Asset Pricing under Tax Rate Uncertainty

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Abstract

In two parts I regard the effects on asset pricing of uncertainty about the tax rate on dividends. In the first part I develop a model in the fashion of the consumption CAPM adding an uncertain dividend tax rate. To model cyclicality of the tax rate, uncertainty about the tax rate is solely driven by deviations of output growth from its long-term mean. When all tax payments are transferred back in a lump sum fashion, a countercyclical tax rate policy increases asset prices and decreases expected returns as well as the equity premium. The opposite is true for a procyclical tax policy. This holds for a certain range of the magnitude of the tax rate cyclicality. Beyond that range effects of tax rate cyclicality on asset pricing decrease again. When taxes are not transferred back, the stochastic discount factor fundamentally changes, and many effects from the case with full transfers do not hold anymore. One important implication is that, for reasonable risk aversion, a procyclical tax rate policy causes an increase in expected equity prices.

In the second part I use a real business cycle (RBC) model with habit formation and adjustment costs for capital to analyze the effects of a volatile tax rate on dividends. Tax rate volatility is purely exogenous. I find that the variability of the tax rate has weak effects on the volatilities of business cycle variables such as consumption growth and investment growth and on the volatilities of asset returns. The equity premium is increased only by a few basis points, when an unreasonably high volatility of tax rate shocks is applied. In turn, the effects on volatilities of business cycle variables and asset returns are very strong in the standard RBC model, i.e., the model without habits and adjustment costs. Effects not related to tax rate volatility are substantial. Introducing a certain tax rate increases the (pre-tax) equity premium, so that after taxes the same is earned as in a world without taxes. Additionally, not paying back taxes as transfer payments greatly increases the equity premium in the habit model with adjustment costs, but there is no effect on the equity premium in the standard RBC model.

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List of Abbreviations

AR	Autoregressive
CAPM	Capital Asset Pricing Model
FRED	Federal Reserve Economic Data
GDP	Gross Domestic Product
lhs	left-hand side
MA	Moving Average
RBC	Real Business Cycle
rhs	right-hand side
SDF	Stochastic Discount Factor

Statement of Original Authorship

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

QUT Verified Signature

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1 Introduction

1.1 Overview and Summary

I use this chapter to introduce two research papers that analyze how uncertainty about the tax rate on dividends affects asset pricing. I use the notions risk and uncertainty interchangeably. That means agents attach certain probabilities to realizations of tax rates. For example Sialm (2006) shows that in the U.S. tax rates on dividends and capital gains changed considerably since their introduction. I make the basic assumption that agents care about those corporate distributions that they can actually consume, i.e., the after-tax dividends. Thus, agents must also care about the tax rate and its stochastic properties. I focus on tax effects on prices and on expected returns on equity and on risk-free bonds, as well as on the respective excess return, i.e., the equity premium.

I present two papers in the following two chapters. Apart from their main contents they both include a literature review and an appendix. The notation differs marginally in some instances, in which I regarded those differences as necessary. For the interested reader, I put the programming code in the general appendix of this thesis. I disregard all other forms of taxes to isolate the effects of the tax rate on dividends.

Chapter 2 presents the first paper. It analyzes a certain form of tax rate uncertainty. The tax rate process is such that it responds to output growth. This way I model procyclical, countercyclical or acyclical tax policies. Two basic cases, which I distinguish, are that taxes are fully transferred back to the investor and that there are no transfer payments at all. I use a consumption CAPM (CCAPM) framework based on the model presented in Sialm (2006). For the first model the implications are that a more countercyclical tax policy increases asset prices and decreases expected returns on equity and the equity premium. A procyclical tax policy does the opposite. Bond returns are not affected. Intuitively, those effects on equity prices and expected returns must be limited. Since the tax rate is bounded from below at zero and from above at one, responses of the tax rate to growth are limited as well. Including those bounds shows limits to price increases and expected return decreases. However, without transfer payments implications fundamentally change. Now, a procyclical tax policy is also able to increase prices. Coming back to the model with full transfers, I address some further issues. I change the logarithm of output growth to a moving average process with a lag of one, i.e., I introduce a simple autocorrelation structure into the output process. I also introduce a time lag of one period for the determination of the new tax rate. Both changes do not alter the direction of the price or return effects of tax rate cyclicality, but they change the magnitude of the effects. I also discuss the limitations of the research paper within a scope that I deem reasonable.

The second paper is included in Chapter 3. It extends several important papers on asset pricing and taxes, namely Jermann (1998), Santoro and Wei (2011), and Sialm (2006), with an uncertain tax rate on dividends. The model features a production economy with habit formation and adjustment costs. I extend the certain tax rate on dividends from Santoro and Wei (2011) to an uncertain tax rate. I also introduce the possibility of funding a public good as in Sialm (2006), so that not all of the taxes are paid back as transfer payments. I find that tax rate uncertainty regularly has an effect on asset returns, but those effects are very small. With the more common parameterization of high habits and high adjustments costs the risk premium is about three basis points higher for an unreasonably high volatility of tax rate shocks of 1.6% on a quarterly basis. For a more realistic volatility the premium is not economically significant at all. Impacts on volatilities of consumption and investment growth, as well as on asset returns are very strong without habits and adjustment costs. However, those effects also become very small with high habits and high adjustment costs. Tax rate uncertainty generates only small additional volatility of the stochastic discount factor so that the increase in the equity premium is small as well. Conversely, the capitalization of taxes into prices and returns has a big impact on the (pre-tax) equity premium. The introduction of a certain tax rate of 32% in the mentioned high habit and high adjustment cost model increases the pre-tax premium by a bit less than 30 basis points, so that after-tax the equity premium is equal to the premium in a world without taxes. Furthermore, not paying back taxes as transfer payments has an economically significant effect in the high habit and high adjustment cost model. Decreasing transfers from 100% to zero can increase the risk premium up to 40 basis points. When there is no risk premium as in the model without habits and without adjustment costs, the risk premium does not respond at all to changes in the share of transfer payments, i.e., it remains zero.

In the conclusion I discuss the limitations of my analysis on a more general level. I also discuss what kind of topics need to be addressed to further uncover the effects of tax rate uncertainty on an economy.

1.2 Context and Motivation of the Thesis

Chapters 2 and 3 contain specific literature reviews in their respective introductions. Here I point out the motivation and the context of both of the papers. The two papers find common ground in the work of Sialm (2006), who analyzes effects of tax rate uncertainty on asset prices using a model with power utility and an exogenous output process. His model taxes dividends and accrual based capital gains at the same tax rate. He models tax rate persistence using a two-state Markov process. He finds that the equity premium is increased with tax rate volatility and that this effect is stronger for higher risk aversion. However, at reasonable coefficients of risk aversion the premium explained by the tax rate volatility is small. Thus, tax rate uncertainty offers at least a partial explanation of the risk premium.

Sialm (2006), as well as Bizer and Judd (1989), argue that the kind of tax rate uncertainty they model is due to shifting powers of interest groups, who, in turn, influence the authorities responsible for setting the tax rate. The idea of interest groups in the process of setting the tax rate is important in its own right, and an early contribution is Becker (1983). However, other forms

that drive tax rate uncertainty can be identified. Vegh and Vuletin (2015) analyze cyclicality of different kinds of tax rates with respect to output for several different developed and developing countries. Thus, uncertainty of tax rates are often at least partly driven by the output process. This finding motivated me to include this observation in a simple model of asset pricing.
(Sialm, 2006, p. 532) points out that his model does not feature real investments, which he deems a viable extension. I take up this idea. However, the power utility model without adjustment costs does not produce any sizeable risk premium (compare Jermann (1998)). For this reason, I use a model based on Jermann (1998) with habit formation and adjustment cost, which both have to be present to generate a significant risk premium.

2 Asset Pricing under Tax Rate Cyclicality

2.1 Introduction

Sialm (2006) analyzes the implications of tax rate uncertainty on asset prices in a simple exchange economy without real investment opportunities. He motivates his work with the frequently changing tax rates on dividends as well as on long-term and short-term capital gains in the U.S. since the beginning of the 20th century. Using a very similar model, I introduce a special form of tax rate uncertainty. In Sialm (2006) tax rate volatility is said to be the product of the varying power of interest groups. Here I analyze tax rate uncertainty that is related to aggregate output, which, in turn, contains uncertainty as well and which moves in business cycles. Looking at tax rates this way is motivated by the results of Vegh and Vuletin (2015), who find that tax rates are set relative to phases of the business cycle and that for different countries different correlations of tax rates with output can be observed. This is referred to as cyclical tax policy.

The fundamental model used herein is based on the consumption CAPM with lognormal consumption growth[1]. The tax base is output, which is paid out as a perishable consumption good. In the absence of real investments and wages, output can also be interpreted as dividends. In the basic model all taxes are transferred back to the representative agent. In a variation of the model none of the taxes are transferred back.

For the model with tax transfers, I find that for reasonable amounts of risk aversion a countercyclical tax policy leads to increased asset prices through decreased discount rates versus the case of an acyclical tax policy. A procyclical tax policy leads to opposite effects in that it decreases prices and increases discount rates. Since bond rates are not influenced by tax rate cyclicality in the model with transfers, increased or decreased discount rates also mean increased or decreased equity risk premiums. The assumption of tax transfers is crucial here. When taxes are not transferred back, a procyclical tax policy tends to increase asset prices. There are mixed results for a countercyclical tax policy. Not transferring back taxes has strong effects on the representative agent's consumption, and therefore, on its discount factor so that relations can fundamentally change versus the case with transfer payments.

I outline possible implications of a time lag for the setting of the tax rate and autocorrelation of output growth using a moving average process of lag one (MA(1)) for the model with transfers. Autocorrelated growth increases or decreases the effect of tax rate cyclicality on asset prices,however, it does not reverse effects. More precisely, if increasing cyclicality decreases asset prices, then, for the MA(1), increasing tax rate cyclicality decreases prices at a higher or lower rate, but does not increase prices. Introducing a lag of one increases or decreases the effect of cyclicality by a greater magnitude, but again does not reverse the effect.

[1] Apart from its use in Sialm (2006), kinds of this model are described in Campbell and Viceira (2003, pp. 39-40) and Cochrane (2005, pp. 10-12).

The model is intentionally set up to allow for analytical solutions. They are bought using various assumptions, mainly the log-normality of output growth. Since the model allows to make inferences in an environment of low complexity, it can be used as a benchmark for more complex models, such as dynamic general equilibrium models that require numerical solutions. It shows at a basic level, how tax policy affects asset pricing and in which direction an unexpected change of the tax policy would sent prices and expected returns. My focus is clearly on asset pricing implications, and the chosen model is useful for this purpose. I do not intend to analyze policy implications in terms of what it should do and I do not look at social welfare.

I contribute to the literature that analyzes the effects of taxes, tax rate uncertainty, and tax rate cyclicality on asset pricing. McGrattan and Prescott (2005) observe large movements of asset prices relative to GDP in the U.S. and the U.K. and found changes in the tax rate on corporate distributions to be their main driver. Sialm (2009) confirms those effects for U.S. data. The effect of tax rate uncertainty is analyzed in Sialm (2006). In his model he taxes dividends and capital gains at a flat uncertain tax rate. He finds that tax rate uncertainty influences asset prices and may increase the equity premium. Moldovan (2010) and Moldovan (2006) look at the effects of a countercyclical tax policy for income taxes in a growth model with monopolistic competition. The studies find countercyclical tax policy to decrease aggregate volatility, whereas this effect is stronger with monopoly power. There is also a growing literature stream that is concerned with fiscal uncertainty in general. A recent work on that is Fernández-Villaverde et al. (2015), who find fiscal volatility shocks to have negative effects on the economy.

Several studies examine the actual presence of tax rate cyclicality. Furceri and Karrast (2011) look at correlations of average effective tax rates on the cyclical components of real GDP. They use total tax rates and the tax rates of different income types of 26 OECD countries from 1965 to 2003. They find that the correlation between those tax rates and the cyclical component is very small and statistically insignificant from zero. Vegh and Vuletin (2015) analyze 62 countries from 1960 to 2013. As measures of tax rates they mainly use the highest marginal income tax rates, corporate tax rates and the value added taxes. They find that tax policy is acyclical in most industrial countries and procyclical in most developing countries. However, the variation is quite large. For example, 14 industrialized countries show a negative (procyclical tax policy) and 6 a positive correlation (countercyclical tax policy) of tax rate changes with the percentage change of GDP. For developing countries 28 have a negative correlation and 10 a positive. They also find that tax and spending policies mostly go hand in hand, i.e., a procyclical tax policy comes along with procyclical spending and a countercyclical tax policy with countercyclical spending.

I will proceed as follows. Section 2.2 presents the assumptions of the model and derives basic pricing equations. Section 2.3 presents and discusses the tax rate process. In the following two sections the model is first analyzed with tax transfers and then without transfers. Section 2.6 provides numerical examples for the two different cases. Section 2.7 discusses possible limitations of the model and Section 2.8 provides a conclusion.

2.2 The Model

2.2.1 Assumptions

Output. As in Sialm (2006) there is a single asset that produces the perishable, risky output Y_t. The output process is exogenously given. Agents hold equity securities, which are claims on that output. That means the equity security pays dividends in the form of the output so that aggregate dividends are equal to output:

$$D_t^S = Y_t. \tag{2.1}$$

I use the term dividends here for corporate distributions as in Sialm (2006). It should be clear that in the absence of other forms of distributions such as wages, those dividends have a more general meaning. I denote output growth by G_t. The natural logarithm (log) of output growth $\ln(G_t) = g_t$ follows the process

$$g_t = \mu^g + \epsilon_t,\ \epsilon_t \sim i.i.d.\, N(0, \sigma^2), \tag{2.2}$$

where μ^g is the unconditional expectation of the log growth rate. This simple random walk with drift is mainly used herein, but I also present an extension to an MA(1) process below. The log of output growth for one period is given as

$$g_{t+1} = \ln(Y_{t+1}) - \ln(Y_t) = \ln(D_{t+1}^S) - \ln(D_t^S),$$

and

$$g_{t,t+i} = g_{t+1} + ... + g_{t+i} = \ln(Y_{t+i}) - \ln(Y_t) = \ln(D_{t+i}^S) - \ln(D_t^S),$$

for i periods. The growth rate $\exp(g_{t+1})$ is a gross rate, i.e., something like 1.05 rather than 5%. The expected dividend as of time t in terms of the growth rate is

$$E_t\left[D_{t+i}^S\right] = D_t^S E_t\left[\exp\left(\sum_{s=1}^{i} g_{t+s}\right)\right] = D_t^S \exp\left(i\mu^g + 0.5i\sigma^2\right) \tag{2.3}$$

$$= D_t^S \exp\left(\mu^g + 0.5\sigma^2\right)^i. \tag{2.4}$$

I also use $E(G) = \exp(\mu^g + 0.5\sigma^2)$ as a shorthand notation for single period expected growth. I leave out time subscripts when they are not necessary.

Financial assets. Households can issue and purchase risk-free bonds with maturity M. The bonds are in zero aggregate net supply. Bonds pay a dividend of one: $D_t^{B,M} = 1$, with maturity $M \in 1, 2, ...$. I assume that those assets are tradable so that prices exist. They are denoted by

p_t^S for the equity security and p_t^B for the bonds. There are no transaction costs or borrowing or short-selling constraints. Ex-dividend prices and dividends can be summarized as column vectors $D_t = (D_t^S\ 1)'$ and $p_t = (p_t^S\ p_t^B)'$. The agent's share in the equity asset x_t^S and holdings of bonds x_t^B are summarized as $x_t = (x_t^S\ x_t^B)'$.

Taxes. I assume that there is no government consumption and that taxes in any period are just transferred from one group to another. The tax base are dividends from the equity asset D_t^S. I abstain from taxing capital gains as Sialm (2006) and concentrate on dividends.[2] Dividends are taxed with a flat but uncertain tax rate[3] τ_t, which depends on output/dividend growth. I explain the tax rate processes used later in detail.

The agent's problem. Consumption in t of the representative agent is

$$C_t = D_t^{\tau\prime} x_{t-1} + p_t'(x_{t-1} - x_t) + Q_t, \tag{2.5}$$

whereas $D_t^{\tau\prime} = (D_t^S(1-\tau_t)\ 1)'$ and Q_t are transfer payments back to the agent. In the aggregate the whole taxes are rebated to the representative investor. Individually, the agent cannot influence the tax rebates. If all investors recovered their taxes exactly as they were paid, there would be no difference to the no-tax case.

The agent chooses his portfolios x_t to maximize expected utility over consumption, where I use power utility with γ being the coefficient of relative risk aversion:

$$\max_{x_{t+i}} \sum_{i=0}^{\infty} E_t \left[\beta^i \frac{C_t^{1-\gamma} + 1}{1-\gamma} \right]. \tag{2.6}$$

I assume the time preference parameter β to be constant over time. I also assume a risk averse representative agent so that $\gamma > 0$. Given his initial endowment and his preferences the agent chooses an after-tax consumption stream $C_t, C_{t+1}, ...$ that maximizes expected utility via choosing how many of the financial assets to hold. The first order conditions lead to the usual Euler equations.

2.2.2 General Pricing Equations

From the first order conditions I derive the pricing equation for the equity security:

$$p_t^S = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} ((1-\tau_{t+1}) D_{t+1}^S + p_{t+1}^S) \right]. \tag{2.7}$$

[2] Sialm (2006) taxes capital gains in an accrual based fashion. Most capital gains taxes are actually realization based, which causes substantial challenges for any analysis. For further reference see for example Viard (2000) and Klein (2001).

[3] I use the terms uncertainty and risk interchangeably, i.e., agents can quantify outcomes of states and attach probabilities to them.

The M-period pricing kernel or stochastic discount factor m_{t+M} is $m_{t+M} = \beta^M \left(\frac{C_{t+M}}{C_t}\right)^{-\gamma}$. The price of a bond with maturity M is the expectation of the stochastic discount factor

$$p_t^{B,M} = E_t[m_{t+M}] = E_t\left[\beta^M \left(\frac{C_{t+M}}{C_t}\right)^{-\gamma}\right]. \tag{2.8}$$

I define gross one-period after-tax returns on equity as

$$R_{t+1}^{S,\tau} = \frac{D_{t+1}^S(1-\tau_{t+1}) + p_{t+1}^S}{p_t^S}, \tag{2.9}$$

and the pre-tax gross return is

$$R_{t+1}^S = \frac{D_{t+1}^S + p_{t+1}^S}{p_t^S}. \tag{2.10}$$

From equation (2.7) it becomes clear that the after-tax gross return has a price of one, whereas this is not generally true for the pre-tax gross return.

The gross return on a bond with $M = 1$ is

$$R_t^{B,1} = \frac{1}{p_t^{B,1}}. \tag{2.11}$$

Since this is a risk-free bond and the return is known at time t, I keep the time subscript at t. Since the equity premium uses pre-tax returns I will focus on the latter return for the analysis of the equity premium. I define the equity premium as the single-period unconditional expected excess pre-tax returns of the equity asset over the bond return:

$$E[R^E] = E[R^S] - E[R^{B,1}]. \tag{2.12}$$

Transfers cannot be influenced per assumption so that they do not play a role in valuation. In the aggregate, consumption is equal to the dividends. Due to monotonicity of preferences the agent is always better off to consume a bit more, making it optimal to consume the whole produce, so that $D_t^S = C_t$. The pricing equations can be restated as

$$p_t^S = E_t\left[\sum_{i=1}^{\infty} \beta^i \left(\frac{C_{t+i}}{C_t}\right)^{-\gamma} (1-\tau_{t+i}) D_{t+i}^S\right] = E_t\left[\sum_{i=1}^{\infty} \beta^i D_t^S \left(\frac{D_{t+i}^S}{D_t^S}\right)^{1-\gamma} (1-\tau_{t+i})\right], \tag{2.13}$$

$$p_t^{B,M} = E_t\left[\beta^M \left(\frac{C_{t+M}}{C_t}\right)^{-\gamma}\right] = E_t\left[\beta^M \left(\frac{D_{t+M}^S}{D_t^S}\right)^{-\gamma}\right]. \tag{2.14}$$

For an expression in terms of growth rates I define $\exp(-\delta) = \beta$, and I obtain

$$p_t^S = E_t\left[\sum_{i=1}^{\infty} D_t^S \exp(-\delta i)\left(\exp\left(\sum_{s=1}^{i} g_{t+s}\right)\right)^{1-\gamma}(1-\tau_{t+i})\right], \tag{2.15}$$

$$p_t^{B,M} = E_t\left[\exp(-\delta M)\left(\exp\left(\sum_{s=1}^{M} g_{t+s}\right)\right)^{-\gamma}\right]. \tag{2.16}$$

As in Sialm (2006) I will frequently use the price dividend ratio as a convenient way to denote equity prices per unit of pre-tax dividend paid:

$$\Psi_t^S = \frac{p_t^S}{D_t^S}. \tag{2.17}$$

Furthermore, I define $p_t^{S,n}$ as the equity price in a world without taxes. In the same way I will use Ψ^S, $E[R^{S,n}]$, and $R^{B,1,n}$ as the price dividend ratio, the expected single-period return on equity and the single-period return on the bond, respectively, in a no-tax world. Those figures show up as a constant in the different pricing equations. They are derived in terms of more fundamental values such as μ^g and σ^2 in Appendix 2.9.2.

2.3 The Tax Rate Process

To focus on the tax rate's dependence on changes in output growth I use a term that accounts for tax rate cyclicality, but I do not include other random variables. For mathematical convenience I determine the process of $1-\tau_t$ instead of τ_t as

$$1-\tau_t = \exp(\mu^{\tau^*} - \phi(g_t - \mu^g) - \phi^2 0.5\sigma^2), \tag{2.18}$$

so that the process for the tax rate is

$$\tau_t = 1 - \exp(\mu^{\tau^*} - \phi(g_t - \mu^g) - \phi^2 0.5\sigma^2). \tag{2.19}$$

That means that $1-\tau_t$ is bounded from below at zero, i.e., that τ_t is bounded from above by one. To minimize any effects of negative τ_t, which is possible in this model, the volatility of the growth rate reflected by the tax term has to be low. An alternative would be to bound τ from below as well using min or max functions. I abstain from that to reduce complexity but discuss this case and its quantitative implications in Section 2.7. The term ϕ determines the strength and the direction of the tax rate cyclicality, i.e. the tax rate reaction on changes in the log of output growth. Notice that the deviations of the log growth rate of output from the long-term mean is ϵ_t. Therefore, the equation can be restated as

$$\tau_t = 1 - \exp(\mu^{\tau^*} - \phi^2 0.5\sigma^2)\left(\frac{1}{\exp(\epsilon_t)}\right)^{\phi}. \tag{2.20}$$

Since I do not want to have the tax rate cyclicality to have an effect on the expected tax term I introduce the term $-\phi^2 0.5\sigma^2$.
It is well known that a constant tax rate will automatically generate countercyclical tax revenues. When the tax rate is a certain percentage of a cyclical tax base, then tax revenues must be lower in the lower phase of the cycle and higher in the higher phase. Vegh and Vuletin (2015) mention that even tax rates that are lower in good states and higher in bad states do not trigger procyclical tax revenues (i.e., lower tax revenues in good states than in bad states), as long as the effect of the higher or lower tax base is greater than the one of the tax rate.
Considering those observations, I follow Vegh and Vuletin (2015) with the following definition of the cyclicality of tax policy:

Definition 2.1. A countercyclical tax policy is a policy in which the tax rate and the cyclical component of output covary positively. A countercyclical procyclical tax policy is a policy in which the tax rate and the cyclical component of output covary negatively.

In the case at hand the shock ϵ_t determines variability of growth and is therefore the cyclical element. I define cyclicality of the tax rate accordingly:

Definition 2.2. A countercyclical tax rate covaries positively with the cyclical component of output. A procyclical tax rate covaries negatively with the cyclical component of output.

Thus, as in Vegh and Vuletin (2015), I use the terms counter- and procyclical with respect to what the government intends to do, since the government sets the tax rate. Its influence on output is much more limited[4] so that I do not look at tax revenues, which is the product of the tax rate and the tax base (output) and might have a different cyclicality.
For $\phi > 0$, a countercyclical tax policy, the tax rate is increased when log output is above its long-term trend, i.e., when $\epsilon_t > 0$ ($\epsilon_t < 0$). Conversely, the tax rate is decreased when the log of output growth is below its long-term trend.
For $\phi < 0$, a procyclical tax policy, the tax rate is decreased when the log of output is above its long-term trend. The tax rate is increased when the log of output is below its long-term trend.
For $\phi = 0$ the tax policy is acyclical. The term μ^{τ^*} is the long-term mean of the log of the tax term $1 - \tau_t$ when $\phi = 0$. The agents make investment decisions at $t-1$ before they know the tax rate at t. This way they do not know the tax rate of the next period when they make their investment decisions. I discuss the implications of time lags between setting the tax rate and its coming into effect in Section 2.7.
The expectation of the tax rate is

$$\begin{aligned} E[\tau_t] &= 1 - E[\exp(\mu^{\tau^*} + \phi\epsilon_t - \phi^2 0.5\sigma^2)] \\ &= 1 - \exp(\mu^{\tau^*}). \end{aligned} \tag{2.21}$$

[4]In this model it is nonexistent because output is exogenous.

Conditional and unconditional expectations are equal here. It holds that $1 - E[\tau] = \exp(\mu^{\tau^*})$, which I will frequently use in the equations that are about to come.
I derive the covariance of the tax rate with the stochastic element of output growth $\exp(\epsilon_t)$ in Appendix 2.9.1. For your convenience I restate the resulting equation here:

$$Cov(\tau_t, \exp(\epsilon_t)) = (1 - E[\tau]) \exp(0.5\sigma^2)(1 - \exp(-\phi\sigma^2)). \tag{2.22}$$

The term $(1 - E[\tau]) \exp(0.5\sigma^2)$ is always positive so that the sign of the covariance depends on the second term $(1 - \exp(-\phi\sigma^2))$. This term is zero for $\phi = 0$, always positive for $\phi > 0$ and always negative for $\phi < 0$. Expectedly, the covariance reflects the desired cyclicality characteristics.

2.4 The Case with Full Tax Transfers

I regard the case when all of the tax payments are transferred back to the agents. That means, in the aggregate, all of the dividends are consumed by the representative agent: $C_t = D_t^S$.
Since consumption is the same as the one without taxes the stochastic discount factor is not affected and therefore bond prices are not affected by taxes. For this reason, I focus on equity. As introduced above, I denote price of untaxed equity by $p_t^{S,n}$. I use the price dividend ratio for the case of no taxes, $\Psi^{S,n}$, to make equations better comparable and to save space.
The full derivation of the following price dividend ratio can be found in Appendix 2.9.3. I present the result here:

$$\Psi^S = \Psi^{S,n}(1 - E[\tau]) \exp(\phi(\gamma - 1)\sigma^2). \tag{2.23}$$

Proposition 2.1. Under the assumptions put forward above and

a) with $1 < \gamma$, Ψ^S is increasing in ϕ,

b) with $1 > \gamma$, Ψ^S is decreasing in ϕ.

Proof. This follows from observation of Equation (2.23) and the fact that σ^2, as well as $\Psi^{S,n}(1 - E[\tau])$, must be positive. □

For $\gamma > 1$ and from Proposition 2.1 a) follows that there is always an increase in the price dividend ratio versus the one for an acyclical tax policy ($\phi = 0$) for positive ϕ, a countercyclical tax policy, and a decrease for $\phi < 0$, a procyclical tax policy. For $\gamma < 1$, i.e., for very small coefficients of risk aversion, which are implied by case b), a $\phi < 0$ increases prices and $\phi > 0$ decreases prices.
The results have the following economic interpretation. For a countercyclical tax policy tax rates are higher in good states and lower in bad states. Tax payments are higher in good states, i.e., the states in which cash flows are less valued by risk averse agents in terms of state prices. In good states, when agents already enjoy a high level of consumption, even more consumption

only adds little utility. Tax payments are lower in bad states, i.e., the states in which any cash flow and any unit of consumption is highly valued by risk averse agents. A tax relief is very welcome in this case because it increases consumption when marginal utility of consumption is high. A countercyclical tax policy insures agents to some extend against low consumption and it smoothens their consumption streams. The net effect is a lower value of tax payments today as compared to taxes with a flat and certain tax rate.

Still, for low risk aversion ($\gamma < 1$) a countercyclical tax policy can decrease the price and a procyclical tax policy can increase the price versus a constant tax. Intuitively, for an agent with low risk aversion a flat consumption stream is less important. To have a closer look at where this ambiguity comes from I look at the cash flow and the discount rate effect.

According to Appendix 2.9.3, conditional after-tax expected dividends are determined by

$$E_t[D^S_{t+1}(1-\tau_{t+1})] = D^S_t E[G](1-E[\tau])\exp(-\phi\sigma^2). \tag{2.24}$$

Proposition 2.2. Under the assumptions put forward above, conditional expected after-tax dividends $E_t[D^S_{t+1}(1-\tau_{t+1})]$ are decreasing in ϕ.

Proof. This follows from inspection of Equation (2.24) and the term $-\phi\sigma^2$, where $\sigma^2 > 0$. □

It follows that with positive ϕ, i.e., with a countercyclical tax policy, after-tax dividends are expected to be lower than with $\phi = 0$, an acyclical tax policy. Positive ϕ means higher tax rates in good states, i.e., states with high pre-tax dividend, and lower ones in bad states. The net effect on the expected value of after-tax dividends is negative. By construction, the cyclicality has no impact on the expectation of the tax term $E[1-\tau_t]$ and on dividends $E[D^S_t]$. But there is an effect on the after-tax dividend $E[D^S_t(1-\tau_t)]$ introduced by the covariance $Cov(D^S_t, 1-\tau_t)$. Since $E[D^S_t(1-\tau_t)] = Cov(D^S_t, 1-\tau_t) + E[D^S_t]E[1-\tau_t] = -Cov(D^S_t, \tau_t) + E[D^S_t]E[1-\tau_t]$, and we know from Equation (2.22) that the covariance $Cov(D^S_t, \tau_t)$ is always positive for positive ϕ and negative for negative ϕ[5], $E[D^S_t(1-\tau_t)]$ is decreased for positive ϕ and increased for negative ϕ versus $E[D^S_t]E[1-\tau_t]$. Agents know about the tax policy and therefore they know that with a countercyclical tax policy, tax rates are, on average, high when dividends are high and low when dividends are low. Therefore, expected after-tax dividends are not just expected dividends multiplied by the expected tax rate, since this would ignore the interaction of the tax rate and dividends.

I turn to expected return or discount rate effects, which reflect risk aversion of agents. The expected return on the equity asset, i.e., on a claim on after-tax dividends, is

$$E[R^{S,\tau}] = \exp(-\gamma\phi\sigma^2)(E[R^S] - E[G]) + E[G], \tag{2.25}$$

[5] The fact that Equation (2.22) does not use dividends but only the random part of dividend growth does not change the sign of the covariance but merely scales it by some factor.

where time subscripts are dropped. The derivation can be found in Appendix 2.9.3.

Proposition 2.3. *For the assumptions put forward above, single-period after-tax expected returns $E[R^\tau]$ are decreasing in ϕ.*

Proof. The proposition follows from equation (2.25). For a positive and finite price of untaxed equity I need $E[R^S] - E[G] > 0$ (see Appendix 2.9.2 for convergence conditions). Taking the derivative of Equation (2.25) with respect to ϕ yields $-\gamma\sigma^2 \exp(-\gamma\phi\sigma^2)(E[R^S] - E[G])$. This expression is always negative under the assumptions made. That means that the expected return strictly monotonously decreases with increasing ϕ or increases with decreasing ϕ. □

Thus, the expected return is increased for $\phi < 0$, a procyclical tax policy, and decreased for $\phi > 0$, a countercyclical tax policy, versus the case with $\phi = 0$. It turns out that the discount rates and the cash flows have opposite effects on the price. With a different tax policy the after-tax dividends have different risk characteristics. For example, when the policy is countercyclical the risk of low consumption in a bad state is lowered so that the agent requires a lower return. Notice that expected after-tax dividends are also expected to be lower with a countercyclical tax policy. With that I come back to the case of a low coefficient of risk aversion. For $\gamma < 1$ the cash flow effect is stronger and for $\gamma > 1$ the discount rate effect is stronger. For log utility ($\gamma = 1$) tax rate cyclicality does not have an effect on the price and the price dividend ratio, since discount rate and cash flow effect exactly cancel each other out.
Eventually, I will have a look at the equity premium. Remember that what is usually referred to as the equity premium is a pre-tax premium. The equation derived in Appendix 2.9.3 is

$$E[R^E] = (1 - E[\tau])^{-1} \exp(\phi\sigma^2(1-\gamma))(E[R^S] - E[G]) + E[G] - R^{B,1}. \tag{2.26}$$

Proposition 2.4. *Under the assumptions put forward above, the equity premium is increasing in ϕ if*

a) $1 > \gamma$.

The equity premium is decreasing in ϕ if

b) $1 < \gamma$.

Proof. This follows from observation of Equation (2.26) and the fact that $\sigma^2, (1 - E[\tau]))$ as well as $E[R^S] - E[G]$ must be positive. The bond rate is not affected by taxes at all. □

Notice that this is a slightly different behavior than the one of the expected return. This is due to the fact that the pre-tax return is used for the equity premium, but the discount rate explicitly accounts for taxes. Equation (2.26) shows that even with $\phi = 0$ the equity premium increases with an increasing expected tax rate. For reasonable risk aversion, i.e., for $\gamma > 1$ the equity premium decreases with ϕ. Thus, a more procyclical tax policy would contribute to explain an elevated equity premium.

With $\gamma > 1$ the propositions suggest that one can increase the equity value just by increasing ϕ ever more. However, the effects of tax rate cyclicality presented here are only valid in certain ranges for ϕ. I discuss those limits in Section 2.7. The assumption of full tax transfers is crucial here for the resulting effects. The next section will show that there are fundamental changes when this assumption is relaxed.

2.5 The Case without Tax Transfers

I regard the case that taxes are not transferred back to the representative agent, so that $Q_t = 0$. This assumption can be justified in several ways. Sialm (2006) uses a similar setting but also includes a public good that enters the representative agent's utility function in a separable way. The separability has the effect that there is no influence on pricing of the financial assets through the public good. In his numerical example he assumes that this good is funded with all of the tax revenues so that transfer payments are zero. Another justification may be that agents that hold financial assets do not receive transfers and agents that do not hold financial assets receive all of the transfer payments, which are nontradable. This way, transfer payments go out of the economy of agents that are involved in asset trading and in determining the asset's price.

