Challenging Mathematically Gifted Middle Years Students:

A Mastery Learning Model

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M.Ed. COGE B.Ed.

Submitted in fulfilment of the requirements for the degree of
Doctor of Philosophy

Faculty of Education
Queensland University of Technology

2019

Keywords

Mastery learning; gifted; learning; social-cognitive theory; case study; mathematics; social constructivism; self-efficacy; autonomy; relatedness; competence; self-determination theory; zone of proximal development; motivation; individualised learning, differentiated instruction, curriculum compacting, acceleration.

Abstract

It is the responsibility of schools to provide learning environments that effectively meet the needs of all individuals. There are many students with abilities which remain underutilised because of a lack of challenge or opportunity to realise their potential. These students might be labelled gifted, talented or high achieving and the heterogeneous nature of this group is often not recognised. Recent studies reveal that while teachers know how to differentiate instruction, many fail to do so. Some studies report how gifted students receive minimal support in the classroom while struggling students receive the lion's share of the teacher's time and attention.

Teachers are faced with a variety of pressures, including having little instruction time, an over-crowded curriculum, and increasing pressures to ensure their students perform well on high stakes tests. These pressures often result in an over-reliance on low-level, drill and recite textbook and teacher centred instruction. This teacher-centric classroom can often leave gifted students lacking adequate challenge and opportunity to extend their learning.

Teachers need to be equipped with strategies that they can apply to their teaching to cater for individual needs of all students in the classroom including gifted students. This study analyses the classroom experiences of mathematically gifted students who participated in a mathematics program guided by the principles of Bloom's (1968) Mastery Learning Model.

An explanatory case study methodology was used to analyse the influence this teaching model had on five mathematically gifted students' achievement, interest and engagement levels within a Queensland Year-8 mathematics classroom. Students' experiences were examined using qualitative and quantitative measures including in-person interviews, in-class video and audio recordings, class test results as well as in-class discussions.

The results highlighted students appreciated having choice and control (autonomy) over their learning, challenge that came with a lack of needless repetition and having access to enrichment tasks in a social learning environment. While successful outcomes were evident, mastery goals coupled with timely feedback allowed students to remain challenged. Revisions to the Mastery Learning Model are proposed along with other suggestions given to improve teaching and learning conditions for gifted students.

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List of Abbreviations

GIM	Gifted Instruction Model
NAPLAN	National Assessment Program – Literacy and Numeracy
PAT Maths	The Progressive Achievement Tests in Mathematics
PSI	Personalised System of Instruction
RtI	Response to Intervention
ZPD	Zone of Proximal Development

Glossary of Terms

Autonomy	According to Ryan and Deci (2017), a person can naturally internalise extrinsic motivations from significant others for autonomy when they experience full volitional control, "characterised by a lack inner conflict and willing engagement" (p. 8).
Competence	In Self-Determination Theory, "competence refers to our basic need to feel effectance and mastery" where the person feels like they can "operate effectively within their environment" (Ryan & Deci, 2017, p. 11).
Differentiation	Differentiation "is the adaptations or changes made to instruction in order to meet all learners' needs" (Lindeman, 2016, p. 422).
Explanatory case study	Yin (2014) stated that explanatory case studies investigate "contemporary phenomenon in depth within its real-world context" (p. 48) to explain causal relationships "where the number of variables far outstrips the number of data points" and the number of participants is limited (p. 281).
Giftedness	Giftedness refers to a person "having a greater awareness, a greater sensitivity and a greater ability to understand and transform perceptions into intellectual and emotional experiences" (Roeper, as cited in Vanderkam, 2012, para. 5). Within the academic and intellectual domains, gifted students can process information that is of a greater level of complexity at faster rates. They typically use less mental effort and therefore need access to challenge taking into consideration their backgrounds and learning preferences (Neubauer, 2009).
Mastery	According to Block (1971), there are no "hard and fast objective rules for setting mastery standards" (p. 68). He suggests that a standard of mastery needs to be established by the teacher (or teaching team). Mastery in the context of this thesis refers to a student achieving more than 85% on a given assessment. This score is in line with the current school's agreement and some research (Damavandi & Kashani, 2010) which proposes percentages between 80-90%.
Online Learning	An online learning management system "provides a place for learning and teaching activities to occur within a seamless environment, one that is not

Management System	dependent upon time and space boundaries. These systems allow educational institutions to manage a large number of fully online or blended/hybrid (part online and part face-to-face) courses using a common interface and set of resources. Face-to-face courses that use an LMS for required or supplemental activities are often referred to as web-enhanced courses" (Piña, 2013, p. 1).
Post-positivism	Post-positivism is "hard to define" (Corman, 2009, p. 776), but has been described as an "adjusted positivism" where claims to knowledge is "tentative" (Phillips, 2014, p. 647) and obtained through "scientific means" (Sullivan, 2009, p. 396). "Warrants can contain evidence of many forms (including) interpretations of human actions (and) statistical analyses of data" (Phillips, 2014, p. 648). Importantly, "postpositivism assumes an intersubjective world where reality is a social construction, and the research aim is to uncover the meaning of this reality as understood by an individual or a group" (Sharma, 2010, p. 702).
Relatedness	Relatedness is an essential psychological need which pertains "to a sense of being integral to social organisations" (p. 11) where the environment is autonomy, and competence supportive.
Self-determination theory	"Self-determination Theory (SDT) is a motivational theory of personality, development, and social processes that examines how social contexts and individual differences facilitate different types of motivation, especially autonomous motivation and controlled motivation, and in turn predict learning, performance, experience, and psychological health. SDT proposes that all human beings have three basic psychological needs – the needs for competence, autonomy, and relatedness – the satisfaction of which are essential nutrients for effective functioning and wellness" (Deci & Ryan, 2015, p. 486).
Zone of Proximal Development	According to Vygotsky (1978), the Zone of Proximal Development (ZPD)"is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers". (p. 86).

Statement of Original Authorship

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

Signature: QUT Verified Signature

Date: May 2019

Acknowledgements

The journey to where this thesis is now, from where I began has been long and filled with unspeakable gratitude for some very special people, who I wish to acknowledge now. First, to two very highly respected and eminent researchers from Queensland's University of Technology. Thank you, Jim and Beth, so truly and sincerely for all the hundreds of hours you have spent in your support of me this thesis. Thank you for your inspiration, mentorship and invaluable help. Words will never be able to thank you both enough.

I heard of Professor James (Jim) Watters from one of the most knowledgeable persons in the field of gifted education in Professor Karen Rogers. She spoke so highly of Professor Watters. I remember praying that he would accept my request to mentor me through the PhD program. You can imagine how happy I was when he said yes. He has taught me so much through a sincere unrivalled gentle and masterful way. It has been the honour of a lifetime to work with this amazing man who is rightly a leader in the field of gifted education.

While I was completing my Master's degree in Education, there was a professor who impacted on me in such a way that instilled in me a growing desire and passion to serve the needs of students on the Autistic Spectrum. I already had a passion for these amazing students, however, Dr Saggers took this passion to a new level to a point where I was inspired to be an advocate for these amazing students in my career. Since participating in her inspirational course, I have endeavoured to make Dr. Saggers proud in my presentations to teacher audiences helping them understand the needs of children on the autistic spectrum and adults just as she had done with me. Beth, please know what an inspiration you were then and are now, and how grateful I have been for your grace, kindness, professionalism and having access to your expertise throughout the last many years.

Second, thanks to my principal Mr. Jeff Davis. He may never know how much he has inspired me. The college which has this amazing man at their helm should be so grateful. Jeff has taught me the value of a servant-hearted leadership. Some principals wear suits, while this amazing educator and thought leader wears a huge heart to see the classroom experience transformed. Jeff, you have inspired me in ways beyond your understanding. Thank you for giving me the time you gave me to finish my PhD. It is my hope that we work together in the future in a way that transforms schools to be places that students love to experience. There are men who chant philosophies, but you are a man who practices what he preaches and has a voice worth listening to in the education sector. I will be indebted to you forever, my friend and mentor. I hope I can do your mentorship proud in growing to a leader who inspires change in every classroom just as you do.

Third, gratitude to the two amazingly precious and beautiful women in my life. Thank you for all your support. Thank you for putting up with me when I had had less than 10 hours sleep in that week. Thank you for allowing me to pursue my dreams, for believing in me and for loving me in a way that is beyond beautiful. I will treasure the beautiful memories and beautiful love you have shown me through the last years.

To my son, Michael, I dedicate this thesis to you, my special amazing son. Michael, you are one of the most amazing young men I have ever met. You were counting to a thousand by the age of three or four and understood the concept of multiplication before entering school. My amazing boy would sit in class after class in the early years of schooling while teachers babbled on with their whole class lessons understanding numbers and "teaching" students as they attempted to do to the best of their abilities. It is my sincere hope that teachers will stop this horrible practice of teaching all students in the same way. I take my hat off to the students who waste hundreds of hours of their learning time, being forced to listen to lessons they already knew before the lesson started. Michael, you have some tough years ahead of you, but if you try, you can achieve whatever your heart desires. I only wonder how much better Michael could be achieving if his mathematical talent could have been catered for in a more meaningful sense from pre-school onwards. Believe in yourself, my amazing young man.

Finally, thanks to the five amazing students in this study. Thank you to you and your parents for letting me learn from and with you. Bree, Oliver, William, Ty and Miley (Pseudonyms) may never read this, but to you, a hearty and sincere thank you.

Chapter 1
Introduction

This thesis explores an educational approach implemented with students perceived as gifted and talented in mathematics. The approach rests on three central ideas. First, the ideas of Vygotsky (1978), whose theory suggested that all learners need to be challenged to the point where they need the assistance of a more capable other to enable learning to take place. Second, Bandura (1986) asserts that cognitive development is dependent on both social (extrinsic), and genetic (intrinsic) factors. Third, Deci and Ryan's (2002) Self Determination Theory contends that a sense of competence, relatedness and autonomy is important if pupils are to be motivated to achieve.

This chapter discusses the nature of the problem, what was investigated, the research aims and the significance of the study. An introduction to the term giftedness is presented, along with the theoretical underpinnings and Bloom's (1968) Mastery Learning Model. A structural outline for the thesis is then provided.

1.1 Background to the Study and Nature of the Problem

Educators argue that school-based tuition should focus on providing opportunities for all students (Gajewski, 2017; Vialle & Rogers, 2012) to achieve learning and social outcomes congruent with their intellectual ability (Bloom, 1971; Deci & Ryan, 2002; Litvack, Ritchie, & Shore, 2011). This task has been challenging for teachers to successfully achieve. Internationally, researchers suggest that teachers are under pressure and find themselves accountable to have students meet state and nationally mandated standards (Hirsch, 2016; Jolly, 2015; Shepard, 2000). Providing optimal opportunities for all children can, therefore, be problematic for stressed teachers with a perceived overcrowded curriculum and time pressures (Skaalvik & Skaalvik 2015).

