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Public and Private Education Expenditures, Variable Elasticity of Substitution and Economic Growth

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Abstract

We explore how aggregate substitutability between public and private education expenditures impacts macroeconomic outcomes using an overlapping generations model. Using a variable elasticity of substitution "education production function" with public and private education inputs, we show that greater aggregate substitutability yields higher long run stocks of human and physical capital and a higher tax rate. Transition towards the locally stable steady state could be monotonic or oscillatory. Specifically, a high share of parental human capital and a low share of education in determining an agent's human capital creates oscillations. Hence, transitional dynamics depend on institutional determinants of these initial conditions and point towards the importance of institutional reforms in creating the right initial conditions that facilitate a smooth transition to long run outcomes.

Keywords: variable elasticity of substitution; education expenditure; stability analysis; optimal policy

JEL codes: E23, E24, I22

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1. INTRODUCTION

Given conventional wisdom regarding the positive externalities associated with education, its provision by the state is often regarded as desirable. Accordingly, there is a voluminous literature in macroeconomics that explores the long run growth outcomes associated with various aspects of public funding of education (see, for instance, Blankenau & Simpson, 2004; Dissou, Didic, & Yakautsava, 2016; Glomm & Ravikumar, 1997; Voyvoda & Yeldan, 2015). In addition to the state, parents also play a critical role in educating their children. According to Article 26 of the 1948 Universal Declaration of Human Rights "parents have a prior right to choose the kind of education that shall be given to their children" (Tooley, 2004). Generally, most decisions about a child's education, at least until the tertiary level, are made by parents.[4] As pointed out by Becker (2009, pp. 367-369) and Bräuninger and Vidal (2000), in the presence of borrowing constraints, children cannot resort to the market to finance their education, and hence, expenditures by parents can play a major role in children's human capital outcomes. Such private education expenditures, which often depend on parental altruism, can exacerbate inequality, as more affluent parents can typically spend more on their children's education. Hence, in addition to considering the role of public education expenditure for human capital accumulation and growth in an economy, and as noted by Glomm (1997) and Das (2007), any modelling construct exploring the macroeconomic impacts of education spending should also consider the role of private education expenditures undertaken by parents.

Studying public and private expenditures in conjunction with one another naturally brings into play another dimension of relevance to this issue: namely the degree to which agents view these expenditures as substitutable or complementary to each other. This paper aims to address this consideration. To this end, we examine, within a theoretical framework to be

[4]At the tertiary level, in some contexts, the presence of higher education income contingent loans, an arrangement that originated in Australia and was adopted by many other countries successfully over time, enables a child to exercise considerable decision-making power relating to her educational choices (Di Gropello, 2011).

described shortly, whether the *degree of perceived substitutability* of the public-private mix of educational expenditures matters for the transitional and long run economic performance of an economy.

To provide some motivation and context, a number of studies explore the public versus private education divide; however, they do so under the assumption that publicly and privately funded education are either substitutes or complements to one another. Studies treating these inputs as substitutes include Glomm & Ravikumar, (1992), Epple & Romano(1996a), de la Croix & Doepke(2004), Goldhaber(1999), among others. On the other hand, private supplementation of publicly provided goods like education and health care, where the state provides a baseline level of service which individuals can *complement* with additional out-of-pocket spending, has been the focus of studies such as Epple and Romano (1996b) and (Gouveia, 1997). Another dimension of complementarity between public and privately funded education, explored in studies such as Kaganovich and Zilcha (1999), Blankenau, Cassou, and Ingram (2007), and Arcalean and Schiopu (2010), is where publicly provided primary and secondary education are a prerequisite for undertaking tertiary education. Tertiary education is privately financed, either by parents or through loans that students are liable to repay once they obtain employment.

However, in practice, these expenditures are neither perfectly substitutable nor complementary. Rather, they are likely to be *imperfect substitutes* for each other, with the individual's ability and willingness to substitute between them depending, among other things, on perceptions of the quality of public vs private education, and the *mix* of the two types of expenditures utilised by the individual to "produce" a certain level of education. Such variations in the *degree of substitutability* between the two types of expenditure could have important implications for human capital accumulation and other macroeconomic outcomes.

As noted by Bearse, Glomm, and Patterson (2005), the extent of research on the macroeconomic impacts associated with the degree of substitutability between public and private education inputs is sparse, possibly due to the lack of empirical evidence relating to this issue. Nevertheless, Bearse et al. (2005) make a notable contribution to the theoretical literature on this subject by explicitly exploring the political economy ramifications associated with different values of the elasticity of substitution between public and private education inputs. These authors incorporate publicly and privately provided educational services into a constant elasticity of substitution (CES) education production function. For the special case of perfect substitutes, their model reveals that parents do not enrol their children in private schools at all. In the general CES case, they demonstrate that a higher elasticity of substitution between public and private education expenditures results in everybody selecting public schooling. However, as the relative efficiency of the private sector declines, although public school enrolment diminishes, agents vote for a higher tax rate to fund public education, resulting in higher public education expenditure per student.

While Bearse et al. (2005) provide several interesting insights into the issue of substitutability between public and private education expenditures, especially in relation to political economy outcomes, the use of a CES education production function implies an *invariant* degree of substitutability between public and private inputs. While this feature may be reasonable in the context of a standard production function with capital and labour as inputs, it is, arguably, applicable to a lesser degree in the case of an education production function. As mentioned earlier, in the context of education, the elasticity of substitution between public and private expenditures can be conditioned by the extent to which people *perceive* these two types of expenditures to be substitutable for one another, as well as the *mix* of public and private expenditures being used to "produce" education for their children. The degree to which people consider these inputs to be substitutable for each other can be impacted by institutional and cultural factors. These factors, in turn, can have an effect upon the specific combination of public and private education expenditures economic agents choose, and consequently, on long run economic outcomes.

In view of these considerations, we develop an overlapping generations model to explore how the degree of substitutability between public and private education can affect the dynamics of human and physical accumulation, as well as optimal policy in an economy in the long run. An individual's adult age human capital is determined by the education she receives in childhood and her parent's human capital. Output in this economy is produced using a standard, constant returns to scale, Cobb-Douglas aggregate production function in which physical and human capital are the inputs. A key point of distinction in our model is that education is "produced" using a *variable* elasticity of substitution (VES) education production function in which the inputs are public and private expenditures. The VES specification, originally discussed in Sato and Hoffman (1968) and Revankar (1971), when applied to the context of education, provides a tractable functional form, in addition to having some appealing properties of relevance in the context of education.

In the VES form, the institutional and cultural factors that influence the representative agent's ability to substitute between public and private education expenditures can be captured by a parameter labelled b, which we refer to as the "aggregate substitutability parameter". The elasticity of substitution between public and private education expenditures is positively related to this parameter. In our interpretation, it captures the relative uniformity of public education compared to private education, with a higher value of b being associated with a greater degree of uniformity between public and private education expenditures. This uniformity is intrinsically related to the quality of both inputs, given that they are only perceived as "uniform" if they are of comparable quality. The parameter is therefore an indicator of the quality of the education system. In addition to the aggregate substitutability parameter, the elasticity of substitution in the VES form is also determined by the mix of public and private expenditures. Consequently, the elasticity of substitution between public and private inputs in education varies not only between input combinations, but also over time.

Several empirical observations lend support to the choice of the VES specification. A good case in point is South Korea, where the cultural emphasis on educational attainment and the intense competition for admission to top universities has led to the development of a large private tutoring industry (Kim & Lee, 2010). Institutional and cultural factors have therefore resulted in South Korean parents perceiving supplementary tutoring to be a vital complement to publicly provided school education, resulting in lower aggregate substitutability between public and private education inputs. However, in recent years, policy reforms, such as improving the quality of public schooling, giving individual schools greater autonomy, and setting national standards for public exams, have been implemented in South Korea in an effort to reduce the demand for private tutoring (Lee, Lee, & Jang, 2010). Such policy reforms have the potential to reduce parental expenditure on supplementary tutoring by impacting on parental perceptions about the extent of substitutability between public and private education, and could also reduce enrolment in private schools. This is evident from the decline in enrolments in private institutions at the secondary level in South Korea from 41.45% in 1998 to 31.14% in 2013 (UNESCO, 2016). The interpretation that is relevant in the context of the VES production function, then, is that such changes have resulted in an increase in aggregate substitutability between public and private education expenditures in addition to altering the mix of public and private expenditures chosen by parents.

The mix of public and private expenditures on education is also closely related to a country's level of development. In practice, a higher degree of uniformity between public and private education is likely to be observed in developed countries where both the public and private education sectors are likely to be of a high quality. Consider Figure 1.1 below, which presents private school enrolment as a percentage of total enrolment in primary (as shown in the top panel) and secondary schools (in the bottom panel) for various groups of countries over time. Data for primary enrolments, shown in the upper panel is available from 1971-2014, while secondary enrolment data shown in the lower panel spans the period 2000-2014. Although it is not possible to observe a clear monotonic relationship between a country's income level and private school enrolment, generally, low-income, lower-middle-income and middle-income countries seem to have higher private school enrolment rates compared to upper-middle-income and high-income countries.

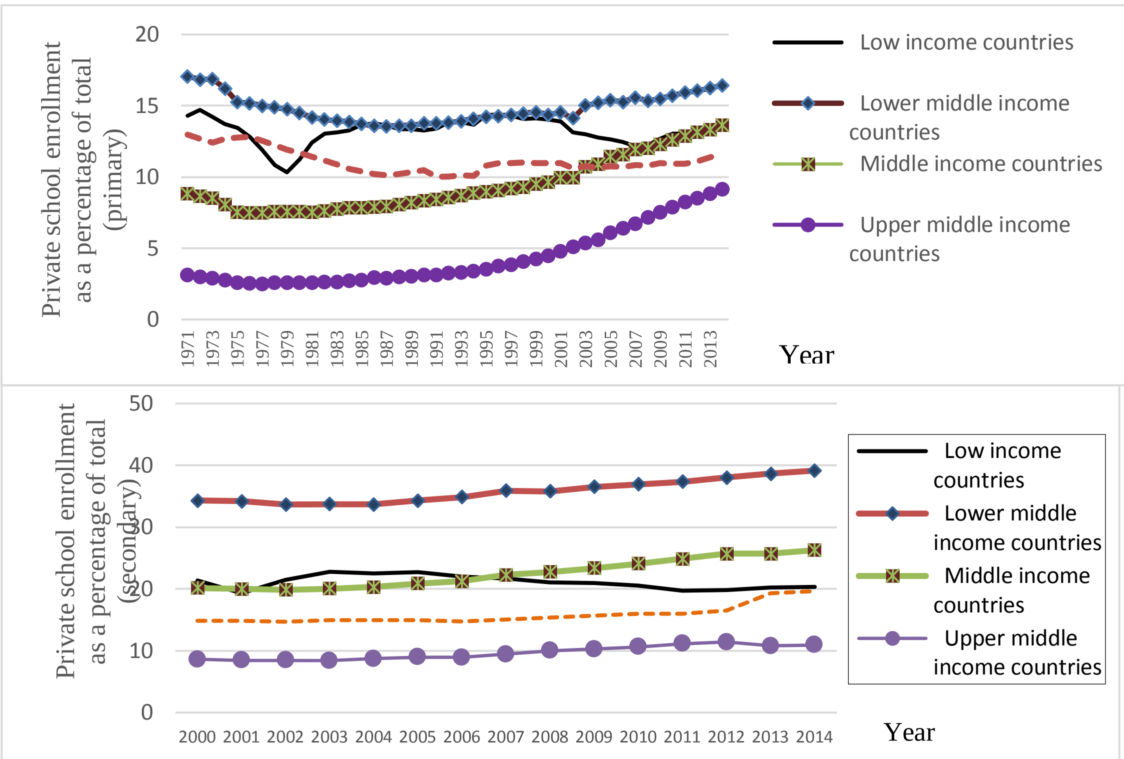

Figure 1.1: Private school enrolment as a percentage of total enrolments for different groups of countries

Source: UNESCO Institute for Statistics

Segregating the data into developed and developing countries provides a clearer picture of this relationship between private school enrolment and development. In the left panel of Figure 1.2, we can see that although private enrolments were higher at the primary level in developed countries from the 1970s to the late 1990s, the trend has clearly reversed since then. With secondary schooling, as shown in the right panel of Figure 1.2, private enrolments have been higher in private schools from 1999 to 2014, the period for which data is available.