An important effect compared to the prior model is that the stochastic discount factor is now influenced by taxes. For example the single-period SDF m_{t+1} in the prior section is $\beta(g_{t+1})^{-\gamma}$ and without tax transfers it changes to $\beta\left(g_{t+1}\frac{1-\tau_{t+1}}{1-\tau_t}\right)^{-\gamma}$. Expanding both SDFs, I obtain $\exp(-\delta)\exp(\mu^g + \epsilon_{t+1})^{-\gamma}$ for the case with transfers and $\exp(-\delta)\exp(\mu^g + \epsilon_{t+1} + \phi(\epsilon_t - \epsilon_{t+1}))^{-\gamma}$ for the case without transfers.[6] The additional term $\phi(\epsilon_t - \epsilon_{t+1})$ stems from the tax rate term, and it covaries with g_{t+1} through $\phi\epsilon_{t+1}$. For negative ϕ (procyclical tax policy) any variability of ϵ_{t+1} is scaled up. For positive ϕ the variability introduced through the growth shocks ϵ_{t+1} can be decreased ($0 < \phi < 1$) or cancelled (for $\phi = 1$), when growth shocks are exactly offset by tax rate movements. However, $-\phi\epsilon_{t+1}$ introduces variability in its own right so that an increase of ϕ by more than one increases variability again. Those effects are reflected in the risk-free rate. Notice also that a valuation at time t is now dependent on the current tax rate τ_t, because this rate is introduced into the SDF. This shows up through the error term ϵ_t. Different from the model with transfers a valuation considers wether the current tax rate is low or high.

Since the stochastic discount factor of the representative agent is now fundamentally changed, bond rates also change:

$$R_t^{B,1} = E_t[m_{t+1}] = R^{B,1,n}\exp(-\gamma^2(\phi^2 - 2\phi)0.5\sigma^2 + \gamma\phi\epsilon_t). \tag{2.27}$$

I refer to Appendix 2.9.4 for the derivation of the equation. The risk-free rate is not constant over time but depends on $\gamma\phi\epsilon_t$. For an interpretation of the term $\gamma\phi\epsilon_t$ I regard the case of a

[6] The constant terms μ^{τ^*} and $\phi^2 0.5\sigma^2$ in the tax terms cancel out.

countercyclical tax rate policy ($\phi > 0$) and a positive growth shock. In this case the current tax rate is increased above its expectation. Taxes reduce consumption by more than the average reduction through taxes. Agents expect to pay less taxes in the future which increases the expectation about consumption tendencially. Since consumption is expected to be higher, again, only from the tax effect, agents require a higher return on the bond to induce them to save and not to consume immediately.

The M-period bond rate is

$$R_t^{B,M} = R^{B,M,n} \exp(-\gamma^2(\phi^2 - 2\phi)0.5\sigma^2 + \gamma\phi\epsilon_t). \tag{2.28}$$

Equation (2.28) shows that the yield curve is not flat anymore. For example the yield for an M-year bond is $(R_t^{B,M})^{1/M} = \exp(\delta + \gamma\mu^g - \gamma^2 0.5\sigma^2)\exp(-\frac{1}{M}\gamma^2(\phi^2 - 2\phi)0.5\sigma^2 + \frac{1}{M}\gamma\phi\epsilon_t)$, which now depends on M through the second term.

To be not arbitrary about the error term at t and since I work towards the risk premium, which is usally referred to as an unconditional expectation, I take unconditional expectations of the single-period bond rate. Notice that $E[\exp(\gamma\phi\epsilon_t)] = \exp(\gamma^2\phi^2 0.5\sigma^2)$ is the convexity effect that is introduced through taking an unconditional expectation. This can be interpreted as an additional premium about uncertainty of the single-period bond rate. This uncertainty is introduced through the tax rate. An investor whose strategy is to invest in consecutive single-period bonds faces this risk. Equation (2.27) turns into

$$E[R^{B,1}] = R^{B,1,n} \exp(\phi\gamma^2\sigma^2). \tag{2.29}$$

Proposition 2.5. Under the assumptions put forward above, the unconditional expected single-period bond rate is increasing in ϕ.

Proof. This follows from inspection of Equation (2.29) and the fact that the gross rate $R^{B,1,n}$ as well as the parameter $\gamma^2\sigma^2$ are strictly positive. □

With a more countercyclical tax policy agents require, on average, a higher return on the single-period bond. The countercyclical tax policy mitigates the risk of a volatile consumption so that agents engage less in precautionary savings. This can be observed from Equation (2.29), in which the cyclicality term is attached to $\gamma^2\sigma^2$, which expresses aversion to volatility of consumption growth σ^2. Risk aversion combined with consumption growth volatility is normally interpreted as a trigger for precautionary savings (Cochrane, 2005, p.12). Here tax rate cyclicality additionally influences precautionary savings.

The derivation of the conditional price dividend ratio can be found in Appendix 2.9.4. I present the unconditional expectation, which factors in a premium for uncertainty about the state of the tax rate. The equation is

$$E[\Psi^S] = \Psi^{S,n}(1 - E[\tau]) \exp(\phi^2(\gamma^2 - \gamma)\sigma^2 - \phi(1-\gamma)^2\sigma^2). \tag{2.30}$$

The exponent is now quadratic in ϕ so that for a given γ the exponent changes the sign of its slope along ϕ.

Proposition 2.6. Under the assumptions put forward above, the unconditional expected price dividend ratio increases in ϕ if

a) $\gamma > 1$ and $\phi > 0.5(1 - 1/\gamma)$ or

b) $\gamma < 1$ and $\phi < 0.5(1 - 1/\gamma)$.

The price dividend ratio decreases in ϕ if

c) $\gamma > 1$ and $\phi < 0.5(1 - 1/\gamma)$ or

d) $\gamma < 1$ and $\phi > 0.5(1 - 1/\gamma)$.

Proof. Taking the first derivative of Equation (2.30) yields $2\phi(\gamma^2 - \gamma) - (1-\gamma)^2$ times a positive constant. A positive slope means $2\phi(\gamma^2 - \gamma) > (1-\gamma)^2$ or $2\phi\gamma(\gamma - 1) > (1-\gamma)^2$. If $\gamma > 1$, this leads to $\phi > 0.5(1 - 1/\gamma)$ (case a)), and for $\gamma < 1$, this leads to $\phi < 0.5(1 - 1/\gamma)$ (case b). A negative slope means $2\phi\gamma(\gamma - 1) < (1-\gamma)^2$. If $\gamma > 1$, this leads to $\phi < 0.5(1-1/\gamma)$ (case c)), and for $\gamma < 1$, this leads to $\phi > 0.5(1 - 1/\gamma)$ (case d). □

Also notice that the zeros of the exponent in Equation (2.30) are at $\phi = 0$ and $\phi = 1 - 1/\gamma$.[7] The slope is zero at $\phi = 0.5(1 - 1/\gamma)$, which is greater than zero for $\gamma > 1$ and less than zero for $\gamma < 1$.

It follows that for $\gamma > 1$ the expected price dividend ratio decreases in ϕ until $\phi = 0$, when it is equal to the expected price dividend ratio without tax rate cyclicality. It decreases further until $\phi = 0.5(1 - 1/\gamma) < 0$. After this point it increases again until $\phi = 1 - 1/\gamma$, where the expected price dividend ratio is again equal to the one without cyclicality, and increases beyond that. For $\gamma < 1$ the expected price dividend ratio is first less than the one without cyclicality. It increases until $\phi = 1 - 1/\gamma < 0$, increases further until $\phi = 0.5(1 - 1/\gamma - 1) < 0$, and then decreases until $\phi = 0$, where it is again equal to the ratio without cyclicality, and decreases beyond that point. Figure 1 in the next section shows an example of this behavior for differences of the ratio with respect to the one at $\phi = 0$ for γ equal to 0.5, 2, and 3 and a range for $\phi \in [-2, 2]$.

For an interpretation I regard the cases with $\gamma > 1$. I simplify the setting a bit and look at the valuation of only one dividend at $t+1$. The SDF at $t+1$ is

$$m_{t+1} = \beta \left(g_{t+1} \frac{1 - \tau_{t+1}}{1 - \tau_t} \right)^{-\gamma} = \exp(-\delta)\exp(-\gamma\mu^g - \gamma\epsilon_{t+1} + \gamma\phi\epsilon_{t+1} - \gamma\phi\epsilon_t).$$

[7] To see this just set $(\phi^2(\gamma^2 - \gamma)\sigma^2 - \phi(1-\gamma)^2\sigma^2)$ equal to zero, divide by ϕ, σ^2 and $\gamma - 1$ and rearrange.

The additional term, due to the fact that there are no transfers, is $\gamma\phi\epsilon_{t+1} - \gamma\phi\epsilon_t$ in the exponent. Now, the additional effect of a postive ϕ (countercyclical tax policy) is that it increases the SDF when shocks are positive, and it decreases the SDF when shocks are negative. Symmetric shocks, due to normality, and the exponential function lead to a higher increase of the SDF for a positive shock than a decrease for a negative shock of the same size. The effect on dividends is exactly in the oppositive way. For example the dividend in $t+1$, i.e., $D^S_{t+1}(1-\tau_{t+1}) = D^S_t \exp(\mu^g + \epsilon_{t+1} + \mu^{\tau^*} - \phi\epsilon_{t+1} - \phi^2 0.5\sigma^2)$, contains the term $-\phi\epsilon_{t+1}$, which leads to smaller after-tax dividends in good states and bigger ones in bad states. However, with $\gamma > 1$ the effect of the tax rate cyclicality on the SDF is scaled up as ϕ increases. The opposite effects due to dividends are not scaled up by γ. Thus, surpassing a certain threshold for ϕ the net effect is an increase in the price of the dividend. This is basically what happens in case a) of the above proposition. Agents marginal utility rapidly increases in good states for a more and more countercyclical tax rate. Therefore agents attach higher and higher values to cash flows in those states. Actually, the high tax rate in states with high pre-tax dividends and which are good states before taxes become bad states with high marginal utility after taxes.

For $\gamma > 1$ the price increases when the tax policy becomes more procyclical. For $\phi < 0$ symmetric shocks and the exponential function lead to a lower decrease of the SDF for a positive shock than an increase for a negative shock of the same size. The effect on the dividend is now in the same direction as for the SDF. Therefore, the increasing effect on the price is more immediate. The examples in Figure 1 show that for any $\phi < 0$ the expected price dividend ratio is higher than with $\phi = 0$. These are the basic mechanics of case c).

Since I have taken an unconditional expectation the risk premium $\exp(\gamma^2\phi^2 0.5\sigma^2)$ for the uncertainty about the conditional price dividend ratio is included in Equation (2.30). The effect of this premium even enforces the effects described before. The premium is increasing for increasing ϕ when $\phi > 0$ and it is increasing for decreasing ϕ when $\phi < 0$.

Notice that there is no change in the after-tax expected dividends compared to the case with full tax transfers. Therefore, I immediately turn to the single-period expected return. The conditional expected return on the equity asset is derived in Appendix 2.9.4. I again analyze its unconditional form and divide it into the expected dividend yield

$$E\left[\frac{D^S_{t+1}(1-\tau_{t+1})}{p^S_t}\right] = \exp((\gamma\phi^2 - 2\gamma\phi + \gamma^2\phi)\sigma^2)(E[R^{S,n}] - E[G]) \tag{2.31}$$

and the expected capital gain

$$E\left[\frac{p^S_{t+1}}{p^S_t}\right] = \exp((\gamma^2\phi^2 - \gamma\phi)\sigma^2)E[G]. \tag{2.32}$$

Proposition 2.7. Under the assumptions put forward above, the unconditional expected single-period after-tax dividend yield increases in ϕ if

a) $\phi > 1 - \gamma/2$,

and decreases in ϕ if

b) $\phi < 1 - \gamma/2$.

Proof. The first derivative of the dividend yield with respect to ϕ is $2\phi\gamma - 2\gamma + \gamma^2$ multiplied by a positive constant. Setting this term greater zero, dividing by the constant and by γ, which are always positive, and rearranging for ϕ leads to a). Setting the term less than zero leads to b). □

The exponent in Equation (2.31) has up to two zeros. Apart from $\phi = 0$ the expected dividend yield is also equal to the equivalent yield without tax rate cyclicality for $\phi = 2 - \gamma$.[8] The expected dividend yield $E[D_{t+1}^S(1 - \tau_{t+1})/p_t^S]$ is decreasing in ϕ until $\phi = 1 - \gamma/2$ and then increasing again. It intersects the abscissa two times, at $\phi = 0$ and at $\phi = 2 - \gamma$, except when $\gamma = 2$, when the function is tangent to the abscissa. An example can be observed in Figure 3a of the next section.
I continue with the expected single-period capital gain.

Proposition 2.8. Under the assumptions put forward above, the unconditional expected single-period capital gain increases in ϕ if

a) $\phi > 1/(2\gamma)$,

and decreases in ϕ if

b) $\phi < 1/(2\gamma)$.

Proof. The first derivative of the capital gain is $2\phi\gamma^2 - \gamma$ times a positive constant. Setting this term greater zero dividing by the positive constant and by positive γ, and rearranging for ϕ leads to a). Setting the term less than zero leads to b). □

Since the exponent in Equation (2.32) has again up to two zeros, which are at $\phi = 0$ and $\phi = 1/\gamma$. In ϕ-cappital gain space the function falls until $\phi = 1/(2\gamma)$ and then increases. It intersects the abscissa two times - at $\phi = 0$ and $\phi = 1/\gamma$. Only for infinite risk aversion the two points fall together. An example for the behavior of the expected capital gain for different parameters of risk aversion can be observed in Figure 3b of the next section.
Looking at both, the expected value of the dividend yield and of the capital gain, I can make some general statements about the expected return outside of certain bounds. For $\phi < \min(1 - \gamma/2, 1/(2\gamma))$ both functions decrease and therefore also the expected return is decreasing in ϕ, and for $\phi > \max(1 - \gamma/2, 1/(2\gamma))$ they are increasing. Looking at the zeros, for $\phi < \min(2 - \gamma, 0, 1/\gamma)$ and for $\phi > \max(2 - \gamma, 0, 1/\gamma)$ both functions and therefore also the expected return are above the abscissa so that the expected return with cyclicality is greater than without it. Notice that the components of the unconditional expected return follow the same U-shape pattern along ϕ as the expected price dividend ratio. For a discount rate one would expect more

[8] To see this, set $\gamma\phi^2 - 2\gamma\phi + \gamma^2\phi$ equal to zero divide by γ and rearrange for ϕ.

something like an inverse relation between the two. This is not happening here because we are not looking exactly at a discount rate but on an unconditional expectation of it. The premium introduced through taking this expectation is again $\exp(\gamma^2\phi^2 0.5\sigma^2)$, which quickly dominates the expected value for increasing ϕ. For the case with transfers the conditional and the unconditional means are the same. Thus, comparing unconditional expected returns of the cases with and without tax transfers leads to big differences. For example, unconditional expected after-tax returns for the no-transfers case can be higher with a countercyclical tax policy, i.e., with $\phi > 0$. For $\gamma = 2$, the function $\max(2-\gamma, 0, 1/\gamma) = \max(0, 0, 0.5) = 0.5$, so that for $\phi > 0.5$ the expected return is definitely higher than with zero cyclicality. It would be lower than without cyclicality in the case with tax transfers.

Eventually, I turn to the equity premium. It contains the pretax expected dividend yield, the expected captal gain and the expected bond return. Above, the after-tax dividend yield was analyzed so that the new variable here is the pretax dividend yield:

$$E\left[\frac{D^S_{t+1}}{p^S_t}\right] = (1-E[\tau])^{-1}\exp((\gamma\phi^2 + \phi(\gamma-1)^2)\sigma^2)(E[R^S] - E[G]) \tag{2.33}$$

This is the first term from Equation (2.64) for the equity premium.[9]

Proposition 2.9. Under the assumptions put forward above, the unconditional expected single-period pre-tax dividend yield increases in ϕ if

a) $\phi > -(1-\gamma)^2/(2\gamma)$,

and decreases in ϕ if

b) $\phi < -(1-\gamma)^2/(2\gamma)$.

Proof. The first derivative of the pre-tax dividend yield is $2\phi\gamma + (\gamma-1)^2$ times a positive constant. Setting this term greater zero, dividing by the positive constant, and rearranging for ϕ leads to a). Setting the term less than zero leads to b). □

The zeros of the exponent of Equation (2.33) are at $\phi = 0$ and $\phi = -(1-\gamma)^2/\gamma$. In ϕ-expected pre-tax dividend yield space this is again a hyperbola with a minimum (now at $\phi = -(1-\gamma)^2/(2\gamma)$) that cuts the abscissa up to two times.

The equity premium adds up the expectations of the pre-tax dividend yield, the capital gain, and subtracts the bond return:

$$E[R^E_{t+1}] = E\left[\frac{D^S_{t+1}}{p^S_t}\right] + E\left[\frac{p^S_{t+1}}{p^S_t}\right] - E[R^{B,1}]. \tag{2.34}$$

Looking at the three components, the expectations of the pre-tax dividend yield, the capital gain, and the bond rate I can again make some statements about the equity premium. Notice

[9]The derivation of the equation can be found in Appendix 2.9.4.

that the bond rate is subtracted in this equation and the negative of the expected bond rate decreases in ϕ.

For $\phi < \min(1 - \gamma/2, -(1 - \gamma)^2/(2\gamma))$ all functions decrease and therefore also the risk premium is decreasing in ϕ. For $\phi < \min(-(1 - \gamma)^2/(\gamma), 0, 1/\gamma)$ all functions, and therefore also the equity premium, are above the abscissa so that the equity premium with a cyclical tax policy is greater than with an acyclical one. For ϕ greater than the respective max functions the expected bond return behaves differently than expected capital gain and dividend yield. However, asymptotically in ϕ the effect of the expected dividend yield and capital gain will outweigh the one of the bond rate due to the quadratic terms of ϕ in the expected dividend yield and capital gain and only a linear one in the expected bond rate.

Bottom line is that for the case without transfers the stochastic discount factor is fundamentally changed by the tax rate, so that some of the relations with respect to a cyclical tax policy for the case with transfers are turned into the opposite direction for the case without transfers. To get a better understanding of the behavior of asset valuation related figures and their magnitudes with respect to tax rate policy, I provide some numerical examples in the next section.

2.6 Numerical Examples

The following examples are provided to gauge magnitudes of tax effects on asset pricing using reasonable parameter values. It is an exercise to learn more about basic asset pricing effects of tax rate cyclicality under the set of assumptions made herein. I will discuss the scope of the models, i.e., the implications and its limitations, in the next section. I continue with the description of the parameter values.

Since the proposed models are very basic aggregate pre-tax dividends, consumption and output are the same in the model with transfers. Without transfers aggregate after-tax divideds, consumption, and after-tax output are the same. Table 1 shows that real annual growth rates for U.S. data from 1929 to 2013 are around 1.8% and 1.9% for all of the three time series. The standard deviation shows more variation among the three variables with, for example, 2.3% for consumption growth and 10.8% for dividend growth. Logarithms of growth rates have a mean as low as 1.2% for dividend growth and as high as 1.8% for output growth. Standard deviations range from 2.3% for consumption growth to 10.9% for dividend growth. I will use 0.018 for the parameter μ^g and 0.046 for σ. As mentioned in Section 2.3, I need low volatility of the growth rate to keep the probability of negative tax rates close to zero. Therefore I choose a relatively low value for σ. I use 0.95 for the time discount rate β.

Table 2 shows the mean of the average marginal tax rate of dividends and the share of tax revenues on aggregate output. A mean farther away from zero also reduces the probability that the tax rate is less than zero. Therefore I choose -0.449 for μ^{τ^*}. That means for $\phi = 0$ and $\epsilon_t = 0$ $\tau_t = 1 - \exp(-0.449) = 1 - 0.638 = 0.362$ or 36.2%. Since everything is multiplicative in this model a higher μ^{τ^*} also leads to stronger reactions of the tax term through shocks of the

Table 1: Mean and standard deviation of growth rates

	$G_t^D - 1$	g_t^D	$G_t^C - 1$	g_t^C	$G_t^Y - 1$	g_t^Y
Mean	0.018	0.012	0.018	0.017	0.019	0.018
Std. Dev.	0.108	0.109	0.023	0.023	0.046	0.046

Source: See Appendix 2.9.5

growth rate. For example for $\phi = -1$ a shock of around 1% would multiply $\exp(0.01) = 1.01$ to $\exp(-0.449) = 0.638$ so that the tax term is decreased by around 0.64%[10]. I show the behavior of asset valuation related variables with respect to ϕ. I choose $\phi \in [-2, 2]$ as the interval for the cyclicality parameter. I always show the differences with respect to an acyclical tax policy. For example Figure 1 shows the percentage differences of (expected) prices for the respective range of ϕ and different parameters for risk aversion. Percentage differences for prices are $(p_t^S(\phi = x) - p_t^S(\phi = 0))/p_t^S(\phi = 0)$. Curves above the abscissa mean that the price is by a certain percentage higher than it would be if the tax policy were acyclical, i.e., the price for $\phi = 0$. Notice that the percentage differences for prices and for price dividend ratios are the same since the dividends would cancel out in computing the percentage difference.

Table 2: Mean of tax rates

	τ^D	$1 - \tau^D$	$\ln(1 - \tau^D)$	τ^Y	$1 - \tau^Y$	$\ln(1 - \tau^Y)$
Mean	0.356	0.644	-0.449	0.151	0.849	-0.165

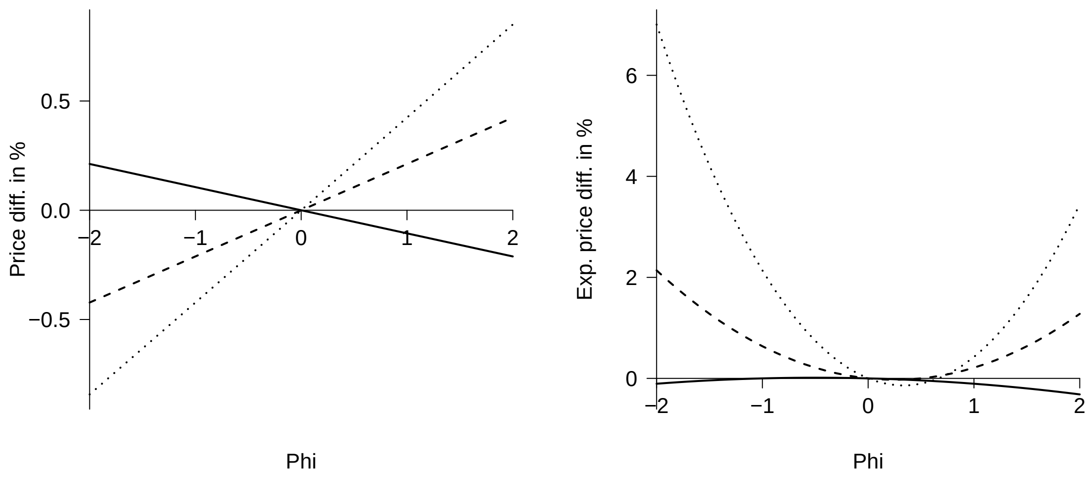
Source: See Appendix 2.9.5

Equation (2.23) shows that the percentage difference of prices must be equal to $\exp(\phi(\gamma - 1)\sigma^2) - 1$ for the case with transfers. Thus price differences increase with risk aversion and a higher ϕ or a higher σ^2 would increase this effect. For the case without transfers and according to Equation (2.30), the percentage differences of prices are, on average, $\exp(\phi^2(\gamma^2 - \gamma)\sigma^2 - \phi(1 - \gamma)^2\sigma^2) - 1$. Panel (a) and (b) in Figure 1 reflect the linear function in ϕ in the exponent for the case with transfers and the quadratic one for the case without transfers. The quadratic funtion leads to very pronounced differences in the expected prices.

According to Equation (2.24), for expected dividends percentage differences from the base case are $\exp(-\phi\sigma^2) - 1$. That means for the dividend parameters that with $\phi = 1$ expected dividends are 4.5% less and with $\phi = -1$ they are 4.71% greater than in the base case.

Figure 2 shows the deviations of expected returns from the base case expected after-tax return. With transfers, the expected returns are increased for a procyclical tax policy and decreased for

[10]The term $\exp(-\phi^2 0.5\sigma^2)$ is very close to one in this case so that its influence can be neglected.



(a) With tax transfers

(b) Without tax transfers

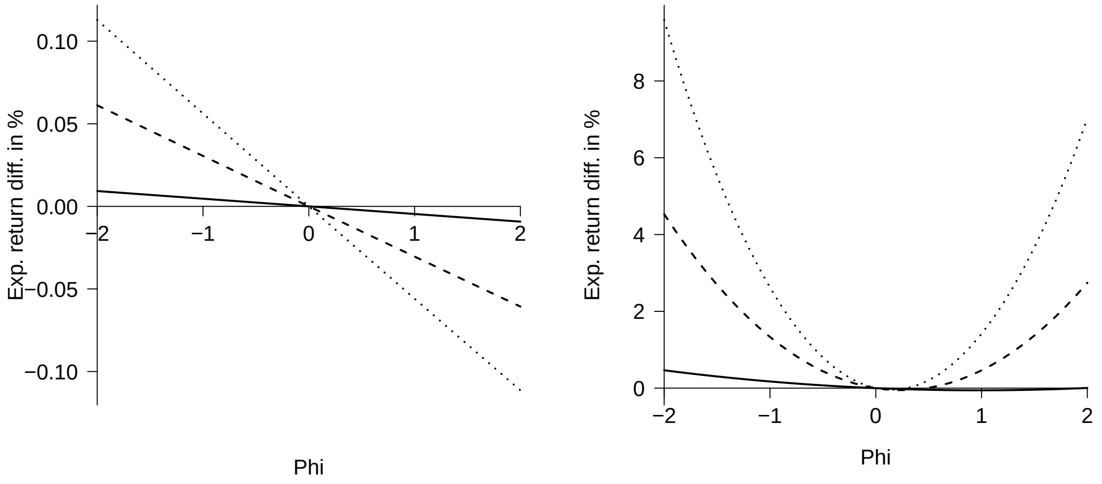
Figure 1: Price deviations from base case price in %

The full line corresponds to $\gamma = 0.5$, the dashed line to $\gamma = 2$, and the dotted line to $\gamma = 3$. The standard parameterization is used.

a countercyclical one. The percentage differences, here computed as $E[R^{S,\tau}(\phi = x)] - E[R^{S,\tau}(\phi = 0)]$, are modest compared to the case without transfers. Notice that for the case without transfers expected price and return differences show the same pattern and not the opposite one. This is because we are not looking at prices and the discount rate but on their unconditional expectations, which are different from the conditional expectation in this case due to the $\gamma\phi\epsilon_t$ term showing up with a negative sign in the price dividend and a positive sign in the conditional expected return equations. Taking the unconditional mean this term shows up as $\phi^2\gamma^2 0.5\sigma^2$ with a positive sign in the exponent. This leads to the unconditional moments sloping up.

Figure 3 shows the two components of the after-tax expected return for the case without transfers for $\gamma = 3$. The graph shows pure differences of the expected dividend yield and capital gain from the acyclical case as for the expected returns. Both functions are hyperbola with a minimum but minima and zeros are not generally the same. The dividend yield shows much smaller deviations from the acyclical dividend yield as the capital gain from its acyclical values.

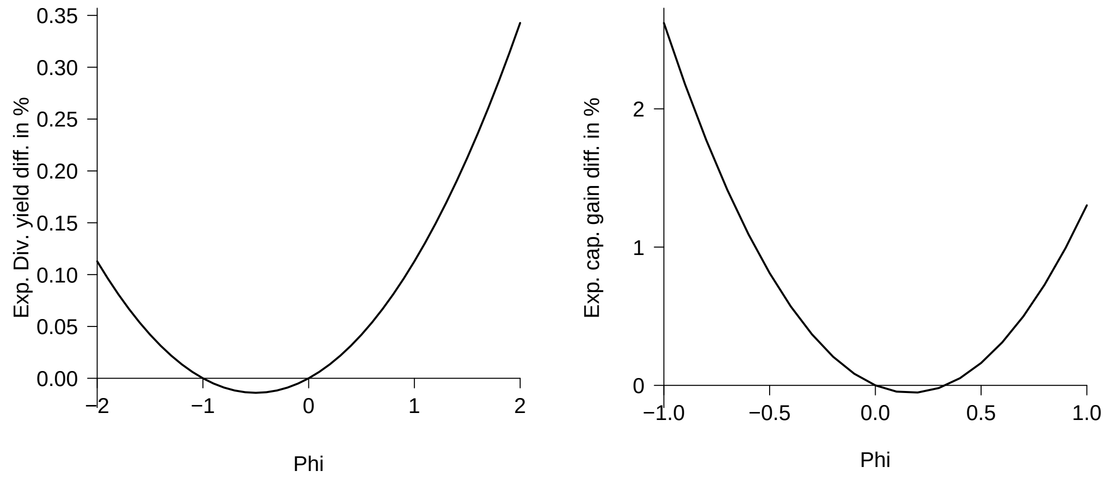
Figure 4 shows the equity premium for both cases. With transfers the equity premium differences very much resemble the graph for the expected return in direction and scale of the functions. However, since the equity premium uses pre-tax values there are slight changes and for $\gamma < 1$, $\gamma = 0.5$ is the example in the figure, the graph is upward sloping. Panel (b) shows the graphs for the case without transfers. Compared to the expected after-tax returns in Figure 2b, for a procyclical tax policy, i.e., for negative ϕ, the equity premiums are very high, and they are low for a countercyclical tax policy. The expected single period bond return is smaller for negative ϕ than for $\phi = 0$ and higher for positive ϕ, which contributes to the shape of the curves compared to after-tax returns.



(a) With tax transfers

(b) Without tax transfers

Figure 2: Deviations of expected returns from the expeted return of the base case
The full line corresponds to $\phi = 0$ (acyclical tax policy), the dashed line to $\phi = 1$ (countercyclical tax policy), and the dotted line to $\phi = -1$ (procyclical tax policy).

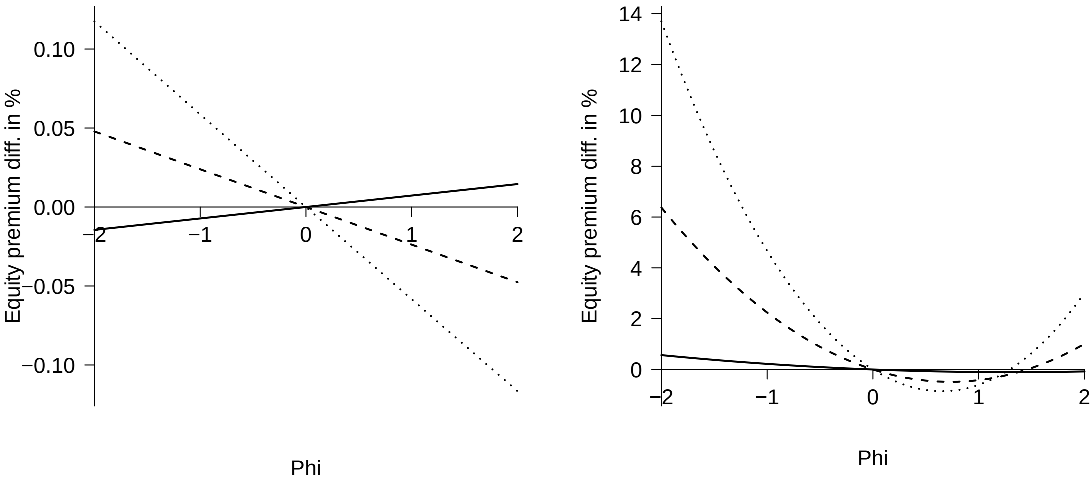


(a) Expected dividend yield

(b) Expected capital gains

Figure 3: Components of the expected return for the case without transfers
The examples use the basic parameterization and $\gamma = 3$. Panel b) uses a closer range for ϕ to make the minimum below the abscissa more visible.

Figure 5 shows conditional bond yields for the case without transfers for the standard parameters $\mu^g = 0.018, \sigma = 0.046$, and for $\epsilon_t = 0$. The additional tax term in equation (2.28) causes an upward sloping yield curve for a procyclical tax policy and a downward sloping curve for a countercyclical tax policy. For the case with transfers bond yields are flat and do not change versus the ones without taxes.



(a) With transfers

(b) Without transfers

Figure 4: Equity risk premium

The examples use the basic parameterization and $\gamma = 3$. Panel b) uses a closer range for ϕ to make the minimum below the abscissa more visible.

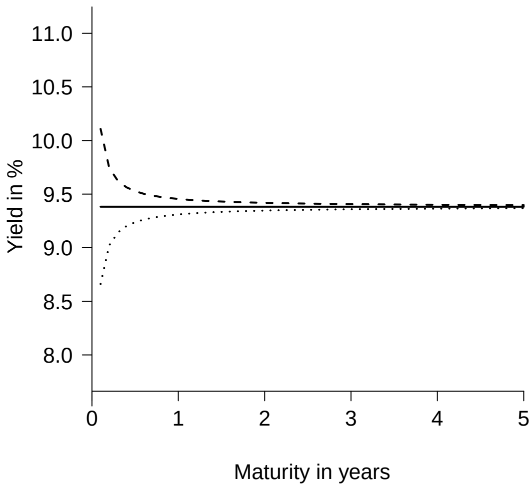


Figure 5: Bond yields for the case of taxes without transfers

The full line corresponds to $\phi = 0$ (acyclical tax policy), the dashed line to $\phi = 1$ (countercyclical tax policy), and the dotted line to $\phi = -1$ (procyclical tax policy). The standard parameterization is used with $\gamma = 2.5$.