One approach to providing appropriate opportunities for all students is termed differentiation (Tomlinson, 2005, 2016). Teachers who practice differentiation in the classroom may identify student needs, provide appropriate content and teach in ways that meet individual learning approaches (Tomlinson, 2005). Manning, Stanford and Reeves (2010), reported that differentiation of instruction is common for students needing additional support for learning difficulties, but is less common for gifted or advanced learners. Many teachers shy away from differentiation for gifted learners (Schmitt, & Goebel, 2015), while others either do not differentiate at all or minimally vary their classroom instruction to meet the needs of all students (Manning et al., 2010; Rotigel & Fello, 2004; Small, & Lin, 2010; Tomlinson, 2016). Rotigel and Fello (2004) suggest that differentiation is still not happening, a point agreed upon by others (Dixon, Yssel, McConnell, & Hardin, 2014; Logan, 2011; Manning et al., 2010; Skaalvik et al., 2015; Suprayogi et al., 2017; Tomlinson, 2005, 2016). Walsh and Jolly (2018) argue that the education of gifted students is "reliant more on the goodwill of principals, and the efforts of a few dedicated teachers and parent advocates, rather than on a well-designed systematic approach" (p. 87). Often, this lack of differentiation results in gifted students underachieving (Hare, 2013, Hill-Wilkinson, 2017). Gifted students are often asked to re-learn already mastered content (Masters, 2015; Vialle & Rogers, 2012). A study by Hare (2013), of 37000 students from Catholic, state and independent schools in Victoria supports this claim. This study demonstrated that students in the top 25% were "flat-lining" and identified the problem as "teaching to the middle and below" (p. 6). Other more recent results (Masters, 2015) reveal that the variability in ability levels in a class can be as high as six-year levels and while in these classes, gifted students are significantly underperforming. In this study, an alternative approach is proposed to what appears general practice with gifted students. This approach suggests that an adoption of Bloom's (1968) Mastery Learning principles should be considered. The principles of Bloom's instructional model will be discussed in detail in Chapter 3. How principles of Mastery Learning framed the instructional program is discussed in Chapter 5.

1.2 Understanding Giftedness

Students who exhibit advanced intellectual characteristics (Appendix A) are often given labels such as gifted, talented, highly able, precocious or superior (Ambrose, & Sternberg, 2016; Reznicek, 2006). These labels pertain to certain qualities or the students' demonstrated ability to process information and understand more challenging concepts in a time efficient manner (Leikin, Leikin, Paz-Baruch, Waisman, & Lev, 2017; McClarty, 2015; Neubauer & Fink, 2009). According to Neihart (2012), these labels can limit our understanding of giftedness which can take many different forms, as elaborated on in Section 2.2.

Chapter 2 provides a list of common characteristics found in mathematically gifted students, which differ from those who are not mathematically gifted (Johnson, 2000; Watt, 2000). According to Johnson (2000), these include the student's ability in the spontaneous formation of problems, flexibility in handling data, the mental agility of fluency of ideas, data organisation ability, originality of interpretation, ability to transfer ideas, and ability to generalise. Adding to this list of traits, Güçyeter, (2015) suggests mathematically gifted students are often good at memorising, for example, remembering formulas and operations.

Vialle and Rogers (2012) suggest that within an Australian context, giftedness is seen as synonymous with "potential" (p. 115). While Renzulli (2012), suggests that giftedness is not fixed but instead describes it as a set of "behaviours (that) can be developed and displayed in certain people, at certain times, and under certain circumstances" (p. 153). Subotnik, Olszewski-Kubilius and Worrell, (2011) suggest gifted individuals often have to accept responsibility for their growth to eminence and receive appropriate psychosocial supports at each stage of their development. This thesis supports a body of literature (Gagné, 2010; Koshy, Ernest, & Casey, 2009; Mrazik & Dombrowski, 2010; Tannenbaum, 2003; Ziegler, 2005) suggesting that students are born with the potential to develop gifts into talents with appropriate environmental and intrapersonal catalysts. This view agrees with Borland's (1997, 2005) conceptualisation suggesting giftedness to be a social and cultural construct.

Given an absence of a widely agreed upon definition of intellectual giftedness, this thesis drew on the Columbus Group's (1992, para. 2) definition of giftedness as:

> Giftedness is asynchronous development in which advanced cognitive abilities and heightened intensity combine to create inner experiences and awareness that are qualitatively different from the norm. This asynchrony increases with higher intellectual capacity. The uniqueness of the gifted renders them particularly vulnerable and requires modifications in parenting, teaching and counselling for them to develop optimally.

My experiences dealing with gifted students are discussed in the next section.

1.3 Researcher Background

In this section, I briefly share my experiences with gifted students as a school's gifted coordinator, as a parent and also as a teacher. I have completed post-graduate training in and worked with students identified as gifted or talented for nearly 20 years, I have noticed a mismatch between classroom practices and what research has suggested is needed. Others (Bain et al., 2003; Dai, Swanson, & Cheng, 2011; Tomlinson, 2005; Walsh & Jolly, 2018) have also noticed a similar difference between theory and practice.

In my role as the school's gifted coordinator, I delivered or saw other trained experts deliver, professional development in understanding and catering for the needs of gifted students in their classrooms. Even after such training, these teachers often still feel unsure on how to best educate these students (Vidergor & Eilam, 2011). I felt a need to bring together these models into one serviceable model that would help students remain challenged in every classroom.

As a teacher of gifted students, I would also regularly hear from parents who revealed how their child was becoming increasingly frustrated with school. They would share how teachers often made them spend much of their time in school re-learning materials they had already understood years prior, a finding echoed by Coleman, Micko and Cross (2015). As a result, they would rarely feel challenged and a growing resentment towards the education sector would emerge.

When my own son went through school, he would often be asked to re-learn concepts he had learned years prior. When he was in kindergarten the students were learning how to count to ten. I shared with his teacher that he could already count to a thousand and had been doing so for a while. Her reaction astounded me, when she exclaimed that she cannot cater for that in her class. As the years progressed, I would ask his teachers to allow my son to focus on more complex problems. In spite of their training in working with gifted students, they continued to ask him to re-learn concepts and skills he had already mastered.

This thesis, therefore, seeks to ameliorate this disparity between what is happening in the classroom and what research suggests should be happening, by providing a usable model for teachers to avoid this needless repetition and meaningfully differentiate their instruction. Among the main reasons for the mismatch, are teachers' desires to meet criteria according to mandated tests or to meet prescribed standards linked to a particular syllabi level or phase of learning. I have sat in mathematics department staff meetings in a variety of schools, where school leaders have instructed teachers' need to teach to the "middle". A discussion then often ensues on which textbook to use, with little regard for considering the unique needs, interests, and abilities of the students in their classrooms. Benjamin Bloom (1968) sort to combat this practice of teaching to the middle with his conception of learning for mastery. With his Mastery Learning Model, he attempted to replicate, in a classroom setting, instruction similar to what a student would receive with one to one tuition.

1.4 An Introduction to Bloom's Mastery Learning Model

Bloom (1968) contended that possibly as many as 90% of all students could achieve mastery given the right educational conditions. He argued against what Rosenholtz and Simpson (1984) coined as the "unidimensional classroom" (p. 21) where all students "work on the same or similar tasks (and) when a small number of different materials and methods are used during instruction" (p. 23). This so-called one-size fits all approach is a practice that some (Suprayogi, Valcke, & Godwin, 2017; Tomlinson, 2017) suggest continues. Figure 1.1 shows the standard distribution curve which has been suggested is often seen in classroom assessment results (Guskey, 2005a; Rust, 2015). Guskey (2005b) indicated that this can be skewed more towards students receiving higher grades when students gain access to the Mastery Learning Model.

Figure 1.1 shows a bell curve that is skewed towards students receiving "A" and "B" grades. Guskey (2005b, 2010) suggests that this curve differs from results expected in a regular class with a normal distribution of grades spread around the average or middle C.

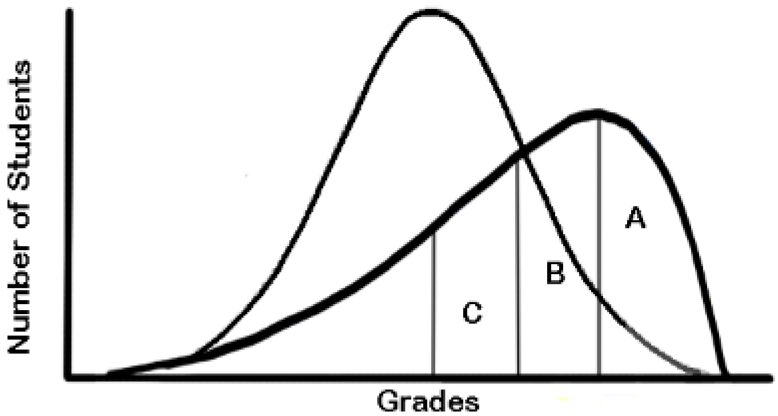


Figure 1.1. Distribution of achievement in mastery classrooms.

Instruction guided by the Mastery Learning Model would see units of work to be smaller in size focussing on one skill and allowing students to master that skill before progressing in their learning. Students would complete pre-post-formative quizzes to help guide their learning. They would then progress to the next smaller unit or enrichment activity after mastery of the smaller units has been achieved (Guskey, 2010).

According to Carrol (1963) and Bloom (1971), the relationship between aptitude and achievement would be high if some element of individualisation of instruction occurs. A point agreed upon by Gonski et al. (2018), who in a review of Australian students' achievement and progress, suggest the need for schools to individualise education, rather than teaching to the middle. A discussion to be elaborated upon in Chapter 3 highlights how Bloom's theory has gained acceptance as a means for increasing performance standards and individualising the education process (Guskey, 2010). The need for a serviceable model underpinned by peer-reviewed research, as proposed, is seemingly needed.

1.5 Theoretical Underpinnings for the Thesis

A Gifted Instruction Model (GIM) is discussed in detail in Section 3.4 to represent the theoretical underpinnings of this thesis, which is framed on the ideas of Vygotsky (1978), Bandura (1989a), Bloom (1968) and Deci and Ryan (2002). At the base of the GIM is an understanding that the gifted student is unique and has an "atypical neuronal" network (Kalbfleisch, 2009, p. 275). According to Beisser, Gillespie and Thacker (2013), the socialisation of gifted students is important. Therefore, the Gifted Instruction Model draws from social constructivist and social cognitive theories in placing the individual learner in a social world where a reciprocal causality exists between the learner, the social environment, and the desired behaviours (Bandura, 1989a). Also, it draws on Bandura's (1989b) Self-Efficacy Theory which suggests that effective feedback from formative assessments can impact positively on students' self-efficacy levels (Foster, 2016) and their ability to self-regulate their learning (Clark, 2012). The design of the study is now explained.

1.6 Research Methodology, Participants and Methods.

The experiences of students during a mathematics program was investigated using an explanatory case study methodology. The focus investigated a "phenomenon within its real-life context" (Yin, 2012, p. 35), how it was "implemented and (what) result(ed)" (Yin, 2014, p. 47) and relied "on multiple sources of evidence, with data needing to converge in a triangulating fashion" (Yin, 2014, p. 49). Triangulation of data was possible through the analysis of interviews, students' work samples, test results and in-class lesson recordings. I acted as both the researcher and teacher throughout the study.

I selected candidates for the present study from an independent Foundation to Year-12 private (non-government) school in Queensland, Australia. As such, the school was required to meet the requirements of the Australian Curriculum, as stipulated by the Queensland Curriculum and Assessment Authority (QCAA, 2014). The study took place over a period of 12 months in one mixed gender streamed Year-8 mathematics class (n=23), with students ranging in age from 11-13 years. The class was a streamed class with students who averaged a B grade or better in mathematics or possessed a superior or very superior intelligence as classified on the Wechsler Intelligence Scale (Weiss, 2006) by using the Slossan Intelligence Test-Revised (Slossan, Nicholson, & Hibpshaman, 2002). The use of the Standard Progressive Matrices - Sets A, B, C, D (Raven, 1989) were also used to identify students for this class. Students in the class scored in the top twentieth percentile when compared with students their age.

The explanatory case study methodology used a mixed methods approach involving both quantitative and qualitative data. Quantitative measures included the use of formative, and summative [both standardised and teacher made] assessments. The qualitative methods included captured images from video footage (Appendix B), student work samples (Appendix C), interviews (Appendix D), audio recordings (Appendix E) and teacher diary entries (Appendix F).