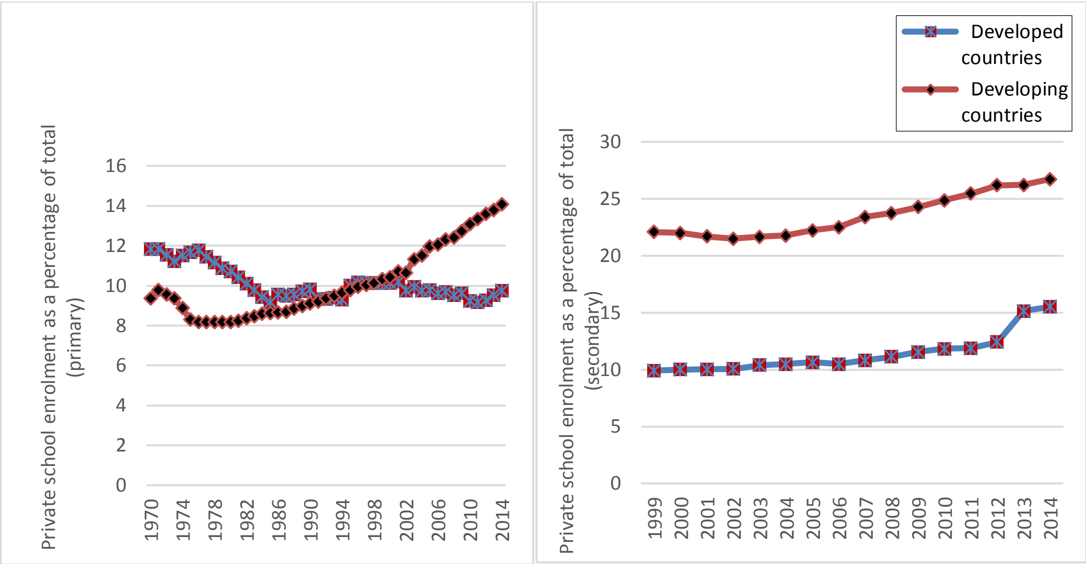

Figure 1.2: Private school enrolment as a percentage of total enrolment for developed and developing countries

Source: UNESCO Institute for Statistics

Relating these observations to our notion of aggregate substitutability, a potential explanation is that public education in developing countries is of a relatively poor quality, which encourages parents to enrol their children in private schools. Although enrolling children in private schools results in those parents opting out of the public system, we wish to emphasise that in our model, public and private education are not considered to be completely independent alternatives for one another. Rather, we abstract from the two extremes of perfect substitutes and perfect complements in the context of the public and private education expenditures and use the VES form to pay attention to a range of intermediate values of the elasticity of substitution that fall between these two extremes.

In most modern societies, parents can choose between sending their children to a public or private school. Furthermore, they can choose how much to spend on supplementing their children's school education with privately provided education services. The aggregate substitutability parameter, which is at the heart of our model, can be interpreted as an amalgam of all these individual decisions, which in turn are influenced by many cultural and institutional parameters. In our model, the representative agent takes this aggregate substitutability parameter as given when she maximises her utility.

As we will describe shortly, the VES form implies that one input is "essential" for production. In other words, whatever the degree of aggregate substitutability may be, no output can be produced without this essential input. In contrast, the CES production function admits a similar feature only in the special case when the two inputs are perfect substitutes, in which instance only the input that can produce the greater marginal output per dollar is used for production.

In the context of the education production function presented in this paper, public education expenditure is considered the essential input, from which the agent cannot opt out.[1] The choice of public education as the essential input is driven by several factors. In any country, usually all taxpayers contribute towards funding the public education system, regardless of whether their children attend public schools or not. Furthermore, in most countries and regions, public sector involvement in education is relatively higher than private sector involvement.[2] This is evident from data on private school enrolment published by the UNESCO Institute of Statistics shows that in many countries, private enrolments account for a relatively small percentage of total enrolments. This observation is also reiterated in Figure 1.2.[3]

The key analytical results of our model point towards the benefits associated with higher aggregate substitutability including higher steady state stocks of human and physical capital per capita, and faster transition towards the steady state. We also conduct stability analysis of the two dimensional discrete dynamical system formed by the human and physical capital accumulation functions. We are able to show that the economy can display monotonic or oscillatory convergence towards a locally stable steady state. The key parameter determining the transitional dynamics is the value of the share of parental human capital in offspring's

[1] We also considered the alternative, somewhat counter-intuitive and counterfactual paradigm in which private education is the essential input. This analysis yielded some intuitively unappealing results in addition to indeterminacies with respect to certain ranges of parameter values. The results of this analysis are available upon request.

[2] There are, however, some exceptions. Countries like Chile, Macao, Equatorial Guinea, Grenada and the United Arab Emirates have very high private school enrolment rates at the primary level while countries like Bangladesh, Belize and Lebanon have very high private enrolments at the secondary level (UNESCO, 2016). However, as noted by the OECD (2017), if private institutions are subsidised by the government directly or through scholarship schemes for students, the share of enrolment in private education appearing in official statistics may be an overestimate. Nonetheless, as such examples are quite rare, we restrict ourselves to a scenario where public education is the essential input.

[3] According to our interpretation, private education expenditure is incurred by parents on education services provided by third parties during a child's formal school years. It does not involve the time and opportunity costs associated with parents educating their children in various ways such as teaching a child to walk and talk. These private efforts by parents, especially during the formative years of a child's life, are an essential determinant of a child's success in later life. However, these private efforts of parents to educate their children are not considered in our model.

human capital. In particular, we observe that oscillatory convergence may occur if the share of parental human capital in determining the child's human capital exceeds a certain threshold value.

Our analysis also reveals that regardless of whether the human capital accumulation function displays increasing, decreasing or constant returns to scale, a low share of education in the human capital accumulation function could result in oscillatory convergence. Since a high share of parental human capital and a low share of education in the determination of a child's human capital outcomes are likely to be linked to the level of institutional development of a country, this result implies that that parental human capital is likely to matter more in the development of a child's human capital when institutional features, such as access to high-quality education, are poorly developed.

As all tax revenues in our model go towards the provision of public education, another interesting consideration is how the degree of substitutability between public and private education can affect political economy outcomes. To this end, we examine how the optimal tax rate set by a benevolent social planner is affected by the aggregate substitutability parameter b. We are able to analytically show that the optimal tax rate is increasing in aggregate substitutability. This implies that more government funding will be allocated towards education in economies where agents perceive public and private education expenditures to be closely substitutable for one another. This outcome is of some empirical relevance, and is similar in spirit to what is observed in the framework in Bearse et al. (2005), where lower relative efficiency of the private sector is associated with higher per capita public education expenditures.

One of the key policy implications emanating from our study is that economies could benefit from policies aimed at improving the degree of aggregate substitutability between public and private education expenditures, and some ways in which this can be achieved are suggested in the concluding section of this paper. The other relates to the economy's behaviour during transition; since the emergence of fluctuations is contingent on certain initial conditions, creating the right institutional environment and getting the relevant parameters "right" could prevent these fluctuations.

The rest of this paper is organised as follows: Section 2 presents the model, Section 3 looks at steady states and stability, Section 4 considers optimal policy, and Section 5 concludes the paper. Several proofs and derivations are supplied in the Appendices.

2. THE MODEL

We consider a three period overlapping generations model where an agent spends her childhood studying, works in adulthood, and spends her old age in retirement. In adult age, the agent gives birth to a single offspring. The population is normalised to unity in each period. Time is discrete and is given by $t = 0,1,2,...$ At time $t = 0$, each adult is endowed with h_0 units of human capital and k_o units of physical capital. There is a representative firm producing a single good, and a government that raises revenue for the purpose of financing public education.

2.1 Human capital and education

Public education expenditure e_t^G and private expenditure on education e_t^P combine to form a child's stock of education e_t. The "education production function" takes a variable elasticity of substitution (VES) form as follows:

$$e_t = \left(e_t^G\right)^a \left(e_t^P + abe_t^G\right)^{1-a} \quad (2.1)$$

The following standard parameter restrictions as discussed in Revankar (1971) hold:
$0 \le a \le 1$, and $\frac{e_t^G}{e_t^P} < \frac{1}{|b|}$ when $b < 0$.[4]

As we mentioned previously, b is a catchall parameter that affects the aggregate degree of substitutability between public and private education expenditures. The parameter a can be interpreted as the "pure" share of public education in the education production function. We abstract from technological progress in the education production function for simplicity and also assume that the education production function is characterised by constant returns to scale. It can be seen that when $e_t^G = 0$, $e_t = 0$. Hence, as mentioned before, e_t^G is the essential input in the education production function.

The elasticity of substitution between private and public expenditures is: [5]

$$\eta = 1 + b\left(\frac{e_t^G}{e_t^P}\right) \tag{2.2}$$

Note that in (2.2), if $b \ge 0$, $\eta \ge 1$ and if $-1 < b \le 0$, $0 \le \eta \le 1$. We can also see that the agent's elasticity of substitution between private and public education expenditures is positively and linearly related to the aggregate substitutability parameter b. The elasticity of substitution is further impacted by the ratio of public to private education expenditures, with an increase in this ratio leading to a rise in the elasticity of substitution if b is positive, and a

[4] As will be shown, in addition to these standard restrictions, further restrictions need to be imposed on the value of b in our model to ensure interior solutions for decision variables.

