Analysis and Correction of Voltage Profile in Low Voltage Distribution Networks Containing Photovoltaic Cells and Electric Vehicles

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Low Voltage Distribution Networks, Voltage Profile, Voltage Unbalance, Photovoltaic Cells, Single-phase rooftop PVs, Plug-in Electric Vehicles, Micro Grid, DSTATCOM, DVR, Sensitivity Analysis, Stochastic Evaluation
Abstract

Voltage drop and rise at network peak and off-peak periods along with voltage unbalance are the major power quality problems in low voltage distribution networks. Usually, the utilities try to use adjusting the transformer tap changers as a solution for the voltage drop. They also try to distribute the loads equally as a solution for network voltage unbalance problem.

On the other hand, the ever increasing energy demand, along with the necessity of cost reduction and higher reliability requirements, are driving the modern power systems towards Distributed Generation (DG) units. This can be in the form of small rooftop photovoltaic cells (PV), Plug-in Electric Vehicles (PEVs) or Micro Grids (MGs). Rooftop PVs, typically with power levels ranging from 1–5 kW installed by the householders are gaining popularity due to their financial benefits for the householders. Also PEVs will be soon emerged in residential distribution networks which behave as a huge residential load when they are being charged while in their later generation, they are also expected to support the network as small DG units which transfer the energy stored in their battery into grid. Furthermore, the MG which is a cluster of loads and several DG units such as diesel generators, PVs, fuel cells and batteries are recently introduced to distribution networks.

The voltage unbalance in the network can be increased due to the uncertainties in the random connection point of the PVs and PEVs to the network, their nominal capacity and time of operation. Therefore, it is of high interest to investigate the voltage unbalance in these networks as the result of MGs, PVs and PEVs integration to low voltage networks. In addition, the network might experience non-standard
voltage drop due to high penetration of PEVs, being charged at night periods, or non-standard voltage rise due to high penetration of PVs and PEVs generating electricity back into the grid in the network off-peak periods.

In this thesis, a voltage unbalance sensitivity analysis and stochastic evaluation is carried out for PVs installed by the householders versus their installation point, their nominal capacity and penetration level as different uncertainties. A similar analysis is carried out for PEVs penetration in the network working in two different modes: Grid to vehicle and Vehicle to grid. Furthermore, the conventional methods are discussed for improving the voltage unbalance within these networks. This is later continued by proposing new and efficient improvement methods for voltage profile improvement at network peak and off-peak periods and voltage unbalance reduction. In addition, voltage unbalance reduction is investigated for MGs and new improvement methods are proposed and applied for the MG test bed, planned to be established at Queensland University of Technology (QUT). MATLAB and PSCAD/EMTDC simulation softwares are used for verification of the analyses and the proposals.
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<tbody>
<tr>
<td>CPD</td>
<td>Custom Power Devices</td>
</tr>
<tr>
<td>DLC</td>
<td>Direct Load Control</td>
</tr>
<tr>
<td>DG</td>
<td>Distributed generation</td>
</tr>
<tr>
<td>DSTATCOM</td>
<td>Distribution Static Compensator</td>
</tr>
<tr>
<td>DVR</td>
<td>Dynamic Voltage Restorer</td>
</tr>
<tr>
<td>FC</td>
<td>Fuel Cell</td>
</tr>
<tr>
<td>G2V</td>
<td>Grid to Vehicle</td>
</tr>
<tr>
<td>IGBT</td>
<td>Insulated Gate Bipolar Transistors</td>
</tr>
<tr>
<td>KCL</td>
<td>Kirchhoff’s Circuit Laws</td>
</tr>
<tr>
<td>LV</td>
<td>Low Voltage</td>
</tr>
<tr>
<td>MODM</td>
<td>Multi–Objective Decision Making</td>
</tr>
<tr>
<td>PCC</td>
<td>Point of Common Coupling</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PEV</td>
<td>Plug–in Electric Vehicle</td>
</tr>
<tr>
<td>PV</td>
<td>Photovoltaic Cells</td>
</tr>
<tr>
<td>THD</td>
<td>Total Harmonic Distortion</td>
</tr>
<tr>
<td>V2G</td>
<td>Vehicle to Grid</td>
</tr>
<tr>
<td>VSC</td>
<td>Voltage Source Converter</td>
</tr>
<tr>
<td>VU</td>
<td>Voltage Unbalance</td>
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</table>
Statement of original authorship

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, this thesis contains no material previously published or written by another person except where due reference is made.

Signature:……………………….

Date:…………………………….
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Chapter 1: Introduction

1.1 Background

The ever increasing energy demand, along with the necessity of cost reduction and higher reliability requirements, are driving the modern power systems towards distributed generation (DG) as an alternative to the expansion of the current energy distribution systems [1]. In particular, small DG systems, typically with power levels ranging from 1 kW to 10 MW, located near the loads are gaining popularity due to their higher operating efficiencies. Photovoltaic cells (PV), Fuel cells (FC), Batteries, micro turbines, etc. are nowadays the most available DGs for generation of power mostly in peak times or in rural areas [2].

It is desirable that the utilities ensure that the customers are supplied with a high power quality. Among the power quality parameters, voltage profile and Voltage Unbalance (VU) are the major concerns in low voltage (LV) distribution networks [3].

Voltage drop can be experienced in network peak hours while voltage rise can be experienced in network off–peak hours with high generation and penetration of DG units [3]. The utilities are responsible for keeping the voltage in their network within the standard limits to prevent malfunction of customer devices.

Voltage unbalance is more common in individual customer loads due to phase load unbalances, especially where large single–phase power loads are used [4]. Although voltages are well balanced at the supply side, the voltages at the customer
ends can become unbalanced due to the unequal system impedances, unequal distribution of single–phase loads or large number of single–phase transformers [4]. Usually, the electric utilities aim to distribute the residential loads equally among the three phases of distribution feeders [5].

An increase in the voltage unbalance can result in overheating and de–rating of all induction motor types of loads [6]. Voltage unbalance can also cause network problems such as mal–operation of protection relays and voltage regulation equipment, and generation of non–characteristic harmonics from power electronic loads [5].

1.1.1 Rooftop PVs

Application of grid–connected Photovoltaic cells (PVs) is increasing in residential low voltage (LV) Distribution Networks around the world. Incentives by different countries promote the development of PVs connected to distribution networks[7]. Several small–scale solar–based neighborhoods are already demonstrated [8]. By generating electricity closer to residential customers, transmission and distribution losses can be reduced in addition to gaining higher benefits from utilizing renewable energies instead of fossil fuels [9].

High penetration of intermittent, customer–owned and non–dispatchable PVs to the existing distribution networks can create technical problems such as voltage rise [10, 11], voltage unbalance [12], power loss and harmonics [9, 13-16].

The PVs are injecting active power based on Maximum Power Point Tracking (MPPT) algorithm in Unity Power Factor (UPF) recommended by IEEE Recommended Practice for Utility Interface of Photovoltaic Systems [7]. It also recommends the PVs to be disconnected from the grid when the network voltage is not within the 88–110 % of its nominal voltage.
Therefore, the distribution networks with PVs have two main voltage problems. In the evenings, network peak hours, the residential load increases while the power output of PVs vanishes. This imposes a voltage drop problem to the network. On the other hand, at noon, the PVs have their highest power generation while the residential load is minimal. The excess of the generated power from PVs, will cause reverse power flow into the grid and hence a voltage rise in the network.

Several methods have already been discussed and investigated for the reduction of voltage rise due to PV penetration. These methods include:

- Automatic distribution transformer tap changing [17]
- Upgrading distribution feeder’s cross-section [17]
- Installing auto-transformer/voltage regulators [17]
- Curtailing the active power output of PVs [9]
- Allowing PVs to inject/absorb reactive power [18-20]

The limitation of the first method is that distribution transformers are not usually capable of on-load tap changing. Upgrading the cross-section of the feeders is a very effective method both for voltage drop and voltage rise but very expensive. Installation of voltage regulators cannot be a permanent solution as the structure of the network might change in future. Active Power Curtailment (APC) of PVs is also effective, but is in contrary to the main purpose of PVs installation that is generating maximum power from sunshine. This can cause dissatisfaction by customers who like to get more financial benefit from electricity sell-back. Therefore, a new voltage control strategy for PVs to improve the voltage profile problems is necessary to improve the power quality within these networks.

On the other hand, the residential rooftop PVs are currently installed randomly across distribution systems. This may lead to an increase in the unbalance index of
the network. This will increasingly cause problems for three–phase loads (e.g. motors for pumps and elevators). References [16, 21] have investigated some technical problems of European and UK distribution networks for maximum allowable number of grid–connected PVs.

A voltage unbalance sensitivity analysis is necessary to be carried out to investigate the effect of PV random location and rating on the voltage unbalance of the feeder. A deterministic analysis may not be suitable given the randomness of PV installations and their intermittent nature of power generation. Monte Carlo method is already applied for analysis of uncertainties in the network in order to study load flow, voltage sag, fault and reliability [22]. Therefore, a stochastic evaluation based on Monte Carlo method is necessary to investigate and predict the network voltage unbalance for the similar uncertainties arising due to rooftop PV power ratings and locations.

1.1.2 Plug–in electric vehicles

In addition to PVs, the technical developments in automotive sector along with environmental concerns and fuel prices have lead to appearance of Plug–in Electric vehicles (PEV). In [23], it is estimated that PEVs market penetration will be about 1.5 million in 2016 in US and over 50 million in 2030 (almost 25% of all new car production). It was also stated that PEVs penetration into market will result in annual 2% increase in network load growth which is equal to double the air conditioning loads.

The PEVs will be charged by drawing current from the network as residential customers return home in the evening to be ready for next day’s travel. Therefore, these will increase the number of single–phase loads in the network considerably. The charging of PEVs are often referred to as Grid–to–Vehicle (G2V). However, it is
expected the PEV battery can inject its stored energy back into the grid as well. In this mode, often referred to as Vehicle–to–Grid (V2G), they can be used as a temporary local dispersed generation units. This means that PEVs can operate as loads or generators [24].

PEV characteristics can impose technical problems to the network and require an expansion or modification to network structure, policies, control and protection. The effects of PEVs penetration on voltage drop, power loss and costs in distribution networks has been already studied in [24-29] through deterministic or probabilistic methods.

Network voltage unbalance has not been addressed in the previous studies. This investigation is of high interest since the random connection point of PEVs in addition to their charging levels, in G2V mode, or output power, in V2G mode, among the three phases of the LV residential network might increase the voltage unbalance of the network.

A deterministic analysis may not be suitable due to the randomness in PEVs penetration level, capacity and connection points in addition to the residential loads. Therefore, a stochastic evaluation based on Monte Carlo method may be necessary to investigate and predict the network voltage unbalance for the uncertainties arising due to PEVs and network loads.

Some conventional improvement methods can be utilized for voltage profile improvement and voltage unbalance reduction. Among them, parallel and series converter–based Custom Power Devices (CPD) are already used widely for power quality improvement [3, 30]. The application of CPDs, in particular, Distribution Static Compensator (DSTATCOM) and Dynamic Voltage Restorer (DVR) are necessary to be investigated for voltage unbalance reduction and voltage profile
correction within these networks. Their optimum installation location, efficacy and rating and multiple applications need to be studied and investigated.

### 1.1.3 Micro grids

Micro grids are systems with clusters of loads and micro sources. To deliver high quality and reliable power, the micro grid should appear as a single controllable unit that responds to changes in the system [31]. The high penetration of DGs, along with different types of loads, always raise concern about coordinated control and power quality issues. In micro grid, parallel DGs are controlled to deliver the desired active and reactive power to the system while local signals are used as feedback to control the converters. The power sharing among the DGs can be achieved by controlling two independent quantities– frequency and fundamental voltage magnitude [32-34]

General introduction on micro grid basics, including the architecture, protection and power management are given in [35]. A review of ongoing research projects on micro grid in US, Canada, Europe and Japan is presented in [36]. Different Power management strategies and controlling algorithms for a micro grid is proposed in [37]. References [38-41] have evaluated the feasibility for the operation of the micro grids during islanding and synchronization. An algorithm was proposed in [42] and used for evaluation of dynamic analysis for grid connected and autonomous modes of the micro grid. In [43], it is shown that a proper control method of distributed resources can improve the power quality of the network. There are still many issues which are needed to be addressed to improve the power quality in a micro grid.

The power quality issues are important as the power electronic converters increase the harmonic levels in the network voltage and current. Unbalanced loads
can cause the current and hence the voltage of the network suffering from high values of negative sequence which can cause problems for all induction motor loads in the network. Nonlinear loads (NL) can increase the harmonic level of the network current and voltage, which will increase the loss and reduce the efficiency of the network [44, 45]. On the other hand, a power electronic converter can mitigate harmonic and unbalanced load or source problems. In [45] a series–shunt compensator is added in micro grid to achieve an enhancement of both the quality of power within the micro grid and the utility grid. The compensator has a series element as well as a shunt element. The series element can compensate for the unwanted positive, negative, and zero sequence voltage during any utility grid voltage unbalance, while the shunt element is controlled to ensure balanced voltages within the micro grid and to regulate power sharing among the parallel–connected DG systems. The proposed method in [45] requires adding other converters, while the same power quality improvement objectives can be achieved by one of the existing converters in the micro grid as proposed and validated in this thesis.

To investigate the operation of all the micro sources together, a micro grid test bed is planned to be established at Queensland University of Technology (QUT) where issues such as decentralized power sharing and enhanced power quality operation will be tested. The QUT conceptual system with the technical parameters of its micro sources was used as one of the test systems in this thesis.

1.1.4 Demand side management

Distribution networks must be designed to supply peak loads to ensure acceptable reliability, despite the fact that these peak loads typically occur for a small fraction of the year [46]. This means that the overall electricity infrastructure cost is largely determined by the peak load on the network. Consequently, there is strong
motivation to minimize peak load growth throughout the electricity network. In many parts of the world peak load growth in residential areas is higher than the consumption growth. As an example, in Queensland Australia, electrical utilities Energex (supplying the high population density south–east) and Ergon Energy (supplying the remainder of Queensland) experience an average annual residential peak load growth of 10–13% compared with an annual residential consumption growth of 3% due to a number of factors including the proliferation of air–conditioning [47, 48]. This has resulted in large annual capital expenditures on system upgrades. In the future the introduction of PEVs (which include plug–in hybrids and battery electric vehicles) is expected to further increase the peak load especially in residential areas [49-51]. This has the potential to significantly impact on the distribution network assets, especially the assets closer to the end user, where the load diversity decreases.

Much work has been historically done on demand management [52-57]. Schemes can generally be classified into either direct or indirect. Direct demand management schemes, often called Direct Load Control (DLC) systems, typically make use of a control signal from the utility to directly control loads. The water heater ripple control systems currently used in many parts of the world are an example of a traditional DLC system. Other more recent schemes often propose using a real time price as the control signal to trigger automated action from home automation controllers [58, 59]. Indirect demand management schemes use price as a control variable to influence consumers’ behavior and thus indirectly control the load. For example, time of use tariffs typically increase the price of power during peak periods thus encouraging consumers to shift their consumption to off–peak [60, 61].
Chapter 1: Introduction

Therefore, an intelligent direct demand management system for low voltage (LV) distribution networks is necessary in order to prevent overloading of distribution and upstream transformers at peak load periods and improve the network voltage profile. A Multi–Objective Decision Making (MODM) process can be used within the system to prioritize the loads to be controlled or delayed. This decision is based on several criteria, each with different weightings. This intelligent direct demand management will indirectly improve the voltage profile of the network.

1.2 Aims and objectives of the thesis

The main objective of this thesis was to analyze and propose new strategies for improving the voltage profile and reducing voltage unbalance problems in the low voltage distribution networks or micro grids with PVs and PEVs. To achieve this goal, the aims of the research project were identified as:

- Analyzing the power quality and sharing within a microgrid
- analyzing the effect of PVs and PEVs on voltage profile and voltage unbalance
- determining the applicability of the existing strategies
- determining the new strategies that are required to achieve appropriate voltage profile and unbalance improvement in a network

1.3 Significance of research

The penetration level of PVs and PEVs in the power distribution network is expected to be very high in the near future. This research will help to improve the voltage profile problems related to a distribution network or micro grid.
1.4 The original contributions of the research

The main objective of this research was to analyze the effects of PV and PEV penetration in low voltage distribution networks and to propose voltage profile improvement strategies to incorporate PVs and PEVs into a distribution network or micro grid by overcoming the identified voltage profile issues. The main contributions of this research can be listed as follows:

- Proposing application of DSTATCOM and DVRs for voltage profile improvement in low voltage distribution networks with PVs or PEVs
- Proposing a new voltage control strategy for PVs in order to improve the voltage profile of distribution networks
- Proposing a new converter control for DG units for voltage unbalance and harmonics reduction in a micro grid
- Proposing a new direct demand side management for low voltage distribution networks with PEVs connected to the network

1.5 Structure of the thesis

This thesis is organized in nine chapters. The research aims and objectives along with need and justification for the research in this field are outlined in Chapter 1. A literature review is carried out to identify the protection issues related to PVs and PEVs connected to the low voltage distribution networks and micro grids.

A new converter control for voltage unbalance reduction and harmonic elimination in a micro grid utilizing one of the DG units is presented in Chapter 2.

In Chapter 3, A voltage unbalance sensitivity analysis is carried out for random location and rating of single–phase rooftop PVs in a low voltage distribution network.
One new improvement method and some conventional methods are discussed in Chapter 4 for voltage unbalance reduction due to random location and ratings of PVs.

In Chapter 5, the application of DSTATCOM and DVRs with a new control algorithm are proposed and studied for voltage profile improvement and voltage unbalance reduction in low voltage distribution networks. In addition, a new converter control is presented for PVs in order to regulate the voltage in peak and off-peak periods along the feeder in Chapter 6.

Chapter 7 discusses the voltage unbalance problem as the result of PEVs running in V2G and G2V modes. The study verifies that similar improvement methods can be used for voltage profile and voltage unbalance improvement when PEVs are connected to low voltage distribution networks.

In chapter 8, a new direct load control is presented for preventing transformer overloading at network peak hours as the results of PEVs connected to the network. This will indirectly improve the network voltage profile at network peak hours.

Conclusions drawn from this research and recommendations for future research are given in Chapter 9.
Chapter 2: Operation and Control of a Hybrid Micro grid with Unbalanced and Nonlinear Loads

In this chapter, the power quality enhanced decentralized power sharing is investigated in an autonomous micro grid with diesel generators and converter interfaced micro sources. To investigate the system response with the dynamics of the DGs, the micro sources and all the power electronic interfaces are modeled in detail. One of the converter interfaced sources is used as the compensator of the nonlinear and unbalanced load while the other DGs share the system load proportional to their rating based on droop control. The compensating DG can work in different operational modes depending on the power requirement of the local nonlinear load from just supplying a part of the nonlinear load to sharing some power of the micro grid loads while functioning as a compensator. Also, the compensation principle is tested on a low voltage residential distribution network that is connected to the micro grid.

2.1 Micro grid structure

The schematic diagram of the micro grid system under consideration is shown in Fig. 2.1. There are four DGs as shown; one of them is an inertial DG (diesel generator) while others are converter interfaced DGs (PV, FC and battery). There are four resistive heater loads and six induction motor loads. A nonlinear load, which is a
Chapter 2: Operation and control of a hybrid micro grid with unbalanced and nonlinear loads

A combination of unbalance and harmonic load, is also connected to BUS 5 in the micro grid. The FC will be used as the compensating DG for power quality improvement in this structure since it is the closest amongst all the converter interfaced DGs to the nonlinear load and connected to the same bus. If the nonlinear load was connected to BUS 3 or 4, the PV or battery should be used as the compensating DG. A discussion on the compensator location and the criteria for its placement is given below. The parameters of the micro grid, loads, DGs and their converters are given in Table 2.1. In this chapter, the autonomous operation mode of the micro grid is studied.

Fig. 2.1 Schematic diagram of the micro grid structure under consideration.

2.2 Effect of compensating DG location

Let us consider a three-phase distribution system with structure shown in Fig. 2.2 where a nonlinear load is connected to BUS 6. BUS 1 is assumed to be stiff and the feeders have impedance. The implications of placing the compensator at various buses of this figure are listed in Table 2.1. It is evident from the table that the compensator can make the voltages of all the buses sinusoidal if it is connected at the same bus in which the nonlinear load is connected.
In Fig. 2.3, the voltage waveforms of BUS 1 and PV output current are shown when the nonlinear load is connected to BUS 1 of micro grid structure of Fig. 2.2. It can be seen that both voltage and current are unbalanced and the distortion in the voltage waveform is obvious. Similar waveforms can be shown for all other buses except BUS 5 at which the compensator is connected.

Table 2.1 Effected buses due to implication of compensator in various buses

<table>
<thead>
<tr>
<th>Compensator at bus</th>
<th>Voltage distortion at buses</th>
<th>Sinusoidal voltage at buses</th>
</tr>
</thead>
<tbody>
<tr>
<td>BUS 2</td>
<td>BUS 4 to 9</td>
<td>BUS 2 to 3</td>
</tr>
<tr>
<td>BUS 3</td>
<td>BUS 2, 4 to 9</td>
<td>BUS 3</td>
</tr>
<tr>
<td>BUS 4</td>
<td>BUS 6 to 9</td>
<td>BUS 2 to 5</td>
</tr>
<tr>
<td>BUS 5</td>
<td>BUS 2 to 4, 6 to 9</td>
<td>BUS 5</td>
</tr>
<tr>
<td>BUS 6</td>
<td>None</td>
<td>All</td>
</tr>
<tr>
<td>BUS 7</td>
<td>BUS 2 to 6</td>
<td>BUS 7 to 9</td>
</tr>
<tr>
<td>BUS 8</td>
<td>BUS 2 to 7, 9</td>
<td>BUS 8</td>
</tr>
<tr>
<td>BUS 9</td>
<td>BUS 2 to 8</td>
<td>BUS 9</td>
</tr>
</tbody>
</table>
2.3 Droop control methods in micro grid

In this section, the power sharing method in the micro grid is discussed. The decentralized power sharing among the DGs is achieved by the use of conventional droop control [32, 33] as

\[
\omega = \omega_s - m (P - P_{\text{rated}}) \\
V = V^* - n (Q - Q_{\text{rated}})
\]  

(2.1)

where \(m\) and \(n\) are the droop coefficients taken proportional to rated power of DGs for power sharing among them, \(\omega_s\) is the synchronous frequency, \(V^*\) is the nominal magnitude of the network voltage, \(V\) is the magnitude of the converter output voltage and \(\omega\) is its frequency, while \(P\) and \(Q\) respectively denote the active and reactive power supplied by the converter, (The suffix \textit{rated} represents the rated power). Thus the frequency and the voltage are being controlled respectively by the active and reactive power output of the DG sources. Therefore, according to [32, 33], the principles of decentralized power sharing in a micro grid is based on keeping proportional power output based on the rating of the DGs and power sharing amongst DGs are given by

\[
\frac{P_1}{P_2} \approx \frac{m_2}{m_1} = \frac{P_{1\text{rated}}}{P_{2\text{rated}}} \quad , \quad \frac{P_3}{P_1} \approx \frac{m_3}{m_1} = \frac{P_{3\text{rated}}}{P_{1\text{rated}}} \quad , \quad \ldots
\]

\[
\frac{Q_1}{Q_2} \approx \frac{n_2}{n_1} = \frac{Q_{1\text{rated}}}{Q_{2\text{rated}}} \quad , \quad \frac{Q_3}{Q_1} \approx \frac{n_3}{n_1} = \frac{Q_{3\text{rated}}}{Q_{1\text{rated}}} \quad , \quad \ldots
\]

(2.2)

where the number suffixes show the number of each DG in the micro grid. The reference angle for the non inertial DGs (genset) is derived from the reference frequency.
Chapter 2: Operation and control of a hybrid micro grid with unbalanced and nonlinear loads

2.4 Compensator control

In this section, the compensator control method and reference generation for the compensating DG is presented. As mentioned before, depending on the power requirement of the local nonlinear load, the compensating DG (FC) can work in different operational modes. If the required power of the local nonlinear load is less than the power rating of the FC, the compensating DG supplies the local nonlinear load totally and then the rest of the power is fed to the micro grid (Mode I). While the power requirement of the local nonlinear load is more than the rating of the compensating DG, the extra power requirement is supplied from the other micro sources in the micro grid (Mode II). In both of the modes, the other three micro sources (genset, PV and battery) always share the power proportional to their rating through the droop control. The most important aim of the compensator is supplying a current to the point of common coupling (PCC) that balances the voltage \( v_p \) at PCC and therefore, a balanced and non harmonic current will be drawn or injected to the micro grid. The schematic structure of the compensator is shown in Fig. 2.4.

Fig. 2.4 Schematic diagram of the compensator.

As has been shown in [62], the compensator current that needs to be supplied to the micro grid is given by
Chapter 2: Operation and control of a hybrid micro grid with unbalanced and nonlinear loads

\[
\begin{bmatrix}
    i_{\text{comp,a}} \\
    i_{\text{comp,b}} \\
    i_{\text{comp,c}} \\
\end{bmatrix} =
\begin{bmatrix}
    i_{NLa} \\
    i_{NLb} \\
    i_{NLc} \\
\end{bmatrix} - \frac{1}{K} \begin{bmatrix}
    3P_{MG}v_{pa} + \sqrt{3}Q_{MG}(v_{pb} - v_{pc}) \\
    3P_{MG}v_{pb} + \sqrt{3}Q_{MG}(v_{pc} - v_{pa}) \\
    3P_{MG}v_{pc} + \sqrt{3}Q_{MG}(v_{pa} - v_{pb}) \\
\end{bmatrix}
\]  

(2.3)

where, as shown in Fig. 2.4, \( P_{MG} \) and \( Q_{MG} \) respectively are the active and reactive power drawn/supplied to the micro grid, \( i_{\text{comp}} \) is the compensator current, \( i_{NL} \) is the nonlinear load current, \( v_p \) is the PCC voltage and the three phases are denoted by the subscripts \( a, b \) and \( c \) and \( K = v_{pa}^2 + v_{pb}^2 + v_{pc}^2 \). From (2.3), we derive the current injection requirements for the two modes, which are discussed below.

2.4.1 Mode I

In this mode, it is assumed that the power demand of the nonlinear load is less than the rated power of the compensating DG. Therefore, the compensator supplies the whole demand of the nonlinear load and a part of the power requirement of the other loads in the micro grid. Therefore, it is expected that the micro grid current \( I_{MG} \), and active and reactive power \( P_{MG} \) and \( Q_{MG} \) shown in Fig. 2.4 are negative. So, the power that can be injected by the compensator to the micro grid will be

\[
-P_{MG} = P_{\text{comp,rated}} - P_{NL} \\
-Q_{MG} = Q_{\text{comp,rated}} - Q_{NL}
\]  

(2.4)

where \( P_{\text{comp,rated}} \) and \( Q_{\text{comp,rated}} \) respectively are the rated active and reactive power output of the FC, which are calculated based on the maximum current that can be supplied by the FC. We can then modify (2.3) to get the following reference currents

\[
\begin{bmatrix}
    i_{\text{comp,a}} \\
    i_{\text{comp,b}} \\
    i_{\text{comp,c}} \\
\end{bmatrix} =
\begin{bmatrix}
    i_{NLa} \\
    i_{NLb} \\
    i_{NLc} \\
\end{bmatrix} + \frac{1}{K} \begin{bmatrix}
    3(P_{\text{comp,rated}} - P_{NL})v_{pa} + \sqrt{3}(Q_{\text{comp,rated}} - Q_{NL})(v_{pb} - v_{pc}) \\
    3(P_{\text{comp,rated}} - P_{NL})v_{pb} + \sqrt{3}(Q_{\text{comp,rated}} - Q_{NL})v_{pc} - v_{pa} \\
    3(P_{\text{comp,rated}} - P_{NL})v_{pc} + \sqrt{3}(Q_{\text{comp,rated}} - Q_{NL})(v_{pa} - v_{pb}) \\
\end{bmatrix}
\]  

(2.5)

Eq. (2.5) remains valid as long as the nonlinear load power demand is less than the rated power of the compensating DG. In case the power requirement is increased
Chapter 2: Operation and control of a hybrid micro grid with unbalanced and nonlinear loads

to more than the compensator rating, the control scheme will change the operation to Mode II.

2.4.2 Mode II

In Mode II, it is assumed that the power demand of the local nonlinear load is more than the rated power of the compensating DG. The compensating DG can supply a part of the total demand of the nonlinear load ensuring power quality improvement, while the rest of the required power is supplied from the micro grid side. This power is shared among the other three micro sources with all other micro grid loads based on droop control as described in Section 2.2.

It is expected that $I_{MG}, P_{MG}$ and $Q_{MG}$ are all positive with respect to the sign convention direction shown in Fig. 2.4. The amount of the nonlinear load power supplied by the compensator can be a fixed fraction of the whole power or equal to the rating of the compensating DG. Therefore, we have:

$$
P_{MG} = P_{Lav} - P_{comp} = P_{Lav} - \lambda_p P_{Lav} = P_{Lav} (1 - \lambda_p)
$$

$$
Q_{MG} = Q_{Lav} - Q_{comp} = Q_{Lav} - \lambda_Q Q_{Lav} = Q_{Lav} (1 - \lambda_Q)
$$

where $P_{Lav}$ and $Q_{Lav}$ are respectively the average active and reactive power demand of nonlinear load and $\lambda_p (0 < \lambda_p < 1)$ and $\lambda_Q (0 < \lambda_Q < 1)$ are respectively fractions of the active and reactive power supplied by compensating DG to the nonlinear load.