1.7 The Research Aims and Questions

This study aimed to explore in what ways a Mastery Learning Model influenced student attitudes toward mathematics, mathematical reasoning, and achievement levels in early adolescent mathematically gifted students.

While some (Potvin & Hasni, 2014) have suggested the two terms interest and attitudes are neighbouring motivational concepts, I drew from Sjöstrand's, (1958) delineation of the two. For him, "interest is connected with an existing drive, while attitude exists, whether the components of this drive are active in the present or not" (p. 408). Levels of interest can, therefore, impact on attitudes and attitudes (positive or negative) can be held on a topic despite interest in it. This thesis predicted that these levels of increased interest would impact positively on students' attitudes towards mathematics through the use of a program that enables control, choice, challenge, complexity and care. Evidence is provided on two key research questions to determine if these predictions were confirmed, which are stated as follows:

1. In what ways does a teaching program informed by the principles of Mastery Learning influence gifted students' mathematical performances?
2. In what ways does a teaching program informed by the principles of Mastery Learning influence gifted students' attitudes, motivation and interest in learning mathematics?

Chapter 2 and Chapter 3 will review research supporting the need for the development of a model that helps gifted children work within their zones of proximal development (ZPD). Research (Bain et al., 2003; Tomlinson, 2005) suggests that teachers who teach the gifted are familiar with and have even received professional development on how to differentiate their programs for gifted, but do not do it for several reasons, suggesting a need for testing such a model. By utilising a model of instruction based on Mastery Learning principles in the classroom, the curriculum can be differentiated; students remained focussed, challenged and motivated, utilising formative assessments, smaller sized mastery goals, and a more personalised autonomy supportive approach to learning actioned.

1.8 The Significance and Implications of the Research

Extensive searching of ERIC, Pro-Quest and PsychINFO failed to identify any research on the effectiveness of implementing Mastery Learning principles with gifted students. Many studies discussed learning to mastery as a concept. However, no peer-reviewed research on the use of the Mastery Learning Model grounded in Bloom's assumptions with gifted students was found despite the perceived benefits suggested by Guskey, (2005a) and Rogers, (2002) that a Mastery Learning approach could have for gifted students.

Mastery Learning is a teaching and learning framework that requires students to master a skill before progressing to the next concept. Benita, Roth and Deci (2014) suggest that with "mastery goals, individuals try to improve their level of competence, develop new skills, or achieve a sense of mastery based on self-referenced (intrapersonal) standards" (p. 258). In the former, mastery goals guide practice, while the latter, mastery goals are set by individuals to help them stay motivated to achieve such goals. There are those like Brown (2012) and Rogers (2007) who advocated the possible benefits in using such models; however, no research was able to be found testing the potential effectiveness of the Mastery Learning Model. This absence of research provides the rationale for this study with gifted students.

Other models such as the Maker Model for differentiation (Maker, 1982, 1986) focused on modifying the learner's environment, product and process in learning. Models such as this and Renzulli's (2005) Schoolwide Enrichment Model speak of ensuring that these students remain challenged but did not emphasise the need for mastery of core content and skills. This mastery could be seen as important by teachers trying to meet nationally mandated standards that are assessed in standardised tests such as National Assessment Program - Literacy and Numeracy (Australian Curriculum Assessment and Reporting Authority, 2016) - (a point to be expanded on further in Chapter 3). While other similar models exist, such as the Response to Intervention (RtI) Model (Brown, 2012), this study adopted the principles of Mastery Learning as a model of instruction. Both the RtI and Mastery Learning models share similarities and differences, with the main difference being time spent on remedial support. RtI seeks to prevent future learning challenges (Callender, 2014), while the Mastery Learning approach exhausts less time in remediation (Guskey & Jung, 2011) believing all or most students can achieve mastery. Both models stress the importance of on-going monitoring of learning, which is accompanied with tailored, high-quality, differentiated tutelage, while the Mastery Learning Model emphasises the importance of setting mastery goals (Guskey & Jung, 2011). This process is important, as it enables students to autonomously identify learning strengths and weaknesses (Miles, 2010). Once mastery is achieved, students can continue with enrichment challenges.

Studies carried out by De Corte, Verschaffel, and Masui, (2004), suggested that deeper learning, which builds on core content, is a crucial feature for effective learning to take place. This inquiry highlighted the need for more resourcing and funding. It also emphasised the point that the practice of differentiating instruction for gifted learners is not commonplace (Dimitriadis, 2012; Johnsen, 2017; Manning et al., 2010; Walsh & Jolly, 2018), often resulting in under-achievement (Vialle & Rogers, 2012). Some research studies (Hirsch, 2016; Rubenstein, Gilson, Bruce-Davis, & Gubbins, 2015; Shepard, 2000; Tomlinson, 2016) have identified the pressures teachers face to meet national standards as a core cause of this lack of differentiation. This argument is central to the contention that this research is both relevant and needed not just to prepare students for national tests, but more importantly, to stop underachievement and repetition in the classroom. Rubenstein, Gilson, Bruce-Davis and Gubbins, (2015) made this case when they showed how teachers were not differentiating instruction, but when given pre-assessment results, they were impelled to differentiate their instruction.

This thesis fills a gap on the effectiveness of a Mastery Learning Model that enables teachers to individualise instruction for gifted mathematics students. This model is different to other models tailored for gifted learners, as it incorporates the ideas of Bandura's Social Cognitive Theory, and Ryan and Deci's (2017) Self Determination Theory in helping such students realise self-efficacy in sub-domains and work autonomously while remaining challenged. This research is also unique, as it documents the students' classroom experiences, the details of the intervention and the students' reactions to the program in their own words.

Implications for students, teachers, parents, schools and education policymakers are considered. These implications suggest the need for further research on the use of this model with the promise of delivering individualised data-driven, challenge appropriate and authentic learning to the classrooms of all students. Research conducted provides a comprehensive body of evidence, arguing in favour of the possible gains in achievement and motivation levels, which can be attained using Bloom's model.

1.9 The Structure of the Thesis

Chapter 1 has outlined the nature of the study, the questions that the research sought answers for, and an introduction to the background of the ideas in the overall thesis. In Chapter 2, the relevant literature on curriculum options for the gifted individual is reviewed. This review includes an examination of the best practices which are related to the enrichment and acceleration options used in this thesis. A review of the research on how to cater for mathematically gifted students using Bloom's Mastery Learning approach is presented in Chapter 3. Chapter 4 then describes the explanatory case study methodology adopted along with the post-positivist theoretical perspective by which the data were collected and analysed. As this is a naturalistic classroom study, it places the researcher in the classroom with opportunities to address the nexus between research and practice (Confrey & Lachance, 2000). The effectiveness of the Mastery Learning Model was able to be evaluated using multiple methods. Chapter 5 provides a critical overview of the Mastery Learning Model while Chapter 6 provides background on the selected students in the study as introduced in Section 1.6.

Chapter 7 presents the raw data in response to the two research questions. Chapter 8 then provides a discussion of the data, their possible implications and meanings. Based on the data generated and analysed, testing conducted on the Mastery Learning Model's influence is noted in this chapter. The outcomes of this study will provide educators with a serviceable programming model alternative, which could guide them in teaching decisions of the mathematically gifted child. The limitations, any potential rival explanations and possible implications for further research are discussed in Chapter 9, while also noting further gaps in this current field.

Chapter 2
Literature Review – Giftedness

2.1 Introduction

Chapter 1 provided a general introduction to this research, identified the research problem, the research aims and the significance of the study. In this chapter, further detail of the literature underpinning the research is discussed. Section 2.2 examines the complex and multifaceted construct of giftedness. I discuss the attributes of mathematically gifted students and the social and cultural context which should be considered in the education process. Curriculum options are then reviewed in Section 2.3. These include acceleration, compacting and other enrichment options.

2.2 Complexities of Giftedness

In this section, I argue that giftedness depends on context, culture, and the social lens through which it is viewed. Terman's (1926) studies of genius sought to elaborate on what it meant to be gifted. His study of 643 gifted subjects intended to find the brightest students who would be in the top 1% of scores, using a revision of Alfred Binet's 1905 scale of intelligence. Terman's studies of genius sort to help us better understand the physical, mental and personality traits of these students. Defining giftedness through a focus on a diversity of traits dominated much of the research in the latter part of the 20th century (e.g., Renzulli, 2012). This thesis, however, argues that being gifted is not just associated with a student possessing a readily identifiable list of traits but views giftedness as having a greater awareness, a greater sensitivity and a greater ability to understand and transform perceptions into intellectual and emotional experiences" (Roeper, as cited in Vanderkam, 2012, para. 5). Within the academic and intellectual domains, gifted students can process information that is of a greater level of complexity at faster rates. They typically use less mental effort on regular learning tasks and therefore need access to challenge, taking into consideration their backgrounds and learning preferences (Neubauer, 2009).

In the next section, I discuss how gifted students may possess some common traits but highlight that giftedness can be complex and context dependent.

2.3 Giftedness

Identification of gifted children is complex. In the 1960's, Drews (1963) advanced Terman's conception of giftedness by proposing four types of gifted students: (i) high-achievers, (ii) social leaders, (iii) creative intellectuals, and (iv) rebels. Drews' conception of giftedness was further advanced by Betts and Neihart (1986), who postulated six different types of gifted learners. The behaviours described in the original research (Betts & Neihart, 1986), outlined six gifted profiles which have been further revised (Neihart, 2012), to include: (i) successful, (ii) creative, (iii) the underground, (iv) those at risk, (v) the twice exceptional, and (vi) the autonomous learner. The profiles provided by Neihart, are, to some extent subjective, but provide a guide to understanding the nature and behaviours commonly seen in gifted students. Neihart highlighted not what giftedness is, but the diversity of profiles that gifted students are observed to assume in school. The problem with defining the multi-faceted gifted learner is trying to include every type of giftedness, including those who may be gifted but are more difficult to identify. If you can somehow fit all of these profiles' characteristics into a definition, you then meaningfully cater for it, taking into consideration the individual context of the child. This complexity suggests giftedness is a complex multi-faceted construct which describes gifted students as individuals who have the potential to excel given an understanding of their uniqueness, along with the provision of suitable services to cater for this (Gagné, 2013; Heald, 2016; Powers, 2008). Despite this complexity, some clarity has been provided by Gagné (2010).

Gagné (1985) initially proposed a talent development model. This model was most recently revised (Gagné, 2013) in 2013. First, his model assumes that giftedness is some innate potential, that predisposes a person to exceptional performance in some domain/s of human endeavour. This is a view supported by others (Squires, 2017; Vialle & Rogers, 2012; Winner & Drake, 2018). Second, Gagné (2010) stresses the impact that certain catalysts (environmental, intrapersonal and developmental), can have on the individual's ability to develop gifts into talent.

The provision of a relevant individualised education, along with an acknowledgment of the catalysts (Gagné, 2010) is important in bringing such giftedness to the fore. His model provides a framework for transforming giftedness into talents, whereby identification happens when a child accesses a "systematic program of activities" (p. 84). This process, therefore, implies a need for identification during instruction.

Further, Betts and Kercher (2004) argue for the use of ongoing identification of giftedness through the instructional phase. This ongoing assessment should include multiple criteria. For example, McGowan, Runge, and Pedersen (2016), suggest that identification procedures should include of the use of a range of comprehensive assessments to identify students' strengths and interests, and not just use a single standardised score to identify giftedness. The teacher should also be familiar with the common gifted traits. This complex process of identifying giftedness (Singer, Sheffield, Freiman, & Brandl, 2016), highlights giftedness as a multi-faceted construct (Heller, 2012) making this group of students hard to identify. It is, however, often the case that gifted performers are identified within the classroom context based on limited criteria. Others can be missed (Baldwin, 2005; Diezmann, 2002), unless the teaching approaches can in some way bring their giftedness to the fore (Betts & Kercher, 2004; Chamberlin, 2006; Dai, Moon, & Feldhusen, 1998). The argument presented in this thesis is that in-class identification on an on-going basis is important and that the program needs to match the students' ability levels to the units of work the students will cover.

