Vibration Characteristics of Shallow Suspension Bridge with Pre-tensioned Cables

By

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Abstract

Cable supported structures offer an elegant and economical solution for bridging over long spans with resultant low material content and ease of construction. These structures, however, tend to be slender and flexible as the structural stiffness mainly depends on the suspending cables. In this paper a cable supported bridge model with pre-tensioned reverse profiled cables is proposed to carry out a conceptual study on the vibration characteristics of shallow suspension pedestrian bridge structures. Effects of some structural parameters such as cable sag, cross sectional area, applied mass and pre-tensions in the reverse profiled cables have been investigated. Numerical results show that for cable supported bridges with shallow sag the lowest frequencies correspond to lateral and torsional vibration modes which are always combined together and become two types of coupled modes: coupled lateral-torsional modes, as well as coupled torsional-lateral modes. The effects of structural parameters are complex and the natural frequencies can be altered by introducing different pre-tensions in the reverse profiled cables.

Keywords cable supported bridge; suspension; pre-tensioned; vibration; natural frequency; reverse profile.

1. Introduction

Cable supported (pedestrian and highway) bridges are aesthetically pleasing and have gained popularity throughout the world. Due to the application of high strength materials and new technology, cable supported pedestrian bridges can be constructed to be much lighter and longer than the other types of bridge structures. Since the structural stiffness of cable supported pedestrian bridges mainly depend on the suspending cables, these bridge structures are often slender and flexible and sensitive to vibration due to the lower structural stiffness and mass. These vibration sensitive structures can exhibit unexpected modes of vibration such as lateral, torsional and longitudinal modes and their combinations under particular patterns of live loading, in addition to the flexural mode usually assumed in bridge design codes.

Dynamic performance and vibration characteristics of bridge structures have been concerns of researchers and bridge engineers for some time and much research work has been carried out on different types of cable supported pedestrian bridge structures. In general, cable supported pedestrian bridges can be classified as suspension, cable stayed or ribbon bridges. Suspension and cable stayed bridges achieve their load and deformation resistance and stability under vibration, oscillation and galloping effects using tension cables with significantly large sags or stayed cables hanging from tall towers interacting with stiffening girders or trusses [1, 2]. Cable supported ribbon bridges on the other hand adopt the principle of suspension bridges and develop it further by using high strength materials and modern engineering technology. They do not require tall towers and can span fairly long distances with shallow profiles [3, 4]. Brownjohn et al [5, 6] have studied the vibration characteristics of suspension footbridges in which the superstructure was composed of suspending cables with large sag, towers and a cross bracing structural system. In these systems, the structural stiffness depends on the suspending cables and cross bracing structure. Nakamura [7] has described some field measurements of lateral vibration on a pedestrian suspension bridge (M-bridge, Japan) and it was found that the bridge vibrated in either the third lateral asymmetric mode or the fourth lateral symmetric mode, depending on the distribution of the pedestrians on the bridge. Fujino et al. [8] have reported their observation of human-induced large-amplitude lateral vibration of an actual pedestrian cable stayed bridge (T-Bridge, Japan) in an extremely congested condition and analysed the walking motions of pedestrians recorded by a video camera. In order to suppress the horizontal vibration, they [9] introduced tuned liquid damper inside the box girder. Ten years later, they [10] studied this bridge again and found that the lateral vibration reduced even if some of the tuned liquid dampers didn't work properly. Stress ribbon bridges were introduced in the sixties of last century and have been built in various countries. The superstructure is generally composed of suspending cables with small sag and prestressed concrete decks. Pirner and Fischer [11, 12] have summarized the dynamic characteristics of three types of stress-ribbon pedestrian concrete bridges and described their experience with the design and response under wind, pedestrian and vandal-excited dynamic excitations. The first type consists of a prestressed ribbon hanging in catenary form; the second type consists of a stretched stress ribbon hung from suspending cables with the centres of curvature of the ribbon and of the suspending cable on opposite sides, under and over the pavements; the third type consists of a stretched stress ribbon supported by a classical arch structure near the mid-span. Tanaka et al [13] have developed two types of hybrid stress ribbon pedestrian bridges, whose superstructures include stayed cables (cable-stayed suspension structure) or suspension cables (stress ribbon suspension structure), with very light steel or a light concrete- steel girder. Dallard et al [14-16] have described the problems and solutions to the synchronous lateral vibration experienced by the Millennium footbridge in London. The Millennium footbridge in London is a tension ribbon bridge in which the cables (with a particular geometry) provide almost all the stiffness of the bridge in both vertical and horizontal directions. During the opening day, the bridge structure suffered strong lateral vibration induced by the pedestrians and had to undergo extensive retrofitting to make it functional.

Though cable supported pedestrian bridges can have different types of superstructures, it is evident that the structural stiffness is mainly provided by the suspending cable system and that these bridge structures are often slender and prone to vibration. However, for cable supported structures, the dynamic properties depend not only on the cable profile, but also on tension force in the cables, particularly for shallow sag cable structures such as cable supported ribbon bridges. In these structures, adjusting the cable tension and cable profiles can alter the dynamic characteristics such as natural frequencies.

In this conceptual study, a cable supported bridge model with pre-tensioned reverse profiled cables in the vertical plane and side pre-tensioned cables in the horizontal plane is proposed to investigate the vibration characteristics of shallow suspension bridges. In this bridge model, the transverse bridge frames with top, bottom and side legs hang from the top suspending cables and further restrained by the reverse profiled pre-tensioned bottom cables and pre-tensioned side cables. The decks are simply supported on the bridge frames. Effects of structural parameters such as cable sag and cross sectional areas and pre-tension on the vibration characteristics will be studied. The results will be helpful to understand the dynamic behaviour of pre-tensioned cable supported bridge structures and the influence of the important parameters on its dynamic response to loads.

2. Pre-tensioned Cable Supported Bridge Model

The proposed pre-tensioned cable supported bridge model is shown in Figure 1. In this bridge model, the cable system is composed of three groups of cables which may have same or different cable profiles: top suspending (or supporting) cables, bottom pre-tensioned cables (Figure 1(a)) and side pre-tensioned cables (Figure 1(b)). The top cables are two parallel suspending cables which have the catenary profiles and provide tension forces to support the whole structural gravity, applied loads and extra internal forces induced by the bottom pre-tensioned cables. Two parallel bottom cables are designed to have reverse profiles in the vertical plane and their function is to introduce pre-tension forces and provide extra internal vertical forces to transverse bridge frames and the top suspending cables. The side cables are a pair of bi-concave cables which have the same cable profiles in the horizontal plane, and their main function is to provide extra internal horizontal forces and horizontal stiffness. When the pre-tensioned bottom and/or side cables are slack, they could carry small tension forces only to support their own gravity and cannot resist any external loads. In this case, they will not be able to contribute stiffness and tension forces to the structure. However, these small tensions can provide sufficient restraining forces to prevent the transverse frames from swaying in the longitudinal direction.

Transverse bridge frames have been designed to support the deck and hold the cables. These frames (Figure 1(c)) comprise cross members (for the support beam and deck), top and bottom vertical legs as well as horizontal side legs and they form a set of spreaders for the cables to create the required profiles. They have in plane stiffness to protect against collapse under in plane forces and contribute very little in the way of longitudinal, lateral and rotational stiffness for the entire system. The transverse bridge frames are hung from the top cables, and further restrained by the lower reversed profile cables as well as the side cables. Two support beams of rectangular section are simply supported on cross members of the adjacent bridge frames, and the deck units are simply supported at the ends on these beams.

In this bridge model, the entire structural stiffness is provided by the cable systems. When the structure is subjected to applied loads, the entire load can be balanced by the tension forces in the cables with deformed cable profiles since these forces can provide components in different directions.