We can substitute (2.6) in (2.3) to obtain

$$
\begin{bmatrix}
i_{comp,a} \\
i_{comp,b} \\
i_{comp,c}
\end{bmatrix} = \begin{bmatrix}
i_{NLa} \\
i_{NLb} \\
i_{NLc}
\end{bmatrix} - \frac{1}{K} \begin{bmatrix}
3P_{Lav} (1 - \lambda_p) \times v_{pa} + \sqrt{3}Q_{Lav} (1 - \lambda_Q) \times (v_{pb} - v_{pc}) \\
3P_{Lav} (1 - \lambda_p) \times v_{pb} + \sqrt{3}Q_{Lav} (1 - \lambda_Q) \times (v_{pc} - v_{pa}) \\
3P_{Lav} (1 - \lambda_p) \times v_{pc} + \sqrt{3}Q_{Lav} (1 - \lambda_Q) \times (v_{pa} - v_{pb})
\end{bmatrix}
$$

2.5 Converter structure

The converter interfaced micro sources like PV, FC and battery are connected to the micro grid through voltage source converters (VSC) as shown in Fig. 2.1.
Output DC voltages of PV and FC are regulated by DC–DC choppers to control the power flow. The VSC structure and control of the micro sources including the compensator is discussed in this section.

2.5.1 Compensator VSC structure

The compensating DG has a VSC structure consisted of three single-phase H–bridges using Insulated Gate Bipolar Transistors (IGBTs) as shown in Fig. 2.5. The outputs of each H–bridge are connected to single–phase transformers and the three transformers are star connected. The VSC is utilizing a closed–loop optimal robust controller based on state feedback. The resistance $R_f$ represents the switching and transformer losses, while the inductance $L_f$ represents the leakage reactance of the transformers and the filter capacitor $C_f$ is connected to the output of the transformers to bypass the switching harmonics.

![Fig. 2.5 Schematic diagram of the VSC for compensating DG.](image)

The single-phase equivalent circuit of the VSC is shown in Fig. 2.6. The rest of the network is represented by voltage source $V_{eq}$ and equivalent resistance ($R_{eq}$) and inductance ($L_{eq}$) as shown in the figure. In this figure, $u \cdot V_{dc}$ represents the converter output voltage, where $u$ is the switching function that can take on $\pm 1$ value.
depending on which pair of the IGBTs is turned on. The main aim of the converter control is to generate $u$.

\[ x^T = \begin{bmatrix} v_{cf} & i_{cf} & i_{comp} \end{bmatrix} \]  

(2.8)

From the circuit of Fig. 2.6, system state space description can be given as

\[ \dot{x} = Ax + B_1 u_c + B_2 V_{eq} \]  

(2.9)

where $u_c$ is the continuous time version of switching function $u$. The discrete–time equivalent of (2.9) is

\[ x(k+1) = Fx(k) + G_1 u_c(k) + G_2 V_{eq}(k) \]  

(2.10)

Based on this model and a suitable feedback control law, $u_c(k)$ is computed. In this thesis, capacitor reference voltage generation is based on measurement of the PCC voltage and calculating the fundamental voltage amplitude and angle. Later, the PCC voltage is fixed at the calculated voltage amplitude and angle by appropriate switching of IGBTs. The switching control laws are given by

\[ u_c(k) = -K\left[x(k) - x_{ref}(k)\right] \]  

(2.11)

where $K$ is a gain matrix and $x_{ref}$ is the reference vector. The gain matrix in this thesis was obtained by LQR based on optimal control which ensures the desired results of the system while the variations of system load and source parameters are within
acceptable limits of reality. From $u_c(k)$, the switching function is generated based on an error level determination generated by

$$
\text{If } u_c > h \text{ then } u = +1 \\
\text{elseif } u_c < -h \text{ then } u = -1
$$

(2.12)

where $h$ shows the error level and has very small value. A more detail on converter control is given in [3].

2.5.2 VSC structure of other DGs

The VSC structure of all the other micro sources are the same as the structure of the compensator but there is an inductance at their output connection point for controlling the amount of active and reactive power injected to the network. The same controlling method in the previous section is being used for generating the switching pulses of the IGBTs in these converters, too.

2.6 Modeling of micro grid

As described in Section 2.1, there are four DGs in the micro grid. The diesel genset is modeled as in [63] and is not discussed here. Other three DG models and associated power electronic controllers are discussed below and their technical data are given in Appendix A.

2.6.1 Fuel Cell (FC)

FCs are emerging as an attractive power supply source for applications such as distributed generation because of their cleanness, high efficiency, and high reliability. A review on the FC technology, characteristics and research area is given in [64]. Usually four types of FCs, named PEMFC, PAFC, MCFC and SOFC, classified based on their electrolyte type, are used in electrical utilities for electric
power generation [65]. A mathematical model for investigating the dynamic performance of a PEMFC was developed in [66]; this model is based on physical laws having clear significance in replicating the FC system and can easily be used to set up different operational strategies. From the empirical point of view, [67] formulated a model that enables simulation of the V–I curve of FCs in typical conditions. Different from the normal PEMFC model, a purely electronic circuit model similar to characteristics of a PEMFC was introduced in [68] that can be used to design and analyze FC power systems using electric circuit elements. As an experiment, the steady–state performance and transient response for hydrogen and oxygen flow in PEMFCs is investigated in [69].

In this chapter, a typical PEMFC with simplified model of Fig. 2.7 and output V–I characteristic of (2.13), verified experimentally and reported in [70] is used. Similar characteristic is given for all FCs in [66-68] where their numerical values differ according to their rating, output voltage and application. The studies in this thesis are based on the output electric power point of view and physical–chemical characteristics of FCs such as hydrogen and oxygen pressure are not investigated. The V–I characteristic of the FC is given by

\[
V(i) = 371.3 - 12.38 \log(i) - 0.2195i - 0.2242e^{0.025i}
\]  

(2.13)

A boost chopper is used at FC output for regulating the necessary DC voltage \(v_c\) across the capacitor. The schematic diagram of the simulated FC model with the output chopper is shown in Fig. 2.7.

FCs have several shortcomings [68] such as no energy storage possibility, slow dynamic response, output voltage fluctuation with load and difficult cold start. Therefore, an electric storage such as battery or ultracapacitor must be accompanied with FC to improve its dynamic characteristics. If the storage is in parallel directly
Chapter 2: Operation and control of a hybrid micro grid with unbalanced and nonlinear loads

with the DC bus, its charge and discharge cannot be controlled [68]; therefore, a bidirectional converter is needed between the DC bus and the electric storage to control its state of charge. A detailed control basics and algorithm for the bidirectional converter of the storage is presented and verified in [71]. The studies carried out in the thesis, show that the FC has a good and acceptable dynamic response for power quality improvement and power sharing objectives and no storage unit on FC was used in this thesis.

![Fuel Cell equivalent circuit](image)

Fig. 2.7 Fuel cell and storage modelled equivalent circuit.

### 2.6.2 Photovoltaic cell (PV)

A series and parallel combination of PV cells constitute a PV array. Fig. 2.8 shows the simplified equivalent circuit where output voltage is a function of the output current while the current is a function of load current, ambient temperature and radiation level [72]. The voltage equation of the PV is calculated by

$$V_{pv} = \frac{AKT}{e} \ln \left( \frac{I_{ph} + I_o - I_c}{I_o} \right) - R_s I_c$$  \hspace{1cm} (2.14)

where

$A$: constant value for curve fitting

$e$: electron charge ($1.602 \times 10^{-19}$ C)

$k$: Boltzmann constant ($1.38 \times 10^{-23}$ J°K)
Chapter 2: Operation and control of a hybrid micro grid with unbalanced and nonlinear loads

$I_c$: output current of PV cell

$I_{ph}$: photocurrent (1 A)

$I_o$: diode reverse saturation current (0.2 mA)

$R_s$: series resistance of PV cell (1 mΩ)

$V_{PV}$: output voltage of PV cell

$T_c$: PV cell reference temperature (25°C)

The output chopper controls the voltage $v_c$ across the capacitor. A Maximum Power Tracking (MPPT) method is used to set the reference voltage of the chopper to achieve maximum power from the PV based on the load or ambient condition changes. The MPPT algorithm used in this thesis is given in [72]. A PI controller is used in the chopper in order to achieve the desired reference voltage set by the MPPT. A battery storage system is connected in parallel with the DC bus of the chopper output through a bidirectional converter which is used to control the charging and discharging the battery. Depending on the terminal voltage of the PV, the battery gets charged or discharged. A more detailed explanation on bidirectional converter control of PV storage system is given in [73].

Fig. 2.8 Equivalent circuit of PV, boost chopper based on MPPT and storage.
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2.6.3 Battery

The battery is assumed to be a constant voltage source with fixed amount of energy and modeled as a constant DC voltage source with series internal resistance where the VSC is connected to its output. The battery has a limitation on the duration of its generated power and depends on the amount of current supplied by it. It is assumed that the battery is charged at the off-peak load periods of the network and is discharged at peak load times through the converter.

2.7 Study case and simulation results

The system is simulated in various operating conditions with different load demand in the micro grid. The simulation results are discussed below.

2.7.1 Compensator principle operation

In this case, the simple structure of Fig. 2.4 is simulated to investigate the compensator effects on power quality improvement of the load. An unbalanced and harmonic load which causes a 9.3% and 1.47% unbalance in the network current and voltage respectively is connected to an ideal voltage source. In addition to that, the nonlinear load has a 9.8% and 8.2% Total Harmonic Distortion (THD) in the current and voltage respectively. The compensator is connected to the network at 0.05 sec. In Fig. 2.9, the network voltage \( V_P \) and current driven from the source \( I_{MG} \) are shown before and after the compensation. As shown in this figure, the compensator is injecting the necessary current to PCC to balance the voltage and therefore, the current drawn from the source is forced to be balanced. In Fig. 2.10, the initial and final values of the unbalance and THD of network current and voltage are shown before and after the compensator connection. The current and voltage unbalance
values are limited to less than 0.2% and 0.05% respectively while the same values for THD are respectively less than 0.4% and 0.2%. Fig. 2.11 illustrates the FFT diagrams and proves the reduction of the harmonic orders of network current and voltage after compensator connection compared to the harmonic order values before the compensation.

The proposed compensator is also capable of complete reactive power compensation and power factor correction by injecting the exact amount of the reactive power demand of the nonlinear load. In Fig. 2.12, the instantaneous voltage and current waveforms are shown together where the phase difference is obvious before the compensator connection which is minimized to zero after the compensation and the power factor is corrected to unity.

Fig. 2.9 Load current, compensator output current, source current and network voltage before and after compensation.
Chapter 2: Operation and control of a hybrid micro grid with unbalanced and nonlinear loads

2.7.2 Power sharing in micro grid

In this case, the micro grid operation has been investigated during autonomous mode. In autonomous mode, total power demand is shared among the DGs proportional to their rating. For investigating the dynamic response of the controller,
two incidents, including load change and power limiting of a DG are studied in the micro grid structure of Fig. 2.1. In this case, it is given that the nonlinear load is not connected to the network and therefore, FC is just working like other micro sources in power sharing. It is assumed that the system is operating in steady state while all the micro sources are connected and supplying all the loads except the 6 kW fan heater load. At 0.4 sec., there is a sudden power limitation on the output power of the PV which is reduced from 2.43 kW to 1 kW. Therefore, the other DG units are responsible for supplying the rest of the required power to the loads. Hence, the output power of the other DG units is increased but still with respect to their rated power. The second incident that occurs is the connection of the 6 kW fan heater load to the network at 0.9 sec. Increase of power demand in the network results in the output power increase of the DG units with respect to their rated values except the PV which is still working in power limit mode. The power response of the DGs and controller in the micro grid are shown in Fig. 2.13 and the numerical values of the power sharing in this case are given in Table 2.2. The dynamic of the step response of the network proves that the micro grid system and controller stabilizes to steady state condition within 5–6 cycles.

The power dispatch among the PV cell and its storage is shown in Fig. 2.14. In this figure, it is assumed that PV cell was generating 2.6 kW which (minus the inverter loss) is feeding into the micro grid. At 0.5 sec., the PV active power generation has increased to 6.4 kW. Therefore, the surplus energy (3.75 kW) is saved into the storage unit. At 1 sec., it is assumed that there is a limitation on the PV power generation but since the storage unit is already charged, the required power is fed into micro grid by the storage. If at 1.5 sec. the PV is again able to produce the required power and hence output power of storage returns to zero.
Chapter 2: Operation and control of a hybrid micro grid with unbalanced and nonlinear loads

Table 2.2 Numerical values of power sharing of the micro sources in micro grid [kW]

<table>
<thead>
<tr>
<th></th>
<th>$P_{\text{SynGen}}$</th>
<th>$P_{\text{FC}}$</th>
<th>$P_{\text{Bat}}$</th>
<th>$P_{\text{PV}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>steady state</td>
<td>10.46</td>
<td>3.28</td>
<td>1.64</td>
<td>2.51</td>
</tr>
<tr>
<td>power limiting in PV</td>
<td>11.1</td>
<td>3.29</td>
<td>1.68</td>
<td>1</td>
</tr>
<tr>
<td>load change</td>
<td>15.1</td>
<td>4.67</td>
<td>2.37</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 2.13 Active power sharing of DGs in micro grid in autonomous mode for load change and PV power limiting.

Fig. 2.14 Active power dispatch among the PV and its storage unit.
2.7.3 Micro grid with nonlinear load

In this case, the FC converter is controlled not only to supply power to the micro grid but also to improve the power quality of the network. The nonlinear load is connected to BUS 5 where the FC is connected and circuit breaker CB_1 is open. The nonlinear load is a combination of unbalanced and harmonic load of Case 1. Depending on the nonlinear load power demand, the compensator can work in Mode I or Mode II. It is assumed that the micro grid is operating in steady state while the compensator is operating in Mode I. In this mode, the FC is supplying the whole power demand of the nonlinear load and the rest of its rated power is fed to the micro grid as discussed in Section 4. At 0.25 sec., the power demand of the nonlinear load is increased to more than the rated power of the FC. Therefore, the operation mode of the converter is changed to Mode II and it is desired that the FC supplies half of the power demand of the nonlinear load ($\lambda_P = \lambda_Q = 0.5$) while the other half is supplied by the micro grid. At 1.25 sec., the power demand of the nonlinear load is decreased to the initial value and the compensator is returned back to Mode I. The power output of the compensator (FC), power from the micro grid to PCC and the power demand of the nonlinear load are shown together in Fig. 2.15. The active power sharing among other micro sources according to the operation mode of the compensating DG is also shown in Fig. 2.16. The numerical results of the active power of all the DG units, nonlinear load and power from micro grid to PCC are given in Table 2.3. The RMS voltage of the network, variation of which is kept in a limit of ± 5%, increase and decrease due to the changes in the nonlinear load power demand, as shown in Fig. 2.17.

In the same case, for investigating the effects of compensator on the voltage and current conditions of the network, the waveforms at PCC are shown separately in
the absence and presence of the compensator in Fig. 2.18. By comparing the waveforms, it is obvious that the compensator has been able to effectively improve the unbalance and THD values. The numerical values of unbalance and THD of current and voltage of the micro grid at PCC with and without compensator are given in Table 2.4.

![Active Power at Compensator PCC](image)

**Fig. 2.15** Active power output of compensator (FC), power from the micro grid to PCC and the power demand of the nonlinear load.

![Active Power Sharing of DG units](image)

**Fig. 2.16** Active power sharing of synchronous generator, PV and battery during Mode I and II operating conditions of the compensator.
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![Bus Voltage RMS](image)

**Fig. 2.17 Micro grid voltage RMS variations.**

![PCC Voltage Before Compensation and Grid Current at PCC Before Compensation](image)

![PCC Voltage After Compensation and Grid Current at PCC After Compensation](image)

**Fig. 2.18 PCC voltage and current instantaneous waveforms of micro grid with and without compensator.**
Chapter 2: Operation and control of a hybrid micro grid with unbalanced and nonlinear loads

<table>
<thead>
<tr>
<th></th>
<th>$P_{\text{SynGen}}$</th>
<th>$P_{\text{PV}}$ (error)</th>
<th>$P_{\text{Bat}}$ (error)</th>
<th>$P_{\text{NL}}$</th>
<th>$P_{\text{comp. (FC)}}$</th>
<th>$P_{\text{MG}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>with compensator (Mode I)</td>
<td>17.99</td>
<td>4.46 (1%)</td>
<td>2.91 (3%)</td>
<td>2.78</td>
<td>3.88</td>
<td>–1.1</td>
</tr>
<tr>
<td>with compensator (Mode II)</td>
<td>18.92</td>
<td>4.52 (4%)</td>
<td>2.93 (5%)</td>
<td>5.17</td>
<td>2.47</td>
<td>2.7</td>
</tr>
<tr>
<td>without compensator</td>
<td>19.47</td>
<td>4.82 (1%)</td>
<td>3.14 (3%)</td>
<td>4.88</td>
<td>0</td>
<td>4.88</td>
</tr>
</tbody>
</table>

Table 2.4 Numerical values of THD and unbalance of current and voltage before and after compensation [%]

<table>
<thead>
<tr>
<th>THD</th>
<th>Before compensation $I_{\text{MG}}$</th>
<th>After compensation $I_{\text{MG}}$</th>
<th>Before compensation $V_p$</th>
<th>After compensation $V_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unbalance</td>
<td>Before compensation $I_{\text{MG}}$</td>
<td>12.7</td>
<td>&lt; 1.5</td>
<td>10.5</td>
</tr>
<tr>
<td></td>
<td>After compensation</td>
<td>&lt; 0.5</td>
<td>0.7</td>
<td>&lt; 0.2</td>
</tr>
</tbody>
</table>

2.7.4 Micro grid supplying single–phase residential loads

Each residential load is usually supplied from one phase of a three–phase system. In addition, any residential customer can have a rooftop PV system, which is also single–phase in nature. These rooftop PVs can have different ratings and they can export power to the micro grid. Therefore, not only the nature of the single–phase loads can increase the unbalance in the current in the network, but also the active power amount and direction can change. Therefore, such residential supply such grid connected PV systems can have much worse effect on the unbalance characteristics of the network.

For this study, the nonlinear load of case 3 has been replaced with a low voltage residential distribution network with the characteristics explained above. The schematic diagram of this network is shown in Fig. 2.19 and its technical data is
Chapter 2: Operation and control of a hybrid micro grid with unbalanced and nonlinear loads
given in Appendix A. For this study, it is assumed that the micro grid with
distribution network is in its steady state where at 0.5 sec. the compensator is
connected to the network and starts functioning in Mode I. The compensator
connection also improves the voltage profile of the network and removes the voltage
sag effect due to high power demand of the loads. It is assumed that at 1.5 sec. a load
change is applied in the distribution network which causes the compensator to start
operating in Mode II. The active power sharing of micro sources (genset, PV and
battery), active power dispatch at PCC (including power generated by compensator,
power from micro grid to PCC and power demand of residential distribution
network) and also the power dispatch at each phase of the residential distribution
network are shown separately in Fig. 2.20. The variation of the voltage RMS of the
network is also shown in Fig. 2.21 which is kept in the acceptable range of 5%. Fig.
2.22 and Fig. 2.23 respectively show the voltage and current instantaneous and
unbalance values at 0.5 sec. in the bus that the compensator is connected to the
network. From these results, the efficacy of the proposed controller for power quality
improvement is evident.

![Schematic diagram of the low voltage residential distribution network.](image)

Fig. 2.19 Schematic diagram of the low voltage residential distribution network.
Chapter 2: Operation and control of a hybrid micro grid with unbalanced and nonlinear loads

Fig. 2.20 Active power sharing of DG units, Active power dispatch at PCC, Active power dispatch at each phase of the residential distribution network.

Fig. 2.21 Micro grid voltage RMS variations.
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2.8 Summary

In this chapter, the power quality improvement in a hybrid micro grid has been discussed. The hybrid micro grid consists of inertial and converter interfaced micro sources where a decentralized power sharing algorithm based on droop control is used. One DG, used as compensator, is able to compensate the nonlinear (unbalanced and harmonic load) of the micro grid while other DGs share the power. The proposed control scheme can change its mode of operation seamlessly depending on the power demand of the nonlinear load. The extension of the nonlinear load to a low voltage residential distribution network shows the possibility of supplying single-phase
residential loads with some PVs. Inclusion of the micro source model ensures proposed power electronic control can work in tandem with the associated dynamics of micro sources. A stable operation in various operating condition shows the efficacy of proposed control scheme.
Chapter 3: Voltage Unbalance Analysis in Residential Low Voltage Distribution Networks with Rooftop PVs

A voltage imbalance sensitivity analysis based on the rating and location of single–phase grid–connected rooftop PVs in a residential LV distribution network is presented in this chapter. A stochastic evaluation based on Monte Carlo method is carried out to investigate and predict the network voltage unbalance for the uncertainties arising due to rooftop PVs.

3.1 Voltage profile and voltage unbalance

The Electricity supplies are nominally 110 V (in US and Canada) or 220–240 V. National standards specify that the nominal voltage at the source should be in a narrow tolerance range of ±5–10% in different countries. Based on Australian Standard Voltages, AS60038–2000, Australian low voltage network is 230 V with a tolerance between +10% and –6% [74].

Voltage unbalance in the three–phase electric system is a condition in which the three phase voltages differ in amplitude and/or does not have its normal 120 degree phase difference. There have been several methods for definition, calculation and interpretation of voltage unbalance as proposed in [75-77]. IEEE Recommended Practice for Monitoring Electric Power Quality defines this as [78]
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\[ VU\% = \left| \frac{V_-}{V_+} \right| \times 100 \quad (3.1) \]

where \( V_- \) and \( V_+ \) are the negative and positive sequence of the voltage, respectively. This will be referred to as percentage voltage unbalance in the thesis.

According to [78], the allowable limit for voltage unbalance is limited to 2% in low voltage and medium voltage networks. Engineering Recommendation P29 in UK not only limits the whole voltage unbalance of the network to 2%, but also limits the voltage unbalance to 1.3% at the load point [79]. ANSI standard for “Electric Power Systems and Equipment Voltage Ratings (60 Hertz)” recommends that electrical supply systems should be designed and operated to limit the maximum voltage unbalance to 3% when measured at the electric utility end points under no–load conditions [80].

IEEE Recommended Practice for Electric Power Systems in Commercial Buildings (Gray Book) [81] indicates that the single–phase power electronics based devices like the computers, entertainment equipment, etc may experience problems if the voltage unbalance is more than 2–2.5% where the voltage amplitudes exceed the limits.

The voltage unbalance has adverse effect on the three–phase power electronic based equipment in the network [82, 83], (e.g. central speed variable air conditioners). It will also have adverse effect on the operating characteristics of three–phase induction motors used in water pumps, elevators, etc in residential apartment complex [84].
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3.2 Voltage unbalance in LV distribution networks with PVs

The utilities try to minimize the unbalance index in their network by distributing single–phase loads equally across all three phases. Probabilistic studies have shown that it is very rare that the residential and small business loads can result in higher values of the voltage unbalance in the network. The measurements done in a LV distribution network in US [82], Brazil [83] and Iran [85] conclude that the probability of the voltage unbalance to be more than 3% in the network is about 2–5%. However, this can only be achieved if the engineering judgments have been applied for selecting the appropriate size of the conductors and cables, transformer ratings and also if the load dispatch among the three phases are re–corrected by later observations and measurements. If the appropriate designs have not been done or there is a non–standard voltage drop in the network, it is highly probable that the network also suffers from higher voltage unbalance.

As mentioned before, rooftop PVs that are currently being installed depend on outlook and financial condition of householders. Therefore, it can be expected that these installations are randomly placed along distribution feeders. For example, it is possible that 80% of customers on a phase have installed PVs, while the other two phases have only 50% and 10% installed. In such a condition, even if the voltage unbalance of the network was within the standard limits without any PVs, it is not guaranteed to remain so. Therefore, the possible PV installation number or rating on such systems must be investigated in a way that the voltage unbalance is still kept within the standard limit.
Chapter 3: Voltage unbalance analysis in residential low voltage distribution networks with rooftop PVs

3.2.1 Network structure

A sample radial LV residential urban distribution network is considered for voltage unbalance investigations. The simplified equivalent single line diagram of one feeder of the network is illustrated in Fig. 3.1. In this model, the PVs are all grid–connected such that the surplus of the electricity generated will flow to the grid. It has been assumed that all the PVs work at unity power factor based on IEEE Recommended Practice for Utility Interface of Photovoltaic Systems [7]. It is also assumed that the neutral conductor is making the pass for unbalance current circulation and the analysis is based on the mutual effect of three phases.

![Fig. 3.1 Schematic single line diagram of one feeder of the studied LV distribution network.](image)

3.2.2 Power flow analysis

For calculating the voltage unbalance, it is necessary that the network to be analysed and the voltages at the desired nodes to be calculated. Based on the Kirchhoff’s Circuit Laws (KCL) on each node of phase $A$, we have for the $k^{th}$ node

$$\frac{\beta(V_{A,PV,k} - V_{A,k})}{X_{A,PV,k}} + \frac{V_{A,k-1} - V_{A,k}}{Z_f} + \frac{V_{A,k+1} - V_{A,k}}{Z_f} + \frac{V_{N,k} - V_{A,k}}{Z_{A,i,k}} = 0 \quad (3.2)$$
where $Z_f$ is the feeder impedance between two adjacent nodes in phase lines, $V_{A,i}$, $i=1,...,n$ is the single–phase voltage of the $i^{th}$ node of phase $A$, $Z_{A,L,k}$ is the load impedance connected to $k^{th}$ node of phase $A$ and and $V_{N,k}$ is the voltage of the neutral wire connected to $k^{th}$ node. $V_{A,PV,k}$ and $X_{A,PV,k}$ are the PV voltage and impedance connected to $k^{th}$ node of phase $A$. Similar equations are valid for phases $B$ and $C$. In (3.2), the controlling constant $\beta$ is equal to 1 when there is a PV connected to $k^{th}$ node, otherwise, it is zero.

KCL for each node on the neutral line is

$$\frac{V_{N,k+1}-V_{N,k}}{Z_a} + \frac{V_{N,k+1}-V_{N,k}}{Z_{A,L,k}} + \frac{V_{N,k}-V_{A,k}}{Z_{B,L,k}} + \frac{V_{N,k}-V_{C,k}}{Z_{C,L,k}} = 0$$

(3.3)

where $Z_n$ is the feeder impedance between two adjacent nodes in neutral line.

The simplified diagram of the PV connection to the grid is shown in Fig. 3.2.

Based on this figure, we have

$$P_{PV,k} = \frac{|V_{PV,k}|}{X_{PV,k}} |V_k| \sin(\delta_{PV,k} - \delta_k)$$

(3.4)

$$Q_{PV,k} = \frac{|V_{PV,k}|}{X_{PV,k}} |V_k| \cos(\delta_{PV,k} - \delta_k) - |V_k|$$

(3.5)

where $P_{PV,k}$ and $Q_{PV,k}$ are respectively the active and reactive power output of the PV connected to $k^{th}$ node. Assuming $P_{PV,k}$ and $Q_{PV,k}$ to be constant and $|V_k|$ and $\delta_k$ are known, $|V_{PV,k}|$ and $\delta_{PV,k}$ can be calculated. Please note that $Q_{PV,k}$ will be zero if the PV operates in Unity Power Factor (UPF).

Fig. 3.2 Schematic diagram of a PV connection to grid.
To calculate $V_k$ from (3.2)–(3.5), an iterative method is required. Starting with a set of initial values, the entire network is solved to determine $V_k$. Once the solution converges, the sequence components are calculated. These sequence components are later used to voltage unbalance calculation given in (3.1).

### 3.2.3 Sensitivity analysis

The voltage at any node can be considered as a function of the location and rating of PV. Therefore the sensitivity of network voltage unbalance to a PV location and rating is expressed as

$$S_k = \frac{\partial VUF}{\partial P_{PV,k}} \frac{P_{PV,k}}{VUF}$$

(3.6)

Since voltages at each node are calculated iteratively, the sensitivity is calculated numerically once the iterations converge as

$$S_k = \frac{VUF(\gamma + 1) - VUF(\gamma)}{P_{PV,k}(\gamma + 1) - P_{PV,k}(\gamma)}$$

(3.7)

where $0 \leq \gamma \leq 4$ defines the rating of the PV (i.e. 1,2,3,4,5 kW).

### 3.2.4 Stochastic evaluation

Monte Carlo method is a technique which obtains a probabilistic approximation to the solution of a problem by using statistical sampling techniques. It is a powerful numerical method of stochastic evaluation based on random input variables [22]. This method is widely used in power system calculations when there are some uncertainties within the parameters. Therefore, a Monte Carlo method is also used in this study.

The inherent characteristic of LV distribution networks includes random variation of residential load demand and PV power generation at different time
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periods. This variation is based on load demand in about 12 hours sunshine daily pattern and summer–winter periods. In addition to this, the random location and nominal power of PVs increase the randomness of the network.