2.3.1. Mathematics Giftedness in the Classroom Context.

The above analysis highlights the complexity faced in attempting to identify giftedness. There are some (Adeyemi, 2010; Chamberlin, 2006; Cho & Suh, 2016) who have argued that identifying a domain-specific mathematical giftedness may be easier. For example, by accepting that scores in the highest standard deviation on a mathematics achievement test might be indicative of mathematical giftedness. This process seems an overly simplistic approach that may see some excluded in the identification process. An example provided by Middleton (2007), talks about one boy named "Wil" who had never scored well on tests. The tests he completed did not, however, specifically assess mathematical ability. Middleton does not explain how Wil came to be identified as "talented" (p. 82), just that "no appropriate programming has yet to be implemented to maximise his talents" (p. 82). The author continued, stating that his school's district used no mathematical talent identification instrument. No teacher in his district was able to identify Wil's ability. Therefore, no appropriate programming was able to be implemented. This case asserted a need for students like Wil to have his specific mathematical talent identified and catered for, and yet, this did not happen.

According to Koshy et al. (2009) and Dai, Moon, and Feldhusen (1998), many students may readily possess a mathematically precocious mind but are difficult to identify. Interestingly, the study by Koshy et al., found that one of the reasons for this difficulty was that students were only achieving average results and that it was also difficult for these teachers to recognise common traits within the context of the regular classroom. For example, in discussing three different types of mathematically gifted students: (a) analytical logico-mathematical; (b) spatial geometric; and (c) both logico-mathematical and geometric thinkers. Diezmann (2002) suggested that a child could potentially be a gifted logico-mathematical thinker and not the other, or indeed both. Similarly, the study by Koshy et al. suggested teacher confusion arises when students who they think are gifted only get average grades. These same students, when given a chance, catch up quickly to a point where they are completing complex tasks after a short period of attendance. Therefore, if the teacher relied on the end of semester tests, it is plausible that gaps in understanding a specific mathematical skill could bring down a potential student's overall grade or percentage score (Rust, 2015).

It is important that the students have access to a program of sufficient complexity which allows for the on-going identification of context-laden strengths, interests and abilities (such as those presented in Table 2.1) as part of the education process in identifying mathematical talent. The list of attributes presented in Table 2.1 is not presented as a comprehensive inventory of every possible attribute. It does, however, accentuate a complex notion of mathematical giftedness, which necessitates a program that allows students gifts to come to the fore.

Further to the identification of attributes associated with giftedness, research (Geake, 2008; Haier & Jung, 2008; Kalbfleisch, 2008) on neuropsychology and brain physiology has provided further insights into the neurological origins of exceptional mathematical performance in people. For example, a neuro-anatomical study (Shaw et al., 2006) of 307 children and adolescents at varying ability levels and ages suggested that neural white matter developmental patterns are different in more intelligent peers. They defined intelligence as "a constellation of skills, including the ability to reason, plan and solve problems, think abstractly and learn quickly and learn from experience" (p. 962). Shaw's (2007) review of neuroimaging studies suggested greater levels of neural activation occur in more intelligent adults (as measured by IQ) and children. According to Shaw (2007), “age-appropriate environmental enrichment can boost academic performance and to a lesser extent IQ” (p. 966). Draganski et al. (2004), suggested that similar changes provide evidence of the malleability of the human brain. This research adds to the argument for providing a differentiated teaching approach for a unique, atypical growing and evolving gifted mind.

This neurological research adds further weight to claims of a complex giftedness that is difficult to identify (Callahan, Renzulli, Delcourt & Hertberg-Davis, 2012) and highlights that no perfect identification system exists. The literature therefore supports that identification of talent should happen in an on-going way and focus on identifying specific strengths and ensure students remain challenged. This complexity is further complicated when one considers students’ social and cultural backgrounds, and the impact this may have on their ability to perform well when using identification instruments.

Table 2.1

Attributes of Gifted Mathematics Students

Attributes	Study where cited
Ability to understand mathematical tasks usually reserved for students in higher year levels	Chamberlin, 2006; Reed, 2005
Can often persist more with challenging problems.	Stepanek, 1999
Experience boredom at lack of challenge in the program.	Chamberlin, 2006; Diezmann & Watters, 2000
An ability for spatial concepts	Koshy et al., 2009; McAllister & Plourde, 2008
A mathematical memory.	Diezmann, 2002; Koshy et al., 2009; Reed, 2005
Pace and flexibility to complete problems faster, more flexible and fluently.	Chamberlin, 2006; Diezmann, 2002; Johnsen & Kendrick, 2005; Koshy, et al., 2009; McAllister & Plourde, 2008; Reed, 2005; Rotigel & Fello, 2004; Wieczerkowski, Cropley, & Prado, 2002
An ability to generalise	Koshy, et al., 2009; Reed, 2005; Wieczerkowski et al., 2002
Ability to think in reverse.	Koshy, et al., 2009; McAllister & Plourde, 2008; Wieczerkowski et al., 2002
Ability to abstract concrete problems.	McAllister & Plourde, 2008; Stepanek, 1999; Wieczerkowski et al., 2002
Possess a creative mathematics ability.	Chamberlin, 2006; Livne & Milgram, 2000; McAllister & Plourde, 2008; Stepanek, 1999
An ability to formalise mathematical learning and make connections between related and not obvious related concepts.	Diezmann, 2002; Koshy et al., 2009; McAllister & Plourde, 2008; Rotigel & Fello, 2004

2.3.2. Social and cultural context of giftedness.

It has been well documented (Bernal, 2010; Callahan et al., 2012; Ford & Grantham, 2003) that different social and ethnic groups have traditionally been underrepresented in gifted programs. For example, a study by Joseph and Ford (2006) directly speaks about identifying and catering for potentially gifted students from diverse social and cultural backgrounds and provides practical strategies that can be used to identify and cater for such students. They recommend the use of culturally fair, non-verbal tests such as the Raven's Progressive Matrices (Raven, 1989). The use of such tests should be used as part of a multiple criteria approach to identify mathematical talent.

2.4 Curriculum and Pedagogy for the Gifted Individual

A recent qualitative study conducted by Cross, Frazier, Kim, and Cross (2018), showed that gifted students often feel isolated, have little access to relevant content, control or choice in their classroom learning, and are often left bored and unchallenged. Other research (Simpson & Adams, 2015) has suggested that many gifted students continue to fail, are bored and disenchanted with the education system, finding it uninteresting and irrelevant and are alienated, and sometimes end up suspended or expelled from school. Many gifted students are still being taught to the textbook and often the same content as the rest of the class and are all too often not receiving access to meaningful challenge (Hill-Wilkson, 2017). This section discusses many of the evidence-based teaching ideas that ensure these students are challenged in a meaningful sense.

McAllister and Plourde (2008) discussed several elements as key to the mathematics curriculum for the gifted learner. These include open-ended problem solving, problem finding (and solving), working on tasks centred on a central theme and receiving access to appropriate levels of challenge. Therefore, a focus should be on understanding, through on-going identification measures such as the effective use of formative assessments and challenging all individual students (Vygotsky, 1962) to achieve mastery (Bloom, 1971) within their zones of proximal development (ZPD) (Vygotsky, 1978).

This practice would enable students to gain "maximal grip" (Dai & Renzulli, 2008). For Dai and Renzulli, this maximal grip is more than just being able to demonstrate mastery of regular core content at a faster rate. The student/s should also be motivated to want to achieve mastery. This motivation then enables students with gifts to work at the "edge of chaos" (p. 122), forming and testing their theories. The authors argue that it is essential that students maintain a deep understanding of the core content (2008). This section suggests, therefore, that gifted students need to have access to choice, with appropriately challenging and complex authentic learning tasks in a scaffolded and supportive learning environment.

2.5 Accelerative Practices

Students should be allowed to move through the curriculum “at a breadth and depth commensurate with their abilities” (Assouline, Colangelo, & VanTassel-Baska, 2015, p. 47) yet the authors share this as an issue shrouded in scepticism. Referring to the term acceleration as a “misnomer”, Lubinski and Benbow (2000) propose an alternate definition for this practice as “appropriate developmental placement” (p. 138). This section, therefore, discusses literature (Colangelo, 2015; Gross, 1992; Guyton, 2013; Southern & Jones, 2015) suggesting the practice commonly referred to as acceleration is an effective strategy to use to help cater for students who possess gifts or talents in mathematics. Research by Assouline and Lupkowski-Shoplik (2005), and Guyton, (2013) revealed that despite concerns for gifted students’ social welfare, acceleration has had positive achievement gains as well as small to moderate social-emotional gains (Rogers, 2015).

According to Colangelo (2015), acceleration is an effective way of educating more precocious students (Colangelo, 2015) even though it is often met with emotional reactions from educators. Diezmann and Watters (2000) suggested, it is often seen as problematic and met with scepticism and reluctance (Southern & Jones, 2015), and its implementation remains a challenge. The discussion that follows addresses issues that may pertain to the forms of acceleration educators use with gifted students.

Often students are accelerated in an ad hoc way, which may leave the student without adequate resources for furthering their knowledge beyond a certain point (Southern & Jones, 2015). This practice may be because the school does not have the required resources or qualified teaching staff. Furthermore, the authors suggested that the student may be placed in a higher-level course, but after placement, they learn that they needed to complete a lower level course to graduate. However, Southern and Jones suggest that it is still possible, that the student’s knowledge may not be able to be catered for in a school. They highlight that as learning becomes increasingly complex and abstract, teachers may lose confidence in their ability to teach this group of students.

Southern and Jones also cite pacing as problematic. They suggest that students might not be able to recognise when they have mastered the topic to a sufficient depth, and similarly, a teacher in the earlier years of schooling might have difficulty convincing a secondary school teacher that the child is ready for instruction at a higher-year level. Lastly, the authors cite age as a common issue. Some school districts do not allow students younger than 4.5 years old into school, while others do not allow a middle school student into a tertiary level institution.

Furthermore, a large-scale (50660 subject accelerated students, and 2811 grade level accelerated students) meta-analytic review conducted by Rogers (2015) of research completed since 2008, revealed greater than moderate effect sizes of subject-based acceleration and moderate to strong positive effects on grade level acceleration. Of particular note in this study was the moderate to strong positive effects on social adjustment in subject and grade accelerative options. Rogers' meta-analysis has produced enough evidence for its use in schools. Positive (0.21 and 0.17) effects were noted for curriculum compacting, +0.72 and +0.24 for online courses, and +0.25 on an individualised curriculum. These results provide promise for this intervention which provides students with components of these accelerative practices.

A number of researchers (e.g. Benbow,1998; Gross, 1992, 2006; Rogers, 2015; Southern & Jones,1991; Steenbergen-Hu, Makel, & Olszewski-Kubilius, 2016) have argued in favour of acceleration practices, suggesting that the child's social and emotional development benefited because they were accelerated. These studies showed that the students were more stimulated, enjoyed closer and more productive social relationships, displayed healthier levels of social self-esteem, and they had more positive attitudes towards school. The pressure to achieve also significantly diminished.

Benbow (1998) reminds us, however, that any one form of acceleration should not be used in isolation. The educator should "develop a combination of accelerative and enrichment options, as well as out-of-school opportunities that reflect the best possible alternative for educating a specific child" (p. 287). Of relevance to this thesis is the use of compacting of the curricula, individualised online education and enrichment options.