In order to simplify the problem, all the transverse bridge frames have been assumed to have the same size, and hence the weight of frame and deck acting on the cables can be considered as equal concentrated loads.

A typical symmetric cable profile with equal concentrated loads is shown in Figure 2. In the following description, different cables and cable profiles in the proposed bridge model are identified by the subscript j. When the subscript j equals to 1, 2 and 3, it represents the top, bottom and side cables as well as their profiles respectively.
For a cable supported bridge model with N uniform segments in the horizontal longitudinal direction, the forces from the N-1 transverse bridge frames can be modelled as N-1 equal concentrated loads acting on the cables. Assuming the horizontal distance between two adjacent transverse bridge frames to be a., the span length will be defined as:

$$L = Na \qquad (1)$$

For the j^{th} symmetric cable, the sag F_j is located at the middle segment or the middle node K. Choosing the local x – y coordinates as shown in Figure 2, the coordinates for the node K can be obtained as:

$$x_{jK} = Ka \quad y_{jK} = F_j \qquad K = \text{int}(N/2) \qquad (2)$$

Where *int*( ) is an integer function. The coordinate of the i^{th} node, j^{th} cable can be determined by

$$x_{ji} = ia \qquad y_{ji} = \alpha_i F_j \qquad i = 0,1,2,\cdots,N \qquad (3)$$

Here the coefficient α_i can be calculated by the following equation.

$$\alpha_i = i(N-i)/[K(N-K)] \qquad (4)$$

The tension force T_{ji} and tensile deformation ΔL_{ji} of the i^{th} segment, j^{th} cable can be obtained by

$$T_{ji} = \beta_{ji} W \quad \Delta L_{ji} = \gamma_{ji} Wa/(E_{ji} A_{ji}) \quad i = 0,1,2,\cdots,N; \quad j = 1,2,3 \qquad (5)$$

E_{ji} and A_{ji} are Young's modulus and area of cross section of the i^{th} cable segment, j^{th} cable. W is the applied equal concentrated load. The coefficients β_{ji} and γ_{ji} are shown to be as follows

$$\beta_{ji} = \frac{1}{2}\sqrt{[K(N-K)(a/F_j)]^2 + [N-2i+1]^2} \qquad (6)$$

$$\gamma_{ji} = \beta_{ji}\sqrt{1+(\alpha_i - \alpha_{i-1})^2 (F_j/a)^2} \qquad (7)$$

In the analysis of the bridge model, all the cables are stretched to keep the designed cable sags or cable profiles and then the decks can be kept in a horizontal plane. This can be done by introducing initial distortions to the cables according their cable sags, cross sectional areas, material properties, loads such as the weight of bridge frame and decks as well as cables, and extra internal forces produced by pre-tensioned reverse profiled cables or horizontal side cables.

Assuming the bottom cables have a diameter D_2, Young's modulus E_2, and cable sag F_2 and if the internal vertical force W_{int} at each bridge frame is induced (Figure 3), the initial distortion ΔL_{2i} introduced to the i^{th} cable segment of one bottom cable can be determined to be:

$$\Delta L_{2i} = -2\gamma_{2i} W_{\text{int}} a / (\pi E_2 D_2^2) \quad (8)$$

The side cables are a pair of bi-concave cables in the horizontal plane which have opposite cable profile to each other. When they are pre-tensioned, only internal horizontal forces can be introduced to the bridge frames. If the side cables have diameter D_3, Young's modulus E_3, and cable sag F_3 (in horizontal plane), and internal horizontal force Q_{int} at each bridge frame is induced by the pair of side cables, the initial distortion ΔL_{3i} introduced to the i^{th} cable segment of one side cable is determined as:

$$\Delta L_{3i} = -4\gamma_{3i} Q_{\text{int}} a / (\pi E_3 D_3^2) \quad (9)$$

When the internal vertical force W_{int} is induced at each bridge frame by pre-tensioned bottom cables, the top suspending cables are subjected to the weight (gravity) of the whole structure and the extra internal vertical forces. If the top suspending cables have diameter D_1, Young's modulus E_1, and cable sag F_1, and the total weight of one bridge frame, the cables and decks between adjacent frames is G , the following initial distortion ΔL_{1i} in the i^{th} cable segment of one top cable should be introduced:

$$\Delta L_{1i} = -2\gamma_{1i} (G + W_{\text{int}}) a / (\pi E_1 D_1^2) \quad (10)$$

After the initial distortions are introduced to the cable systems, the cable profiles can have the designed cable sags and the bridge deck will be kept in the horizontal plane before it is subjected to the applied loads.

The structural analysis package software MICROSTRAN [17] is adopted in the numerical study. In the bridge model, stainless steel (Young's modulus 2.0×10^{11} N/m^2 and density 7850 kg/m^3) is chosen for the transverse bridge frame and support beams, and Aluminium (Young's modulus 6.5×10^{10} N/m^2 and density 2700 kg/m^3) is chosen for the deck units to reduce the weight of the bridge structure. All members of the transverse bridge frames have a uniform rectangular cross sectional area of 250×300 mm^2 and the support beams have a uniform rectangular cross section of 200×250 mm^2. 8 deck units with dimensions 4000×500×50 mm^3 are simply supported on the support beams between the adjacent transverse bridge frames. Stainless steel cables are chosen for all the cable systems and the material properties are the same as those of bridge frames. In the numerical analysis, the span length is set to 80m, the horizontal distance between the adjacent bridge frames is set to 4m and the width of the deck for applied loads is set to 4m. The cable profiles (sags), cable sectional areas (diameters) and pre-tension are important structural parameters and can be changed for the parameter study. Two types of cable supported bridge models will be treated. Pre-tensioned bridge refers to a cable supported bridge model with pre-tensioned bottom and/or side cables. Un-pre-tensioned bridge (UPTB), on the other hand, refers to a cable supported bridge model with slack bottom and side cables which have no contribution to the structural stiffness but carry small tension forces to support their own gravity loads and prevent the transverse bridge frames from swaying in the longitudinal direction. To make the bottom and side cables slack, a small initial distortion (extension 0.01m) is introduced to these cables before the loads are applied.

3. Vibration Mode Shapes

In general, cable supported bridge structures and in particular suspension bridges, have four main types of vibration modes [18]: lateral, vertical, torsional and longitudinal modes. A cable supported bridge structure (pre-tensioned or un-pre-tensioned) with shallow cable sag will also have these four main kinds of vibration modes. However, numerical results show that the lateral modes and torsional modes do not always appear as pure lateral or torsional vibration modes. Often, they are combined together and form two types of coupled vibration modes: coupled lateral-torsional modes (LmTn or L-T) and coupled torsional-lateral modes (TmLn or T-L), where L and T represent Lateral and Torsional modes respectively, m and n are the number of half waves. In order to illustrate the typical vibration modal shapes, an un-pre-tensioned bridge model with the cable sag of 1.8m and cable diameter of 240 mm for all the cables ($F_1=F_2=F_3=1.8$ m, $D_1=D_2=D_3=240$, $W_{int}=0$, $Q_{int}=0$) is initially adopted for the free vibration analysis. In such bridges, the early modes of vibration, i.e. the modes with lower frequencies are usually coupled lateral – torsional (L-T) or coupled torsional-lateral (T-L) modes.

Coupled Lateral-Torsional Vibration Modes

When the bridge structure vibrates with the coupled lateral-torsional (L-T) vibration modes, one of the top suspending cables exhibits lateral and downward movements while the other top suspending cable has lateral and upward movement. In such situations the deck exhibits lateral movement and in addition appears to rotate and sway about a point above the deck. To some extent, the coupled lateral-torsional vibration modes can be called as lateral sway vibration modes. Figure 4 shows the first three coupled lateral-torsional vibration modes of the deck.

