COVER SHEET

CONNECTING ALGEBRAIC DEVELOPMENT TO MATHEMATICAL PATTERNING IN EARLY CHILDHOOD

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Pattern exploration is advocated as an essential element of young children's mathematical development. However, past research has shed little light on the effect that mathematical patterning experiences can have on the development of children's understanding of specific mathematical concepts. This paper explores the content of mathematical patterning experiences that were observed in a multi-site case study conducted in Australian preparatory and preschool classrooms with similarly aged children. These experiences were analysed to ascertain the potential contribution they make to algebraic development. From the results, it appears that the content of these mathematical patterning experiences in prior-to-school environments, provide limited connections to algebraic thinking.

PATTERNING AND MATHEMATICAL LEARNING IN THE EARLY YEARS

Mathematics and patterning are closely interrelated. Mathematics has been described as "the science of patterns" and "the search for patterns" (National Research Council, 1989), while pattern exploration has been identified as a central construct of mathematical inquiry (Heddens & Speer, 2001; NCTM, 2000). Patterning involves "observing, representing and investigating patterns and relationships in social, and physical phenomena, and between mathematical objects themselves" (Australian Education Council, 1991, p. 4).

Globally, there has been increased interest in two research arenas where children's pattern exploration features significantly – early childhood education and algebraic thinking in the early years. Algebra is "a generalization of the ideas of arithmetic where unknown values and variables can be found to solve problems" (Taylor-Cox, 2003, p. 14).

The value of patterning in the early years has been endorsed by many researchers. Owen (1995) suggests that an affinity with and understanding of repeating patterns offers younger children access to "elements of mathematical thought which are not available to them through any other medium in mathematics" (p. 126). Williams and Shuard (1982) also endorse the mathematical value of patterning for young children: "The search for order and pattern … is one of the driving forces of all mathematical work with young children" (p. 330). Hence, from children's earliest years, patterning is foundational to learning because it assists children to make sense of their everyday world. Prior to attending school, children recognise, compare, and analyse patterns in daily events, chants, nursery rhymes, movement, and physical objects.

Research from the past twenty years has concluded that young children are capable of mathematical insights and inventions which exceed our expectations and necessary groundwork and foundations are laid for future mathematics learning (Ferrini-Mundy & Lappan, 1997). Additionally, research has increased our expectations of young children's learning. In the early years, the study of patterns is a productive way of developing algebraic reasoning (Ferrini-Mundy & Lappan, 1997). Steen (1988) has suggested that observations of patterns and relationships lie at the heart of acquiring deep understanding of mathematics – algebra and function in particular.

There is an interrelationship between patterns and algebra in content groupings of curriculum and in research agendas. Patterns, functions and algebra comprise one of the strands in the Principles and Standards for School Mathematics (National Council of Teachers of Mathematics [NCTM], 2000). Members of the algebra working party, which was established at PME27 (2003), include the study of patterning in their research agenda (e.g., Warren, 2005).

The NCTM (2000) also suggests that students need to be prepared for success in algebra by teaching them to think algebraically in the early years. However, if the study of patterns is an effective way of developing foundations in algebraic reasoning, it needs to be accommodated in programming students' early educational opportunities in prior-to-school settings. The development of appropriate curricula to support mathematical learning in the early years environment has received much attention (Clements, Sarama, & DiBiase, 2000; National Association for the Education of Young Children [NAEYC]; & NCTM). However, it is essential that these curricula provide adequate guidance for teachers to support the development of patterning knowledge and algebraic reasoning in young children.

THE STUDY

This paper reports on one aspect of a multi-site case study (Yin, 2003) that investigated the nature of patterning in the pre-compulsory years of schooling. This paper examines the mathematical patterning activities designed and implemented by two teachers in prior-to-school settings and the possible opportunities that these activities provide for developing the foundations of algebraic reasoning.

Setting and participants

This study was conducted in a preschool and preparatory setting because these sites are typical examples of Queensland children's learning environments in the year prior to the commencement of compulsory schooling. The two schools chosen for involvement in the study were located in the inner city suburbs of Brisbane. These schools were geographically close and shared similar socio-economic clientele. The preschool was in a state school and operated a five day per fortnight program. The preparatory class was in a private school and conducted a full-time program of five days per week.

By coincidence both settings had 13 female and 12 male students. The students in each of these programs were required to turn five by 31 December of the preceding year to be eligible to attend. The preparatory class teacher, Mrs Jones, had 12 years experience in early primary classes and was experiencing her first year in a preparatory setting. The preschool class was staffed by Mrs Smith, a four-year trained early childhood teacher, who was experienced in teaching in preschool.