[5] Equation (2.2) is derived by simplifying the standard expression for the elasticity of substitution between two inputs, which in the case of the education production function with public and private expenditures, can be expressed as: $\frac{d\ln\left(\frac{e_t^G}{e_t^P}\right)}{d\ln\left(\frac{\partial e_t / \partial e_t^G}{\partial e_t / \partial e_t^P}\right)}$. For a detailed explanation of the derivation of the elasticity of substitution in the standard VES production function with labour and capital as the inputs, see Revankar (1971).

reduction in the elasticity of substitution if b is negative. As there is no empirical evidence regarding the link between the ratio of public-private expenditures and the degree of substitutability between them, the VES form allows the flexibility to explore both of these possibilities by varying the value of b.[6] The human capital production function is of the Cobb-Douglas form, with education acquired in childhood and parental human capital forming an adult's stock of human capital h_{t+1} as follows:

$$h_{t+1} = e_t^{\gamma} h_t^{\delta}, \qquad 0 < \gamma < 1, \ 0 < \delta < 1; \tag{2.3}$$

In (2.3), parental human capital affects the child's human capital directly through the inclusion of parental human capital in the human capital formation equation, as well as indirectly through education, as higher human capital enables the parent to earn more, thereby making it possible for the parent to invest more in her child's education. We do not make an assumption about the returns to scale to human capital because, as we shall see later, transitional dynamics could differ depending on whether the human capital formation function displays increasing, decreasing or constant returns to scale. We also abstract from time spent studying in order to improve analytical tractability.

2.2 Production

Production in this economy follows a Cobb Douglas functional form with human and physical capital as the inputs like in Boldrin (2005), and takes the following intensive form:

$$y_t = k_t^{\lambda} h_t^{1-\lambda} \tag{2.4}$$

The competitive interest rate and wage rate per unit of human capital are given by:

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[6] This variation in the elasticity of substitution at different points along a particular isoquant is consistent with the idea proposed earlier that the two inputs are imperfect substitutes for each other.

$$r_t = \lambda \left(\frac{k_t}{h_t} \right)^{\lambda - 1} \tag{2.5}$$

$$w_t = (1 - \lambda) \left(\frac{k_t}{h_t} \right)^{\lambda} \tag{2.6}$$

From equation (2.6) above, an adult's earnings in period t is her wage per unit of human capital w_t times her stock of human capital h_t which is accumulated according to equation (2.3).

2.3 Government

The government imposes a proportional tax τ in each period on the adult agent's earnings to finance public education expenditure and runs a balanced budget. Hence,

$$e_t^G = \tau w_t h_t \tag{2.7}$$

2.4 The agent's problem

Assuming logarithmic preferences, the agent's utility at time t is given by:

$$U = \ln(c_{t+1}) + \beta \ln(c_{t+2}) + \theta \ln(h_{t+2}) \ , \ 0 \le \beta \le 1, \ \ \theta > 0. \tag{2.8}$$

The agent derives utility from consumption in adulthood c_{t+1} and consumption in old age c_{t+2}. In old age, the agent also derives utility from her child's human capital h_{t+2}. The parameter β represents the discount factor. The parameter θ is the product of the discount factor β and another "warm glow" parameter, which measures the satisfaction a parent receives from her child's human capital.

This agent faces the following budget constraints in adulthood and old age respectively:

$$c_{t+1} + e_{t+1}^P + s_{t+1} = w_{t+1} h_{t+1} (1 - \tau) \tag{2.9}$$

$$c_{t+2} = R_{t+2} s_{t+1} \tag{2.10}$$

According to (2.9) above, in adulthood, the agent inelastically supplies h_{t+1} units of human capital, and receives a wage of w_{t+1} per unit of human capital supplied. The government levies a proportional tax of τ on her labour income to finance public education. Hence, her after tax income is $w_{t+1}h_{t+1}(1-\tau)$. She utilises her after tax income on consumption c_{t+1}, private expenditure on her child's education e^{P}_{t+1} and savings s_{t+1}. According to (2.10), in old age, the agent uses her savings that have accumulated a gross real return of $R_{t+2} = 1 + r_{t+1}$ to finance her consumption c_{t+2}.

Thus, the agent's utility maximisation problem involves maximising (2.8) subject to (2.9) and (2.10). This yields the following FOCs:

$$\frac{c_{t+2}}{c_{t+1}} = \beta R_{t+2} \tag{2.11}$$

$$\frac{h_{t+2}}{c_{t+1}} = \theta \tag{2.12}$$

Equation (2.11) is the standard consumption Euler equation shows that the consumer cannot improve her utility by shifting consumption between periods when she maximises utility. Equation (2.12) captures the idea that the agent has to forego utility from adult age consumption in order to invest in her child's education and enjoy a higher utility from her child's stock of human capital in old age. When the agent maximises utility, the marginal cost of a unit of private education spending in terms of the foregone adult age consumption should be equal to the marginal benefit the agent enjoys in terms of the additional human capital her child acquires.

Before we define the competitive equilibrium, we note that in any given period it is essentially the middle-aged agent's decision making that is non-trivial; in the first period of life all decisions are undertaken by the parent while the last period of life is spent consuming savings carried over from the previous period, inclusive of returns. Markets for physical capital clear in the sense that old agents inelastically supply all of their savings in the form of physical capital to the representative firm ($s_t = k_{t+1}$) and receive a return as described in (2.5). Markets for human capital also clear in a similar sense; as human capital is determined by the parent's investment in the offspring's education and the parent's human capital, the agent in middle-age inelastically supplies this endowment in the labour market, receiving a wage described by (2.6). The Walras Law applied to period *t* then ensures goods market equilibrium.

2.5 Competitive equilibrium

A competitive equilibrium in this environment is a sequence of consumption, savings and private education expenditure $\{c_{t+1}, c_{t+2}, s_{t+1}, e^P_{t+1}\}_{t=0}^{t=\infty}$ chosen by the agent that satisfies the FOCs (2.11) and (2.12), taking factor prices represented by (2.5) and (2.6) as given; a sequence of input and output choices $\{y_{t+1,} k_{t+1}, h_{t+1}\}_{t=0}^{t=\infty}$ made by the representative firm according to (2.4) so as to maximise profits, taking factor prices and government policy as given; a sequence of factor prices $\{r_{t+1}, w_{t+1}\}_{t=0}^{t=\infty}$ as given by (2.5) and (2.6), such that the markets for physical capital, human capital and aggregate output clear; and a sequence of government policies $\{e^G_{t+1}, \tau_{t+1}\}_{t=0}^{t=\infty}$ such that the government's budget as given by (2.7) is balanced in each period. Equations (2.13)-(2.16) below are the optimal solutions to the agent's decision problem:[7]

$$c_{t+1} = \frac{w_{t+1} h_{t+1} (1 - \tau + ab\tau)}{1 + \beta + \theta\gamma(1 - a)} \tag{2.13}$$

$$e^P_{t+1} = w_{t+1} h_{t+1} \left[\frac{\theta\gamma(1 - a)(1 - \tau) - ab\tau(1 + \beta)}{1 + \beta + \theta\gamma(1 - a)} \right] \tag{2.14}$$

[7] The definition of competitive equilibrium given here is similar to that found in De La Croix and Michel (2002) and Acemoglu (2008). As decision-making in the economy is staggered, the decisions in competitive equilibrium hold for all generations and all resource constraints are satisfied for these optimal values.

$$s_{t+1}=\frac{\beta w_{t+1}h_{t+1}\left(1-\tau+ab\tau\right)}{1+\beta+\theta\gamma\left(1-a\right)} \tag{2.15}$$

$$c_{t+2}=\frac{\beta R_{t+2}w_{t+1}h_{t+1}\left(1-\tau+ab\tau\right)}{1+\beta+\theta\gamma\left(1-a\right)} \tag{2.16}$$

Let $m=-\frac{(1-\tau)}{a\tau}$ and $n=\frac{\theta\gamma(1-a)(1-\tau)}{a\tau(1+\beta)}$. Interiority of the above solutions occur only in the range $m<b<n$. However, in any given period, the elasticity of substitution between public and private education inputs η, given by equation (2.2), should be bounded from below so that $\eta>0$. Hence, the above range of values for b needs to be modified to incorporate this constraint. Given $p=\frac{-\theta\gamma(1-\tau)}{\tau[(1+\beta)+\theta\gamma]}$, the range of values of b necessary to ensure interiority *and* a positive elasticity of substitution between public and private inputs is: $p<b<n$. A detailed derivation of this range is provided in Appendix C. The analyses presented in the remainder of the paper assume that the parameter b conforms to this range.

From (2.14), we can see that if $b>\frac{-\theta\gamma(1-a)}{a(1+\beta)}$, public education expenditure crowds out private education spending. This is because a higher value of the aggregate substitutability parameter b means that there is a greater degree of similarity between private and public education inputs in terms of access and quality. Thus, as indicated by equations (2.13) to (2.16), a higher value of the aggregate substitutability parameter b causes the agent to undertake lower private investment in education, causing consumption in both periods and savings to rise. Hence, from (2.8), we have:

Proposition 1: *A higher value of b results in a higher utility for the agent.*

The intuition behind Proposition 1 lies in the idea that a higher value of the aggregate substitutability parameter b, which impacts the elasticity of substitution positively, implies that private education spending is a closer substitute for public spending. This reduces the incentive for a parent who already supports the public education system by paying a mandatory tax to undertake private expenditure. Typically, parents would be keen to undertake more private expenditure on education only if private education services can augment a child's learning experience because they are of a superior quality to public expenditure, implying a lower degree of substitutability between the two inputs. In such an instance, private education services play a critical role in enhancing and augmenting the child's stock of education, and parents would therefore be keen to spend on such services in order to compensate for the low quality of public education their children receive. On the other hand, when aggregate substitutability is high, parents would undertake lower private education expenditures which enables them to undertake higher consumption and savings, leading to a higher level of utility from consumption during both adulthood and old age, thereby resulting in a higher lifetime utility.

3. STEADY STATE AND STABILITY ANALYSIS

In this section, we first derive and explore the behaviour of the steady state values of per capita physical and human capital. Then we explore the dynamic properties of the model and the conditions under which the steady state is stable.