Results show that these vibration modes are dominated by the lateral vibration modes in conjunction with the torsional vibration. For the first coupled lateral-torsional vibration mode, the component of torsional movement will decrease and the mode can reduce to a pure lateral mode if the sag of the top suspending cables increases or a large pre-tension force is introduced to the pre-tensioned bottom cables. For the other coupled modes, the lateral vibration modes are always combined with torsional vibration.

Coupled Torsional-lateral Vibration Modes

In the particular bridge models treated here, the next higher modes are coupled torsional –lateral modes (T-L). These coupled torsional-lateral vibration modes can be considered as the reverse of coupled lateral-torsional vibration modes (Figure 5). One of the top suspending cables also has the lateral and downward movement and the other has the lateral and upward movement. However, the deck has lateral movement like the cables, and it rotates and sways about a point below the bridge deck. This type of vibration mode can be called reverse lateral swaying vibration mode. The dominant contributions are from torsional vibration and the coupled modes sometimes will reduce to pure torsional ones.

Vertical Vibration Modes

Vertical flexural modes assumed in bridge design codes for providing Dynamic Load Allowance (DLA) for design usually have higher frequencies than the early coupled L-T and T-L modes. When the bridge structure vibrates with the vertical vibration modes, the cables and deck have same upwards and downwards movement. These are common vibration modes for bridge structures. In general, most vertical vibration modes appear as pure vertical modes, without corresponding lateral or torsional ones. Figure 6 shows the elevation of the first three pure vertical vibration modes. Here, V represents the vertical modes, and the number represents the half waves. However, for cable supported bridge structures, coupled vertical vibration modes may exist. The most common coupled mode is the symmetric half wave vertical mode (V1) corresponding with the symmetric one and half wave mode (V3). Coupled asymmetrical vertical modes were not obtained in this investigation.

Longitudinal Vibration Modes

Longitudinal vibration modes exist in most cable supported bridge structures. When cable supported bridge structures vibrate in this kind of vibration modes, the bridge frames sway in the longitudinal direction. This kind of vibration modes can hence be called longitudinal swaying (LSW) vibration modes. As mentioned before, the longitudinal modes can be classified as pure longitudinal modes and modes associated with other vibrations. The pure longitudinal modes have distinct frequencies and modal configurations, while the other longitudinal modes participate in vertical modes as well. Figure 7 shows the first three longitudinal vibration modes. The first mode (LSW1) and the third mode (LSW3) are almost pure longitudinal modes, while the second mode is coupled with the first symmetric vertical vibration mode. It should be mentioned that the longitudinal vibration modes mainly depend on the connection between the bridge frames. When the connection is weak, these modes will correspond to low frequencies. Otherwise, they may correspond to high frequencies or disappear. For example, for a pre-tensioned cable supported bridge model, these modes will disappear when the pre-tension force in the bottom cables increase to a high level. Hence, in real bridges, these modes may not be significant.

4. Natural Frequencies and Effects of Structural Parameters

Natural frequencies and their corresponding vibration mode shapes are important vibration properties for dynamics of structures and they are affected by many factors such as structural stiffness, mass and structural geometry. Cable supported bridge structures are always flexible and slender with low natural frequencies, since the structural stiffness is mainly provided by the cable systems. In this section, the effects of important structural parameters on the natural frequencies and their corresponding vibration modal shapes have been investigated. These structural parameters include cable sag, cable cross sectional area (diameter), applied mass and pre-tensions (extra internal vertical and horizontal forces) in the pre-tensioned bottom and side cables. When the effect of cable cross sectional area is investigated, the total gravity load of the entire bridge structure is kept the same by changing the dimeters of the side cables in the following numerical analysis.

Effects of Cable Sag and Cross Sectional Area of the Top Suspending Cables (Un-pre-tensioned Bridge Model)

Cable sag and cross sectional area (diameter) have great effects on the structural stiffness. They also have evident effects on the natural frequencies and the corresponding modal shapes, particularly on the vertical vibration modes. As mentioned before, when an un-pre-tensioned cable supported bridge model is treated, the bottom and side cables are let to be slack and they can carry only small tension forces to support their own gravity loads and have no contribution to the structural stiffness. Numerical results show that these small tension forces will mainly affect the natural frequencies in the longitudinal direction and have only slight effect on the other vibration modes.

Table 1 shows the first twenty natural frequencies and their corresponding vibration modal shapes when the cable sag of the top suspending cables (as well as the slack bottom and side cables) is set to 1.2m, 1.8m and 2.4m. It can be seen that when the cable sag increases, the maximum tension force in the top suspending cables (T_1) decreases. All the frequencies decrease, except the first coupled lateral-torsional vibration mode which changes slightly and the frequency corresponding to the symmetric half wave vertical mode increases. This illustrates that the cable sag has a significant effect on the natural frequencies, since it changes the tension force in the top suspending cables and thereby influences the stiffness k. It can also be seen that the cable sag will change the order of vibration modes and the modal shapes. When the cable sag is set to 1.2m and 1.8m, the half wave symmetric vertical mode (V1) is the lowest vertical mode, and when the cable sag increases to 2.4m, the two half waves asymmetric vertical mode (V2) has become the first vertical mode. Also the first coupled torsional-lateral mode has reduced to a pure torsional vibration mode, while the other coupled modes retain the same modal shapes.

Table 2 shows the natural frequencies and their corresponding vibration modal shapes when the cables have different cross sectional area (diameter). Here the cable sag is set to 1.8m for all the cable systems. From this table, it can be seen that when the sectional area (diameter) increases, the frequencies for vertical modes, coupled torsional-lateral modes and longitudinal modes increase. The effect on coupled lateral-torsional modes (L-T) is interesting. The lowest frequency for the first coupled lateral-torsional mode (L1T1) increases, but the other frequencies of this same type of vibration modes decrease. All the frequencies are arranged in the Tables, according to number of half waves of their vibration modes.

The Effects of Applied Mass (Un-pre-tension Bridge Model)

In general, natural frequencies will decrease when the structural mass increases; but for cable supported bridge structures, effect of structural mass could be a little more complicated. Table 3 has shown the effect of applied mass on the natural frequencies. Here, the cable sag of the bridge model is set to 1.8m and diameter of the cables is let to be 240mm (L=80m, $F_1=F_2=F_3=1.8$m, $D_1=D_2=D_3=240$mm). The applied mass is assumed to be uniformly distributed on the deck and is modelled as lumped masses on the supporting beams. After the extra applied mass is applied, the top suspending cables are stretched to the required cable sag to keep the deck in horizontal plane. Numerical results show that the applied mass has great effect on the half wave symmetric vertical (fundamental) frequency and only a slight effect on the other vertical frequencies. When the applied mass increases, the first vertical fundamental frequency will reduce while the others vary slightly. The applied mass also has some effect on the frequencies corresponding to the other vibration modes. When the applied mass increases, the lowest frequency corresponding to the coupled lateral-torsional mode or coupled torsional-lateral mode decreases, however, the others go up slightly. Frequencies of all the longitudinal modes decrease.

Effects of Pre-tensioned Reverse Profiled Bottom and Side Cables (Extra Internal Vertical and Horizontal Forces)

In pre-tensioned cable supported bridge structures, if the pre-tension forces are introduced to the reverse profiled bottom cables, the tension forces in the top suspending cables will increase due to the extra internal vertical forces induced by the pre-tensioned bottom cables. If the side cables have been pre-tensioned, the horizontal structural stiffness will be improved greatly. Both the extra internal vertical forces and horizontal forces will enhance the connection between the transverse bridge frames, and therefore, the frequencies corresponding to the longitudinal vibration modes increase or the longitudinal modes disappear from the first twenty frequencies. Frequencies corresponding to all the other vibration modes will increase when the tension forces in the cable systems increase.