For investigating the voltage unbalance in the network when there is a random combination of rooftop PVs with different ratings and at different location on all phases and feeders on the network, a stochastic evaluation based on Monte Carlo method has been carried out. Three random inputs of the stochastic evaluation are the number of householders with installed rooftop PVs, the ratings of the PVs and their location in the different phases and feeders. The flowchart of Monte Carlo method is shown in Fig. 3.3. In this figure, the inputs are network (load, feeder and transformer) data. The random number generation and selection of the other parameters for the Monte Carlo method is explained in Section 3.3.3. For each set of data, the load flow is carried out and voltage unbalance is calculated.

The expected VU at the calculation node $VU_j$ from each trial $1 \leq k \leq N$, is calculated by

$$\overline{VU}_j = \frac{1}{N} \sum_{k=1}^{N} VU_k$$

(3.8)

The unbiased sample variance for VU at the calculated nodes (beginning or end of feeder) is as follows:

$$Var(VU_j) = \frac{1}{N-1} \sum_{k=1}^{N} (VU_k - \overline{VU}_j)^2$$

(3.9)

The stopping rule of the Monte Carlo method is chosen based on achieving an acceptable convergence for $\overline{VU}$ and $Var(VU)$. In this study, the number of Monte Carlo trials is chosen as $N=10,000$ to achieve an acceptable convergence. This is explained in Section 3.3.3.
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The voltage unbalance results as the output of the Monte Carlo simulations are used to calculate the Probability Density Function (PDF) and the average (mean value) of all VUs which is shown as $\lambda$ in the thesis.

![Monte Carlo flowchart for stochastic evaluation.](image)

### 3.3 Numerical results

A sample radial LV (415 V) residential urban distribution network is considered for voltage unbalance investigations. This network is supposed to supply electricity to a combination of residential and small business customers. It has three
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feeders, each three–phase and 4–wire system with equal length (400 m) and equal number of customers on each phase and feeder. The poles are located at a distance of almost 40 meters from each other. At each pole, 2 houses are supplied from each phase. The feeders and their cross–section are also designed appropriately based on the amount of power and the voltage drop. The technical data of the network is given in the Table 3.1.

<table>
<thead>
<tr>
<th>Technical Parameters of the Studied LV Distribution Network</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Transformer</strong></td>
</tr>
<tr>
<td><strong>MV Feeder</strong></td>
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<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>LV Feeder</strong></td>
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<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td><strong>Residential Load Type 1</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Residential Load Type 2</strong></td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td><strong>Residential Load Type 3</strong></td>
</tr>
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<tr>
<td></td>
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<tr>
<td><strong>Rooftop PV</strong></td>
</tr>
</tbody>
</table>

It is assumed that the total electric demand of the network is almost 1 MVA including the LV network under consideration that is supplying a total demand of 360 kW. It is also assumed that during the period of study the loads of phase \( A, B \)
and $C$ are 60, 120 and 180 kW, respectively. The rest of the network load (the portion not included in this study) is considered as a lumped load. The rooftop PVs installed by the householders have an output power in the range of 1–5 kW working in unity power factor. Several studies are performed, which are discussed below.

The utilities usually measure and monitor the voltage unbalance at the beginning of the feeder (i.e. secondary side of the distribution transformer). As described previously, the probabilistic studies of the measurement results show that there is low chance of having a voltage unbalance beyond 2% at this location. Let us assume that the length of the three phases and number of customers per phase are the same. Even then, since the power consumption of the loads are different, there will be different voltage drops along the feeder. This will result in different voltage amplitudes and angles at different locations along the feeder. This phenomenon can result in high voltage unbalance especially at the end of the feeders. To keep the voltage drop within the limit all along the feeder, the utilities install single–phase pad mounted capacitors or increase the cross–section of the feeder. However, voltage unbalance at the end of the feeder still remains higher than the beginning of the feeder. This might cause a problem if some three–phase induction motors or power electronics based equipment are located far from the beginning of the feeder.

For example, in the network under consideration, the voltage amplitude at the beginning of the feeder is 0.975, 0.970 and 0.96 pu for phases $A$, $B$ and $C$ respectively. These values are decreased to 0.95, 0.92 and 0.90 pu at the end of the feeder, respectively. Therefore, voltage unbalance at the beginning of the feeder has increased from 0.45 % to 1.62 % at the end.

Due to time varying characteristic of residential load demand and PV output power, a study is included to investigate the voltage unbalance variation during a
specific time period. Let us assume some PVs with the power output profile as shown in Fig. 3.4(a) and the residential loads with load profile as shown in Fig. 3.4(b) are considered [86, 87]. The time varying characteristic of VU at the beginning and end of the feeder is shown in Fig. 3.4(c). VU will have a time varying characteristic but its value might increase depending on the location and ratings of the PVs.

![PV Power Generation Profile](a)

![Residential Load Profile](b)

![Residential Network Voltage Imbalance Variation](c)

Fig. 3.4 (a) Power generation profile of a 2 kW rooftop PV, (b) 10 different types of residential loads profiles, (c) Time varying characteristic of voltage unbalance with constant location for PVs in the network.

### 3.3.1 Sensitivity analysis of a single PV on voltage unbalance

The voltage unbalance variation due to the location of one PV with a constant rating will depend on the total load of the phase in which it is installed. Usually rooftop PVs can have ratings up to 5 kW for urban residential customers. It is also important to define the point at which the voltage unbalance will be measured.
It is expected that the voltage profile will improve in the phase the PV is installed. Let us consider either a 1 kW or a 5 kW PV installation at the beginning, middle and end of a feeder. In Fig. 3.5, the voltage profile of phase A is shown. As expected, the voltage amplitude increases with PV of higher ratings or when it is installed at the end.

The installation of a PV in a low load phase (phase A in this case) results in the increase in voltage difference and hence voltage unbalance at the end of the feeder while having minor effect at the beginning of the feeder. This voltage unbalance is more if the PV is installed at the end of the feeder or if the rating of the PV is high. The sensitivity analysis of voltage unbalance (calculated at the end of the feeder) versus the location and rating of rooftop PV installed in low load phase A is shown in Fig. 3.6(a).

The voltage difference between the phases reduces if the PV is installed in high load phase (phase C). The decrease is more pronounced at the end of the feeder than at the beginning. This decrease at the end of the feeder is more if the PV is installed at the end of the feeder or if the rating of the PV is high. The sensitivity analysis of voltage unbalance (calculated at the end of the feeder) versus the location and rating of rooftop PV installed in low load phase C is illustrated in Fig. 3.6(b).
Chapter 3: Voltage unbalance analysis in residential low voltage distribution networks with rooftop PVs

Fig. 3.5 Variation of phase $A$ voltage profile versus the location and rating of the PV in phase $A$.

Fig. 3.6 Voltage unbalance sensitivity analysis versus PV location and rating in (a) low load phase– Phase $A$, (b) high load phase– Phase $C$. 
These results prove that a rooftop PV (with a rating of less than 5 kW) can cause network voltage unbalance to increase by 0.1% when installed at the beginning of the feeder and by 25% when installed at the end of the feeder, specifically when the feeder supplies up to 1 MW load. For example, in the worst case, the VU figure of 1.62% without any PV increased to 2.02% (i.e., a 25% rise) when a 5 kW PV was installed at the end of the feeder. Even in this worst case, the VU, at the end of the feeder, is not significant since it still does not lead to non–standard voltage unbalance. However, this may not be true when more than one PV get installed in the network.

3.3.2 Mutual effect of PVs on voltage unbalance

In this part, it is assumed that a number of PVs are installed only in one phase of the network. Note that there are three feeders and the PVs will be installed in only the low load phase ($A$) in each of these feeders.

Fig. 3.7(a) shows the voltage unbalance in Feeder–1 both at the beginning and end of the feeder. We first add PVs to phase $A$ of Feeder–1, one at a time with the maximum number being 10. Then the PVs in the other two feeders are added in the same manner. During this, the size of the PVs, varying from 1 kW to 5 kW, is assumed to be the same. It can be seen that VU in Feeder–1 rises rapidly as the PVs are added. However, adding PVs in the other two feeders does not cause a significant increase in the VU in Feeder–1. Also it can be seen that VU increases with the size of the PVs. Moreover, note that VU at the beginning of the feeder does not change much.

Fig. 3.7(b) shows VU at the end of all three feeders when the 5 kW PVs are installed in phase $A$. This figure overlays the plot similar to those shown in Fig.
3.7(a) for all the three feeders. It is evident that VU in each of the feeders increase significantly when PVs are installed in the feeder itself and rises at a slow rate when the PVs are installed in the other feeders.

Fig. 3.8, shows the effects of adding PVs in the high load phase (phase C). It can be seen from this figure that for smaller PVs (1 and 2 kW), the VU decreases continuously. However for PVs of rating 3 kW and higher, VU decreases up to a point, beyond which it rises sharply. The reason for this is that as the PV ratings increase, the power generated becomes higher than the load demand of the phase. This causes the phase voltage angle to differ from 120° resulting in higher VU.

Another study is performed to find out the effects of number of PVs installed on a phase of a feeder while their total power injection to the grid remains constant, either at 10 kW or 20 kW. The results are shown in Table 3.2 which highlight the importance of the location of PV installation. For example, if 2×5 kW PVs are installed on nodes 1,10 or 2,10 or 5,10, they will result in different values of voltage unbalance. Now if the PVs are chosen 5×2 kW, the voltage unbalance might have increased or decreased compared to the previous situation. Therefore, making a general conclusion about voltage unbalance for different numbers of PVs on one phase but with constant total injected power seems to be impossible without taking into account their locations. Hence, through Fig. 3.6 and Table 3.2, it can be concluded that the voltage unbalance is greater if PVs with constant total power injection are installed at the end of the feeder comparing to when installed at the beginning.
Fig. 3.7 Network voltage unbalance variations based on the location and rating effects of the PVs in phase $A$ of all three feeders, (a) calculated in Feeder–1, (b) calculated on all feeders.
Fig. 3.8 Network voltage unbalance variations based on the location and rating effects of the PVs in phase C of all three feeders, (a) in the beginning of Feeder–1, (b) at the end of Feeder–1
Table 3.2 Numerical voltage unbalance

<table>
<thead>
<tr>
<th>PV installed in</th>
<th>Low Load Phase</th>
<th>High Load Phase</th>
<th>Total PV Power</th>
<th>Feeder beginning</th>
<th>Feeder End</th>
<th>Feeder beginning</th>
<th>Feeder End</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Feeder beginning</td>
<td>Feeder End</td>
<td>Feeder beginning</td>
<td>Feeder End</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 kW</td>
<td>2×5</td>
<td>0.42</td>
<td>1.87</td>
<td>0.36</td>
<td>1.34</td>
<td>2×5</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>2×5</td>
<td>0.43</td>
<td>2.00</td>
<td>0.43</td>
<td>0.35</td>
<td>1.27</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>5×2</td>
<td>0.43</td>
<td>2.00</td>
<td>0.45</td>
<td>0.35</td>
<td>1.22</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>5×2</td>
<td>0.43</td>
<td>1.95</td>
<td>0.41</td>
<td>0.36</td>
<td>1.32</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>10×1</td>
<td>0.42</td>
<td>1.83</td>
<td>0.36</td>
<td>1.32</td>
<td>10×1</td>
<td>0.41</td>
</tr>
<tr>
<td>20 kW</td>
<td>2×5</td>
<td>0.47</td>
<td>2.22</td>
<td>0.34</td>
<td>1.22</td>
<td>4×5</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>4×5</td>
<td>0.51</td>
<td>2.81</td>
<td>0.33</td>
<td>1.27</td>
<td>4×5</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>4×5</td>
<td>0.54</td>
<td>3.02</td>
<td>0.32</td>
<td>1.37</td>
<td>5×4</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>5×4</td>
<td>0.51</td>
<td>2.73</td>
<td>0.33</td>
<td>1.23</td>
<td>5×4</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>5×4</td>
<td>0.51</td>
<td>2.73</td>
<td>0.33</td>
<td>1.23</td>
<td>10×2</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>10×2</td>
<td>0.51</td>
<td>2.73</td>
<td>0.33</td>
<td>1.23</td>
<td>20×1</td>
<td>0.53</td>
</tr>
</tbody>
</table>

3.3.3 Stochastic evaluation of voltage unbalance

A stochastic evaluation is carried out for investigating the uncertainties in the network. In this study, it is assumed the PVs (with rating of 1, 2, 3, 4, 5 kW) have equal probability of 20% each, as shown in Fig. 3.3. The uncertainty in the PV rating is modelled by drawing a random number $U_1$ distributed uniformly under $[0, 1]$. Using $U_1$, the instantaneous output power of each PV is selected in 0–5 kW range during day–night period. The uncertainty of PV location along the feeder, [0– 400 m], is modelled by drawing a random number $U_2$ distributed uniformly under $[0, 1]$. This is done for all phases and feeders of the studied network independently.
Chapter 3: Voltage unbalance analysis in residential low voltage distribution networks with rooftop PVs

The number of the householders with installed rooftop PVs is assumed to be \(\frac{1}{4}\), \(\frac{1}{2}\) and \(\frac{3}{4}\) of the total number of householders as 3 different scenarios, shown in Fig. 3.3. For selecting 1 out of these 3 scenarios, a random number \(U_3\) distributed uniformly under \([0,1]\) is used. If \(U_3 < 0.33\) then scenario 1 is selected, if \(0.33 \leq U_3 \leq 0.66\) then scenario 2 is selected and if \(U_3 > 0.66\) then scenario 3 is selected.

This study was carried out for several times with minimum number of \(N=10,000\) trials. A sample result for the scenario of \(\frac{1}{2}\) householders having PV are shown in Fig. 3.9(a). It can be seen that the voltage unbalance calculated at the beginning of the feeder always remain about 0.8%. This value for the end of the feeder varies between 1 and 3%.

![Voltage Imbalance Graph](a)

![Probability Density Graph](b)

Fig. 3.9 (a) Voltage unbalance for 10,000 scenarios of random location and ratings of PVs. (b) Probability density function of voltage unbalance.
The PDFs for the cases when $\frac{1}{2}$ of the householders have PVs is shown in Fig. 3.9(b). The PDFs for all the three cases have mean value ($\lambda$) equal to 0.61% at the beginning of the feeder and 1.80% at the end of the feeder.

The stopping rule of the Monte Carlo method is chosen based on achieving an acceptable convergence for $\overline{VUF}$ and $Var(VU)$. In this study, the program was rerun for several trial numbers. The $Var(VU)$ at the beginning and end of feeder in addition to Failure Index ($F_I$ %) for different trial numbers is listed in Table 3.3. From this table, it can be seen that the mean, variance and failure index values do not vary much after N=10,000 trials. The error value in $Var(VU)$ is given in the last column of the table assuming the base case of 10,000 trials. It can be seen that an increase in trial number from 10,000 does not increase the error in variance significantly. Therefore this value is chosen as the stopping rule.

<table>
<thead>
<tr>
<th>N (Trial Number)</th>
<th>Failure Index (FI) [%]</th>
<th>Var(VU) at beginning of the feeder [%]</th>
<th>Var(VU) at the end of feeder [%]</th>
<th>Error [%] of end Var(VU) to other trial numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>4.9000</td>
<td>0.1692</td>
<td>5.9295</td>
<td>26.28</td>
</tr>
<tr>
<td>5,000</td>
<td>4.3400</td>
<td>0.1714</td>
<td>6.0554</td>
<td>24.71</td>
</tr>
<tr>
<td>10,000</td>
<td>6.8900</td>
<td>0.1905</td>
<td>8.0436</td>
<td>0</td>
</tr>
<tr>
<td>20,000</td>
<td>6.9900</td>
<td>0.1883</td>
<td>7.9531</td>
<td>1.12</td>
</tr>
<tr>
<td>30,000</td>
<td>7.0367</td>
<td>0.1900</td>
<td>8.0176</td>
<td>0.32</td>
</tr>
<tr>
<td>50,000</td>
<td>7.0700</td>
<td>0.1886</td>
<td>8.1231</td>
<td>0.98</td>
</tr>
<tr>
<td>75,000</td>
<td>6.9693</td>
<td>0.1896</td>
<td>8.0754</td>
<td>0.39</td>
</tr>
<tr>
<td>100,000</td>
<td>7.0040</td>
<td>0.1896</td>
<td>8.0561</td>
<td>0.15</td>
</tr>
</tbody>
</table>

From Fig. 3.9(b), it can be seen that, there is a high probability that the voltage unbalance at the end of the feeder is more than the 2% standard limit. This
The probability is referred to as the failure index ($F_i\%$) which is the frequency of the cases in the shaded area in probability density function and is calculated as

$$F_i\% = shaded\ area \times 100$$  \hspace{1cm} (3.10)

While voltage unbalance failure index is zero at the beginning of the feeder, it is about 30.19% at the end of the feeder.

The customer load consumption is different at different times. Therefore, the residential loads also have an effect on the VU. This phenomenon is included as the fourth uncertainty condition for Monte Carlo method. The results of this analysis are given in Table 3.4 for different load consumption levels in the network during different times. It can be seen that when the loads are almost balanced, $\lambda$ and failure index decrease while they increase if the loads are highly unbalanced.

<table>
<thead>
<tr>
<th>Residential Load Status</th>
<th>Highly Unbalanced</th>
<th>Lightly Unbalanced</th>
<th>Almost Balanced</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ at beginning of feeder</td>
<td>0.66</td>
<td>0.58</td>
<td>0.46</td>
</tr>
<tr>
<td>$\lambda$ at end of feeder</td>
<td>1.92</td>
<td>1.70</td>
<td>1.37</td>
</tr>
<tr>
<td>Failure Index ($F_i%$)</td>
<td>46.0</td>
<td>22.3</td>
<td>6.0</td>
</tr>
</tbody>
</table>

The influence of the location of the PVs (at the beginning or end of the feeder) on voltage unbalance was discussed previously. In the previous studies, it was assumed the PVs are distributed randomly along the feeder. It is of high interest to investigate the case when the majority of the PVs are installed at the beginning or at the end of the feeder. Therefore, another Monte Carlo study is carried out to investigate this phenomenon. The results of this study given are in Table 3.5. It can
be seen that generally when the majority of the PVs are installed at the end of the feeder, the failure index and \( \lambda \) increase at the end points of the network.

Table 3.5 \( \lambda \) and Failure Indices of voltage unbalance of the studied network considering majority of PVs installed at beginning or end of the feeder

<table>
<thead>
<tr>
<th>Majority of PVs at</th>
<th>Beginning of Feeder</th>
<th>Middle of Feeder</th>
<th>End of Feeder</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda ) at beginning of feeder</td>
<td>0.61</td>
<td>0.61</td>
<td>0.62</td>
</tr>
<tr>
<td>( \lambda ) at end of feeder</td>
<td>1.88</td>
<td>1.97</td>
<td>2.06</td>
</tr>
<tr>
<td>Failure Index (( F_{%} ))</td>
<td>16.2</td>
<td>51.8</td>
<td>67.9</td>
</tr>
</tbody>
</table>

3.4 Summary

In this chapter, a voltage imbalance sensitivity analysis and stochastic evaluation based on the rating and location of single–phase grid–connected rooftop PVs in a residential LV distribution network were presented. Through the studies, it is proved that rooftop PV installation will have minor effect on the voltage imbalance at the beginning of a LV feeder designed with engineering judgments. However, it might increase at the end of the feeder to more than the standard limit. It is also proved that depending on the load of the phase in which the PV is installed, the voltage imbalance will increase or decrease based on the location and rating of the PVs. If a PV is installed on one feeder, the voltage imbalance will be modified on all the other LV feeders of the network. Based on the numerical results, a generalized characteristic of voltage imbalance of LV residential networks due to rooftop PV installation is presented. The stochastic simulation demonstrated that the failure index of non–standard voltage imbalance in these networks is high (30.19%).
Chapter 4: Voltage Unbalance Improvement Methods

Based on the probabilistic and numerical results of Chapter 3, it can be concluded that the voltage unbalance at the beginning of the feeder, regardless of the location, number and rating of the installed rooftop PVs, is likely to be less than 1%. However, the voltage unbalance at the end of the feeder can be more than 2% for 30.19% of the cases. To reduce this unbalance, four improvement methods are discussed below, including one proposed PV converter control method.

4.1 Methods

The four improvement methods are discussed below.

4.1.1 Increasing feeder cross-section

This will result in reducing the voltage drop along the feeder and therefore, there will be little difference among the voltage amplitude of three phases of a feeder at the end.

4.1.2 Capacitor installation

The second effective method is the installation of pad mounted switched capacitors in the LV feeders. Based on IEEE Guide for Application of Shunt Power Capacitors for uniformly distributed loads, the capacitor should be placed two-thirds
of the distance from the distribution transformer [88]. It is important to note that if a three–phase capacitor is installed on a LV feeder, the voltage unbalance will almost remain the same. But if instead, three single–phase switched capacitors utilized, where just a single–phase capacitor is connected to the phase with voltage amplitude less than 0.95 pu, with the help of proper operation command of capacitor regulator, the voltage profile on that phase can be shifted up and made closer to the voltage amplitude of other phases. In this case, the voltage unbalance can be improved effectively.

4.1.3 Cross–section increase and capacitor installation

The third improvement method is combination of the previous two methods, i.e. increasing the cross–section of the feeder while installing three switched single–phase capacitors on the feeder. The voltage unbalance improvement in this method has much effective results compared to when each of them employed separately.

4.1.4 New control scheme for PV converters

The previous three methods mentioned above will increase the cost of installations significantly. Instead a new converter control strategy is presented below which will incur very little additional cost. If the PV converter is controlled appropriately to regulate the voltage of the feeder, it can result in voltage profile improvement which can reduce VU. In general, PV converters are controlled such that they inject constant active power with zero reactive power (termed as constant PQ mode here). Instead, the voltage can be regulated if reactive power can also be supplied by these converters.

The proposed scheme controls the injected amount of reactive power of the PV based on the amplitude of the voltage of the feeder at the point of common coupling.
For this purpose, it is necessary to control the voltage amplitude ($|V_{PV,k}|$) and angle ($\delta_{PV,k}$) at the output of the converter (as shown in Fig. 3.2). The controller should generate $|V_{PV,k}|$ and $\delta_{PV,k}$ accurately so that the injected $Q_{PV,k}$ will lead to increase of feeder voltage amplitude $|V_k|$ to a desired value (let us say 0.95–0.96 pu) while its active power output $P_{PV,k}$ remains equal to maximum PV active power output. Let us assume

$$|V_{PV,k}| = |V_{ref}| + nQ_{PV,k} \quad \text{(4.1)}$$

where $|V_{ref}|$ is equal to $|V_{PV,k}|$ when the PV operates in UPF mode with $Q_{PV,k}=0$ and $n$ is a coefficient. Substituting (4.1) in PV power flow equations (3.3) and (3.4), we get

$$P_{PV,k} X_{PV,k} = |V_k| \left( |V_{ref}| + nQ_{PV,k} \right) \sin(\delta_{PV,k} - \delta_k) \quad \text{ (4.2)}$$

$$Q_{PV,k} = \frac{|V_k| \left( |V_{ref}| + nQ_{PV,k} \right) \cos(\delta_{PV,k} - \delta_k) - |V_k|^2}{X_{PV,k}} \quad \text{ (4.3)}$$

Denoting angle difference $\delta_d = \delta_{PV,k} - \delta_k$, from (4.1), $Q_{PV,k}$ can be solved as

$$Q_{PV,k} = \frac{|V_k| \left| V_{ref} \right| \cos \delta_d - |V_k|^2}{X_{PV,k} - n|V_k| \cos \delta_d} \quad \text{ (4.4)}$$

Substituting (4.4) in (4.2), we get

$$P_{PV,k} X_{PV,k} = |V_k| \left( |V_{ref}| + nQ_{PV,k} \right) \sin \delta_d$$

$$= |V_k| \times \left( \frac{|V_{ref}| + n|V_k|^2}{X_{PV,k} - n|V_k| \cos \delta_d} \right) \sin \delta_d$$

$$= |V_k| \times \left( \frac{X_{PV,k} - n|V_k|^2}{X_{PV,k} - n|V_k| \cos \delta_d} \right) \sin \delta_d \quad \text{ (4.5)}$$

For small values of $\delta_d$, we have $\cos(\delta_d)=1$ and $\sin(\delta_d)=\delta_d$. Therefore, from (4.5) we get

$$\delta_d = \frac{P_{PV,k} X_{PV,k} (X_{PV,k} - n|V_k|^2)}{|V_k| \left| V_{ref} \right| X_{PV,k} - n|V_k|^2} \quad \text{ (4.6)}$$

where the approximation has error less than 1% for $\delta_d \leq 15^\circ$. 

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4.2 Numerical results

To verify the efficacy of these methods, another set of stochastic studies are carried out.

For method (1) mentioned in Section 4.1.1, the feeder cross–section is increased to 95 mm$^2$. The results of the stochastic analysis are shown in Fig. 4.1(a). As it can be seen from this figure, $\lambda$ at the beginning of the feeder remains almost constant ($\lambda=0.36\%$) but decreases from 1.84 to 1.56% at the end of the feeder. The failure index reduces to 8.6%.

For method (2) mentioned in Section 4.1.2, a 15 kVAr capacitor is installed at the 2/3rd distance from the beginning of the feeder. The result of this analysis is shown in Fig. 4.1(b) and indicate that $\lambda$ is decreased from 0.36 to 0.28% at the beginning and from 1.84 to 1.41% at the end of the feeder when a 15 kVar capacitor is installed. The failure index in this case reduces to 3.5%.

For method (3) mentioned in Section 4.1.3, both feeder cross–section has been increased to 95 mm$^2$ and a 15 kVAr capacitor is installed at the 2/3rd distance from the beginning of the feeder. In this case, $\lambda$ decreases from 0.36 to 0.28% at the beginning and from 1.84 to 1.18% at the end of the feeder, as shown in Fig. 4.1(c) and the failure index is zero.
Fig. 4.1 Probability density function of voltage unbalance at the beginning and end of the feeder (a) for cross–section increase of LV feeder (b) for capacitor installation in LV feeder (c) for cross–section increase of LV feeder combined with capacitor installation.
As mentioned earlier, the previous three methods will increase the cost of installation. This can be avoided by using the proposed converter control scheme. For method (4) mentioned in 4.1.4, let us assume the presence of three 1 kW rooftop PVs with proposed control scheme at the end points of one of the feeders. Since the voltage amplitude of the three phases where about 0.95, 0.92 and 0.90 pu at the end nodes, all the PVs have worked in the proposed control scheme to increase it to 0.96 pu on all phases. In this way, the voltage amplitude at the end node of for all three phases increase to 0.96 pu and VU decreases to 0.23%.

A stochastic study has also been conducted to verify its performance for different scenarios of PVs installed in different locations and ratings in the network while three 1 kW rooftop PVs are installed always at the end of the three phases. The result of this stochastic analysis is shown in Fig. 4.2. It can be seen that $\lambda$ reduces significantly from 0.61 to 0.16% at the beginning of the feeder and from 1.80 to 0.23% at the end of the feeder. The failure index for this case is zero. The failure index and $\lambda$ values for the network with four different improvement methods are given in Table 4.1. In this, the nominal case indicates when the feeder cross–section is 70 mm$^2$ and no capacitors are installed in the system which are obtained from Fig. 3.9(b). The efficacy of the discussed improvement methods is obvious from this table. Comparing the results in Table 4.1, it can be concluded that the proposed converter method has the highest effect on voltage unbalance reduction along the whole feeder with minimum costs applied to the network.
Chapter 4: Voltage Unbalance Improvement Methods

Table 4.1 $\lambda$ and Failure Indices of Voltage Unbalance of the Studied LV Distribution Network for Three Improvement Methods

<table>
<thead>
<tr>
<th></th>
<th>Nominal Case</th>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
<th>Method 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ at beginning of feeder</td>
<td>0.61</td>
<td>0.61</td>
<td>0.47</td>
<td>0.47</td>
<td>0.16</td>
</tr>
<tr>
<td>$\lambda$ at end of feeder</td>
<td>1.80</td>
<td>1.58</td>
<td>1.37</td>
<td>1.20</td>
<td>0.23</td>
</tr>
<tr>
<td>Failure Index (FI %)</td>
<td>30.19</td>
<td>8.6</td>
<td>3.5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig. 4.2 Probability density function of voltage unbalance with the proposed control scheme.

The main drawback of the proposed converter control method is the need to increase the capacity of the converter. Since PVs are owned and operated by the customer, this will increase the investment cost. Therefore, it is important to investigate an optimum capacity for the converters (i.e. the amount of reactive power that can be supplied by the converter to the grid). A study has been carried out to investigate the effect of limiting the capacity of the converter on the VU. This result is shown in Table 4.2. It can be seen that with a 16.62% increase in the capacity of these converters, failure index can be decreased from 30.19% to 1.8%.
Chapter 4: Voltage Unbalance Improvement Methods

<table>
<thead>
<tr>
<th>Injected Q/ Rated P</th>
<th>Converter Capacity Increase (%)</th>
<th>Failure Index (FI %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>30.19</td>
</tr>
<tr>
<td>0.2</td>
<td>1.98</td>
<td>23.0</td>
</tr>
<tr>
<td>0.4</td>
<td>7.70</td>
<td>12.8</td>
</tr>
<tr>
<td>0.6</td>
<td>16.62</td>
<td>1.8</td>
</tr>
<tr>
<td>0.8</td>
<td>28.06</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>41.42</td>
<td>0</td>
</tr>
</tbody>
</table>

The Simulations have taken into account the real network conditions and technical parameters including network, PV, converter and load pattern. Therefore, the analytical extrapolation is expected to have low uncertainty.