While the above shows consistently positive effects noted with an accelerated curriculum, it also suggests various problems associated with it, if not administered carefully (Rogers, 2015). The use of the Mastery Learning Model's systematic and comprehensive approach ensures that this academic acceleration happens naturally as a result of a natural progression through a compacted series of shortened topic-based courses as outlined in Chapter 5. In the next section, research on the use of curriculum compaction as an accelerative option is discussed.

2.5.1. Curriculum compacting.

This section examines what curriculum compacting is, and then discusses the research regarding its use as one form of acceleration. Why compacting is not used as a strategy for differentiation for gifted learners is reviewed, and how it can be used as part of a Mastery Learning approach to teaching, discussed.

Curriculum compacting, according to Renzulli and Reis (2009) is where teachers use pre-assessments to determine which students can move through the prescribed curriculum at a faster pace. It (curriculum compacting) enables the elimination of work students already understand (Phillipson et al., 2009). Phillipson et al. suggest that compacting should facilitate the clustering of ideas together so that students can identify key ideas and make connections with enrichment tasks that follow the compacted unit of instruction.

Compacting the regular curriculum can be done in at least two ways. First, students can initially be given the most difficult problems related to a topic. Second, students can be required to complete only a portion of the required amount of work covering a range of questions of varying difficulty.

2.5.2. Research on curriculum compacting.

Gifted students require less repetition than other students and should not be (re)learning content and skills they have already mastered (Stanley, 2000; Tsai, 2007). The challenge for educators is “to find a match between the child’s abilities and the curriculum while balancing academic, social and physical activities” (Lupkowski-Shoplik & Assouline, 1994, p. 144). To meet this challenge, the research discussed below suggests curriculum compacting should be used to ensure the gifted learner remains challenged.

A quasi-experimental matched-pair design study (Stamps, 2004) discussed the use of curriculum compacting with 70 Year-1 mixed gender gifted students in a rural Alabama school district. Stamps research shared how the intention of the use of curriculum compacting was for students who had already “mastered” (p. 31) the curriculum content. The outcomes of the study showed positive effects for teachers, parents and students. Positive effects included the ability for students to progress to enrichment tasks in such a way as to keep them motivated and interested in their learning. It (curriculum compacting) reduced students' frustration levels with the less challenging non-compacted curriculum. Parent satisfaction levels were also high because of the use of a compacted curriculum. The results of Stamps' study tended to focus on the results of the teacher and parent attitudes more so than on the students. While Stamps' results were interesting, only this small segment of her findings suggested that the curriculum compacting component of the Mastery Learning Model may have some benefit for gifted learners. Her research outcomes were also consistent with other research (Reis et al., 1993; Reis, Westberg, Kulikowich & Purcell, 1998) on the effectiveness of using curriculum compacting in the classroom.

Searches of the relevant education databases (ProQuest, PsycINFO, Academic Search Elite, EBSCO eBook, Education Source, ERIC, MAS Ultra, Primary Search) up to and including 2018, mirrored that of Stamps (2004) who found a scarcity of recent research on the effectiveness of curriculum compacting. One later meta-analytic best-evidence synthesis of research (Rogers, 2015) on the array of accelerative options noted a slight (+0.20), but positive effect of curriculum compacting on achievement. The author noted that these data came from 18 studies but did not cite the actual studies. There is enough research (Rogers, 2007; Rotigel & Fello, 2004; Sutton, 2001; Tsai, 2007), however, which asserts its effectiveness for use with gifted students. It is worth noting, however that all of these articles draw from the study by Reis et al., (1998) which discusses one nationwide study conducted across the United States of America.

Reis et al. (1998) conducted an experimental study of 336 students from heterogeneous Year-2 to Year-6 classrooms across rural, suburban and urban areas using all subscales of the pre-post-test Iowa Tests of Basic Skill. They aimed to test whether results of gifted students who learned with a compacted curriculum differed from those with non-compacted units of study. Three multivariate analyses of covariance found their results were statistically significant; however, there was no difference between treatment and control groups. They suggested that gifted students' results were not affected in a curriculum compacted by as much as 40-50%. The research of Coleman, Micko and Cross (2015) re-iterate these claims in their article which argues for the importance of a faster-paced curriculum to cater for students who learn at faster rates so that these students are not bored.

Coleman et al. presented a synthesis of 25 years of phenomenological qualitative studies on lived experiences of gifted students in schools. While this review discussed a plethora of issues, I specifically focus on their findings on the impact of pacing on interest levels in learning. The study did not discuss the total number of students or studies. They revealed that it is common for gifted students to have to wait for new learning opportunities “because school is designed for the masses” (p. 367). As a result of waiting, students would often turn to their own devices and waste time in class daydreaming, looking around the classroom, and generally feel “bored” (p. 368).

Neurological research provides a possible reason for such boredom in the classroom, suggesting that gifted students have the cognitive ability to understand less complex cognitive tasks more efficiently, or with less mental effort (Brown, 2012; Leikin, Leikin, Paz-Baruch, Waisman, & Lev, 2017; McGlonn-Nelson, 2005; Neubauer & Fink, 2009). The neuroscience confirms a need to allow students to spend less time on the simpler overly repetitious tasks (Lee & Olszewski-Kubilius, 2006; Reis et al., 1993) and thereby allowing more time for them to focus on complex mathematical problems that can be implemented with the use of the Mastery Learning Model. Supporting this notion, a case study documenting the educational experiences of four mathematically talented students conducted by Lupkowski-Shoplik and Assouline, (1994) noted that “instruction should begin at the point where they have mastered the content and are ready for new material” (p. 149).

2.5.3. Challenges to curriculum compacting,

Yuen et al. (2018) documented the results of a training course run with 106 Hong Kong based teachers providing them with strategies to differentiate instruction for gifted students in the mixed ability classroom. Their study asserted that gifted students “benefit greatly from opportunities to work through the curriculum at a faster pace” (p. 36) instead of spending time learning concepts they already understand. Despite this assertion, their training program utilised a vast range of curriculum differentiation strategies but did not employ either curriculum compacting or any form of acceleration. They stated that it was because teachers “do not find it easy to differentiate their lessons in this way” (p. 36).

While teachers are aware of the positive effects of using curriculum compacting, many do not use it as a strategy for differentiation of curriculum (Lee & Olszewski-Kubilius, 2006; Reis & Renzulli, 1992; Reis, Westberg, Kulikowich, & Purcell, 1998; Taylor & Frye, 1988). Reis and Renzulli suggest that this is to the detriment of highly able students who are often rewarded for their best efforts with more endless repetitive work on a “dumbed down” textbook (p. 52).

The dilemma for educators is to ensure that curriculum compacting is used along with formative assessments to ensure gaps do not appear in learning and students are learning at appropriately challenging levels (Chatterji, Koh, Choi, & Iyengar, 2009; Gregory & Herndon, 2010). Chatterji et al. suggest the effective use of formative assessments can help detect gaps in learning as well as identify what students already understand. They contrast the use of high-stakes testing with the use of formative assessment. According to Chatterji et al., high-stakes testing places pressure on teachers causing them to teach to the test while the use of formative assessments should be used to teach students content they do not already understand. However, Herman, Osmundson, Dai, Ringstaff and Timms (2015), have suggested that teachers’ use of formative assessments was sporadic, and teacher analysis of student work was not consistent. Furthermore, students’ responses were not analysed in much depth. This research proposed that teachers’ use of formative assessments does not necessitate a change in the way instruction is delivered, and even though research (VanTassel-Baska & Wood, 2010; Stamps, 2004; Reis et al., 1993; Reis et al., 1998) has confirmed the effectiveness of curriculum compacting for use with gifted students, this is not being used with formative assessments effectively.

2.5.4. Summary.

The argument presented in the research cited above, has suggested that the use of acceleration can pose problems for educators, but if done carefully, it could create benefits for students who process more complex information at faster rates. Within a Mastery Learning framework, therefore and in application to this thesis, I acknowledge that demonstration of mastery would be a precursor to moving on to the next topic in mathematics. If done this way, the chances of creating gaps in knowledge and skills become less evident, and the teacher is then able to meaningfully acknowledge students’ strengths and abilities within the framework of an assessment-driven program.

2.5.5. Enrichment.

This section discusses the literature in relation to enrichment. It is proposed as an effective strategy to use with gifted students. Kim (2016), suggested that in the “early days of gifted education”, acceleration was the main educational provision made for gifted students. While many definitions exist around the term enrichment, I draw from Kim's meta-analytic review of enrichment programs, where she described them as follows:

> Enrichment programs provide exploratory activities, in-depth materials on a topic, materials for the development of higher-level thinking processes and skills, self-selected independent projects, or authentic products or services for a real-world audience. Enrichment programs have emphasised the importance of profound knowledge and skills within a subject to develop students' higher mental processes and creative production. (p. 103)

I have drawn from this review the importance of independent (and cooperative) projects that involve a profound and deeper knowledge (mastery) of the curriculum that is coupled with authentic, real-world exploratory activities to develop authentic products. Limited current research exists on the effects of such enrichment.

Kim's (2016) meta-analysis of 13 studies examined the pre/post-test effect sizes of enrichment (in and out of school) programs on gifted students' academic achievement. These studies noted varying effects on academic achievement; however, when moderators such as program type and year level were considered, the effect sizes were significantly greater than zero. Of note to this study, the mean effect size for middle school students (ages not defined), was 1.37 standard deviations.

The author of this meta-analysis referred to some complications with accurately calculating a statistical effect size. They noted a significantly positive effect of enrichment in high schools, but the study was only done in one school setting. The author suggested that other studies did not include "detailed characteristics of the programs in terms of (an) enrichment program definition, types of intervention, detailed participants' demographic information and duration of programs" (Kim, 2016, p. 112). Kim's study suggested that enrichment programs "influence middle school students the most in socio-emotional development" (p. 113). The author highlighted this as significant, as it is this age that students often make important career decisions on their futures. While her meta-analysis was limited, and despite limitations cited above, Kim was able to assert that enrichment programs have a positive effect on students' academic and socioemotional development. The following research provides an example of how such enrichment looks in a qualitative sense within the mathematical domain.

Diezmann and Watters (2000) discussed enrichment within the mathematics context, suggesting that boredom "stems from a lack of challenge in academic tasks and a perception by these students of the limited value of the learning experience" (p. 14). The authors synonymously refer to enrichment tasks as challenging. These tasks are characterised by "authenticity", "complexity", having an "obstacle to a ready-made solution and the need for high-level thinking and reasoning" (p. 14). Enrichment also provides students with the opportunities "to emulate the practices of mathematicians, at a less-sophisticated level" (p. 14).

Diezmann and Watters suggested these tasks help develop autonomy and motivation, but students still require support from the teacher in the form of appropriate scaffolding, modelling and time allocation. The authors shared how the work needs to be complex and set at an appropriate level. The example Diezmann and Watters provided, demonstrated how a student struggled with a challenging task, but the teacher provided clues along the way to help prompt the student to understand the problem, and steps to follow to solve it. The following research highlights why teachers do not give enrichment opportunities to gifted students.

A study by Koshy et al., (2009) found that when given mathematical enrichment investigations, similar to those elaborated upon in Chapter 5, students often sought to understand content often reserved for students from older year levels. This research suggested that teachers often resist availing students to enrichment options because identified gifted students were not able to demonstrate an understanding of the regular curriculum for the respective year level. Enrichment tasks could be in the form of problems involving divergent thinking, individual projects, and group activities that would connect learning completed in class to real-world events and happenings (Rogers, 2007; Rotigel & Fello, 2004). While quantitative data from test results were not collected in the Koshy et al. study, questionnaire survey results did find up to 94% of students were more interested and engaged in the learning of mathematics as a result of being able to participate in these investigations. Similar to research cited by Diezmann and Watters (2000), Koshy et al., found that prior to the use of such enrichment tasks, up to 74% of students found mathematics classroom work to be "boring, repetitive and too easy" (p. 225). Likewise, another study (Matsko & Thomas, 2014) shared research which revealed the misconception that mathematics is often construed as a "black and white, rigid and boring subject" (p. 154). Matsko and Thomas advise that this is in spite of the fact that it “stands near the top of any hierarchical list of intellectual domains ordered according to the extent to which creativity is evident” (p. 154).