Table 4 shows the natural frequencies and their corresponding vibration modes with different extra internal vertical forces induced by the pre-tensioned bottom cables. Table 5 shows the dynamic properties with different extra internal horizontal forces when the side cables have been pre-tensioned. Here in the bridge model, the cable sags of all the cable systems are set to 1.8m and the diameters of all the cables are 240mm (L=80m, $F_1=F_2=F_3=1.8$m, $D_1=D_2=D_3=240$mm). It can be seen that the longitudinal modes have disappeared from the first twenty natural frequencies and all the frequencies corresponding to the other vibration modes increase when the extra internal forces increase. Since the main function of the pre-tensioned bottom cables is to improve the vertical structural stiffness, as mentioned before, the frequencies corresponding to the vertical vibration modes increase rapidly when the extra internal vertical forces increase, although the other frequencies also increase at the same time. After the side cables have been pre-tensioned, the frequencies corresponding to the coupled lateral-torsional modes as well as the coupled torsional-lateral modes, increase rapidly when the pre-tension forces in the side cables increase.

Table 6 shows the effect of the cross sectional area (diameter) of the pre-tensioned bottom cables. When the sectional area increase, the frequency corresponding to the first coupled lateral-torsional mode almost retain the same value while the other frequencies corresponding to this kind of modes decrease. All the frequencies of vertical modes and coupled torsional-lateral modes increase, since large bottom cables with the same extra internal vertical forces can carry more pre-tension force than the small ones.

5. Conclusion and Discussion

Cable supported bridges are slender and flexible structures and their dynamic properties are affected by many structural parameters. Numerical results show that for cable supported bridges with shallow cable sags, lowest frequencies usually correspond to the lateral and torsional vibration modes which are always combined together and become two coupled vibration modes: coupled lateral-torsional modes and coupled torsional-lateral modes. The effects of structural parameters such as cable sag, cross sectional area etc. are more complex. When the sag of suspending cables increases, most of the frequencies decrease except the symmetric half wave vertical vibration mode, since the tension forces in the suspending cables decrease but the static stiffness in vertical direction increases. Cross sectional area of the suspending cables has great effect on the symmetric half wave vertical mode as well as on the symmetric half wave coupled vibration modes, but has slight effect on the other modes. Applied mass has some effect on the symmetric half wave vertical mode and only slight effect on the other modes. When pre-tension forces are introduced to the reverse profiled bottom and/or side cables, the tension forces in the cable systems increase and hence all the natural frequencies increase.

For cable supported bridge structures, the natural frequencies mainly depend on the tension forces in the bridge structures. It is very difficult to change the natural frequencies for a traditional cable supported bridge structure, since the tension forces in the suspending cables depend mainly on the cable profiles and structural gravity loads. When the cable profiles are retained in the required forms, any change in the structural gravity will cause a change in both the tension forces in the suspending cables and structural mass at the same time, and therefore, the natural frequencies will change only slightly. However, when pre-tensioned cables are introduced to a cable supported bridge structure, the tension forces in the bridge structure can be adjusted by introducing different pre-tensions in the pre-tensioned cables, while the cable profiles and structural gravity are kept the same. In other words, the natural frequencies can be altered by the pre-tensioned cables, independent of mass. It can be seen that by introducing pre-tensioned cables to a cable supported bridge, the natural frequencies can be controlled on some extent.
The research information generated in this study will be useful in understanding the dynamic response of such bridges and will facilitate the development of design guidance.

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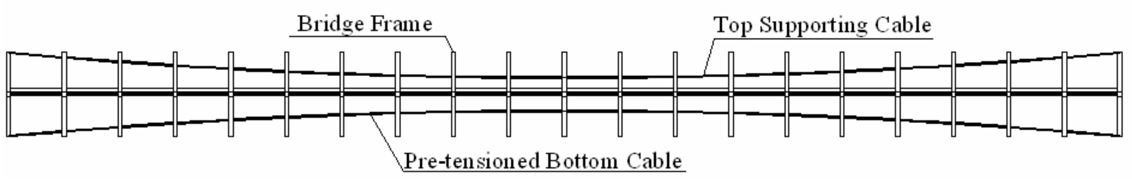
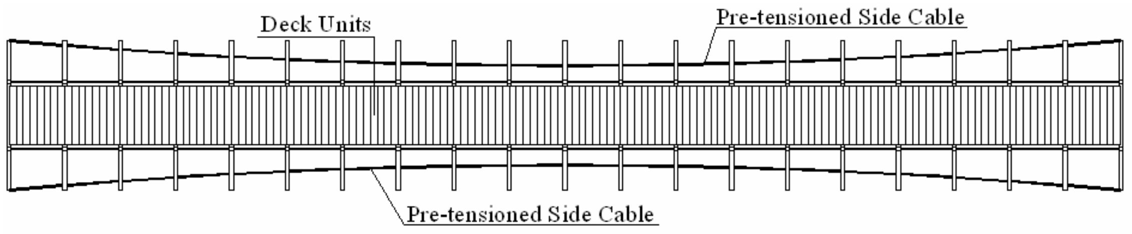
Figure Captions:

Table List

Figures



(a)



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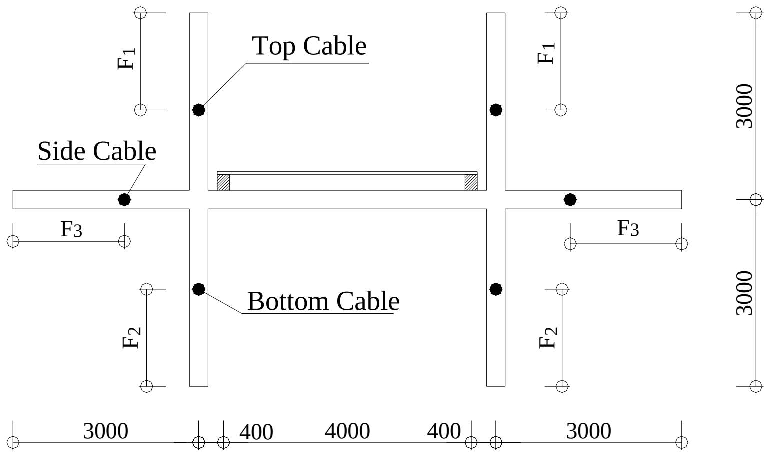


(c)

Figure 1 Pre-tensioned cable supported bridge model: (a) – elevation; (b) – top view; (c) – middle transverse bridge frame