4.3 Summary

In this chapter, several conventional and one new improvement methods were studied for the voltage unbalance reduction which was caused by the random location and ratings of rooftop single-phase PVs. It was shown that increasing the LV feeder cross section or installing LV capacitors can reduce the voltage unbalance however; the new PV converter control method was the most efficient method. It was shown that by 16% increase in the converter capacity, the voltage unbalance failure index can be effectively reduced from 30.19% to 1.8%.
Chapter 5: Application of Custom Power Devices for Voltage Unbalance Reduction in Low Voltage Distribution Networks with Rooftop PVs

The applications of DSTATCOM and DVR for voltage unbalance reduction are presented in this chapter. The stochastic analyses are used for studying the improvement effect for random load and PV rating and location scenario. In addition, multiple applications of DVR are also investigated for longer radial LV lines.

5.1 Network under consideration

A sample radial LV residential urban distribution network is considered for investigations, the single line diagram of which is shown in Fig. 3.1. For analyzing this network, the effect of the neutral conductor that creates a path for the return current is taken into consideration.

For calculating the voltage unbalance, it is necessary that the network to be analysed and the voltages at the desired nodes to be calculated. Based on the KCL on each node of phase $A$, we have for the $k^{th}$ node

$$\frac{\beta(V_{A,PT,k} - V_{A,k})}{X_{A,PV,k}} + \frac{V_{A,k-1} - V_{A,k}}{Z_f} + \frac{V_{A,k+1} - V_{A,k}}{Z_f} + \frac{V_{PV,k} - V_{A,k}}{Z_{A,L,k}} = 0$$

(5.1)

where $Z_f$ is the feeder impedance between two adjacent nodes in phase lines, $V_{A,i}$, $i=1,...,n$ is the single–phase voltage of the $i^{th}$ node of phase $A$, $Z_{A,L,k}$ is the load
impedance connected to $k^{th}$ node of phase $A$ and and $V_{N,k}$ is the voltage of the neutral wire connected to $k^{th}$ node. $V_{A,PV,k}$ and $X_{A,PV,k}$ are the PV voltage and impedance connected to $k^{th}$ node of phase $A$. In (5.1), the controlling constant $\beta$ is equal to 1 when a PV is connected to $k^{th}$ node, otherwise, it is zero. Similar equations are valid for phases $B$ and $C$ and the neutral line.

![Diagram of PV connection to grid](image)

Fig. 5.1 Single line diagram of PV connection to grid.

Based on this simplified diagram of the PV connection to the grid, we have

$$P_{PV,k} = \frac{|V_{PV,k}|}{X_{PV,k}} |V_k| \sin(\delta_{PV,k} - \delta_k)$$

$$Q_{PV,k} = \frac{|V_k|}{X_{PV,k}} \left| V_{PV,k} \cos(\delta_{PV,k} - \delta_k) - |V_k| \right|$$

where $P_{PV,k}$ and $Q_{PV,k}$ are respectively the active and reactive power output of the PV connected to $k^{th}$ node. Assuming $P_{PV,k}$ and $Q_{PV,k}$ to be constant and $|V_k|$ and $\delta_k$ are known, $|V_{PV,k}|$ and $\delta_{PV,k}$ can be calculated. Please note that $Q_{PV,k}$ will be zero if the PV operates in Unity Power Factor (UPF).

To calculate $V_k$ from (5.1)–(5.3), an iterative method is required. Starting with a set of initial values, the entire network is solved to determine $V_k$. Once the solution converges, the sequence components are calculated. These sequence components are later used to voltage unbalance calculation.
It is to be noted that VU is to be measured at 10 minute time intervals [78]. Therefore, the transient and intermittent characteristic of PVs and loads are not of interest in VU studies.

### 5.2 Custom power devices

In this thesis, the CPDs are applied to fix the voltage at Point of Common Coupling (PCC) by forcing the three phase voltages to be of the same desired magnitude, while their phases are separated by 120°.

#### 5.2.1 DSTATCOM

A DSTATCOM is connected in parallel to the network to fix the voltage magnitude at PCC to the desired value of $E_{DSTAT}$ ($0.94 \leq E_{DSTAT} \leq 1$ pu) by injecting/absorbing a required amount of reactive power. Hence the DSTATCOM can be supplied by only dc capacitor.

Let us assume that a three–phase DSTATCOM is installed at the $k^{th}$ node, which is assumed to be a voltage controlled node (i.e., with constant active power and voltage magnitude). The sum of active power injected by DSTATCOM in three phases is set to be zero ($P_{DSTAT,k}=0$) and the voltage magnitude for all three phases is set to be $E_{DSTAT}$. The single–line diagram of the DSTATCOM connection is shown in Fig. 5.2.

The amount of reactive power to be injected/absorbed by DSTATCOM is given by

$$Q_{DSTAT,k} = -\text{Im}\left\{ V_k^* \times \left[ V_k \times \left( \frac{2}{Z_f} + \frac{1}{Z_{L,k}} \right) \right] + \left( \frac{V_{k-1} + V_{k+1}}{Z_f} + \frac{V_{N,k}}{Z_{L,k}} \right) \right\}$$  \hspace{1cm} (5.4)

Based on the calculated $Q_{DSTAT,k}$, PCC voltage ($V_k$) will be modified as
\[ V_k = \frac{1}{Y_{ak}} \left[ P_{\text{DSTAT},k} - jQ_{\text{DSTAT},k} - \left( \frac{V_{k-1} + V_{k+1}}{Z_f} + \frac{V_{N,k}}{Z_{L,k}} \right) \right] \]

\[ Y_{ak} = \frac{2}{Z_f} + \frac{1}{Z_{L,k}} \]

Equations (5.4)–(5.5) are used in the network analysis iterative method along with equations (5.1)–(5.3), for the node in which the DSTATCOM is installed.

When a DSTATCOM is installed in a node, it will inject/absorb reactive power to fix the voltage in that node to the desired value. By changing the PCC voltage, the voltages of all the nodes will be improved.

Fig. 5.2 Schematic diagram of DSTATCOM application in the studied LV residential distribution network.

5.2.2 DVR

DVR is connected in series within the network as shown in Fig. 5.3, where the DVR buses are indicated with voltages of \( V_{in} \) and \( V_{ref} \). The DVR adds/subtracts a small amount of voltage in series such that the voltage magnitude such that the magnitude of \( V_{ref} \) is equal to a desired value \( E_{DVR} \) (0.94 \( \leq E_{DVR} \leq 1 \) pu). In Fig. 5.3, 0 \( \leq \gamma \leq 1 \) represents the location of DVR between two adjacent buses \( k \) and \( k + 1 \). Unlike a DSTATCOM, a DVR needs to inject/absorb both active and reactive power. However, as will be shown later, its rating is much smaller than that of a DSTATCOM.
The amount of necessary voltage to be added by the DVR to phase–A of the network is

\[ V_{DVR,A} = V_{ref,A} - V_{in,A} \]  \hspace{1cm} (5.6)

The desired voltages at the output of DVR for all three phases are based on same desired magnitude \( E_{DVR} \) and are displaced 120° from each other. These reference voltages are set based on the angle of one of the phases of the voltage \( V_{in} \) as

\[ V_{ref,A} = E_{DVR} \angle \delta_{in,A} \]
\[ V_{ref,B} = E_{DVR} \angle (\delta_{in,A} - 2\pi/3) \]
\[ V_{ref,C} = E_{DVR} \angle (\delta_{in,A} + 2\pi/3) \]  \hspace{1cm} (5.7)

The selection of \( E_{DVR} \) is based on the location of the DVR along the feeder and will have a significant effect on the rating of the DVR. To optimize the rating while satisfying the voltage and VU conditions, \( E_{DVR} \) needs to have a higher value if the DVR is installed close to the beginning of feeder and have a lower value if it is installed at far end. For network analysis, (5.6) is used in the iterative method of equations (5.1)–(5.3), at the DVR connection point. Unlike a DSTSTCOM, which improves the voltages of all nodes, a DVR installation will improve the voltages of all the nodes downstream of the DVR.
5.2.3 Structure and connection type

The proposed CPD is consisted of three single–phase H–bridge VSC using IGBTs as shown in Fig. 5.4. The outputs of each H–bridge are connected to single–phase transformers. In DTSTATCOM structure, as shown in Fig. 5.4(a), the three transformers are star connected to the neutral wire. In DVR structure, as shown in Fig. 5.4(b), each transformer is in series with one of the phases of the network. In this figure, the resistance $R_f$ represents the switching and transformer losses, while the inductance $L_f$ represents the leakage reactance of the transformer and the filter capacitor $C_f$ is connected to the output of the transformers to bypass the switching harmonics.

5.2.4 VSC control

The single–phase equivalent circuit of VSC for DSTATCOM and DVR is shown in Fig. 5.4(c). Here it is assumed that both of them are operating in voltage control mode. In this figure, $u V_{dc}$ represents the converter output voltage, where $u$ is the switching function that can take on ±1 value depending on which pair of the IGBTs is turned on. The main aim of the converter control is to generate $u$. Let the state vector be defined by

$$x^T = [v_{cf} \quad i_f]$$ (5.8)

From the circuit of Fig. 5.4(c), state space description of the system can be given as

$$\dot{x} = Ax + Bu_c$$
$$y = v_{cf} = [1 \quad 0]x$$ (5.9)

where $u_c$ is the continuous–time version of switching function $u$. The discrete–time equivalent of (5.9) is

$$x(k + 1) = Fx(k) + Gu_c(k)$$ (5.10)

The system of (5.10) is converted into an equivalent difference equation as [30]
Chapter 5: Application of Custom Power Devices for Voltage Unbalance Reduction in Low Voltage Distribution Networks with Rooftop PVs

\[ F(z^{-1})y(k) = G(z^{-1})u_c(k) \]  

where \( F \) and \( G \) are polynomials in the backward shift (delay) operator \( z^{-1} \). Let the control law be given by

\[ u_c(k) = \frac{S(z^{-1})}{R(z^{-1})} \{ y_{ref}(k) - y(k) \} \]  

Fig. 5.4 (a) Schematic structure of DSTATCOM, (b) Schematic structure of DVR, (c) Single–phase equivalent circuit of VSC at PCC.
where $y_{ref}$ is the reference value. The controller parameters are then obtained by the solution of the following equation.

$$F(z^{-1})R(z^{-1}) + G(z^{-1})S(z^{-1}) = T(z^{-1})$$ (5.13)

In (14), $T(z^{-1})$ is the closed–loop system characteristic equation obtained after radially shifting the open–loop poles to more stable locations [30]. The inverter switching logic is then

\[
\begin{align*}
\text{if} & \quad u_c > h \quad \text{then} \quad u = +1 \\
\text{elseif} & \quad u_c \leq -h \quad \text{then} \quad u = -1 \\
\end{align*}
\] (5.14)

where $h$ is a small positive constant that defines the hysteresis band. This converter control strategy is used in Section VI for dynamic simulation studies to generate $E_{DSTAT}$ (for DSTATCOM) and $E_{DVR}$ (for DVR).

### 5.3 Numerical analysis

It is assumed that one 11 kV overhead line is feeding several distribution transformers of voltage rating 11kV/415V. Only one radial LV (415 V) residential feeder is considered, the total load demand of which is 120 kW. The feeder length is taken as 400 meters. The poles are located at a distance of 40 meters from each other. At each pole, 2 houses are supplied from each phase. The feeders and their cross–section are also designed appropriately based on the nominal power drawn and voltage drop. The technical data of the network is given in Table 5.1.
Table 5.1 Technical Parameters of the Studied LV Distribution Network

<table>
<thead>
<tr>
<th>Component</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transformer</td>
<td>11/0.415 kV, 500 kVA, Δ/Υ grounded, x=0.04 pu</td>
</tr>
<tr>
<td>MV Feeder</td>
<td>Three–phase 11 kV radial</td>
</tr>
<tr>
<td></td>
<td>Supplying 4 transformers with total demand of 1MVA</td>
</tr>
<tr>
<td></td>
<td>50 mm² ACSR, 2 km overhead line, (z=1.08+j×0.302) Ω/km</td>
</tr>
<tr>
<td>LV Feeder</td>
<td>3 feeders, each 3–phase 4–wire, 415 V</td>
</tr>
<tr>
<td></td>
<td>Each with a length of 400 m long</td>
</tr>
<tr>
<td></td>
<td>Aerial Bundle Cable 70 mm² (z=0.551+j×0.088) Ω/km for Phase wires</td>
</tr>
<tr>
<td></td>
<td>and 35 mm² (z=0.65+j×0.09) Ω/km for Neutral wire</td>
</tr>
<tr>
<td>Residential Load</td>
<td>20 one kW residential loads on each feeder (All on phase A)</td>
</tr>
<tr>
<td>Load Type 1</td>
<td>cosφ=0.95, z=51.9840+j×17.0863 Ω</td>
</tr>
<tr>
<td></td>
<td>Two loads connected to each pole, next pole distance 40 m</td>
</tr>
<tr>
<td>Residential Load</td>
<td>20 two kW residential loads on each feeder (All on phase B)</td>
</tr>
<tr>
<td>Load Type 2</td>
<td>cosφ=0.95, z=25.9920+j×8.5432 Ω</td>
</tr>
<tr>
<td></td>
<td>Two loads connected to each pole, next pole distance 40 m</td>
</tr>
<tr>
<td>Residential Load</td>
<td>20 three kW residential load on each feeder (All on phase C)</td>
</tr>
<tr>
<td>Load Type 3</td>
<td>cosφ=0.95, z=17.3280+j×5.6954 Ω</td>
</tr>
<tr>
<td></td>
<td>Two loads connected to each pole, next pole distance 40 m</td>
</tr>
<tr>
<td>Rooftop PV</td>
<td>1–5 kW, unity power factor, L=5mH</td>
</tr>
</tbody>
</table>

It is assumed that during the period of study, the loads of phase A, B and C are 20, 40 and 60 kW, respectively. Other distribution transformers with their loads in the 11 kV network are considered as one single lumped load. The rooftop PVs installed by the householders have an output power in the range of 1–3 kW working at UPF. Several studies are performed, some of which are discussed below.

5.3.1 Nominal case

Let us first assume there is no rooftop PV installed in the network. The voltage magnitude at the beginning of the feeder is 0.98, 0.97 and 0.97 pu for phases A, B and C, respectively. However, these values decrease to 0.96, 0.94 and 0.92 pu,
respectively, at the end of the feeder. VU along the feeder has increased from 0.32% at the beginning of feeder to 1.31% at the end.

Let us now consider the case in which rooftop PVs are installed in such a way that very high VU results at the end of the feeder. For this purpose, the total power generation of rooftop PVs are assumed to be 40, 5, 1 kW in phases A, B and C, respectively.

The effect of PVs on feeder voltage profile is shown in Fig. 5.5(a). Now, in phase A, the power generation of PVs is 40 kW while the load demand is 20 kW. This results in reverse active power flow in phase A from PVs to the transformer. Therefore, as expected, the voltage profile of phase A decreases from the end of the feeder to the beginning of the feeder. VU profile along the feeder is shown in Fig. 5.5(b). As expected, VU has increased to 2.56% at the end of the feeder due to unequal distribution of PVs in the network.

![Voltage Profile](image)

![Voltage Unbalance Profile](image)

Fig. 5.5 (a) LV feeder voltage profile, (b) VU versus the length of feeder.

### 5.3.2 DSTATCOM application

Let us now assume a DSTATCOM is installed at location that is at a distance of 280 m (2/3rd) from the beginning of the feeder. The DSTATCOM is controlled to
fix the PCC voltage magnitude to $E_{DSTAT} = 0.98$ pu. The voltage profile of the three phases of the network is shown in Fig. 5.6. In this figure, the dashed lines show the voltage profile when there was no DSTATCOM installed, while the solid lines show that with the DSTATCOM installed. It is clearly evident that the DSTATCOM is capable of fixing the magnitude of all three phases to $E_{DSTAT}$ by injecting reactive power to phases $B$ and $C$ and absorbing reactive power from phase $A$. VU profile along the feeder is shown in Fig. 5.7. In this figure, the effect of DSTATCOM in VU reduction is compared with the two nominal case. Maximum value of VU in the network after DSTATCOM installation is 0.55 % at the end of the feeder. This value is even smaller than the case when no PVs were installed in the network. This verifies the high efficiency of DSTATCOM application for VU reduction and voltage profile improvement in these networks.

Fig. 5.6 LV feeder voltage profile before and after DSTATCOM installation at 2/3 of feeder beginning.
Fig. 5.7 LV feeder VU profile before and after DSTATCOM installation at 2/3 of feeder beginning.

For investigating the effect of DSTATCOM location in VU reduction, another study is carried out. In this study, the DSTATCOM is installed in different nodes along the feeder and the VU profiles are compared. In Fig. 5.8, VU profile along the feeder is shown when the DSTATCOM is installed at 1/3rd of feeder from the transformer, midpoint, 2/3rd of feeder and at the end. Comparing the VU profiles for these four cases, it can be concluded that DSTATCOM installation is not efficient near the beginning of the feeder. Also, when the DSTATCOM is installed exactly at the end of the feeder, the nodes around the midpoint can suffer from higher VU values. While if the DSTATCOM is installed somewhere around 2/3 of feeder, it will have the most optimum values all along the feeder. From this figure, we can conclude that the installation of DSTATCOM is optimum anywhere between the midpoint and 2/3rd of the feeder.
5.3.3 DVR application

Instead of a DSTATCOM, let us now assume a DVR is installed in series with the LV feeder, at location that is $1/3^{rd}$ of the line length from the feeder beginning (160 m from the transformer). The DVR is applied to fix its output voltage ($V_{\text{ref}}$) magnitude to $E_{\text{DVR}} = 0.975$ pu. The voltage profile of the three phases of the LV feeder is shown in Fig. 5.9 when rooftop PVs are installed. In this figure, the dashed lines are the three–phase voltage profiles when there was no DVR in the network, while the solid lines represent the voltage profiles when the DVR is installed. As is evident from this figure, the DVR is capable of fixing the magnitude of all three phases to $E_{\text{DVR}}$ by adding some positive voltage to phases $B$ and $C$ and adding some negative voltage to phase $A$. 
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The VU profile along the feeder is shown in Fig. 5.10. In this figure, the solid line represents the VU when the DVR is installed. The two other lines represent the VU for the network with and without PVs. Maximum value of VU in the network after DVR installation is 1.21% at the end of the feeder. Although DVR has reduced VU even at the end of the feeder; it is not as efficient as DSTATCOM. Nevertheless, it must be noted that the applied DVR has a very smaller rating compared to the DSTATCOM.

Fig. 5.10 LV feeder VU profile before and after DVR installation at 1/3 of feeder beginning.
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For investigating the effect of DVR location in VU reduction, another study is carried out. In this study, the DVR is installed in different locations along the feeder and the VU profiles are compared. In Fig. 5.11, VU profile along the feeder is shown when the DVR is installed in series at very beginning, 1/3rd of feeder, middle and far end of feeder. Comparing the VU profiles for these four cases, it can be concluded that DVR installation is not efficient at the very beginning or far end of the feeder. Also, when the DVR is installed exactly at the middle of the feeder, there is a high value of VU in the middle of feeder (DVR input side). However, when the DVR is installed at 1/3 of feeder beginning, VU value is smaller all along the feeder compared to other locations.

![Voltage Unbalance Profile](image)

Fig. 5.11 Comparing LV feeder VU profile when DVR is installed in series in four different locations along the feeder.

5.3.4 Stochastic analysis

The distribution network load and PV output power generation are time variant. They vary within a 24–hour period and also for winter/summer seasons. Therefore, for a specific network, there might be hundreds of different load demand and PV output power generation patterns. In addition to this, the number of PVs installed by householders within the network, their ratings and location have random...
values. Therefore, a stochastic analysis is necessary to investigate all these different random data for a network [89].

For investigating the efficacy of the CPDs for the network at any other combination of loads and PVs, a stochastic analysis based on the Monte Carlo method presented in Section 3.2.4 is carried out. The four uncertainties in this study are: instantaneous residential load demands, number of householders with installed rooftop PVs, instantaneous active power output of PVs and the location of PVs in different phases and along the feeder. The details of the uncertainties modelling are given in Table 5.2.

In the first study, it is assumed that a DSTATCOM is installed at 2/3rd distance of feeder beginning. VU is only calculated at the end of the feeder since it was seen that it has its highest value at this point. The Probability Density Function (PDF) of VU at feeder end is shown in Fig. 5.12(a). In this figure, the dashed line represents PDF before DSTATCOM installation and the solid line represents PDF after DSTATCOM installation. The average value of VU at feeder end has reduced from 1.71 % to 0.23 %. On the other hand, the probability of VU at feeder end to be more than 2% standard limit is zero with DSTATCOM.

In the second study, it is assumed that instead of DSTATCOM, a DVR is installed at 1/3rd distance of feeder beginning. Maximum of VU is calculated all along the feeder. PDF of highest VU is shown in Fig. 5.12(b). In this figure, the dashed line represents PDF before DVR installation and the solid line represents PDF after DVR installation. The average value of highest VU along the feeder has reduced from 1.71 % to 1.04 %. On the other hand, the probability of VU at feeder end to be more than 2% standard limit is zero with DVR. Comparing this figure with Fig. 5.12(a), it is shown that DSTATCOM is more efficient for VU reduction.
Chapter 5: Application of Custom Power Devices for Voltage Unbalance Reduction in Low Voltage Distribution Networks with Rooftop PVs

Table 5.2 Parameters of the Stochastic Analysis

<table>
<thead>
<tr>
<th>Uncertainty 1 (Load Demand)</th>
<th>Random Number $U_1$ distributed uniformly under [0, 5 kW] for each load in each phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty 2 (Number of Householders with rooftop PVs)</td>
<td>Random Number $U_2$ distributed uniformly under [0, 1]:</td>
</tr>
<tr>
<td></td>
<td>if $U_2 &lt; 0.33$ then No. of installed PVs = $0.25 \times$ No. of householders</td>
</tr>
<tr>
<td></td>
<td>if $0.33 \leq U_2 &lt; 0.66$ then No. of installed PVs = $0.50 \times$ No. of householders</td>
</tr>
<tr>
<td></td>
<td>if $U_2 \geq 0.66$ then No. of installed PVs = $0.75 \times$ No. of householders</td>
</tr>
<tr>
<td>Uncertainty 3 (Nominal Rating of rooftop PVs)</td>
<td>Equal probability of 20% each for 1, 2, 3, 4 and 5 kW chosen by random Number $U_3$ distributed uniformly under [0, 1]:</td>
</tr>
<tr>
<td></td>
<td>if $U_3 &lt; 0.2$ then $P_{PV,k} = 1$ kW</td>
</tr>
<tr>
<td></td>
<td>if $0.2 \leq U_3 &lt; 0.4$ then $P_{PV,k} = 2$ kW</td>
</tr>
<tr>
<td></td>
<td>if $0.4 \leq U_3 &lt; 0.6$ then $P_{PV,k} = 3$ kW</td>
</tr>
<tr>
<td></td>
<td>if $0.6 \leq U_3 &lt; 0.8$ then $P_{PV,k} = 4$ kW</td>
</tr>
<tr>
<td></td>
<td>if $U_3 \geq 0.8$ then $P_{PV,k} = 5$ kW</td>
</tr>
<tr>
<td>Uncertainty 4 (Location of PVs along the feeder in each Phase)</td>
<td>Random Number $U_4$ distributed uniformly under [0, 400 m]</td>
</tr>
<tr>
<td>Monte Carlo Stopping Rule</td>
<td>N=10,000 trials leads to satisfactory convergence of mean and variance of VU in studied nodes.</td>
</tr>
</tbody>
</table>
Fig. 5.12 (a) Comparing PDF of VU at feeder end before and after DSTATCOM installation, (b) Comparing PDF of highest VU all along the feeder before and after DVR installation.

### 5.3.5 Semi–urban LV network

LV networks are usually designed to satisfy voltage drop standards. Urban LV lines are not very long (typically 400 meters in length). However, geographical issues in addition to cost analysis might lead some utilities to install longer LV networks. This happens more frequently in rural/semi urban/low load density areas.
To investigate this, we have chosen a feeder with a length of 800 m. Four different cases are studied as follows:

1. A DSTATCOM is installed at 2/3rd of the feeder length,
2. A DVR is installed at 1/3rd of the feeder length,
3. Two DVRs installed at 1/4th and midpoint of the feeder length.
4. Three DVRs installed at 1/0th, 1/4th and midpoint of the feeder length.

The VU profiles of the four cases are shown in Fig. 5.13. The VU at the feeder end in nominal case without either DSTATCOM or DVR reaches 4.5%. However it reduces to 0.68% by using the DSTATCOM (case 1). When one single DVR is used (case 2), the VU has a maximum of 2.3% just before DVR location and 2.58% at the feeder end. This is beyond VU standard limit. By installing two DVRs (case 3), the VU is 1.9% and 1.51% just before the first and second DVR locations, respectively and reaches 1.45% at the end of feeder. If three DVRs are installed, maximum VU is 1.45% at the end of the feeder and it less than 1.2% before all the DVR locations along the feeder. We can therefore surmise that the VU values for cases (1), (3) and (4) are within the standard limits of 2%. From Fig. 5.13, it is obvious that a DSTATCOM is capable of fixing voltage even in longer LV lines, but may require higher rating. It can also be seen that a single DVR is not capable of fixing the voltage in longer LV lines. However, when multiple DVRs are installed with correct rating and at correct locations, the VU along the line can be kept within the standard limit even in longer LV lines.
5.4 Simulation results

The load demand and PV output power vary with time. For studying the dynamic characteristics of DSTATCOM and DVR in LV networks, another case study is carried out in PSCAD/EMTDC. It is assumed that initially the total power generation of rooftop PVs are 40, 5, 1 kW in phases A, B and C, respectively, while the loads in these phases are 20, 40 and 60 kW, respectively.

5.4.1 DSTATCOM dynamic performance

Let us assume a DSTATCOM is installed at the 2/3rd point of the feeder, however it is not connected to the feeder. Then at $t=1$ sec the DSTATCOM is connected to the network. The PCC instantaneous and RMS voltage are shown in Fig. 5.14(a) and Fig. 5.14(b). It can be seen from these figures, the three–phase voltage waveform seems more balanced and the RMS values of the three phases are shifted up close to the desired value ($E_{DSTAT} = 0.98$ pu).
With the system operating in the steady state with the DSATCOM being connected, the PV generation is increased by 13 kW and 1 kW in phases A and B, respectively at $t_1=0.05$ sec. Subsequently at $t_2=0.35$ sec, a load change is created in the network. A total 4 kW load is reduced from phase A, 8 kW increased in Phase B and 12 kW increased in phase C. Furthermore at $t_3=0.55$ sec, another PV generation and load variation occur. A total 8 kW of PV output is reduced from phase A while 2 kW and 6 kW PV output is increased in phases B and C, respectively. At the same time, the load in phase A is increased by 2 kW while the load in phases B and C is decreased by 4 kW and 6 kW, respectively. The power flow from the transformer to the LV network is shown separately for each phase in Fig. 5.14(c). This variation is due to load demand or PV output changes as explained above.

The variation in VU at the feeder end before and after DSTATCOM installation is shown in Fig. 5.14(d). Comparing the VU results in this figure, the efficacy of DSTATCOM application is verified. The output reactive power of the DSTATCOM is shown in Fig. 5.14(e). As seen from this figure, the DSTATCOM will vary the amount of its injected reactive power to the network based on network load and power parameters to fix the PCC voltage magnitude to the desired value.
Chapter 5: Application of Custom Power Devices for Voltage Unbalance Reduction in Low Voltage Distribution Networks with Rooftop PVs

Fig. 5.14 (a) PCC instantaneous voltage before and after DSTATCOM connection, (b) RMS voltage of PCC before and after DSTATCOM connection, (c) Power demand variation for three phases of studied LV network, (d) Voltage unbalance variation at LV feeder end before and after DSTATCOM installation, (e) Reactive power injected by DSTATCOM at PCC.
5.4.2 DVR dynamic performance

A similar study is carried out to investigate DVR dynamic performance. Let us assume a DVR is installed at 1/3rd point along the feeder. In Fig. 5.15(a) and Fig. 5.15(b), the PCC instantaneous and RMS voltage are shown for cases before and after DVR connection. As it can be seen from these figures, the three–phase voltage waveform seems more balanced and the RMS values of the three phases are shifted up to the desired value of $E_{DVR} = 0.975$ pu.

The VU at the end of the network before and after DVR installation is shown in Fig. 5.15(c) for the same PV generation and load variation discussed in the previous sub–section. The RMS of the injected voltage by DVR to each phase of the LV feeder is shown in Fig. 5.15(d). As seen from Fig. 5.15(d), the DVR will vary the amount of its injected voltage to each phase based on network load and power parameters to fix its output voltage to the desired value. Comparing the VU in Fig. 5.15(c) with those of Fig. 5.14(d), it is obvious that the DSTATCOM reduces the VU more than the DVR. The required DSTATCOM and DVR power requirements for different time periods are given in Table 5.3.
Chapter 5: Application of Custom Power Devices for Voltage Unbalance Reduction in Low Voltage Distribution Networks with Rooftop PVs

Fig. 5.15 (a) PCC instantaneous voltage before and after DVR application, (b) PCC voltage RMS before and after DVR application, (c) Voltage unbalance variation at LV feeder end before and after DVR installation, (d) DVR injected voltage to each phase of LV feeder.

