—Short-term traffic flow data is characterized by rapid and dramatic fluctuations. It reflects the nature of the frequent congestion in the lane, which shows a strong nonlinear feature. Traffic state estimation based on the data gained by electronic sensors is critical for much intelligent traffic management and the traffic control. In this paper, a solution to freeway traffic estimation in Beijing is proposed using a particle filter, based on macroscopic traffic flow model, which estimates both traffic density and speed. Particle filter is a nonlinear prediction method, which has obvious advantages for traffic flows prediction. However, with the increase of sampling period, the volatility of the traffic state curve will be much dramatic. Therefore, the prediction accuracy will be affected and difficulty of forecasting is raised. In this paper, particle filter model is applied to estimate the short-term traffic flow. Numerical study is conducted based on the Beijing freeway data with the sampling period of 2 min. The relatively high accuracy of the results indicates the superiority of the proposed model.

Index Terms—short-term traffic flow; particle filter; traffic estimation; Beijing freeway

I. INTRODUCTION

Increased traffic flow on roadways results in congestion. Congestion has been becoming a serious problem on urban network, and we have traded congestion in the urban core for congestion at the urban periphery in many cities of the world. Congestion leads to delays, decreasing flow rate, higher fuel consumption and thus has negative environmental effects. Among kinds of the traffic flow data, the short term data is characterized by rapid and dramatic fluctuations. It reflects the nature of the frequent congestion in the lane, which shows a strong nonlinear feature. Traffic state estimation based on the data gained by electronic sensors is critical for much intelligent traffic management and the traffic control.

II. PROBLEM DESCRIPTION

Many of the proposed solutions for estimating of aggregated traffic variables are based on the application of Extended Kalman Filtering (EKF) in macroscopic models [4-5]. Furthermore, most of them assume Gaussian distribution of the noises and are based on linearization of the state and observation models. Therefore, it is characterized as: computationally cheap, but relying on a linearization of the state and measurement models which can cause filter divergence. However, traffic flow data shows a strong nonlinear feature. Particle Filter (PF), a powerful and scalable approximate approach that has recently been developed, calculates the posterior density function of the state by an empirical histogram obtained from samples generated by a Monte Carlo simulation. And it is appropriate for traffic state estimation because it can cope with large and highly nonlinear models as well as non-Gaussian signals. In [1], a solution to high way traffic estimation is proposed by using particle filter, based on stochastic and macroscopic model. The freeway is considered as a network of components, and the time period of measurement is 10s, and It is shown in [6] that the macroscopic model works pretty accurately with segment lengths in the order of 500m (or less) and model time step in the order of 10s.

The time interval of traffic flow data collected by the sensor in Beijing freeway is 2 min, so that the volatility of the traffic state curve is more significant. And the state curve will appear instantaneous mutation in some congested traffic periods and busy sections. All above makes the traffic estimation challenging. In this paper, a solution to freeway traffic estimation in Beijing is proposed using a particle filter, based on macroscopic traffic flow model, which estimates both traffic density and speed.

The outline of the paper is as follows. Section III presents the estimation framework; a problem of traffic state estimation is formulated in the Section. Section IV proposed the estimation model. The model we are using is macroscopic traffic flow model and a particle filter traffic state estimation is designed. The particle filters performance evaluation is presented in Section V. Finally, conclusions are highlighted in the last section.

III. ESTIMATION FRAMEWORK

A freeway network may be represented as a directed graph. More precisely, bifurcations, junctions, on-ramps, and off-ramps are represented by the nodes, whereas the freeway stretches between these locations are represented by the links of the graph. The two directions of a freeway stretch are modeled as separate links with opposite directions. Inside each link homogeneous geometric characteristics such as number of lanes, slopes, curvatures. The traffic flow is modeled as a stochastic hybrid system, with continuous and discrete states, interacting with states from neighboring sections, and sensors are located at the first and last sections. It is explained and illustrated in Fig. 1.
Fig. 1 Freeway way links and measurement points