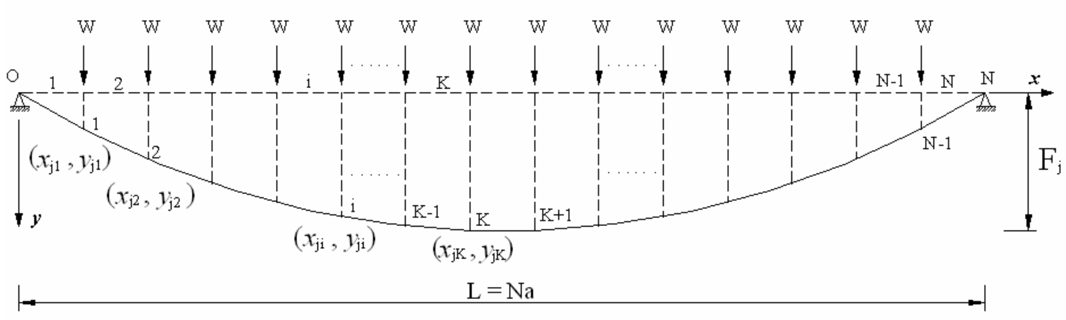


Figure 2 Typical cable profile

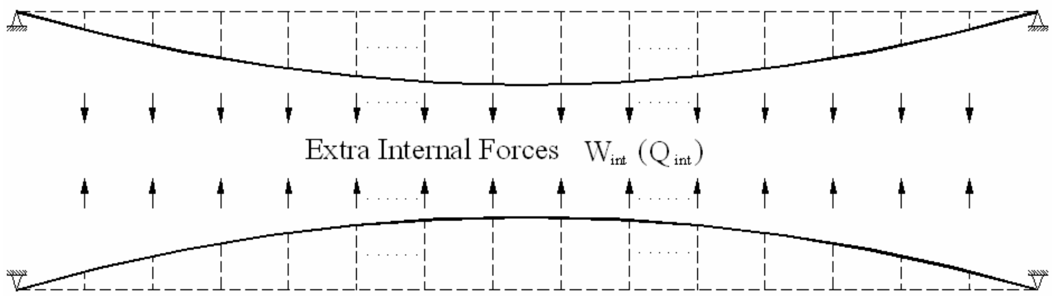
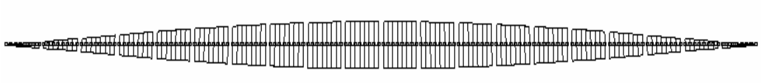
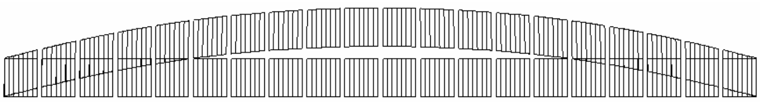


Figure 3 Extra internal forces in cables



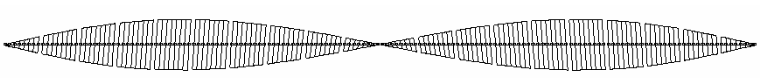
(a)

(c)

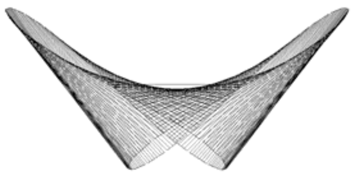


L1T1 (0.37312 Hz)

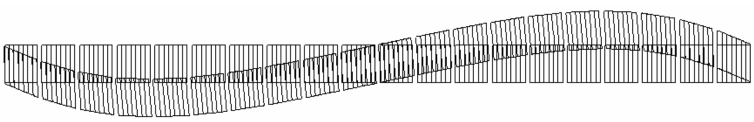
(b)



(a)

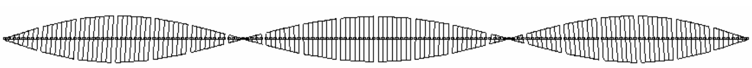


(c)

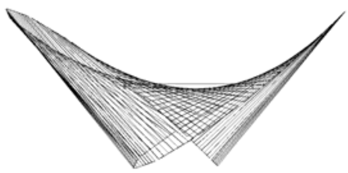


L2T2 (0.53434 Hz)

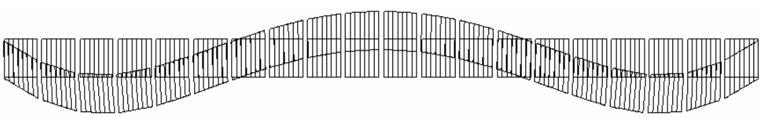
(b)



(a)



(c)



L3T3 (0.79064 Hz)

(b)

Figure 4 Coupled lateral-torsional vibration modes
(a) -- elevation; (b) -- top view; (c) -- side view

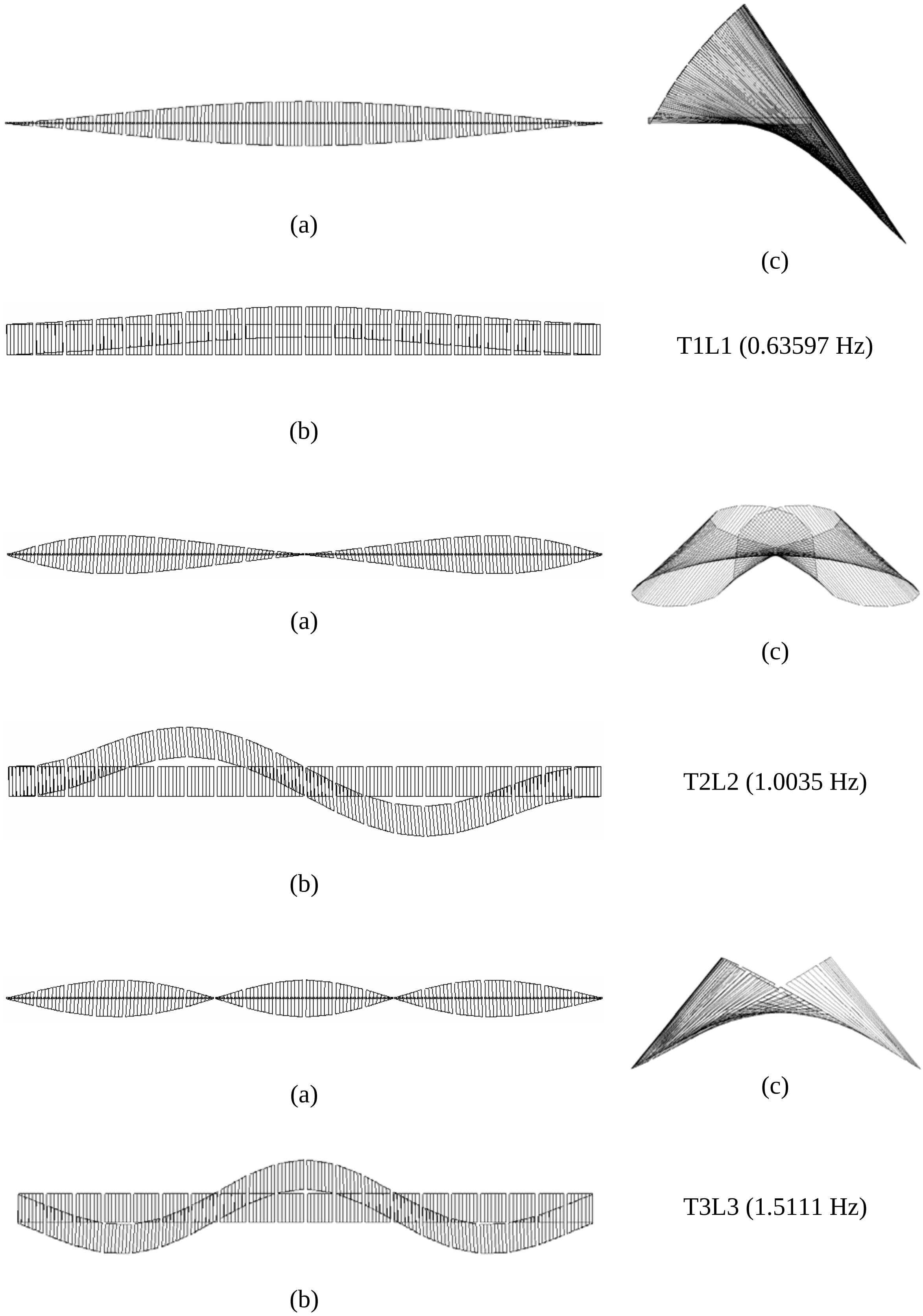
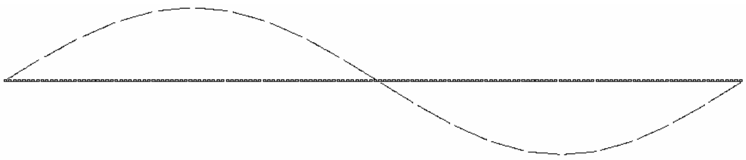
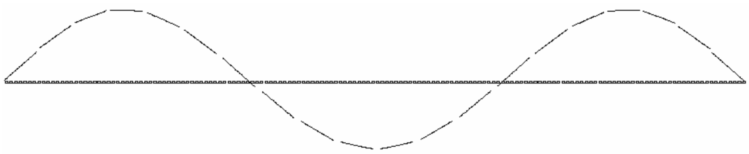