2.7 Discussion of the Limitations and Possible Extensions of the Analysis

I come back to the model with transfers and discuss some further issues. Proposition 2.1 suggests that increasing ϕ, i.e., having a more cyclical tax policy, can increase the price ever more. However, the tax rate process, or more specifically, the process $1 - \tau_t$ is only bounded from below at zero but not from above. A remedy would be to define

$$1 - \tau_t = \min(\exp(\mu^{\tau^*} - \phi\epsilon_t - \phi^2 0.5\sigma^2), 1) \tag{2.35}$$

$$= \exp(\mu^{\tau^*} - \phi\epsilon_t - \phi^2 0.5\sigma^2) - \max(\exp(\mu^{\tau^*} - \phi\epsilon_t - \phi^2 0.5\sigma^2) - 1, 0). \tag{2.36}$$

The price dividend ratio becomes then

$$\Psi_t^S = \sum_{i=1}^{\infty} \beta^i E_t \left[\left(\frac{D_{t+i}^S}{D_t^S} \right)^{1-\gamma} (\exp(\mu^{\tau^*} - \phi\epsilon_{t+i} - \phi^2 0.5\sigma^2) - \max(\exp(\mu^{\tau^*} - \phi\epsilon_{t+i} - \phi^2 0.5\sigma^2) - 1, 0) \right], \tag{2.37}$$

which is basically the same equation as in Section 2.4, where a series of call option prices is subtracted. The equation can be written as

$$\Psi_t^S = \Psi^{S,n}(1 - E[\tau]) \exp(\phi(\gamma - 1)\sigma^2) - \sum_{i=1}^{\infty} \beta^i E_t \left[\left(\frac{D_{t+i}^S}{D_t^S} \right)^{1-\gamma} \max(\exp(\mu^{\tau^*} - \phi\epsilon_{t+i} - \phi^2 0.5\sigma^2) - 1, 0) \right], \tag{2.38}$$

where I use Equation (2.23). I only look at the option part and since growth innovations are i.i.d. this can be stated as

$$\begin{aligned}
&\sum_{i=1}^{\infty} \beta^i E_t[G_{t,t+i-1}^{1-\gamma}] E_t[G_{t+i}^{1-\gamma} \max(\exp(\mu^{\tau^*} - \phi\epsilon_{t+i} - \phi^2 0.5\sigma^2) - 1, 0)] \\
&= \beta E_t[\exp(\mu^g + \epsilon_{t+i})^{1-\gamma} \max(\exp(\mu^{\tau^*} - \phi\epsilon_{t+i} - \phi^2 0.5\sigma^2) - 1, 0)] \sum_{i=0}^{\infty} \beta^i E_t[G_{t,t+i}^{1-\gamma}] \\
&= \beta E_t[\exp(\mu^g + \epsilon_{t+i})^{1-\gamma} \max(\exp(\mu^{\tau^*} - \phi\epsilon_{t+i} - \phi^2 0.5\sigma^2) - 1, 0)] \frac{E[R^{S,n}]}{E[R^{S,n}] - E[G]}.
\end{aligned} \tag{2.39}$$

The second equality shifts the index of the sum and the third equality traces the equation back to an infinite geometric series that starts with power of zero. I simulate 100,000 $N(0, \sigma^2)$ distributed shocks ϵ for different γ and compute the above expression. I use the basic parameterization with $\mu^g = 0.018$ and $\sigma = 0.046$. Figure 6 shows the percentage price deviations versus the case when $\phi = 0$. The increase of the absolute value of ϕ increases the variability of the cyclicality term $\phi\epsilon_t$ in the tax term. The increased variability leads to a higher percentage of tax terms $1 - \tau_t$ greater one, i.e., a higher percentage of $\tau_t < 0$. This increases the option value that is deducted from the actual price dividend ratio. The grey lines show this effect for different γ. It becomes clear that the positive effects of a countercyclical tax policy only are effective in a certain range of ϕ. The countercyclical tax policy reduces the value of tax payments because higher taxes are paid in good states, which have a low value (low state prices), and it increases tax payments in bad states, which have a high value (high state prices). Increasing ϕ in absolute terms sends the tax rate in more and more states below zero, but this is countered by the option. Thus, a higher ϕ in absolute terms has less and less effect in the bad states and more effect in the good states. There are less value decreases and more values increases of tax payments, which contributes to increasing the overall value tax payments.

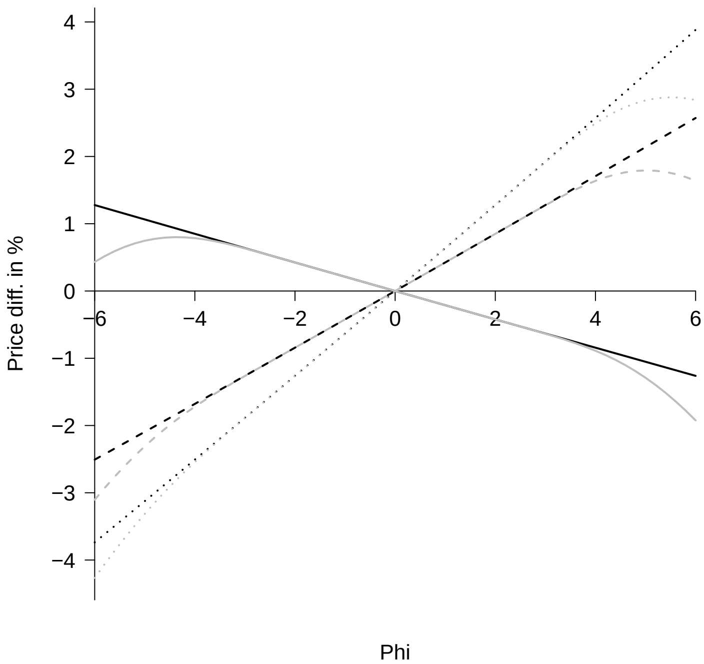


Figure 6: Price differences from base case price in %
Black lines correspond to the case without the option, grey lines show values computed accounting for the option. The full lines use $\gamma = 0$, the dashed lines $\gamma = 3$ and the dotted lines $\gamma = 4$. Furthermore, $\mu = 0.018$ and $\sigma = 0.108$.

The tax rate is set after the output growth is observed, i.e., immediately prior to the payout of the dividends. A reasonable variation is that the tax rate is set with a lag. In the model with uncorrelated i.i.d. shocks this will not have great repercussions on asset pricing except that, for example for a lag of one, the tax rate in the next period is known today. With autocorrelated growth g_t this becomes more involved. For simplicity I regard the pricing of a single dividend that arrives at $t+i$. Log growth is an invertible MA(1) process of the form

$$g_t = \mu^g + \epsilon_t + \theta\epsilon_{t-1},\ \epsilon_t \sim i.i.d.\, N(0, \sigma^2), \tag{2.40}$$

with $|\theta| < 1$. The MA(1) process has the advantage that beyond lag one there is no autocorrelation, which simplifies the math. The process of the tax term is

$$1 - \tau_t = \exp(\mu^{\tau^*} - \phi(\epsilon_{t-l} + \theta\epsilon_{t-1-l}) - \phi^2(1+\theta^2)0.5\sigma^2), \tag{2.41}$$

where l is the time lag for setting the tax rate and the term $-\phi^2(1+\theta^2)0.5\sigma^2$ takes effects of ϕ out of the expected value of the tax term. I show the pricing implication of this process as an example and to get some intuition about consequences of autocorrelated growth, but I do not claim that its implications are generally valid for any kind of autocorrelated process. For the price of a dividend that arrives at $t+i$ I obtain[11]

$$
\begin{aligned}
p_t^S(D_{t+i}^\tau) &= D_t^S \beta^i E_t\left[\left(\frac{D_{t+i}^S}{D_t^S}\right)^{1-\gamma}(1-\tau_{t+i})\right] \\
&= D_t^S E_t\Big[\exp(-\delta i)\exp(i\mu_g+\epsilon_{t+1}+\theta\epsilon_t+\epsilon_{t+2}+\theta\epsilon_{t+1}...+\epsilon_{t+i-1}+\theta\epsilon_{t+i-2}+\epsilon_{t+i}+\theta\epsilon_{t+i-1})^{1-\gamma} \\
&\quad \exp(\mu^{\tau^*}-\phi\epsilon_{t+i-l}-\phi\theta\epsilon_{t+i-1-l}+\phi^2(1+\theta^2)0.5\sigma^2)\Big].
\end{aligned}
\tag{2.42}
$$

For a zero lag the shocks at times $t+i-1$ and $t+i$ are important for tax effects. This leads to

$$
\begin{aligned}
p_t^S(D_{t+i}^\tau) &= D_t^S \beta^i E_t\left[\left(\frac{D_{t+i}^S}{D_t^S}\right)^{1-\gamma}(1-\tau_{t+i})\right] \\
&= D_t^S E_t\Big[\exp(-\delta i)\exp((1-\gamma)(i\mu_g+\theta\epsilon_t)+(1-\gamma)(1+\theta)(\epsilon_{t+1}+...+\epsilon_{t+i-2}) \\
&\quad +((1-\gamma)(1+\theta)-\phi\theta)\epsilon_{t+i-1}+((1-\gamma)-\phi)\epsilon_{t+i}+\mu^{\tau^*}-\phi^2(1+\theta^2)0.5\sigma^2)\Big] \\
&= D_t^S E_t\Big[\exp(-\delta i)\exp((1-\gamma)(i\mu_g+\theta\epsilon_t)+(1-\gamma)^2(1+\theta)^2(i-2)0.5\sigma^2 \\
&\quad +((1-\gamma)(1+\theta)-\phi\theta)^2 0.5\sigma^2+((1-\gamma)-\phi)^2 0.5\sigma^2+\mu^{\tau^*}-\phi^2(1+\theta^2)0.5\sigma^2)\Big] \\
&= D_t^S E_t\Big[\exp(-\delta i)\exp((1-\gamma)(i\mu_g+\theta\epsilon_t)+(1-\gamma)^2(1+\theta)^2(i-1)0.5\sigma^2 \\
&\quad +(1-\gamma)^2 0.5\sigma^2+\phi(\gamma-1)(\theta^2+\theta+1)\sigma^2+\mu^{\tau^*})\Big].
\end{aligned}
\tag{2.43}
$$

I focus on the term involving ϕ, which is $\phi(\gamma-1)(\theta^2+\theta+1)\sigma^2$. The term $\theta^2+\theta+1$ is greater one for $0<\theta<1$ and less than one for $-1<\theta<0$. Thus autocorrelation can increase or decrease the effect of tax rate cyclicality in this case but it does not reverse it, i.e., the sign of the effect of ϕ is not changed. For a lag of one the shocks at times $t+i-2$ and $t+i-1$ are important for tax effects. The pricing equation changes to

$$
\begin{aligned}
p_t^S(D_{t+i}^\tau) =& D_t^S E_t\Big[\exp(-\delta i)\exp((1-\gamma)(i\mu_g+\theta\epsilon_t)+(1-\gamma)(1+\theta)(\epsilon_{t+1}+...+\epsilon_{t+i-3})+(1-\gamma)\epsilon_{t+i} \\
&\quad +((1-\gamma)(1+\theta)-\phi\theta)\epsilon_{t+i-2}+((1-\gamma)(1+\theta)-\phi)\epsilon_{t+i-1}+\mu^{\tau^*}-\phi^2(1+\theta^2)0.5\sigma^2)\Big] \\
=& D_t^S E_t\Big[\exp(-\delta i)\exp((1-\gamma)(i\mu_g+\theta\epsilon_t)+(1-\gamma)^2(1+\theta)^2(i-1)0.5\sigma^2 \\
&\quad +(1-\gamma)^2 0.5\sigma^2+\phi(\gamma-1)(1+\theta)^2\sigma^2+\mu^{\tau^*})\Big].
\end{aligned}
\tag{2.44}
$$

[11] I use the price as an operator here to make clear that only one dividend is valued.

The term involving ϕ is now $\phi(\gamma-1)(1+\theta)^2\sigma^2$. Thus, the only difference to the equation with lag zero is that here we have $(1+\theta)^2 = \theta^2+2\theta+1$, which is one θ more than above where the term is $\theta^2+\theta+1$. Thus, θ between minus one and zero would decrease the effect on $\phi(\gamma-1)\sigma^2$ a bit and for positive θ it would increase the effect versus the lag zero case.

Notice that the equations for lag zero and one are almost identical. However, to price equity one has to add up all cash flows and as was mentioned above, the tax rate at $t+1$ would be certain for lag one so that this effect has to considered as well.

Eventually, for the MA(1) the cyclicality effect of the tax rate is not reversed for any of the lags but it can be increased or decreased for $0 < \theta < 1$ (positively correlated g_t) and $-1 < \theta < 0$ (negatively correlated g_t, respectively.

Further questions of the effects of tax rate cyclicality may involve how matters change with real investment opportunities, a suggestion also made by Sialm (2006) for his model with an uncertain tax rate, and non-zero government deficits. Real business cycle models including those features may be a tool to answer those questions. The current model is most compatible with a linear production technology, full depreciation, and a constant share of investment in output. In this case consumption and investments are both constant shares of output so that all the three variables grow at the same rates. I will give a short outline on this case. Consider a representative firm with a linear production technology that produces output according to

$$Y_t = K_t \exp(\mu^K + \epsilon_t), \tag{2.45}$$

where K_t is the capital stock, $\exp(\mu+\epsilon_t)$ the marginal rate of return of capital, with μ^K as the mean of its natural logarithm and ϵ_t a shock as defined before. A represenative firm determines the evolution of the capital stock to be

$$K_{t+1} = K_t \exp(\mu^g + \epsilon_t), \tag{2.46}$$

and investments $I_t = K_{t+1}$, which implies that K_t fully depreciates. The representative firm finances investments only through retained earnings. The equation also implies that the representative firm chooses investments to be a constant share of output: $I_t/Y_t = K_t\exp(\mu^g + \epsilon_t)/(K_t\exp(\mu^K+\epsilon_t)) = \exp(\mu^g-\mu^K)$. Since consumption $C_t = Y_t - I_t$, consumption is also a constant share of output. Dividends are again the same as consumption, since they are the remaining cash flow after investment is done. Using Equations (2.45) and (2.46) the log of output growth is now $\ln(Y_{t+1}) - \ln(Y_t) = \ln(K_t\exp(\mu^g+\epsilon_t)\exp(\mu^K+\epsilon_{t+1})) - \ln(K_t\exp(\mu^K+\epsilon_t)) = \mu^g + \epsilon_{t+1}$, which is the same is Equation (2.2). The consumers problem is the same so that its solution leads back to the same pricing equations.

2.8 Conclusion

Building on the model in Sialm (2006) I analyzed tax rate cyclicality of taxes on output and its effects on asset pricing in a model where output and dividends are the same. The basic model features full tax transfers, whereas in a variation the taxes are not redistributed to capital market participants or invested in a public good that enters utility in a separable fashion. For the model with tax transfers reasonable amounts of risk aversion and a countercyclical tax policy lead to increased asset prices through decreased discount rates versus the case of an acyclical tax policy. A procyclical tax policy has the opposite effect. Prices are decreased, and discount rates are increased. Bond rates are not influenced by tax rate cyclicality in the model with transfers. For reasonable risk aversion the equity risk premium is decreased for a countercyclical tax policy and increased for a procyclical one. All of those effects are limited to certain ranges of the cyclicality parameter, since the tax rate is bounded at zero and one so that the tax rate cannot scaled up or down without limit. Fundamental is the assumption of tax transfers. Without transfers the stochastic discount factor is fundamentally changed, which has various effects on values and rates. For example for the case without transfers both, a procyclical and a countercyclical tax policy, can increase asset prices. Additionally, I sketched some of the possible extensions of the basic model with tax transfers. I showed the effects of a time lag for the setting of the tax rate and autocorrelation of output growth using an MA(1) process. The impact of autocorrelated growth is an increase or decrease of the effect of tax rate cyclicality on asset prices however it does not reverse effects. That is if rising cyclicality decreases asset prices then tax rate cyclicality with the MA(1) for output growth decreases prices at a higher or lower rate, but does not increase prices. Introducing a lag of one in the MA(1) model increases or decreases the effect of cyclicality by a greater magnitude but again does not reverse the effect of the tax policy versus the effect without a lag and without the MA(1).

The scope for possible extensions is large. Different tax bases, the introduction of real investment, government debt, and heterogeneous agents with respect to transfer payments may be fruitful ways to follow.

2.9 Appendix to Chapter 2

2.9.1 Covariance of the Tax Rate with the Stochastic Part of Output

The covariance between the tax rate and the stochastic part of output growth can be derived as follows:

$$\begin{aligned}
Cov(\tau_t, \exp(\epsilon_t)) &= Cov\left(1 - \exp(\mu^{\tau^*} - \phi^2 0.5\sigma^2)\left(\frac{1}{\exp(\epsilon_t)}\right)^{\phi}, \exp(\epsilon_t)\right) \\
&= -\exp(\mu^{\tau^*} - \phi^2 0.5\sigma^2) Cov(\exp(-\phi\epsilon_t), \exp(\epsilon_t)) \\
&= -\exp(\mu^{\tau^*} - \phi^2 0.5\sigma^2)\left(E[\exp((1-\phi)\epsilon_t)] - E[\exp(-\phi\epsilon_t)]E[\exp(\epsilon_t)]\right) \\
&= -\exp(\mu^{\tau^*} - \phi^2 0.5\sigma^2)(\exp((1 - 2\phi + \phi^2)0.5\sigma^2) - \exp((1+\phi^2)0.5\sigma^2)) \\
&= -\exp(\mu^{\tau^*} + 0.5\sigma^2)(\exp(-\phi\sigma^2) - 1) \\
&= \exp(\mu^{\tau^*} + 0.5\sigma^2)(1 - \exp(-\phi\sigma^2)) \qquad (2.47) \\
&= (1 - E[\tau])\exp(0.5\sigma^2)(1 - \exp(-\phi\sigma^2)) \qquad (2.48)
\end{aligned}$$

2.9.2 Pricing without Taxes

The n in the superscript stands for a world without taxes. Using the relation from equation (2.15) with zero tax rates I obtain

$$\begin{aligned}
p_t^{S,n} &= E_t\left[\sum_{i=1}^{\infty} D_t^S \exp(-\delta i)\left(\exp\left(\sum_{s=1}^{i} g_{t+s}\right)\right)^{1-\gamma}\right] \\
&= D_t^S \sum_{i=1}^{\infty} \exp\Big(-\delta i + (1-\gamma)i\mu^g + (1-\gamma)^2 0.5 i\sigma^2\Big) \qquad (2.49)
\end{aligned}$$

for equity. I put all the terms related to risk aversion and time discounting into the denominator. In a second step I use the fact that this is an infinite geometric series:

$$\begin{aligned}
p_t^{S,n} &= D_t^S \sum_{i=1}^{\infty} \frac{\exp(\mu^g + 0.5\sigma^2)^i}{\exp(\delta + \gamma\mu^g + (2\gamma - \gamma^2)0.5\sigma^2)^i} \\
&= D_t^S \frac{\exp(\mu^g + 0.5\sigma^2)}{\exp(\delta + \gamma\mu^g + (2\gamma - \gamma^2)0.5\sigma^2) - \exp(\mu^g + 0.5\sigma^2)}. \qquad (2.50)
\end{aligned}$$

The single-period discount rate or expected return is i.i.d. so that I leave out time subscripts:

$$E(R^{S,n}) = \exp(\delta + \gamma\mu^g + (2\gamma - \gamma^2)0.5\sigma^2). \qquad (2.51)$$

The price dividend ratio is

$$\Psi^{S,n} = \frac{p_t^{S,n}}{D_t^S} = \frac{E[G]}{E[R^{S,n}] - E[G]}, \qquad (2.52)$$

which is time independent in this case.

The pricing equation for bonds is

$$p_t^{B,M,n} = E_t\left[\exp(-\delta M)\left(\exp\left(\sum_{s=1}^{M} g_{t+s}\right)\right)^{-\gamma}\right]$$
$$= \exp(-\delta M - \gamma M \mu^g + \gamma^2 0.5 M \sigma^2)$$
$$= \exp(\delta + \gamma\mu^g - \gamma^2 0.5\sigma^2)^{-M}. \tag{2.53}$$

The single-period risk-free rate is

$$R^{B,1,n} = \exp(\delta + \gamma\mu^g - \gamma^2 0.5\sigma^2). \tag{2.54}$$

Convergence. For equity prices to exist, i.e., to be finite, the sum in equation (2.49) must converge (transversality condition). That means the price of cash flows must go to zero as time extends into the future. I look at

$$\lim_{i\to\infty} E_t\left[\exp(-\delta i)\left(\exp\left(\sum_{s=1}^{i} g_{t+s}\right)\right)^{1-\gamma}\right] = \lim_{i\to\infty}\left(\frac{E[G]}{E[R^{S,n}]}\right)^i,$$

which converges to zero only if $E[G]/E[R^{S,n}] < 1$ or $E[G] < E[R^{S,n}]$.

2.9.3 Equations for the Case with Taxes and Transfers

The expected value of after-tax dividends arriving at $t+i$ from the viewpoint of time t is equal to

$$E_t[D_{t+i}(1-\tau_{t+i})] = D_t^S E_t[\exp(i\mu^g + \sum_{s=1}^{i}\epsilon_{t+s} + \mu^{\tau^*} - \phi\epsilon_{t+i} - \phi^2 0.5\sigma^2)]$$
$$= D_t^S \exp(i\mu^g + \mu^{\tau^*} + 0.5 i\sigma^2 - \phi\sigma^2)$$
$$= D_t^S E[G]^i (1 - E[\tau]) \exp(-\phi\sigma^2) \tag{2.55}$$

To derive this result I use the fact that $E_t[\exp(\epsilon_{t+i}(1-\phi) - \phi^2 0.5\sigma^2)] = \exp((1-\phi)^2 0.5\sigma^2 - \phi^2 0.5\sigma^2) = \exp((1-2\phi)0.5\sigma^2)$.

For the price dividend ratio I obtain:

$$
\begin{aligned}
\Psi_t^S &= \sum_{i=1}^{\infty} \beta^i E_t \left[\left(\frac{D_{t+i}^S}{D_t^S} \right)^{1-\gamma} (1-\tau_{t+i}) \right] \\
&= \sum_{i=1}^{\infty} E_t \left[\exp(-\delta i) \exp((1-\gamma)(i\mu_g + \sum_{s=1}^{i} \epsilon_{t+s}) + \mu^{\tau^*} - \phi \epsilon_{t+i} - \phi^2 0.5\sigma^2) \right] \\
&= \sum_{i=1}^{\infty} \exp(-\delta i + (1-\gamma) i\mu_g + 0.5(1-\gamma)^2 i\sigma^2 + \mu^{\tau^*} + (\gamma-1)\phi\sigma^2) \\
&= \sum_{i=1}^{\infty} \frac{\exp(i\mu^g + 0.5 i\sigma^2 + \mu^{\tau^*} - \phi\sigma^2)}{\exp(\delta + \gamma\mu^g + (2\gamma - \gamma^2)0.5\sigma^2)^i \exp(-\gamma\phi\sigma^2)} \\
&= (1 - E[\tau]) \frac{E[G]}{E[R^{S,n}] - E[G]} \frac{\exp(-\phi\sigma^2)}{\exp(-\gamma\phi\sigma^2)} \\
&= \Psi^{S,n} (1 - E[\tau]) \frac{\exp(-\phi\sigma^2)}{\exp(-\gamma\phi\sigma^2)}.
\end{aligned}
\tag{2.56}
$$

For the transition from the second to the third equality, notice that at $t+i$ there are the terms $(1-\gamma)\epsilon_{t+i}$ and $-\phi\epsilon_{t+i}$. That leads to the combined term $\epsilon_{t+i}(1-\gamma-\phi)$. Taking the variance I obtain $((1-\gamma)^2 - (1-\gamma)2\phi + \phi^2)0.5\sigma^2$ and I still subtract the term $\phi^2 0.5\sigma^2$. The fourth equality divides into expected taxes according to equation (2.55) and the discount rate. The fifth equality separates the no-tax equity price dividend ratio. The last equality shows the tax effects whereas I kept in the numerator the effects on cash flows and in the denominator the effects on the discount rate. This price dividend ratio is also independent from time so that the time subscript may be omitted.

To derive the single-period after-tax expected return $E_t[R_{t+1}^{S,\tau}]$ I define:

$$
B = (1 - E[\tau]) \frac{\exp(-\phi\sigma^2)}{\exp(-\gamma\phi\sigma^2)}.
$$

The expected return is then

$$
\begin{aligned}
E_t[R_{t+1}^{S,\tau}] &= E_t \left[\frac{D_{t+1}^S (1-\tau_{t+1}) + p_{t+1}^S}{p_t^S} \right] \\
&= D_t^S E_t \left[\frac{\exp(g_{t+1})(1-\tau_{t+1}) + \exp(g_{t+1}) \Psi^{S,n} B}{D_t^S \Psi^{S,n} B} \right] \\
&= E[G](1 - E[\tau]) \exp(-\phi\sigma^2)(\Psi^{S,n} B)^{-1} + E[G] \\
&= \exp(-\gamma\phi\sigma^2)(E[R^{S,n}] - E[G]) + E[G].
\end{aligned}
\tag{2.57}
$$

The equity premium is

$$
\begin{aligned}
E[R^E] &= E\left[\frac{D^S_{t+1}+p^S_{t+1}}{p^S_t}\right] - R^{B,1,n} \\
&= E\left[D^S_t \frac{\exp(g_{t+1})+\exp(g_{t+1})\Psi^{S,n}B}{D^S_t\Psi^{S,n}B}\right] - R^{B,1,n} \\
&= E[G](\Psi^{S,n}B)^{-1} + E[G] - R^{B,1,n} \\
&= \frac{\exp(-\gamma\phi\sigma^2)}{(1-E[\tau])\exp(-\phi\sigma^2)}(E[R^{S,n}]-E[G]) + E[G] - R^{B,1,n} \\
&= (1-E[\tau])^{-1}\exp(\phi\sigma^2(1-\gamma))(E[R^{S,n}]-E[G]) + E[G] - R^{B,1,n}. \qquad (2.58)
\end{aligned}
$$

2.9.4 Equations for the Case with Taxes and without Transfers

As in Sialm (2006) aggregate consumption changes to $C_t = D^S_t(1-\tau_t)$, which changes the stochastic discount factor. For the price dividend ratio I obtain:

$$
\begin{aligned}
\Psi^S_t &= (1-\tau_t)\sum_{i=1}^{\infty}\beta^i E_t\left[\left(\frac{D^S_{t+i}(1-\tau_{t+i})}{D^S_t(1-\tau_t)}\right)^{1-\gamma}\right] \\
&=(1-\tau_t)\sum_{i=1}^{\infty}E_t\left[\exp(-\delta i)\exp(i\mu_g+\sum_{s=1}^{i}\epsilon_{t+s}-\phi\epsilon_{t+i}+\phi\epsilon_t)^{1-\gamma}\right] \\
&= \exp(\mu^{\tau^*}-\gamma\phi\epsilon_t-\phi^2 0.5\sigma^2)\sum_{i=1}^{\infty}\exp(-\delta i+(1-\gamma)i\mu_g+(1-\gamma)^2(i-1)0.5\sigma^2+(1-\gamma)^2(1-\phi)^2 0.5\sigma^2) \\
&= \exp(\mu^{\tau^*}-\gamma\phi\epsilon_t-\phi^2 0.5\sigma^2)\sum_{i=1}^{\infty}\exp(-\delta i+(1-\gamma)i\mu_g+(1-\gamma)^2 i 0.5\sigma^2+(1-\gamma)^2(-2\phi+\phi^2)0.5\sigma^2) \\
&= \exp(\mu^{\tau^*}-\gamma\phi\epsilon_t-\phi^2 0.5\sigma^2)\sum_{i=1}^{\infty}\frac{\exp(\mu^g+0.5\sigma^2)^i\exp((1-\gamma)^2(\phi^2-2\phi)0.5\sigma^2)}{\exp(\delta+\gamma\mu^g+(2\gamma-\gamma^2)0.5\sigma^2)^i} \\
&= \exp(\mu^{\tau^*}-\gamma\phi\epsilon_t+(\gamma^2-2\gamma)(\phi^2-2\phi)0.5\sigma^2-\phi\sigma^2)\frac{E[G]}{E[R^{S,n}]-E[G]} \\
&= \Psi^{S,n}(1-E[\tau])\exp(-\gamma\phi\epsilon_t+(\gamma^2-2\gamma)(\phi^2-2\phi)0.5\sigma^2-\phi\sigma^2).
\end{aligned}
$$

(2.59)

In the second equality I use $(1-\tau_t) = \exp(\mu^{\tau^*}-\phi\epsilon_t-\phi^2 0.5\sigma^2)$. The constant terms in the tax terms $\phi^2 0.5\sigma^2$ and μ^{τ^*} cancel out. In the third equality I use the fact that $(1-\tau_t)\exp((1-\gamma)\phi\epsilon_t) = \exp(\mu^{\tau^*}-\gamma\phi\epsilon_t-\phi^2 0.5\sigma^2)$.

To derive the single-period expected return $E_t[R^{S,\tau}_{t+1}]$ I define

$$C = (1-E[\tau])\exp((\gamma^2-2\gamma)(\phi^2-2\phi)0.5\sigma^2-\phi\sigma^2),$$

which is the factor in the pricing equation, where I exclude the time dependent term

$\exp(-\gamma\phi\epsilon_t)$. The unconditional single-period expected return is then

$$
\begin{aligned}
E[E_t[R^{S,\tau}_{t+1}]] &= E\left[E_t\left[\frac{D^S_{t+1}(1-\tau_{t+1})}{p^S_t}\right] + E_t\left[\frac{p^S_{t+1}}{p^S_t}\right]\right]\\
&= E\left[E_t\left[\frac{\exp(g^S_{t+1})(1-\tau_{t+1})}{\exp(-\gamma\phi\epsilon_t)\Psi^{S,n}C}\right] + E_t\left[\frac{\exp(g^S_{t+1})\exp(-\gamma\phi\epsilon_{t+1})\Psi^{S,n}C}{\exp(-\gamma\phi\epsilon_t)\Psi^{S,n}C}\right]\right]\\
&= E\left[\exp(\gamma\phi\epsilon_t + (2\gamma-\gamma^2)(\phi^2-2\phi)0.5\sigma^2)(E[R^{S,n}]-E[G]) + \exp(\gamma\phi\epsilon_t + \mu^g + (1-\gamma\phi)^2 0.5\sigma^2)\right]\\
&= \exp((\gamma\phi^2 - 2\gamma\phi + \gamma^2\phi)\sigma^2)(E[R^{S,n}]-E[G]) + \exp((\gamma^2\phi^2-\gamma\phi)\sigma^2)E[G],
\end{aligned}
\tag{2.60}
$$

where the first term is the unconditional expected after-tax dividend yield and the second term the unconditional expected capital gain for one period.

Since the stochastic discount factor changes versus the no-tax world, bond prices changes as well:

$$
\begin{aligned}
p^{B,M}_t =& E_t\left[\exp(-\delta M)\left(\frac{D^S_{t+i}(1-\tau_{t+i})}{D^S_t(1-\tau_t)}\right)^{-\gamma}\right]\\
&= E_t\left[\exp(-\delta M)\exp(M\mu_g + \sum_{s=1}^{M}\epsilon_{t+s} - \phi\epsilon_{t+M} + \phi\epsilon_t)^{-\gamma}\right]\\
&= \exp(-\delta M)\exp(-\gamma M\mu_g + \gamma^2(M-1)0.5\sigma^2 + \gamma^2(1-\phi)^2 0.5\sigma^2 - \gamma\phi\epsilon_t)\\
&= \exp(-\delta M)\exp(-\gamma M\mu_g + \gamma^2 M 0.5\sigma^2 + \gamma^2(-2\phi+\phi^2)0.5\sigma^2 - \gamma\phi\epsilon_t).
\end{aligned}
\tag{2.61}
$$

The single period risk free rate is therefore

$$
\begin{aligned}
R^{B,1}_t &= \exp(\delta + \gamma\mu_g - \gamma^2 0.5\sigma^2 - \gamma^2(\phi^2-2\phi)0.5\sigma^2 + \gamma\phi\epsilon_t)\\
&= R^{B,1,n}\exp(-\gamma^2(\phi^2-2\phi)0.5\sigma^2 + \gamma\phi\epsilon_t)
\end{aligned}
\tag{2.62}
$$

The M-period bond rate is

$$
R^{B,M}_t = (R^{B,1,n})^M\exp(-\gamma^2(\phi^2-2\phi)0.5\sigma^2 + \gamma\phi\epsilon_t).
\tag{2.63}
$$

The equity premium is

$$
\begin{aligned}
E[R^E] = E&\left[\frac{\exp(g^S_{t+1})}{\exp(-\gamma\phi\epsilon_t)\Psi^{S,n}C} + \frac{\exp(g^S_{t+1})\exp(-\gamma\phi\epsilon_{t+1})\Psi^{S,n}C}{\exp(-\gamma\phi\epsilon_t)\Psi^{S,n}C} - R^{B,1}_t\right]\\
&= \exp(\gamma^2\phi^2 0.5\sigma^2)[\exp(-\mu^{\tau^*} - (\gamma^2-2\gamma)(\phi^2-2\phi)0.5\sigma^2 + \phi\sigma^2)(E[R^S]-E[G])\\
&+ \exp(\mu^g + (1-\gamma\phi)^2 0.5\sigma^2)] - R^{B,1,n}\exp(\gamma^2\phi\sigma^2)\\
&= (1-E[\tau])^{-1}\exp((\gamma\phi^2 + \phi(\gamma-1)^2)\sigma^2)(E[R^S]-E[G])\\
&+ E[G]\exp((\gamma^2\phi^2-\gamma\phi)\sigma^2) - R^{B,1,n}\exp(\gamma^2\phi\sigma^2).
\end{aligned}
\tag{2.64}
$$

2.9.5 Data Sources for Numerical Examples

I use annual data from 1929 to 2013 for the U.S. I use dividends for the S&P 500 from Robert Shillers website[12]. Furthermore, I use nominal consumption per capita of nondurables and services provided by FRED (Federal Reserve Economic Data) for the period from 1929 to 2013.[13] Nominal GDP per capita from 1929 to 2013 is also provided by FRED.[14] I deflate all of the nominal values and compute the growth rates of the real values.

For 1947 to 2013 I compute consumption deflator series from nominal and real consumption per capita from data provided by FRED.[15] For the time prior to 1947 I use the consumption deflator series from Grossman and Shiller (1981).[16]

Due to those non-flat tax rates and the optimization at the margin the marginal tax rates are appropriate here[17]. For average marginal tax rates on dividends, I use the data provided by NBER TAXSIM from 1979 to 2013. For tax rate from 1930 to 1978 I use the data from Sialm (2009)[18]. I am confident to use time series from two sources since during the overlap period they are very close with correlation coefficients of over 0.9.

[12] The data can be found under `http://www.econ.yale.edu/~shiller/data.htm`.

[13] The data can be retrieved under `https://research.stlouisfed.org/fred2`, series A796RC0A052NBEA and A797RC0A052NBEA.

[14] The respective time series is A939RC0A052NBEA.

[15] From FRED I use the annual real consumption series A796RX0Q048SBEA and A797RX0Q048SBEA.

[16] During the overlapping time of the timer series from 1947 to 1992 the derived inflation rates are virtually the same and have a correlation coefficient of 0.99.

[17] For the use of marginal tax rates see for example Sialm (2009) in an empirical paper or Brennan (1970) in a theoretical asset pricing context.