Table 5.3 Power requirement of DSTATCOM and DVR

<table>
<thead>
<tr>
<th></th>
<th>Apparent Power [KVA]</th>
<th>Between $t_1$ and $t_2$</th>
<th>Between $t_2$ and $t_3$</th>
<th>After $t_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSTATCOM</td>
<td>70.0</td>
<td>77.5</td>
<td>76.6</td>
<td></td>
</tr>
<tr>
<td>DVR</td>
<td>2.1</td>
<td>2.9</td>
<td>2.3</td>
<td></td>
</tr>
</tbody>
</table>

5.4.3 Multi–DVRs in semi–urban networks

For studying the dynamic characteristics of multiple DVRs in series for semi–urban networks, another study is carried out. The network of the previous subsection with a similar PV power generation and residential load pattern is assumed to have a length of 1000 m. The VU at the end of the feeder varies between 4% and 5% at
different time intervals. Let us assume now that 3 DVRs are installed at 1/10th, 1/4th and midpoint of the feeder. These points were shown to have the best result on VU reduction all along the feeder in Section 5.3.5. The VU at the end of the network and before the location of each DVR (DVR input) is shown in Fig. 5.16. From this figure, it can be seen that VU along the feeder is kept below 2% at all periods. This verifies the efficacy of the proposed method and the accuracy of the numerical analysis. The voltage RMS injected by each DVR to each phase in addition to the total power requirement of each DVR for each period is given in Table 5.4. The results from this table illustrate that 3 DVRs each with a rating of less than 8 kVA connected in series at right locations can successfully reduce a 5% VU to below standard level.

![Fig. 5.16 Voltage unbalance variation at the end of the feeder and before each DVR’s location in a semi urban network with 3 DVRs installed in series.](image-url)
Table 5.4 Power requirement of DVRs and their injected power for multi–DVR case.

<table>
<thead>
<tr>
<th></th>
<th>Between $t_1$ and $t_2$</th>
<th>Between $t_2$ and $t_3$</th>
<th>After $t_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Apparent Power</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[KVA] for the</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DVR installed at</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/10th of feeder</td>
<td>6.29</td>
<td>7.91</td>
<td>6.58</td>
</tr>
<tr>
<td>1/4th of feeder</td>
<td>6.56</td>
<td>8.09</td>
<td>6.81</td>
</tr>
<tr>
<td>midpoint</td>
<td>5.48</td>
<td>6.46</td>
<td>5.33</td>
</tr>
<tr>
<td></td>
<td>Phase A</td>
<td>Phase B</td>
<td>Phase C</td>
</tr>
<tr>
<td>1/10th of feeder on</td>
<td>0.004</td>
<td>0.003</td>
<td>0.004</td>
</tr>
<tr>
<td>Phase A</td>
<td>0.025</td>
<td>0.029</td>
<td>0.024</td>
</tr>
<tr>
<td>Phase B</td>
<td>0.042</td>
<td>0.048</td>
<td>0.044</td>
</tr>
<tr>
<td>Phase C</td>
<td>0.014</td>
<td>0.013</td>
<td>0.014</td>
</tr>
<tr>
<td>1/4th of feeder on</td>
<td>0.033</td>
<td>0.037</td>
<td>0.032</td>
</tr>
<tr>
<td>Phase A</td>
<td>0.049</td>
<td>0.054</td>
<td>0.051</td>
</tr>
<tr>
<td>Phase B</td>
<td>0.017</td>
<td>0.017</td>
<td>0.018</td>
</tr>
<tr>
<td>Phase C</td>
<td>0.038</td>
<td>0.041</td>
<td>0.035</td>
</tr>
<tr>
<td>Midpoint on</td>
<td>0.051</td>
<td>0.061</td>
<td>0.056</td>
</tr>
</tbody>
</table>

**5.5 Summary**

In this chapter, the applications of DSTATCOM and DVR were investigated for voltage unbalance reduction. It has been shown that a DSTATCOM is more efficient for voltage profile improvement and VU reduction in comparison with a DVR. However, a DVR has a much smaller rating than a DSTATCOM. The stochastic analyses prove that for any random load and PV rating and location scenario, the discussed methods are successful in reducing voltage unbalance to standard limits. Multiple applications of DVR are also possible for longer radial LV lines. Through extensive analysis and simulation carried out using PSCAD/EMTDC and MATLAB, the efficacy of the discussed methods is verified.
Chapter 6: Decentralized Local Voltage Support of Low Voltage Distribution Networks with a New Control Strategy of PVs

In this chapter, the capability of PV converter to regulate the voltage of a specific load connected is investigated for voltage support in LV distribution networks. A decentralized local voltage support method is illustrated for a distribution network using the proposed operational strategy for the PVs installed by the householders. This method is capable of preventing voltage drop and voltage rise at network peak and off-peak periods, respectively. The proposed strategy is verified through numerical analysis, carried out by MATLAB and dynamic simulations, carried out by PSCAD/EMTDC.

6.1 Analysis

Let us assume a typical 3–phase radial balanced LV distribution feeder with 3–phase PVs installed as shown in Fig. 6.1. The rating and tap changing of distribution transformers and feeder cross–sections are designed in such a way that during normal operation conditions, the voltage profile is within the standard limits. However, practical measurements indicate that, a voltage drop to figures below the lower limit at network peak hours is frequent especially at the end of the feeders [85]. Network
peak period is usually in the evening when PVs do not generate any active power. Therefore, voltage support by active power generation of PVs cannot be used.

At noon, the network is roughly in its off–peak load condition while the PVs generate their maximum active power. This results in voltage rise along the feeder.

![Diagram of the LV distribution network under consideration](image1.png)

Consider the system shown in Fig. 6.2 in which $E_S$ and $Z_F$ are the Thevenin equivalents looking at the left of Point of Common Coupling (PCC). It is assumed that a PV and a load $Z_L$ are connected at the PCC bus. From this figure, we have

$$ E_{PCC} = \frac{Z_L}{Z_F + Z_L} (E_S + Z_F I_{PV}) \quad (6.1) $$

In (6.1), $E_{PCC}$ has a higher value when $I_{PV} > 0$ than when $I_{PV} = 0$. $E_{PCC}$ can be also greater than $E_S$ if $I_{PV} > E_S / Z_L$. This shows how PVs can result in voltage rise in the network. For keeping the network voltage below an upper limit (i.e. $E_{PCC} \leq \alpha E_S$ where $\alpha \leq 1.1$) then acceptable range of PV output current is

$$ I_{PV} \leq \frac{E_S}{Z_L} + \frac{\alpha - 1}{Z_F} \quad (6.2) $$

![Circuitical representation of a distribution network, load and PV.](image2.png)
Based on this concept, in [12] it was shown that voltage rise in a typical distribution feeder is greater at the end of the feeder and if the PV has a higher rating.

In Fig. 6.2, let us assume \( P = P_{PV} - P_L \) and \( Q = Q_{PV} - Q_L \) and \( \Delta V \) is the voltage difference between beginning of feeder (\( E_S \)) and PCC voltage (\( E_{PCC} \)) then

\[
\Delta V = \frac{P_R + Q_X}{E_{PCC}} \quad (6.3)
\]

During the off-peak period at noon, let us assume that \( P_L \) and \( Q_L \) are constant but low, while the PV is working in UPF (\( Q_{PV} = 0 \)). Let the PV be injecting such an amount of active power (\( P_{PV} \)) that \( E_{PCC} \) is equal to its maximum limit (=1.1×\( E_S \)). Now, if based on MPPT, the PV starts to increase its injected active power by \( \Delta P \), \( \Delta V \) will rise. In this situation, \( E_{PCC} \) can be kept constant only if \( \Delta V \) remains constant by making the PV to absorb some reactive power (\( Q_{PV} \)) without operating in UPF. This amount of reactive power is

\[
Q_{pv} = -\Delta P \frac{R}{X} \quad (6.4)
\]

This required amount of reactive power is dependent on the feeder \( R/X \) ratio.

In the evening, at the peak load period, the PV cannot inject active power (\( P_{PV} = 0 \)). Let us assume \( P_L \) and \( Q_L \) are such that the PCC voltage drops to its minimum acceptable limit (=0.94×\( E_S \)). If \( P_L \) and \( Q_L \) now increase by \( \Delta P \) and \( \Delta Q \), respectively, the PCC voltage will fall below the acceptable limit. This can be solved through reactive power injection at PCC. The required reactive power to be injected is

\[
Q_{pv} = \Delta P \frac{R}{X} + \Delta Q \quad (6.5)
\]

Comparing (6.4) and (6.5), it can be seen that more reactive power exchange is needed for voltage drop improvement than voltage rise reduction for the same amount of \( \Delta P \) change.
Chapter 6: Decentralized Local Voltage Support of Low Voltage Distribution Networks with a New Control Strategy of PVs

6.2 PV Control strategies

As shown by (6.4) and (6.5), the PVs are capable of regulating their PCC voltage and prevent voltage rise or drop in the feeder. A PV can be operated in any of the three following strategies:

- **Strategy 1: UPF strategy**
- **Strategy 2: Constant PQ strategy**
- **Strategy 3: Voltage Control strategy**

The schematic diagrams of these strategies are shown in Fig. 6.3(a) while their detailed control block diagrams are shown in Fig. 6.4.

6.2.1 Strategy–1: UPF strategy

In this strategy, the PV regulates its output voltage such that the MPPT or APC generated active power flows into PCC in UPF. Let us consider the single–line diagram shown in Fig. 6.2. Let the PCC voltage magnitude and angle respectively be $|E_{\text{PCC}}|$ and $\delta_{\text{PCC}}$. Let us assume that the PV is required to inject active power $P_{\text{PV,ref}}$, while reactive power is $Q_{\text{PV,ref}} = 0$. From Fig. 6.2, the power flow equations between the PV and PCC is

$$P_{\text{PCC,ref}} = \frac{|E_{\text{PV}}| |E_{\text{PCC}}|}{\omega L_1} \sin(\delta_{\text{PV}} - \delta_{\text{PCC}})$$  \hspace{1cm} (6.6)

$$Q_{\text{PCC,ref}} = \frac{|E_{\text{PV}}| |E_{\text{PCC}}|}{\omega L_1} \left( |E_{\text{PV}}| \cos(\delta_{\text{PV}} - \delta_{\text{PCC}}) - |E_{\text{PCC}}| \right)$$  \hspace{1cm} (6.7)

where $L_1$ is the filter inductance in the output of PV converter. From (6.6) and (6.7) $|E_{\text{PV}}|$ and $\delta_{\text{PV}}$ can be calculated for any reference value of PV active power and the PCC voltage.
6.2.2 Strategy–2: Constant PQ strategy

In this strategy, the PCC voltage is monitored to maintain it between two threshold values. When it goes above the higher threshold, the PV starts absorbing a constant amount of reactive power, while it starts injecting a constant amount of reactive power if the PCC voltage falls below a lower–threshold. The MPPT or APC methods determine the active power injection in this strategy. Note that, the operation of the PV can be changed from Strategy–1 to Strategy–2 with the help of a selector which changes the input of the $Q_{PV,ref}$ (shown in Fig. 6.4) to a constant value.

6.2.3 Strategy–3: Voltage control strategy

The general approach for this strategy is based on fixing the PCC voltage of each PV along the feeder to a desired value. The desired values of each node are based on a voltage droop line. A PV, while the sun is shining can act as a power source. However during the evenings, the converter and capacitor connected at the output of the PV can act as a Distribution Static Compensator (DSTATCOM).

For this purpose, in the distribution feeder of Fig. 6.1 with $n$ nodes each with a 3–phase PV installed, each PV will fix its PCC voltage to the assigned value by the droop controller by injecting or absorbing the required amount of reactive power ($Q_{PV,ref}$).

Two major ideas for defining $Q_{PV,ref}$ are based on:

- Minimization PCC reactive power flow from grid
- Minimization PCC voltage error

In the first idea, each PV will try to generate the reactive power requirement of the load connected to that PCC, i.e.

$$Q_{PV,ref} = Q_L$$  \hspace{1cm} (6.8)

This will improve the power factor at each PCC to unity.
For the second idea, $Q_{PV, ref}$ is calculated based on the difference between PCC actual voltage and the voltage reference as

$$Q_{PV, ref} = m(V_{PCC, ref} - V_{PCC})$$  \hspace{1cm} (6.9)

where $m$ is a coefficient. This is shown in Fig. 6.3(b). For both of these methods, the PV converter must be capable of injecting/absorbing the calculated reactive power in (6.8) or (6.9). Therefore, at each time, this value must be in the acceptable range of

$$-\sqrt{S_{PV, max}^2 - P_{PV}^2} \leq Q_{PV, ref} \leq \sqrt{S_{PV, max}^2 - P_{PV}^2}$$  \hspace{1cm} (6.10)

where $S_{PV, max}$ is the maximum apparent power of the PV converter and $P_{PV}$ is the instantaneous active power generation of the PV. If the required $Q_{PV, ref}$ is beyond its maximum injection or absorption capability, it will run on the maximum limits. For voltage drop improvement, if $Q_{PV, ref}$ is more than the converter capacity, no further improvement is possible. However for voltage rise reduction, if $Q_{PV, ref}$ is more than the converter capacity, further improvement can be carried out by APC of the PV [9].

For the second method, the PVs sometimes need to inject large amount of reactive power, which may result in large reverse reactive power flow into the distribution transformer ($Q_{Feeder}$). This may cause increased power loss in the network. To prevent this, a feedback from the $Q_{Feeder}$ will be added to $Q_{ref}$ in the case $Q_{Feeder} < 0$. This limits the reverse reactive power flow back to the distribution transformer.

The PV active power reference ($P_{PV, ref}$) is already defined from MPPT for day times or adjusted by APC controller and is equal to zero when the PV is not generating any active power in the evening or at night during DSTATCOM mode of operation. $P_{PV, ref}$ will be controlled by the appropriate angle control of converter output capacitor ($\delta_f$) [3].
Chapter 6: Decentralized Local Voltage Support of Low Voltage Distribution Networks with a New Control Strategy of PVs

6.3 Numerical and dynamic modeling

Consider the radial LV residential urban distribution network, the single–line diagram of which is shown in Fig. 6.1. This study focuses on the effectiveness and feasibility of the proposed PV strategies on 3–phase balanced network where a similar 3–phase PV is installed at each node on the network. We shall present a study with unbalanced loads later.

6.3.1 Load flow analysis

Load flow analysis is carried out first to investigate the voltage profile of the network for the different operation strategies of PVs.
6.3.2 UPF strategy

For numerical analysis of the network, the PV and converter combination is modeled as a voltage source. Assuming the PVs operating in UPF, applying Kirchoff’s current law (KCL) at the $k^{th}$ node of Fig. 6.1, we have

$$\beta (V_{PV,k} - V_k) - \frac{V_{k-1} - V_k}{Z_f} + \frac{V_{k+1} - V_k}{Z_f} + \frac{-V_k}{Z_{L,k}} = 0$$

where $Z_f$ is the feeder impedance between two adjacent nodes, $V_i$ ($1 \leq i \leq N$) is the voltage of the $i^{th}$ node, $Z_{L,k}$ is the load impedance connected to $k^{th}$ node, $V_{PV,k}$ and $X_{PV,k}$ are PV voltage and impedance connected to $k^{th}$ node. In (6.11), $\beta = 1$ when a PV is connected to $k^{th}$ node; otherwise, it is zero.

In UPF strategy, the voltage source is assumed to inject a constant active power into grid and is modeled based on (6.6) and (6.7) from where $V_{PV,k}$ is calculated. To
calculate $V_k$ from (6.11) and (6.6)–(6.7), an iterative method is applied in which, starting with a set of initial values, the entire network is solved to determine $V_k$.

### 6.3.3 Constant PQ strategy

Consider the PV is working in UPF strategy. Let us assume now $V_i$ exceeds an upper limit (let us say $V_i > 1$ pu) or it falls below a lower limit (let us say $V_i < 0.94$ pu). In this case, the PV can change its operation strategy from UPF to constant PQ and starts to respectively absorb or inject a constant amount of reactive power (let us say $Q_{PV,ref} = \pm 0.5 \times P_{PV,ref}$ which just needs a 10% increase in converter rating). Now $Q_{PV,ref}$ needs to be updated in (6.7) and used in the load flow iterations.

### 6.3.4 Voltage control strategy

Let us assume the 3–phase PV is installed at the $k^{th}$ node of Fig. 6.1. This node is assumed to be a voltage controlled node (i.e., with constant active power and voltage magnitude). The constant active power is defined from MPPT or APC at day times and is zero at evening/night.

The amount of reactive power to be injected/absorbed by the PV is given by

$$Q_{PV,ref,k} = -\text{Im}\left[ V_k^* \times \left[ V_k - \left( \frac{2}{Z_f} + \frac{1}{Z_{L,k}} \right) \right] + \left( \frac{V_{k-1} + V_{k+1} + V_{N,k}}{Z_f} \right) \right]$$

(6.12)

Based on the calculated $Q_{PV,ref,k}$, PCC voltage ($V_k$) will be modified as

$$V_k = \frac{1}{Y_{ik}} \left[ P_{PV,ref,k} - jQ_{PV,ref,k} - \left( \frac{V_{k-1} + V_{k+1} + V_{N,k}}{Z_f} \right) \right]$$

$$Y_{ik} = \frac{2}{Z_f} + \frac{1}{Z_{L,k}}$$

(6.13)

Equations (6.12)–(6.13) are used in the network load flow analysis iterative method along with (6.11) for the nodes in which a PV is installed. As the PCC voltage is updated in each iteration, the voltages of all the nodes will be updated accordingly.
6.3.5 PV and converter dynamic modeling and MPPT algorithm

For dynamic studies, the PV is modeled as shown in Fig. 6.5 where the photo current \( I_{ph} \) depends on sun radiation \( G \) and environment temperature \( T \) as \[90\]

\[ I_{ph} = \frac{G}{G_{ref}} I_{sc}(T_{ref}) \left( 1 + I_{sc}(T) - I_{sc}(T_{ref}) \right) \] \hspace{1cm} (6.14)

where \( G_{ref} \) and \( T_{ref} \) are reference radiation and temperature respectively and \( I_{sc} \) is the short circuit current. The diode current \( I_D \) is calculated based on its saturation current \( I_0 \), its thermal voltage \( V_T \) and the PV cell output voltage \( V_{cell} \) and output current \( I_{cell} \) as

\[ I_D = I_0 \left[ \exp \left( \frac{V_{cell} + I_{cell} R_s}{V_T} \right) - 1 \right] \] \hspace{1cm} (6.15)

A PV module output voltage \( V_{PV} \) and output current \( I_{PV} \) is then calculated based on the number of PV cells connected in series \( N_s \) and number of PV cells connected in parallel \( N_p \) as

\[ V_{PV} = N_s V_{cell} \quad , \quad I_{PV} = N_p I_{cell} \] \hspace{1cm} (6.16)

Hence, the total output power of the PV module is

\[ P_{PV} = N_s N_p V_{cell} I_{cell} \] \hspace{1cm} (6.17)

The MPPT flowchart algorithm [91], shown in Fig. 6.5(b), is applied to the PV in order to calculate the reference for PV output voltage \( V_{dc,ref} \) in order to adjust \( V_{PV} \) for getting the maximum of its output active power. For this purpose, \( P_{PV} \) is calculated at each time step, compared to its value at the previous time step, an appropriate \( \pm \Delta V \) is added to \( V_{dc,ref} \) as shown in the flowchart.

The PV converter control is as discussed in Section 5.2.4.
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Fig. 6.5 (a) PV Equivalent circuit, (b) MPPT algorithm flowchart [91].

6.4 Numerical analysis

The numerical studies are carried out for a 3–phase balanced network with 3–phase PVs at different periods of the day. In this section, network peak time is referred to as evening period with high electric demand and zero active power generation from PVs. The off–peak period is referred to as noon that electric load demand is minimal and active power generation from PVs is maximum.

It is assumed that one 11 kV overhead line is feeding several distribution transformers of voltage rating 11kV/415V. Only one radial LV (415 V) residential feeder is considered for study, the schematic diagram of which is shown in Fig. 6.1. It is assumed that the residential loads are located at a distance of 40 meters from each other and distributed uniquely along the feeder. The residential loads are
assumed 3–phase balanced and each has an installed 12 kW, 3–phase PV. It is assumed that the total electric demand of the studied LV feeder is 9 kW in off–peak and 81 kW in peak. The feeders and their cross–section are also designed appropriately based on the nominal power drawn and voltage drop. The technical data of the network is given below.

Table 6.1 Technical Parameters of the Studied LV distribution network.

<table>
<thead>
<tr>
<th>Component</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transformer</td>
<td>11/0.415 kV, 50 Hz, 100 kVA, Δ/Y grounded, $Z_l = 5%$</td>
</tr>
<tr>
<td>Feeder</td>
<td>70 mm$^2$ AAC Bare Conductor ($Z = 0.452 + j \times 0.270$) $\Omega$/km</td>
</tr>
<tr>
<td></td>
<td>50 mm$^2$ ACSR, 2 km overhead, ($Z = 1.08 + j \times 0.302$) $\Omega$/km</td>
</tr>
<tr>
<td>PV</td>
<td>DC Bus Capacitor 400 V 2000 µF</td>
</tr>
<tr>
<td></td>
<td>$G_{ref} = 1000 \text{ W/m}^2$, $T_{ref} = 25 ^\circ\text{C}$, $I_0 = 4.05 \times 10^{-7}$ A</td>
</tr>
<tr>
<td></td>
<td>$V_{cell} = 21.70$ V, $I_{cell} = 3.45$ A, $P_{PV} = 12$ kW</td>
</tr>
<tr>
<td>Filter</td>
<td>$R_f = 0.1$ $\Omega$, $C_f = 25$ µf, $L_f = 5$ mH</td>
</tr>
<tr>
<td>Loads</td>
<td>Power Factor = 0.95</td>
</tr>
</tbody>
</table>

This network is studied for different PV operation strategies at different periods of the day as follows:

6.4.1 Off–peak period

In this period, it is assumed that total electric demand of the LV feeder is 9 kW while total active power generation from PVs, running in UPF strategy, is 60 kW.

The voltage profile of the network in this case is shown in Fig. 6.6. From this figure, it can be seen that the voltage amplitude of the feeder increases from 0.99 pu at the beginning of the feeder to 1.01 pu at the end of the feeder as the result of reverse power flow.
Now let us assume that the PVs are running in constant PQ strategy. In this case, the PV absorb a constant amount of reactive power equal to 3 kVAR (i.e. $0.25 \times P_{PV}$). This will result in the voltage profile to shift down as shown in Fig. 6.6.

Now let us assume that in the same network, the PVs are running in voltage control strategy, based on a voltage droop control which tried to fix the voltage amplitude of all nodes to 1 pu. In this case, the study is carried out for the condition with and without considering the maximum and minimum limits on the reactive power capacity of the PVs. In this case, each PV has to absorb some amount of reactive power to change the voltage profile as shown in Fig. 6.6.

The numerical results of above mentioned cases are given in Table 6.2. In this table, $Q_{Grid}$ and $Q_{PV\text{-}i}$ ($1 \leq i \leq 5$) respectively refer to the reactive power injected from the distribution transformer and each PV into the LV feeder. From this table, it can be concluded that when the PVs are assumed with no reactive power limit, the network voltage amplitudes can reach the desired levels; however, there is a reverse reactive power flow back into the grid. When the reactive power capacity limit is applied, there is a small error between the actual voltages and the desired values but this can result in minimizing the reverse reactive power flow.

### 6.4.2 Peak period

In this period, it is assumed that total electric demand of the LV feeder is 81 kW while PVs generate no active power. The voltage profile of the network in this case is shown in Fig. 6.7. From this figure, it can be seen that the voltage amplitude of the feeder decreases from 0.97 pu at the beginning of the feeder to 0.92 pu at the end of the feeder as the result of heavy loading.
Now let us assume that the PVs are running in constant PQ strategy (i.e. $P_{pv} = 0$ and $Q_{pv} = Q_{max} = 12$ kVAr). In this case, the PVs with their PCC voltage amplitude less than 0.94 pu, will start injecting a constant amount of reactive power equal to 12 kVAr. This will result in voltage profile to be improved as shown in Fig. 6.7.

Now let us assume that in the same network, the PVs are running in voltage control strategy, based on a voltage droop control which tried to fix the voltage amplitude of all nodes to 1 pu. In this case, each PV has to inject some amount of reactive power and hence, the voltage profile will change as shown in Fig. 6.7.

The numerical results of above mentioned cases are given in Table 6.3. The voltage control strategy is carried out for the conditions with and without reactive power limit on PVs. As expected, the PVs require exchanging large amounts of reactive power in the case of no limit, however, by applying the limit, this figures reduce while there is a small error between the actual values of the node voltages and the desired values.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>$Q_{Grid}$</th>
<th>$Q_{PV-1}$</th>
<th>$Q_{PV-2}$</th>
<th>$Q_{PV-3}$</th>
<th>$Q_{PV-4}$</th>
<th>$Q_{PV-5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No PVs</td>
<td>3.24</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>UPF</td>
<td>3.65</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Constant PQ</td>
<td>18.64</td>
<td>3.00</td>
<td>3.00</td>
<td>3.00</td>
<td>3.00</td>
<td>3.00</td>
</tr>
<tr>
<td>Voltage Control</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• No Qmax limit</td>
<td>−2.61</td>
<td>8.65</td>
<td>4.55</td>
<td>6.01</td>
<td>8.90</td>
<td>−6.55</td>
</tr>
<tr>
<td>• With Qmax limit</td>
<td>1.25</td>
<td>6.00</td>
<td>−2.78</td>
<td>6.00</td>
<td>−1.16</td>
<td>−3.22</td>
</tr>
</tbody>
</table>
Table 6.3 Reactive Power Injection from Grid and PVs in Peak [kVAr]

<table>
<thead>
<tr>
<th>Strategy</th>
<th>$Q_{Grid}$</th>
<th>$Q_{PV-1}$</th>
<th>$Q_{PV-2}$</th>
<th>$Q_{PV-3}$</th>
<th>$Q_{PV-4}$</th>
<th>$Q_{PV-5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No PVs / UPF</td>
<td>41.76</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Constant PQ</td>
<td>-15.19</td>
<td>12.00</td>
<td>12.00</td>
<td>12.00</td>
<td>12.00</td>
<td>12.00</td>
</tr>
<tr>
<td>Voltage Control</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>• No Qmax limit</td>
<td>-26.33</td>
<td>-81.8</td>
<td>39.37</td>
<td>33.30</td>
<td>29.37</td>
<td>41.22</td>
</tr>
<tr>
<td>• With Qmax limit</td>
<td>-26.93</td>
<td>4.92</td>
<td>12.00</td>
<td>12.00</td>
<td>12.00</td>
<td>12.00</td>
</tr>
</tbody>
</table>

Fig. 6.6 Voltage Profile of LV feeder in off-peak period for different PV operational strategies.

Fig. 6.7 Voltage Profile of LV feeder in peak period for different PV operational strategies.
6.5 Dynamic simulations

The dynamic modeling of the network was explained in Section 6.3. This analysis is carried out in several cases as explained below.

For studying the dynamic behavior of the proposed strategies, a 3–phase balanced network with the schematic diagram of Fig. 6.1 is considered. In this study, the PVs are assumed as 3–phase and identical with a 10 kVA converter. This analysis is carried out at peak and off–peak periods.

6.5.1 Peak period

Let us assume the network in peak period. Initially, it is assumed that the PVs are operating in voltage control strategy without any limit. At \( t = 1.5 \) sec, it is assumed that the reverse reactive power flow limit is applied to prevent large reactive power flow back into the grid. At \( t = 2.5 \) sec, the network load is decreased by 40% (32 kW). At \( t = 3.5 \) sec, the PVs are assumed to be shut down.

The desired voltage amplitude for each PCC is chosen as 1 pu. Similar to the previous case, the droop controller will decide the required reactive power exchange and the final value of each PCC voltage.

The PCC RMS voltage for each node along the feeder is shown in Fig. 6.8(a). The active and reactive power output of the PVs is shown in Fig. 6.8(b) and Fig. 6.8(c), respectively. In addition, the reactive power flow in the feeder, at the beginning of the feeder and along the feeder before each PV PCC is shown in Fig. 6.8(d). Active and reactive power exchange of a sample PV in voltage control mode in peak period (DSTATCOM) is also shown in Fig. 6.8(e).

At \( t < 1.5 \) when the PVs run in voltage control mode without any limitation, from Fig. 6.8(a), it can be seen that based on the voltage droop controller, the nodes
voltage reduce from 1 pu at the beginning of feeder to 0.979 pu at the end of feeder. This is achieved by injecting the reactive power amounts as shown in Fig. 6.8(b) for $t < 1.5$. In this time interval, there is a 35 kVAR reverse reactive power flow into the grid, as seen from Fig. 6.8(c).

For $1.5 < t < 2.5$ when the PVs run in voltage control mode with reverse reactive power flow limit, less reactive power is generated by each PV which causes the node voltages to reduce.

The successful dynamic operation of the PVs for a sample load change in the network is also shown in Fig. 6.8 at $2.5 < t < 3.5$.

For $t > 3.5$, the PVs are shut down. At this time, the node voltages reduce from 0.973 pu at the beginning of feeder to 0.956 pu at the end of the feeder. This proves the efficacy of this strategy in improving the voltage profile in network peak periods.