$q_i$ is the average number of vehicles at the boundary between sections $i$ and $i+1$. In this paper, state vector are described as $\mathbf{x}_{i,k}$, the average speed of the vehicles counted in section $i$, and $d_{i,k}$, the average density of these vehicles. The whole state of link $m$ at time $k$ is described by the vector $\mathbf{x}_{k,m} = [x_{i,k}, x_{i+1,k}, ..., x_{N,k}]^T$, containing local state vectors $x_{i,k} = (v_{i,k}, d_{i,k})^T$ of $i$th section forming this link.

Noisy measurements of mean speed of the vehicles crossing the boundary between section $i$ and section $i+1$ during the time interval $(t_i, t_{i+1})$, together with noisy measurements of mean density of these vehicles.

The evolution from one sample time to the next sample time is described by the update equation:

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{v}_k, \mathbf{d}_k, q_{i,k}, \mathbf{z}_k, p_k)$$

$p_k$ denotes the vector of all time-varying parameters, such as road conditions, or number of available lanes. In Fig.1 variables are not indicated (input and output flows within section boundaries $0$ and $N$).

Within the intervals $(t_i, t_{i+1})$ several state update steps are required, and the observation equation is given by

$$y_k = h[\mathbf{x}_k, \mathbf{z}_k, k, \in (t_i, t_{i+1})]$$

IV. PROPOSED ESTIMATION MODEL

The framework of PF to estimate the traffic flow is explained below.

A. Traffic Flow Model

A second-order validated macroscopic traffic flow model [6-8] is employed to describe the dynamic behavior of traffic flow along a freeway stretching terms of Appropriate aggregated traffic variables, which are traffic density, space mean speed, and traffic flow. For the convenience of computation, the macroscopic model is usually presented in a space-Time discretized form. More specifically, a considered way stretch is subdivided into a number $N$ of segments with lengths $L_i, i = 1, 2, ..., N$ while the time discretization is based on a time step $T$ and the discrete time indices $k = 0, 1, 2...$. The macroscopic aggregated traffic variables are denoted in this discrete space-time frame as follows:

(a) Traffic density $d_{i,k}$ (veh/km/lane) is the number of vehicles in segment $i$ at time $kT$, divided by the segment length $L_i$ and lane number $\lambda$.

(b) Space mean speed $v_{i,k}$ (km/h) is the average speed of all vehicles included in segment $i$ at time $kT$.

(c) Traffic flow $Q_{i,k}$ (veh/h) is the number of vehicles leaving segment $i$ during the time period $[kT, (k+1)T]$, divided by $T$. For a segment $i$, the stochastic nonlinear difference equations of the second-order macroscopic traffic flow model are as follows:

$$d_{i,k+1} = d_{i,k} + \frac{1}{L_i} \left[ q_{i-1,k} - q_i,k + r_i(k) - s_i(k) \right]$$

$$s_i(k) = \beta_i(k) \cdot q_{i-1,k}$$

$$v_{i,k+1} = v_{i,k} + \frac{T}{L_i} \left[ v(d_{i,k}) - v_{i,k} \right] + \frac{T}{L_i} v_{i,k} \left[ q_{i-1,k} - q_{i,k} \right] - \frac{\delta T}{L_i} r_i(k) v_{i,k}^2$$

$$\frac{\delta T}{L_i} r_i(k) v_{i,k}^2 + \xi_{i,k}$$

$$v(d) = v_f \exp\left[-\frac{1}{\alpha} \left( \frac{d}{d_p} \right)^\gamma \right]$$

$$q_{i,k} = d_{i,k} \cdot v_{i,k} \cdot l_i + \xi_{i,k}$$

Where (3), (5), (6) and (7) are the well-known conservation equation, dynamic speed equation, stationary speed equation,
and flow equation, respectively: \( \tau, v, \delta, \kappa, v_f, \rho_c \) and \( a \) are model parameters, in particular, \( \tau, v, \delta, \kappa \) are given the same values for all segments. While \( v_f \) (free speed), \( \rho_c \) (critical density), and \( a \) (exponent parameter) address each individual link. \( \xi_i(k) \) and \( \xi_f(k) \) denote zero-mean Gaussian white noise acting on the empirical speed equation and the approximate flow equation, respectively, to reflect the modeling in accuracies.