[18] The data and the data appendix can be found under `https://www.aeaweb.org/articles?id=10.1257/aer.99.4.1356`. During the overlap of the data from Sialm (2009) and the TAXSIM data, the time series are very close.

3 Asset Pricing under Tax Rate Uncertainty in a Real Business Cycle Model

3.1 Introduction

I combine the ideas of Sialm (2006), who considers uncertain tax rates in a model similar to the consumption CAPM (CCAPM), and Santoro and Wei (2011), who consider certain tax rates on dividends and corporate profits in a real business cycle (RBC) model with habits and adjustment costs for capital. Also using an RBC model with habits and adjustment cost, I analyze the effect of an uncertain tax rate on dividends on business cycle variables and moments of asset returns. I also include the feature in Sialm (2006) that a part of the taxes are invested in a public good, whose consumption goes into the investor's utility function in a separable fashion.

As in Santoro and Wei (2011) I assume mature firms, which finance investments out of retained earnings and abstain from share issues or repurchases. I regard dividend taxes in isolation and concentrate on the effects of uncertainty about the tax rate.

I confirm that a certain and constant tax rate on dividends has no distortionary effects on investments at least as long as all tax payments are transferred back to the investor. This result can be extended to an uncertain tax rate on dividends as long as certain assumptions on the expected value and correlations of the tax rate with other variables of the firm's first order condition hold. However, those assumptions are very restrictive so that tax rate uncertainty will regularly have an effect on the firm's investment decisions.

I conduct several numerical experiments. Shocks on the tax rate are uncorrelated with productivity shocks. I compare parameterizations without taxes, with a certain tax rate, without and with autocorrelated uncertain tax rates. I do this for different groups of models, in which I vary habit and adjustment cost parameters.

In the numerical experiments I find that tax rate uncertainty has a strong effect on business cycle variables in the standard RBC model without habits and adjustment costs. In the first set of experiments all of the taxes are paid back as transfer payments. Variability of consumption and investment growth are strongly increased with tax rate volatility. Volatilies of asset returns are increased as well. However, those effects diminish greatly with the introduction of adjustment costs and decrease even more with higher habit parameters. An unexpected increase in the tax rate leads to a decrease in corporate payouts and consumption and an increase in investments. With adjustment costs and habits, tax shocks affect consumption and investment much less. For the equity premium it turns out that the tax capitalization effect is much more important than effects through tax rate volatility. A certain tax rate of 32% increases the quarterly pre-tax equity premium by a little bit less than 0.3% in all of the parameterizations. The equity premium, which is usually determined using pre-tax mean returns, reflects capitalized taxes so that after-tax, an investor earns the same as in a world without taxes. Introducing tax rate volatility does not increase the equity premium in an economically significant way. I obtain the greatest effect for the standard RBC model with an increase of the equity premium by 6 basis points versus the case with a certain tax rate. However this is achieved with an unreasonably high volatility of the tax rate shocks. With a more realistic volatility, the increase is only two basis points. For high adjustment costs and high habits increases are even less. Without habits and adjustment costs tax rate shocks are levelled out by strong investment reactions and then do not cause enough volatility in the stochastic discount factor to increase the equity premium a lot. With adjustment costs and habits small adjustments of investments with small changes in consumption have already enough effect on the stochastic discount factor to bring first order conditions back to equality. This additional volatility of the discount factor is too small to cause sizeable effects of tax rate volatility.

I also regard the case that not all of the taxes are transferred back, but are invested in a public good. Consumption of this good enters the utility function in a separable fashion. I find that reducing transfer payments has a big impact on the equity premium when habits are strong and adjustment costs are high. Reducing transfer payments to zero and investing all tax revenues in the public good increases the equity premium by around 40 basis points. Making a part of output unavailable for consumption and regular investment creates extreme additional discount factor volatility that leads to a much higher equity premium.

I contribute to the literature in which models are developed that explain asset pricing in real business cycle models and that include taxes as important determinants. Jermann (1998) lays the foundations to explain the equity premium in RBC models without taxes. He finds that both, habit formation and adjustment cost of capital, are necessary to generate non-trivial equity premiums. In turn, standard business cycle models as in Rouwenhorst (1995), which rest on power utility, are not able to produce substantial risk premiums. Santoro and Wei (2011) look at flat tax rates on corporate profits and dividends. However, they only find interesting effects for corporate taxes. A flat dividend tax rate is found to have no effect on investment decisions. Sialm (2006) considers uncertain tax rates explicitly. He taxes accrual capital gains and dividends at the same flat, but uncertain, tax rate. He finds that tax rate uncertainty can at least partly explain the equity premium. The model he uses features an exogenous output process without real investment opportunities so that it very much resembles an endowment economy in the fashion of the CCAPM. Due to this limitation, Sialm (2006) adds that a model with endogenous investments would be a fruitfull extension. Croce et al. (2012) analyze uncertain corporate tax rates in a production-based model with recursive preferences. They find that tax rate increases through a tax rate shock lead to less investment and more consumption. I find that the opposite is true for an increase in dividend taxes through a tax shock.

McGrattan and Prescott (2005) also see strong ties between taxation and asset prices. They observe large movements of asset prices relative to GDP in the U.S. and in the U.K. They found changes in the tax rate on corporate distributions to be their main driver. Bizer and Judd (1989) build a model with random taxes on capital gains. One finding is that the randomization of capital income taxation will raise revenue at a relatively low efficiency cost, showing that naive arguments about a stable tax policy may be misplaced. Hasset and Metcalf (1999) test tax effects on investment behaviour, whereas taxes are subject to different stochastic processes: a stationary and discrete jump process and a random walk. Among other results they find that different stochastic processes lead to opposite conclusions about investment behavior. Pastor and Veronesi (2012) derive a general equilibrium model in a Bayesian fashion. The paper is remarkable in that it provides a new view on the matter using parameter uncertainty. Empirically, Sialm (2009) finds evidence that effective tax rates on dividends and capital gains are negatively related to equity valuation, which supports the tax capitalization hypothesis. Poterba and Summers (1984) outline and test several hypotheses of effects of dividend taxation on investments and asset prices for the UK. They reject the tax capitalization hypothesis in favor of the traditional view, in which taxes on dividends are seen as an additional tax on corporate capital income. They also state that the different views may well coexist since for different firms different payout and investment policies may apply. They see the tax capitalization view as most applicalbe to mature firms, which fund investments mostly out of retained earnings.

3.2 The Model

The basic model follows the one in Jermann (1998), which was extended by Santoro and Wei (2011) with certain and flat tax rates on corporate profits and dividends. I also introduce a public good that enters the utility function and a separable way as in Sialm (2006). My main focus is on dividend taxation and not on corporate taxes, since the latter ones are already analyzed in Croce et al. (2012).

Utility and budget constraints. The representative investor is infinitely-lived and maximizes expected utility, $E_0[u(\cdot)]$, with respect to consumption C_t and portfolio weights x_t, at time zero:

$$\max_{\{C_t\}_{t=0}^{\infty}\{x_t\}_{t=0}^{\infty}} E_0\left[\sum_{t=0}^{\infty}\beta^t[u(C_t, C_{t-1}) + v(G_t)\right]. \tag{3.1}$$

Here G_t represents the quantity of the public good at time t, and $v(\cdot)$ is the utility function for this good. The households budget constraint for $t = 0, 1.., \infty$ reads

$$C_t = p_t'(x_{t-1} - x_t) + D_t^{\tau\prime} x_{t-1} + w_t L_t + (1-\omega)Q_t, \tag{3.2}$$

Furthermore, $p_t = (p_t^S \;\; p_t^B)'$ is a vector of asset prices, with p_t^S the price of the equity asset and p_t^B the price of a bond that pays out one unit of the consumption good in the next period. $D_t^\tau = (D_t^S(1-\tau_t) \;\; 1)'$ is a vector of after-tax payouts of the equity asset, in which D_t^S are dividends on the equity asset, and τ_t is the tax rate on dividends. The bond is not taxed. Similarly, the vector of portfolio weights is $x_t = (x_t^S \;\; x_t^B)'$. Bonds are in zero net supply. Equity is in positive net supply so that the sum of all weights of the equity asset is one. I assume, as in Santoro and Wei (2011), that the firm does not issue further equity but finances investments out of retained earnings. As mentioned in the introduction, this assumption makes most sense for mature firms. The investor works L_t units per period and receives a wage rate w_t for one unit of labor provided. The term Q_t stands for tax revenues and $(1-\omega)Q_t$ for transfer payments, in which $0 \leq \omega \leq 1$, is the flat and deterministic share of the whole tax revenues that is invested in the public good.

As in Sialm (2006), I assume that the government has a zero deficit at all times so that $G_t = \omega Q_t$.

First order conditions for the representative investor. I write the Lagrangian as

$$\mathcal{L} = E_0\Bigg[\sum_{t=0}^{\infty}\beta^t\Bigg(u(C_t,C_{t-1})+v(G_t)+\Lambda_t\big(p_t'(x_{t-1}-x_t)+D_t^{\tau}{}'x_{t-1}+w_tL_t+(1-\omega)Q_t-C_t\big)\Bigg)\Bigg], \tag{3.3}$$

in which Λ_t is a Lagrange multiplier. Heer and Maussner (2009, pp. 312-317) provide a derivation for the case without taxes. Santoro and Wei (2011) show some results with a certain dividend tax rate.

The derivatives with respect to C_t, x_t^S, and x_t^B conditional on the information at time t are, respectively,

$$0 = u_{t,1} + \beta E_t[u_{t,2}] - \Lambda_t, \tag{3.4}$$

$$0 = -p_t^S\Lambda_t + E_t\Big[\beta\Lambda_{t+1}\big(D_{t+1}^S(1-\tau_{t+1})+p_{t+1}^S\big)\Big] \tag{3.5}$$

$$0 = -p_t^B\Lambda_t + E_t\Big[\beta\Lambda_{t+1}\Big]. \tag{3.6}$$

I denote $u_{t,1}$ and $u_{t,2}$ the derivatives with respect to the first argument of the function at time t and the second argument at time t, respectively. For example, $u_{t,1}$ is the derivative of $u(\cdot)$ with respect to C_t. I suppress arguments of functions when they are not needed for clarity.

The representative firm. The firm produces output Y_t using capital K_t and labor L_t and a production technology represented by a production function, which is homogeneous of degree one,

$$Y_t = Z_tF(K_t, A^tL_t). \tag{3.7}$$

Output is subject to productivity shocks Z_t. Furthermore, $A^t = (1+a)^t$ is a deterministic growth rate. Per assumption investments I_t are equal to retained earnings. Therefore, dividends are

$$D_t^S = Y_t - w_tL_t - I_t. \tag{3.8}$$

The capital stock evolves according to

$$K_{t+1} = \phi(K_t, I_t) + (1-\delta)K_t, \quad \delta \in (0,1], \tag{3.9}$$

where δ is the depreciation rate of capital and the function $\phi(K_t, I_t)$ accounts for adjustment costs. Any produce cannot costlessly converted into installed capital, and this function accounts for that fact. The firm maximizes its value through the optimal choice of investment and capital:

$$\max_{\{I_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} E_0 \left[\beta^t \frac{\Lambda_t}{\Lambda_0} (Z_t F(K_t, A^t L_t) - w_t L_t - I_t)(1-\tau_t) \right]. \tag{3.10}$$

With that maximization problem I also follow Santoro and Wei (2011) in that the firm maximizes the present value of after-tax dividends. The argument in favor of this assumption is that to maximize shareholder value the firm must also consider personal taxes. I write the problem as a Lagrangian subject to the firms budget constraint (3.9):

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} \left[(1-\tau_t) \big[Z_t F(K_t, A^t L_t) - w_t L_t - I_t \big] + q_t \big[\phi(K_t, I_t) + (1-\delta) K_t - K_{t+1} \big] \right] \right\}. \tag{3.11}$$

The derivative with respect to capital K_{t+1} leads to

$$E_0 \left\{ \beta^{t+1} \frac{\Lambda_{t+1}}{\Lambda_0} \left[(1-\tau_{t+1}) Z_{t+1} F_{t+1,1} + q_{t+1} \big[\phi_{t+1,1} + 1 - \delta \big] \right] + \beta^t \frac{\Lambda_t}{\Lambda_0} q_t (-1) \right\} = 0. \tag{3.12}$$

From the viewpoint of time t and with some cancellations and rearrangements, I obtain an expression with respect to q_t:

$$q_t = E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left[(1-\tau_{t+1}) Z_{t+1} F_{t+1,1} + q_{t+1} \big[\phi_{t+1,1} + 1 - \delta \big] \right] \right\}. \tag{3.13}$$

Eventually, the derivative with respect to investments I_t at time t yields

$$q_t = \frac{1-\tau_t}{\phi_{t,2}}. \tag{3.14}$$

Substituting equation (3.14) into (3.13) leads to the firm's first order condition:

$$\frac{1-\tau_t}{\phi_{t,2}} = E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} (1-\tau_{t+1}) \left[Z_{t+1} F_{t+1,1} + \frac{\phi_{t+1,1} + 1 - \delta}{\phi_{t+1,2}} \right] \right\}. \tag{3.15}$$

The derivative with respect to labor leads to

$$w_t = Z_t F_{t,2} A^t, \tag{3.16}$$

i.e., the wage rate is equal to the marginal product of labor.

Market equilibrium. Many of the results can be found in Sialm (2006). However, the consumption budget constraint (3.2) can be rewritten using the firm's dividend payment from Equation (3.8) and the fact that $x_{t-1} - x_t = 0$ in the aggregate. Since labor or leisure does not enter the households utility, it is optimal for households to provide as much labor as possible, i.e., $L_t = 1$. No equity is issued or bought back, so that aggregate tax revenues are equal to tax payments:

$$Q_t = \tau_t D_t^S. \tag{3.17}$$

In the aggregate consumption is equal to

$$\begin{aligned} C_t &= (1 - \omega\tau_t) D_t^S + w_t \\ &= (1 - \omega\tau_t)(Y_t - w_t - I_t) + w_t. \end{aligned} \tag{3.18}$$

Government spending on the public good is

$$G_t = \omega\tau_t D_t^S = \omega\tau_t (Y_t - w_t - I_t). \tag{3.19}$$

The remainder of the tax revenue is transfer payments. Thus, all of the produce is either consumed, invested in capital or invested in the public good:

$$Y_t = C_t + I_t + G_t. \tag{3.20}$$

Using the prior two equations and Equation (3.16), I can restate investments in equilibrium as

$$I_t = Y_t - \frac{C_t - \omega\tau_t Z_t F_{t,2} A^t}{1 - \omega\tau_t}. \tag{3.21}$$

Substituting Equations (3.18) and (3.19) into Equation (3.20), investments can also be written as

$$I_t = Y_t - (D_t^S + w_t), \tag{3.22}$$

in which $D_t^S + w_t$ is the pre-tax income of the representative investor.

First results. In contrast to Santoro and Wei (2011) I extend the model with an uncertain tax rate on dividends. For their Proposition 1 Santoro and Wei (2011) use a constant dividend tax rate and find that it does not have an effect on investment decisions since the tax terms cancel out. In this case, the equality of the marginal cost and marginal benefit of investment is not dependent on the dividend tax rate. With respect to an uncertain tax rate on dividends, I propose the following:

Proposition 3.1. In equilibrium and with full tax transfers ($\omega = 0$), an uncertain tax rate on dividends has no effects on the firm's investment decisions as long as the dividend tax rate is uncorrelated with the remaining variables on the right side of equation (3.15) and as long as the (conditional) expected value of the dividend tax rate is equal to the current tax rate: $\tau_t = E_t[\tau_{t+1}]$.

Proposition 3.2. In equilibrium but without full tax transfers ($0 < \omega \leq 1$), a tax rate on dividends has an effect on the firm's investment decisions even when it is certain and constant.

Proof. Dividing Equation (3.15) by $1 - \tau_t$ it becomes clear that the firm's decisions dependent on $\frac{1-\tau_{t+1}}{1-\tau_t}$. With τ_{t+1} uncorrelated with all other rhs variables and with $\tau_t = E_t[\tau_{t+1}]$ Equation (3.15) becomes

$$\frac{1}{\phi_{t,2}} = E_t\left\{\beta\frac{\Lambda_{t+1}}{\Lambda_t}\left[F_{t+1,1} + \frac{\phi_{t+1,1} + 1 - \delta}{\phi_{t+1,2}}\right]\right\}\frac{1 - E_t[\tau_{t+1}]}{1 - \tau_t}, \tag{3.23}$$

where $(1 - \tau_t)$ and $(1 - E_t[\tau_{t+1}])$ cancel out for $\tau_t = E_t[\tau_{t+1}]$, so that taxes do not play a role. Apart from the optimality condition I also have to consider the equilibrium conditions that are derived from the budget constraint. Equations (3.18) to (3.21) shows that for $\omega \neq 0$ the tax rate has an effect on the budget constraints, i.e., it has an effect on the feedback relations between investment and consumption, whereas for $\omega = 0$ the tax rate disappears from the budget constraints. □

In other cases, i.e., with tax rates correlated with other variables or a conditional expectation of the tax rate different from the current tax rate or both, tax effects do not cancel out of the firm's first order conditions, so that the tax rate regularly has an effect on the firm's investment decisions.

3.3 Choice of Processes and Functional Forms

I model the tax rate τ_t as a process that includes an AR(1) component:

$$\begin{aligned} \tau_t &= \mu^\tau + \tau_t^* \\ \tau_t^* &= \rho^{\tau^*}\tau_{t-1}^* + \sigma_u u_t, \quad u_t \sim N(0,1). \end{aligned} \tag{3.24}$$

I use μ^τ as the mean of the tax rate, ρ^{τ^*} as the persistence parameter of the AR(1) part of the process, and σ_u as the volatility parameter of the shocks u_t. Theoretically, this specification comes along with the problem that tax rates can be greater than one and less than zero. However, with a small variance and a mean not close to zero or one, as it will be used herein, this is not a practical problem. As explained further in the Appendix 3.10.1, for normal distributions good numerical approximations exist that very much shorten computation time.
I use a Cobb-Douglas production function of the form

$$F(K_t, A^t L_t) = K_t^\alpha (A^t L_t)^{1-\alpha} \quad \alpha \in (0,1). \tag{3.25}$$

Very similar to the function in Santoro and Wei (2011) and Jermann (1998), I use the following specification for adjustment costs:

$$\phi(K_t, I_t) = \left(\frac{(a+\delta)^\eta}{1-\eta} \left(\frac{I_t}{K_t} \right)^{1-\eta} - \frac{\eta(a+\delta)}{1-\eta} \right) K_t, \tag{3.26}$$

in which $\eta \neq 1$ is the curvature parameter of the function. The derivatives with respect to K_t and I_t are, respectively:

$$\phi_{t,1} = \left(\frac{(a+\delta)^\eta}{1-\eta} \left(\frac{I_t}{K_t} \right)^{1-\eta} - \frac{\eta(a+\delta)}{1-\eta} \right) - \left(\frac{I_t}{(a+\delta)K_t} \right)^{-\eta} \left(\frac{I_t}{K_t} \right), \tag{3.27}$$

$$\phi_{t,2} = \left(\frac{I_t}{(a+\delta)K_t} \right)^{-\eta}. \tag{3.28}$$

The logarithm of factor productivity $\ln(Z_t) = z_t$ follows the AR(1) process:

$$z_t = \rho^z z_{t-1} + \sigma_\epsilon \epsilon_t, \ \ \epsilon_t \sim N(0,1). \tag{3.29}$$

I assume that the shocks to productivity and to the tax rate are uncorrelated and bivariate normal. Utility is given by

$$u(C_t) = \frac{(C_t - bC_{t-1})^{1-\gamma}}{1-\gamma}, \tag{3.30}$$

where b is a parameter that determines the habit with respect to prior consumption. The derivatives of the production function with respect to capital and labor are, respectively,

$$\begin{aligned} F_{t,1}(K_t, A^t L_t) &= \alpha K_t^{\alpha-1} (A^t L_t)^{1-\alpha} \\ F_{t,2}(K_t, A^t L_t) A^t &= (1-\alpha) K_t^\alpha (A^t L_t)^{-\alpha} A^t. \end{aligned} \tag{3.31}$$

The derivative of the utility function with respect to consumption C_t is

$$u_{t,1}(C_t, C_{t-1}) = (C_t - bC_{t-1})^{-\gamma}.$$

The langrange multiplier Λ_t becomes

$$\Lambda_t = (C_t - bC_{t-1})^{-\gamma} - \beta b E_t[(C_{t+1} - bC_t)^{-\gamma}]. \tag{3.32}$$

I expand the first order condition (3.23) to

$$\left(\frac{I_t}{(a+\delta)K_t}\right)^{\eta} = E_t\Bigg\{\beta\frac{\Lambda_{t+1}}{\Lambda_t}\frac{1-\tau_{t+1}}{1-\tau_t}\Bigg[\alpha Z_{t+1}K_{t+1}^{\alpha-1}(A^t L_{t+1})^{1-\alpha}+ \\ \frac{\left(\frac{(a+\delta)^{\eta}}{1-\eta}\left(\frac{I_{t+1}}{K_{t+1}}\right)^{1-\eta}-\frac{\eta(a+\delta)}{1-\eta}\right)-\left(\frac{I_{t+1}}{(a+\delta)K_{t+1}}\right)^{-\eta}\left(\frac{I_{t+1}}{K_{t+1}}\right)+1-\delta}{\left(\frac{I_{t+1}}{(a+\delta)K_{t+1}}\right)^{-\eta}}\Bigg]\Bigg\}. \tag{3.33}$$

I extract the investment capital ratio from the last fraction and rewrite the equation to

$$q_t = E_t\left\{\beta\frac{\Lambda_{t+1}}{\Lambda_t}\left[(1-\tau_{t+1})\frac{\alpha Y_{t+1}-I_{t+1}}{K_{t+1}}+q_{t+1}\left(\frac{\phi(K_{t+1},I_{t+1})}{K_{t+1}}+1-\delta\right)\right]\right\}. \tag{3.34}$$

Since the production function is homogeneous of degree one, Euler's theorem holds: $Y_t = Z_tF(K_t,A^tL_t) = Z_tK_tF_{t,1}(K_t,A^tL_t)+Z_tA^tL_tF_{t,2}(K_t,A^tL_t)$. I use this result and Equations (3.8) and (3.16) to obtain $D_t^S = Y_t - Z_tA^tL_tF_{t,2}(K_t,A^tL_t) - I_t = Z_tK_tF_{t,1} - I_t = \alpha Y_t - I_t$. I use this result in Equation (3.34). Following Heer and Maussner (2009, pp. 314-317) and Rouwenhorst (1995, pp. 299-303), I derive the relation of the price of the equity asset to the captial stock. I multiply the equation by K_{t+1} and use the fact that $K_{t+2} = \phi(K_{t+1},I_{t+1}) + (1-\delta)K_{t+1}$ from Equation (3.9) to write

$$K_{t+1}q_t = E_t\left\{\beta\frac{\Lambda_{t+1}}{\Lambda_t}\left[(1-\tau_{t+1})D_{t+1}^S + q_{t+1}K_{t+2}\right]\right\}. \tag{3.35}$$

Setting $p_t^s = K_{t+1}q_t = K_{t+1}\frac{1-\tau_t}{\phi_{t,2}}$, I obtain a simple expression for the equity price[19], which can be used to compute asset returns and expected returns.

3.4 Transformation to Stationary Values and the Deterministic Case

Following Heer and Maussner (2009, pp. 39-40), I transform the business cycle variables into stationary variables. I denote stationary values in lower case letters. I define $c_t = C_t/A^t$, $\lambda_t = \Lambda_t(A^t)^{\gamma}$ and $\tilde{\beta} = \beta A^{1-\gamma}$. I use those definitions and take out $(A^t)^{\gamma}$ from both sides of Equation (3.32) and divide by it so that it turns to:

$$\lambda_t = (c_t - \frac{b}{A}c_{t-1})^{-\gamma} - \tilde{\beta}\frac{b}{A}E_t[(c_{t+1}-\frac{b}{A}c_t)^{-\gamma}]. \tag{3.36}$$

Defining $k_t = K_t/A^t$ and dividing Equation (3.9) by A^t leads to the new formulation of the capital evolution function

$$Ak_{t+1} = \phi(k_t,i_t) + (1-\delta)k_t. \tag{3.37}$$

[19] Compare also Heer and Maussner (2009, pp. 314-317), Restoy and Rockinger (1994) and Cochrane (1991).

I obtain this result since the adjustment cost equation is also homogeneous of degree one. This implies $\phi(K_t/A^t, I_t/A^t) = \phi(K_t, I_t)/A^t$.

I use the equilibrium condition for investments (3.21) and restate it with stationary investment $i = I_t/A^t$. Thus, I divide Equation (3.21) by A^t to obtain:

$$i_t = y_t - \frac{c_t - \omega\tau_t F_{t,2}(k_t, L_t)}{1 - \omega\tau_t}. \tag{3.38}$$

Since $F_{t,2}(K_t, A^t L_t)$ from Equation (3.16) is homogeneous of degree zero, $F_{t,2}(K_t, A^t L_t) = F_{t,2}(K_t/A_t, L_t) = F_{t,2}(k_t, L_t)$ holds. The variable q_t from Equation (3.14) can be expressed as a function homogeneous of degree zero in its arguments so that using k_t and i_t instead of K_t and I_t changes nothing.

Equation (3.34) in stationary values reads

$$q_t = E_t\left\{\beta\frac{1}{A^\gamma}\frac{\lambda_{t+1}}{\lambda_t}\left[(1-\tau_{t+1})\frac{\alpha y_{t+1} - i_{t+1}}{k_{t+1}} + q_{t+1}\left(\frac{\phi(k_{t+1}, i_{t+1})}{k_{t+1}} + 1 - \delta\right)\right]\right\}. \tag{3.39}$$

Multiplying the equation by k_{t+1}, using $\tilde{\beta} = \beta A^{1-\gamma}$ and since $Ak_{t+2} = \phi(k_{t+1}, i_{t+1}) + (1-\delta)k_{t+1}$ I restate

$$k_{t+1}q_t = E_t\left\{\frac{\tilde{\beta}}{A}\frac{\lambda_{t+1}}{\lambda_t}\left[(1-\tau_{t+1})d^S_{t+1} + q_{t+1}Ak_{t+2}\right]\right\}. \tag{3.40}$$

I multiply by A to obtain

$$Ak_{t+1}q_t = E_t\left\{\tilde{\beta}\frac{\lambda_{t+1}}{\lambda_t}\left[(1-\tau_{t+1})d^S_{t+1} + q_{t+1}Ak_{t+2}\right]\right\}. \tag{3.41}$$

Dividing by $Ak_{t+1}q_t$ I obtain gross returns, where the deterministic growth rate is taken out:

$$\frac{(1-\tau_{t+1})d^S_{t+1} + q_{t+1}Ak_{t+2}}{Ak_{t+1}q_t} = \frac{(1-\tau_{t+1})D_{t+1}/A^{t+1} + p^S_{t+1}/A^{t+1}}{p^S_t/A^t} = \frac{R^{S\tau}_{t+1}}{A}. \tag{3.42}$$

In this equation $R^{S\tau}_{t+1}$ is the total single-period after tax return

$$R^{S\tau}_{t+1} = \frac{D^s_{t+1}(1-\tau_{t+1}) + p^S_{t+1}}{p^S_t}. \tag{3.43}$$

Pre-tax returns on the equity asset are defined as

$$R^S_{t+1} = \frac{D^s_{t+1} + p^S_{t+1}}{p^S_t}. \tag{3.44}$$

The (total) return can be split up into the dividend yield $\frac{D^s_{t+1}}{p^S_t}$ and the capital gain $\frac{p^S_{t+1}}{p^S_t}$. The return on a single-period bond is

$$R^B_t = \frac{A\lambda_t}{E_t[\tilde{\beta}\lambda_{t+1}]}, \tag{3.45}$$

which is the price of a certain cash flow of one unit arriving at $t+1$.

As in King et al. (1988) the only two changes when the stationary values are used are with respect to the impatiance factor and the capital accumulation equation.

With the stationary values I compute the stationary solutions for the deterministic case. I denote the solutions using the same letters as before but without time subscripts. Let X_t be any variable such as capital, investments etc., then $X = X_t = X_{t+1} = ...$ for the deterministic stationary case. Furthermore, productivity is equal to one, i.e., $Z_t = 1$, and the tax rate is set equal to μ^τ. From Equation (3.37) follows that in the deterministic stationary state $k(a+\delta) = \phi(k,i)$. This holds for steady state investments $i = k(a+\delta)$. That leads to $q = (1-\mu^\tau)$. Using that in Equation (3.41) together with $d^S = \alpha k^\alpha - k(a+\delta)$ and rearranging for k leads to

$$k = \left(\frac{\alpha\tilde{\beta}}{A - \tilde{\beta}(1-\delta)}\right)^{\frac{1}{1-\alpha}}. \tag{3.46}$$

The dividend tax plays no role for the value of the deterministic stationary capital. Using Equation (3.18) with stationary values and the stationary solution for capital, I derive stationary consumption

$$c = (1-\omega\mu^\tau)(\alpha k^\alpha - k(a+\delta)) + (1-\alpha)k^\alpha. \tag{3.47}$$

As long as $\omega = 0$, i.e., as long as all of the taxes are transfer payments, the tax rate also has no effect on the steady state of consumption. However with ω greater zero the steady state consumption decreases. To complete the picture, steady state output is

$$y = k^\alpha, \tag{3.48}$$

for which I divide Equation (3.25) by A^t, and I use the fact that $L_t = 1$.

3.5 Quantitative Analysis

3.5.1 Method and Parameterization

I use a second order approximation of the policy functions, and I run simulations to obtain averages of business cycle variables and asset returns. Appendix 3.10.1 describes the numerical methods and why they were chosen more in detail. I point out, that I do not use the assumption of joint log normality of the stochastic discount factor and asset returns as in Jermann (1998). I drop this assumption and follow mostly Heer and Maussner (2009). With equal parameters this leads to higher risk premiums as pointed out by Heer and Maussner (2012).

The benchmark parameters are given in Table 3. The parameters unrelated to taxes are from Heer and Maussner (2009, p. 322). For tax rates on dividends, I use average marginal tax rates. Due to tax rates being not flat and due to the optimization at the margin, the marginal tax rates are appropriate here[20]. For average marginal tax rates on dividends, I use the data provided by NBER TAXSIM from 1979 to 2013. For tax rates from 1913 to 1978, I use the data from Sialm (2009)[21]. I am confident to use time series from two sources since during the overlap period they are very close, with correlation coefficients of over 0.9. The mean of the tax rate is 0.32. I use this value in the quantitative analysis as well. To obtain the AR(1) process of τ^*, I take out the mean μ^τ and estimate an AR(1) process for the remaining series. I obtain a coefficient ρ^{τ^*} of 0.93 and a standard deviation for the shocks σ_u of 0.03.[22] The tax rate data is at annual frequency. I follow Croce et al. (2012) and disaggregate the tax rate data to a quarterly frequency. This way I can keep the non-tax parameters at quarterly frequency, which makes them better comparable to other analyses within the literature, which most of the time do not differ that much in basic parameterizations. This implies that $\sigma_u = 0.008$ and $\rho^{\tau^*} = 0.98$[23]. For ρ^{τ^*} I choose a value of 0.9. This is a bit lower than the implied value of 0.98, but still a high persistence. I decrease persistence a bit to reduce the probability that the simulations produce tax rates below zero or above one.

I analyze the standard RBC model with $\eta = 0$ and $b = 0$, since this was also used in Sialm (2006). Furthermore, I use the adjustment cost parameter from Jermann (1998) with $\eta = 1/0.23$ and different habit parameters b. I also use different specifications of the tax rate process to analyse the sensitivity to those parameters. Later, I analyze the sensitivity to different parameters such as growth or risk aversion.

Before I present and interpret the numerical results I will outline the basic mechanisms of the habit model.

3.5.2 The Basic Mechanisms in the Habit Model with Adjustment Costs

Table 4 shows the results of the simulations for different model specifcations. To address the results more conveniently, rows are numbered from (1) to (20) and the columns from (a) to (j). The table shows ratios of volatilities of real values (column (a) and (b)) and asset return moments in percent in the columns (c) to (j). The parameterizations are divided into groups, and the groups differ with respect to the tax rate process. The first parameterization in each group has no taxes, the second includes a certain tax rate, and the following ones use different degrees of volatilities of shocks and autocorrelation of the AR(1) part of the tax rate pocess. All values are quarterly values. When necessary, I will additionally provide graphs of impulse repsonses to make sense of some of the results from the table.

As Jermann (1998) notes, stronger habits, i.e., an increase in the parameter b leads to a decrease

[20] For the use of marginal tax rates see for example Sialm (2009) in an empirical paper or Brennan (1970) in a theoretical asset pricing context.

[21] The data and the data appendix can be found under `https://www.aeaweb.org/articles?id=10.1257/aer.99.4.1356`. During the overlap of the data from Sialm (2009) and the TAXSIM data, the time series are very close.

[22] The parameters μ^τ, ρ^{τ^*}, and σ_u are rounded to the second decimal.

[23] An AR(1) at quarterly frequency implies $(\rho^{\tau^*}(quart))^4 = \rho^{\tau^*}(ann)$. For the variance of the shocks the relation is $\sigma_u(quart)(1 + \rho^{\tau^*}(quart) + (\rho^{\tau^*}(quart))^2 + (\rho^{\tau^*}(quart))^3) = \sigma_u(annually)$.

Table 3: Benchmark parameters

Parameter	**Notation**	**Value**
Deterministic growth rate	a	0
Rate of depreciation	δ	0.011
Output elasticity of capital	α	0.27
Time discount factor	$\tilde{\beta}$	0.994
Utility curvature parameter	γ	2
Persistence (productivity)	ρ^z	0.9
Standard deviation of prod. shocks	σ_ϵ	0.0072
Mean of the tax rate	μ^τ	0.32
Persistence (AR part of tax rate process)	ρ^{τ^*}	0.9
Standard deviation of tax rate shocks	σ_u	0.008

The parameters that are unrelated with the dividend tax rate are from Heer and Maussner (2009, p. 322). Tax rate parameters are based on the data in Sialm (2009) and TAXSIM. The persistence of the AR(1) part of the tax rate process is decreased to 0.9 from a higher implied value of around 0.98. All values are at quarterly frequency.

in consumption volatility. People do not want to deviate from their habitual consumption, so that consumption becomes smoother. Agents very much smooth consumption because deviations from consumption have strong effects on marginal utility in the habit model. When investment is not endogenous, this is enough to produce a sizeable equity premium. However, with endogenous investment agents use the investment channel to flatten any shocks introduced by productivity shocks, which, in turn, are immediately reflected in output changes. This can best be observed comparing column (a) of the first two groups of parameterizations in the additional table, Table 7, in the appendix. One can close the retreat to the investment channel in making investment costly. This is done through adjustment costs of capital. Since those adjustments are costly the volatility of investments decreases. Output cannot be converted into capital one to one anymore. This can be observed by comparing column (b) of the last two groups of parameterizations in Table 7, in the appendix. Table 7 also shows that adjustment costs tend to increase consumption volatility, which, together with habits, influences marginal utility strongly. For habits the investment-output volatility ratio is decreased when the habit parameter is increased.