In their study, Matsko and Thomas, (2014) examined the impact of creating original mathematical problems had on 82 Year-10, 11 and 12 gifted mathematics students’ levels of motivation and challenge. This study found that the students who were completing these problems found them to be more difficult than questions asked by the instructor. Interestingly, the students shared that the use of this style of instruction had no impact on their motivation to learn mathematics. The authors suggested that because students may not have understood what was meant by the term motivation, they looked for evidence of an “enhanced sense of efficacy, deeper engagement, feelings of ownership, and intrinsic motivation” (p. 162). Their study found a shift in some students’ motivation to learn mathematics from getting a good grade to one of exploring topics due to an interest in understanding the content or skills better.

According to Renzulli and Reis (n.d.), some enrichment is more suited to learners who have higher levels of task commitment and high levels of interest and ability in a given academic area. This research by Renzulli and Reis proposes that enrichment should be threaded into the regular classroom curriculum, as is the case in this proposed model. This integration of enrichment allows students to complete investigations related to their possible areas of interest as “first-hand enquirer(s)” (Olenchak & Renzulli, 1989, p. 37).

These investigations, therefore, may help to facilitate a deeper level of understanding and interest in the current curriculum by completing such individual or group projects (as elaborated upon in the fifth phase of the Mastery Learning model in Chapter 5). With the teacher’s assistance, appropriate scaffolding and sufficient time allocations, the students could potentially learn time management, self-directed learning skills and the ability to self-evaluate their work. The research cited suggested the positive effects this can have on students’ academic achievement and social-emotional development. This implies that mastery of the curriculum is necessary to afford students with the content knowledge to apply to these more complex tasks.

The use of these enrichment and acceleration options allowed for the unique and diverse student to remain at the centre of the education process. When enrichment is carried out in the above manner, it counters criticism levelled by Willis

(2007) who suggested that it is often the case that children are given worksheets with more challenging questions with no real relevance to their lives, interests or passions. Students were given the option of a range of investigations that may take shorter/longer time periods to complete, depending on what the teacher recommended. Such a framework also catered to the individual, as they are in control of their learning, have the choice of topic, the challenge of a rich investigation, the relevant complexity that often comes with authentic real-world tasks and the care of a supportive and collaborative learning environment.

2.6 Summary

Chapter 2 has provided a theoretical analysis of the literature about what giftedness and mathematical giftedness is. When the uniqueness of gifted students is established as a basis for understanding their learning needs, the need for a more individualised program can be emphasised as discussed in Chapter 3. This supports a view that the educator should adjust their planning to ensure that the talented mathematics students can remain on an optimal developmental trajectory.

I have argued with Bain, et al. (2003), Dixon, Yssel, McConnell, and Hardin (2014), Marotta-Garcia (2011), Tomlinson (2005, 2017) and Walsh & Jolly (2018), that while teachers understand how to differentiate their instruction, they may not implement this in practice for the reasons stated in this chapter. These include factors such as time, pressure to have students perform well on state tests and a crowded curriculum. A serviceable model that allows meaningful differentiation to occur is therefore needed. This differentiation can occur in the form of providing an individualised program within the structure of a Mastery Learning Model. Using this teaching approach, students can gain access to a compacted program. This program should cater for their advanced neural network where they can master regular content at a faster pace and spend more time on appropriately challenging, complex and often authentic enrichment tasks as is discussed in Chapter 3.

Chapter 3
Literature Review – Mastery Learning

3.1 Introduction

Chapter 1 identified the purpose of the study; the research aims and its significance. Chapter 2 reviewed the literature on catering for a mathematically gifted student who requires access to a more individualised program. It is proposed that such a program can draw on the principles of mastery learning. In this chapter, a critical review of mastery learning is presented. Section 3.3 discusses the role of assessment in mastery learning. Section 3.4 provides a theoretical framework that informs the teaching program while section 3.5 briefly shares the impact learning preferences can have on an interest in learning.

3.2 Mastery Learning

There is a large body of research that has demonstrated the positive effects of the mastery learning approach on student learning (Bloom, 1984; Guskey, 2010; Kulik, Kulik & Bangert-Drowns, 1990; Postlethwaite & Haggarty, 1998; Slavin, 1990). The background and critical summary of this research is presented in this section.

3.2.1. Origins of Mastery Learning.

In 1968, Bloom set out to explore ways to individualise instruction in a group setting, drawing on the ideas of the Winnetka Plan (Corcoran, 1927) and the University of Chicago Laboratory School experiments. Corcoran described how Carleton Washburne, a superintendent of schools in the Winnetka School District in Chicago, implemented a program where a core component was made available to all students. An extension or creative component could then be selected by the students who mastered the core program. Students had to demonstrate mastery at the end of the lesson before they could move on to new instructional material. Bloom (1968) also drew from the ideas of Carroll's (1963) Model of School Learning, in noting that all students learn differently, requiring more, or less time to demonstrate mastery of a given concept/s.

According to Quick (2010), research into the use of Mastery Learning decreased in the 1980s, but there has been a resurgence of interest with its use with “e-learning”. Lalley and Gentile (2009) also argued that there is a need for a resurgence of Bloom’s Mastery Learning approach given a contemporary emphasis on inclusive policies. Lalley and Gentile suggest Mastery Learning is a model of instruction that is suited to the “faster” (p. 34) students; however they did not complete any research testing this contention in the classroom setting. Therefore, this resurgence has prompted interest in this model as a feasible way of providing for the individual in the classroom. The following section now examines the historical and contemporary research on the implementation of Mastery Learning.

3.2.2. The effectiveness of the Mastery Learning Model.

The success of the Mastery Learning Model has been well documented (Corbett & Anderson, 1994; Guskey & Pigott, 1988; Kulik et al., 1990). A range of recent studies (Bautista, 2012; Ihendinihu, 2013; Shafie, Shahdan, & Liew, 2010; Wambugu & Changeiywo, 2008) have shown improvements in academic achievement, where mastery learning approaches have been used.

Research carried out by Wambugu and Changeiywo (2008) noted the positive effects of utilising the Mastery Learning Model with 161 secondary school physics students. Using a non-equivalent control group design, the authors employed the Physics Achievement Test (Stephanou & Lindsey, 2011) to measure the effectiveness of the teaching approach. An ANCOVA of the post-test PAT scores highlighted a significant statistical difference between the control and experiment groups $F (3,156) = 85.12$, $p < 0.05$. They concluded that the Mastery Learning treatment effect yielded better academic performance results than the non-Mastery Learning groups. They argued the effects could be attributed to the teaching approach. This study countered the argument, discussed in Section 3.3.4 by Slavin (1987), who suggested that studies previously carried out did not use standardised instruments and ones that did, noted a zero effect. The study did not report whether any gifted or talented students were present in the study. However, similar research conducted by Bautista (2012) and Idendinihu (2013) did report results for both low and high achieving students.

Bautista (2012) implemented a quasi-experimental (pre-test-post-test design) study of 52 college level biology students (location of college not given). The author explored the effectiveness of mastery learning approaches, drawing on both Bandura’s principles of self-efficacy and Bloom’s Mastery Learning approach. One purpose of this study was to find the effectiveness of the teaching approach and uncover correlations between motivational levels towards the subject and achievement levels. The study found that students responded well, as measured on post-test scores. Bautista did not reveal details of what specific strategies that were implemented in the classroom. The author cited gains in achievement of 7.63 points in the experimental group (N=27) and 2.56 points in the control group (N=25), showing a statistically significant result with a "t-value of 4.760 and p-value of <.001 at 0.05 level of significance" (p. 29). Attitudes towards the intervention were also measured using the "Motivation towards Science Learning Questionnaire" (Tuan, Chin, & Shieh, 2005) by determining the motivation level of the respondents in learning biology. These results, while not being elaborated upon in the study, did show higher motivational levels in the experimental group (M=4.55) when compared with the control group (M=4.21). Details of the performance of low and high ability students were not given. The study is useful as it points towards the possible effectiveness of a program that encourages self-regulated learning under Bandura's self-efficacy framework.

A six-week quasi-experimental pre-post-test non-equivalent group study (Ihendinihu, 2013), investigated the effectiveness of the Mastery Learning approach with 150 randomly selected secondary mathematics students from three different schools in the Umuahia Education Zone in Nigeria. The ages, gender and students' year levels were not given, and no information was offered on the nature of the instructional methods used. Results from the Mathematics Achievement Test with a "reliability index of 0.87" (p. 848) found that the use of the Mastery Learning approach enhanced student achievement and reduced the "gap between students with high and low abilities in mathematics" (p. 848).

The Ihendinihu (2013) study did not note any difference in the effectiveness of using or not using collaborative learning with the Mastery Learning Model or whether the learning experiences were differentiated. The author suggested that there were both high and low ability students in the groups; however, he did not elaborate on whether these students were gifted. He did suggest that the high and low ability students were determined with the results from the pre-test. However, no more information was given on just what constituted high or low ability. The study noted a mean standardised pre-post-test mean improvement of 25.66% in the high ability students, and a 35.85% improvement with the "low-ability students" (p. 852). The author does not mention whether the learning experiences were differentiated for the high or low ability students. The study points to the effectiveness of the Mastery Learning Model. The brevity of the study, however, has left the reader wondering why the low-ability students achieved greater gains, and whether one can accurately ascertain the ability level of a child on a single pre-test.

Another study carried out by Shafie, Shahdan and Liew (2010) also noted the success of using a Mastery Learning Model with university-level mathematics education degree courses. The authors argued the benefits for low aptitude students as 70% of the students (N=30) in this study achieved a mark of “A”. Ten per cent also failed to score a pass mark because of their negative attitudes and failure to treat the subject seriously. This research is useful because it reports students’ positive and negative impressions of the Mastery Learning Model, with most students suggesting that they enjoyed learning under this model. Questions remain, however, as the authors neither reported ability levels of the students involved, nor did they discuss any quantitative findings at length.

They failed to discuss the nature of the intervention, the teachers, or their teaching approaches. That is, the authors suggested a level of success as measured by student interest and academic grades but failed to elaborate meaningfully on the details of the intervention as it happened in the classrooms. An example of this is given in Table 3.1 where the reader is left wanting more information. This table provides a summary of three recent Mastery Learning studies that have relevance to the present study. Davrajoo, Tarmizi, Nawawi, and Hassan (2010), also suggest the effectiveness of the Mastery Learning approach. Their research is not discussed here, as little relevance to this study could be ascertained. It considered the effects of utilising the Mastery Learning Model with low achieving students who struggled significantly with anxiety in mathematics.

Table 3.1

Summary of Effectiveness of Recent Mastery Learning Studies

Study	Number of Participants	Effectiveness	Limitations and implications
Wambugu & Changeiywo (2008)	161 secondary school physics students.	$F(1,69)=85.60$, $p<0.05$ showed highly significant treatment effect on mastery group compared to the non-treatment group.	Students' ability levels not specifically stated.
Shafie, Shahdan & Liew (2010)	30 Students.	Benefits low aptitude learners 70% of learners received an A grade. 10% Failed to pass.	Smaller scale study. Intervention not described in detail.
Bautista, (2012)	52 Biology students. Control group (N=25) Experimental group (N=27)	t-value of 4.760 and p-value of <0.001 at 0.05 level of significance showing significance in gains made in post-test results after treatment.	The sample size is smaller (N=52). No standardised test was given – Teacher made assessments used.
Ihendinihu (2013)	150 randomly selected secondary mathematics students	25.66% improvement in the high achieving students, and a 35.85% improvement with the low-achieving students	Used one pre-test to identify high and low ability students. Did not discuss what happened in the classroom.