The effectiveness of the converter control method can be verified by observing its tracking error as shown in Fig. 6.9(a) and the stability of the DC capacitor voltage magnitude and AC capacitor voltage angle can be verified from Fig. 6.9(b).
Chapter 6: Decentralized Local Voltage Support of Low Voltage Distribution Networks with a New Control Strategy of PVs

(a)

(b)

(c)

(d)
Fig. 6.8 (a) PCC RMS voltage, (b) Injected reactive power from each PV working in voltage control mode, (c) Active and reactive power supply from Distribution network into LV feeder, (d) Reactive power flow along the feeder, at the beginning and before each PCC, (e) Active and reactive power exchange of a sample PV in voltage control mode in peak period (DSTATCOM).

Fig. 6.9 (a) Tracking error of one phase of the converter, (b) DC capacitor voltage magnitude and AC capacitor voltage angle variation.
6.5.2 Off–peak period

Now, let us assume the network in off–peak period and the PVs initially running in voltage control strategy. At \( t = 1.5 \text{ sec} \) the network load is increases by 0.65 kW (i.e. 30% increase). At \( t = 2 \text{ sec} \), the PVs are assumed to be changing their strategy from voltage control to UPF.

The desired voltage amplitude for each PCC is chosen as 1 pu. Similar to the previous case, the droop controller will decide the required reactive power exchange and the final value of each PCC voltage.

The PCC RMS voltage for each node is shown in Fig. 6.10(a). The active and reactive power supplied by the distribution transformer is shown in Fig. 6.10(b). In Fig. 6.10(c), the reactive power exchange in a sample PV along with the voltage error at its PCC is shown.

For \( t < 1.5 \) when the PVs run in voltage control mode, from Fig. 6.10(a), it can be seen that based on the voltage droop controller, the nodes voltage are with the range of 0.986 and 0.996 pu. This is achieved by PVs absorbing the reactive power.

The successful dynamic operation of the PVs for a sample load change in the network is also shown in Fig. 6.10 at \( 1.5 < t < 2 \).

For \( t > 2 \), the PVs are running in UPF mode. At this time, the node voltages increase from 1.003 pu at the beginning of feeder to 1.033 pu at the end of feeder. This proves the efficacy of this strategy in improving the voltage profile in network off–peak periods.
Fig. 6.10 (a) PCC RMS voltage, (b) Active and reactive power supply from Distribution network into LV feeder, (c) Voltage error and reactive power exchange by a sample PV in its PCC.
6.6 Summary

In this chapter, a decentralized local voltage support method was proposed for voltage profile improvement of the distribution feeder in network peak and off-peak periods. For this purpose, the PVs are controlled to exchange reactive power with the network to regulate their PCC voltage to 1 pu. This strategy can accompany methods such as MPPT and APC and can control its reactive power injection based on droop control or based on improving PCC power factor. The numerical and dynamic simulations carried out in MATLAB and PSCAD/EMTDC verified the efficacy of the proposed method.
Chapter 7: Predicting Voltage Unbalance Impacts of Plug–in Electric Vehicles Penetration in Residential LV Distribution Networks: Analysis and Improvement

In this chapter, a voltage unbalance sensitivity analysis and stochastic evaluation based on the connection point and charging/discharging capacities of Plug-in Electric Vehicles (PEVs) connected to a residential low voltage distribution network in two Grid-to-Vehicle (G2V) and Vehicle-to-Grid (V2G) modes are presented. Later, five improvement methods are discussed and their efficacy is investigated for voltage unbalance reduction in these networks.

7.1 Plug–in electric vehicles

PEVs are expected as significant loads for future residential distribution networks, especially when high clusters of PEVs occur in a certain area. They can be charged in two general locations: individual charging points (in residential and small business places) and charging stations. Rapid charging (in 0.5–1 hr) is expected in charging stations which are being fed from three–phase networks. However, normal charging (in 6–8 hrs) from single–phase LV outlets is expected in individual charging points [25].
It is expected that the PEVs start being charged as customers return back home (immediately or with a delay, considering the off-peak tariff possibility) until the PEVs are fully charged or the householder departs. Therefore, most of PEVs will be charged in evening or night periods. In addition, some of the PEVs parked at home during day can work in V2G mode if they have enough stored energy.

For G2V mode, the normal uncontrolled charging from Australian standard 10, 15 and 20 A single-phase 240 V residential outlets are considered in the rest of the paper [29]. The efficiency of the chargers are assumed to be 90% [49]. In addition, a 10% and 30% penetration level of PEVs (for short-term) and 50% (for long-term) is considered [25, 26]. Even in the case of short-term small penetration level, high localized concentrations of PEVs are possible.

7.2 Modeling and analysis

The studied network consists of one 11 kV overhead line feeding several distribution transformers. Each distribution transformer (11kV/400V) supplies residential customers. One distribution transformer is selected for the rest of the study and the other transformers are modelled as a lumped load of the 11 kV network. The studied transformer has three feeders, each three-phase, with equal length of 400 m and equal number of customers on each phase. The feeders and their cross-section are designed appropriately based on nominal power and voltage drop. The network technical data is given in Table 7.1. The single line diagram of one feeder of the distribution transformer is shown in Fig. 3.1. It is noted that the neutral conductor is making the pass for unbalance current circulation and the analysis is based on the mutual effect of the three phases. Since Voltage Unbalance (VU) is to be measured at 10 minute time intervals based on [78], the transient and intermittent...
characteristic of PEVs and loads are not of interest in VU studies. Thereupon, the
PEVs are assumed as constant current load in G2V mode. For V2G mode, they are
assumed as battery storage devices with constant output power. The schematic
diagram of the PEV in these two modes is shown in Fig. 7.1(a) and Fig. 7.1(b).

Table 7.1 Technical Parameters of the Studied LV Distribution Network

<table>
<thead>
<tr>
<th>Component</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transformer</td>
<td>11 kV/ 415 V, 250 kVA, ∆/ Y grounded, Zf = 4%</td>
</tr>
<tr>
<td>Feeders</td>
<td>3×70+35 mm² AAC, 400 m overhead line for LV feeder</td>
</tr>
<tr>
<td></td>
<td>3×50 mm² ACSR, 2 km overhead line for MV feeder</td>
</tr>
<tr>
<td>PEVs</td>
<td>1–5 kW, unity power factor, L=5mH in V2G mode</td>
</tr>
<tr>
<td></td>
<td>10, 15 and 20 A constant current load in G2V mode</td>
</tr>
<tr>
<td>Residential Loads</td>
<td>1 kW, (cos\varphi = 0.95, z = 51.9840 + j\times17.0863 \ \Omega)</td>
</tr>
<tr>
<td></td>
<td>2 kW, (cos\varphi = 0.95, z = 25.9920 + j\times8.5432 \ \Omega)</td>
</tr>
<tr>
<td></td>
<td>3 kW, (cos\varphi = 0.95, z = 17.3280 + j\times5.6954 \ \Omega)</td>
</tr>
</tbody>
</table>

Fig. 7.1 (a) Single line diagram of one phase of the studied LV distribution feeder,
(b) Schematic diagram of PEV in G2V mode, (b) Schematic diagram of PEV in V2G mode.

7.2.1 Load flow and sensitivity analysis

For calculating the VU, it is necessary that the network to be analysed and the
voltages at the desired nodes to be calculated. Based on the KCL on the \(k^{th}\) node of
phase \(A\), we have

\[
\beta_1 I_{A,PEV,k} + \frac{\beta_1 (V_{A,PEV,k} - V_{A,k})}{X_{A,PEV,k}} + \frac{V_{A,k-1} - V_{A,k}}{Z_f} + \frac{V_{A,k+1} - V_{A,k}}{Z_f} + \frac{V_{N,k} - V_{A,k}}{Z_{A,k,k}} = 0
\]  

(7.1)
where $Z_f$ is the feeder impedance between two adjacent nodes in phase lines, $V_{A,i}$ ($i = 1, ..., n$) is the single-phase voltage of the $i^{th}$ node of phase $A$, $Z_{A,k}$ is the load impedance connected to $k^{th}$ node of phase $A$ and $V_{N,k}$ is the voltage of the neutral wire connected to $k^{th}$ node. In (7.2), $\beta_1 = -1$ when a PEV, running in G2V mode, is connected to $k^{th}$ node of phase $A$, otherwise, it is zero. Also, $\beta_2 = +1$ when a PEV, running in V2G mode, is connected to $k^{th}$ node of phase $A$, otherwise, it is zero. In G2V mode, $I_{A,PEV,k}$ is the charging level of the PEV connected to that node and in V2G mode, the $V_{A,PEV,k}$ and $X_{A,PEV,k}$ are the PEV voltage and impedance connected to that node. Similar KCL equations are valid for phases $B$ and $C$ and the neutral wire.

For the PEV in V2G mode, we have

$$P_{PEV,k} = \frac{|V_{PEV,k}|}{X_{PEV,k}} |V_{i}| \sin(\delta_{PEV,k} - \delta_k)$$

$$Q_{PEV,k} = \frac{|V_{i}|}{X_{PEV,k}} (|V_{PEV,k}||\cos(\delta_{PEV,k} - \delta_k)| - |V_{i}|)$$

where $P_{PEV,k}$ and $Q_{PEV,k}$ are respectively the active and reactive power output of the PEV connected to $k^{th}$ node. Assuming $P_{PEV,k}$ and $Q_{PEV,k}$ to be constant and $|V_k|$ and $\delta_k$ are known, $|V_{PEV,k}|$ and $\delta_{PEV,k}$ can be calculated. Please note that $Q_{PEV,k}$ is assumed to be zero and the PEV only injects active power into the grid.

To calculate $V_k$ from (7.1)–(7.3), an iterative method is required. Starting with a set of initial values, the entire network is solved to determine $V_k$. Once the solution converges, the sequence components are calculated. These sequence components are later used for VU calculation.

The voltage at any node can be considered as a function of PEV location and charging/discharging capacity. Since voltages at each node are calculated iteratively, once the iterations converge, the sensitivity is calculated numerically as
where $\alpha = 1$ for the analysis in G2V mode and is equal to zero for the analysis in V2G mode. In (7.4), $\gamma$ defines the charging capacity of PEV. For G2V mode, $0 \leq \gamma \leq 2$ represents the three charging levels of 10, 15 and 20 A. For V2G mode, $0 \leq \gamma \leq 4$ represents the output active power of PEVs (i.e. 1, 2, 3, 4, 5 kW).

### 7.2.2 Stochastic analysis

The inherent characteristic of LV distribution networks includes random variation of residential load demand and PEV power consumption and generation at different time periods. This variation is based on load demand in about 24–hour daily pattern. In addition to this, the random location and nominal charging/discharging capacity of PEVs in addition to their operation time increases the randomness of the network.

For investigating the uncertainty effects of the VU on the network, a Monte Carlo based stochastic evaluation is carried out. The considered uncertainties include PEV penetration level, their charging and discharging capacities, connection points on all phases and the residential load demand. The flowchart of the Monte Carlo method is shown in Fig. 7.2.

This study is carried out for G2V and V2G modes separately. In this study, it is assumed the PEVs have equal probability of 33% each for 10, 15 and 20 A charging levels in G2V mode. For selecting 1 out of these 3 charging levels, a random number $U_1$ distributed uniformly under [0, 1] is used. If $U_1 < 0.33$ then 10 A charging level is selected, if $0.33 \leq U_1 \leq 0.66$ then 15 A charging level is selected and if $U_1 > 0.66$ then 20 A charging level is selected. For V2G mode, it is assumed that PEVs have equal
probability of 20% each for 1, 2, 3, 4 and 5 kW output active power being fed into grid. Therefore, it is assumed that

\[
\begin{align*}
\text{if } U_1 < 0.2 & \quad \text{then } P_{PEV,k} = 1 \text{ kW} \\
\text{if } 0.2 \leq U_1 < 0.4 & \quad \text{then } P_{PEV,k} = 2 \text{ kW} \\
\text{if } 0.4 \leq U_1 < 0.6 & \quad \text{then } P_{PEV,k} = 3 \text{ kW} \\
\text{if } 0.6 \leq U_1 < 0.8 & \quad \text{then } P_{PEV,k} = 4 \text{ kW} \\
\text{if } U_1 \geq 0.8 & \quad \text{then } P_{PEV,k} = 5 \text{ kW}
\end{align*}
\]

The uncertainty of PEV location along the feeder, [0, 400 m], is modelled by drawing a random number \( U_2 \) distributed uniformly under [0, 1]. This is carried out for all 3 phases and 3 feeders of the studied network independently.

The number of PEVs in a network is selected based on the penetration levels of PEV for different short and long term periods. This value is chosen by a random number \( U_3 \) which has a normal distribution of \( \text{N}(\mu, \sigma) \) with mean value of \( \mu = \text{penetration level [\%]} \times \text{number of households in the feeder} \) and variance of \( \sigma = 0.02 \).

The network load demand is chosen by random number \( U_4 \) distributed uniformly under [0, 5 kW] for each residential house.

The stopping rule of the Monte Carlo method is chosen based on achieving an acceptable convergence for \( \bar{VU} \) and \( Var(VU) \). In this study, the number of Monte Carlo trials is chosen as \( N=10,000 \) to achieve an acceptable convergence.

The VU results as the output of the Monte Carlo simulations are used to calculate the Probability Density Function (PDF) and the average (mean value) of all VUs which is shown as \( \lambda \) in the paper.
7.3 Analysis numerical results

For the network described in Section 7.2, it is assumed that the total electric demand of the 11 kV network is almost 1 MVA. The distribution transformer studied is assumed to have a demand of 360 kW. The poles are located at a distance of almost 40 meters from each other. At each pole, 2 houses are supplied from each phase. It is assumed that during the period of study the loads of phase $A$, $B$ and $C$ are 60, 120 and 180 kW, respectively. The rest of the network load (the portion not included in this study) is considered as a lumped load.
In the network under consideration, the voltage amplitude at the beginning of the feeder is 0.98, 0.97 and 0.96 pu for phases A, B and C respectively. These values decrease to 0.95, 0.93 and 0.90 pu at the end of the feeder, respectively. Therefore, VU at the beginning of the feeder has increased from 0.88 % to 1.84 % at the end.

Several studies are performed, some of which discussed below. These studies are carried out for G2V and V2G modes in two separate scenarios as their timings are different.

### 7.3.1 Sensitivity analysis of a single PEV on VU

The VU profile variation in a feeder as the result of one PEV connection will depend on the total load of the phase in which it is connected, its operation mode and the point at which it is connected.

In G2V mode, it is expected that the voltage amplitude and profile will be reduced in the phase which the PEV is connected to. Let us consider a PEV with 20 A charging level is connected to the beginning, middle and end of a feeder. In Fig. 7.3(a), the voltage profile of phase A is shown. As expected, the voltage amplitude decreases when the PEV is connected and the impact is more when it is connected to the end of the feeder.

In G2V mode, the connection of a PEV in a low load phase (phase A in this case) results in the reduction in voltage difference and hence VU at the end of the feeder decreases while having minor effect at the beginning of the feeder. This VU reduction is more if the PEV is connected to the far end nodes of the feeder or if the charging level of the PEV is higher. The sensitivity analysis of VU (calculated at the end of the feeder), versus the location and charging level of one PEV in G2V mode, connected to low load phase A, is shown in Fig. 7.3(b).
For V2G mode, it is expected that the voltage amplitude and profile will be increased in the phase which the PEV is connected to. Therefore a PEV running in V2G mode, when connected to a low load phase, will result in increasing the VU. Again, the impact is more when the PEV output power is higher or when it is connected to the far end nodes of the feeder. The sensitivity analysis of VU (calculated at the end of the feeder), versus the location and output power of one PEV in V2G mode, connected to low load phase A, is shown in Fig. 7.3(c).

These results prove that, in the worst case, the VU figure of 1.84%, without any PEV, increases to 1.96% (i.e. a 6.3% rise) when a PEV with charging level of 20 A is connected to the end of the low load phase, specifically when the feeder supplies up to 1 MW load. The VU of 1.96% at the end of the feeder is not significant since it still is within the standard limit. However, this may not be true when more than one PEV is connected in the network.

When the PEV is connected to high load phase (phase C), in G2V mode, the voltage difference between the phases increases. This increase is more if the PEV is connected to the far end nodes of the feeder or if it has a higher charging level. However, in V2G mode, the voltage difference between the phases decreases. This decrease is more if the PEV is connected to the far end nodes of the feeder or if it has a higher output power.
Chapter 7: Predicting Voltage Unbalance Impacts of Plug-in Electric Vehicles Penetration in Residential LV Distribution Networks: Analysis and Improvement

Fig. 7.3 (a) Variation of phase A voltage profile versus the location and charging level of the PEV, running in G2V mode, connected to phase A, (b) VU sensitivity analysis versus one PEV location and charging level, running in G2V mode, when connected to low load phase A, (c) VU sensitivity analysis versus one PEV location and output power, running in V2G mode, when connected to low load phase A.
7.3.2 Mutual effect of PEVs on VU

In this part, it is assumed that a number of PEVs are connected to only one phase of the network. Note that there are three feeders and the PEVs will be connected to only the low load phase (A) in each of these feeders.

Fig. 7.4(a) and Fig. 7.4(b) show the VU in Feeder–1 both at the beginning and end of the feeder for V2G and G2V modes, respectively. We first add PEVs to phase A of Feeder–1, one at a time with the maximum number being 10. Then the PEVs in the other two feeders are added in the same manner. During this, the charging or discharging capacity of the PEVs (i.e. charging level of 10, 15 and 20 A in G2V mode and constant output power of 1, 2, 3, 4 and 5 kW in V2G mode), is assumed to be the same.

From Fig. 7.4(a), it can be seen that VU in Feeder–1 rises rapidly as the PEVs working in V2G mode are added. However, adding PEVs in the other two feeders does not cause a significant increase in the VU in Feeder–1. Also it can be seen that VU increases with the output power of the PEVs. Moreover, note that VU at the beginning of the feeder does not change much.

Fig. 7.4(b) shows that VU in Feeder–1 decreases as the PEVs working in G2V mode are added. Also it shows that VU decreases more if the charging level of PEVs is higher. Similar to V2G mode, VU at the beginning of the feeder does not change much.

The same study is carried out for high load phase (C) and the results are shown in Fig. 7.4(c) and Fig. 7.4(d). From Fig. 7.4(c), it can be seen that VU in Feeder–1 decreases rapidly as the PEVs working in V2G mode are added. However, adding PEVs in the other two feeders will lead to an increase in voltage profile of phase C and after a point, the VU will start to increase. This increase is more obvious for
PEVs with higher output power (i.e. 4 and 5 kW in this study). However, the VU at the beginning of the feeder decreases slightly while all the PEVs are being added.

Fig. 7.4(d) shows that VU in Feeder–1 decreases very slightly up to a point as the PEVs working in G2V mode are added in Feeder–1. After a point, the number of PEVs and their load is highly increased and this results in a reduction in the voltage profile of Phase C. This leads to an increase in VU at the end of the network, while a slight variation is reflected at the beginning of the feeder.

Another study is performed to find out the effects of number of PEVs connected to one phase on VU, while their total power consumption (in G2V mode) or power injection (in V2G mode) remains constant. This study is carried out for either at 10 kW or 20 kW total power injection in V2G mode and total power consumption of 100 A in G2V mode, when the PEVs are connected to low load phase A. The results are shown in Table 7.2 and Table 7.3, respectively for G2V and V2G modes. The results highlight the importance of the location of PEV connection.

For example, if $2 \times 5$ kW PEVs in V2G mode are connected to nodes 1, 10 or 2, 10 or 5, 10 of phase A, they will result in different values of VU. Now if the PEVs are chosen $5 \times 2$ kW, the VU might have increased or decreased compared to the previous situation. Therefore, making a general conclusion about VU for different numbers of PEVs in V2G mode on one phase but with constant total injected power seems to be impossible without taking into account their locations. Hence, from Table 7.3, it can be concluded that the VU is greater if PEVs with constant total power injection are installed at the end of the feeder comparing to when installed at the beginning. Similar discussion can be carried out for PEVs in G2V mode. In this case, it can be concluded that VU is greater if PEVs with higher charging levels are connected to the end of the feeder comparing to when installed at the beginning.
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Fig. 7.4 VU at the beginning and end of Feeder 1 when PEVs are connected to different locations in all three feeders for (a) different charging levels in G2V mode when connected to phase $A$, (b) different constant output powers in V2G mode when connected to phase $A$, (c) different charging levels in G2V mode when connected to phase $C$, (d) different constant output powers in V2G mode when connected to phase $C$.

Table 7.2 VU values of several cases with total power consumption of 100 A by PEVs in G2V mode.

<table>
<thead>
<tr>
<th>Total PEV Power Consumption from Grid</th>
<th>PEV connected to Low Load Phase</th>
<th>PEV No × Charging level</th>
<th>Low Load Phase</th>
<th>High Load Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Feeder beginning</td>
<td>Feeder End</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Feeder beginning</td>
<td>Feeder End</td>
</tr>
<tr>
<td>100 A</td>
<td></td>
<td></td>
<td>0.79</td>
<td>1.56</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.74</td>
<td>1.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.70</td>
<td>1.45</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.70</td>
<td>1.45</td>
</tr>
</tbody>
</table>
A stochastic evaluation is carried out for investigating the uncertainties in the network based on the explanation in Section 7.2.2. This study was carried out for several times each with minimum number of $N=10,000$ trials. A sample result for the VU calculated at the beginning and the end of the feeder for a 30% penetration level of PEVs working in G2V mode is shown in Fig. 7.5(a). From this figure, it can be seen that the VU calculated at the beginning of the feeder always remain less than 1.2%. This value for the end of the feeder varies between 1.2% and 2.6%. The PDF of VU for the 30% penetration level of PEVs in G2V mode is shown in Fig. 7.5(b).
The PDF for this case has a mean value ($\lambda$) equal to 0.95% at the beginning of the feeder and 1.89% at the end of the feeder.

Fig. 7.5 (a) Monte Carlo results of VU for PEVs in G2V mode for N=10,000 trials for penetration level of 30%, (b) Probability density function of VU for PEVs in G2V mode for penetration level of 30%.

From Fig. 7.5(b), it can be seen that, there is a high probability that the VU at the end of the feeder to be more than the 2% standard limit. This probability, which shows the frequency of the cases in the shaded area, is referred to as the Failure Index and is calculated by $F_I\% = \text{Shaded Area} \times 100$. While $F_I$ of VU is zero at the beginning of the feeder, it is about 34.1% at the end of the feeder.
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This study is carried out for different penetration levels of PEVs in both G2V and V2G modes. The $\lambda$ at the beginning and end of the feeder and $F_I$ results of this study is given in Table 7.4. Comparing the data given in Table 7.4, it can be seen that as the PEV penetration level increases from 0 to 50%, the probability of non-standard VU at the end of the feeder increases from 0 to 36.5% for G2V mode and from 0 to 28% for V2G mode.

Table 7.4 Stochastic analysis based $\lambda$ and $F_I$ of VU in the studied LV distribution network for different PEV penetration levels

<table>
<thead>
<tr>
<th>Penetration Level [%]</th>
<th>0</th>
<th>10</th>
<th>30</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>PEV Mode</td>
<td>G2V</td>
<td>V2G</td>
<td>G2V</td>
<td>V2G</td>
</tr>
<tr>
<td>$\lambda$ at the beginning of the feeder</td>
<td>0.88</td>
<td>0.96</td>
<td>0.95</td>
<td>0.96</td>
</tr>
<tr>
<td>$\lambda$ at the end of the feeder</td>
<td>1.84</td>
<td>1.90</td>
<td>1.89</td>
<td>1.89</td>
</tr>
<tr>
<td>Failure Index ($F_I$ %)</td>
<td>27.1</td>
<td>15.9</td>
<td>34.1</td>
<td>26.0</td>
</tr>
</tbody>
</table>

The customer load consumption is time variant. Therefore, the residential loads also have an effect on the VU. This phenomenon is included as the fourth uncertainty condition for Monte Carlo method. The results of this analysis are given in Table 7.5 for different load consumption levels in the network assuming a constant level of 30% for PEV penetration. It can be seen that when the loads are almost balanced, $\lambda$ and $F_I$ decrease while they increase if the loads are highly unbalanced.

Table 7.5 Stochastic analysis based $\lambda$ and $F_I$ of VU in the studied LV distribution network for different residential load levels

<table>
<thead>
<tr>
<th>Residential Load Status</th>
<th>Highly Unbalanced</th>
<th>Lightly Unbalanced</th>
<th>Almost Balanced</th>
</tr>
</thead>
<tbody>
<tr>
<td>PEV Mode</td>
<td>G2V</td>
<td>V2G</td>
<td>G2V</td>
</tr>
<tr>
<td>$\lambda$ at the beginning of the feeder</td>
<td>1.03</td>
<td>1.02</td>
<td>0.90</td>
</tr>
<tr>
<td>$\lambda$ at the end of the feeder</td>
<td>2.03</td>
<td>2.00</td>
<td>1.79</td>
</tr>
<tr>
<td>Failure Index ($F_I$ %)</td>
<td>64.3</td>
<td>67.0</td>
<td>25.3</td>
</tr>
</tbody>
</table>
The influence of the location of the PEVs (at the beginning or end of the feeder) on VU is discussed before. In the previous studies, it was assumed the PEVs were distributed randomly along the feeder. It is of high interest to investigate the case when the majority of the PEVs are connected to the beginning or to the end of the feeder. Therefore, another Monte Carlo study is carried out to investigate this phenomenon. The results of this study assuming a 30% PEV penetration level are given in Table 7.6. From this table, it can be seen that generally when the majority of the PEVs are connected to the end of the feeder, the $F_I$ at the end points of the network is more than when the majority of PEVs are connected to the beginning or middle of the feeder. This is valid for both G2V and V2G modes.

Table 7.6 Stochastic analysis based $\lambda$ and $F_I$ of VU in the studied network with majority of PEVs connected to beginning or end of the feeder

<table>
<thead>
<tr>
<th>Majority of PEVs installed at</th>
<th>Beginning of Feeder</th>
<th>Middle of Feeder</th>
<th>End of Feeder</th>
</tr>
</thead>
<tbody>
<tr>
<td>PEV Mode</td>
<td>G2V</td>
<td>G2V</td>
<td>G2V</td>
</tr>
<tr>
<td></td>
<td>V2G</td>
<td>V2G</td>
<td>V2G</td>
</tr>
<tr>
<td>$\lambda$ at the beginning of the feeder</td>
<td>0.96</td>
<td>0.97</td>
<td>0.96</td>
</tr>
<tr>
<td>$\lambda$ at the end of the feeder</td>
<td>1.89</td>
<td>1.91</td>
<td>1.89</td>
</tr>
<tr>
<td>Failure Index ($F_I$, %)</td>
<td>12.2</td>
<td>5.5</td>
<td>23.0</td>
</tr>
</tbody>
</table>

7.4 Improvement methods

Based on the numerical and stochastic results of Section 7.3, it can be concluded that the VU at the beginning of the feeder, regardless of the location, number and charging/discharging capacity of the connected PEVs, is likely to be less than 1.2%. However, the VU at the end of the feeder can be more than 2% standard limit for 34.1% of the cases. To reduce the VU, five conventional improvement methods, discussed in Section 4.1 and Section 5.2, are studied for this case and their efficacy is investigated. The methods are:
(1) Increasing feeder cross-section: This will result in reducing the voltage drop along the feeder and therefore, there will be little difference among the voltage amplitude of three phases of a feeder at the end.

(2) Installing capacitors: Installation of pad mounted switched capacitors in the LV feeders. It is important to note that if a three-phase capacitor is installed on a LV feeder, the voltage imbalance will almost remain the same. However, if a capacitor is connected only on a phase at a point where the voltage is below 0.95 pu, the voltage profile of the phase can be improved.

(3) Both feeder cross section increase and capacitor installation: This is a combination of the above two methods.

(4) Installing DSTATCOM: Shunt installation of a DSTATCOM at 2/3\textsuperscript{rd} distance from feeder beginning.

(5) Installing DVR: Series Installation of a DVR with the LV feeder at 1/3\textsuperscript{rd} of feeder beginning.

To verify the efficacy of these methods, another set of stochastic studies are carried out assuming a 30% penetration level for PEVs and the results are given in Table 7.7. In this, the nominal case indicates when the feeder cross-section is 70 mm\textsuperscript{2} and no capacitors are installed in the system. For method (1) mentioned above, the feeder cross-section is increased to 95 mm\textsuperscript{2}. The $F_I$ reduces to 8.6%. For method (2), a 15 kVAR capacitor is installed at the 2/3\textsuperscript{rd} distance from the beginning of the feeder. The $F_I$ reduces to 3.5%. For method (3), both feeder cross-section has been increased to 95 mm\textsuperscript{2} and a 15 kVAR capacitor is installed at the 2/3\textsuperscript{rd} distance from the beginning of the feeder. In this case, the $F_I$ is zero. For method (4), a DSTATCOM is connected in parallel to the 2/3\textsuperscript{rd} distance of feeder beginning. The DSTATCOM is intended to fix the voltage of its point of common coupling to a
desired value of 0.98 pu by injecting a required amount of reactive power. In this method, the $F_I$ reduces to zero. For method (5), a DVR is connected in series to the $1/3^{rd}$ distance of feeder beginning. The DVR fixes the voltage of its downstream side to a desired value of 0.98 pu by adding a small amount of voltage in series with the LV feeder. In this method, the $F_I$ reduces to zero. The results given in Table 7.7 prove the efficacy of the discussed improvement methods. In this table, PEV Mode G and V stand for G2V and V2G modes, respectively.