3.1 Steady state analysis

First, note that savings in period $t+1$ are used to build the stock of physical capital in period $t+2$. Hence, in general equilibrium,

$$s_{t+1} = k_{t+2} \tag{3.1}$$

Using equation (3.1), and setting $j = \frac{(1-\tau+ab\tau)}{1+\beta+\theta\gamma(1-a)}$ the physical capital accumulation equation is:

$$k_{t+2} = \phi^1(k_{t+1}, h_{t+1}) = \beta j k_{t+1}^{\lambda} h_{t+1}^{1-\lambda} \tag{3.2}$$

Similarly, using (2.3), the human capital accumulation function is:

$$h_{t+2} = \phi^2(k_{t+1}, h_{t+1}) = k_{t+1}^{\lambda\gamma} h_{t+1}^{\delta+(1-\lambda)\gamma} (1-\lambda)^{\gamma} \left\{\tau^{a}\left[\theta\gamma(1-a)j\right]^{1-a}\right\} \tag{3.3}$$

(3.2) and (3.3) constitute of a two-dimensional system of non-linear first order difference equations. A steady state equilibrium of this system is a pair of values $(\bar{k}, \bar{h})$ such that $k_{t+2} = k_{t+1} = \bar{k}$ and $h_{t+2} = h_{t+1} = \bar{h}$. The steady state stocks of physical and human capital are therefore given by:[8]

$$\bar{h} = \left\{(1-\lambda)^{\gamma} \tau^{a\gamma} j^{\frac{\lambda\gamma+(1-a)(1-\lambda)\gamma}{1-\lambda}} \beta^{\frac{\lambda\gamma}{1-\lambda}} \left[\theta\gamma(1-a)\right]^{(1-a)\gamma}\right\}^{\frac{1-\lambda}{1-\lambda\gamma-(1-\lambda)[\delta-(1-\lambda)\gamma]}} \tag{3.4}$$

$$\bar{k} = (j\beta)^{\frac{1}{1-\lambda}} \left\{(1-\lambda)^{\gamma} \tau^{a\gamma} j^{\frac{\lambda\gamma+(1-a)(1-\lambda)\gamma}{(1-\lambda)}} \beta^{\frac{\lambda\gamma}{1-\lambda}} \left[\theta\gamma(1-a)\right]^{(1-a)\gamma}\right\}^{\frac{1}{1-\lambda\gamma-(1-\lambda)[\delta-(1-\lambda)\gamma]}} \tag{3.5}$$

Before exploring the stability of this non-linear system, we examine the manner in which the steady state stocks of human and physical capital are affected by the value of the parameter b. The following proposition captures how the value of the aggregate substitutability parameter b impacts on the steady state stocks of human and physical capital:

Proposition 2: *A higher value of b enables an economy to reach a higher steady state level of physical and human capital.*

---

[8] The steady state values given by equations (3.4) and (3.5) are less than 1 because, firstly, all the parameters here are less than 1, and $j < 1$ for all values in the range $p < b < n$ that was established earlier. This observation is useful for the subsequent stability analysis.

Note that j is always positive and monotonically increasing in b. As it is evident from equations (3.4) and (3.5) that the steady state stocks of human and physical capital are monotonically rising in j, it follows that they are also increasing in b. In the case of the conventional CES production function typically embedded in growth models, a higher elasticity of substitution between labour and capital has been demonstrated theoretically and empirically as a means of achieving a higher output per capita (see, for instance, Irmen, 2011; Karagiannis et al., 2005; Klump & De La Grandville, 2000; Mallick, 2012a). Proposition 2 is a somewhat heuristic extension of this notion, in that a higher elasticity of substitution between public and private education expenditures impacts an individual's stock of human and physical capital positively, thereby yielding a higher output per capita. As is evident from the education production function given by equation (2.3), the agent's stock of education, and thereby human capital, is increasing in the aggregate substitutability parameter. In terms of physical capital, a higher elasticity of substitution means that it is easier for an agent to substitute between public and private education. This means that a given level of human capital can be achieved with lower private spending, thereby enabling agents to save more in adulthood as is evident from equation (2.15), which shows that savings are increasing in the aggregate substitutability parameter b, which yields a higher stock of physical capital in the next period for any given level of human capital. This is made clear if we look at how the ratio of physical to human capital at steady state is affected by the value of the aggregate substitutability parameter b. Dividing (3.4) by (3.5) gives us the following ratio of physical to human capital:

$$\frac{\bar{k}}{\bar{h}} = \left(\bar{h}\right)^{\frac{\lambda}{1-\lambda}} \left(j\beta\right)^{\frac{1}{1-\lambda}} \tag{3.6}$$

As $\bar{h}$ is monotonically increasing in b, it is clear that $\frac{\bar{k}}{\bar{h}}$ is also increasing in the aggregate substitutability parameter b. Thus, higher aggregate substitutability between public and private education expenditures causes the stock of physical capital to rise relative to the stock of human capital in steady state.

3.2 Stability analysis

Now, we analyse the stability of the system characterised by (3.2) and (3.3) which is of the form $X_{t+1} = \phi(X_t)$ where $X = \begin{pmatrix} k \\ h \end{pmatrix}$. We use the methods for studying the stability of non-linear first order discrete dynamical systems described in Galor (2007). First, we lag the two dimensional system characterised by equations (3.2) and (3.3) by one period, and we then employ a Taylor series expansion to linearise them around the steady state, which enables us to rewrite the system in the following form:

$$k_{t+1} = \phi^1(\bar{k},\bar{h}) + \frac{\partial \phi^1(\bar{k},\bar{h})}{\partial k_t}(k_t - \bar{k}) + \frac{\partial \phi^1(\bar{k},\bar{h})}{\partial h_t}(h_t - \bar{h}) \tag{3.7}$$

$$h_{t+1} = \phi^2(\bar{k},\bar{h}) + \frac{\partial \phi^2(\bar{k},\bar{h})}{\partial k_t}(k_t - \bar{k}) + \frac{\partial \phi^2(\bar{k},\bar{h})}{\partial h_t}(h_t - \bar{h}) \tag{3.8}$$

Upon evaluating the partial derivatives at the steady state values and substituting them into (3.7) and (3.8), and also setting $l = (1-\lambda)^{\gamma}\left\{\tau^{a}\left[\theta\gamma(1-a)j\right]^{1-a}\right\}$, the linearised system in matrix form can be expressed as follows:

$$\begin{bmatrix} k_{t+1} \\ h_{t+1} \end{bmatrix} = \begin{bmatrix} \beta j\lambda\left(\frac{\bar{k}}{\bar{h}}\right)^{\lambda-1} & \beta j(1-\lambda)\left(\frac{\bar{k}}{\bar{h}}\right)^{\lambda} \\ l\lambda\gamma\bar{k}^{\lambda\gamma-1}\bar{h}^{\delta+(1-\lambda)\gamma} & l[\delta+(1-\lambda)\gamma]\bar{k}^{\lambda\gamma}\bar{h}^{\delta+(1-\lambda)\gamma-1} \end{bmatrix}\begin{bmatrix} k_t \\ h_t \end{bmatrix} + \begin{bmatrix} 0 \\ l(1-\delta-\gamma)\bar{k}^{\lambda\gamma}\bar{h}^{\delta+(1-\lambda)\gamma} \end{bmatrix} \tag{3.9}$$

We have now approximated our original non-linear two-dimensional discrete dynamical system with a new system which is linear around the steady state that conforms to the form $X_{t+1} = AX_t + B$, where A is the Jacobian matrix of $\phi(X)$ evaluated at the steady state $\bar{X}$. The eigenvalues of A can be used to obtain a characterisation of the system.[9] We capture the different characterisations of the steady state as follows:

Proposition 3: Let $z_1 = \beta j \lambda \left(\frac{\bar{k}}{\bar{h}} \right)^{\lambda - 1} + l[\delta + (1-\lambda)\gamma]\bar{k}^{\lambda\gamma}\bar{h}^{-\delta + (1-\lambda)\gamma - 1}$,

and $z_2 = \beta j \lambda l \delta \bar{k}^{\lambda(1+\gamma)-1}\bar{h}^{\delta + \gamma - \lambda(1+\gamma)}$.

The eigenvalues μ_1 and μ_2 of the Jacobian matrix A are given by:

$$\mu_1 = \frac{z_1 + \sqrt{z_1^2 - 4z_2}}{2} \tag{3.10}$$

$$\mu_2 = \frac{z_1 - \sqrt{z_1^2 - 4z_2}}{2} \tag{3.11}$$

Given these eigenvalues, the possible characterisations of the steady state are as follows:

(i) If $z_1^2 > 4z_2$ and $\frac{z_1 + \sqrt{z_1^2 - 4z_2}}{2} < 1$, i.e. $z_1 - 1 < z_2 < z_1^2/4$, we have distinct, real eigenvalues $0 < \mu_2 < \mu_1 < 1$, and the steady state is stable, resulting in monotonic convergence to the steady state. [10]

(ii) If $z_1^2 = 4z_2$, we have repeated, real eigenvalues such that $0 < \mu_2 = \mu_1 < 1$, the steady state is stable, and both human and physical capital converge to the steady state at an equal rate.

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[9] These characterisations derived from the linearised system provide an analysis of the local stability conditions.

[10] Note that this range is continuous only if $z_1 > 2$.

(iii) If $z_1^2 < 4z_2$, μ_1 and μ_2 are distinct complex eigenvalues with a positive real part. Since $0 < \frac{z_1}{2} < 1$, and since the condition $|\mu_1| < 1$ is always satisfied, there is will oscillatory convergence towards a locally stable steady state. [11]

In Proposition 3 above, case (i) resembles a long run outcome akin to that which occurs in the Solow growth model. In the ranges for which the conditions for monotonic convergence towards the steady state are satisfied, if the initial value of human or physical capital is below the steady state, its value would monotonically increase until its steady state value is reached, while an initial value below the steady state leads to a monotonic decrease in the value until steady state is reached. As μ_1 is the eigenvalue associated with physical capital per capita k_t, the eigenvalues $0 < \mu_2 < \mu_1 < 1$ imply that physical capital per capita converges to its steady state value at a relatively faster rate than human capital per capita. However, once convergence to the steady state has occurred, the growth rate of physical and human capital per capita, as well as output per capita is 0, which is the outcome we observe in the standard Solow-Swan model. The phase diagram associated with this steady state is illustrated in Figure 3.1. Notice that in Figure 3.1, all the arrows point towards the steady state $(\bar{k}, \bar{h})$. The dotted straight lines indicate that if the initial stock of physical or human capital is equal to the steady state value, the value of the other variable whose initial value is above or below the steady state value will monotonically converge towards the steady state. The curved solid lines show the trajectory of the equilibrium path when the initial stocks of both human and physical capital are above or below the steady state. The small arrows on the sides, which point towards the dotted and solid

[11] Note that if the condition $\sqrt{\frac{z_1^2}{4} + \frac{z_1^2 - 4z_2}{4}} > 1$ could be satisfied when $z_1^2 < 4z_2$, oscillatory divergence, or a spiral source as it is sometimes called would have occurred. However, these two conditions can never be satisfied simultaneously, and we can therefore rule out the case of the spiral source.

lines, indicate that the two equilibrium trajectories can take only one of these two forms depending on the initial stocks of human and physical capital.