Further to these studies, Guskey and Gates' (1985) meta-analytic review of 38 group-based Mastery Learning studies involved interventions that lasted between two and twelve weeks. They found an effect size of 0.76 improvement in test scores, and 0.74 in studies lasting longer than 18 weeks. From a total of 36 Mastery Learning studies, Kulik et al., (1990) concluded how the effects of Mastery Learning programs would notice scores rise by 0.59 standard deviations. According to Cohen (1988), such an effect size would be significant. That is, such an effect would be bordering between medium and large, with a medium effect to be around 0.50 and a large effect size being approximately 0.80.

A search of the relevant education databases also found similar effects when the Mastery Learning Model was applied to the mathematics classroom. Guskey and Pigott (1988) and Bloom (1968) however, suggested that effects in mathematics classes would vary, as the results were dependent on entry-level knowledge. Due to the developmental nature of the subject of mathematics, some students enter the classroom having learned or understood more mathematics concepts which gave them a head start on the other students who may not have learned or understood such content. Many of the studies carried out in these reviews were completed more than 20 years ago. In this respect, the studies cited within are not elaborated upon in depth, other than to point out such far-reaching claims. Guskey (2010) suggests that Mastery Learning classes which have been well-implemented, average “higher levels of achievement and develop(ed) greater confidence in their ability to learn” (p. 54). The next section will extend the critical review of Mastery Learning and the implications this may have in this present thesis.

3.3 The Mastery Learning Model

The core principles of a Mastery Learning Model are given in Table 3.2 and discussed here. How they informed the teaching program is presented in further detail in Chapter 5. The approach to the Mastery Learning Model used in this study is informed by the specific characteristics of gifted students as discussed in Chapter 2. Pre-assessments were used before the first unit of instruction while learning in the subsequent unit of instruction was compacted according to the results of these pre-assessments. Students were given the option to work on related accelerated content following mastery of the core skills on given formative or pre-assessment tasks.

The teacher was the main agent for the delivery of instruction in the original Mastery Learning Model, while in this study, the students learned from each other, their teacher, information obtained via the internet, and their parents. Students in this study were in control of their learning under the guidance of a teacher who monitored their progress which ensured they were on track with their learning. Formative interactions, which included diagnostic, formative and summative assessments, individualised goal setting and feedback were an integral component of the Mastery Learning Model.

Table 3.2

Principles Adopted Within the Mastery Learning Framework for Gifted Students

Principles of Mastery Learning Model (Guskey, 2007)	Principles adopted for gifted students
• Pre-assessments used to ascertain what students know.	• Diagnostic (pre-assessments) are used to ensure students have access to appropriately challenging compacted units of instruction.
• Student learning is teacher directed	• Students work is self-directed
• The teacher sets the pace of the units.	• Students control the pace of their learning.
• Initial unit is taught, and then the students complete their first formative assessment.	• Various modes of online, peer, parent and teacher-delivered instruction.
• The teacher is the main agent for delivery of instruction.	• Formative interactives were dialogic and assured students of mastery of key concepts.
• Results on tests guided future instruction.	• Students also have the option of working on accelerated content after mastery has been achieved.

3.3.1. Assessment.

This section focusses on assessing student learning within a mastery framework. I first examine literature about the use of standardised assessments. A discussion then continues on the use of formative interactions, which includes diagnostic, formative and summative (standardised and teacher-made tests) assessments, feedback and goal setting.

3.3.1.1. The use of standardised assessments.

The Progressive Achievement Tests in Mathematics (PATMaths Plus) (Australian Council for Educational Research, 2011) is a battery of tests which are widely used as a diagnostic tool to assess general student knowledge. However, the use of standardised assessments is a “deeply contested issue” (Guadalupe, 2017, p.

326). I contend that these assessments should not be used as the sole measure of success of a learning program and that this success should be measured through assessments based on the content and skills taught within the unit. This practice is in line with recommendations made by Gipps (1994) who suggested the need to keep the purpose in mind with the use of assessments. Therefore, teacher made assessment instruments were used in parallel with the PATMaths test.

The use of standardised tests may give general feedback on overall improvement in the curriculum area, but this may not always correlate well with results observed in teacher-made assessments, which are directly relevant to what was taught in the units (Slavin, 1987). Guskey and Gates (1985) reviewed research where the students in mastery conditions did equally as well as students from non-mastery settings on standardised measures while also achieving significantly higher on teacher-made assessments.

Content validity measures suggested the tests measure what they purport to measure. Research conducted by Fogarty (2007) with 513 boys and 292 girls from a regional Queensland school, showed high levels of predictive validity of the Pat Maths Plus assessment. That study cited a high correlation between school achievement scores and results on the Pat Maths Plus assessment measures and concluded "… students who do well on the PAT battery tend to get better school grades" (Fogarty, 2007, p. 15).

According to Hosp and Ardoin (2008), the assessment instrument should include criteria that are related to intended outcomes. Airasian and Madaus (1983), suggested that rarely is the correlation between standardised assessments and teacher-made assessments high, because the "low correlations are in part a function of differences between the method of measurement and the method of instruction and learning" (p. 105). This finding implies that standardised instruments, which are often multiple-choice tasks, ask students to pick best answers. In contrast, curriculum-aligned teacher-made instruments, if well-constructed, require students to give their responses while also showing their reasoning.

The implications from the studies (Guskey & Gates, 1985; Gipps, 1994) are that researchers need to use authentic assessment methods that purposefully align with the researcher's intent. This practice would, therefore, allow the researcher to document improvements (as measured by the teacher-made assessment/s) and provide a level of external validity of the results by pointing to the scores on the standardised instruments. I now elaborate on the research on the use of formative interactions that were an integral part of the Mastery Learning Model.

3.3.1.2. Formative interactions.

Black and Wiliam (2009) broadly define formative interaction as "one in which an interactive situation influences cognition". They suggest that

> practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited. (p. 9)

This definition is apt for its application to this thesis as it suggests assessments should be used by students and teachers to enable students to take the next steps in their learning. Black and Wiliam (2009) suggested that the teacher should not "undermine the creation of student autonomy" and therefore this guided formative interaction also involves "activating students as owners of their own learning" (p. 8). This process also includes students setting their own goals in learning which is discussed later in this section.

Black and Wiliam's (2009) research asserts that assessment is a dialogic process, in that students are active participants in receiving, responding to and using corrective instruction to guide their learning. This formative interactive process seeks to do five things: (a) understand where the learner is at, using diagnostic assessments; (b) show them what they need to learn with individualised interactive feedback and subsequent goal setting; (c) provides student-teacher, student to student dialogic and scaffolded guidance and feedback during learning moving them closer to achieving set goals; (d) shows students how much or how well they have learned what they needed to understand (formative and summative assessments); and then finally (e) allows further revision, if necessary, which in turn asks them to make revised goals. This interactive process, I refer to as formative interactions. In keeping with Black and Wiliam's recommendations, these formative interactions are on-going throughout all phases of learning.

This interaction includes assessments that shape both what is taught as well as how it is taught. Within a constructivist epistemology, it involves an understanding of the individual learner in such a way so that the educator can provide appropriate learning experiences that are in “advance of the child’s development” (p. 18). Hence, diagnostic, formative and summative assessments using both standardised PATMaths and teacher-made tests provide a responsive way to monitor and guide individualised instruction.

Diagnostic, formative and summative assessments: Formative interactions begin with designing learning activities based on the results of appropriately challenging diagnostic assessments. According to Sia and Lim (2018) “diagnostic assessment is an instrument that can help make formative inferences on students’ cognitive strengths and weaknesses in a specific topic” (p. 124). In this context, diagnostic assessment is primarily used to collect information on what students already know about a topic that is about to be taught. In this thesis, the term “pre-assessment” is used synonymously with diagnostic assessment. As instruction continues, the formative interaction would also include the provision of formative and summative feedback to both guide student learning and provide feedback in the form of summative achievement.

This process of formative interaction is contrasted with more traditional forms of summative assessment, which, according to Chappuis and Chappuis, (2007) is used to make a judgement or determine a grade. Chappuis and Chappuis speak of the “confusion” (p. 14) between the two terms. For this thesis, the term formative assessments refer to evidence of student learning (which includes, but is not limited to quizzes, tests, responses to challenges, assignments and enrichment tasks), used to guide students’ future learning. Summative assessments, which should also guide future learning are considered as a more traditional form of formal pre-post standardised or non-standardised tests used with formative assessments to evaluate and report on student learning (Black & Wiliam, 2009).

Formative Feedback: All assessments, whether summative or formative should provide prompt, informative feedback for each student (Feinstein, 2014). Research discussed in this section suggests that a range of factors can moderate the effects of feedback on achievement. They can include the kind or type of feedback, its timing, who is giving it, how it is given, the value attached to the feedback or whether it is accepted or rejected (Hattie & Timperley, 2007; Maier, Wolf & Randler, 2016).

Hattie and Timperley conceptualise feedback "as information provided by an agent (e.g., teacher, peer, book, parent, self, experience) regarding aspects of one's performance or understanding" (p. 81). They suggest that the purpose of feedback is to bridge a gap between students' current understanding and their desired level of understanding and that the timing of this feedback is important.

The best feedback is corrective in nature, is prompt, and enables learners to make corrections to the way they approach similar problems and not continue to get further problems incorrect (Feinstein, 2014). Hattie and Timperley seemingly agree when they suggest that feedback is more effective when it "builds on changes from previous trials" (p. 85) and the effects of the timing of feedback is variable. They cite some studies which suggest the correction of errors can "result in faster rates of acquisition" (p. 98), while their meta-analysis of 53 studies found immediate feedback during instruction is effective (0.36). This process level feedback guides students' thinking. By learning from their errors from task-related feedback, students develop "error detection skills, which leads to their self-feedback aimed at reaching a goal" (p. 86).

According to Hattie and Timperley, the learner's willingness to engage in feedback will depend on the cost to the learner. They suggest that the student's self-belief can impact on the effectiveness of feedback as can the teaching approach influence the types of feedback used in the classroom. For example, a social constructivist approach would see the teacher participate in a type of guiding dialogue to help the student understand the language of the question, the methods to solve similar questions, or guidance that is provided after an incorrect solution is found. Feinstein (2014) adopts a similar stance, suggesting that less instruction and more corrective feedback can be "more academically beneficial" (p. 209) if given promptly, so that students' understanding of incorrect processes is not reinforced. The effects of the different forms of feedback can be "positive or negative", either of which "could enhance motivation" or "may be accepted as a challenge" to trigger self-regulation (Black & Wiliam, 2009, p. 24).

Goal Setting. Hattie (2012) identified goal setting of high importance in ensuring academic growth in student learning. According to Ryan and Deci (2017), a person's level of intrinsic motivation can be affected by a feeling of competence when accompanied with a sense of autonomy. The two are linked, as one could argue that by receiving ongoing task feedback, the student would feel an increased sense of competence, which would in turn impact positively on student's personal beliefs about their capabilities to be successful at school. An argument also supported by Bandura's Social Cognitive Theory regarding the impact of success on a student's sense of self-efficacy in specific learning areas.

Research conducted by Locke and Latham (2002, 2006) found a number of factors to mediate the effects of goal setting. They stated that goal difficulty levels, self-efficacy beliefs, goal specificity and goal commitment will impact on the effects of goal setting. Other factors such as completing tasks in groups, unconscious motivators (e.g., motivational posters beside a desk) and goal satisfaction were also shown to mediate the effects of goal setting.