The expected return equation and a simple example additionally help to pin down why the equity premium is small for the standard RBC model. I use Equation (3.6), divide it by the price p_t^S to obtain returns and rearrange it to

$$\begin{aligned} E_t[R_{t+1}^S] &= \frac{1}{E_t[m_{t+1}]}\left(1 - Cov_t\left(m_{t+1}, R_{t+1}^S\right)\right) \\ &= R_t^B\left(1 - \sigma_t\left(m_{t+1}\right)\sigma_t(R_{t+1}^S) Corr_t\left(m_{t+1}, R_{t+1}^S\right)\right), \end{aligned} \tag{3.49}$$

in which I use $m_{t+1} = \frac{\beta\Lambda_{t+1}}{\Lambda_t}$ for the stochastic discount factor (SDF). The SDF and the return on equity determine the conditional equtiy premium. This covariance can be decomposed into the standard deviations $\sigma_t(\cdot)$ of its parts and their correlation coefficient $Corr_t(\cdot)$. To obtain a higher equity premium one can try to strongly increase the volatility of the stochastic discount factor, i.e., to increase $\sigma_t\left(m_{t+1}\right)$. This is seen as the central mechanism to resolve the equity premium puzzle (Cochrane, 2005, p. 455). Cochrane (2005, p. 459) states further that the discount factor volatility can come either from the volatility of its conditional expected value, i.e., from variations in $E_t\left[m_{t+1}\right]$, or from the variation in the unexpected part of the stochastic discount factor: $Var(m) = Var(E_t[m]) + Var(m - E_t[m])$.[24] However, the power model and the habit model with adjustment costs both increase SDF volatility though increasing the variability of its conditional expectation. Apart from a convexity effect the variance of the bond rate in column (f) indicates the variance of the conditional expected value of the SDF. In all of the tables herein a comparison of column (f) and (g) or (j) show that a higher variability of the SDF or the bond rate goes together with a higher risk premium. The same can be observed in the Table 1 in Jermann (1998). Thus, the habit model increases SDF volatility and it makes investments costly so that the investment channel stops to be a good way to smoothen consumption again. A sizeable equity premium can be generated and the bond rate is relatively low. However, the risk-free rate puzzle with respect to its volatility cannot be resolved by this model
I continue with the case of full tax transfers, i.e., $\omega = 0$, and then treat the case in which not all of the taxes are paid out as transfer payments. I will use numerical examples as well as the equilibrium conditions to pin down the tax effects.

3.5.3 The Case with Full Tax Transfers

After those general remarks I come back to Table 4. I start with the group of the standard RBC model, i.e., the parameterizations that feature power utility (no habits and no adjustment costs). Paramerization (par.) (1) includes no taxes and par. (2) a constant tax rate. Expectedly, and as confirmed in Jermann (1998) and Santoro and Wei (2011), the equity premium (column (g)) in this group of parameterizations is virtually zero. For par. (2) the (pre-tax) equity premium increases to 0.28% per quarter. However, the after-tax equity premium in (j) is the same as the equity premium in the no-tax model. Taxes are capitalized in returns, so that after-tax the expected return is the same as the expected return without taxes. This can be observed in all of the different groups of parameterizations. The ratios of standard deviations in (a) and (b) are also the same for the no-tax and the certain tax model of any group. This reflects that investments are not affected by the presence of a constant tax rate on dividends. Par. (3) introduces a (high) volatility of 1.6% per quarter for the tax rate shocks, but no autocorrelation. The results in (a) and (b) increase markedly. Both consumption growth and investment growth become

[24]The equation for the variance decomposition can be found in (Cochrane, 2005, p.459).

Table 4: Results for business cycle variables and asset returns for the case with full tax transfers

Group	**N.**	**Tax parameters**	$\frac{\sigma_{\Delta c}}{\sigma_{\Delta y}}$ (a)	$\frac{\sigma_{\Delta i}}{\sigma_{\Delta y}}$ (b)	$E[r^S]$ (c)	σ_{r^S} (d)	$E[r^B]$ (e)	σ_{r^B} (f)	$E[r^S - r^B]$ (g)	$E[r^{S,\tau}]$ (h)	$\sigma_{r^{S,\tau}}$ (i)	$E[r^{S,\tau} - r^B]$ (j)
Stand. RBC	(1)	$\mu^\tau = 0, \sigma_u = 0, \rho^{\tau^*} = 0$	0.15	5.10	0.60	0.03	0.60	0.03	0.00	NA	NA	NA
$\eta = 0, b = 0$	(2)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	0.15	5.10	0.89	0.02	0.60	0.03	0.28	0.60	0.03	0.00
	(3)	$\mu^\tau = 0.32, \sigma_u = 0.016, \rho^{\tau^*} = 0$	2.25	11.88	0.94	3.37	0.60	2.36	0.34	0.66	3.35	0.05
	(4)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	1.13	7.38	0.90	1.69	0.60	1.18	0.30	0.62	1.68	0.01
	(5)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	0.70	6.06	0.89	1.22	0.60	0.27	0.29	0.61	1.21	0.01
Low habit,	(6)	$\mu^\tau = 0, \sigma_u = 0, \rho^{\tau^*} = 0$	1.09	0.56	0.62	1.83	0.59	0.49	0.03	NA	NA	NA
high adj.	(7)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	1.09	0.56	0.90	1.83	0.59	0.48	0.32	0.62	1.83	0.03
$b = 0.1$,	(8)	$\mu^\tau = 0.32, \sigma_u = 0.016, \rho^{\tau^*} = 0$	1.11	1.07	0.90	1.83	0.59	0.56	0.32	0.62	1.87	0.03
$\eta = 1/0.23$	(9)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	1.10	0.72	0.90	1.84	0.59	0.50	0.32	0.62	1.84	0.03
	(10)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	1.09	0.64	0.90	1.84	0.59	0.48	0.32	0.62	1.84	0.03
Med. habit,	(11)	$\mu^\tau = 0, \sigma_u = 0, \rho^{\tau^*} = 0$	0.95	1.41	0.71	4.57	0.55	2.32	0.16	NA	NA	NA
high adj.	(12)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	0.95	1.41	0.99	4.59	0.55	2.32	0.44	0.71	4.58	0.16
$b = 0.5$,	(13)	$\mu^\tau = 0.32, \sigma_u = 0.016, \rho^{\tau^*} = 0$	0.95	1.57	1.00	4.77	0.55	2.51	0.45	0.72	4.77	0.16
$\eta = 1/0.23$	(14)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	0.95	1.45	0.99	4.63	0.55	2.38	0.44	0.71	4.62	0.16
	(15)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	0.95	1.44	0.99	4.61	0.55	2.32	0.44	0.71	4.60	0.16
High habit,	(16)	$\mu^\tau = 0, \sigma_u = 0, \rho^{\tau^*} = 0$	0.58	3.54	1.26	11.58	0.24	5.45	1.02	NA	NA	NA
high adj.	(17)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	0.58	3.54	1.54	11.55	0.24	5.83	1.30	1.26	11.54	1.02
$b = 0.8$,	(18)	$\mu^\tau = 0.32, \sigma_u = 0.016, \rho^{\tau^*} = 0$	0.58	3.54	1.58	11.88	0.25	5.83	1.33	1.29	11.87	1.05
$\eta = 1/0.23$	(19)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	0.58	3.54	1.55	11.63	0.24	5.57	1.31	1.26	11.62	1.02
	(20)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	0.58	3.54	1.54	11.55	0.24	5.50	1.30	1.26	11.54	1.02

All values are at quarterly frequency. I denote $\sigma_{\Delta c}$ the standard deviation of consumption growth, $\sigma_{\Delta y}$ the standard deviation of output growth, and $\sigma_{\Delta i}$ the standard deviation of investment growth. Expected returns and standard deviations are given in percent. Furthermore, $E[r^S]$ and σ_{r^S} are the unconditional expectation and the standard deviation, respectively, of the single-period return on equity. The superscript B indicates the same for a single-period bond returns. The equity premium is the unconditional mean $E[r^S - r^B]$. The last three columns show mean equity returns, standard deviation and the equity premium for after-tax returns. All values are obtained from 40000 simluations.

much more volatile versus output growth than without the volatility of the tax rate shocks. Expected returns on equity increase by five basis points. The bond rate does not change so that the increase in the expected return on equity also increases the equity premium one for one. Volatilities of returns increase strongly. Thus, the representative agent reacts to tax rate shocks even though taxes are fully transferred back. However, the representative agent stands for a single agent, who cannot influence the lump sum tax transfers so that the decisions are made with respect to the after-tax income. With par. (4), which is different to par. (3) only in the lower volatility, the results decrease again. In par. (5) the tax rate is strongly persistent but the volatility of the tax rate shocks is the same as in par. (4). The persistence decreases the effect of the tax rate volatility on the different results. The after-tax equity premium remains close to zero.

The impulse responses in Figure 7 and the equilibrium conditions, especially Equation (3.33), help to explain the tax effects. Figure 7 panel (a) and (b) show the impulse responses of par. (4) and (5), respectively, to a 1% productivity shock (lhs of each panel) and a 1% tax rate shock (rhs of each panel). The figures show deviations in percent from deterministic stationary values, i.e., something like $y_t/y-1$ for output and $c_t/c-1$ for consumption. The responses to productivity shocks are equal in both cases. The positive productivity shock increases output, consumption and investment. Dividend yields are decreased through lower pre-tax dividends, which, in turn, are decreased through higher investment. Capital gains increase due to the productivity shock, which has a positive impact on investment and capital.

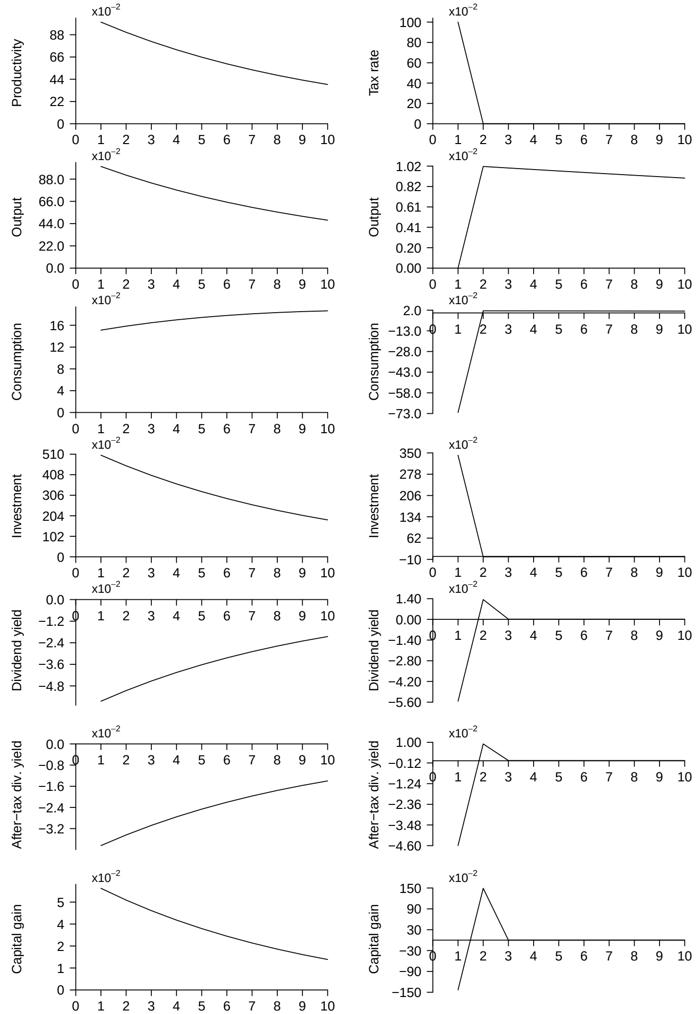
The responses to a 1% tax rate shock are different. Output reactions must be zero in the period when the shock arrives, since capital is determined before the tax rate changes. Even after the first period the reactions of output to a tax rate shock are very small compared to the reactions on the 1% productivity shock. Notice that in panel (a) the tax rate shock increases the tax rate in period 1 but the tax rate goes back to its prior level in the second period. Reactions to consumption and investments are very pronounced in both panels. A positive tax rate shock decreases consumption and increases investment. This effect is already described in Poterba and Summers (1984). They state that firms would avoid paying out dividends when dividend taxes are increased. Instead they invest the funds even though the projects may be of low productivity (Poterba and Summers, 1984, p. 18). This can also be derived from equilibrium conditions. For your convenience I rewrite Equation (3.33) with $a=0$ and $L_{t+1}=1$ and some simplifications as

$$\left(\frac{I_t}{\delta K_t}\right)^{\eta} = E_t\left\{\beta\frac{\Lambda_{t+1}}{\Lambda_t}\frac{1-\tau_{t+1}}{1-\tau_t}\left[\alpha Z_{t+1}K_{t+1}^{\alpha-1}+\frac{I_{t+1}}{K_{t+1}}\frac{\eta}{1-\eta}+\left(\frac{I_{t+1}}{\delta K_{t+1}}\right)^{\eta}\left(1-\delta\frac{1}{1-\eta}\right)\right]\right\}. \tag{3.50}$$

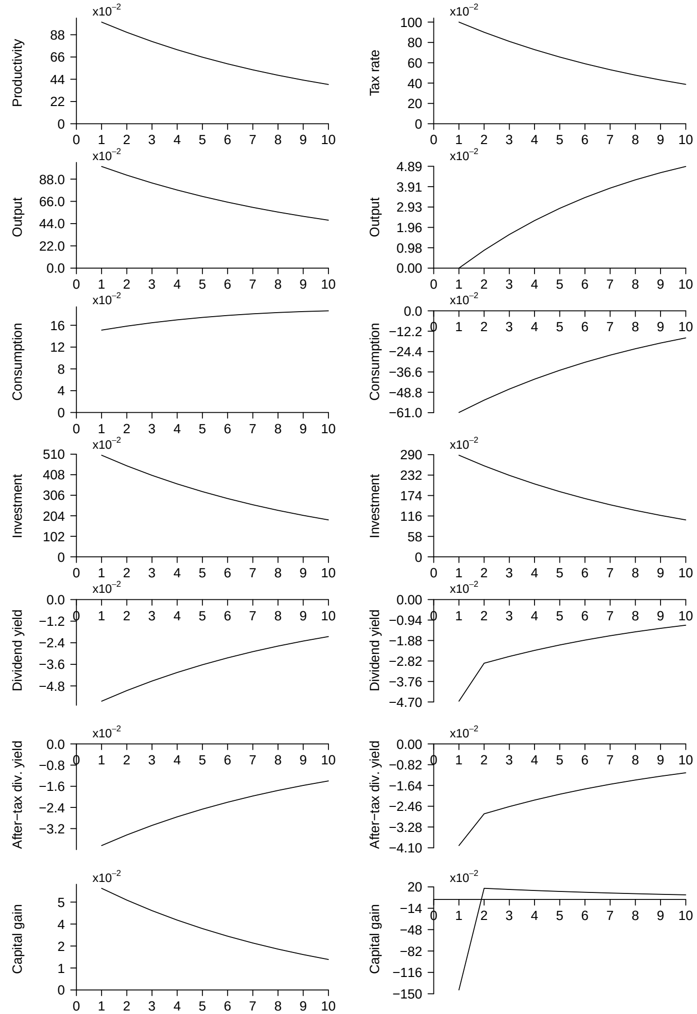
A tax rate shock, which is not persistent, at t increases τ_t and therefore increases the ratio $\frac{1-\tau_{t+1}}{1-\tau_t}$ through the decrease of the denominator. Since the shock is not persistent agents expect the tax rate to be back to normal in the next period. The increase in the tax rate ratio increases the rhs of Equation (3.50) and brings it out of equilibrium. Output at t is given so that the agent can act through an increase or decrease of investment, which would decrease or increase consumption at t. To bring Equation (3.50) back to equality investments I_t have to be increased, which increases the lhs. The decrease in consumption through the increase in investments additionally increases the ratio of marginal utilities on the rhs since the denominator its increased. Without a habit motive, consumption and investment are back to normal in the next period as can be observed in Figure 7 panel (a). With a persistent tax shock agents know that the tax rate in the next period τ_{t+1} will probably still be high. Thus, agents expect the ratio $\frac{1-\tau_{t+1}}{1-\tau_t}$ to increase only slightly. Therefore, any adjustements of the optimality condition through investments and consumption are lower but also persistent. That leads to lower volatility of consumption and investment than for not persistent shocks. This can be observed in Figure 7, comparing panel (a) with par. (4) than panel (b) with par. (5).

The same pattern can be observed in the other groups of models. One exception can be found in par. (29) and (30) in Table 7, where the ratios in (a) and (b) increase with persistence. In this case there are no adjustment costs but the habit parameter is high. Habits lead to persistent consumption and investment responses also for the temporary tax shock. With additional persistence of the tax rate, consumption smoothing dictates that consumption must be even farther away from its stationary state and that for a longer time. This leads to a stronger initial response of consumption and investments. Adding adjustment costs counters this effect again in that they decrease responses of investment to tax rate shocks. This way investment reactions to tax rate shocks become very small compared to responses to productivity shocks. For example in par. (19) and (20) of Table 3 there is no change at all of the ratios in (a) and (b). The respective impulse responses in Figure 8 show that responses of consumption and investment are stronger for tax rate persistence, but they are small compared to responses to productivity shocks of the same magnitude. Tax effects have a much smaller impact on the volatility ratios in (a) and (b) and the volatilities in (c), (f) and (i) in all of the other groups of parameterizations. Table 7 in the appendix shows that both, habits and adjustment cost contribute to this effect. The ratios in (a) and (b) show more variability for the first and for the third group than for the second and the forth. Increasing the parameter b or η lead both to less variability in the volatility ratios (a) and (b) within a group. As already mentioned, tax shocks that increase the tax rate induce firms to invest more and pay less dividends and vice versa. When people care about volatility of consumption, as with habits, firms react less over the investment channel so that dividends and therefore also consumption are less affected.

The impulse responses also show that the dividend yield reactions are only very small as compared to capital gain reactions. That contributes to that fact that pre- and after-tax returns have only minute differences in their volatilities (columns (d) and (i)), since they are mainly driven by the capital gain. This can also be observed for any other parameterization. Tax shocks cause dividend yields to decrease in the first period. The sudden increase of investment decreases



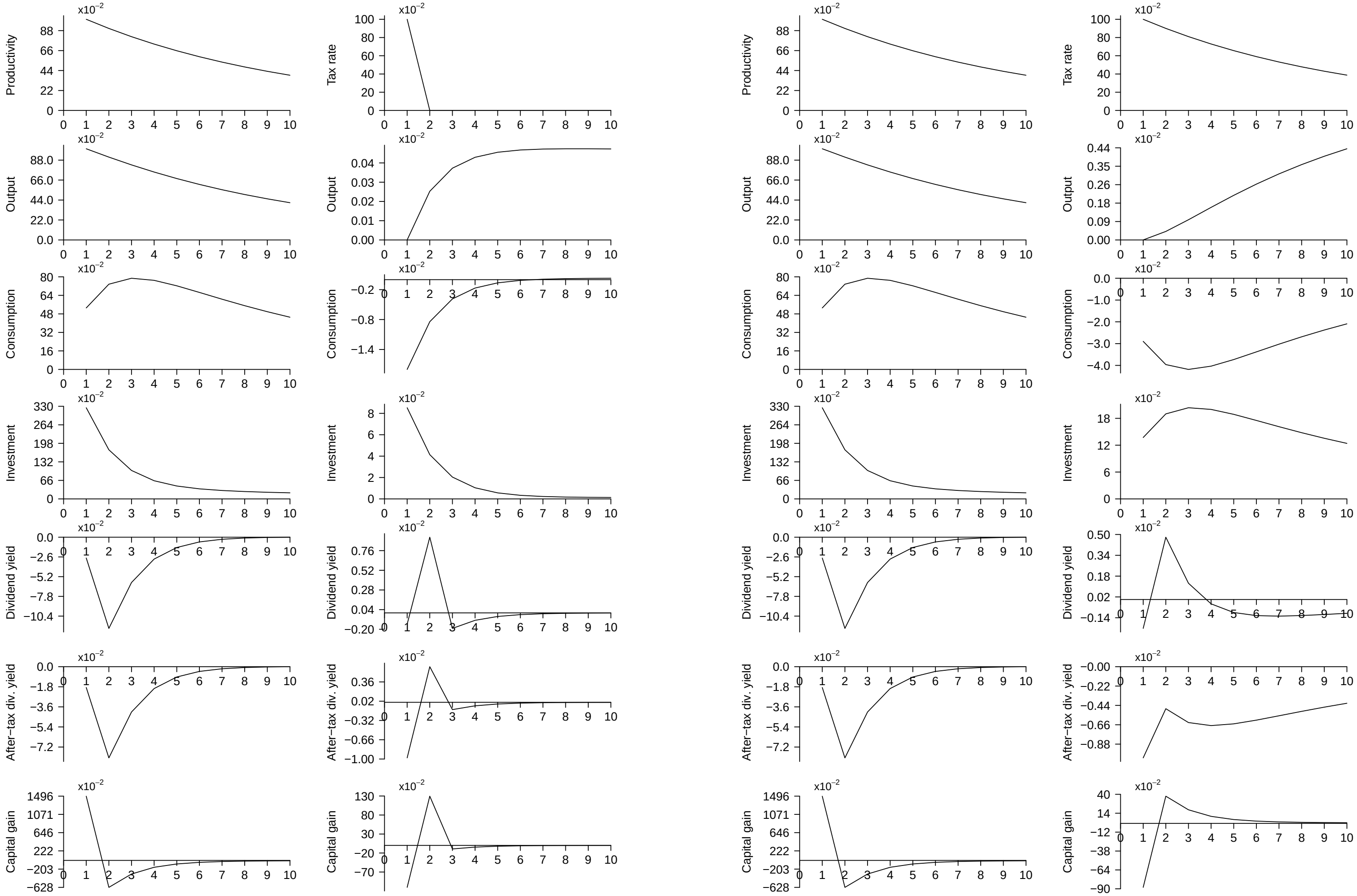
(a) Par. (4) ($\mu^{\tau} = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$)



(b) Par. (5) ($\mu^{\tau} = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$)

Figure 7: Impulse responses for the standard RBC model ($\eta = 0, b = 0$)

The impulse responses show the percentage deviations for productivity, the tax rate, ouput, consumption, and investment from the deterministic stationary state in percent. For pre- and after-tax dividend yields and the single-period capital gain the graphs show the difference to the deterministic stationary value in percent. The abscissa shows quarters.



(a) Par. (19) ($\mu^{\tau} = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$)

(b) Par. (20) ($\mu^{\tau} = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$)

Figure 8: Impulse responses for high habits and high adjustment costs ($\eta = 1/0.23, b = 0.8$)

The impulse responses show the percentage deviations for productivity, the tax rate, ouput, consumption, and investment from the deterministic stationary state in percent. For pre- and after-tax dividend yields and the single-period capital gain the graphs show the difference to the deterministic stationary value in percent. The abscissa shows quarters.

dividends. Capital gains also decrease in the first period due to a decrease in the equity price after the tax rate increase. In the second period the tax rate is back to its original level so that prices are also equal to the ones prior to the tax rate shock. Therefore, dividend yields and capital gains show a reverse reaction in period 2 and then take on their steady state levels again. With adjustment costs investments become expensive so that again tax shocks lead to a weaker response through investments and consumption. The conversion of investments into capital becomes more expensive so that investments are still increased but by a much lower amount. At the same time the decrease in consumption after a tax shock that increases the tax rate is less with adjustment cost.

With higher habits the representative agent avoids tax shock induced changes in after-tax dividends. Again, investment volatility must decrease as well.

The risk premium is higher for the models with a high habit parameter b. The high habit group produces substantial risk premiums of more than 1.3% per quarter. The introduction of tax rate volatility increases expected equity returns in some cases or they are not notably affected. The risk free-rate shows close to no responsiveness at all to tax rate volatility or even to the introduction of a certain tax rate. Increases of the expected return on equity and the equity premium are small for all groups. For the high habit group the pre-tax risk premium for par. (17) with a certain tax rate is 1.30%. For the models with an uncertain tax rate the premiums vary from 1.30% to 1.33%, i.e., zero to three basis points higher. For the standard RBC model the (pre-tax) equity premium increases up to four basis points, the after-tax premium up to five basis points. With habits or adjustment costs only small adjustments of investments are necessary to bring the first order conditions back to equality after a tax rate shock. This small adjustment does not produce much additional volatility of the SDF and, therefore, does not increase the equity premium substantially.

Eventually, the tax capitalization effect is much more important to explain the (pre-tax) equity premium than the volatility of the tax rate itself. Economically, the effect of tax rate volatility on the equity premium is small for all of the parameterizations.

3.5.4 The Case without Full Tax Transfers

I regard two groups of parameterizations, the standard RBC and the high habit group. Table 5 shows the results for the two groups. There is an additional column showing the values of the parameter ω, the share of tax payments invested in the public good. For ease of comparability, I restate the results for some parameterizations from the prior case, i.e., for $\omega = 0$. I am mainly interested in the effect of the variation in ω. The table shows that the ratio of consumption growth and output growth volatility increases slightly with increasing ω. The ratio of investment growth and output growth volatility increases substantially.

In the standard RBC model ω has no perceivable effect on expected returns and volatilities. From the budget constraints we know that ω has an effect on consumption and investment. This

Table 5: Results for business cycle variables and asset returns for the case without full tax transfers (sensitivity to ω)

Group	**N.**	**Tax parameters**	ω	$\frac{\sigma_{\Delta c}}{\sigma_{\Delta y}}$ (a)	$\frac{\sigma_{\Delta i}}{\sigma_{\Delta y}}$ (b)	$E[r^S]$ (c)	σ_{r^S} (d)	$E[r^B]$ (e)	σ_{r^B} (f)	$E[r^S - r^B]$ (g)	$E[r^{S,\tau}]$ (h)	$\sigma_{r^{S,\tau}}$ (i)	$E[r^{S,\tau} - r^B]$ (j)
Stand. RBC	(2)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	0	0.15	5.10	0.89	0.02	0.60	0.03	0.28	0.60	0.03	0.00
$\eta = 0, b = 0$	(2i)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	0.5	0.17	5.69	0.89	0.03	0.60	0.03	0.28	0.60	0.03	0.00
	(2ii)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	1	0.19	6.55	0.89	0.04	0.60	0.03	0.28	0.60	0.04	0.00
	(4)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	0	1.13	7.38	0.90	1.69	0.60	1.18	0.30	0.62	1.67	0.01
	(4i)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	0.5	1.13	8.06	0.90	1.68	0.60	1.18	0.30	0.62	1.68	0.01
	(4ii)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	1	1.13	9.08	0.90	1.68	0.60	1.18	0.30	0.62	1.67	0.01
	(5)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	0	0.70	6.06	0.89	1.22	0.60	0.27	0.29	0.61	1.22	0.01
	(5i)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	0.5	0.70	6.65	0.89	1.22	0.60	0.26	0.29	0.61	1.22	0.01
	(5ii)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	1	0.71	7.52	0.89	1.22	0.60	0.27	0.29	0.61	1.22	0.01
High habit,	(17)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	0	0.58	3.54	1.54	11.55	0.24	5.46	1.30	1.26	11.54	1.02
high adj.	(17i)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	0.5	0.60	3.84	1.65	12.54	0.20	6.13	1.46	1.37	12.53	1.17
$b = 0.8$,	(17ii)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	1	0.62	4.26	1.83	13.95	0.13	7.01	1.70	1.55	13.93	1.41
$\eta = 1/0.23$	(19)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	0	0.58	3.54	1.55	11.63	0.24	5.57	1.31	1.26	11.62	1.02
	(19i)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	0.5	0.60	3.86	1.69	12.86	0.30	6.39	1.49	1.41	12.85	1.21
	(19ii)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	1	0.63	4.33	1.92	14.65	0.15	7.71	1.77	1.64	14.64	1.49
	(20)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	0	0.59	3.54	1.54	11.55	0.24	5.50	1.30	1.26	11.54	1.02
	(20i)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	0.5	0.60	3.85	1.67	12.70	0.16	6.06	1.52	1.39	12.69	1.23
	(20ii)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	1	0.63	4.28	1.86	14.18	0.12	7.08	1.74	1.58	14.17	1.45

The parameter ω indicates the share of taxes that is invested in the public good, i.e. $1-\omega$ is the share of tax payments that is paid out as transfer payments. For better comparability, I also show some of the cases with $\omega = 0$, which correspend to parameterizations from Table 4. All values are at quarterly frequency. I denote $\sigma_{\Delta c}$ the standard deviation of consumption growth, $\sigma_{\Delta y}$ the standard deviation of output growth, and $\sigma_{\Delta i}$ the standard deviation of investment growth. Expected returns and standard deviations are given in percent. Furthermore, $E[r^S]$ and σ_{r^S} are the unconditional expectation and the standard deviation, respectively, of the single-period return on equity. The superscript B indicates the same for a single-period bond returns. The equity premium is the unconditional mean $E[r^S - r^B]$. The last three columns show mean equity returns, standard deviation and the equity premium for after-tax returns. All values are obtained from 40000 simluations.

can be seen in the increase in the variability of investments. However, investments can still be used to smoothen consumption and the stochastic discount factor. Therefore, the equity premium remains largely unaffected. However, for the habit model all ratios and moments of returns increase in ω in an economically significant way. For example, from par. (17) with $\omega = 0$ to (17ii) with $\omega = 1$ the pre-tax equity premium increases by 40 basis points from 1.30% to 1.70%. For a volatile tax rate the increase is about the same magnitude. Since tax rate volatility does not seem to have a first order effect, I only look at the impulse responses to a productivity shock. Figure 9 shows that with $\omega = 1$ (panel (b)) investments and capital gains react much stronger than with $\omega = 0$ (panel (a)), whereas little volatlity is added to consumption.

For this effect of changes in ω on the equity premium to happen an equity premium is necessary in the first place. Thus, it is again a combination of habits and adjustment costs that leads to the higher equity premium when ω is increased. An unreported experiment shows that for low habits changing ω from zero to one does not have a substantial effect. Since a substantial amount of consumption is lost to the public good the investment channel has even more work to do to smoothen consumption. With adjustment costs this channel cannot be fully utilized without incurring additional costs and utility loss. Thus, this serves as another way to trigger volatility of the SDF. Column (d) with the volatility of the bond rate is a indirect indicator for the additional volatility of the SDF.

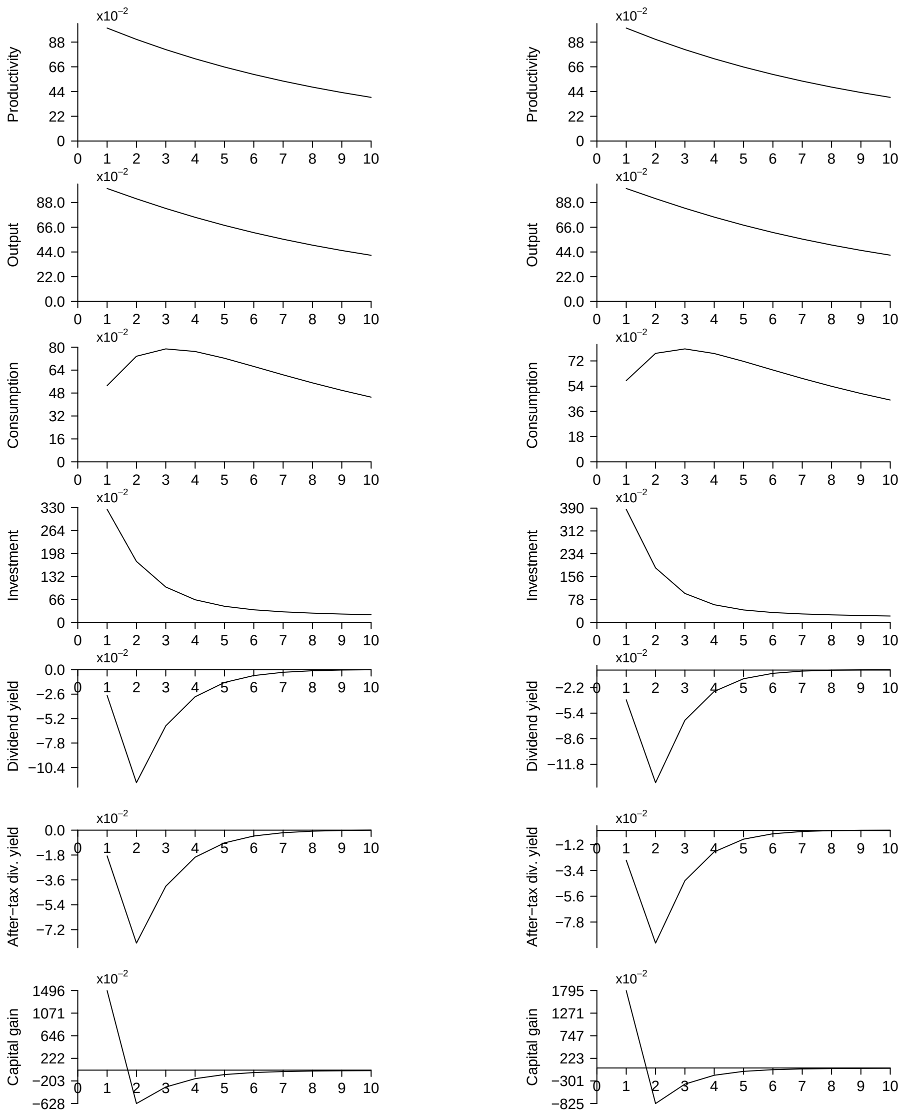
Eventually, increases in the equity premium are substantial when taxes cannot be used for regular consumption and when habits and adjustment costs are present. This effect is big as long as the fraction of transfer payments of the whole tax revenues is small and investments in the public good are high.

3.6 Extensions and Limitations of the Analysis

I analyze the impact of several parameters that were left unchanged in the prior analyses. I give a short comparison of the effects of a volatile tax rate on dividends to effects of a volatile tax rate on corporate profits. I also discuss the limitations of the model as it is used herein.