In [5-6] a model is introduced that also represents the evolution of the average speed in each section, and the density is upstate according to [3]. In addition, Note that (3) is not corrupted by noise as it describes the conservation of vehicles, which holds strictly in any case. The model parameters are normally unknown and may vary with environmental conditions.

### B. Observation Equations

Consider a traffic detector installed at the boundary of two adjacent segments \( i \) and \( i + 1 \), as illustrated in Fig.1. For the speed, we have the measurement

\[
Z_{v,i}(k+1) = v_f(k+1) + \xi_{v,i}
\]

(8)

Where denotes the mean speed measurement during the time period and denotes the corresponding speed measurement noise. For the density, we have the measurement

\[
Z_{d,i}(k+1) = d_i(k+1) + \xi_{d,i}
\]

(9)

Where denotes the mean density measurement during the time period and denotes the corresponding density measurement noise.

### V. PARTICLE FILTER FOR URBAN TRAFFIC

#### A. Particle Filter Theory

According to the Bayesian theory, all information about the states of interest can be obtained from the posterior state distribution. Within the Bayesian framework, Particle filters are used in order to sequentially update a priori knowledge about predetermined state variable \( x_k \) by using the observation data \( Z_k \). The optimized state estimation is obtained via calculating the degree of confidence of \( p(x_k \mid Z_k) \) in different states. Moreover, the conditional density \( p(x_k \mid Z_k) \) is recursively updated according to:

\[
p(x_{k+1} \mid Z_k) = \int_{z_k} p(x_{k+1} \mid x_k) p(x_k \mid Z_k) dx_k
\]

(10)

\[
p(x_k \mid Z_k) = \frac{p(z_k \mid x_k) p(x_k \mid Z_k^{k-1})}{p(z_k \mid Z_k^{k-1})}
\]

(11)

Where \( p(z_k \mid Z_k^{k-1}) \) is a normalized constant, \( p(x_k \mid Z_k) \) is proportional to \( p(Z_k \mid x_k) p(x_k \mid Z_k^{k-1}) \), but the analytical expression for propagating \( p(x_k \mid Z_k) \) through (11) is difficult.

Particle filtering is particularly useful in dealing with nonlinear and non-Gaussian problems. And particle filtering technique [3] is a sequential Monte Carlo methodology where the basic idea is the recursive computation of relevant probability distributions using the concepts of importance sampling and approximation of probability distributions with discrete random measures [1]. In this view, it provides an approximate solution to this discrete-time recursive updating of the posterior probability density function \( p(x_k \mid Z_k) \).

It’s assumed that \( x_{0:k} = \{x_j, j = 0,\ldots,k\} \) is the all state during the time period \([0,k]\), \( \{x_{0:k}, i = 0,\ldots,n\} \) is collection of \( N \) particles(samples), and each particle has an assigned relative weight, \( \{w_i, i = 0,\ldots,N\} \), such that the sum of all weights is unity. The particle filter updates the value of the particles location and their weights according to Bayes’ rule, after the arrival of a new observation \( Z_k \). The cloud of particles evolves with time and depending on the observations, so that the particles represent with sufficient accuracy the true conditional density of the state. \( p(x_{0:k} \mid y_{0:k}) \) is defined as the posterior probability density function, so that it is approximated by

\[
p(x_{0:k} \mid y_{0:k}) = \sum_{i=1}^{N} w_i \delta(x_{0:k} - x_{0:k}^i)
\]

(12)

Where \( \delta \) is the delta-Dirac function.