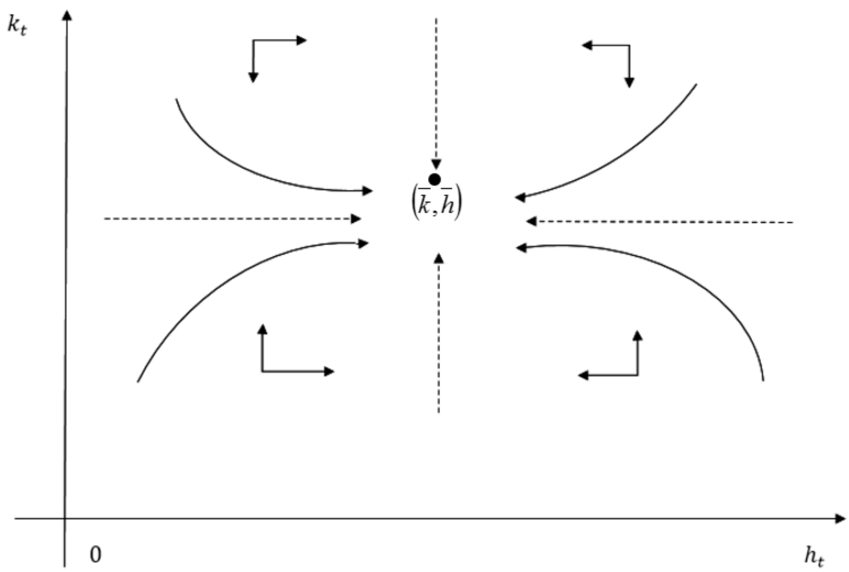

Figure 3.1: Phase diagram for the stable steady state

The steady state associated with the second case outlined in Proposition 3 is commonly referred to as a "focus." In this instance, both human and physical capital converge linearly to the steady state. This convergence occurs at an identical rate for both variables. Hence, the only difference between cases (i) and (ii) is the speed of convergence of the two variables to their steady state values. The phase diagram in this instance is depicted in Figure 3.2 below. In this instance, regardless of the initial stocks of human and physical capital, the family of straight lines surrounding the steady state combination of human and physical capital $(\bar{k}, \bar{h})$ indicates that there is systematic convergence to the steady state.

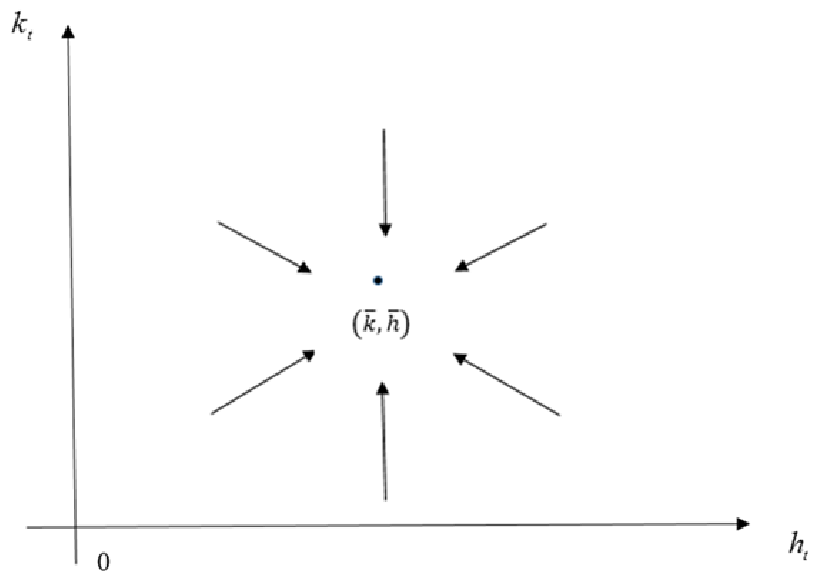

Figure 3.2: Phase diagram for the focus

It is also interesting to observe that the eigenvalues are monotonically increasing in the parameter b in cases (i) and (ii), implying that a higher degree of aggregate substitutability between public and private education inputs enables an economy to converge to the steady state faster. This observation suggests that a higher degree of aggregate substitutability benefits an economy in two main ways: firstly by enabling the economy to transition towards a higher steady state and secondly by contributing towards the speed of this transition. This observation is also consistent with extant literature that suggests that in the context of a standard production function with capital and labour, a higher degree of factor substitutability enables an economy to transition to the steady state faster (see Irmen, 2011 for a detailed discussion).

When the eigenvalues are complex, as stated in case (iii), oscillatory convergence towards a locally stable steady state, commonly known as a spiral sink, will occur. In the case of a spiral sink, both physical and human capital per capita are characterised by oscillatory convergence to the steady state. These oscillations create fluctuations in the stocks of human and physical capital over time as the economy converges to the steady state. However, the steady state is asymptotically stable in this instance, implying that as $t \to \infty$, $k_t \to \bar{k}$ and

$h_t \to \bar{h}$. We know that the spiral convergence to the steady state in this instance occurs in clockwise motions as the real part $\frac{\mu_1}{2}$ of both eigenvalues is positive. The phase diagram depicting the manner in which human and physical capital converge to the steady state is given in Figure 3.3. As we can see, regardless of whether the initial stocks of human and physical capital are higher or lower than the steady state values, the arrows that point inwards indicate that they converge over time to the steady state. Therefore, this outcome is also quite similar to cases (i) and (ii). The only difference in this instance is the non-monotonicity of the convergence path. The converging spiralling motions indicate that although both the stocks of human and physical capital are characterised by oscillations, the size of these oscillations grow smaller over time, and decay off once the steady state is reached.

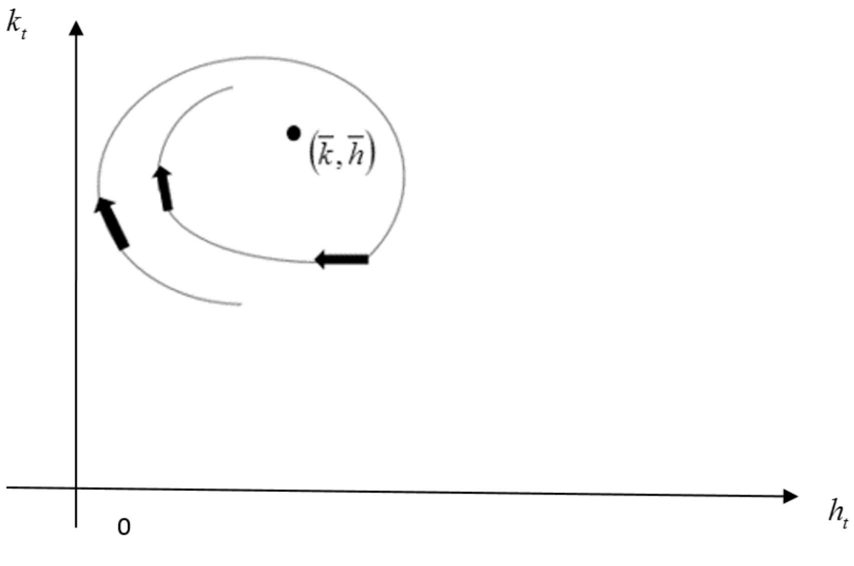

Figure 3.3: Phase diagram for the standard spiral sink

However, there is an aspect to the transitional dynamics that warrants further discussion. Recall that our model comprises of restrictions on the parameter b; as established earlier, for the elasticity of substitution between public and private education inputs to be positive and for interiority of the solutions to the agent's lifetime utility maximisation problem, the range $p < b < n$ should hold. This range for b also implies that there is a corresponding range of values for the stocks of human and physical capital in any given period given by equations (3.2) and (3.3). This implies that in any given period, both forms of capital should conform to certain ranges of positive values $k_{min} < k_t < k_{max}$ and $h_{min} < h_t < h_{max}$ as determined by these bounds for b.

While these bounds on human and physical capital are present across all the cases outlined in Proposition 3, they do not impact the convergence paths in the case of the *stable steady state* and the *focus*. The initial stock of human and physical capital should conform to these ranges, and the presence of these upper and lower bounds do not affect the monotonic convergence towards the steady state that occurs in these instances. However, with the *spiral sink*, these bounds create transitional dynamics that differ from the standard cases.

With the spiral sink, the phase portrait given in Figure 3.4 shows how the upper and the lower bounds affect the economy's convergence path. At point A, the standard convergence path falls outside the maximum human capital stock. Therefore, the economy moves along the upper bound until point B, where it meets the usual transitional path once again. Similarly, between points C and D, the standard path results in human capital stocks below h_{min}. Hence, the economy moves vertically along the bound given by h_{min} until it meets the usual convergence path at point D. Nevertheless, the changes to the transition path that occur due to the presence of the upper and lower bounds do not prevent the economy from converging to the steady state in the long run.

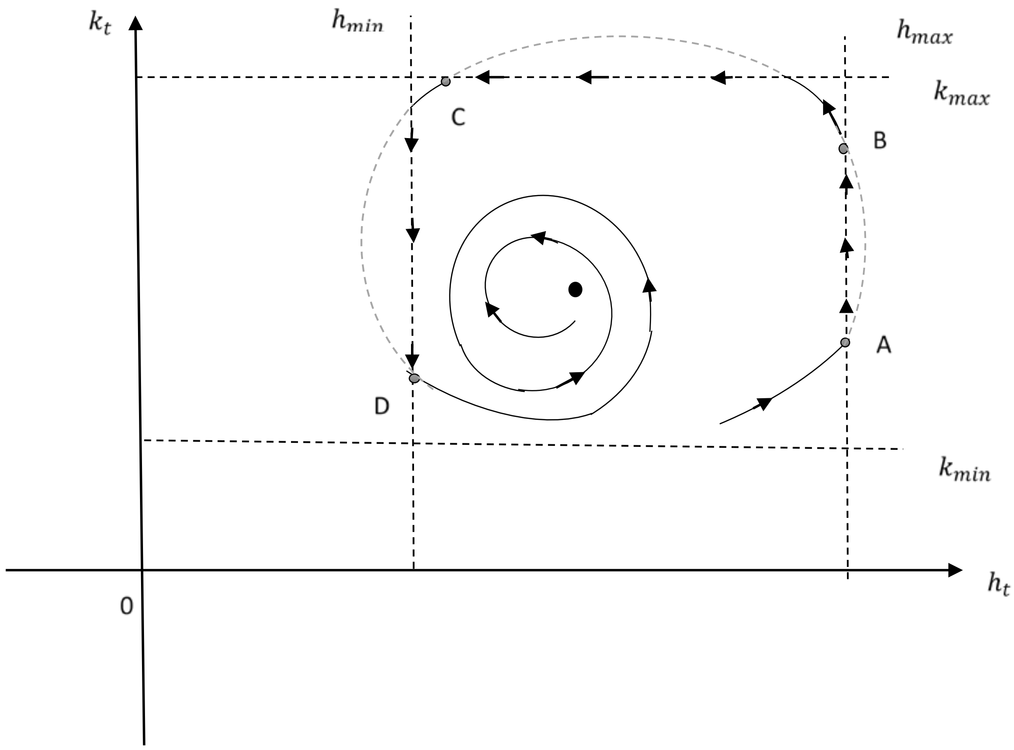

Figure 3.4: Modified phase diagram for the spiral sink

The spiral sink is suggestive of cyclical behaviour of private investments in education. The fluctuations in private school enrolments experienced by some countries may be indicative of the empirical relevance of this outcome. For instance, although the evidence is by no means clear-cut, Figure 3.5 below, which shows the private enrolment rates at the secondary level for Spain, USA, Syria and Mexico is indicative of a mildly cyclical pattern, as well as a downward trend in private enrolment rates over time.