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nominal</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>G  V</td>
<td>G  V</td>
<td>G  V</td>
<td>G  V</td>
</tr>
</tbody>
</table>

| $\lambda$ | at the beginning of the feeder | 0.95 0.96 | 0.75 0.75 | 0.73 0.66 | 0.96 0.96 |
| $\lambda$ | at the end of the feeder       | 1.89 1.89 | 1.48 1.48 | 0.73 0.59 | 0.95 0.96 |
| Failure Index ($F_I%$) | 34.1 26.0 | 0.4 0 | 0 0 | 0 0 | 0 0 |

### 7.5 Summary

In this chapter, a VU sensitivity analysis and stochastic evaluation based on the connection point and charging/discharging capacities of PEVs connected to a residential LV distribution network in two G2V and V2G modes were presented. Through the studies, it is proved that PEVs will have minor effect on the VU at the beginning of a LV feeder designed with engineering judgments. However, it might increase at the end of the feeder to more than the standard limit. It is also proved that depending on the load of the phase in which the PEV is connected to, the VU will
increase or decrease based on its connection point and charging level (in G2V mode) and output power (in V2G mode). The failure index result of the stochastic simulations demonstrated that the probability of non-standard VU in these networks is high (36.5%). Five conventional improvement methods were discussed and their efficacy was verified by the stochastic and numerical results.
Chapter 8: Smart Distributed Demand Side Management of LV Distribution Networks Using Multi–Objective Decision Making

In this chapter, an intelligent direct demand management system is proposed for low voltage (LV) distribution networks in order to improve the network voltage profile. The main objective of the proposed method is to prevent overloading of distribution and upstream transformers at peak load periods and indirectly improving the voltage profile. This method works based on instantaneous load levels and requires installation of smart controllers with local communications capability. A Multi–Objective Decision Making (MODM) process is used within the system to prioritize the loads to be controlled or delayed. This decision is based on several criteria, each with different weightings.

8.1 Network modeling and analysis

Fig. 8.1 shows the schematic of a typical radial distribution network in a suburban area. The network supplies several residential and small business loads as well as a hospital. Three 11kV/400V 100kVA transformers supply residential feeders. Additional dedicated 100kVA transformers supply small business loads and a small hospital.
The network assets are traditionally chosen for a certain peak load level based on standardized After Diversity Maximum Demand (ADMD) figures [46]. For example, Ergon Energy, the distribution utility in regional Queensland Australia, uses an ADMD of 5kVA per detached dwelling for suburban houses. This figure is based on historical peak load data (and obviously neglects PEVs). Transformer and conductor ratings are also calculated using these ADMD values along with the associated voltage drop.

Fig. 8.1 Sample structure of radial LV distribution networks.

In practice, the peak load value in residential and small business networks is season and area dependent but generally occurs in the evening (6–7pm). Uncontrolled Level I charging of PEVs (from a house outlet at 230V 10/15A) is expected to be added to this peak since it is thought that customers will generally plug in their vehicles for charging as they return back home.

If the traditional network design methodology is followed, penetration of PEVs will have significant implications on the network infrastructure cost.
8.1.1 Residential type load modeling

To investigate these potential implications a detailed system model is required. The load models were built up in Matlab from individual device/appliance models which later were aggregated to form a house and then a residential feeder. Seventeen different appliance types were modeled. Each device was allocated a power rating and power factor as well as a time usage pattern. The time usage pattern of many appliances is linked to seasonal sunrise and sunset times as well as temperature. Therefore a simple “climate model” was constructed to vary the temperature throughout each day.

Each house was assigned a floor area according to a Gaussian distribution with mean 240m$^2$ and standard deviation 20m$^2$ corresponding to the Australian 2008 new house data [92]. The floor area is used in the calculation of many appliance loads below.

The power ratings of appliances were generally determined using mean values based on the ratings of common models in Australian market while usage patterns were typically derived from an informal assessment of typical behavior patterns. The detailed models of the main devices/appliances, including all parameters used in the study, are presented in Appendix B and are briefly discussed below:

**Lighting**

The total lighting load of each house was assigned according to a Gaussian distribution with the mean proportional to the house floor area. The morning and evening turn–on and turn–off times were based on informal observations as well as accounting for sunrise and sunset times (which vary seasonally).
Chapter 8: Smart distributed demand side management of LV distribution networks using multi-objective decision making

**Fridges and Freezers**

The number of fridges in a house was related to the house size, with the probability of more than one fridge being proportional to the floor area. Similarly the probability of a separate freezer is proportional to the house size.

In reality, refrigerators and freezers operate at ON/OFF duty cycles determined by hysteresis temperature controllers. In this study, in order to limit the computational complexity, the thermal dynamic model is neglected and both fridges and freezers are modeled as loads operating at ambient temperature dependant duty cycles.

**Cooking appliances**

Stove-top cookers, conventional ovens and microwave ovens were modeled separately. Every house was assumed to have one stove top cooker and one oven, however a certain percentage of these are assumed to be gas powered. In addition, each house was assumed to have a microwave oven.

**Air Conditioners (AC)**

ACs can have a major impact on residential peak load, so they were modeled in some detail. In this study, a residential market penetration of 70% was assumed, split between traditional hysteresis ACs and inverter ACs as described in Appendix B. A small percentage of the inverter ACs was assumed to represent ducted air conditioning (air conditioning the entire dwelling).

All ACs are modeled as closed loop temperature controlled devices (either hysteresis or proportional), attempting to force internal house temperature to follow a set point. A second order dynamic thermal model of the house, taken from [93], was
used and is explained briefly in Appendix B. The ACs only operate if the internal temperatures rise above preset thresholds.

**Clothes washers, Dryers and Dishwashers**

All houses were assumed to have clothes washers, with 30% assumed to have clothes dryers and 50% dishwashers. Clothes dryer operation was assumed dependent on weather (i.e. a rain parameter) and operated only once clothes washing were completed. All washers and dryers are modeled as constant power loads; the variation of power through different washing and/or drying cycles was not modeled.

**Water heaters**

Electric water heaters are still common in many parts of the world including in Australia, although certain plans are in place to phase them out over a number of years. In this study a market penetration of 85% was assumed. The thermal dynamics of the water heater are modeled and it is assumed that they are not operating under a pre–existing direct load control system.

**Swimming pool pumps**

Swimming pool pumps represent large loads that operate for long period of time (e.g. 1.5kW for 6– 8 hours per day). In this study a 50% penetration of swimming pool pumps was assumed. The pool pumps were assumed to operate each day with the total operation time varying from a mean of 6 hours in winter to a mean of 8 hours in summer.

**Electric vehicles**

Electric vehicles are widely thought to be a significant future load for residential power systems, especially when high clusters of PEV owners occur in a certain area. In this study a total of 15 EVs (25% penetration) were modeled with a
mean battery capacity of 20kWh (corresponding to the average of the Chevy Volt and Nissan Leaf battery capacities). It was assumed that the vehicles travel an average of 50km per day and the batteries charge at a constant rate of 15A at 230V until either fully charged or the householder departs.

**Televisions and PCs**

The number of Televisions (TVs) and Personal Computers (PCs) per household was assumed to be in relation the house floor area. The operational times are assumed to be a small amount in the morning and much larger durations in the afternoon and evening to model both adult and children occupants.

**Stand–by power and miscellaneous appliances**

A small random constant power was allocated to each house to account for stand–by power and miscellaneous continuously operating appliances (e.g. broadband modems). This amount was changed each day.

**Period of no occupancy**

During week days, there is a high probability that many of the houses will be unoccupied for significant periods of the day. During these unoccupied periods it is highly unlikely that ACs, lighting, cooking appliances, TVs and PCs will be operated (albeit that occasionally some people may leave some loads on). To simulate this effect 90% of the houses were allocated a stochastic “no occupant” period during each week day where operation of the aforementioned appliances was excluded.
8.1.2 Small business and hospital type load modeling

Several small office and shops were modeled on a small business feeder. This included Takeaways, Restaurants, Coffee shops, Pharmacies, a Bookstore, a Grocery, a Fruit shop, a Florist, Clothes Shops, a Bank, Offices, Bars, a 24–hr shop, a Butcher, a Tailor, a Bakery and a Laundry. Their modeling is based on detailed modeling of the main appliances used within each shop/office. The models used are identical to the residential appliances, only with different parameters (rating, number and operation time) as given in Appendix B. Each device was allocated a power rating as well as a time usage pattern. The time usage pattern of many appliances is linked to shop/office working hours. The same type of modeling is also applied for the hospital with the parameters given in Appendix B.

Having discussed the main appliances modeled, the next section describes the developed software program to simulate the LV network.

8.2 Analysis method

A Matlab–based simulation was developed to simulate the LV distribution network with an arbitrary number of houses/offices, each containing a number of the above mentioned electric appliances with different time usage patterns. Since most appliances turn ON, run for a certain time at constant power/impedance and then turn OFF, the simulation was made event–based. The simulation comprised a main routine plus a number of appliance modules which simulate the power characteristics of all appliances of that type. In addition, a calculation is made of the timing of next switching event of that appliance type, and that appliance module is run at that time. The active and reactive power consumptions (or load impedances) are maintained
constant between switching events. The simulation also has a fixed time step clock which generates regular events at a user defined interval (typically 5 minutes). All continuous models (such as the “climate model” and inverter AC) are run at this fixed time step. Note that at each event, only the appliance module which contains the next switching device is run since the power of other appliances is not changed at that time instant. This makes the simulation computationally efficient.

The flowchart of the program is shown in Fig. 8.2.
Chapter 8: Smart distributed demand side management of LV distribution networks using multi-objective decision making

Fig. 8.2 Flowchart of the analysis and simulation method.
8.3 Proposed control scheme

The main requirements of the proposed controller are:

- **Effectiveness**: the control system should limit peak load while assuring customer satisfaction.
- **Low cost**: the system should utilize low cost hardware.
- **Scalable**: the system should be easily scalable to larger networks.
- **Robustness**: the system should be fault tolerant.

Consider the distribution network depicted in Fig. 8.1 where a 33/11 kV substation is feeding several 11 kV/400 V distribution transformers. The main objective of the control system is to ensure that all the transformers and conductors do not exceed their ratings while minimizing the negative impact on consumers. If a transformer loading is below its rating, no control action is taken.

It is proposed that a microprocessor-based intelligent controller is installed in each house/shop/hospital, called end-user controller, to measure and control loads and communicate with the controllers located at the transformers. All controllers have low-bandwidth two-way communication capability. The end-user controllers measure the power consumption of each device in the house/shop and send that information to the relevant transformer controller every 2 minutes. The schematic diagram of the proposed control system is shown in Fig. 8.3. When the transformer controller detects an overload in a transformer, it decides which load(s) should be controlled and sends a command to the end-user controller in the related house/shop/hospital to delay/adjust that load. The load selection procedure is discussed in the following section. After the selected load is delayed or adjusted, the end-user controller sends back a confirmation command to the transformer controller. Upon receiving this, the transformer controller re-measures the total load.
If the load is now below the threshold, no further action is taken. If the loading still exceeds the threshold, the transformer controller will again choose another load to be delayed or adjusted. This process continues until all controllable loads are delayed or reduced.

The specific control action depends on the load type. Most loads (e.g., pool pumps, washers/dryers, water heaters, and electric vehicle chargers) are delayed by 15 minutes and then reconnected, while the set point of locally controlled loads (such as inverter ACs and water heaters) are adjusted for 15 minutes and then reset.

If all the distribution (11kV/400V) transformers run within their nominal rating but the total network load still exceeds the rating of the upstream (33/11 kV) transformer, then the controller in the upstream transformer will request a load reduction from the downstream transformer controllers. In this way the system can be scaled up the distribution network.

Several incentive methods can be used to encourage the householder or business to participate in this control scheme. While not the focus of this thesis, this could be as simple as free installation of controllers along with rebates or discounts on monthly energy bills.
8.4 Multi–Objective Decision Making (MODM) process

When the controller of a distribution transformer determines that the total load of that transformer exceeds its rating (i.e. the transformer is overloaded), the controller will decide which load(s) on that transformer should be delayed or controlled.

The first level of control action, called low level control, is based on the controllers at the distribution transformers (11kV/400V) to reduce the total amount of their load to below their nominal ratings.

However, if all the distribution transformers operate at or below their nominal ratings and the upstream transformer (33/11 kV) is still overloaded, then a higher
level control will be carried out by the controller in the upstream transformer. There are a few possible methods of the high level control, two of which are described below:

- **Simple high level control:** In this method, the overloaded 33/11 kV transformer will request an equal load reduction from all of the downstream distribution transformers. This method requires simple low bandwidth communication and is suited to networks with many downstream distribution transformers. The controllers in the distribution transformers will then request this load reduction from their controlled loads.

- **Accurate high level control:** In this method, there is a hierarchy of control from distribution transformers to upstream network. The controller in the upstream transformer will be able to select a specific load from a specific feeder using the same MODM procedure described in the low level control. This method is more applicable for networks with limited number of distribution transformers.

Both of these high level control methods effectively reduces the total network load. For the rest of the study, the accurate high level control method is utilized.

In this study, eight controllable loads are considered as alternatives for delay or control by the distribution transformer controller. In residential feeders, swimming pool pumps, PEV, electric water heaters, dish washers, clothes washers, dryers and ACs (both hysteresis type and inverter type) are assumed as controllable devices. In business feeders, only ACs are assumed as controllable devices and in hospital feeder, water heater and AC loads are assumed as the controllable devices. The inverter type ACs and water heaters can be adjusted by changing their temperature
set point while the delay type of control (ON/OFF) will be applied to other alternatives.

The decision making process consists of the following stages:

**8.4.1 Defining criteria and weighting**

A multi–objective decision making process is utilized in the control system. In this way, it is possible to consider the effect of several criteria, each with a different weighting value on prioritizing the loads to be delayed or adjusted. The criteria which have effect on prioritizing the loads are:

**User priority**

Customers may set the general priority of load delay or adjustment i.e. the order in which their appliances are delayed or controlled during peak load periods. The user priority is converted to a numerical value in range of [0, 1] for their eight controllable devices, where 1 and 0 show the highest and lowest priority for delay/control, respectively.

**Flexibility**

Inherent characteristics of different load types result in different flexibilities for disconnection and reconnection of appliances. For example, a swimming pool pump can work any time of the day if it satisfies its desired total hours of operation per day. Therefore, it has a high flexibility.

Clothes/ dish washers have a lower flexibility. This is due to this fact that if they are disconnected while working, the heated water inside will cool down; needing to be warmed up again next time it starts.

Water heaters and ACs are considered to have a high flexibility (subject to their satisfaction criterion discussed next).
Satisfaction

A satisfaction index is defined to represent how close a device/appliance is to its optimum state of operation. The index is dynamic, being updated every 5 minutes. This index is calculated differently for the appliances as follows and is shown schematically in Fig. 8.4.

- AC satisfaction index depends on how close the room temperature is to its set point.
- PEV satisfaction depends linearly on battery state of charge.
- Clothes washer, dryer and dishwasher satisfaction depends on the ratio of remaining operational time and available time. Available time is based on constraints set by the user i.e. washing/drying must be finished by a certain time.
- Swimming pool satisfaction depends on total operational time in the last 24 hours compared with set time.

![Flowchart of the control scheme including MODM process.](image)

Fig. 8.4 Flowchart of the control scheme including MODM process.
Chapter 8: Smart distributed demand side management of LV distribution networks using multi–objective decision making

**Power similarity**

It is more desirable for the transformer controller to delay a load which closely matches the required power decrease than one which is highly dissimilar. Therefore, the controller calculates a dynamic power similarity criterion for each load at each decision making step. All loads are normalized in range of [0 1] where 1 shows the power consumption of a specific load is very close to the required power reduction.

**High power consumption**

Assuming all customers are on the same tariff, it is desirable for the controller to first control an appliance from the house/shop with higher total electric power consumption rather than with lower total power, since the houses/shops with the highest consumptions are the biggest contributors to the overload. Therefore, at each decision making step, a numerical value in range [0 1] is allocated for all controllable devices in each house/shop expressing the ratio of total power of the house/shop compared to other houses/shops, where 1 and 0 show the house/shop with highest and lowest total power consumption at that moment, respectively.

Each of the above mentioned criteria have a different weighting. These weightings (in range [0 1]) are defined based on which criterion is more important than others when selecting a load to be delayed or adjusted.

### 8.4.2 Defining decision making matrix

A numerical value in range [0 1] is allocated for all alternatives (each controllable device in the network) based on the different criteria. All these data are set in a matrix as shown in Table 8.1.

In this matrix, $B_j$ represents the weighting values for $j^{th}$ criterion, $H(i,j)$ the rank of $i^{th}$ alternative among other alternatives from the $j^{th}$ criterion point of view and $D_i$,
the control priority value for $i^{th}$ criterion is calculated as:

$$D_i = \sum_{j=1}^{s} H(i, j) \times B_j$$

(8.1)

The alternative with highest $D_i$ will be chosen as the first alternative for control (delay/adjustment).

The flowchart of the proposed control scheme including MODM process is shown in Fig. 8.5. This flowchart is used for each transformer (low level and high level control) within the network individually.

Fig. 8.5 Flowchart of the control scheme including MODM process.
Table 8.1 Decision Making Matrix

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>House No</th>
<th>Shop</th>
<th>Criteria</th>
<th>Priority</th>
<th>Flexibility</th>
<th>Satisfaction</th>
<th>Power</th>
<th>High Similarity</th>
<th>Control Power</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>Water heater</td>
<td>B1</td>
<td>H(1,1)</td>
<td>H(1,2)</td>
<td>H(1,3)</td>
<td>H(1,1)</td>
<td>H(1,2)</td>
<td>D1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>Swimming pool</td>
<td>B2</td>
<td>H(2,1)</td>
<td>H(2,2)</td>
<td>H(2,3)</td>
<td>H(2,1)</td>
<td>H(2,2)</td>
<td>D2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>PEV</td>
<td>B3</td>
<td>H(3,1)</td>
<td>H(3,2)</td>
<td>H(3,3)</td>
<td>H(3,1)</td>
<td>H(3,2)</td>
<td>D3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>AC (hysteresis)</td>
<td>B4</td>
<td>H(4,1)</td>
<td>H(4,2)</td>
<td>H(4,3)</td>
<td>H(4,1)</td>
<td>H(4,2)</td>
<td>D4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>AC (inverter)</td>
<td>B5</td>
<td>H(5,1)</td>
<td>H(5,2)</td>
<td>H(5,3)</td>
<td>H(5,1)</td>
<td>H(5,2)</td>
<td>D5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>Dish washer</td>
<td>B6</td>
<td>H(6,1)</td>
<td>H(6,2)</td>
<td>H(6,3)</td>
<td>H(6,1)</td>
<td>H(6,2)</td>
<td>D6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>Clothes washer</td>
<td>B7</td>
<td>H(7,1)</td>
<td>H(7,2)</td>
<td>H(7,3)</td>
<td>H(7,1)</td>
<td>H(7,2)</td>
<td>D7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>Dryer</td>
<td>B8</td>
<td>H(8,1)</td>
<td>H(8,2)</td>
<td>H(8,3)</td>
<td>H(8,1)</td>
<td>H(8,2)</td>
<td>D8</td>
<td></td>
</tr>
</tbody>
</table>

2 9 Water heater | ... | ... | ... |
10 Swimming pool | ... | ... | ... |
11 PEV | ... | ... | ... |
12 AC (hysteresis) | ... | H(i,j) | ... |
13 AC (inverter) | ... | ... | ... |
8.5 Simulation results

For verifying the efficacy of the proposed control, several cases were studied of which a few sample results are given below. It is assumed the network consists of a 400 kVA 33/11 kV transformer feeding 3 residential feeders, 1 business feeder and 1 hospital feeder. Each residential feeder is fed through a 100 kVA 11kV/400V pole-mounted transformer and supplies 20 houses (i.e. designed with an ADMD of 5kVA per house). Two similar transformers are used to feed separately two business and hospital feeders. There are 24 different types of small business/shop/offices assumed on the business feeder. The load parameters of the residential, business and hospital feeders are given in the Appendices F, G and H. The simulation also included a total of 15 PEVs (25% penetration level) which are plugged–in as the owners return their houses. The simulation was over a 48–hr summer week–day.

The weighting of the MODM criteria assumed in this study is given in Table 8.2, although this can of course be changed depending on stakeholder feedback. In addition, the flexibility assumed for each controllable device is listed in Table 8.3. In this table, each device is given a number of 1–8 which represents that device in the simulation results figures. Note that the final device selection in each house is dependent on the priority (randomly generated), flexibility and criteria weightings.

<table>
<thead>
<tr>
<th>Priority</th>
<th>Flexibility</th>
<th>Satisfaction</th>
<th>Power similarity</th>
<th>High Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0.5</td>
<td>0.7</td>
<td>0.7</td>
<td>0.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Controllable Device</th>
<th>Water Heater</th>
<th>Pool Pump (hys.)</th>
<th>AC (hys.)</th>
<th>AC (inv.)</th>
<th>PEV</th>
<th>Dish Washer</th>
<th>clothes Washer</th>
<th>Dryer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Flexibility</td>
<td>0.5</td>
<td>0.9</td>
<td>0.7</td>
<td>0.7</td>
<td>0.9</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 8.2 Weighting of MODM criteria

Table 8.3 Controllable device number allocation and flexibility
Fig. 8.6 shows the total aggregated apparent power supplied by a 100 kVA residential feeder transformer in the presence of PEVs. Since the specific load shape will vary somewhat stochastically from day to day and from area to area, no attempt was made to quantitatively match the resulting load shape to a measured one. However the load shape roughly matches measured profiles in Ergon Energy as well as published profiles [57] which provide confidence in the bottom up modeling approach. As seen from this figure, although the average apparent power supplied by the transformer is around 60 kVA per day, it reaches a peak of 150 kVA. The transformer is overloaded by 50% for a 2–hr period per day.

The total apparent power of the upstream 400 kVA transformer without the controller is shown in Fig. 8.7. As seen in this figure, the transformer has a peak value of 550 kVA and overloaded for about 5 hrs during the second day.

Let us now assume that the proposed control system is applied to the network. The total aggregated apparent power supplied by the residential distribution transformer is shown in Fig. 8.8. As seen in this figure, the peak load power is limited to 100 kVA hence verifying the efficacy of the proposed control system in controlling load. The total aggregated apparent power supplied by the upstream transformer is shown in Fig. 8.9. As seen from this figure, the peak load power is now limited to 400 kVA and this verifies the efficacy of the proposed high level control system in controlling load. Comparing Fig. 8.7 and Fig. 8.9, a load increase is seen after the control operation period which is the result of the delay/adjustment of specific loads by the controller.
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Fig. 8.6 Total apparent power of one of the residential distribution transformers including 25% penetration level of PEVs.

Fig. 8.7 Total apparent power of the studied network without any control.

Fig. 8.8 Total apparent power of one of the residential distribution transformers with the proposed control system.

Fig. 8.9 Total apparent power of the studied network with the proposed control system.
It is important for the control scheme to have a minimal impact on consumers. To verify this, sample operation states of PEVs and swimming pools as (delayed) controllable loads are given below. Fig. 8.10 shows the ramp waveforms indicating the battery charging state of a few PEVs of the network. The increasing section shows that they are being charged from an arbitrary initial value to full (100%) before their departure time. This value is reset to an arbitrary value at 12 pm everyday and they start being charged at an arbitrary time in the evening when the cars return back home. The flat sections of the traces show that they are disconnected by the controller at some stages and again reconnected, nevertheless all vehicles complete charging. This figure verifies that even though the controller has delayed the charging for some of them, they have all been fully charged before departure time.

Fig. 8.11 shows the swimming pool pump remaining operation time for a few houses in the network, which is reset every day. As seen in this figure, each swimming pool has a randomized required operational time per day (around 7–9 hrs) and they have all completed operation (i.e. they fall to zero) within each 24 hour period. All other controllable devices have the same operational characteristic.

Inverter type ACs and water heaters are large loads in the network and have adjustable characteristics based on changing the set points of room temperature and tank water temperature.

Based on assumption in this study, inverter ACs will operate when the internal house/shop temperature exceeds its set point providing it is occupied. If an inverter AC is selected, the transformer controller will increase the temperature set point by 1°C. Fifteen minutes later this set point will be reset to its original value, thus preventing set point divergence from the householders desired level. In Fig. 8.12(a),
the AC inverter set point increase and reset is shown for three sample houses. The set point increase results in less electric power consumption by the AC as shown in Fig. 8.12(b).

However, this set point change should not appreciably worsen the customer satisfaction. This is investigated in Fig. 8.13. In Fig. 8.13(a), the ambient and internal temperature in 48–hr period is shown. As it can be seen the internal temperature is kept around 25 °C (set point). In Fig. 8.13(b), the electric power consumption of the AC is shown. It can be seen that when the internal temperature rises beyond the set point limit, the AC turns on and turns off when the internal temperature is reduced to the set point. In Fig. 8.13 (c) the AC satisfaction is shown. When the people are away, the AC is off so “satisfaction” is decreased but when the people are at home, it is kept close to 100%. In Fig. 8.13(d), the set point and its variation based on the controller command is shown. It can be seen that around 43:00 hr, there are several commands for set point increase.

The same behavior is evident for other ACs and water heaters in the network.

![PEV Charging States](image)

Fig. 8.10 PEVs battery charging states.
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Fig. 8.11 Swimming pool pump operation characteristic.

Fig. 8.12 (a) Temperature set point change for sample residential inverter ACs in network, (b) Apparent power consumption of sample residential inverter ACs versus their set point variation.
Fig. 8.13 (a) Ambient and house internal temperature variation, (b) AC electric power consumption, (c) AC satisfaction, (d) AC set point.

It is of interest to study the number of the low level and high level control commands, described in Section IV which are applied by the transformers’ controller. In Fig. 8.14, the number of control commands applied to each controllable device is shown for 4 sample houses in a residential feeder. The controllable device number was given in Table 8.3. From this figure, it can be seen that the number of control commands differs from house to house based on the status of any controllable device running in each house, as well as their objectives (power, satisfaction, priority and flexibility).

In Fig. 8.15, the total number of the low level control commands applied for each customer in each feeder individually and for each type of controllable device in that feeder is shown. It can be seen that the number of commands for load control differs from a house to house. It is also shown that inverter type ACs experience the
highest number of control actions. The same parameters are shown for high level control in Fig. 8.16.

A comparison between the total number of low level commands for each feeder and also a comparison for total number of high and low level commands within the whole studied network is shown in Fig. 8.17. It can be seen that since residential feeders had more controllable devices than hospital and business feeders, and the highest contributor to network peak load was from residential feeders, there is a larger number of control commands applied to residential loads. In addition, it is seen that high level control command number is much smaller than low level control command number.

For investigating the effect of the proposed control method on the upstream network another case study is carried out. In this study the network in Fig. 8.1 is simulated assuming the 33/11 kV substation supplies 20 transformers each with 100 kVA nominal rating.

Fig. 8.18 shows the total apparent power of the substation in 48–hr period. As it can be seen from this figure, in the uncontrolled case, the substation transformers experience an overload around 30% for a period of 3–hours. However, if the proposed control method is applied for all the residential loads which are supplied by each distribution transformer, the peak load is limited to 2 MVA as shown in this figure. This demonstrates the potential for savings in the upstream network electrical infrastructure. The slight difference in the load profiles following the peak is a result of some peak loads being transferred to off peak periods.
Fig. 8.14 Number of the low level control commands applied for different controllable devices in 4 sample houses of residential feeder 2 in 48-hr period.

Fig. 8.15 Number of the low level control commands applied for each customer in a feeder individually (left), Number of the low level commands applied for a specific controllable device in each feeder (right).

Fig. 8.16 Number of the higher level control commands applied for each customer in the network individually (top), Number of the higher level commands applied for a specific controllable device in the network (bottom).
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Fig. 8.17 The total number of the low level commands applied for each feeder individually (left), Comparison of the total number of the low level and higher level commands applied for the loads (right).

Fig. 8.18 Total apparent power of substation feeding 100 distribution transformers in an area.

8.6 Summary

In this chapter, a novel intelligent direct demand side management system has been proposed for peak load reduction in LV distribution networks. This will indirectly improve the voltage profile of the network at network peak period. The system utilizes low cost controllers with two–way communications capability, installed in each house/office and at the supply transformers, to monitor and control the loads. A multi–objective decision making process has been proposed to select which load(s) to be delayed or reduced. The algorithm uses several criteria, each with different weightings to reduce the load while minimizing the impact on consumers.
Chapter 9: Conclusions and recommendations

In this chapter, the general conclusions of the thesis and recommendations for future research are presented.

9.1 Conclusions

The general conclusions of the thesis are:

(1) Integration of PVs and connection of PEVs can cause voltage profile and voltage unbalance problems for the LV distribution network or Micro grid due to their random location and rating.

(2) The power quality improvement in a hybrid micro grid can be improved utilizing one of the DG units to function as a compensator. The compensator can be controlled in such a way to reduce voltage unbalance problems and harmonics caused by nonlinear and unbalanced loads.