Data Collection and Analysis

A case study was undertaken to gain an understanding of the nature and occurrence of mathematical patterning in pre-compulsory settings. Briefly, this study involved ongoing observations of the pre-compulsory settings until a full day of activities had been observed. This data collection period spanned 4 weeks. Typical of a case study, multiple sources of data were collected. These data comprised a semi-structured interview with each teacher (outside of class times), copies of their programs and video-taped observations of the classes. Analysis of a total of approximately 80 hours of video observations collected in the two classrooms revealed ten mathematical patterning episodes. These comprised of three teacher-planned, four teacher-initiated, two child-initiated and one teacher intervention episode. This paper focuses on the three episodes identified as teacher-planned. Teacher-planned episodes were events containing mathematical patterning, which the teacher planned for the children. The activity appeared in the teacher's daily plans and may have been confined to a verbal dialogue or required the creation of an end product. A discussion of the child-initiated episodes is reported elsewhere (Fox, 2004, 2005).

FINDINGS

Three teacher-planned episodes were analysed to identify the nature of mathematical patterning within the activities. The first episode occurred in the preschool site and involved tessellations. The children created tessellations (the arrangement of shapes to form spatial patterns) using *pattern blocks* on the carpet as one of their small group rotational activities. Mrs Smith questioned the children. "How do you make tessellating patterns?" to which children variously responded "make it grow". The teacher further probed the children's understanding, "What is the difference between a tessellating pattern and one you make in a line?" Children made different responses, such as "It goes by itself", "It goes out" and "It goes round." After the children gave their ideas, Mrs Smith shared her definition of a tessellating pattern by stating "You do the same on both sides."

The children began creating tessellating patterns on the carpet. Four of the five children began their designs with a central shape and then added shapes around the centre point (see Figure 1). One child, Sam, was the only child to create a random linear design (see Figure 2). He made a line of hexagons and red rectangles which were placed on either side of a central shape. When he ran out of hexagons he added shapes in a second layer on top of the first line. Mrs Smith identified this design to

the group as a symmetrical pattern. Sam had randomly placed the pattern blocks onto his design and as he ran out of one shape or colour he substituted it for another.

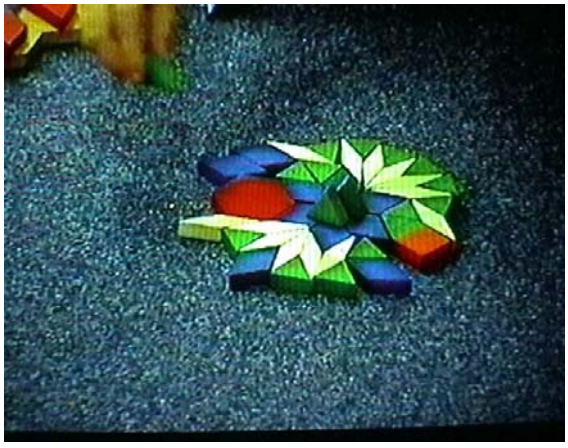

Figure 1. Example of tessellating pattern.

Figure 2. Example of a random linear pattern.

Tessellations involve patterning skills but also knowledge of shape, space and angle. However, this is not the definition that was articulated to the children. The teacher's knowledge of tessellating patterns was not clear and no connection was made to relationships, generalisations, or any other algebraic notions. Working with patterns should encourage children to identify relationships and form generalisations (NCTM, 2000); however this episode incorporated limited references to repeating cycles of shapes or recurring segments.

The second teacher-planned episode required the students in the preparatory setting to create a pattern on a school uniform for a paper doll. The wearing of school uniforms is often regarded by children of this age in Australia as a rite of passage to school. To introduce the patterning activity, Mrs Jones showed the students various items of clothing to demonstrate patterns. The designs on the clothing were a mixture of shapes, colours, flowers, stripes, checks, hearts, and stars. These examples demonstrated random designs and it was very difficult to identify any regularities. It is the repetitive nature of pattern that distinguishes it from random arrangement or design. Mrs Jones mentioned the need for repetition when discussing a floral dress, when she observed the "same pattern over and over again in lines" (Figure 3). However, Mrs Jones did not focus on the identification of repeating elements. She suggested the children could also use "lovely patterns" like stripes, flower patterns, different shapes, or checked patterns on their doll uniforms.

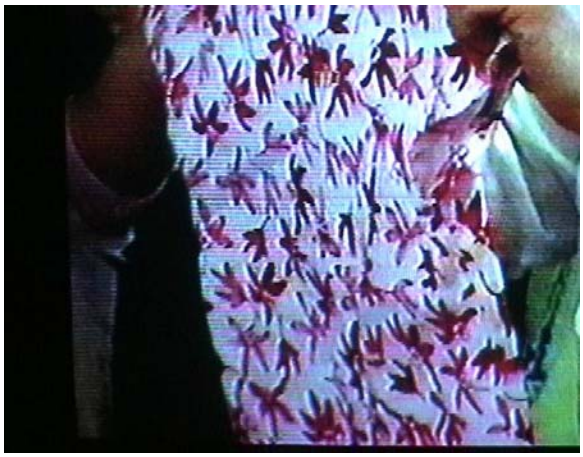

Figure 3. Floral dress.