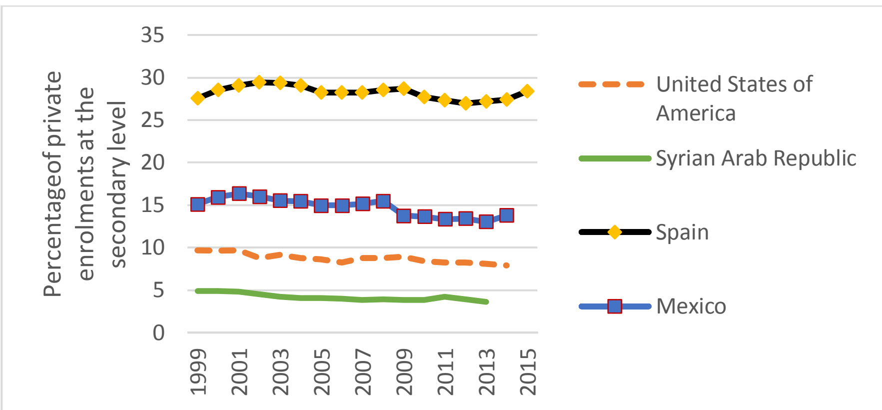

Figure 3.5: Cyclical movements in private secondary school enrolment in selected countries

Source: UNESCO Institute for Statistics

Next, we look at *when* these different scenarios are likely to occur. Interestingly, we find that there are three cases depending on the fulfilment or non-fulfilment of a condition relating to the described in the Appendix G. While the analysis is largely technical, the occurrence of the scenarios in proposition 3 can be characterised according to how the parameter δ, which represents the share of parental human capital in the child's human capital production as determined by equation (2.3), compares with $(1-\lambda)\gamma$, the product of the share of human capital in output production and the share of education in the child's human capital production and several other threshold values of δ that lie above $(1-\lambda)\gamma$.[12] As such, this analysis is somewhat amenable to economic interpretation. Intuitively δ represents the extent to which *inherited* human capital matters in contributing to an agent's human capital, while $(1-\lambda)\gamma$ is a composite parameter reflecting both direct and indirect returns to investment in education. The derivations presented in Appendix G reveal three possibilities:

The first possibility is where a spiral sink never occurs; in this instance, monotonic convergence occurs when $\delta < (1-\lambda)\gamma$, while a focus could occur when δ exceeds $(1-\lambda)\gamma$. Nevertheless, there are two cases where a spiral sink may transpire, and in these instances, we are able to identify certain threshold values of δ that determine the ranges of this parameter for which the economy is able to transition monotonically towards the steady state, In the first of these cases, monotonic convergence occurs when $\delta < (1-\lambda)\gamma$, a focus occurs in a range $(1-\lambda)\gamma \le \delta < \delta_A$ and a spiral sink occurs when $\delta_A \le \delta \le 1$. Hence, in this instance, δ_A can be

[12] These threshold values capture the instances where certain equality conditions on the eigenvalues of the dynamical system are satisfied.

interpreted as a threshold value below which the economy can achieve a stable transition towards the steady state. The second possibility, which occurs only if a certain threshold value δ_B is below 1, is where monotonic convergence occurs when $\delta < (1-\lambda)\gamma$, a focus occurs in a range $(1-\lambda)\gamma \leq \delta < \delta_B$, a spiral sink occurs when $\delta_B \leq \delta < \delta_C$ and a focus could transpire once again in the range $\delta_C \leq \delta \leq 1$. In this instance, since the economy is subject to oscillatory convergence in the range $\delta_B \leq \delta < \delta_C$, δ_B has an interpretation similar to the threshold value in the second scenario: as long as $\delta < \delta_B$, the economy is able to transition monotonically towards the steady state. Furthermore, δ_C represents another threshold value *above* which the economy can achieve monotonic convergence consistent with the focus.

The preceding analysis shows that generally, the economy could display oscillatory convergence towards the steady state for sufficiently large values of the share of parental human capital δ. In addition to the value of δ in itself, returns to scale to human capital, given by $\delta + \gamma$, also have a bearing upon the type of transitional dynamics that are likely to occur. Recall that we did not restrict ourselves to a particular returns-to-scale scenario when we introduced the human capital accumulation function given by equation (2.3). The occurrence of each of the three possible transition paths depends on whether the returns to scale to human capital are increasing, constant or decreasing. By combining each returns to scale scenario with the ranges of δ for which the different transition paths occur, we obtain the following results: with increasing returns to scale, i.e. $\delta + \gamma > 1$, if $\gamma > 1/(2-\lambda)$, all three outcomes are possible but when $\gamma < 1/(2-\lambda)$, the only possible outcomes are the focus and the spiral sink. With decreasing returns to scale where $\delta + \gamma < 1$, if $\gamma > 1/(2-\lambda)$ only monotonic convergence can occur while any of the three outcomes when $\gamma < 1/(2-\lambda)$. Finally, under constant returns to scale where $\delta + \gamma = 1$, if $\gamma > 1/(2-\lambda)$, only monotonic convergence can occur, but when $\gamma < 1/(2-\lambda)$, the only dynamics possible are the focus or oscillatory convergence. From this analysis, it is clear that oscillatory convergence is likely to occur when $\gamma < 1/(2-\lambda)$. The presence of such a threshold level of the share of education beyond which the economy can achieve a stable transition path has some interesting policy implications as the role of education in the creation of human capital is often determined by the allocation of resources by policymakers. Disparities in resourcing across income groups and regions could undermine the share of education in the creation of human capital.

The foregoing results shows that fluctuations during transition would occur if the share of parental human capital in determining offspring's human capital is sufficiently high and the share of education in the human capital accumulation function is below a certain threshold level. The roles of parental human capital and education in determining an individual's human capital are often dependent upon the broader level of institutional development in a country. Government spending on health, education and welfare can reduce the strength of intergenerational transmission of human capital and increase returns to education by improving equity and contributing towards the upward socio-economic mobility of children from poor families (Guo, Hu, Liu, & Yin, 2016). Furthermore, countries where families have less children on average tend to exhibit higher returns to education (Zhu, Whalley, & Zhao, 2014). Bargaining power of females within the typical household also improves returns to schooling for children (Weir, 2000). Generally however, it is likely that parental human capital would play a more important role relative to education in the determination of the human capital of offspring in developing countries where there are limited opportunities for children of poor parents to attain a good education, and one's lineage and connections play a key role in success.

Hence, a possible interpretation of our results is that these institutional factors, through their bearing on the relative importance of determinants of human capital, may contribute towards the instability of growth paths of many developing countries.

4. OPTIMAL POLICY

So far, we assumed that the tax rate τ was given. We now look for the optimal value of τ a benevolent social planner may select in order to enable the representative agent to attain maximum utility. This is done by substituting the utility maximising values of c_{t+1}, e^{P}_{t+1} and c_{t+2} given by (2.13), (2.14) and (2.16) respectively into the agent's utility function (2.8) and differentiating it with respect to τ (the derivation is outlined in Appendix H). This results in the following optimal solution for τ:[13]

$$\tau^{*} = \frac{\gamma\theta a}{(1+\beta+\gamma\theta)(1-ab)} \tag{4.1}$$

The optimal tax rate can be expressed solely in terms of the parameters in the model, which means that it is recursive, i.e. the tax rate remains the same in each period. For the solution to τ^{*} to be interior, the condition $b < \frac{1}{a}$ must be satisfied. When $b > \frac{1}{a}$ a corner solution where $\tau^{*} = 0$ occurs. This leads to $h_{t+2} = 0$. On the other hand, if $b < \frac{-\theta\gamma}{(1+\beta+\gamma\theta\beta)} + \frac{1}{a}$, $\tau^{*} = 1$. However, in the range $p \le b < n$, which was established previously as required to ensure that the solutions to the decision problem are interior and the elasticity of substitution at steady state is greater than 0, neither of these corner solutions will emerge. Thus, in this range, a higher value of *b* results in a higher value of τ^{*}. Hence we have:

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[13] Time invariance of the optimal tax rate in our model is an artefact of log utility. With a utility function of more general form, such as $u(c) = c^{1-\psi}/(1-\psi)$, the model does not allow a closed form solution, although it is clear that tax rates are implicitly related to income, and hence time.

Proposition 4: *The tax rate which enables the agent to maximise utility is increasing in the value of b and hence the elasticity of substitution.*

Proposition 4 shows that when public and private education inputs are closely substitutable, people are willing to pay more tax and strengthen the public education system. The intuition behind this result is that, given public education expenditure is the essential input in the education production function, if an additional dollar on either of these expenditures provide a comparable benefit for the child, parents would be willing to pay more tax and improve the public education system. If the public education system is of a poor quality, it would be an inefficient substitute for private education, and this would encourage parents to supplement public education with private education services. In such an instance, parents would not be willing to pay high taxes towards the maintenance of the inefficient public education system, as this money would be better spent on buying high quality private educational services for the child.

As a key motivation for our work is the idea that aggregate substitutability is likely to be higher in developed countries, this result suggests that governments in developed countries would spend more on education. Table 4.1 gives government expenditures on education as a percentage of GDP for a number of countries at various stages of development. In general, in high income countries such as Sweden, Switzerland, Finland and Denmark and New Zealand a relatively higher proportion of GDP is spent on education when compared to developing nations such as Bangladesh, Cambodia, Ethiopia, Nepal and Pakistan.

Country	Government expenditure on education as a percentage of GDP (latest year available)
Afghanistan	3.37581
Bangladesh	2.17913
Brazil	5.99795
Cambodia	1.90083
Denmark	8.61116
Ethiopia	4.49855
Finland	7.15848
Germany	4.94445
India	3.84184
Mozambique	6.48009
Nepal	3.7473
New Zealand	6.34961
Pakistan	2.66067
Portugal	5.27787
Republic of Moldova	7.46451
Rwanda	5.02795
Sweden	7.71774
Switzerland	5.06686
United States of America	4.94379

Table 4.1: Government expenditure as a percentage of GDP for selected countries

Source: Bank. (2017)

5. CONCLUSION

We develop an overlapping generations model to explore how public and private expenditures on education impact long run macroeconomic outcomes. Using a variable elasticity of substitution “education production function,” in which we assume public education to be the essential input, we show analytically that steady state utility, human capital, physical capital, output, and the human to physical capital ratio are increasing in aggregate substitutability between the two inputs, which, in our view, captures the degree of similarity in quality between public and private education expenditure. We also show that higher substitutability is associated with a higher tax rate to fund public education and simultaneously, lowers private education expenditures. Furthermore, the economy could display monotonic or oscillatory convergence towards the steady state depending on certain parameter values. In particular, a high share of parental human capital combined with a low share of education in human capital formation could lead to oscillations during transition.