Figure 5 Coupled torsional-lateral vibration modes
(a) -- elevation; (b) -- top view; (c) -- side view

V1 (0.66259 Hz)



V2 (0.82865 Hz)



V3 (1.24940 Hz)

Figure 6 Vertical vibration modes

LSW1 (0.63597 Hz)

LSW2 (1.3214 Hz)

LSW3 (1.9700 Hz)

Figure 7 Longitudinal swaying vibration modes

Table 1 Natural frequencies and corresponding modes with the cable sag

Model	L=80 m, N=20, a=4 m; D1=D2=D3=240 mm					
Internal Forces	W_{int} (kN)	0	W_{int} (kN)	0	W_{int} (kN)	0
	Q_{int} (kN)	0	Q_{int} (kN)	0	Q_{int} (kN)	0
Cable Diameters	D_1 (mm)	240	D_1 (mm)	240	D_1 (mm)	240
	D_2 (mm)	240	D_2 (mm)	240	D_2 (mm)	240
	D_3 (mm)	240	D_3 (mm)	240	D_3 (mm)	240
Cable Sags	F_1 (mm)	1200	F_1 (mm)	1800	F_1 (mm)	2400
	F_2 (mm)	1200	F_2 (mm)	1800	F_2 (mm)	2400
	F_3 (mm)	1200	F_3 (mm)	1800	F_3 (mm)	2400
Cable Tensions	T_1 (N)	2.23E+07	T_1 (N)	1.49E+07	T_1 (N)	1.12E+07
	T_2 (N)	5.77E+04	T_2 (N)	5.80E+04	T_2 (N)	5.83E+04
	T_3 (N)	5.74E+04	T_3 (N)	5.76E+04	T_3 (N)	5.78E+04
Modes	Frequencies		Frequencies		Frequencies	
Coupled Lateral-Torsional	0.36066	L1T1	0.37312	L1T1	0.35474	L1T1
	0.59589	L2T2	0.53434	L2T2	0.50658	L2T2
	0.87993	L3T3	0.79064	L3T3	0.75538	L3T3
	1.15080	L4T4	1.03740	L4T4	0.97815	L4T4
	1.41100	L5T5	1.27330	L5T5	1.20520	L5T5
	1.65400	L6T6	1.49970	L6T6	1.42160	L6T6
	1.87910	L7T7	1.71350	L7T7	1.62770	L7T7
	2.08490	L8T8	1.91180	L8T8	1.82110	L8T8
	2.27070	L9T9				
	2.43660	L10T10				
Coupled Torsional-Lateral	0.66666	T1L1	0.59431	T1L1	0.64282	T1L1 (T1)
	1.29070	T2L2	1.00350	T2L2	0.82500	T2L2
	1.92240	T3L3	1.51110	T3L3	1.25300	T3L3
			1.99430	T4L4	1.64500	T4L4
Vertical	0.61541	V1	0.66259	V1	0.72073	V2
	1.01160	V2	0.82865	V2	0.75684	V1
	1.51560	V3	1.24940	V3	1.10360	V3
	1.99930	V4	1.64020	V4	1.42380	V4
		V5	2.03840	V5	1.77460	V5
Longitudinal Swaying	0.68305	LSW1	0.63597	LSW1	0.48824	LSW1
	1.37390	LSW2	1.32140	LSW2	1.15930	LSW2
	2.04790	LSW3	1.97000	LSW3	1.68340	LSW3

Notes:

LmTn – Coupled lateral-torsional modes;
TmLn – Coupled torsional-lateral modes
Vm – Vertical modes;
LSWm – Longitudinal swaying modes;
L – Lateral modes;
T – Torsional modes
m, n – Number of half wave

Table 2 Natural frequencies and corresponding modes with the cross sectional area (diameter)

Model	L=80 m, N=20, a=4 m; F1=F2=F3=1.8 m							
Internal Forces	W_{int} (kN)	0	W_{int} (kN)	0	W_{int} (kN)	0	W_{int} (kN)	0
	Q_{int} (kN)	0	Q_{int} (kN)	0	Q_{int} (kN)	0	Q_{int} (kN)	0
Cable Diameters	D_1 (mm)	120	D_1 (mm)	169.7	D_1 (mm)	240	D_1 (mm)	339.4
	D_2 (mm)	240	D_2 (mm)	240	D_2 (mm)	240	D_2 (mm)	190
	D_3 (mm)	317	D_3 (mm)	294	D_3 (mm)	240	D_3 (mm)	190
Cable Sags	F_1 (mm)	1800	F_1 (mm)	1800	F_1 (mm)	1800	F_1 (mm)	1800
	F_2 (mm)	1800	F_2 (mm)	1800	F_2 (mm)	1800	F_2 (mm)	1800
	F_3 (mm)	1800	F_3 (mm)	1800	F_3 (mm)	1800	F_3 (mm)	1800
Cable Tensions	T_1 (N)	1.49E+07	T_1 (N)	1.49E+07	T_1 (N)	1.49E+07	T_1 (N)	1.53E+07
	T_2 (N)	5.68E+04	T_2 (N)	5.68E+04	T_2 (N)	5.68E+04	T_2 (N)	3.57E+04
	T_3 (N)	1.01E+05	T_3 (N)	8.64E+04	T_3 (N)	5.76E+04	T_3 (N)	3.62E+04
Modes	Frequencies		Frequencies		Frequencies		Frequencies	
Coupled Lateral-Torsional	0.31304	L1T1	0.34081	L1T1	0.37312	L1T1	0.39713	L1T1
	0.51605	L2T2	0.52191	L2T2	0.53434	L2T2	0.55892	L2T2
	0.76104	L3T3	0.77030	L3T3	0.79065	L3T3	0.83762	L3T3
	0.99456	L4T4	1.00180	L4T4	1.03740	L4T4	1.08530	L4T4
	1.22800	L5T5	1.24250	L5T5	1.27330	L5T5	1.33700	L5T5
	1.44610	L6T6	1.46320	L6T6	1.49970	L6T6	1.57540	L6T6
	1.65120	L7T7	1.67110	L7T7	1.71350	L7T7	1.80210	L7T7
	1.84130	L8T8	1.86380	L8T8	1.91180	L8T8	2.01350	L8T8
	2.01620	L9T9						
Coupled Torsional-Lateral	0.51212	T1L1	0.53557	T1L1	0.59431	T1L1	0.70211	T1L1
	1.01830	T2L2	1.01980	T2L2	1.00350	T2L2	0.97555	T2L2
	1.51800	T3L3	1.51550	T3L3	1.51110	T3L3	1.47130	T3L3
	2.00420	T4L4	2.00080	T4L4	1.99430	T4L4	1.93670	T4L4
Vertical	0.49129	V1	0.55534	V1	0.66259	V1	0.81798	V1
	0.82990	V2	0.82949	V2	0.82865	V2	0.82610	V2
	1.23990	V3	1.24290	V3	1.24940	V3	1.27150	V3
	1.63970	V4	1.64000	V4	1.64020	V4	1.63380	V4
	2.03330	V5	2.03520	V5	2.03840	V5	2.03780	V5
Longitudinal	0.66574	LSW1	0.65630	LSW1	0.63597	LSW1	0.53799	LSW1
	1.37850	LSW2	1.36030	LSW2	1.32140	LSW2	1.12020	LSW2
			2.01700	LSW3	1.95960	LSW3	1.66990	LSW3