3.7 Growth

For simplicity, I have assumed a zero growth rate: $a = 0$. Increasing a, one can obtain higher asset returns. One can also generate higher equity premiums without sending the risk-free rate below zero (Heer and Maussner, 2009, p. 323). I do not present a table on the results, since I cannot find a different pattern or different magnitudes of effects caused by the introduction of a volatile tax rate. I use the high habit parameterization and introduce a quarterly growth rate of 0.5%. For the no-tax case this leads to a risk premium of 0.70%. With the introduction of taxes and volatile taxes nothing changes with respect to the already observed patterns or magnitudes. The after-tax equity premium with a volatile tax rate is 0.73% for $\sigma_u = 0.016$, which is about three basis points higher than the equity premium with certain taxes. For $\sigma_u = 0.008$, the risk



(a) Par. (17) ($\omega = 0$) (b) Par. (17ii) ($\omega = 1$)

Figure 9: Impulse responses for high habits and high adjustment costs ($\eta = 1/0.23, b = 0.8, \mu^{\tau} = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$)

The impulse responses show the percentage deviations for productivity, the tax rate, ouput, consumption, and investment from the deterministic stationary state in percent. For pre- and after-tax dividend yields and the single-period capital gain the graphs show the difference to the deterministic stationary value in percent. The abscissa shows quarters.

premium does not change notably.

3.8 Risk Aversion

Table 6 shows the sensitivity with respect to the risk aversion parameter γ for high habits and high adjustment costs. The table shows that higher risk aversion pushes down consumption volatility. More risk averse agents prefer less volatile consumption streams. To achieve that investments become more volatile. Expected returns of the equity asset increase and the ones for the bond decrease, so that the risk premiums increase. With higher risk aversion tax rate volatility seems to add some more basis points to the risk premium. For $\gamma = 3$ par. (18gi) generates a (pre-tax) risk premium of 1.62%, which is three basis points higher than the one with a certain tax rate (par. (17gi)) of 1.59%. For $\gamma = 0.5$ this difference is only one basis point.

3.8.1 Risk Aversion

Table 6 shows the sensitivity with respect to the risk aversion parameter γ for high habits and high adjustment costs. The table shows that higher risk aversion pushes down consumption volatility. More risk averse agents prefer less volatile consumption streams. To achieve that investments become more volatile. Expected returns of the equity asset increase and the ones for the bond decrease, so that the risk premiums increase. With higher risk aversion tax rate volatility seems to add some more basis points to the risk premium. For $\gamma = 3$ par. (18gi) generates a (pre-tax) risk premium of 1.62%, which is three basis points higher than the one with a certain tax rate (par. (17gi)) of 1.59%. For $\gamma = 0.5$ this difference is only one basis point.

3.8.2 Corporate Taxes

The corporate tax rate can also be analyzed in this framework. The first order conditions and the steady state values need to be adjusted a bit (compare for example Santoro and Wei (2011)). Santoro and Wei (2011) find that the effect of a certain corporate tax rate on business cyclce variables and asset returns is due to changes in steady state values induced by the corporate tax rate. I will not go to deep into the details of the effects of a corporate tax rate because this is out of the scope of this work. However, I will point at some differences to the analysis of the uncertain dividend tax rate. One is that with corporate taxes the (mean) tax rate shows up in the steady state value for capital (compare Equation (16) in Santoro and Wei (2011)), whereas the (mean) tax rate for dividends does not affect steady state value for capital (compare Equation (3.46)). With corporate taxes, the Lagrangian from Equation (3.11) turns to

$$\mathcal{L} = E_0\left\{\sum_{t=0}^{\infty}\beta^t\frac{\Lambda_t}{\Lambda_0}\Big[(1-\tau_t)\big[Z_tF(K_t, A^tL_t) - w_tL_t\big] - I_t + q_t\big[\phi(K_t, I_t) + (1-\delta)K_t - K_{t+1}\big]\Big]\right\}. \tag{3.51}$$

Since taxes are subtracted before investments are made, the derivative with respect to investments does not involve a tax term. The final first order condition of the firm turns to

$$\frac{1}{\phi_{t,2}} = E_t\left\{\beta\frac{\Lambda_{t+1}}{\Lambda_t}\left[(1-\tau_{t+1})Z_{t+1}F_{t+1,1} + \frac{\phi_{t+1,1}+1-\delta}{\phi_{t+1,2}}\right]\right\}. \tag{3.52}$$

The result can also be found in Santoro and Wei (2011) Equation (15) for a certain tax rate on corporate profits. Notice that the tax rate at time t does not show up in any equilibrium or first order condition. Only the future tax rate at $t+1$ matters. It follows that for any effects of tax rate volatility the characteristics of the stochastic part of the tax rate process, i.e., the autcorrelation of the autoregressive part of the tax rate process and covariance of tax rate shocks with

Table 6: Results for business cycle variables and asset returns for the case with full tax transfers (sensitivity to risk aversion)

Group	**N.**	**Tax parameters**	$\frac{\sigma_{\Delta c}}{\sigma_{\Delta y}}$ (a)	$\frac{\sigma_{\Delta i}}{\sigma_{\Delta y}}$ (b)	$E[r^S]$ (c)	σ_{r^S} (d)	$E[r^B]$ (e)	σ_{r^B} (f)	$E[r^S - r^B]$ (g)	$E[r^{S,\tau}]$ (h)	$\sigma_{r^{S,\tau}}$ (i)	$E[r^{S,\tau} - r^B]$ (j)
High habit,	(16g)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	0.86	2.06	0.82	6.66	0.54	3.92	0.29	NA	NA	NA
high adj.	(17g)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	0.86	2.06	1.11	6.69	0.54	3.91	0.57	0.82	6.69	0.29
$b = 0.8$,	(18g)	$\mu^\tau = 0.32, \sigma_u = 0.016, \rho^{\tau^*} = 0$	0.87	2.12	1.12	6.92	0.54	4.16	0.58	0.84	6.91	0.30
$\eta = 1/0.23$	(19g)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	0.87	2.07	1.11	6.74	0.54	3.98	0.57	0.83	6.73	0.29
$\gamma = 0.5$	(20g)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	0.86	2.08	1.11	6.71	0.54	3.91	0.57	0.83	6.71	0.29
High habit,	(16)	$\mu^\tau = 0, \sigma_u = 0, \rho^{\tau^*} = 0$	0.58	3.54	1.26	11.58	0.24	5.45	1.02	NA	NA	NA
high adj.	(17)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	0.58	3.54	1.54	11.55	0.24	5.46	1.30	1.26	11.54	1.02
$b = 0.8$,	(18)	$\mu^\tau = 0.32, \sigma_u = 0.016, \rho^{\tau^*} = 0$	0.58	3.54	1.58	11.88	0.25	5.83	1.33	1.29	11.87	1.05
$\eta = 1/0.23$	(19)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	0.58	3.54	1.55	11.63	0.24	5.57	1.31	1.26	11.66	1.02
$\gamma = 2$	(20)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	0.59	3.54	1.54	11.55	0.24	5.50	1.30	1.26	11.54	1.02
High habit,	(16gi)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	0.50	3.92	1.40	12.79	0.10	5.59	1.30	NA	NA	NA
high adj.	(17gi)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	0.50	3.92	1.69	12.83	0.10	5.57	1.59	1.41	12.82	1.30
$b = 0.8$,	(18gi)	$\mu^\tau = 0.32, \sigma_u = 0.016, \rho^{\tau^*} = 0$	0.50	3.92	1.73	13.18	0.11	5.97	1.62	1.45	13.17	1.34
$\eta = 1/0.23$	(19gi)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	0.50	3.93	1.70	12.93	0.10	5.66	1.60	1.42	12.92	1.32
$\gamma = 3$	(20gi)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	0.50	3.92	1.69	12.84	0.10	5.59	1.59	1.40	12.83	1.30

All values are at quarterly frequency. I denote $\sigma_{\Delta c}$ the standard deviation of consumption growth, $\sigma_{\Delta y}$ the standard deviation of output growth, and $\sigma_{\Delta i}$ the standard deviation of investment growth. Expected returns and standard deviations are given in percent. Furthermore, $E[r^S]$ and σ_{r^S} are the unconditional expectation and the standard deviation, respectively, of the single-period return on equity. The superscript B indicates the same for a single-period bond returns. The equity premium is the unconditional mean $E[r^S - r^B]$. The last three columns show mean equity returns, standard deviation and the equity premium for after-tax returns. All values are obtained from 40000 simluations.

productivity shocks, are important drivers. Furthermore, the workings of an uncertain corporate tax rate have already been analyzed by Croce et al. (2012). They use recursive preferences and several additional complexities such as interest deductability of taxes, influences of taxes on productivity, and government debt. They only regard a persistent tax rate. With impulse repsonses they show, that a tax shock that increases the tax rate decreases consumption, increases investments, and decreases ex-post returns on equity. This is exactly the opposite to a tax shock on dividends. Figure 10 confirms those reactions for the model used herein, in which I changed the dividend tax rate for the tax rate on corporate profits. The impulse response uses the standard parameterization as well as $b = 0.8, \eta = 1/0.23$ and $\omega = 0$. One can observe that the effects of the one percent tax rate shock on output, consumption, investment, dividend yields and capital gain are very small compared to a productivity shock. Thus, the additional effect of tax rate volatility on expected returns and volatilities can also expected to be small.

3.8.3 Limitations

As every model, the one presented here is limited by its assumptions, which abstract it from reality. However, this abstraction is necessary to be able isolate and to understand causes and effects. Heer and Maussner (2009, p. 323) mention that the fixed labor supply is a major assumption to cause a risk premium. With variable labor, the representative agent would again have a channel to compensate shocks apart from the investment channel. Constraints to the variability of labor supply as in Boldrin et al. (2001) are necessary to establish a risk premium again. Furthermore, the government either redistributes funds or invests a fixed fraction of the tax revenue. Actions of the government can be endogenized and the possibility of government debt can be introduced. With government debt the need for (future) tax revenues becomes more pressing to manage the government debt service. For example Croce et al. (2012) assume a more complex set of fiscal policies. The approach taken here, with an exogenous tax rate is more in line with the view that interest groups influence tax rates in one or the other way as mentioned in Sialm (2006) and Bizer and Judd (1989). This influence would probably make up only a part of the variability of tax rates. Croce et al. (2012) also regard the impact of the tax rate on productivity. At the same time it would be possible to introduce a cyclicality effect into the tax rate. Thus, there are different possible drivers of tax rate uncertainty. As Santoro and Wei (2011)
I assume that the firm maximizes the after-tax value of dividends, and shapes its investment policy accordingly. This is related to numerous practical problems, so that a maximization of the pre-tax value maybe closer to reality. In this case the tax rate terms would not show up in the FOC for the firm so that investments would not be influenced. However, shareholders still can only consume the after-tax dividends, so that changes in the tax rate matter for them and matter for the valution of the firm. Eventually, the uncovered theoretical effects need to be addressed in econometric tests.

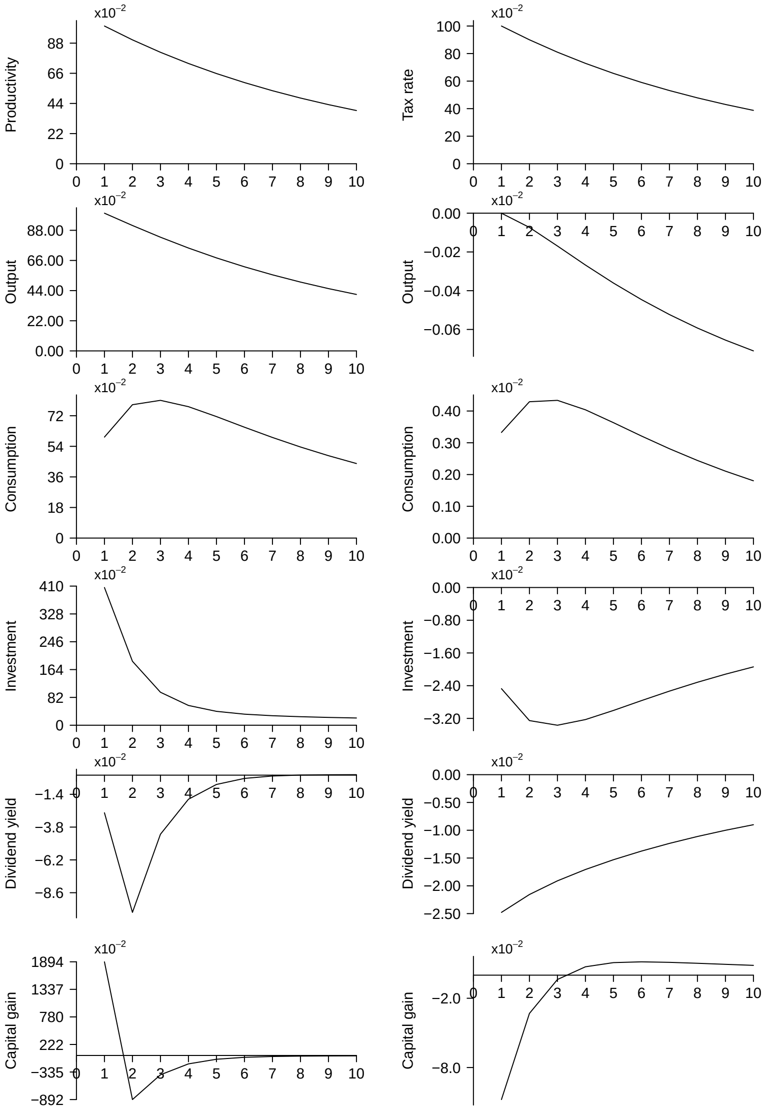


Figure 10: Impulse responses for high habits and high adjustment costs ($\eta = 1/0.23, b = 0.8, \mu^{\tau} = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$) to a 1% shock on the tax rate on corporate profits

The impulse responses show the percentage deviations for productivity, the tax rate, ouput, consumption, and investment from the deterministic stationary state in percent. For pre- and after-tax dividend yields and the single-period capital gain the graphs show the difference to the deterministic stationary value in percent. The abscissa shows quarters.

3.9 Conclusion

I analyze the impact of an uncertain tax rate on dividends on business cycle variables and asset returns. For the standard RBC model without habits and without adjustment costs, I find that tax rate volatility substantially increases volatilies of business cycle variables, such as consumption and investment growth. However, this effect disappears with a high habit parameter and high adjustment costs. Tax rate volatility has only very limited potential to increase the equity premium. Even with an unreasonably high volatility of tax rate shocks and for any parameterization, the increase in the equity premium is never more than a few basis points. Effects of tax rate capitalization in the pre-tax equity premium are much stronger than effects of tax rate volatility. With a certain tax rate, the representative agent earns an after-tax equity premium that is as high as the equity premium in a no-tax world. To earn that premium the agent requires a substantially higher pre-tax equity premium. For a certain tax rate of 32% used herein the capitalization effect increases the pre-tax equity premium by about 28 basis points.

When tax rates are not fully transferred back, steady state consumption is decreased. Investment growth becomes more volatile versus output growth the lesser the amount that is transferred. In the standard RBC model there is no effect at all on asset returns. However, for high habits and high adjustment costs expected returns on equity increase, the risk-free rate decreases and volatilities increase with less transfer payments. The equity premium increases as well, and depending on the share of taxes not transferred the increases can be substantial with more than 40 basis points for some parameterizations and a decrease in transfer payments from 100% to zero.

I conduct several sensitivity analyses, which do not reveal any changes to the observed patterns. Volatility of the corporate tax rate is also not likely to have a major impact on asset pricing. Sources of tax rate volatility can be manifold. I suggest to identify those sources through empirical test.

3.10 Appendix to Chapter 3

3.10.1 Some Remarks on the Numerical Procedure

I use a quadratic approximation of the policy function of the model as described in Heer and Maussner (2009, pp. 106-131). I adapt the algorithms provided in the book and the code provided on `https://www.wiwi.uni-augsburg.de/de/vwl/maussner/dge_buch/dge_book_2ed/downloads_2nd/` in R programming language. The usefulness and the limitations of a second order approximation are described in Schmitt-Grohé and Uribe (2004).

State variables are k_t, s_t, Λ_t, z_t, and τ_t^*, whereas $s_t = c_{t-1}$ captures the lagged consumption and is used to reduce the variables in the equilibrium conditions to variables at t and $t+1$. The variables k_t and s_t are predetermined state variables, i.e., they are already known at the beginning of period t, whereas Λ_t is a non-predetermined variable. Exogenous variables are the natural logarithm of productivity $z_t = \ln(Z_t)$ and the demeaned tax rate τ_t^*. I approximate the conditional expectation $E_t[\Lambda_{t+1}]$ that is necessary to obtain the risk-free rate using a six point Gauss Hermite Quadrature approximation as described in Heer and Maussner (2009, p. 600). Therefore, I use the fact that all of the error terms are normally distributed.

I use the policy functions to compute 40,000 simulations. Repeating the simulations shows that the means and standard deviations are stable and do not change in any economically significant way. I compute means and standard deviations of real variables and of asset returns and provide them in the respective tables above.

I choose a second order approximation because of relative speed versus other methods and its suficient precision for the purpose of this paper. I tested the model with the parameterization used in (Heer and Maussner, 2009, p. 322). For the parameterizations in Table 6.4 in (Heer and Maussner, 2009, p. 322) I obtain values that are zero to three basis points off. The biggest differences arise for the very nonlinear model with a high parameter b. The values computed in the table in Heer and Maussner (2009, p. 322) are obtained using a projection method as opposed to the quadratic approximation used herein.

Using the parameters for the computations in Table 3 of the Appendix in Jermann (1998) I can match all the ratios of the standard deviations of the real values except for the last parameterization, where I obtain a value for the ratio of the standard deviation of consumption growth and output growth that is about 0.16 greater and a value that is about 0.27 lower for the ratio of standard deviations of investment and output growth. For the second and forth parameterization I obtain much higher values for expected returns and risk premiums[25]. This is due to the different method applied by Jermann (1998) in computing asset returns. More specificall, Jermann (1998) describes his approach as using a loglinear or, in the appendix, a nonlinear approach to determine the real values of the model. In a second step he computes moments of asset returns

[25] For the first and third parameterization there is substantiallly no risk premium, which is in line with my observations.

using analytical results that rest on the assumption that the stochastic discount factor and asset returns are jointly log-normally distributed. Heer and Maussner (2012) show in numerical experiments that this assumption leads to risk premiums that are about one third less than using non-linear approximations of the Euler equations. Since I do not rely on the assumption of joint log-normality, their finding explains the higher differences.

3.10.2 Additional Table

The additional table shows sensitivities to habit and adjustment cost parameters. It is displayed on the following page.

Table 7: Results for business cycle variables and asset returns for the case with full tax transfers (sensitivities to habits and adjustment costs)

Group	**N.**	**Tax parameters**	$\frac{\sigma_{\Delta c}}{\sigma_{\Delta y}}$	$\frac{\sigma_{\Delta i}}{\sigma_{\Delta y}}$	$E[r^S]$	σ_{r^S}	$E[r^B]$	σ_{r^B}	$E[r^S - r^B]$	$E[r^{S,\tau}]$	$\sigma_{r^{S,\tau}}$	$E[r^{S,\tau} - r^B]$
			(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
Low habit,	(21)	$\mu^\tau = 0, \sigma_u = 0, \rho^{\tau^*} = 0$	0.14	5.71	0.60	0.03	0.60	0.03	0.00	NA	NA	NA
no adj. cost	(22)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	0.14	5.71	0.89	0.03	0.60	0.03	0.28	0.60	0.03	0.00
$\eta = 0, b = 0.1$	(23)	$\mu^\tau = 0.32, \sigma_u = 0.016, \rho^{\tau^*} = 0$	1.75	9.80	0.94	3.38	0.60	2.37	0.34	0.66	3.37	0.06
	(24)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	0.88	6.64	0.90	1.68	0.60	1.18	0.30	0.62	1.67	0.01
	(25)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	0.62	5.94	0.89	1.22	0.60	0.27	0.29	0.61	1.21	0.01
High habit,	(26)	$\mu^\tau = 0, \sigma_u = 0, \rho^{\tau^*} = 0$	0.06	5.66	0.60	0.03	0.60	0.03	0.00	NA	NA	NA
no adj. cost	(27)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	0.06	5.66	0.89	0.03	0.60	0.03	0.28	0.60	0.03	0.00
$\eta = 0, b = 0.8$	(28)	$\mu^\tau = 0.32, \sigma_u = 0.016, \rho^{\tau^*} = 0$	0.09	5.67	0.94	3.36	0.60	2.37	0.34	0.66	3.37	0.06
	(29)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	0.07	5.67	0.90	1.68	0.60	1.18	0.30	0.62	1.68	0.01
	(30)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	0.15	5.71	0.89	1.21	0.60	0.27	0.30	0.61	1.21	0.01
Low adj. cost,	(31)	$\mu^\tau = 0, \sigma_u = 0, \rho^{\tau^*} = 0$	1.01	0.96	0.61	1.43	0.59	0.34	0.02	NA	NA	NA
no habit	(32)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	1.01	0.96	0.90	1.44	0.59	0.33	0.30	0.61	1.44	0.02
$\eta = 2, b = 0$	(33)	$\mu^\tau = 0.32, \sigma_u = 0.016, \rho^{\tau^*} = 0$	1.08	2.07	0.90	1.56	0.59	0.53	0.31	0.62	1.56	0.02
	(34)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	1.03	1.33	0.90	1.47	0.59	0.39	0.30	0.61	1.47	0.02
	(35)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	1.02	1.14	0.90	1.47	0.59	0.33	0.30	0.61	1.47	0.02
High adj. cost,	(36)	$\mu^\tau = 0, \sigma_u = 0, \rho^{\tau^*} = 0$	1.10	0.53	0.62	1.60	0.59	0.36	0.02	NA	NA	NA
no habit	(37)	$\mu^\tau = 0.32, \sigma_u = 0, \rho^{\tau^*} = 0$	1.10	0.53	0.90	1.60	0.59	0.36	0.31	0.62	1.60	0.02
$\eta = 4, b = 0$	(38)	$\mu^\tau = 0.32, \sigma_u = 0.016, \rho^{\tau^*} = 0$	1.12	1.14	0.90	1.64	0.59	0.43	0.31	0.62	1.63	0.03
	(39)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0$	1.10	0.74	0.90	1.61	0.59	0.38	0.31	0.62	1.61	0.02
	(40)	$\mu^\tau = 0.32, \sigma_u = 0.008, \rho^{\tau^*} = 0.9$	1.10	0.63	0.90	1.62	0.59	0.36	0.31	0.62	1.61	0.02

All values are at quarterly frequency. I denote $\sigma_{\Delta c}$ the standard deviation of consumption growth, $\sigma_{\Delta y}$ the standard deviation of output growth, and $\sigma_{\Delta i}$ the standard deviation of investment growth. Expected returns and standard deviations are given in percent. Furthermore, $E[r^S]$ and σ_{r^S} are the unconditional expectation and the standard deviation, respectively, of the single-period return on equity. The superscript B indicates the same for a single-period bond returns. The equity premium is the unconditional mean $E[r^S - r^B]$. The last three columns show mean equity returns, standard deviation and the equity premium for after-tax returns. All values are obtained from 40000 simulations.

4 Conclusion

The results of my analyses were stated explicitly several times so that I will not repeat them here. I use this part to give a more general view on the limitations of the two parts of my research and possible implications for future research.

4.1 Limitation of the Thesis

The two presented papers show two different drivers of tax rate uncertainty. One driver is output growth together with a cyclical tax policy. The other one is a process of exogenous tax rate shocks, which may arise from the varying power of different interest groups. Thus, the analyses are limited to those two forms of tax rate uncertainty. There may be other drivers such as government debt. A higher debt level might lead to pressure to increase tax rates to be able to pay interest and principal to the debtholders.

Not only government debt but also the degree of leverage of the firm can lead to additional effects. Together with taxes on corporate profits, outstanding debt leads to tax savings on interest payments. Leverage increases the expected return on equity that, in turn, has an increasing effect on the equity premium.

Thus, the papers do not take into account alternative drivers of tax rate volatility and alternative drivers of the equity premium. I do not claim that those limitations are complete. One can think of many changes such as another utility function or the inclusion of labor. However, the limitations mentioned are closest to the topic. They also give scope for further research. Apart from formal representative agent RBC models, I can also identify other interesting research areas, some of which I will address in the following.

4.2 Possible Areas for Future Research

The possibility that interest groups are responsible for changes in tax rates is stated in Bizer and Judd (1989) and Sialm (2006). As an area for further research, I think it is important to find out to which extend interest groups influence the process of setting tax rates. The presence of interest groups poses many new question related to equity, justice and also to economic efficiency since resources are used up unproductively in this rent-seeking process.

Portfolio theory is intimately related to asset pricing. However, representative agent models are not useful to state something about an optimal portfolio in a general equilibrium context. Partial equilibrium models that consider taxes and tax rate movements can be helpful for investors in a normative way. Models in the style of Campbell and Viceira (2003) may be a possible way to go. Heterogeneous agent models may be able to explain portfolio holdings in a general equilibrium model. For asset pricing under tax rate uncertainty investment horizons are important. For a long-term investor tax rates, which change at an annual frequency, are very important. The short-term investor may already have sold all portfolios holdings before any changes of the tax rate take place. A long-term investor, who takes tax rate movements into account, may hold a different portfolio from someone who disregards tax rate changes. All of those considerations may play an important role when taxes are included in portfolio problems.

To find out whether tax rate movements are really taken into account is an econometric task. The challenge is here to make sense of the aggregate data that is mostly only available. For example for the U.S. there are different tax brackets for different income classes (Sialm (2006)), a fact that already complicates the analyses.

Eventually, taxes are often forms of redistributing income. Welfare considerations, which often come along with taxes, can also be extended to a tax rate that is not constant over time. Bizer and Judd (1989) made some comments on that issue, but they can only be considered a starting point.

References

Becker, G. S., August 1983. A theory of competition among pressure groups for political influence. The Quarterly Journal of Economics 98 (3), 371–400.

Bizer, D. S., Judd, K. L., May 1989. Taxation and uncertainty. The American Economic Review 79 (2), 331.336.

Boldrin, M., Chirstiano, L. J., Fisher, J. D. M., 2001. Habit persistence, asset returns, and the business cycle. American Economic Review 91 (1), 149–166.

Brennan, M. J., 1970. Taxes, market valuation and corporate financial policy. National Tax Journal 23 (4), 417–427.

Campbell, J. Y., Viceira, L. M., 2003. Strategic Asset Allocation: Portfolio Choice for Long-Term Investors. Oxford University Press.

Cochrane, J. H., 1991. Production-based asset pricing and the link between stock returns and economic fluctuations. The Journal of Finance 46 (1), 209–237.

Cochrane, J. H., 2005. Asset Pricing / Revised Edition. Princeton University Press.

Croce, M. M., Kung, H., Nguyen, T. T., Schmid, L., September 2012. Fiscal policy and asset prices. The Review of Financial Studies 25 (9), 2635–2672.

Fernández-Villaverde, J., Guerrón-Quintana, P., Kuester, K., Rubio-Ramírez, J., 2015. Fiscal volatility shocks and economic activity. American Economic Review 105 (11), 3352–3384.

Furceri, D., Karrast, G., 2011. Average tax rate cyclicality in oecd countries: A test of three fiscal policy theories. Southern Economic Journal 77 (4), 958–972.

Grossman, S. J., Shiller, R. J., 1981. The determinants of the variability of stock market prices. American Economic Review 71, 222–227.

Hasset, K. A., Metcalf, G. E., July 1999. Investment with uncertain tax policy: Does random tax policy discourage investment? The Economic Journal 109, 372–393.

Heer, B., Maussner, A., 2009. Dynamic General Equilibrium Modeling: Computational Methods and Applications, 2nd Edition. Springer.

Heer, B., Maussner, A., 2012. Log-normal approxmation of the equity premium in the production model. Applied Economics Letters 19 (5), 407–412.

Jermann, U. J., 1998. Asset pricing in production economies. Journal of Monetary Economics 41, 257–275.

King, R. G., Plosser, C. I., Rebelo, S. T., 1988. Production, growth and business cycles. Journal of Monetary Economics 21, 195–232.

Klein, P., 2001. The capital gain lock-in effect and long-horizon return reversal. Journal of Financial Economics 59, 33–62.

McGrattan, E. R., Prescott, E. C., 2005. Taxes, regulations, and the value of u.s. and u.k. corporations. The Review of Economics Studies 72 (3), 767–796.

Moldovan, I., 2010. Countercyclical taxes in a monopolicitcally competitive environment. European Economic Review 54 (5), 692–717.

Moldovan, I. R., August 2006. Countercyclical tax policies. Ph.D. thesis, Indiana University.
URL http://media.proquest.com

Pastor, L., Veronesi, P., August 2012. Uncertainty about government policy and stock prices. The Journal of Finance 67 (4), 1219–1264.

Poterba, J. M., Summers, L. H., May 1984. The economic effects of dividend taxation, nBER Working Paper No. 1353.
URL https://ideas.repec.org/p/nbr/nberwo/1353.html

Restoy, F., Rockinger, G. M., 1994. On stock market returns and returns on investment. The Journal of Finance 49 (2), 543–556.

Rouwenhorst, K. G., 1995. Asset Pricing Implications of Business Cycle Models. Princeton University Press, Ch. 10, pp. 294–330.

Santoro, M., Wei, C., 2011. Taxation, investment and asset pricing. Review of Economic Dynamics 14, 443–454.

Schmitt-Grohé, S., Uribe, M., 2004. Solving dynamic general equilibrium models using a second-order approximation to the policy function. Journal of Economic Dynamics & Control 28, 755–775.

Sialm, C., 2006. Stochastic taxation and asset pricing in dynamic general equilibrium. Journal of Economics Dynamics & Control 30 (3), 511–540.

Sialm, C., 2009. Tax changes and asset pricing. American Economic Review 99 (4), 1356–1383.

Vegh, C. A., Vuletin, G., 2015. How is tax policy conducted over the business cycle? American Economic Journal 7 (3), 327–370.

Viard, A. D., 2000. Dynamic asset pricing effects and incidence of realization-based capital gains taxes. Journal of Monetary Economics 46, 465–48.