Especially in developing countries, if public and private education are of comparable quality, it would encourage more parents to enrol their children in public schools, thereby reducing parental expenditure on education, and improving access to education for the poor. On the other hand, if the two inputs are less substitutable for one another, even if parents send their children to public schools, they would have to undertake complementary private expenditures on activities such as out-of-school tutoring, which may deter poor parents from educating their children.

Another benefit from the presence of public and private education that are of comparable quality is that it gives parents a greater choice with regard to the type of education they wish to provide to their children. One possible approach to achieving greater uniformity between public and private education inputs is to develop private participation in the education industry by offering greater support, possibly in the form of tax benefits, training educators in the private sector, and developing a common curriculum that can be used by both the public and private sectors. The quality of private provision can be ensured through arrangements such as registration, quality audits, and reviews undertaken by government authorities that provide parents with the confidence to consider the private education sector to be a close competitor to the public sector.

The fluctuations in economic activity during transition that can occur with a high share of parental human capital and low share of education in the human capital accumulation function relates to the idea that an economy's transition path depends on certain institutional factors. Typically such a combination of parameters is indicative low government spending on social services and infrastructure, a poor regulatory environment and the presence of high levels of corruption which restricts intergenerational social mobility. Hence, these results emphasise the importance role of institutions in enabling economies to achieve stable growth.

Empirical evidence regarding the effect of school type—whether publicly or privately funded—on student achievement is mixed (for a recent survey of the literature, see Hanushek & Woessmann, 2014). Similarly, evidence regarding the effect of private tutoring on academic achievement is also inconclusive (for instance, see Dang, 2007; Hof, 2014; Zhang, 2013). However, an unaccounted factor in most of the studies relating to both the public-private school divide and the private supplementation of public education is quality. While we have made an effort to capture relative equality implicitly using the aggregate substitutability parameter in the VES form, developing a measure for relative quality of the private and public components in an education system could be useful in empirically testing the results presented in the paper, and could contribute towards gleaning deeper insights into the variations in the public-private mix of educational expenditures across countries. Furthermore, as we consider a framework in which intra-generational heterogeneity is absent, we do not account for the impacts on inequality associated with greater aggregate substitutability. Distributional considerations are, however, an important dimension of this issue, and consequently an important direction of future research.

The positive relationship between the ratio of physical to human capital and the degree of aggregate substitutability is also an interesting result with implications for inter-generational redistribution. In our model, middle-aged agents own human capital and the old agents own physical capital. Therefore, despite a higher aggregate substitutability between public and private education expenditures directly contributing towards strengthening the stock of human capital, the model suggests that by helping to raise savings, it provides a greater contribution towards physical capital accumulation in the economy. As suggested by Bertola (1996), modelling these considerations in a continuous time OLG framework may enable one to explore these intergenerational redistribution concerns in greater depth. Thus, exploring both intra and intergenerational aspects of the model within a continuous time OLG framework might be a further extension of the model.

APPENDICES

Appendix A: Optimal solutions to the agent's decision problem

Under the assumption of log utility, (2.11) simplifies to:

$$c_{t+2} = \beta R_{t+2} c_{t+1} \tag{A1}$$

Using (2.1), (2.3) and (2.8), and assuming logarithmic preferences, we get:

$$\frac{\partial V(h_{t+2})}{\partial e^P_{t+1}} = \frac{1}{h_{t+2}} \times \frac{\gamma h_{t+2}}{e_{t+1}} \times \frac{(1-a)e_{t+1}}{e^P_{t+1} + abe^G_{t+1}} \tag{A2}$$

Simplifying (A2) and substituting into (2.12) yields:

$$\frac{\theta\gamma(1-a)}{e^{P}_{t+1}+ab\tau w_{t+1}h_{t+1}}=\frac{1}{c_{t+1}} \tag{A3}$$

(A3) can be rearranged to yield:

$$e^{P}_{t+1}=\theta\gamma(1-a)c_{t+1}-ab\tau w_{t+1}h_{t+1} \tag{A4}$$

Using equations (A1) and (A4), we can derive the optimal solutions to the agent's decision problem given by equations (3.13) to (3.16).

Appendix B: Proof of Proposition 1

From (2.13), we get:

$$U(c_{t+1})=\ln\left[\frac{w_{t+1}h_{t+1}(1-\tau+ab\tau)}{1+\beta+\theta\gamma(1-a)}\right] \tag{B1}$$

Then,

$$\frac{\partial U(c_{t+1})}{\partial b}=\frac{a\tau}{1-\tau+ab\tau} \tag{B2}$$

Similarly, from (4.2.16) we get: $\dfrac{\partial U(c_{t+2})}{\partial b}=\dfrac{a\tau}{1-\tau+ab\tau}$.

In the case of human capital, substituting (2.7) and (2.14) into (2.1) and then substituting into (2.3) gives us:

$$U(h_{t+2})=\ln\left\{h^{\delta}_{t+1}\left[w_{t+1}h_{t+1}\tau^{a}\left[\frac{\theta\gamma(1-a)(1-\tau+ab\tau)}{1+\beta+\theta\gamma(1-a)}\right]^{1-a}\right]^{\gamma}\right\}. \tag{B3}$$

Differentiating (B3) with respect to b gives us: $\dfrac{\partial U(h_{t+2})}{\partial b}=\dfrac{a\tau}{1-\tau+ab\tau}$.

As all three components of the utility function are increasing in b, the consumer's entire utility is increasing in b.

Appendix C: Derivation of the upper and lower bounds on b

First, note from (2.13), (2.15) and (2.16) that for c_{t+1}, s_{t+1} and c_{t+2} to be interior, the condition $1-\tau+ab\tau>0$ should hold. This simplifies to $b>m$, where $m=-\frac{(1-\tau)}{a\tau}$.

From (2.14), for $e^P_{t+1}>0$, we need $\theta\gamma(1-a)(1-\tau)-ab\tau(1+\beta)>0$. This condition simplifies to: $b<n$, where $n=\frac{\theta\gamma(1-a)(1-\tau)}{a\tau(1+\beta)}$

To get the modified upper bound for b that is needed to ensure that in any given period, the elasticity of substitution between public and private education expenditures is positive, we first substitute the expression for e^G_{t+1} from (2.7) and the optimal value for e^P_{t+1} given by (2.14) into (2.2) which yields the following inequality:

$$\eta=1+\frac{b\tau w_{t+1}h_{t+1}}{w_{t+1}h_{t+1}\left[\frac{\theta\gamma(1-a)(1-\tau)-ab\tau(1+\beta)}{1+\beta+\theta\gamma(1-a)}\right]}>0 \qquad \text{(C1)}$$

Noting that $\eta\geq 0$ and making b the subject of the resulting inequality yields the lower bound of $b>p$. However, the condition necessary for interior solutions for the optimal values of the decision variables was $b>\max\{-1,m\}$. Given $p=\frac{-\theta\gamma(1-\tau)}{\tau(1+\beta+\theta\gamma)}$ and $m=\frac{-(1-\tau)}{a\tau}$, p is binding if $p>m$. The required condition for $p>m$ is: $\theta\gamma a<1+\beta+\theta\gamma$, which is always satisfied since $0\leq a\leq 1$. Therefore, the range of values for b yielding interior solutions *and* satisfying the condition $\eta>0$ is $p<b<n$.

Appendix D: Derivation of the eigenvalues

The eigenvalues μ_1 and μ_2 are given by the following equations.

$$\mu_1 + \mu_2 = trA \qquad \text{(D1)}$$

$$\mu_1 \mu_2 = detA \qquad \text{(D2)}$$

Upon substituting the expressions for the trace and the determinant of the Jacobian matrix A into (D1) and (D2), we get:

$$\mu_1 + \mu_2 = \beta j \lambda \left(\frac{\bar{k}}{\bar{h}}\right)^{\lambda - 1} + l[\delta + (1-\lambda)\gamma]\bar{k}^{\lambda\gamma}\,\bar{h}^{\delta + (1-\lambda)\gamma - 1} \qquad \text{(D3)}$$

$$\mu_1 \mu_2 = \beta j \lambda l \delta \bar{k}^{\lambda(1+\gamma)-1}\,\bar{h}^{\delta + \gamma - \lambda(1+\gamma)} \qquad \text{(D4)}$$

By combining the expressions and setting $\mu_1 = \beta j \lambda \left(\frac{\bar{k}}{\bar{h}}\right)^{\lambda - 1} + l[\delta + (1-\lambda)\gamma]\bar{k}^{\lambda\gamma}\,\bar{h}^{\delta + (1-\lambda)\gamma - 1}$ and $\mu_2 = \beta j \lambda l \delta \bar{k}^{\lambda(1+\gamma)-1}\,\bar{h}^{\delta + \gamma - \lambda(1+\gamma)}$, we get the solutions for μ_1 and μ_2 given in (3.10) and (3.11).

Appendix E: Proof of proposition 3

To limit the possible characterisations of the steady state to cases (i) to (iii), we need to prove that $0 < \mu_1 < 1, \quad 0 < \mu_2 < 1$.

Note that $j < 1$ if $b < \frac{1 + \beta + \theta\gamma(1-a) + \tau - 1}{a\tau}$. Given the upper bound on *b*, which was derived earlier to be $n = \frac{\theta\gamma(1-a)(1-\tau)}{a\tau(\beta+1)}$, this condition is always satisfied. Now, it is clear that all the coefficients of the expressions in (3.4) and (3.5) are below 1. Hence, $0 < \bar{k}, \bar{h} < 1$. As $0 < \bar{h} < 1$, it is clear from (3.6) that $0 < \frac{\bar{k}}{\bar{h}} < 1$. Hence, it follows that $0 < \mu_1 < 1, \quad 0 < \mu_2 < 1$.

This limits the number of possible characterisations of the steady state to cases (i) to (iii) outlined in Proposition 3.

Appendix F: Derivation of the sufficient condition for the emergence of case (i) of proposition 3

Expanding the term inside the square root in equation (3.10) or (3.11), we get:

$$\left\{\beta j\lambda\left(\frac{\overline{k}}{\overline{\overline{h}}}\right)^{\lambda-1}+l\left[\delta+(1-\lambda)\gamma\right]\overline{k}^{\lambda\gamma}\,\overline{h}^{\delta+(1-\lambda)\gamma-1}\right\}^{2}-4\beta j\lambda l\delta\overline{k}^{\lambda(1+\gamma)-1}\,\overline{h}^{\delta+\gamma-\lambda(1+\gamma)} \qquad \text{(F1)}$$

If we ignore the squared terms, the following is a sufficient condition to ensure that (F1) is positive:

$$2\beta j\lambda l\left[\delta+(1-\lambda)\gamma\right]\overline{k}^{\lambda(1+\gamma)-1}\,\overline{h}^{\delta+\gamma-\lambda(1+\gamma)}-4\beta j\lambda l\delta\overline{k}^{\lambda(1+\gamma)-1}\,\overline{h}^{\delta+\gamma-\lambda(1+\gamma)}>0 \qquad \text{(F2)}$$

This simplifies to $\sigma<(1-\lambda)\gamma$.