#### B. Estimation Framework

The framework of Particle Filter to estimate the traffic flow is illustrated in Fig.2.

![Fig. 2 Estimation framework](image)

### VI. NUMERICAL STUDY

#### A. Simulation Problem

In this paper, we estimate the freeway traffic flow in Beijing by using west of Peace Bridge to west of San Yuan Bridge as a link. The link considered consists of three sections, with measurements received only at the boundaries of the first and third sections. The boundary conditions (in flow and speed at entrance and density in section 4) are known, not estimated.

Hence, the augmented state vector is \( x_k = (v_{1:k}, d_{1:k}, v_{2:k}, d_{2:k}, v_{3:k}, d_{3:k}) \) and the measurement vector is \( y_k = (z_{1:k}, z_{2:k}) \). The initial section states is \( x_0 = [60, 4, 52, 6, 55, 8]^T \) and the size of the sample set is \( M = 500 \) particles.

The parameters of the state model are chosen as follows:
$t=20s, \ v=35km/s, \ \kappa =13veh/km/lane, \ \nu_f=90km/h, \ \rho_s=40veh/km, \ \alpha=2$. Traffic flow curve of the entrance section is shown as follows:

In this paper, the Root-mean square errors (RMSE) and relative Root-mean square errors (RMSER) are used to measure the performance of PF,

$$RMSE_\lambda = \sqrt{\frac{1}{KN} \sum_{k=0}^{KN} \sum_{i=1}^{N} (x_i(k) - \hat{x}(k/k-1))^2}$$  \hspace{1cm} (13)

$$RMSER_\lambda = \frac{RMSE_\lambda}{\frac{1}{KN} \sum_{k=0}^{KN} \sum_{i=1}^{N} x_i(k)}$$  \hspace{1cm} (14)

Where, $K$ is the simulation time span, $N$ is the number of divided sections, $x'$ is the estimated value of the filter, and $(k)$ is the corresponding true value. In addition, the error analysis will estimate the density and velocity estimates considered separately.

**B. Filter Performance**

The measurements are supplied to the filter every minute, similarly to a possible update with real data, whereas the state prediction is performed also in the intermediate time instants. Density and speed are used to evaluate the filter performance.

Each component of the state vector are presented in Fig 4 and Fig 5, (Performance value of each section is shown in table 1) and all figures are plotted from the estimated states. According to these results the particle filter can accurately estimate the traffic states, and for the very brief increase in error during the fast transients, particle filter can also approximate estimate of its trends. Addition, as can be known from the figure.
Fig 7 Density RMSER

Fig 6 and Fig 7 show relative RMSE_R of each component of the state vector, and the RMSE_R of speed and densities in all segments are respectively 9.691% and 11.90%. According to above results, the RMSE_R of section 3 is larger compared with sections 1 and sections 2. Section 3 is located at the joint between East Third Ring Road and North Third Ring Road, namely the corner of Third Ring Road. In addition, there is an intersection of the city loop and Beijing-Chengde expressway. This particular environment of the road affects the stability of traffic flow significantly so that the accuracy of the results is also influenced. These cases lead to the large error in sections 3.

<table>
<thead>
<tr>
<th>Variable Index</th>
<th>Density</th>
<th>Speed</th>
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<td></td>
<td>RMSEA</td>
<td>RMSE_R</td>
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<td>Section 1</td>
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<tr>
<td>Average</td>
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<td>11.90%</td>
</tr>
</tbody>
</table>

VII. CONCLUSION

In the experiment, the particle filter and macro-stochastic traffic flow model are combined to estimate the short-term freeway traffic flow of Beijing. According to all simulation results, the particle filters performance efficiently in modeling the dynamics of the status section. It is noticeable that the data sampling interval is 2 minutes, namely the frequency of sampling is higher than the conventional ones. The use of particle filters in estimating the significant volatility traffic flow data in relatively high sampling frequency is regarded as the major contribution of this paper.

REFERENCES