(3) The voltage unbalance can increase from the beginning till the end of a feeder due to PV and PEV presence within the network. This increase is more obvious at the end of the feeder rather than the beginning. It is also more for higher ratings of PVs and PEVs. It also depends on the load of the phase in which they are connected. If they are installed on one feeder, the voltage imbalance will be modified on all the other LV feeders of the network. The stochastic simulation demonstrated that the failure index of non–standard voltage imbalance in these networks is high.
Chapter 9: Conclusions and recommendations

(4) Feeder cross-section increase or switching single-phase capacitor installation in LV feeders can reduce the voltage unbalance and improve the voltage profile.

(5) Voltage control strategy of PVs seems the most suitable way of voltage profile correction and voltage unbalance reduction in LV distribution networks in both network peak and off-peak periods.

(6) DSTATCOM and DVR can be used for voltage unbalance reduction and voltage profile improvement in LV distribution networks. In a specific network, the utilized DVR has a smaller rating than a DSTATCOM; however for longer feeders, multiple DVRs are necessary.

(7) An intelligent direct demand side management system can be utilized for controlling the controllable loads within a network in order to prevent overloading of network transformers by adjusting or delaying some of the loads to other periods. This will indirectly result in better voltage profile in the network peak hour period.

9.2 Recommendations for future research

The scope for future research are:

9.2.1 Studying the dynamic behavior of PEVs

In this research, only the steady state behavior of PEVs was investigated for voltage profile and unbalance studies. Although the studies and improvement methods were studied in detail but the dynamic behavior was not addressed in this thesis and can be a topic for future research.
9.2.2 Voltage control strategy for single–phase PVs and unbalanced networks

In this research, the voltage control strategy was only proposed and studied for balanced and three–phase networks. This can be also continued for single–phase PVs and the networks with unbalanced loads. Therefore, voltage control strategy for single–phase and unbalanced loads can be a topic for future research.

9.2.3 Detailed demand side management

In this research, the demand side management only was carried out from the total energy consumption of the loads and from a network company point of view. However, some other parameters such as network losses and transformer thermal modeling can also be included. In addition, the energy cost, which is an important factor for energy retail companies, can also be included in this study. These ideas can be included in the future research.
References


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Publications arising from the thesis

Journal papers


Conference papers


Appendix–A
Technical data and parameters

Table A.1
Grid and Load Types in the Micro grid

<table>
<thead>
<tr>
<th>Grid</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>415 V L–L RMS</td>
</tr>
<tr>
<td>Frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Line Impedance</td>
<td>$R =0.02 \ \Omega$, $L =0.001 \ \text{H}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Load Type</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fan Heater</td>
<td>3 three–phase Resistive Load each $P =4.5 \ \text{kW}$</td>
</tr>
<tr>
<td>Fan Heater</td>
<td>1 three–phase Resistive Load $P =6 \ \text{kW}$</td>
</tr>
<tr>
<td>Induction Motor</td>
<td>6 three–phase each $P =1.5 \ \text{kW}$</td>
</tr>
</tbody>
</table>

Table A.2
PV, Boost Chopper, Converter and Controller

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of PV cells in series</td>
<td>2</td>
</tr>
<tr>
<td>No. of PV cells in parallel</td>
<td>3</td>
</tr>
<tr>
<td>Output voltage of PV cell</td>
<td>0.1 V DC</td>
</tr>
<tr>
<td>Rated output power</td>
<td>3.06 kW</td>
</tr>
<tr>
<td>Radiation level</td>
<td>1100</td>
</tr>
<tr>
<td>Ambient Temperature</td>
<td>30 $^\circ$C</td>
</tr>
<tr>
<td>Output voltage of Chopper</td>
<td>250 V DC</td>
</tr>
<tr>
<td>Boost Chopper Parameters</td>
<td>$L =10 \ \text{mH}$, $C = 5 \ \text{mF}$</td>
</tr>
<tr>
<td>Boost Chopper Controller</td>
<td>Hysteresis Voltage Control, $k_p=0.0001$, $\text{Hysteresis bandwidth} =0.0002$</td>
</tr>
<tr>
<td>Converter Structure</td>
<td>3 Single–Phase H–Bridge Inverter</td>
</tr>
<tr>
<td>Converter Loss</td>
<td>$R =0.1 \ \Omega$ per phase</td>
</tr>
<tr>
<td>Transformer</td>
<td>0.25/0.415 kV, 0.5 MVA, $L_r=4.4 \ \text{mH}$</td>
</tr>
<tr>
<td>LC Filter</td>
<td>$L_f=49.8 \ \text{mH}$, $C_f=50 \ \mu\text{F}$</td>
</tr>
<tr>
<td>Hysteresis Constant</td>
<td>$10^{-5}$</td>
</tr>
</tbody>
</table>
### Table A.3
Battery, Converter and Controller

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of battery units in series</td>
<td>10</td>
</tr>
<tr>
<td>No. of battery units in parallel</td>
<td>2</td>
</tr>
<tr>
<td>Output voltage of battery unit</td>
<td>12 V DC</td>
</tr>
<tr>
<td>Rated output power</td>
<td>2 kW, 226 A.hr</td>
</tr>
<tr>
<td>Converter Structure</td>
<td>3 Single–Phase H–Bridge Inverter</td>
</tr>
<tr>
<td>Converter Loss</td>
<td>$R = 0.1 , \Omega$ per phase</td>
</tr>
<tr>
<td>Transformer</td>
<td>0.12/0.415 kV, 0.5 MVA, $L_r = 4.4 , \text{mH}$</td>
</tr>
<tr>
<td>LC Filter</td>
<td>$L_f = 76.2 , \text{mH}, C_f = 50 , \mu F$</td>
</tr>
<tr>
<td>Hysteresis Constant</td>
<td>$10^{-5}$</td>
</tr>
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</table>

### Table A.4
Fuel Cell, Boost Chopper, Converter and Controller

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel cell rated power</td>
<td>4 kW</td>
</tr>
<tr>
<td>Boost Chopper Parameters</td>
<td>$L = 1 , \text{mH}, C = 1 , \text{mF}, f_{sw} = 10 , \text{kHz}$</td>
</tr>
<tr>
<td>Boost Chopper Controller</td>
<td>Open loop control, Switch duty cycle=10%</td>
</tr>
<tr>
<td>Converter Structure</td>
<td>3 Single–Phase H–Bridge Inverter</td>
</tr>
<tr>
<td>Converter Loss</td>
<td>$R = 1.5 , \Omega$ per phase</td>
</tr>
<tr>
<td>Transformer</td>
<td>0.4/0.415 kV, 0.25 MVA, $L_r = 0.54 , \text{mH}$</td>
</tr>
<tr>
<td>LC Filter</td>
<td>$L_f = 38.1 , \text{mH}, C_f = 50 , \mu F$</td>
</tr>
<tr>
<td>Hysteresis Constant</td>
<td>$10^{-5}$</td>
</tr>
</tbody>
</table>

### Table A.5
Diesel Generator Set

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure</td>
<td>Internal Combustion Engine + Exciter + 3 phase Synchronous Generator</td>
</tr>
<tr>
<td>Rated power</td>
<td>14 kVA</td>
</tr>
<tr>
<td>Rated voltage</td>
<td>415 V L–L RMS</td>
</tr>
<tr>
<td>Rated Frequency</td>
<td>50 Hz, 1500 rpm</td>
</tr>
</tbody>
</table>
Table A.6
Droop Controller Coefficients

<table>
<thead>
<tr>
<th>DG Type</th>
<th>Active Power–Angle [rad/MW]</th>
<th>Reactive Power–Voltage [kV/Mvar]</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV</td>
<td>441</td>
<td>0.196</td>
</tr>
<tr>
<td>FC</td>
<td>337.5</td>
<td>0.15</td>
</tr>
<tr>
<td>Battery</td>
<td>675</td>
<td>0.30</td>
</tr>
<tr>
<td>Syn. Generator</td>
<td>112.5</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table A.7
Residential Low voltage Distribution Network

**Grid**
- Voltage: 415 V L–L RMS
- Frequency: 50 Hz
- Line Impedance: $R = 0.02 \, \Omega$, $L = 0.001 \, H$ (between each load)

**Loads [kW] (values at the time of simulation)**
- $L_{a1}= 1.9$, $L_{a2}= 2.3$, $L_{a3}= 2.3$, $L_{a4}= 2.3$, $L_{b1}= 2.4$, $L_{b2}= 3.8$
- $L_{b3}= 2.5$, $L_{b4}= 2.2$, $L_{b5}= 1.7$, $L_{b6}= 1.9$, $L_{b7}= 2.3$, $L_{b8}= 2.3$
- $L_{c1}= 1.9$, $L_{c2}= 2.8$, $L_{c3}= 2.2$, $L_{c4}= 2.3$, $L_{c5}= 2.5$

**Single–Phase PVs [kW]**
- $PV_1= 1$, $PV_2= 2$, $PV_3= 1$
- $PV_4= 1$, $PV_5= 3$, $PV_6= 3$
Appendix–B
Residential, Business, Hospital Load Modeling

(i) Residential Load Data
Residential load modeling data was taken from manufacturers’ websites. Extensive use was made of the Gaussian or Normal distribution to generate appliance power data and usage times. Specifically $N(\mu, \sigma)$ denotes the Normal (or Gaussian) random function generating a value according to a Normal distribution with mean $\mu$ and standard deviation $\sigma$.

B.1 Lighting
The total lighting load of each house was determined by:

$$P_{li,i} = \frac{A_i}{240} \cdot N(322,20)$$

(B.1)

where $P_{li,i}$ is the lighting load in kW of house $i$ and $A_i$ is the floor area of house $i$ in $\text{m}^2$. The power factor is assumed to be unity. The mean morning and night lighting loads were calculated as 50% and 80% of the total lighting load. The turn–on and turn–off times were determined as follows:

Table B.1
Lighting Load Turn–on and Turn–off Times

<table>
<thead>
<tr>
<th>TURN–ON TIME $T_{ON}$</th>
<th>TURN–OFF TIME $T_{OFF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AM</strong></td>
<td></td>
</tr>
<tr>
<td>$N(06:00,1:00)$ or no turn–on if $T_{SUNRISE}&gt;T_{ON}$</td>
<td>Earliest of: $T_{ON} + N(02:00,0:20)$ or $T_{SUNRISE}$</td>
</tr>
<tr>
<td><strong>PM</strong></td>
<td></td>
</tr>
<tr>
<td>Latest of: $N(18:00,0:30)$ or $T_{SUNSET}$</td>
<td>$N(23:00,01:00)$</td>
</tr>
</tbody>
</table>

B.2 Fridges and Freezers
Each house was assumed to have at least one fridge. The probability of second fridge was related to house floor area as:
\[ P(fr2,i) = \min(0.1 + (A_i - 240) / 240, 1) \]
\[ N_{fr,i} = 1 + (\text{rand} < P(fr2,i)) \]  \hspace{1cm} (B.2)

where \( \min(.) \) is the minimum function, \( \text{rand} \) is a uniformly distributed random variable and \( \text{rand}<P(fr2,i) = 1 \) if true or 0 if false. The fridge period and duty cycle were

\[ \text{Per}_{fr,i} = \mathcal{N}(0.50, 0.05) \]
\[ D_{fr,i} = \mathcal{N}(0.18, 0.02).T_{amb}/298 \]  \hspace{1cm} (B.3)

where \( \text{Per}_{fr,i} \) is the period in minutes, \( D_{fr,i} \) is the duty cycle and \( T_{amb} \) is the ambient temperature in Kelvin. The turn–on time of the fridge within the period was assumed to be random.

The probability of each house having a separate freezer was similarly related to the floor area. In addition, the freezers duty cycle and period were similarly modeled except the mean period was 60min and the mean duty cycle 0.16.

The power ratings of the fridges and freezers (in kW) were determined by

\[ P_{fr,i} = \mathcal{N}(0.47, 0.04) \]
\[ P_{fr,i} = \mathcal{N}(0.4, 0.04) \]  \hspace{1cm} (B.4)

The power factor was assumed to be 0.85.

\textit{B.3 Cooking Appliances}

The probability of any house having an electrical stove top cooker and oven was taken as 70% and 90% respectively (assuming 30% and 10% gas stove top and oven market penetrations respectively). Each house was assumed to have a microwave oven. The table below shows the calculation of the power ratings and times of use.
Table B.2
Cooking Appliance Power Ratings and Times of Use

<table>
<thead>
<tr>
<th></th>
<th>STOVETOP COOKER</th>
<th>CONVENTIONAL OVEN</th>
<th>MICROWAVE OVEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>POWER RATING</td>
<td>N(3.0,0.3)</td>
<td>N(2.4,0.2)</td>
<td>N(1.4,0.14)</td>
</tr>
<tr>
<td>T_{ON}(AM)</td>
<td>N(07:00,0:30)</td>
<td>N(07:00,0:30)</td>
<td>N(07:00,0:30)</td>
</tr>
<tr>
<td>Probability</td>
<td></td>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>T_{OFF}(AM)</td>
<td>T_{ON}(AM) + N(0:30,0:03)</td>
<td>T_{ON}(AM) + N(0:30,0:03)</td>
<td>T_{ON}(AM) + N(0:05,0:015)</td>
</tr>
<tr>
<td>T_{ON}(PM)</td>
<td>N(18:30,0:30)</td>
<td>N(18:30,0:30)</td>
<td>N(18:30,1:00)</td>
</tr>
<tr>
<td>T_{OFF}(PM)</td>
<td>T_{ON}(PM) + N(01:00,0:15)</td>
<td>T_{ON}(PM) + N(0:45,0:10)</td>
<td>T_{ON}(AM) + N(0:15,0:02)</td>
</tr>
</tbody>
</table>

B.4 ACs

For the 60 house study presented, a total of 42 houses (70%) were assumed to be using AC during the hottest part of the day. The assumed split between different types of AC is presented in the table below. Temperatures $T_{0}, T_{100}$ are expressed in Kelvin.

Table B.3
AC Parameters

<table>
<thead>
<tr>
<th></th>
<th>HYSTERESIS AC</th>
<th>INVERTER AC</th>
<th>DUCTED AC</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO OF HOUSES</td>
<td>Total 9 houses:</td>
<td>Total 30 houses:</td>
<td>Total 3 house</td>
</tr>
<tr>
<td></td>
<td>– 6 in a single room</td>
<td>– 18 in a single room</td>
<td></td>
</tr>
<tr>
<td></td>
<td>– 3 in 2 rooms</td>
<td>– 12 in 2 rooms</td>
<td></td>
</tr>
<tr>
<td>RATING</td>
<td>N(2.0,0.2) per room</td>
<td>N(2.3,0.2) per room</td>
<td>N(5.2,0.2)</td>
</tr>
<tr>
<td>CYCLE</td>
<td>N(0:30,0:05)</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

B.5 Clothes washers, Dryers and Dishwashers

Parameters for washers and dryers were as follows.
### Table B.4
Washer Operating Parameters

<table>
<thead>
<tr>
<th></th>
<th>CLOTHES WASHER</th>
<th>CLOTHES DRYER</th>
<th>DISH WASHER</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RATING</strong></td>
<td>N(1.0,0.1)</td>
<td>N(2.8,0.3)</td>
<td>N(2.0,0.2)</td>
</tr>
<tr>
<td><strong>T&lt;sub&gt;ON&lt;/sub&gt;</strong></td>
<td>N(8:30,0:30)</td>
<td>Clothes washer T&lt;sub&gt;OFF&lt;/sub&gt;</td>
<td>N(07:30,0:30) or N(20:00,0:30) + N(0:15,0:01)</td>
</tr>
<tr>
<td><strong>T&lt;sub&gt;OFF&lt;/sub&gt;</strong></td>
<td>T&lt;sub&gt;ON&lt;/sub&gt; + N(01:00,0:10)</td>
<td>T&lt;sub&gt;ON&lt;/sub&gt; + N(01:00,0:10)</td>
<td>T&lt;sub&gt;ON&lt;/sub&gt; + N(01:00,0:02)</td>
</tr>
</tbody>
</table>

### B.6 Electric Water Heaters

Electric water heaters were modeled as dependent on ambient temperature and water consumption rate. The following parameters were used in the study.

Table B.5
Electric Water Heater Parameters

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>POWER RATING</strong></td>
<td>N(3.6,0.3)</td>
</tr>
<tr>
<td><strong>SLOPE OF TEMPERATURE DEPENDENCY</strong></td>
<td>N(–0.0125,0.001)</td>
</tr>
</tbody>
</table>

### B.7 Swimming Pool Pumps

Swimming pool pumps were assumed to operate for a total mean daily period of 8 hours (during summer). The operation was assumed to be split into two equal periods of operation occurring randomly during the day. The parameters used are listed as follows.

Table B.6
Swimming Pool Parameters

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>POWER RATING</strong></td>
<td>N(1.5,0.1)</td>
</tr>
<tr>
<td><strong>T&lt;sub&gt;ON(AM)&lt;/sub&gt;</strong></td>
<td>12:00×rand</td>
</tr>
<tr>
<td><strong>T&lt;sub&gt;ON(PM)&lt;/sub&gt;</strong></td>
<td>12:00 + 12:00×rand</td>
</tr>
<tr>
<td><strong>T&lt;sub&gt;OFF&lt;/sub&gt;</strong></td>
<td>T&lt;sub&gt;ON&lt;/sub&gt; + N(8–2×sin(2π×d/730),0.8)</td>
</tr>
</tbody>
</table>

where \(d\) is the number of days into the year.
B.8 Plug-in Electric Vehicles (PEVs)

For uncontrolled charging, the PEVs were assumed to start charging at a constant rate from when the customer arrives home until the battery is fully charged or the customer departs (whichever occurs first). An average travel distance of 50km per day was assumed with an economy of 20kWh per 100km [29] and an average charge/discharge efficiency of 85%. Parameters used were as follows.

Table B.7
PEV Parameters

<table>
<thead>
<tr>
<th>CHARGING POWER RATING</th>
<th>230V, 15A, unity power factor = 3.45kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAILY REQUIRED CHARGE</td>
<td>N(11.8kWh, 2kWh)</td>
</tr>
<tr>
<td>TARRIVAL</td>
<td>N(17:00, 1:00)</td>
</tr>
<tr>
<td>TDEPARTURE</td>
<td>N(07:30:0:30)</td>
</tr>
</tbody>
</table>

B.9 Television and Personal Computers

The following parameters were used in this study.

Table B.8
Consumer Electronics Parameters

<table>
<thead>
<tr>
<th></th>
<th>TVs</th>
<th>PCs</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO. DEVICES PER HOUSE</td>
<td>round(N(1.5,0.3) × A/240)</td>
<td>ceil(N(1.5,0.3) × A/240)</td>
</tr>
<tr>
<td>POWER RATING PER DEVICE</td>
<td>N(0.18, 0.02)</td>
<td>N(0.26, 0.02)</td>
</tr>
<tr>
<td>TON(AM)</td>
<td>N(07:00, 0:30)</td>
<td>N(07:00, 0:30)</td>
</tr>
<tr>
<td>Probability</td>
<td>= 0.5</td>
<td>= 0.5</td>
</tr>
<tr>
<td>TOFF(AM)</td>
<td>TON(AM) + N(0:30, 0:03)</td>
<td>TON(AM) + N(0:30, 0:03)</td>
</tr>
<tr>
<td>TON(PM)</td>
<td>N(16:00, 2:00)</td>
<td>N(16:00, 2:00)</td>
</tr>
<tr>
<td>TOFF(PM)</td>
<td>TON(PM) + N(2:00, 0:30)</td>
<td>TON(AM) + N(2:30, 0:30)</td>
</tr>
</tbody>
</table>

where round is a function rounding to the nearest integer and ceil is a function rounding up to the next highest integer.
(ii) Small Business Data

Shops/offices load modeling data was made based on assumptions of their working hours and different kind of electric devices used. Similar to residential loads, Gaussian or Normal distributions are used to model their working hours, electric devices ratings and number. The businesses assumed in this study with their working hours are listed in Table B.9. The floor area of the Restaurants and Bars are assumed to be around 1000 m² while for all others it is assumed around 250 m². Their lighting was calculated similar to (B.1) based on their floor area during working hours. All shops/offices are assumed to have an AC and PC. A loss factor of 10% is applied for the AC operation for considering shops doors opening and closing. The electric power consumption of most of the devices (including ACs, and lighting) was based on working hours for each business. However, the ACs for Drug store, Grocery store, Fruit shop, Florist and 24–hr shop are assumed to be operating 24 hours. The cooking devices in Restaurants, Take–away and Bakery are assumed to be working on Gas (non–electric). The main electric appliances for each business after lighting and ACs are listed in Table B.9. The power rating of these devices for each business is given in Table B.10.
Table B.9  
Business Loads, their Working Hours and Main Electric Devices

<table>
<thead>
<tr>
<th><strong>TYPE AND NO.</strong></th>
<th><strong>WORKING HOURS</strong></th>
<th><strong>MAIN DEVICES AND NO</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Restaurant (3)</td>
<td>N(10,0.5) to N(22,0.5)</td>
<td>Freezer, Fridge, Dish washer, TV</td>
</tr>
<tr>
<td>Coffee shop (2)</td>
<td>N(7,0.5) to N(18,0.5)</td>
<td>Frozen, Dish washer</td>
</tr>
<tr>
<td>Take–away (1)</td>
<td>N(10,0.5) to N(23,0.5)</td>
<td>Frozen, Fridge, Dish washer</td>
</tr>
<tr>
<td>Drugstore (1)</td>
<td>N(9,0.5) to N(17,0.5)</td>
<td></td>
</tr>
<tr>
<td>Bookstore (1)</td>
<td>N(9,0.5) to N(17,0.5)</td>
<td></td>
</tr>
<tr>
<td>Grocery store (1)</td>
<td>N(9,0.5) to N(17,0.5)</td>
<td>Frozen, Fridge, Dish washer, TV</td>
</tr>
<tr>
<td>Fruit shop (1)</td>
<td>N(9,0.5) to N(17,0.5)</td>
<td>Fridge (2)</td>
</tr>
<tr>
<td>Florist (1)</td>
<td>N(9,0.5) to N(17,0.5)</td>
<td></td>
</tr>
<tr>
<td>Clothes shop (3)</td>
<td>N(9,0.5) to N(17,0.5)</td>
<td></td>
</tr>
<tr>
<td>Offices (3)</td>
<td>N(9,0.5) to N(17,0.5)</td>
<td>PC (4), Printer</td>
</tr>
<tr>
<td>Bars (2)</td>
<td>N(18,0.5) PM to N(02,0.5) AM</td>
<td>Fridge (3), TV (2), Dish washer</td>
</tr>
<tr>
<td>24–7 shop (1)</td>
<td>24 hours</td>
<td>Fridge (3), Freezer</td>
</tr>
<tr>
<td>Butcher (1)</td>
<td>N(9,0.5) to N(17,0.5)</td>
<td>Fridge (3), Freezer</td>
</tr>
<tr>
<td>Tailor (1)</td>
<td>N(9,0.5) to N(17,0.5)</td>
<td></td>
</tr>
<tr>
<td>Bakery (1)</td>
<td>N(9,0.5) to N(17,0.5)</td>
<td>Dish washer</td>
</tr>
<tr>
<td>Laundry (1)</td>
<td>N(9,0.5) to N(17,0.5)</td>
<td>Clothes washer, Dryer</td>
</tr>
</tbody>
</table>
Table B.10  
Power Rating of Main Electric Devices for Shops/Offices

<table>
<thead>
<tr>
<th>TYPE</th>
<th>BUSINESS</th>
<th>RATING</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freezer</td>
<td>Restaurant, Take–away, Grocery store,</td>
<td>N(0.8,0.04)</td>
</tr>
<tr>
<td></td>
<td>Butcher, 24–hr shop</td>
<td></td>
</tr>
<tr>
<td>Fridge</td>
<td>Restaurant, Coffee shop, Takeaway</td>
<td>N(0.47,0.04)</td>
</tr>
<tr>
<td></td>
<td>Grocery store, Fruit store</td>
<td>N(0.94,0.04)</td>
</tr>
<tr>
<td></td>
<td>Butcher, Bar, 24–7 shop</td>
<td>N(2.82,0.04)</td>
</tr>
<tr>
<td>Dish washer</td>
<td>Restaurant, Coffee shop, Takeaway, Bar</td>
<td>N(5.0,1)</td>
</tr>
<tr>
<td>TV</td>
<td>Restaurant, Bar</td>
<td>N(1.0,1)</td>
</tr>
<tr>
<td>PC–Printer</td>
<td>All</td>
<td>N(1.0,1)</td>
</tr>
<tr>
<td>AC</td>
<td>Restaurant, Bar</td>
<td>N(4.6,0.1)</td>
</tr>
<tr>
<td></td>
<td>Other shops</td>
<td>N(9.2,0.1)</td>
</tr>
<tr>
<td>Clothes washer</td>
<td>Laundry</td>
<td>N(10,0.1)</td>
</tr>
<tr>
<td>Dryer</td>
<td>Laundry</td>
<td>N(14,0.3)</td>
</tr>
</tbody>
</table>

(iii) Hospital Data

Hospital load modeling data was made based on assumptions of different kind of electric devices used. Similar to residential loads, Gaussian or Normal distributions are used to model their working hours, electric devices ratings and number. The main data of the electric devices inside a hospital are given in Table B.11. The floor area of the hospital is assumed to be around 1500 m². The hospital working hours is assumed to be from 7 AM to 17 PM. Central ducted AC and a part of the lighting is assumed to be operating 24 hours. A loss factor of 10% is applied for the AC operation for considering shops doors opening and closing. A large standby power of around 7 kW is representing the standby lighting, ventilation and standby power of electrical instrumentations. The lighting is calculated similar to (B.1) based on their floor with a fixed part (running for 24 hrs) and a temporary part running at. The cooking devices in are assumed to be working on Gas.
Table B.11  
Hospital Electric Devices Parameters

<table>
<thead>
<tr>
<th>TYPE AND NO.</th>
<th>POWER RATING PER DEVICE TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACs</td>
<td>N(22,2)</td>
</tr>
<tr>
<td>Clothes Washer</td>
<td>N(10,0.1)</td>
</tr>
<tr>
<td>Dryer</td>
<td>N(14,0.3)</td>
</tr>
<tr>
<td>Dish Washer</td>
<td>N(10,0.2)</td>
</tr>
<tr>
<td>Water Heater</td>
<td>N(14.4,1)</td>
</tr>
<tr>
<td>TVs</td>
<td>round(N(10,0.5)×A/1500)</td>
</tr>
<tr>
<td>PCs–electronic eq.</td>
<td>ceil(N(20,2) ×A/1500)</td>
</tr>
<tr>
<td>X–RAYs</td>
<td>N(7,0.1)</td>
</tr>
<tr>
<td>Fridges</td>
<td>N(4.7,5)</td>
</tr>
<tr>
<td>Freezers</td>
<td>N(1.6,5)</td>
</tr>
</tbody>
</table>

(iv) Indoor Thermal Modeling

The second order dynamic thermal model of a house was presented in [93]. This model takes into account the effect of AC output power, sun radiation, ambient temperature, door and window size, and floor area of the house. All ACs were modeled as closed loop temperature controlled devices attempting to keep the internal temperature around a set point. The ACs only run if the internal temperature rises above a threshold. This is shown in Fig. B.1 and calculated in (B.5). The parameters of the indoor thermal model for houses, shops and hospital are given in Table B.12.

\[
\begin{bmatrix}
\frac{dT_m}{dt} \\
\frac{dT_{int}}{dt}
\end{bmatrix} =
\begin{bmatrix}
-1 & 1 \\
-\frac{1}{r_{int}C_{m}} & -1 - \frac{1}{r_{int}C_{m}}
\end{bmatrix}
\begin{bmatrix}
T_m \\
T_{int}
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\frac{T_{amb}}{\phi_c} \\
\frac{T_{s}}{\phi_s}
\end{bmatrix}
\]  

(B.5)
Fig. B.1 Indoor thermal modeling and temperature closed loop control.

where

\( r_a \) = Outside ambient/Internal air thermal resistance [°C/kW]

\( r_{int} \) = Internal air/House slab thermal resistance [°C/kW]

\( C_{int} \) = House internal air thermal capacity [kWh/°C]

\( C_m \) = House slab thermal capacity [kWh/°C]

\( \varphi_c \) = air conditioning cooling [kW]

\( \varphi_s \) = solar radiation [kW/m²]

\( T_m \) = House slab temperature [°C]

\( T_{int} \) = Internal temperature [°C]

\( T_{amb} \) = Outside Ambient temperature [°C]

\( T_{set} \) = Air conditioner set point [°C]

\( A_w \) = Effective window area [m²]

\( LF \) = loss factor due to door opening and closing [%]
Table B.12
Indoor Thermal Modeling Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{m,i}$</td>
<td>(N(4,0.4) \cdot A_i/240)</td>
<td></td>
</tr>
<tr>
<td>$C_{\text{int},i}$</td>
<td>(N(1.2,0.12) \cdot A_i/240)</td>
<td></td>
</tr>
<tr>
<td>$R_{a,i}$</td>
<td>(N(29.4,2.9))</td>
<td></td>
</tr>
<tr>
<td>$R_{\text{int},i}$</td>
<td>(N(0.48,0.048))</td>
<td></td>
</tr>
</tbody>
</table>
| $A_{w,i}$ | $N(2.9,0.29)$ for Houses  
$N(5.8,0.29)$ for Shops  
$N(17.4,0.29)$ for Hospital |  |
| $LF_i$ | 0% for houses, 10% for shops and hospital |  |