Appendix G: Derivation of the conditions under which each of the cases in Proposition 4 occur

Case (i):monotonic convergence

In the range $z_1-1<z_2<z_1^2/4$, expanding the upper bound and rearranging gives:

$$\beta j\lambda\left(\frac{\overline{k}}{\overline{\overline{h}}}\right)^{\lambda-1}+l\left[\delta+(1-\lambda)\gamma\right]\overline{k}^{\lambda\gamma}\,\overline{h}^{\delta+(1-\lambda)\gamma-1}+2\beta j\lambda l\left[\delta+(1-\lambda)\gamma\right]\overline{k}^{\lambda(1+\gamma)-1}\,\overline{h}^{\delta+\gamma-\lambda(1+\gamma)}>4\beta j\lambda l\delta\overline{k}^{\lambda(1+\gamma)-1}\,\overline{h}^{\delta+\gamma-\lambda(1+\gamma)}$$

(G1)

Since the exponents of $\overline{k}$ and $\overline{h}$ on the LHS and the third term on the RHS are identical, a sufficient condition for this inequality to be satisfied is:

$$4\beta j\lambda l\delta<2\beta j\lambda l\left[\delta+(1-\lambda)\gamma\right]\rightarrow\ \delta<(1-\lambda)\gamma \qquad \text{(G2)}$$

Case (ii): focus

Upon substituting for z_1 and z_2 the condition for a focus becomes:

$$\beta j\lambda\left(\frac{\bar{k}}{\bar{h}}\right)^{\lambda-1}+l[\delta+(1-\lambda)\gamma]\bar{k}^{\lambda\gamma}\bar{h}^{\delta+(1-\lambda)\gamma-1}+2\beta j\lambda l[\delta+(1-\lambda)\gamma]\bar{k}^{\lambda(1+\gamma)-1}\bar{h}^{\delta+\gamma-\lambda(1+\gamma)}=4\beta j\lambda l\delta\bar{k}^{\lambda(1+\gamma)-1}\bar{h}^{\delta+\gamma-\lambda(1+\gamma)} \qquad \text{(G3)}$$

Since all terms in equation H3 are positive, clearly, a necessary condition for this equality to hold is:

$$4\beta j\lambda l\delta>2\beta j\lambda l[\delta+(1-\lambda)\gamma]\rightarrow\delta>(1-\lambda)\gamma \qquad \text{(G4)}$$

Case (iii): oscillatory convergence

For the emergence of complex eigenvalues, expanding the required condition $z_1^2<4z_2$ yields:

$$(\beta j\lambda)^2\left(\frac{\bar{k}}{\bar{h}}\right)^{2(1-\lambda)}+l^2[\delta+(1-\lambda)\gamma]^2\bar{k}^{2\lambda\gamma}\bar{h}^{2[\delta+(1-\lambda)\gamma-1]}+2\beta j\lambda l[\delta+(1-\lambda)\gamma]\bar{k}^{\lambda(1+\gamma)-1}\bar{h}^{\delta+\gamma-\lambda(1+\gamma)}<4\beta j\lambda l\delta\bar{k}^{\lambda(1+\gamma)-1}\bar{h}^{\delta+\gamma-\lambda(1+\gamma)}$$

$$\rightarrow 2[\delta-(1-\lambda)\gamma]>\frac{\beta j\lambda\bar{k}^{2(1-\lambda)-\lambda(1+\gamma)+1}\bar{h}^{\lambda(1+\gamma)-\delta-\gamma-2(1-\lambda)}}{l}+\frac{[\delta+(1-\lambda)\gamma]^2\bar{k}^{2\lambda\gamma-\lambda(1+\gamma)+1}\bar{h}^{\lambda(1+\gamma)+2[\delta+(1-\lambda)\gamma-1]-\delta-\gamma}}{\beta j\lambda} \qquad \text{(G5)}$$

The first derivative of the RHS of the above inequality is positive, hence it is monotonically increasing in δ. However, it is not possible to clearly determine the sign of the second derivative. Hence the RHS could either be concave or convex.

Before looking at the transitional dynamics for each possible scenario, we need to establish that the oscillations are always convergent in nature. For this, for all parameter values in the specified ranges, $\left|\frac{z_1+\sqrt{z_1^2-4z_2}\,i}{2}\right|<1\rightarrow z_1^2<2(1+z_2)$. As $z_2<1$, always $4z_2<2(1+z_2)$. Hence, any complex value associated with the linearised system will have an absolute value below 1. Hence, a spiral source (oscillatory divergence) never occurs.

Going back to the inequality G5, Figures G1, G2 and G3 respectively depict the convex case where the inequality is never satisfied (i.e the RHS is always greater than LHS), the convex case and the convex case that leads to two intersections respectively.

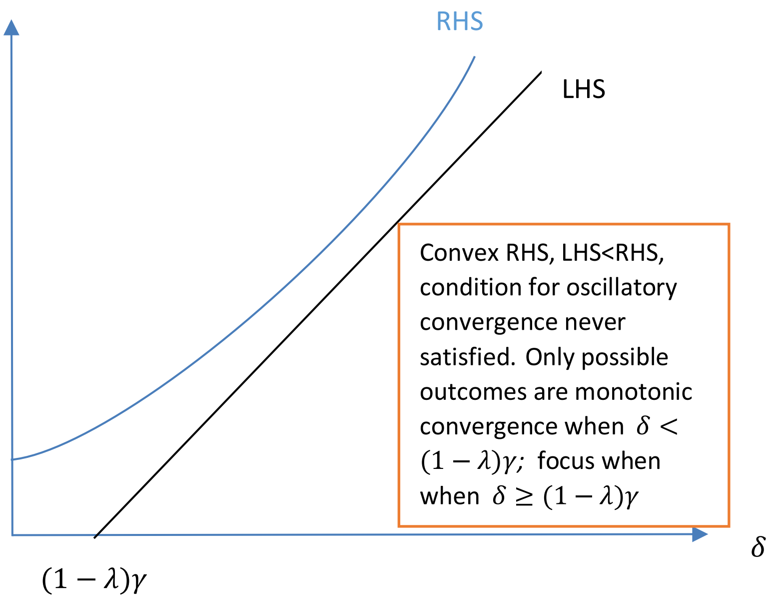

Figure G1: Convex RHS, LHS<RHS

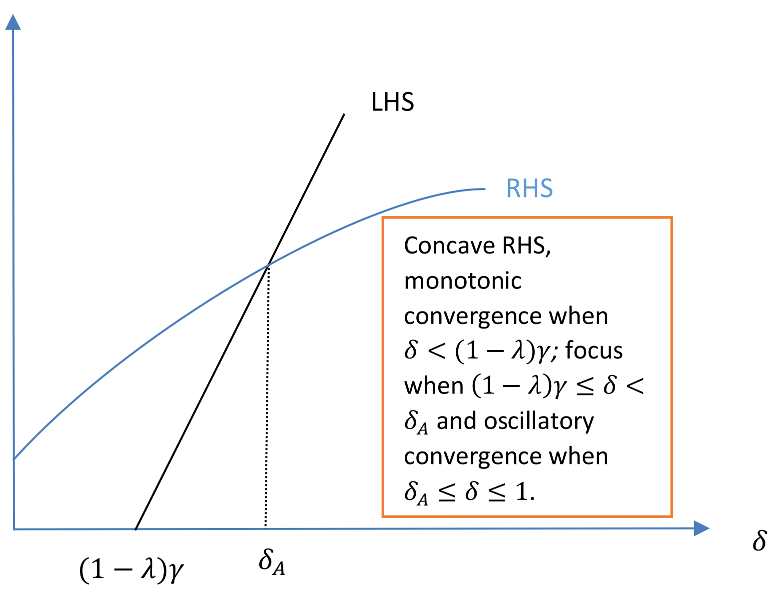

Figure G2: Concave RHS

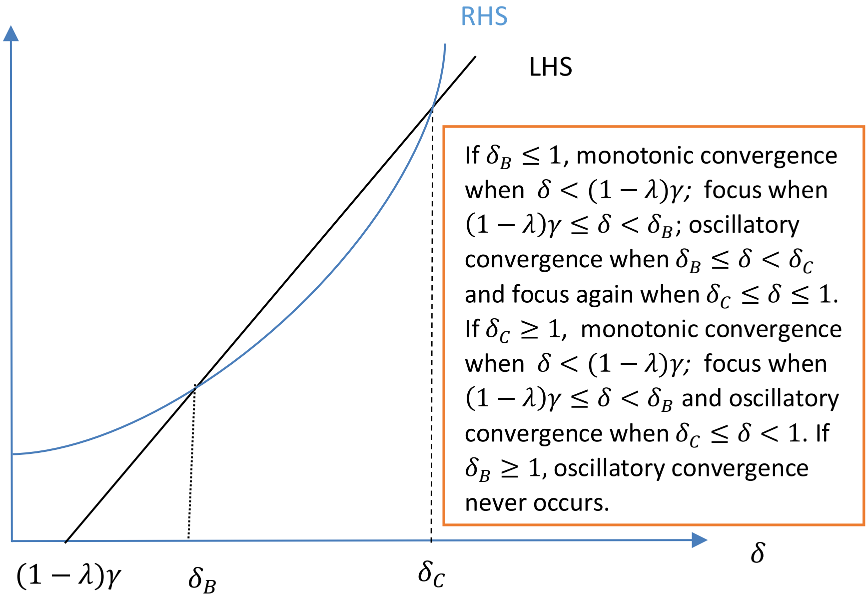

Figure G3: Convex RHS, two intersections

Appendix H: Derivation of the optimal value of τ

Let I be the indirect utility function.

$$I = U\left(c_{t+1}^{*}\right) + \beta U\left(c_{t+2}^{*}\right) + \theta V\left(h_{t+2}^{*}\right) \tag{H1}$$

From (2.13), (2.14) and (2.16) we have:

$$\frac{\partial U\left(c_{t+1}^{*}\right)}{\partial \tau} = \frac{\partial U\left(c_{t+2}^{*}\right)}{\partial \tau} = \frac{ab-1}{1-\tau+ab\tau} \tag{H2}$$

$$\frac{V\left(h_{t+2}^{*}\right)}{\partial \tau} = \theta\gamma\left[\frac{a}{\tau} + \frac{(1-a)(ab-1)}{1-\tau+ab\tau}\right] \tag{H3}$$

Substituting these expressions into the derivative of (H1), we can obtain the optimal value τ^* given by equation (4.1).

The second derivatives associated with (H2) and (H3) are negative, which confirms that at τ^* indirect utility is maximised.

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