A R Code for Chapter 2

A.1 Code for the Graphs for the Case with Full Transfers

```
# Graphs for the case of with full transfers
# Marko Krause
# January 2017

rm(list=ls())
mu_g<-0.018        #0.017
sigma<-0.046       #0.023
beta<-0.95
delta<--log(beta)
mu_taust<--0.449

#convergence
mgam_1<-(-mu_g/(0.5*sigma^2)+((mu_g/(0.5*sigma^2))^2+4*delta/(0.5*sigma
    ^2))^0.5)/2
mgam_2<-(-mu_g/(0.5*sigma^2)-((mu_g/(0.5*sigma^2))^2+4*delta/(0.5*sigma
    ^2))^0.5)/2
gam1<-(mgam_1-1)*-1
gam2<-(mgam_2-1)*-1
print(gam1)
print(gam2)
gamn<-3
gam<-matrix(c(0.5,2,3),nrow=5,ncol=1)

phimin<--2
phimax<-+2
int<-0.1
N<-(phimax-phimin)/int
Psi_S<-matrix(0,nrow=gamn,ncol=N+1)
Psi_S0<-matrix(0,nrow=gamn,ncol=N+1)
Psi_Sdiff<-matrix(0,nrow=gamn,ncol=N+1)

ER<-matrix(0,nrow=gamn,ncol=N+1)
ERdiff<-matrix(0,nrow=gamn,ncol=N+1)
ERP<-matrix(0,nrow=gamn,ncol=N+1)
ERP0<-matrix(0,nrow=gamn,ncol=N+1)
```

```
ERPdiff<-matrix(0,nrow=gamn,ncol=N+1)
Rb<-matrix(0,nrow=gamn,ncol=N+1)

zeros<-rep(0,N+1)
phi<-vector(length=N+1)

for (j in 1:gamn){
  phi[1]<-phimin
  for (i in 1:(N+1)){
    EG<-exp(mu_g+0.5*sigma^2)
    ERn<-exp(delta+gam[j,1]*mu_g+(2*gam[j,1]-gam[j,1]^2)*0.5*sigma^2)
    Rb[j,i]<-exp(delta+gam[j,1]*mu_g-gam[j,1]^2*0.5*sigma^2)
    Psi_Sn<-EG/(ERn-EG)

    Psi_S[j,i]<-Psi_Sn*exp(mu_taust)*exp(phi[i]*(gam[j,1]-1)*sigma^2)
    Psi_S0[j,i]<-Psi_Sn*exp(mu_taust)
    Psi_Sdiff[j,i]<-Psi_S[j,i]/Psi_S0[j,i]-1

    ER[j,i]<-exp(-gam[j,1]*phi[i]*sigma^2)*(ERn-EG)+EG

    ERdiff[j,i]<-ER[j,i]-ERn

    ERP[j,i]<-exp(-mu_taust)*exp(phi[i]*(1-gam[j,1])*sigma^2)*(ERn-EG)+
        EG-Rb[j,i]
    ERP0[j,i]<-exp(-mu_taust)*(ERn-EG)+EG-Rb[j,i]

    ERPdiff[j,i]<-ERP[j,i] - ERP0[j,i]
    if (i<=N) {phi[i+1]<-phi[i]+int}
  }
}

dev.off()
#par(mar=c(4,4,2,2))
par(mfrow=c(1,1))
#Price deviations

ypos<-seq(floor(min(Psi_Sdiff)),ceiling(max(Psi_Sdiff)),by=0.005)
ymax<-max(Psi_Sdiff)
```

```
ymin<-min(Psi_Sdiff)

plot(phi[1:(N+1)],Psi_Sdiff[1,], type="l",pch=20, cex=0.5, col="black",
    lwd=2,mgp=c(2.3,0,0),xlab="Phi",ylab="Price diff. in %",lty=1,ylim=
    c(ymin,ymax),yaxt="n",xaxt="n",bty="n")
axis(2, at=ypos,labels=sprintf("%1.1f", ypos*100),pos=-2,las=2)
axis(1,pos=0)
lines(phi[1:(N+1)],Psi_Sdiff[2,], type="l", col="black",lwd=2,lty=2)
lines(phi[1:(N+1)],Psi_Sdiff[3,], type="l", col="black",lwd=2,lty=3)



ypos<-seq(floor(min(ERdiff)),ceiling(max(ERdiff)),by=0.0005)
ymax<-max(ERdiff)
ymin<-min(ERdiff)

plot(phi[1:(N+1)],ERdiff[1,], type="l",pch=20, cex=0.5, col="black",lwd
    =2,mgp=c(2.8,0,0),xlab="Phi",ylab="Exp. return diff. in %",lty=1,
    ylim=c(ymin,ymax),yaxt="n",xaxt="n",bty="n")
axis(2, at=ypos,labels=sprintf("%1.2f", ypos*100),pos=-2,las=2)
axis(1,pos=0)
lines(phi[1:(N+1)],ERdiff[2,], type="l", col="black",lwd=2,lty=2)
lines(phi[1:(N+1)],ERdiff[3,], type="l", col="black",lwd=2,lty=3)

#legend("topleft", c("Phi=1", "Phi=0","Phi=-1"), lty=c(2,1,3),text.col
    = "black",cex=0.8,y.intersp = 0.5, x.intersp=0.2)

ypos<-seq(floor(min(ERPdiff)),ceiling(max(ERPdiff)),by=0.0005)
ymax<-max(ERPdiff)
ymin<-min(ERPdiff)

plot(phi[1:(N+1)],ERPdiff[1,], type="l",pch=20, cex=0.5, col="black",
    lwd=2,mgp=c(2.8,0,0),xlab="Phi",ylab="Equity premium diff. in %",
    lty=1,ylim=c(ymin,ymax),yaxt="n",xaxt="n",bty="n")
axis(2, at=ypos,labels=sprintf("%1.2f", ypos*100),pos=-2,las=2)
axis(1,pos=0)
lines(phi[1:(N+1)],ERPdiff[2,], type="l", col="black",lwd=2,lty=2)
lines(phi[1:(N+1)],ERPdiff[3,], type="l", col="black",lwd=2,lty=3)
```

A.2 Code for Graphs for the Case without Transfers

```
# Graphs for the case of no transfers
# Marko Krause
# January 2017

rm(list=ls())
mu_g<-0.018        #0.017
sigma<-0.046       #0.023
beta<-0.95
delta<--log(beta)
mu_taust<--0.449

#convergence
mgam_1<-(-mu_g/(0.5*sigma^2)+((mu_g/(0.5*sigma^2))^2+4*delta/(0.5*sigma
    ^2))^0.5)/2
mgam_2<-(-mu_g/(0.5*sigma^2)-((mu_g/(0.5*sigma^2))^2+4*delta/(0.5*sigma
    ^2))^0.5)/2
gam1<-(mgam_1-1)*-1
gam2<-(mgam_2-1)*-1
print(gam1)
print(gam2)

gamn<-3

gam<-matrix(c(0.5,2,3),nrow=5,ncol=1)

phimin<--2
phimax<-+2
int<-0.1
N<-(phimax-phimin)/int
Psi_S<-matrix(0,nrow=gamn,ncol=N+1)
Psi_S0<-matrix(0,nrow=gamn,ncol=N+1)
Psi_Sdiff<-matrix(0,nrow=gamn,ncol=N+1)

ER<-matrix(0,nrow=gamn,ncol=N+1)

Dy<-matrix(0,nrow=gamn,ncol=N+1)
Dypt<-matrix(0,nrow=gamn,ncol=N+1)
```

```
Cg<-matrix(0,nrow=gamn,ncol=N+1)
Dydiff<-matrix(0,nrow=gamn,ncol=N+1)
Dyptdiff<-matrix(0,nrow=gamn,ncol=N+1)
Cgdiff<-matrix(0,nrow=gamn,ncol=N+1)

ERdiff<-matrix(0,nrow=gamn,ncol=N+1)
ERP<-matrix(0,nrow=gamn,ncol=N+1)
ERP0<-matrix(0,nrow=gamn,ncol=N+1)
ERPdiff<-matrix(0,nrow=gamn,ncol=N+1)
Rbn<-matrix(0,nrow=gamn,ncol=N+1)
Rb<-matrix(0,nrow=gamn,ncol=N+1)

zeros<-rep(0,N+1)
phi<-vector(length=N+1)

for (j in 1:gamn){
  phi[1]<-phimin
  for (i in 1:(N+1)){
    EG<-exp(mu_g+0.5*sigma^2)
    ERn<-exp(delta+gam[j,1]*mu_g+(2*gam[j,1]-gam[j,1]^2)*0.5*sigma^2)
    Rbn[j,i]<-exp(delta+gam[j,1]*mu_g-gam[j,1]^2*0.5*sigma^2)
    Psi_Sn<-EG/(ERn-EG)

    Psi_S[j,i]<-Psi_Sn*exp(mu_taust)*exp(phi[i]^2*(gam[j,1]^2-gam[j,1])
        *sigma^2-phi[i]*(1-gam[j,1])^2*sigma^2)
    Psi_S0[j,i]<-Psi_Sn*exp(mu_taust)
    Psi_Sdiff[j,i]<-Psi_S[j,i]/Psi_S0[j,i]-1

    Dy[j,i]<-exp((gam[j,1]*phi[i]^2-2*gam[j,1]*phi[i]+gam[j,1]^2*phi[i
        ])*sigma^2)*(ERn-EG)
    Cg[j,i]<-exp((gam[j,1]^2*phi[i]^2-gam[j,1]*phi[i])*sigma^2)*EG
    Dydiff[j,i]<-Dy[j,i]-(ERn-EG)
    Cgdiff[j,i]<-Cg[j,i]-EG
    Dypt[j,i]<-exp(-mu_taust)*exp((gam[j,1]*phi[i]^2+phi[i]*(gam[j
        ,1]-1)^2)*sigma^2)*(ERn-EG)
    ER[j,i]<-Dy[j,i]+Cg[j,i]

    ERdiff[j,i]<-ER[j,i]-ERn
```

```
    Rb[j,i]<-Rbn[j,i]*exp(phi[i]*gam[j,1]^2*sigma^2)

    ERP[j,i]<-Dypt[j,i]+Cg[j,i]-Rb[j,i]
    ERP0[j,i]<-exp(-mu_taust)*(ERn-EG)+EG-Rbn[j,i]

    ERPdiff[j,i]<-ERP[j,i] - ERP0[j,i]
    if (i<=N) {phi[i+1]<-phi[i]+int}
  }
}

dev.off()
#par(mar=c(4,4,2,2))
par(mfrow=c(1,1))
#Price deviations

ypos<-seq(floor(min(Psi_Sdiff)),ceiling(max(Psi_Sdiff)),by=0.02)
ymax<-max(Psi_Sdiff)
ymin<-min(Psi_Sdiff)

plot(phi[1:(N+1)],Psi_Sdiff[1,], type="l",pch=20, cex=0.5, col="black",
    lwd=2,mgp=c(2.3,0,0),xlab="Phi",ylab="Exp. price diff. in %",lty=1,
    ylim=c(ymin,ymax),yaxt="n",xaxt="n",bty="n")
axis(2, at=ypos,labels=sprintf("%1.0f", ypos*100),pos=-2,las=2)
axis(1,pos=0)
lines(phi[1:(N+1)],Psi_Sdiff[2,], type="l", col="black",lwd=2,lty=2)
lines(phi[1:(N+1)],Psi_Sdiff[3,], type="l", col="black",lwd=2,lty=3)



# expected after-tax return differences
ypos<-seq(floor(min(ERdiff)),ceiling(max(ERdiff)),by=0.02)
ymax<-max(ERdiff)
ymin<-min(ERdiff)

plot(phi[1:(N+1)],ERdiff[1,], type="l",pch=20, cex=0.5, col="black",lwd
    =2,mgp=c(2.3,0,0),xlab="Phi",ylab="Exp. return diff. in %",lty=1,
    ylim=c(ymin,ymax),yaxt="n",xaxt="n",bty="n")
axis(2, at=ypos,labels=sprintf("%1.0f", ypos*100),pos=-2,las=2)
axis(1,pos=0)
```

```
lines(phi[1:(N+1)],ERdiff[2,], type="l", col="black",lwd=2,lty=2)
lines(phi[1:(N+1)],ERdiff[3,], type="l", col="black",lwd=2,lty=3)

#legend("topleft", c("Phi=1", "Phi=0","Phi=-1"), lty=c(2,1,3),text.col
    = "black",cex=0.8,y.intersp = 0.5, x.intersp=0.2)


# equity premium differences
ypos<-seq(floor(min(ERPdiff)),ceiling(max(ERPdiff)),by=0.02)
ymax<-max(ERPdiff)
ymin<-min(ERPdiff)

plot(phi[1:(N+1)],ERPdiff[1,], type="l",pch=20, cex=0.5, col="black",
    lwd=2,mgp=c(2.3,0,0),xlab="Phi",ylab="Equity premium diff. in %",
    lty=1,ylim=c(ymin,ymax),yaxt="n",xaxt="n",bty="n")
axis(2, at=ypos,labels=sprintf("%1.0f", ypos*100),pos=-2,las=2)
axis(1,pos=0)
lines(phi[1:(N+1)],ERPdiff[2,], type="l", col="black",lwd=2,lty=2)
lines(phi[1:(N+1)],ERPdiff[3,], type="l", col="black",lwd=2,lty=3)

# div yield
ypos<-seq(floor(min(c(Dydiff))),ceiling(max(c(Dydiff))),by=0.0005)
ymax<-max(c(Dydiff))
ymin<-min(c(Dydiff))

plot(phi[1:(N+1)],Dydiff[3,], type="l",pch=20, cex=0.5, col="black",lwd
    =2,mgp=c(2.5,0,0),xlab="Phi",ylab="Exp. Div. yield diff. in %",lty
    =1,ylim=c(ymin,ymax),yaxt="n",xaxt="n",bty="n")
axis(2, at=ypos,labels=sprintf("%1.2f", ypos*100),pos=-2,las=2)
axis(1,pos=0)

# cap gain
ypos<-seq(floor(min(c(Cgdiff))),ceiling(max(c(Cgdiff))),by=0.01)
ymax<-max(c(Cgdiff))
ymin<-min(c(Cgdiff))

plot(phi[1:(N+1)],Cgdiff[3,], type="l",pch=20, cex=0.5, col="black",lwd
    =2,mgp=c(2.3,0,0),xlab="Phi",ylab="Exp. cap. gain diff. in %",lty
```

```
    =1,ylim=c(ymin,ymax),yaxt="n",xaxt="n",bty="n")
axis(2, at=ypos,labels=sprintf("%1.0f", ypos*100),pos=-1,las=2)
axis(1,pos=0)

# bond stuff
M<-5
int<-0.1
N<-M/int
gam<-2.5
Rb_hi<-vector(length=N)
Rb_lo<-vector(length=N)
Rb<-vector(length=N)
phi_hi<-1
phi_lo<--1
ep<-0
time<-vector(length=N)
time[1]<-int
for (i in 1:N){
  Rb_hi[i]<-exp(delta+gam*mu_g-gam^2*0.5*sigma^2)*exp(-gam^2*(phi_hi-2*
      phi_hi)*0.5*sigma^2+gam*phi_hi*ep)^(1/i)-1
  Rb_lo[i]<-exp(delta+gam*mu_g-gam^2*0.5*sigma^2)*exp(-gam^2*(phi_lo-2*
      phi_lo)*0.5*sigma^2+gam*phi_lo*ep)^(1/i)-1
  Rb[i]<-exp(delta+gam*mu_g-gam^2*0.5*sigma^2)-1
  if (i<N){time[i+1]<-time[i]+int}
}
# yield curve
ypos<-seq(floor(min(Rb_hi,Rb_lo,Rb)-0.01),ceiling(max(Rb_hi,Rb_lo,Rb)
    +0.01),by=0.005)
ymax<-max(Rb_hi,Rb_lo,Rb)+0.01
ymin<-min(Rb_hi,Rb_lo,Rb)-0.01

plot(time,Rb, type="l",pch=20, cex=0.5, col="black",lwd=2,mgp=c(3,0,0),
    xlab="Maturity in years",ylab="Yield in %",lty=1,ylim=c(ymin,ymax),
    yaxt="n",xaxt="n",bty="n")
axis(2, at=ypos,labels=sprintf("%1.1f", 100*ypos),pos=0,las=2)
axis(1,pos=ymin)
lines(time,Rb_hi, type="l", col="black",lwd=2,lty=2)
lines(time,Rb_lo, type="l", col="black",lwd=2,lty=3)
```

A.3 Code for the Graphs for the Case with Full Transfers and with Options

```
# Graphs for the case of no transfers, including option
# Marko Krause
# January 2017

rm(list=ls())
mu_g<-0.018        #0.017
sigma<-0.046       #0.023
beta<-0.95
delta<--log(beta)
mu_taust<--0.449

#convergence
mgam_1<-(-mu_g/(0.5*sigma^2)+((mu_g/(0.5*sigma^2))^2+4*delta/(0.5*sigma
    ^2))^0.5)/2
mgam_2<-(-mu_g/(0.5*sigma^2)-((mu_g/(0.5*sigma^2))^2+4*delta/(0.5*sigma
    ^2))^0.5)/2
gam1<-(mgam_1-1)*-1
gam2<-(mgam_2-1)*-1
print(gam1)
print(gam2)

gamn<-3

gam<-matrix(c(0,3,4),nrow=5,ncol=1)

phimin<--6
phimax<-+6
int<-0.2
N<-(phimax-phimin)/int
Psi_S<-matrix(0,nrow=gamn,ncol=N+1)
Psi_S0<-matrix(0,nrow=gamn,ncol=N+1)
Psi_Sdiff<-matrix(0,nrow=gamn,ncol=N+1)
Psi_Sno<-matrix(0,nrow=gamn,ncol=N+1)
Psi_Snodiff<-matrix(0,nrow=gamn,ncol=N+1)

zeros<-rep(0,N+1)
phi<-vector(length=N+1)
```

```
M<-100000
epsilon<-rnorm(M,0,sigma)
for (j in 1:gamn){
  phi[1]<-phimin
  for (i in 1:(N+1)){
    EG<-exp(mu_g+0.5*sigma^2)
    ERn<-exp(delta+gam[j,1]*mu_g+(2*gam[j,1]-gam[j,1]^2)*0.5*sigma^2)
    Psi_Sn<-EG/(ERn-EG) # mean(max(0,exp(mu_taust-0.5*phi_hi^2*sigma^2-
        phi_hi*epsilon)-1))
    vec<-exp(-delta)*exp(mu_g+epsilon)^(1-gam[j,1])*pmax(0,exp(mu_taust
        -0.5*phi[i]^2*sigma^2-phi[i]*epsilon)-1)
    opt<-mean(vec)*(ERn/(ERn-EG))

    Psi_S0[j,i]<-Psi_Sn*exp(mu_taust)
    Psi_S[j,i]<-Psi_Sn*exp(mu_taust+phi[i]*(gam[j,1]-1)*sigma^2)-opt
    Psi_Sno[j,i]<-Psi_S[j,i]+opt
    Psi_Sdiff[j,i]<-Psi_S[j,i]/Psi_S0[j,i]-1
    Psi_Snodiff[j,i]<-Psi_Sno[j,i]/Psi_S0[j,i]-1
    if (i<=N) {phi[i+1]<-phi[i]+int}
    }
}
dev.off()
#par(mar=c(4,4,2,2))
par(mfrow=c(1,1))

#Price deviations

ypos<-seq(floor(min(c(Psi_Sdiff,Psi_Snodiff))),ceiling(max(c(Psi_Sdiff,
    Psi_Snodiff))),by=0.01)
ymax<-max(c(Psi_Sdiff,Psi_Snodiff))
ymin<-min(c(Psi_Sdiff,Psi_Snodiff))

plot(phi[1:(N+1)],Psi_Snodiff[1,], type="l",pch=20, cex=0.5, col="black
    ",lwd=2,mgp=c(2.3,0,0),xlab="Phi",ylab="Price diff. in %",lty=1,
    ylim=c(ymin,ymax),yaxt="n",xaxt="n",bty="n")
axis(2, at=ypos,labels=sprintf("%1.0f", ypos*100),pos=-6,las=2)
axis(1,pos=0)
```

```
lines(phi[1:(N+1)],Psi_Snodiff[2,], type="l", col="black",lwd=2,lty=2)
lines(phi[1:(N+1)],Psi_Snodiff[3,], type="l", col="black",lwd=2,lty=3)
lines(phi[1:(N+1)],Psi_Sdiff[1,], type="l", col="darkgrey",lwd=2,lty=1)
lines(phi[1:(N+1)],Psi_Sdiff[2,], type="l", col="darkgrey",lwd=2,lty=2)
lines(phi[1:(N+1)],Psi_Sdiff[3,], type="l", col="darkgrey",lwd=3,lty=3)
```