Table 3 Applied mass and the natural frequencies and their corresponding modes

Model	**L=80 m, N=20, a=4 m; D1=D2=D3=240 mm**							
Internal Forces	W_{int} (kN)	0	W_{int} (kN)	0	W_{int} (kN)	0	W_{int} (kN)	0
	Q_{int} (kN)	0	Q_{int} (kN)	0	Q_{int} (kN)	0	Q_{int} (kN)	0
Cable Diameters	D_1 (mm)	240	D_1 (mm)	240	D_1 (mm)	240	D_1 (mm)	240
	D_2 (mm)	240	D_2 (mm)	240	D_2 (mm)	240	D_2 (mm)	240
	D_3 (mm)	240	D_3 (mm)	240	D_3 (mm)	240	D_3 (mm)	240
Cable Sags	F_1 (mm)	1800	F_1 (mm)	1800	F_1 (mm)	1800	F_1 (mm)	1800
	F_2 (mm)	1800	F_2 (mm)	1800	F_2 (mm)	1800	F_2 (mm)	1800
	F_3 (mm)	1800	F_3 (mm)	1800	F_3 (mm)	1800	F_3 (mm)	1800
Applied Mass	m (kg/m^2)	0	m (kg/m^2)	200	m (kg/m^2)	600	m (kg/m^2)	1000
Cable Tensions	T_1 (N)	1.49E+07	T_1 (N)	1.67E+07	T_1 (N)	2.02E+07	T_1 (N)	2.37E+07
	T_2 (N)	5.80E+04	T_2 (N)	5.80E+04	T_2 (N)	5.80E+04	T_2 (N)	5.80E+04
	T_3 (N)	5.76E+04	T_3 (N)	5.76E+04	T_3 (N)	5.76E+04	T_3 (N)	5.76E+04
Modes	Frequencies		Frequencies		Frequencies		Frequencies	
Coupled Lateral-Torsional	0.37312	L1T1	0.37160	L1T1	0.36893	L1T1	0.36667	L1T1
	0.53434	L2T2	0.54400	L2T2	0.55896	L2T2	0.56987	L2T2
	0.79064	L3T3	0.80458	L3T3	0.82616	L3T3	0.84181	L3T3
	1.03740	L4T4	1.05700	L4T4	1.08720	L4T4	1.10890	L4T4
	1.27330	L5T5	1.28760	L5T5	1.33290	L5T5	1.35720	L5T5
	1.49970	L6T6	1.52690	L6T6	1.57140	L6T6	1.59980	L6T6
	1.71350	L7T7	1.74730	L7T7	1.79690	L7T7	1.82890	L7T7
	1.91180	L8T8	1.91190	L8T8	2.00720	L8T8	2.04310	L8T8
Coupled Torsional-Lateral	0.59431	T1L1	0.59420	T1L1	0.59387	T1L1	0.59351	T1L1
	1.00350	T2L2	1.01330	T2L2	1.03020	T2L2	1.04420	T2L2
	1.51110	T3L3	1.52860	T3L3	1.55420	T3L3	1.57650	T3L3
	1.99430	T4L4	2.01560	T4L4	2.05090	T4L4	2.07830	T4L4
Vertical	0.66259	V1	0.64169	V1	0.60904	V1	0.58467	V1
	0.82865	V2	0.82855	V2	0.82838	V2	0.82818	V2
	1.24940	V3	1.24760	V3	1.25330	V3	1.25020	V3
	1.64020	V4	1.64400	V4	1.64880	V4	1.65090	V4
	2.03840	V5	2.04610	V5	2.05600	V5	2.06100	V5
Longitudinal Swaying	0.63597	LSW1	0.62166	LSW1	0.59557	LSW1	0.57240	LSW1
	1.32140	LSW2	1.29720	LSW2	1.21790	LSW2(V3)	1.16510	LSW2(V3)
	1.97000	LSW3	1.95060	LSW3	1.81180	LSW3	1.72590	LSW3

Table 4 Internal vertical forces and the natural frequencies and their corresponding modes

Model	**L=80 m, N=20, a=4 m; D1=D2=D3=240 mm**							
Internal Forces	W_{int} (kN)	0	W_{int} (kN)	10	W_{int} (kN)	20	W_{int} (kN)	30
	Q_{int} (kN)	0	Q_{int} (kN)	0	Q_{int} (kN)	0	Q_{int} (kN)	0
Cable Diameters	D_1 (mm)	240	D_1 (mm)	240	D_1 (mm)	240	D_1 (mm)	240
	D_2 (mm)	240	D_2 (mm)	240	D_2 (mm)	240	D_2 (mm)	240
	D_3 (mm)	240	D_3 (mm)	240	D_3 (mm)	240	D_3 (mm)	240
Cable Sags	F_1 (mm)	1800	F_1 (mm)	1800	F_1 (mm)	1800	F_1 (mm)	1800
	F_2 (mm)	1800	F_2 (mm)	1800	F_2 (mm)	1800	F_2 (mm)	1800
	F_3 (mm)	1800	F_3 (mm)	1800	F_3 (mm)	1800	F_3 (mm)	1800
Cable Tensions	T_1 (N)	1.49E+07	T_1 (N)	1.56E+07	T_1 (N)	1.60E+07	T_1 (N)	1.66E+07
	T_2 (N)	5.80E+04	T_2 (N)	7.02E+05	T_2 (N)	1.17E+06	T_2 (N)	1.70E+06
	T_3 (N)	5.76E+04	T_3 (N)	5.76E+04	T_3 (N)	5.76E+04	T_3 (N)	5.76E+04
Modes	Frequencies		Frequencies		Frequencies		Frequencies	
Coupled Lateral-Torsional	0.37312	L1T1	0.40813	L1T1	0.42359	L1T1	0.43836	L1T1
	0.53434	L2T2	0.62016	L2T2	0.65486	L2T2	0.69166	L2T2
	0.79064	L3T3	0.88525	L3T3	0.94023	L3T3	0.99736	L3T3
	1.03740	L4T4	1.14920	L4T4	1.22120	L4T4	1.29790	L4T4
	1.27330	L5T5	1.41530	L5T5	1.50510	L5T5	1.59620	L5T5
	1.49970	L6T6	1.66950	L6T6	1.77760	L6T6	1.89150	L6T6
	1.71350	L7T7	1.91180	L7T7	2.03780	L7T7	2.17030	L7T7
	1.91180	L8T8	2.13950	L8T8	2.28320	L8T8	2.43390	L8T8
			2.35120	L9T9	2.51240	L9T9	2.68140	L9T9
Coupled Torsional-Lateral	0.59431	T1L1	0.68580	T1L1	0.71579	T1L1	0.72983	T1L1
	1.00350	T2L2	1.04940	T2L2	1.06910	T2L2	1.09060	T2L2
	1.51110	T3L3	1.55760	T3L3	1.58770	T3L3	1.62460	T3L3
	1.99430	T4L4	2.04850	T4L4	2.08690	T4L4	2.13010	T4L4
			2.53350	T5L5	2.58090	T5L5	2.63340	T5L5
Vertical	0.66259	V1	0.79349	V1	0.83294	V1	0.84947	V1
	0.82865	V2	0.86172	V2	0.88590	V2	0.91274	V2
	1.24940	V3	1.31590	V3	1.35540	V3	1.39500	V3
	1.64020	V4	1.70890	V4	1.75670	V4	1.80970	V4
	2.03840	V5	2.12510	V5	2.18470	V5	2.25030	V5
			2.52150	V6	2.59150	V6	2.66930	V6
Longitudinal Swaying	0.63597	LSW1						
	1.32140	LSW2						
	1.97000	LSW3						