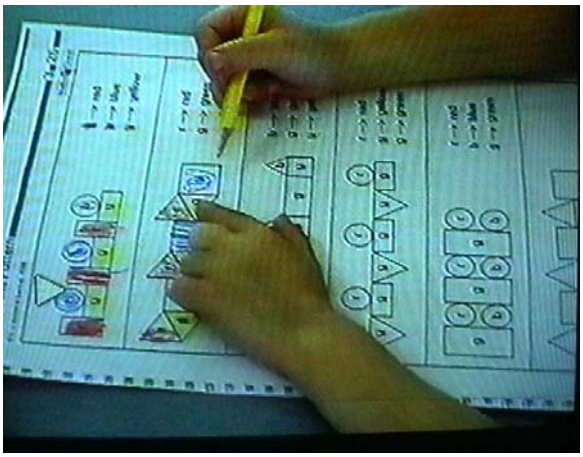

Figure 4. Patterning worksheet.

During the teacher's introduction, three children discovered patterns on their own clothes. The examples shown by both the teacher and the children were a combination of patterns (i.e., repeating designs), line symmetry and random designs. However, all were labelled by the teacher (or children) as patterns. At the end of the activity, only one of the children's doll uniforms depicted a repeating design. The child had drawn stripes on the uniform using an ABC pattern and another child had copied this pattern. The other 11 children who participated drew uniforms of random designs with no identifiable repeating elements. The teacher had not provided the students with consistent examples of pattern features. Thus, children might have been operating from a variety of interpretations of the term 'pattern'.

The third teacher-planned episode, which occurred in the preparatory setting, was to complete a patterning worksheet. The worksheet indicated via a colour code, which colour was to be used in which space, and when completed correctly it would create a pattern (Figure 4). Mrs Jones did not discuss with the children what a pattern was or what she actually expected them to create. The children were unable to decipher the colour code and their attempts to create patterns largely failed. The children did not identify the repetitious nature of the shapes nor did the teacher suggest any prediction strategies. An identification of regularity makes it possible to predict what lies ahead, however these strategies were not identified by the teacher. Essential components of linear patterns were neither verbalised to the children, nor were examples given.

DISCUSSION

The three episodes planned by the teachers had the potential to be meaningful learning opportunities for the students. However, apparent weaknesses in the teachers' knowledge together with the nature of the activities chosen reduced the learning opportunities within the episodes.

Mrs Smith (preschool) designed episodes that explored the concept of tessellation. Tessellations follow the principles of shape and space and incorporate the use of inquiry, discussion and reflection. Students developing tessellation knowledge also require experience with pattern and angle. The guidelines given by Mrs Smith were "make it grow…on all sides." However, these directions did not fully describe the concept of tessellations.

The episodes developed by Mrs Jones (preparatory) required the students to create their own repeating linear patterns. The objective of the second episode *(uniform activity)* encouraged children to make their own patterns, whilst the third episode *(pattern worksheet)* was to use colour to create a pattern. The teacher's instructions to the students did not provide consistent information on the development of patterns. Whilst Mrs Jones used pattern terminology such as 'repeat', 'over and over' and 'over and over again', she did not discuss key components of patterns. Furthermore, the teacher did not offer consistent definitions or examples to the student or make explicit features of mathematical patterning. Mrs Jones' restricted knowledge of patterning or the limited knowledge she shared with the children effectively contributed to the limited opportunities for learning. The promotion of mathematical patterning in pre compulsory settings relies heavily on the teacher's ability to identify concepts and convey them to the students (Fox, 2005).

CONCLUSIONS

Various forms of patterns, from basic repetition through to spatial surface patterns were documented in the observed patterning experiences. Warren's (2005) work also showed that children in Australian early childhood classrooms explore simple repeating and growing patterns using shapes, colours, and movement. These forms of patterning activities have the potential to expose children to the beginning notions of algebraic thinking. It was evident however in this study, that both the teachers and the children had limited understanding of the types, levels and complexity of patterns. Experiences with identifying, creating, extending and generalising patterns, recognising relationships, making predictions, and abstracting rules provide foundations for future algebraic development. However, the powerful contribution patterning can have to both mathematical development and algebraic foundations, appears to be largely unrealised in pre-compulsory years classrooms. The NAEYC and NCTM joint statement (2002) clearly stated that patterning, as a component of algebra "merits special mention because it is accessible and interesting to young children" (p. 9) and most importantly patterning "grows to undergird all algebraic thinking" (p .9).

Whilst it is believed that young children are capable of thinking both algebraically and functionally (Blanton & Kaput, 2004) and that work with patterns is valuable in "fostering logical reasoning and algebraic thinking" (Ginsburg, Cannon, Eisenband, & Pappas, in press, p. 12), teachers play an important role in drawing connections and creating explicit learning opportunities. NAEYC and NCTM (2000) claimed that

making connections needs special attention: “teaching concepts and skills in a connected integrated fashion tends to be particularly effective” (p. 8). Teachers who are better informed and more knowledgeable about mathematical patterning and algebraic development can provide children with appropriate, meaningful and powerful mathematical foundations. This study demonstrates that opportunities for children to explore mathematical patterning do occur in pre-compulsory settings. However, there is a need for teachers to have a deep understanding of the nature and power of mathematical patterning. Understanding what to teach, when to teach, and how to teach will provide the opportunity for children to engage in rich patterning experiences and to promote meaningful algebraic foundations.

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