B R code for Chapter 3

```
# Quad apprxomation, habit, adjustment costs, stoch tax rate of
    dividends
# Created by Marko Krause
# Mostly adapted from Heer & Maussner (2009), Dynamic General
    Equilibrium Modelling
# January 2017
# rm(list=ls())
#   library(matrixcalc) # for vec and vech
#   library(numDeriv)   # for Hessian Jacobian
#   library(Matrix)     # for Kronecker
#   library(geigen)     # for generalized Schur
#   library(MASS)       # for bivariate normal distribution
#   library(beepr)      # beep signal
# index functions

ix1<-function(i,j,k){
  ix1<-(i-1)*nx^2+(j-1)*nx+k
  return(ix1)
}

iy1<-function(i,j,k){
  iy1<-nx^3+ix1(i,j,k)
  return(iy1)
}

ix2<-function(i,j,k){
  ix2<-(i-1)*(nz*nx)+(j-1)*nz+k
  return(ix2)
}

iy2<-function(i,j,k){
  iy2<-nx^2*nz+ix2(i,j,k)
  return(iy2)
}

ix3<-function(i,j,k){
```

```
  ix3<-(i-1)*nz^2+(j-1)*nz+k
  return(ix3)
}

iy3<-function(i,j,k){
  iy3<-nx*nz^2+ix3(i,j,k)
  return(iy3)
}

Geta<-function(i,j,Lyz,Omat) {
  n<-ncol(Lyz)
  fx=0
  for (s in 1:n) {
    for (q in 1:n) {
      fx<-fx+Lyz[i,q]*Lyz[j,s]*(t(Omat[q,])%*%Omat[s,]) # r seems to
          save extracted rows also as columns
    }
  }
  return(fx)
}

Getab<-function(i,j,Lyz,Omat) {
  n<-ncol(Lyz)
  fx=0
  for (s in 1:n) {
    fx<-fx+Lyz[i,s]*(t(Omat[s,])%*%Omat[j,])
  }
  return(fx)
}
# policy functions
fcthk<-function(kb,sb,zb,taub,sig){
  vec<-matrix(c(kb,sb,zb,taub,sig),nrow=5,ncol=1)
  fcthk<-kst+t(vec[1:nx])%*%Lxx[1,]+t(vec[(nx+1):(nx+nz)])%*%Lxz[1,]+
      secord*0.5*(t(vec)%*%xcube[[1]]%*%vec)
  return(fcthk)
}

fcths<-function(kb,sb,zb,taub,sig){
```

```
  vec<-matrix(c(kb,sb,zb,taub,sig),nrow=5,ncol=1)
  fcths<-s+t(vec[1:nx])%*%Lxx[2,]+t(vec[(nx+1):(nx+nz)])%*%Lxz[2,]+
      secord*0.5*(t(vec)%*%xcube[[2]]%*%vec)
  return(fcths)
}

fcthLAM<-function(kb,sb,zb,taub,sig){
  vec<-matrix(c(kb,sb,zb,taub,sig),nrow=5,ncol=1)
  fcthLAM<-LAM+t(vec[1:nx])%*%Lyx[1,]+t(vec[(nx+1):(nx+nz)])%*%Lyz[1,]+
      secord*0.5*(t(vec)%*%ycube[[1]]%*%vec)
  return(fcthLAM)
}

fcthc<-function(kb,sb,zb,taub,sig){
  vec<-matrix(c(kb,sb,zb,taub,sig),nrow=5,ncol=1)
  fcthc<-c+t(vec[1:nx])%*%Lyx[2,]+t(vec[(nx+1):(nx+nz)])%*%Lyz[2,]+
      secord*0.5*(t(vec)%*%ycube[[2]]%*%vec)
  return(fcthc)
}
# equilibrium conditions

fct_q<-function(kt,it){
  fct_q<-(it/(kt*apd))^eta
  return(fct_q)
}

fct.1<- function(x){
  kt<-x[1]
  st<-x[2]
  LAMt<-x[3]
  ct<-x[4]
  Zt<-x[5]
  taustt<-x[6]
  k1t<-x[7]
  s1t<-x[8]
  LAM1t<-x[9]
  c1t<-x[10]
  Z1t<-x[11]
```

```
  taust1t<-x[12]
  yt<-exp(Zt)*kt^alpha
  it<-yt-(ct-omega*(c_tau+taustt)*onemal*exp(Zt)*kt^alpha)/(1-omega*(
      c_tau+taustt)) # from eq conditions
  phit<-(a1*(it/kt)^onemeta-a2)*kt
  fct.1<- phit+onemdel*kt-k1t*A
  return(fct.1)
}
fct.2<-function(x){
  kt<-x[1]
  st<-x[2]
  LAMt<-x[3]
  ct<-x[4]
  Zt<-x[5]
  taustt<-x[6]
  k1t<-x[7]
  s1t<-x[8]
  LAM1t<-x[9]
  c1t<-x[10]
  Z1t<-x[11]
  taust1t<-x[12]
  yt<-exp(Zt)*kt^alpha
  y1t<-exp(Z1t)*k1t^alpha
  it<-yt-(ct-omega*(c_tau+taustt)*onemal*exp(Zt)*kt^alpha)/(1-omega*(
      c_tau+taustt)) # from eq conditions
  i1t<-y1t-(c1t-omega*(c_tau+taust1t)*onemal*exp(Z1t)*k1t^alpha)/(1-
      omega*(c_tau+taust1t))
  phit<-(a1*(it/kt)^onemeta-a2)*kt
  phi1t<-(a1*(i1t/k1t)^onemeta-a2)*k1t
  q1t<-(1-(c_tau+taust1t))*(i1t/(apd*k1t))^(eta)
  qt<-(1-(c_tau+taustt))*(it/(apd*kt))^(eta)
  fct.2<-LAMt-LAM1t*betati/A*((1-(c_tau+taust1t))*(alpha*y1t-i1t)/k1t+
      q1t*(phi1t/k1t+onemdel))/qt
  return(fct.2)
}
fct.3<-function(x){
  ct<-x[4]
  s1t<-x[8]
```

```
  fct.3<-s1t-ct
  return(fct.3)
}

fct.4<-function(x){
  kt<-x[1]
  st<-x[2]
  LAMt<-x[3]
  ct<-x[4]
  Zt<-x[5]
  taustt<-x[6]
  k1t<-x[7]
  s1t<-x[8]
  LAM1t<-x[9]
  c1t<-x[10]
  Z1t<-x[11]
  taust1t<-x[12]
  fct.4<-LAMt-((ct-st*b/A)^(-gamma)-b*betati/A*(c1t-ct*b/A)^(-gamma))
  return(fct.4)
}

sys<-function(x){
  fx<-matrix(nrow=4,ncol=1)
  fx[1]<-fct.1(x)
  fx[2]<-fct.2(x)
  fx[3]<-fct.3(x)
  fx[4]<-fct.4(x)
  return(fx)
}

# parameters
alpha<-0.27
betati<-0.994
delta<-0.011
rho<-0.9
sigma<-1 # scaling parameter
sigz<-0.0072
```

```
# tax rate parameters
c_tau<-0.32
sigtau<-0.016#0.008
rhotau<-0#0.9

#risk averion
gamma<-3

b<-0.8
eta<-1/0.23

Rmat<-matrix(c(rho,0,0,rhotau),nrow=2,ncol=2)

Omat<-matrix(c(sigz,0,0,sigtau),nrow=2,ncol=2)
Omat2<-matrix(c(sigz^2,0,0,sigtau^2),nrow=2,ncol=2)

a<-(1+0.00)^4-1
A<-1+a

omega<-0 # if zero nothing is invested in public good and all is
    transferred

nsim<-1
Tmax<-40000
nanflag=0
haltflag=FALSE
secord<-1 # if zero onlz first order approximation is used

# Gauss Hermite definitions
ghx<-matrix(c(-2.35060497367, -1.33584907401, -0.436077411928,
    0.436077411928, 1.33584907401 , 2.35060497367),nrow=6,ncol=1)
ghw<-matrix(c(0.00453000990551, 0.157067320323, 0.724629595224,
    0.724629595224, 0.157067320323, 0.00453000990551),nrow=6,ncol=1)

#static terms
a1<-((a+delta)^eta)/(1-eta)
a2<-eta*(a+delta)/(1-eta)
apd<-a+delta
```

```
onemal<-1-alpha
onemdel<-1-delta
onemeta<-1-eta

ksa<-vector(length=Tmax)
csa<-vector(length=Tmax)
isa<-vector(length=Tmax)
Zsa<-vector(length=Tmax)
ysa<-vector(length=Tmax)
wsa<-vector(length=Tmax)
ssa<-vector(length=Tmax)
fsa<-vector(length=Tmax)
tausa<-vector(length=Tmax)
taustsa<-vector(length=Tmax)
Zbarsa<-vector(length=Tmax)
msa<-vector(length=Tmax)
LAMsa<-vector(length=Tmax)
ELAMsa<-vector(length=Tmax)
psa<-vector(length=Tmax)
dsa<-vector(length=Tmax)
Rsa<-vector(length=Tmax)
Rsatau<-vector(length=Tmax)
Rfsa<-vector(length=Tmax)
dysa<-vector(length=Tmax)
dytausa<-vector(length=Tmax)
cgsa<-vector(length=Tmax)

Ers<-vector(length=nsim)
sdrs<-vector(length=nsim)
Erf<-vector(length=nsim)
sdrf<-vector(length=nsim)
Erstau<-vector(length=nsim)
sdrstau<-vector(length=nsim)
rf<-vector(length=nsim)
dZ<-vector(length=Tmax)
di<-vector(length=Tmax)
dc<-vector(length=Tmax)
dy<-vector(length=Tmax)
```

```
dk<-vector(length=Tmax)

sddi<-vector(length=nsim)
sddc<-vector(length=nsim)
sddy<-vector(length=nsim)
sddk<-vector(length=nsim)

# stationary solutions

kst<-(alpha*betati/(A-betati*(1-delta)))^(1/(1-alpha))
c<-(1-omega*c_tau)*(alpha*kst^alpha-(a+delta)*kst)+(1-alpha)*kst^alpha
y<-kst^alpha
ist<-kst*(a+delta)
Z<-0
s<-c
LAM<-(1-b*betati/A)*((1-b/A)*c)^(-gamma)
taust<-0
dyst<-(alpha*kst^alpha-ist)/((1-c_tau)*kst)
dytaust<-dyst*(1-c_tau)
cgst<-A

# sizes
nx<-2
ny<-2
nu<-0
nz<-2

# obtain numerical derivatives at stationary solution
xvec<-rbind(kst,s)
yvec<-rbind(LAM,c)
zvec<-rbind(Z,taust)
vec<-matrix(c(xvec,yvec,zvec,xvec,yvec,zvec),nrow=2*(nx+ny+nz),ncol=1)
J<-jacobian(sys,vec)

Cz.zeros<-matrix(0,nrow=nx+ny,ncol=nz)
Cu.zeros<-matrix(0,nrow=nx+ny,ncol=nx+ny)
Cxy.zeros<-matrix(0,nrow=nx+ny,ncol=nx+ny)
Du.zeros<-matrix(0,nrow=nx+ny,ncol=nx+ny)
```

```
Fu.zeros<-Du.zeros
# fill out the matrices

Cu<-0
Cxy<-Cxy.zeros
Cz<-Cz.zeros
Dxy<-J[,(nx+ny+nz+1):((nx+ny)*2+nz)]
Fxy<-J[,1:(nx+ny)]
Du<-Du.zeros
Fu<-Fu.zeros
Dz<--J[,(2*(nx+ny)+nz+1):(2*(nx+ny+nz))]
Fz<--J[,(nx+ny+1):(nx+ny+nz)]

if (Cu==0) {Cu.inf<-Cu.zeros} else {Cu.inf<-solve(cu)}

W<--solve(Dxy-Du%*%Cu.inf%*%Cxy)%*%(Fxy-Fu%*%Cu.inf%*%Cxy)
R<-solve(Dxy-Du%*%Cu.inf%*%Cxy)%*%((Dz+Du%*%Cu.inf%*%Cz)%*%Rmat+(Fz+Fu
    %*%Cu.inf%*%Cz))

Schur.W<-gqz(W,diag(nx+ny),"S")  # Schur decomposition  A = Q S t(Q)
    with ordered eigenvalues (the one less then one in the first row)
Trans<-solve(Schur.W$Q) # T^-1
Tm<-Schur.W$Q # T
S<-Schur.W$S
Q<-Trans%*%R

# obtain blocks
Sxx<-S[1:nx,1:nx]
Sxy<-S[1:nx,(nx+1):(nx+ny)]
Syy<-S[(nx+1):(nx+ny),(nx+1):(nx+ny)]

Qx<-Q[1:nx,1:nz]
Qy<-Q[(nx+1):(nx+ny),1:nz]

Rx<-R[1:nx,1:nz]

Trxx<-Trans[1:nx,1:nx]
Trxy<-Trans[1:nx,(nx+1):(nx+ny)]
```

```
Tryx<-Trans[(nx+1):(nx+ny),1:nx]
Tryy<-Trans[(nx+1):(nx+ny),(nx+1):(nx+ny)]

Txx<-Tm[1:nx,1:nx]
Txy<-Tm[1:nx,(nx+1):(nx+ny)]
Tyx<-Tm[(nx+1):(nx+ny),1:nx]
Tyy<-Tm[(nx+1):(nx+ny),(nx+1):(nx+ny)]

Wxx<-W[1:nx,1:nx]
Wxy<-W[1:nx,(nx+1):(nx+ny)]
Wyx<-W[(nx+1):(nx+ny),1:nx]
Wyy<-W[(nx+1):(nx+ny),(nx+1):(nx+ny)]

# obtain Phi
vecQy<-matrix(Qy)
vecPhi<-solve(kronecker(t(Rmat),diag(1,ny))-kronecker(diag(1,nz),Syy))
    %*%vecQy

Phi<-matrix(vecPhi,nrow=ny,ncol=nz)

#matrics for policy function of yt

Lyx<--1*solve(Tryy)%*%Tryx

Lyz<-solve(Tryy)%*%Phi

# policy function for xt

Lxx<-Txx%*%Sxx%*%solve(Txx) # the not transposed T is used here

Lxz<-Wxy%*%solve(Tryy)%*%Phi+Rx

# policy function for ut

Lux<-Cu.inf%*%Cxy%*%rbind(diag(1,nx),Lyx)

Luz<-Cu.inf%*%Cxy%*%rbind(matrix(0,nrow=nx,ncol=nz),Lyz)+Cu.inf%*%Cz
```

```
# quad part

# create general Hessian matrices
Hg<-list() #array(dim=c(nx+ny,2*(nx+ny+nz),2*(nx+ny+nz)))
Hg[[1]]<-hessian(func=fct.1,x=vec,method.args=list(eps=1e-4, d=0.0001,
     zero.tol=sqrt(.Machine$double.eps/7e-7), r=4, v=2, show.details=
     FALSE))
Hg[[2]]<-hessian(func=fct.2,x=vec,method.args=list(eps=1e-4, d=0.0001,
     zero.tol=sqrt(.Machine$double.eps/7e-7), r=4, v=2, show.details=
     FALSE))
Hg[[3]]<-hessian(func=fct.3,x=vec,method.args=list(eps=1e-4, d=0.0001,
     zero.tol=sqrt(.Machine$double.eps/7e-7), r=4, v=2, show.details=
     FALSE))
Hg[[4]]<-hessian(func=fct.4,x=vec,method.args=list(eps=1e-4, d=0.0001,
     zero.tol=sqrt(.Machine$double.eps/7e-7), r=4, v=2, show.details=
     FALSE))


# numbers for indexes for example ys means the starting index number
     for y end ye the ending number, yps is the starting number for y
     prime
nx<-ncol(Lxx)
nz<-ncol(Lxz)
ny<-nrow(Lyx)

# Hxx

nall<-(nx+ny)*nx^2
amat<-matrix(0,nrow=nall,ncol=nall)
bvec<-matrix(0,nrow=nall,ncol=1)
zi<-0
for (j in 1:nx){
  for(k in 1:nx){
    for (i in 1:(nx+ny)){
      zi<-zi+1
      for (l1 in 1:ny){
        amat[zi,iy1(l1,j,k)]<-J[i,nx+l1]
      }
```

```
      for (l1 in 1:nx){
        amat[zi,ix1(l1,j,k)]<-J[i,nx+ny+nz+l1]
      }
      for (l1 in 1:ny) {
        for (l2 in 1:nx) {
          amat[zi,ix1(l2,j,k)]<-amat[zi,ix1(l2,j,k)]+J[i,2*nx+nz+ny+l1
              ]*Lyx[l1,l2]
        }
      }
      for (l1 in 1:ny) {
        for (l2 in 1:nx) {
          for (l3 in 1:nx) {
            amat[zi,iy1(l1,l2,l3)]<-amat[zi,iy1(l1,l2,l3)]+J[i,2*nx+nz+
                ny+l1]*Lxx[l2,j]*Lxx[l3,k]
          }
        }
      }
      hvecxj<-matrix(c(1,Lyx[,j],Lxx[,j],(Lyx%*%Lxx[,j])),nrow=ny+nx+ny
          +1,ncol=1)
      hvecxk<-matrix(c(1,Lyx[,k],Lxx[,k],(Lyx%*%Lxx[,k])),nrow=ny+nx+ny
          +1,ncol=1)
      indz<-c(j,(nx+1):(nx+ny),(nx+ny+nz+1):(2*(nx+ny)+nz))
      inds<-c(k,(nx+1):(nx+ny),(nx+ny+nz+1):(2*(nx+ny)+nz))
      bvec[zi]<-t(hvecxj)%*%Hg[[i]][indz,inds]%*%hvecxk
    }
  }

}

xvec1<--solve(amat)%*%bvec
# hx_xz and hy_xz

nall<-(nx+ny)*nx*nz
amat<-matrix(0,nrow=nall,ncol=nall)
bvec<-matrix(0,nrow=nall,ncol=1)

zi<-0
for (j in 1:nx){
```

```
for(k in 1:nz){
  for (i in 1:(nx+ny)){
    zi<-zi+1
    for (l1 in 1:ny){
      amat[zi,iy2(l1,j,k)]<-J[i,nx+l1]
    }
    for (l1 in 1:nx){
      amat[zi,ix2(l1,j,k)]<-J[i,nx+ny+nz+l1]
    }
    for (l1 in 1:ny) {
      for (l2 in 1:nx) {
        amat[zi,ix2(l2,j,k)]<-amat[zi,ix2(l2,j,k)]+J[i,2*nx+nz+ny+l1
          ]*Lyx[l1,l2]
      }
    }
    for (l1 in 1:ny) {
      for (l2 in 1:nx) {
        for (l3 in 1:nz) {
          amat[zi,iy2(l1,l2,l3)]<-amat[zi,iy2(l1,l2,l3)]+J[i,2*nx+nz+
            ny+l1]*Lxx[l2,j]*Rmat[l3,k]
        }
        for (l3 in 1:nx){
          bvec[zi]<-bvec[zi]+J[i,2*nx+ny+nz+l1]*xvec1[iy1(l1,l2,l3)]*
            Lxx[l2,j]*Lxz[l3,k]
        }
      }
    }
    hvecxj<-matrix(c(1,Lyx[,j],Lxx[,j],(Lyx%*%Lxx[,j])),nrow=ny+nx+ny
      +1,ncol=1)
    hveczk<-matrix(c(Lyz[,k],Lxz[,k],(Lyx%*%Lxz[,k]+Lyz%*%Rmat[,k])
      ,1,Rmat[,k]),nrow=ny+nx+ny+1+nz,ncol=1)
    indz<-c(j,(nx+1):(nx+ny),(nx+ny+nz+1):(2*(nx+ny)+nz))
    inds<-c((nx+1):(nx+ny),(nx+ny+nz+1):(2*(nx+ny)+nz),(nx+ny+k),(2*
      nx+2*ny+nz+1):(2*(nx+ny+nz)))
    bvec[zi]<-bvec[zi]+t(hvecxj)%*%Hg[[i]][indz,inds]%*%hveczk
  }
 }
}
```

```
xvec2<-solve(amat)%*%bvec
# hx_zz and hy_zz

nall<-(nx+ny)*nz^2
amat<-matrix(0,nrow=nall,ncol=nall)
bvec<-matrix(0,nrow=nall,ncol=1)

zi<-0
for (j in 1:nz){
  for(k in 1:nz){
    for (i in 1:(nx+ny)){
      zi<-zi+1
      for (l1 in 1:ny){
        amat[zi,iy3(l1,j,k)]<-J[i,nx+l1]
      }
      for (l1 in 1:nx){
        amat[zi,ix3(l1,j,k)]<-J[i,nx+ny+nz+l1]
      }
      for (l1 in 1:ny) {
        for (l2 in 1:nx) {
          amat[zi,ix3(l2,j,k)]<-amat[zi,ix3(l2,j,k)]+J[i,2*nx+nz+ny+l1
              ]*Lyx[l1,l2]
        }
      }
      for (l1 in 1:ny) {
        for (l2 in 1:nz) {
          for (l3 in 1:nz) {
            amat[zi,iy3(l1,l2,l3)]<-amat[zi,iy3(l1,l2,l3)]+J[i,2*nx+nz+
                ny+l1]*Rmat[l2,j]*Rmat[l3,k]
          }
          for (l3 in 1:nx){
            bvec[zi]<-bvec[zi]+J[i,2*nx+ny+nz+l1]*Rmat[l2,j]*xvec2[iy2(
                l1,l3,l2)]*Lxz[l3,k] # order of ls different here
          }
        }
        for (l2 in 1:nx) {
          for (l3 in 1:nx) {
```

```
          bvec[zi]<-bvec[zi]+J[i,2*nx+ny+nz+l1]*Lxz[l2,j]*xvec1[iy1(
            l1,l2,l3)]*Lxz[l3,k]
        }
        for (l3 in 1:nz) {
          bvec[zi]<-bvec[zi]+J[i,2*nx+ny+nz+l1]*Lxz[l2,j]*xvec2[iy2(
            l1,l2,l3)]*Rmat[l3,k]
        }
      }
    }
    hveczj<-matrix(c(Lyz[,j],Lxz[,j],(Lyx%*%Lxz[,j]+Lyz%*%Rmat[,j])
      ,1,Rmat[,j]),nrow=ny+nx+ny+1+nz,ncol=1)
    hveczk<-matrix(c(Lyz[,k],Lxz[,k],(Lyx%*%Lxz[,k]+Lyz%*%Rmat[,k])
      ,1,Rmat[,k]),nrow=ny+nx+ny+1+nz,ncol=1)
    indz<-c((nx+1):(nx+ny),(nx+ny+nz+1):(2*(nx+ny)+nz),(nx+ny+j),(2*
      nx+2*ny+nz+1):(2*(nx+ny+nz)))
    inds<-c((nx+1):(nx+ny),(nx+ny+nz+1):(2*(nx+ny)+nz),(nx+ny+k),(2*
      nx+2*ny+nz+1):(2*(nx+ny+nz)))
    bvec[zi]<-bvec[zi]+t(hveczj)%*%Hg[[i]][indz,inds]%*%hveczk
  }
 }
}

xvec3<--solve(amat)%*%bvec

# Hss

amat<-matrix(0,nrow=nx+ny,ncol=nx+ny)
bvec<-matrix(0,nrow=nx+ny,ncol=1)

zi<-0
for (i in 1:(nx+ny)){
 for (j in 1:ny){
    amat[i,nx+j]<-J[i,nx+j]+J[i,2*nx+ny+nz+j]
 }
 for (j in 1:nx){
   amat[i,j]<-J[i,nx+ny+nz+j]
   for (l1 in 1:ny){
    amat[i,j]<-amat[i,j]+J[i,2*nx+ny+nz+l1]*Lyx[l1,j]
```

```
    }
  }
  for (j in 1:ny) {
    for (l1 in 1:nz){
      for (l2 in 1:nz) {
        bvec[i]<-bvec[i]+J[i,2*nx+ny+nz+j]*xvec3[iy3(j,l1,l2)]*(t(Omat[
            l1,])%*%Omat[l2,])
      }
    }
  }
  indz<-c((2*nx+ny+nz+1):(2*(nx+ny+nz)))
  temp1<-Hg[[i]][indz,indz]
  temp2<-matrix(0,nrow=ny+nz,ncol=ny+nz)
  for (l1 in 1:ny) {
    for (l2 in 1:ny) {
      temp2[l1,l2]<-Geta(l1,l2,Lyz,Omat)
    }
    for (l2 in 1:nz) {
      temp2[l1,ny+l2]<-Getab(l1,l2,Lyz,Omat)
      temp2[ny+l2,l1]<-temp2[l1,ny+l2]
    }
  }
  for (l1 in 1:nz) {
    for (l2 in 1:nz) {
      temp2[ny+l1,ny+l2]<-t(Omat[l1,])%*%Omat[l2,]
    }
  }
  bvec[i]<-bvec[i]+sum(diag(temp1%*%temp2))
}

xvec4<--solve(amat)%*%bvec

# put everything in coefficient matrix
xcube<- list()
ycube<- list()

# for x
for (i in 1:nx){
```

```
  xcube[[i]]<-matrix(0,nrow=nx+nz+1,ncol=nx+nz+1)
  Hxx<-matrix(xvec1[ix1(i,1,1):ix1(i,nx,nx)],nrow=nx,ncol=nx)
  Hxz<-t(matrix(xvec2[ix2(i,1,1):ix2(i,nx,nz)],nrow=nz,ncol=nx)) #
      stuff needs to go in row wise
  Hzz<-matrix(xvec3[ix3(i,1,1):ix3(i,nz,nz)],nrow=nz,ncol=nz)
  Hss<-xvec4[i]
  xcube[[i]][1:nx,1:nx]<-Hxx
  xcube[[i]][1:nx,(nx+1):(nx+nz)]<-Hxz
  xcube[[i]][(nx+1):(nx+nz),1:nx]<-t(Hxz)
  xcube[[i]][(nx+1):(nx+nz),(nx+1):(nx+nz)]<-Hzz
  xcube[[i]][nx+nz+1,nx+nz+1]<-Hss
}

# for y
for (i in 1:ny){
  ycube[[i]]<-matrix(0,nrow=nx+nz+1,ncol=nx+nz+1)
  Hxx<-matrix(xvec1[iy1(i,1,1):iy1(i,nx,nx)],nrow=nx,ncol=nx)
  Hxz<-t(matrix(xvec2[iy2(i,1,1):iy2(i,nx,nz)],nrow=nz,ncol=nx))# stuff
      needs to go in row wise
  Hzz<-matrix(xvec3[iy3(i,1,1):iy3(i,nz,nz)],nrow=nz,ncol=nz)
  Hss<-xvec4[nx+i]
  ycube[[i]][1:nx,1:nx]<-Hxx
  ycube[[i]][1:nx,(nx+1):(nx+nz)]<-Hxz
  ycube[[i]][(nx+1):(nx+nz),1:nx]<-t(Hxz)
  ycube[[i]][(nx+1):(nx+nz),(nx+1):(nx+nz)]<-Hzz
  ycube[[i]][nx+nz+1,nx+nz+1]<-Hss
}

#making a time series for the optimal capital path
    *****************************************************
l=1
while (l<=nsim) {
  ksa[1]<-kst
  Zsa[1]<-1
  ysa[1]<-y
  isa[1]<-ist
  ssa[1]<-s
  tausa[1]<-c_tau
```

```
Zbarsa[1]<-0
taustsa[1]<-0
shocks<-mvrnorm(n = Tmax, c(0,0), Omat2, empirical = TRUE)
err<-shocks[,1]
urr<-shocks[,2]
csa[1]<-fcthc(ksa[1]-kst,ssa[1]-s,Zbarsa[1],taustsa[1],sigma)
LAMsa[1]<-fcthLAM(ksa[1]-kst,ssa[1]-s,Zbarsa[1],taustsa[1],sigma)
for (t in 2:Tmax){
  ksa[t]<-fcthk(ksa[t-1]-kst,ssa[t-1]-s,Zbarsa[t-1],taustsa[t-1],
     sigma)
  ssa[t]<-fcths(ksa[t-1]-kst,ssa[t-1]-s,Zbarsa[t-1],taustsa[t-1],
     sigma) # which csa[t-1]
  Zbarsa[t]<-(Zbarsa[t-1])*rho+err[t]
  taustsa[t]<-(taustsa[t-1])*rhotau+urr[t]
  Zsa[t]<-exp(Zbarsa[t])
  tausa[t]<-(c_tau+taustsa[t])
  LAMsa[t]<-fcthLAM(ksa[t]-kst,ssa[t]-s,Zbarsa[t],taustsa[t],sigma) #
      which csa[t+1]
  csa[t]<-fcthc(ksa[t]-kst,ssa[t]-s,Zbarsa[t],taustsa[t],sigma)
  ysa[t]<-Zsa[t]*ksa[t]^alpha
  isa[t]<-ysa[t]-(csa[t]-omega*(c_tau+taustsa[t])*onemal*Zsa[t]*ksa[t
     ]^alpha)/(1-omega*(c_tau+taustsa[t]))
  psa[t-1]<-ksa[t]*fct_q(ksa[t-1],isa[t-1])*(1-tausa[t-1])
  #wsa[t]<-Zsa[t]*(1-alpha)*ksa[t]^(alpha)
  # conditional expectation for LAM
  ELAMsa[t-1]<-0
  for (j in 1:6){
    Zbarsa2<-(Zbarsa[t-1])*rho+sqrt(2)*sigz*ghx[j]
    for (i in 1:6) {
      taustsa2<-(taustsa[t-1])*rhotau+sqrt(2)*sigtau*ghx[i]
      LAM2<-fcthLAM(ksa[t]-kst,ssa[t]-s,Zbarsa2,taustsa2,sigma)
      ELAMsa[t-1]<-ELAMsa[t-1]+LAM2*ghw[j]*ghw[i]
    }
  }
  ELAMsa[t-1]<-ELAMsa[t-1]/pi # 2 dim case
  Rfsa[t-1]<-LAMsa[t-1]/(betati/A*ELAMsa[t-1])
  if (t>2){
    dsa[t-1]<-alpha*ysa[t-1]-isa[t-1]
```

```
      Rsa[t-1]<-(dsa[t-1]+psa[t-1]*A)/psa[t-2]
      Rsatau[t-1]<-(dsa[t-1]*(1-tausa[t-1])+psa[t-1]*A)/psa[t-2]
      if (is.nan(Rsa[t-1])){
        nanflag=nanflag+1
        l=l-1
        haltflag=TRUE
        break
      }
    }
    di[t]<-isa[t]/isa[t-1]
    dc[t]<-csa[t]/csa[t-1]
    dy[t]<-ysa[t]/ysa[t-1]
    dk[t]<-ksa[t]/ksa[t-1]
  }
  if(haltflag==FALSE){
    Ers[l]<-mean(Rsa[2:(Tmax-1)])-1
    Erstau[l]<-mean(Rsatau[2:(Tmax-1)])-1
    sdrstau[l]<-sd(Rsatau[2:(Tmax-1)])
    sdrs[l]<-sd(Rsa[2:(Tmax-1)])
    Erf[l]<-mean(Rfsa[2:(Tmax-1)])-1
    sdrf[l]<-sd(Rfsa[2:(Tmax-1)])
    # annualized values
    # business cyclce stats
    sddi[l]<-sd(di[2:(Tmax-1)])
    sddy[l]<-sd(dy[2:(Tmax-1)])
    sddc[l]<-sd(dc[2:(Tmax-1)])
    sddk[l]<-sd(dk[2:(Tmax-1)])
  }
  if (l%%50==0){print(l)}
  haltflag=FALSE
  l=l+1
}
sprintf("%.2f",c(mean(Ers)*100, mean(sdrs)*100,mean(Erf)*100,mean(sdrf)
    *100,mean(Ers-Erf)*100,nanflag))
sprintf("%.2f",c(mean(Erstau)*100, mean(sdrstau)*100,mean(Erf)*100,mean
    (sdrf)*100,mean(Erstau-Erf)*100))
sprintf("%.2f",c(mean(sddy)*100,mean(sddc)/mean(sddy),mean(sddi)/mean(
    sddy),mean(sddk)*100,mean(sddc)*100))
```

```
beep()
#sprintf("%.3f",J)
#sprintf("%.3f",c(c,kst))

# impulse responses for productivity shock
    ****************************************************************
sigma<-0 # set scalar for volatility to zero so that it does not have
    an effect on stationary values
Tmax<-20

Zbarsa[1]<-0
Zsa[1]<-exp(Zbarsa[1])
Zbarsa[2]<-(Zbarsa[1])*rho+0.01
Zsa[2]<-exp(Zbarsa[2])
for (t in 2:(Tmax-1)){
  Zbarsa[t+1]<-(Zbarsa[t])*rho
  Zsa[t+1]<-exp(Zbarsa[t+1])
}

ksa[1]<-kst
Zsa[1]<-1
ysa[1]<-y
isa[1]<-ist
ssa[1]<-s
tausa[1]<-c_tau
taustsa[1]<-0
csa[1]<-fcthc(ksa[1]-kst,ssa[1]-s,Zbarsa[1],taustsa[1],sigma)
LAMsa[1]<-fcthLAM(ksa[1]-kst,ssa[1]-s,Zbarsa[1],taustsa[1],sigma)

for (t in 2:Tmax){
    ksa[t]<-fcthk(ksa[t-1]-kst,ssa[t-1]-s,Zbarsa[t-1],taustsa[t-1],
        sigma)
    ssa[t]<-fcths(ksa[t-1]-kst,ssa[t-1]-s,Zbarsa[t-1],taustsa[t-1],
        sigma) # which csa[t-1]
    taustsa[t]<-(taustsa[t-1])*rhotau
    tausa[t]<-(c_tau+taustsa[t])
    LAMsa[t]<-fcthLAM(ksa[t]-kst,ssa[t]-s,Zbarsa[t],taustsa[t],sigma) #
          which csa[t+1]
```

```
    csa[t]<-fcthc(ksa[t]-kst,ssa[t]-s,Zbarsa[t],taustsa[t],sigma)
    ysa[t]<-Zsa[t]*ksa[t]^alpha
    isa[t]<-ysa[t]-(csa[t]-omega*(c_tau+taustsa[t])*onemal*Zsa[t]*ksa[t
       ]^alpha)/(1-omega*(c_tau+taustsa[t]))
    psa[t-1]<-ksa[t]*fct_q(ksa[t-1],isa[t-1])*(1-tausa[t-1])
    # conditional expectation for LAM
    ELAMsa[t-1]<-0
    for (j in 1:6){
      Zbarsa2<-(Zbarsa[t-1])*rho+sqrt(2)*sigz*ghx[j]
      for (i in 1:6) {
        taustsa2<-(taustsa[t-1])*rhotau+sqrt(2)*sigtau*ghx[i]
        LAM2<-fcthLAM(ksa[t]-kst,ssa[t]-s,Zbarsa2,taustsa2,sigma)
        ELAMsa[t-1]<-ELAMsa[t-1]+LAM2*ghw[j]*ghw[i]
      }
    }
    ELAMsa[t-1]<-ELAMsa[t-1]/pi # 2 dim case
    Rfsa[t-1]<-LAMsa[t-1]/(betati/A*ELAMsa[t-1])
    if (t>2){
      dsa[t-1]<-alpha*ysa[t-1]-isa[t-1]
      Rsa[t-1]<-(dsa[t-1]+psa[t-1]*A)/psa[t-2]
      cgsa[t-1]<-psa[t-1]*A/psa[t-2]-cgst
      dysa[t-1]<-dsa[t-1]/psa[t-2]-dyst
      dytausa[t-1]<-dsa[t-1]*(1-tausa[t-1])/psa[t-2]-dytaust
      Rsatau[t-1]<-(dsa[t-1]*(1-tausa[t-1])+psa[t-1]*A)/psa[t-2]
      if (is.nan(Rsa[t-1])){
        nanflag=nanflag+1
        l=l-1
        haltflag=TRUE
        break
      }
    }
    dZ[t]<-Zsa[t]-1
    di[t]<-isa[t]/ist-1
    dc[t]<-csa[t]/c-1
    dy[t]<-ysa[t]/y-1
}
dev.off()
par(mai=c(0.2,1.1,0.2,0.2))
```

```
par(mfcol=c(7,1))
Tp<-10
labsc<-1
# Productivity growth
fl<-floor(min(dZ[1:(Tp+1)]*1000))/1000
ce<-ceiling(max(dZ[1:(Tp+1)]*1000))/1000
ypos<-seq(fl,ce,by=(ce-fl)/5)
ymax<-max(dZ[1:(Tp+1)])
ymin<-min(dZ[1:(Tp+1)])
plot(dZ[2:(Tp+1)], type="l", col="black",lwd=1,ylab="Productivity",xlab
    ="Period", ylim=c(ymin,ymax),xaxt="n",yaxt="n",bty="n",mgp=c(6,0,0)
    )
axis(2, at=ypos,labels=sprintf("%1.0f", 10000*ypos),pos=0,las=2,cex.
    axis=labsc)
axis(1,pos=0,xpd=TRUE,at=c(0:Tp),cex.axis=labsc)
mtext(expression("x"*10^-2),adj=0,padj=0,outer=FALSE,cex=0.6)

# Output growth
fl<-floor(min(dy[1:(Tp+1)]*1000))/1000
ce<-ceiling(max(dy[1:(Tp+1)]*1000))/1000
ypos<-seq(fl,ce,by=(ce-fl)/5)
ymax<-max(dy[1:(Tp+1)])
ymin<-min(dy[1:(Tp+1)])

plot(dy[2:(Tp+1)], type="l", col="black",lwd=1,ylab="Output",xlab="
    Period", ylim=c(ymin,ymax),xaxt="n",yaxt="n",bty="n",mgp=c(6,0,0))

axis(2, at=ypos,labels=sprintf("%1.1f", 10000*ypos),pos=0,las=2,cex.
    axis=labsc)
axis(1,pos=0,xpd=TRUE,at=c(0:Tp),cex.axis=labsc)
mtext(expression("x"*10^-2),adj=0,padj=0,outer=FALSE,cex=0.6)

# Consumption growth
fl<-floor(min(dc[1:(Tp+1)]*1000))/1000
ce<-ceiling(max(dc[1:(Tp+1)]*1000))/1000
ypos<-seq(fl,ce,by=(ce-fl)/5)
ymax<-max(dc[1:(Tp+1)])
ymin<-min(dc[1:(Tp+1)])
```

```
plot(dc[2:(Tp+1)], type="l", col="black",lwd=1,ylab="Consumption",xlab
    ="Period", ylim=c(ymin,ymax),xaxt="n",yaxt="n",bty="n",mgp=c(6,0,0)
    )
axis(2, at=ypos,labels=sprintf("%1.0f", 10000*ypos),pos=0,las=2,cex.
    axis=labsc)
axis(1,pos=0,xpd=TRUE, at=c(0:Tp),cex.axis=labsc)
mtext(expression("x"*10^-2),adj=0,padj=0,outer=FALSE, cex=0.6)

# Investment growth
fl<-floor(min(di[1:(Tp+1)]*1000))/1000
ce<-ceiling(max(di[1:(Tp+1)]*1000))/1000
ypos<-seq(fl,ce,by=(ce-fl)/5)
ymax<-max(di[1:(Tp+1)])
ymin<-min(di[1:(Tp+1)])

plot(di[2:(Tp+1)], type="l", col="black",lwd=1,ylab="Investment",xlab="
    Period", ylim=c(ymin,ymax),xaxt="n",yaxt="n",bty="n",mgp=c(6,0,0))
axis(2, at=ypos,labels=sprintf("%1.0f", 10000*ypos),pos=0,las=2,cex.
    axis=labsc)
axis(1,pos=0,xpd=TRUE, at=c(0:Tp),cex.axis=labsc)
mtext(expression("x"*10^-2),adj=0,padj=0,outer=FALSE, cex=0.6)

# Pre-tax div yield
fl<-floor(min(dysa[1:(Tp+1)]*10000))/10000
ce<-ceiling(max(dysa[1:(Tp+1)]*10000))/10000
ypos<-seq(fl,ce,by=(ce-fl)/5)
ymax<-max(dysa[1:(Tp+1)])
ymin<-min(dysa[1:(Tp+1)])

plot(dysa[2:(Tp+1)], type="l", col="black",lwd=1,ylab="Dividend yield",
    xlab="Period", ylim=c(ymin,ymax),xaxt="n",yaxt="n",bty="n",mgp=c
    (6,0,0))
axis(2, at=ypos,labels=sprintf("%1.1f", 10000*ypos),pos=0,las=2,cex.
    axis=labsc)
axis(1,pos=0,xpd=TRUE, at=c(0:Tp),cex.axis=labsc)
mtext(expression("x"*10^-2),adj=0,padj=0,outer=FALSE, cex=0.6)
```

```
# After-tax div yield
fl<-floor(min(dytausa[1:(Tp+1)]*10000))/10000
ce<-ceiling(max(dytausa[1:(Tp+1)]*10000))/10000
ypos<-seq(fl,ce,by=(ce-fl)/5)
ymax<-max(dytausa[1:(Tp+1)])
ymin<-min(dytausa[1:(Tp+1)])

plot(dytausa[2:(Tp+1)], type="l", col="black",lwd=1,ylab="After-tax div
    . yield",xlab="Period", ylim=c(ymin,ymax),xaxt="n",yaxt="n",bty="n
    ",mgp=c(6,0,0))
axis(2, at=ypos,labels=sprintf("%1.1f", 10000*ypos),pos=0,las=2,cex.
    axis=labsc)
axis(1,pos=0,xpd=TRUE,at=c(0:Tp),cex.axis=labsc)
mtext(expression("x"*10^-2),adj=0,padj=0,outer=FALSE,cex=0.6)

# Capital gain
fl<-floor(min(cgsa[1:(Tp+1)]*10000))/10000
ce<-ceiling(max(cgsa[1:(Tp+1)]*10000))/10000
ypos<-seq(fl,ce,by=(ce-fl)/5)
ymax<-max(cgsa[1:(Tp+1)])
ymin<-min(cgsa[1:(Tp+1)])

plot(cgsa[2:(Tp+1)], type="l", col="black",lwd=1,ylab="Capital gain",
    xlab="Period", ylim=c(ymin,ymax),xaxt="n",yaxt="n",bty="n",mgp=c
    (6,0,0))
axis(2, at=ypos,labels=sprintf("%1.0f", 10000*ypos),pos=0,las=2,cex.
    axis=labsc)
axis(1,pos=0,xpd=TRUE,at=c(0:Tp),cex.axis=labsc)
mtext(expression("x"*10^-2),adj=0,padj=0,outer=FALSE,cex=0.6)

# impulse responses for tax rate shock

taustsa[1]<-0
tausa[1]<-(c_tau+taustsa[1])
taustsa[2]<-(taustsa[1])*rhotau+0.01
tausa[2]<-(c_tau+taustsa[2])
for (t in 2:(Tmax-1)){
  taustsa[t+1]<-(taustsa[t])*rhotau
```

```
  tausa[t+1]<-(c_tau+taustsa[t+1])
}

Zbarsa[1]<-0
Zsa[1]<-exp(Zbarsa[1])
ksa[1]<-kst
Zsa[1]<-1
ysa[1]<-y
isa[1]<-ist
ssa[1]<-s
csa[1]<-fcthc(ksa[1]-kst,ssa[1]-s,Zbarsa[1],taustsa[1],sigma)
LAMsa[1]<-fcthLAM(ksa[1]-kst,ssa[1]-s,Zbarsa[1],taustsa[1],sigma)

for (t in 2:Tmax){
  ksa[t]<-fcthk(ksa[t-1]-kst,ssa[t-1]-s,Zbarsa[t-1],taustsa[t-1],sigma)
  ssa[t]<-fcths(ksa[t-1]-kst,ssa[t-1]-s,Zbarsa[t-1],taustsa[t-1],sigma)
      # which csa[t-1]
  Zbarsa[t]<-(Zbarsa[t-1])*rho
  Zsa[t]<-exp(Zbarsa[t])
  LAMsa[t]<-fcthLAM(ksa[t]-kst,ssa[t]-s,Zbarsa[t],taustsa[t],sigma) #
     which csa[t+1]
  csa[t]<-fcthc(ksa[t]-kst,ssa[t]-s,Zbarsa[t],taustsa[t],sigma)
  ysa[t]<-Zsa[t]*ksa[t]^alpha
  isa[t]<-ysa[t]-(csa[t]-omega*(c_tau+taustsa[t])*onemal*Zsa[t]*ksa[t]^
     alpha)/(1-omega*(c_tau+taustsa[t]))
  psa[t-1]<-ksa[t]*fct_q(ksa[t-1],isa[t-1])*(1-tausa[t-1])
  # conditional expectation for LAM
  ELAMsa[t-1]<-0
  for (j in 1:6){
    Zbarsa2<-(Zbarsa[t-1])*rho+sqrt(2)*sigz*ghx[j]
    for (i in 1:6) {
      taustsa2<-(taustsa[t-1])*rhotau+sqrt(2)*sigtau*ghx[i]
      LAM2<-fcthLAM(ksa[t]-kst,ssa[t]-s,Zbarsa2,taustsa2,sigma)
      ELAMsa[t-1]<-ELAMsa[t-1]+LAM2*ghw[j]*ghw[i]
    }
  }
  ELAMsa[t-1]<-ELAMsa[t-1]/pi # 2 dim case
  Rfsa[t-1]<-LAMsa[t-1]/(betati/A*ELAMsa[t-1])
```

```
  if (t>2){
    dsa[t-1]<-alpha*ysa[t-1]-isa[t-1]
    Rsa[t-1]<-(dsa[t-1]+psa[t-1]*A)/psa[t-2]
    cgsa[t-1]<-psa[t-1]*A/psa[t-2]-cgst
    dysa[t-1]<-dsa[t-1]/psa[t-2]-dyst
    dytausa[t-1]<-dsa[t-1]*(1-tausa[t-1])/psa[t-2]-dytaust
    Rsatau[t-1]<-(dsa[t-1]*(1-tausa[t-1])+psa[t-1]*A)/psa[t-2]
    if (is.nan(Rsa[t-1])){
      nanflag=nanflag+1
      l=l-1
      haltflag=TRUE
      break
    }
  }
  dZ[t]<-Zsa[t]-1
  di[t]<-isa[t]/ist-1
  dc[t]<-csa[t]/c-1
  dy[t]<-ysa[t]/y-1
}

# Tax rate
fl<-floor(min(taustsa[1:(Tp+1)]*1000))/1000
ce<-ceiling(max(taustsa[1:(Tp+1)]*1000))/1000
ypos<-seq(fl,ce,by=(ce-fl)/5)
ypos<-seq(floor(min(taustsa[1:(Tp+1)])),ceiling(max(taustsa[1:(Tp+1)]))
    ,by=0.002)
ymax<-max(taustsa[1:(Tp+1)])
ymin<-min(taustsa[1:(Tp+1)])

plot(taustsa[2:(Tp+1)], type="l", col="black",lwd=1,ylab="Tax rate",
    xlab="Period", ylim=c(ymin,ymax),xaxt="n",yaxt="n",bty="n",mgp=c
    (6,0,0))
axis(2, at=ypos,labels=sprintf("%1.0f", 10000*ypos),pos=0,las=2,cex.
    axis=labsc)
axis(1,pos=0,xpd=TRUE,at=c(0:Tp),cex.axis=labsc)
mtext(expression("x"*10^-2),adj=0,padj=0,outer=FALSE,cex=0.6)

# Output growth
```

```
fl<-floor(min(dy[1:(Tp+1)]*1000000))/1000000
ce<-ceiling(max(dy[1:(Tp+1)]*1000000))/1000000
ypos<-seq(fl,ce,by=(ce-fl)/5)
ymax<-max(dy[1:(Tp+1)])
ymin<-min(dy[1:(Tp+1)])

plot(dy[2:(Tp+1)], type="l", col="black",lwd=1,ylab="Output",xlab="
    Period", ylim=c(ymin,ymax),xaxt="n",yaxt="n",bty="n",mgp=c(6,0,0))
axis(2, at=ypos,labels=sprintf("%1.2f", 10000*ypos),pos=0,las=2,cex.
    axis=labsc)
axis(1,pos=0,xpd=TRUE,at=c(0:Tp),cex.axis=labsc)
mtext(expression("x"*10^-2),adj=0,padj=0,outer=FALSE,cex=0.6)

# Consumption growth
fl<-floor(min(dc[1:(Tp+1)]*10000))/10000
ce<-ceiling(max(dc[1:(Tp+1)]*10000))/10000
ypos<-seq(fl,ce,by=(ce-fl)/5)
ymax<-max(dc[1:(Tp+1)])
ymin<-min(dc[1:(Tp+1)])

plot(dc[2:(Tp+1)], type="l", col="black",lwd=1,ylab="Consumption",xlab
    ="Period", ylim=c(ymin,ymax),xaxt="n",yaxt="n",bty="n",mgp=c(6,0,0)
    )
axis(2, at=ypos,labels=sprintf("%1.1f", 10000*ypos),pos=0,las=2,cex.
    axis=labsc)
axis(1,pos=0,xpd=TRUE,at=c(0:Tp),cex.axis=labsc)
mtext(expression("x"*10^-2),adj=0,padj=0,outer=FALSE,cex=0.6)

# Investment growth
fl<-floor(min(di[1:(Tp+1)]*1000))/1000
ce<-ceiling(max(di[1:(Tp+1)]*1000))/1000
ypos<-seq(fl,ce,by=(ce-fl)/5)
ymax<-max(di[1:(Tp+1)])
ymin<-min(di[1:(Tp+1)])

plot(di[2:(Tp+1)], type="l", col="black",lwd=1,ylab="Investment",xlab="
    Period", ylim=c(ymin,ymax),xaxt="n",yaxt="n",bty="n",mgp=c(6,0,0))
axis(2, at=ypos,labels=sprintf("%1.0f", 10000*ypos),pos=0,las=2,cex.
```

```
    axis=labsc)
axis(1,pos=0,xpd=TRUE,at=c(0:Tp),cex.axis=labsc)
mtext(expression("x"*10^-2),adj=0,padj=0,outer=FALSE,cex=0.6)

# Pre-tax div yield
fl<-floor(min(dysa[1:(Tp+1)]*100000))/100000
ce<-ceiling(max(dysa[1:(Tp+1)]*100000))/100000
ypos<-seq(fl,ce,by=(ce-fl)/5)
ymax<-max(dysa[1:(Tp+1)])
ymin<-min(dysa[1:(Tp+1)])

plot(dysa[2:(Tp+1)], type="l", col="black",lwd=1,ylab="Dividend yield",
    xlab="Period", ylim=c(ymin,ymax),xaxt="n",yaxt="n",bty="n",mgp=c
    (6,0,0))
axis(2, at=ypos,labels=sprintf("%1.2f", 10000*ypos),pos=0,las=2,cex.
    axis=labsc)
axis(1,pos=0,xpd=TRUE,at=c(0:Tp),cex.axis=labsc)
mtext(expression("x"*10^-2),adj=0,padj=0,outer=FALSE,cex=0.6)

# After-tax div yield
fl<-floor(min(dytausa[1:(Tp+1)]*100000))/100000
ce<-ceiling(max(dytausa[1:(Tp+1)]*100000))/100000
ypos<-seq(fl,ce,by=(ce-fl)/5)
ymax<-max(c(dytausa[1:(Tp+1)]))
ymin<-min(c(dytausa[1:(Tp+1)]))

plot(dytausa[2:(Tp+1)], type="l", col="black",lwd=1,ylab="After-tax div
    . yield",xlab="Period", ylim=c(ymin,ymax),xaxt="n",yaxt="n",bty="n
    ",mgp=c(6,0,0))
axis(2, at=ypos,labels=sprintf("%1.2f", 10000*ypos),pos=0,las=2,cex.
    axis=labsc)
axis(1,pos=0,xpd=TRUE,at=c(0:Tp),cex.axis=labsc)
mtext(expression("x"*10^-2),adj=0,padj=0,outer=FALSE,cex=0.6)

# Capital gain
fl<-floor(min(cgsa[1:(Tp+1)]*1000))/1000
ce<-ceiling(max(cgsa[1:(Tp+1)]*1000))/1000
ypos<-seq(fl,ce,by=(ce-fl)/5)
```

```
ymax<-max(cgsa[1:(Tp+1)])
ymin<-min(cgsa[1:(Tp+1)])

plot(cgsa[2:(Tp+1)], type="l", col="black",lwd=1,ylab="Capital gain",
    xlab="Period", ylim=c(ymin,ymax),xaxt="n",yaxt="n",bty="n",mgp=c
    (6,0,0))
axis(2, at=ypos,labels=sprintf("%1.0f", 10000*ypos),pos=0,las=2,cex.
    axis=labsc)
axis(1,pos=0,xpd=TRUE,at=c(0:Tp),cex.axis=labsc)
mtext(expression("x"*10^-2),adj=0,padj=0,outer=FALSE,cex=0.6)
```