Table 5 Internal horizontal forces and the natural frequencies and Their corresponding modes

Model	**L=80 m, N=20, a=4 m; D1=D2=D3=240 mm**							
Internal Forces	W_{int} (kN)	0	W_{int} (kN)	0	W_{int} (kN)	0	W_{int} (kN)	20
	Q_{int} (kN)	0	Q_{int} (kN)	10	Q_{int} (kN)	20	Q_{int} (kN)	10
Cable Diameters	D_1 (mm)	240	D_1 (mm)	240	D_1 (mm)	240	D_1 (mm)	240
	D_2 (mm)	240	D_2 (mm)	240	D_2 (mm)	240	D_2 (mm)	240
	D_3 (mm)	240	D_3 (mm)	240	D_3 (mm)	240	D_3 (mm)	240
Cable Sags	F_1 (mm)	1800	F_1 (mm)	1800	F_1 (mm)	1800	F_1 (mm)	1800
	F_2 (mm)	1800	F_2 (mm)	1800	F_2 (mm)	1800	F_2 (mm)	1800
	F_3 (mm)	1800	F_3 (mm)	1800	F_3 (mm)	1800	F_3 (mm)	1800
Cable Tensions	T_1 (N)	1.49E+07	T_1 (N)	1.49E+07	T_1 (N)	1.49E+07	T_1 (N)	1.60E+07
	T_2 (N)	5.80E+04	T_2 (N)	5.80E+04	T_2 (N)	5.80E+04	T_2 (N)	1.17E+06
	T_3 (N)	5.76E+04	T_3 (N)	1.17E+06	T_3 (N)	2.23E+06	T_3 (N)	1.17E+06
Modes	Frequencies		Frequencies		Frequencies		Frequencies	
Coupled Lateral-Torsional	0.37312	L1T1	0.55257	L1T1	0.57335	L1T1	0.64353	L1T1
	0.53434	L2T2	0.60216	L2T2	0.65811	L2T2	0.75219	L2T2
	0.79064	L3T3	0.90682	L3T3	0.99061	L3T3	1.04990	L3T3
	1.03740	L4T4	1.17160	L4T4	1.28450	L4T4	1.34350	L4T4
	1.27330	L5T5	1.44680	L5T5	1.58800	L5T5	1.66080	L5T5
	1.49970	L6T6	1.70540	L6T6	1.87400	L6T6	1.95320	L6T6
	1.71350	L7T7	1.95450	L7T7	2.15020	L7T7	2.24110	L7T7
	1.91180	L8T8	2.18680	L8T8	2.40870	L8T8	2.51260	L8T8
			2.40360	L9T9	2.65280	L9T9	2.76860	L9T9
Coupled Torsional-Lateral	0.59431	T1L1	0.68081	T1L1	0.70218	T1L1	0.74982	T1L1
	1.00350	T2L2	1.07680	T2L2	1.10820	T2L2	1.12710	T2L2
	1.51110	T3L3	1.57360	T3L3	1.62130	T3L3	1.64290	T3L3
	1.99430	T4L4	2.06480	T4L4	2.12910	T4L4	2.15650	T4L4
			2.55380	T5L5	2.63270	T5L5	2.66590	T5L5
Vertical	0.66259	V1	0.67261	V1	0.68202	V1	0.84195	V1
	0.82865	V2	0.85713	V2	0.88501	V2	0.91408	V2
	1.24940	V3	1.29640	V3	1.33770	V3	1.39640	V3
	1.64020	V4	1.70150	V4	1.75750	V4	1.81420	V4
	2.03840	V5	2.11540	V5	2.18560	V5	2.25730	V5
			2.51440	V6	2.59900	V6	2.68020	V6
Longitudinal Swaying	0.63597	LSW1						
	1.32140	LSW2						
	1.97000	LSW3						

Table 6 Cross sectional area (diameter) of the pre-tensioned bottom cables and the natural frequencies and their corresponding modes

Model	**L=80 m, N=20, a=4 m; F1=F2=F3=1.8 m**							
Internal Forces	W_{int} (kN)	30	W_{int} (kN)	30	W_{int} (kN)	30	W_{int} (kN)	30
	Q_{int} (kN)	0	Q_{int} (kN)	0	Q_{int} (kN)	0	Q_{int} (kN)	0
Cable Diameters	D_1 (mm)	240	D_1 (mm)	240	D_1 (mm)	240	D_1 (mm)	240
	D_2 (mm)	120	D_2 (mm)	169	D_2 (mm)	240	D_2 (mm)	339
	D_3 (mm)	317	D_3 (mm)	294	D_3 (mm)	240	D_3 (mm)	0
Cable Sags	F_1 (mm)	1800	F_1 (mm)	1800	F_1 (mm)	1800	F_1 (mm)	1800
	F_2 (mm)	1800	F_2 (mm)	1800	F_2 (mm)	1800	F_2 (mm)	1800
	F_3 (mm)	1800	F_3 (mm)	1800	F_3 (mm)	1800	F_3 (mm)	1800
Cable Tensions	T_1 (N)	1.65E+07	T_1 (N)	1.65E+07	T_1 (N)	1.66E+07	T_1 (N)	1.67E+07
	T_2 (N)	1.67E+06	T_2 (N)	1.68E+06	T_2 (N)	1.70E+06	T_2 (N)	1.84E+06
	T_3 (N)	1.01E+05	T_3 (N)	8.64E+04	T_3 (N)	5.76E+04	T_3 (N)	0.00E+00
Modes	Frequencies		Frequencies		Frequencies		Frequencies	
Coupled Lateral-Torsional	0.43196	L1T1	0.43487	L1T1	0.43836	L1T1	0.44246	L1T1
	0.69280	L2T2	0.69324	L2T2	0.69166	L2T2	0.69217	L2T2
	0.99903	L3T3	0.99791	L3T3	0.99736	L3T3	1.00680	L3T3
	1.30550	L4T4	1.30240	L4T4	1.29790	L4T4	1.30010	L4T4
	1.61320	L5T5	1.61090	L5T5	1.59620	L5T5	1.60320	L5T5
	1.90210	L6T6	1.89790	L6T6	1.89150	L6T6	1.89390	L6T6
	2.18330	L7T7	2.17810	L7T7	2.17030	L7T7	2.17200	L7T7
	2.44910	L8T8	2.44310	L8T8	2.43390	L8T8	2.43440	L8T8
	2.69860	L9T9	2.69190	L9T9	2.68140	L9T9	2.68000	L9T9
Coupled Torsional-Lateral	0.63003	T1L1	0.66424	T1L1	0.72983	T1L1	0.83620	T1L1
	1.05130	T2L2	1.06400	T2L2	1.09060	T2L2	1.15570	T2L2
	1.55820	T3L3	1.57480	T3L3	1.62460	T3L3	1.72220	T3L3
	2.05760	T4L4	2.07990	T4L4	2.13010	T4L4	2.25720	T4L4
	2.54150	T5L5	2.56990	T5L5	2.63340	T5L5	2.79410	T5L5
Vertical	0.73632	V1	0.77783	V1	0.84947	V1	0.91820	V2
	0.91242	V2	0.91227	V2	0.91274	V2	0.94295	V1
	1.37980	V3	1.38430	V3	1.39500	V3	1.42100	V3
	1.80540	V4	1.80650	V4	1.80970	V4	1.82340	V4
	2.23970	V5	2.24290	V5	2.25030	V5	2.27190	V5
	2.65460	V6	2.65930	V6	2.66930	V6	2.69590	V6