First, students who set higher goals noted better results than those who set easier goals. Research by Kluger and DeNisi's (1996) showed a 0.51 effect on student achievement when students were set with difficult goals which is similar to the effect (0.46) noted by Hattie and Timperley (2007). Research by Matsui, Okada and Mizuguchi (1981) which focussed on student's expectancy levels, found that when students saw the rewards of achieving harder goals the rewards for achieving them were worthwhile. Interestingly, in a separate study by Plante, O'Keefe and Théorêt (2013), students who were set mastery goals achieved better than those who were set performance goals.

Second, studies by Locke and Latham (2002) and by Plant, O'Keefe and Théorêt propose that a person's self-belief affects their ability to achieve a difficult goal. Locke and Latham suggest that the person such as a leader (or teacher) who set the goals, implicitly places a belief that the person could achieve the goals, which in turn impacts on their self-belief (or self-efficacy) in their ability to complete the goals successfully. The students, therefore, see the reward of achieving the goal to be worth the effort expended.

Third, Locke and Latham, and Hattie and Timperley (2007) draw from a variety of studies which revealed how workers were given very specific and difficult goals which they would achieve, such as carrying more logs on a truckload.

Last, by setting specific goals, the person can remain committed to that goal. Hattie and Timperley agree, suggesting that if students combine goals, they can blur the original intent of goal setting and therefore make the process of achieving them more difficult. This research by Locke and Latham and by Hattie and Timperley, therefore, supports the use of smaller sized units of instruction, enabling students to set very specific goals based on the specific topic they are trying to understand.

3.3.1.3. Summary.

This section has discussed the literature in relation to assessments, feedback and goal setting. I furthered Black and Wiliam's notion of formative interaction which included diagnostic, formative and summative assessments, feedback and goals related to this feedback and results. These assessments should ideally include the use of both standardised and non-standardised assessments. The goal of the teacher is complex, which necessitates the use of online computer programs to provide instant feedback and other more capable others to help students understand the increasingly advanced content. At all times, the instruction is scaffolded with all participants involved in the learning process (which includes computer-assisted instruction) helping students achieve set goals to understand the appropriately challenging content.

3.4 Theoretical Framework for Teaching

The theoretical framework proposed in this thesis places the developing gifted mind as central to the process of a developing mathematical competence. The multi-faceted giftedness discussed in Section 2.2, requires an individualised approach to the education of such students. This section provides a critical look at the theoretical underpinnings of the Mastery Learning Model. I argue that the teacher should not provide an accelerated program in the hope that the student can lift beyond what they can handle. The importance of using scaffolded smaller sized units is considered in Section 3.4.3. Section 3.4.4 then elaborates on Ryan and Deci's (2017) Self-Determination Theory, as related to the use of the Mastery Learning Model to facilitate engagement and interest in learning mathematics.

3.4.1. Bandura's Social Cognitive Theory's with the Mastery Learning Model.

A Gifted Instruction Model (GIM) is presented diagrammatically in Figure 3.1. This model stresses the importance of catalysts in the form of a social learning environment, mastery goals, scaffolded instruction and feedback guiding ability appropriate challenge. The GIM proposes that these catalysts are needed to encourage optimal mathematical self-efficacy levels. When provided with co-jointly set learning goals and prompt feedback, learners can be motivated to learn (Bandura, 1989b; Brookhart, 2008; Guskey, 2010). Students remain challenged, as work is set at a level which necessitates collaboration with a capable other, based on results from such formative assessments. According to Bandura (1989b), the development of the learner emerges from an interplay between the individual and a need to stay motivated, to apply effort (behaviour), in an environment which is dynamic, to realise and develop some optimal level of self-efficacy.

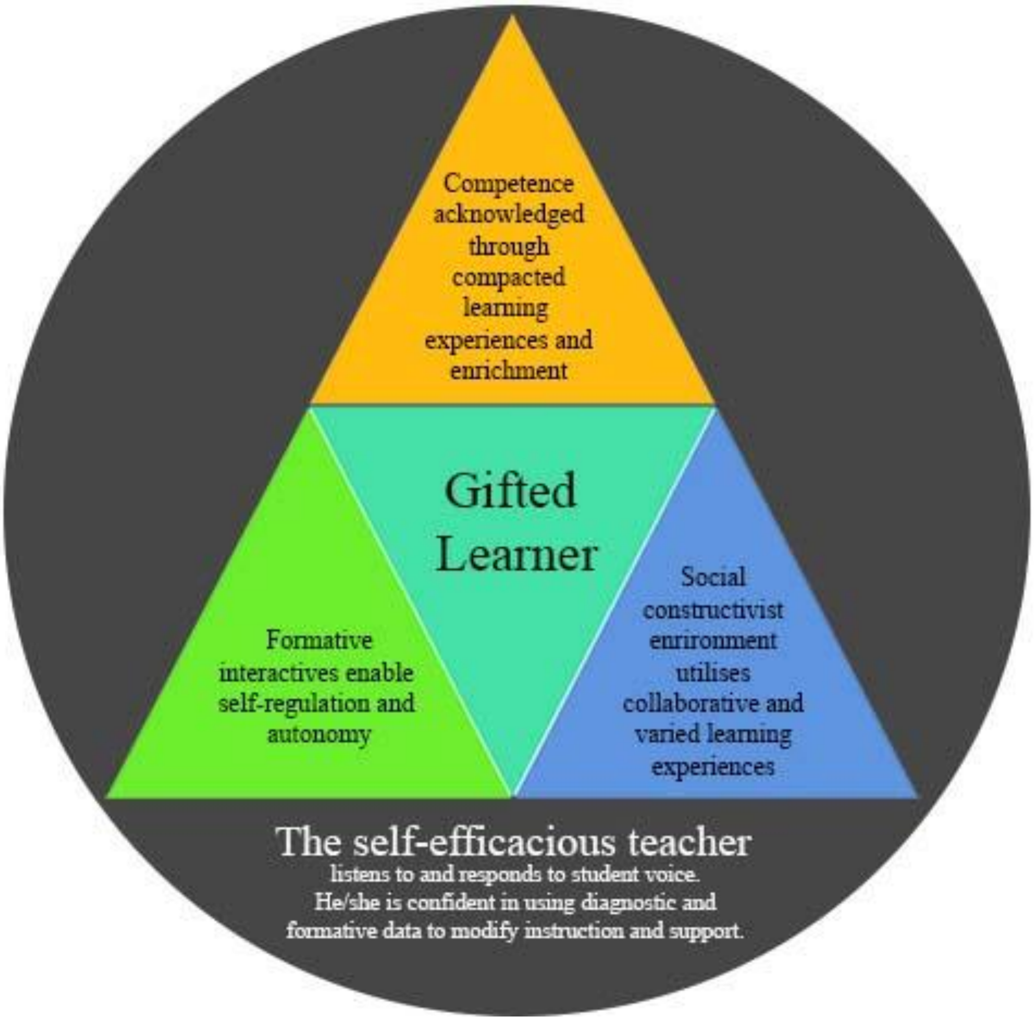


Figure 3.1. Gifted Instruction Model (GIM).

This development of belief in their abilities helps produce optimal social and cognitive growth. Burney's (2008) interpretation of and application of Bandura's Social Cognitive Theory to a gifted context would see students believing that their ability levels are not static, but rather, that they can "be enhanced through the use of cognitive strategies, self-regulation, and effort" (p. 135). These ideas are similar to Dweck's (2008) notion of possessing a growth mindset, achieving success in a given endeavour through effort, persistence and willingness to learn from failures. To help achieve this goal of self-regulation, Schunk (2008) advocates the need for “social models, corrective feedback, strategy instruction and practice, goal setting and self-evaluations of learning progress” (p. 127). An application of Bandura’s Social Cognitive Theory (1986) in a social context of the self-contained gifted mathematics classroom would seemingly support the arguments of Guskey (2010) and Bloom (1968).

3.4.2. The Zone of Proximal Development (ZPD) and scaffolded instruction.

The teacher can differentiate their instruction in a meaningful sense when they understand what their students’ actual ability levels are. The goal is to make sure the students are appropriately challenged within their ZPD (Vygotsky, 1978) rather than only relying on what the syllabus might dictate. Vygotsky suggests that the core premise of the ZPD is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers.

Vygotsky suggested that optimal challenge takes place when students’ learning tasks are set at a sufficient challenge level where they need access to a tutor for help. If the student can successfully solve increasingly difficult tasks after mastering content and skills, then the gifted student’s self-efficacy beliefs should remain at optimal levels. These accomplishments help to maintain their beliefs about their capabilities in the mathematics classroom.

Vygotsky asserted that what a student “can do with the assistance of others might be in some sense even more indicative of their mental development than what they can do alone” (p. 85). Vygotsky, a Marxist thinker (Bruner, 1984), was concerned with a collective and abler society helping those who are less able, to be equipped with the tools than the less able person to grow in knowledge. Sternberg and Williams (2010) suggested that “many psychologists have used and expanded on Vygotsky’s ideas to understand better how children learn and think” (p. 53). While Vygotsky highlighted the social nature of learning, Bruner’s research would elaborate on how this could be achieved through the use of scaffolded instruction.

3.4.3. Scaffolded instruction.

In 1973, Bruner argued that development of higher order skills in infants develops by constructing knowledge from earlier learned skills that encompass the initially mastered skill as a “subroutine” (Bruner, 1973, p. 4). This development of Bruner’s constructivist thinking led to his work with others (Wood, Bruner & Ross., 1976) in discussing the role of the tutor (the more capable other) in helping the student by scaffolding instruction, taking them from what they did not know to be able to solve unfamiliar problems. For Wood et al., the “child’s success or failure at any point in time thus determined the tutor’s next level of instruction” (p. 92). Bruner was concerned with helping students understand the structure of the problem (Bruner, 1963) in a way that enabled them to solve a similar or related problem. The tutor enables the learner to advance through a series of steps progressing from enlisting interest in the challenge through to the successful completion of this challenge that they were seemingly unable to do unassisted. The job of the tutor or teacher according to Bruner, therefore, was to allow the learner to do as much as they could without assistance (self-paced learning then followed by formative evaluation). The teacher or tutor then helped them understand the nature of the problem, what it entails and guide them through to such a successful solution when needed. Some research (Hidi & Renninger, 2006; Koshy et al., 2009) that provide examples of this scaffolding are now discussed.

A social, cultural approach to the teaching and learning of 330 mathematically gifted and talented students in 11 Education authorities in the UK highlighted several points about the teaching of more capable mathematics students (Koshy et al., 2009). This social construction of learning placed the teacher as the more capable other. Meanwhile, the students (who construct their idiosyncratic understanding of the concepts and content covered), received scaffolded instruction in such a way to allow for the gradual withdrawal of support as they completed tasks which were initially outside their zones of proximal development (i.e., outside what they could do on their own unaided).

Students in the Koshy et al. (2009), action research study, were challenged with tasks requiring higher processes of thinking, as suggested by Bloom in his taxonomy of educational objectives (analysis, synthesis and evaluation) (Bloom, 1956). In this project the responsibilities of peer groups and adults were stressed as important to the “effective construction of knowledge…during training sessions” (p.

220) where students' questions and group discussions were considered important in such a construction of knowledge and understanding. The study revealed that students found mathematics to be "boring, repetitive and too easy" (p. 20).

Teachers who received extra professional development found teaching and identifying gifted students to be a difficult job, especially when the students seemingly knew more than they did in that subject area. Many external factors were noted to come to bear on the student achievement levels that had previously prevented students from performing to a level that teachers felt was more commensurate with their levels of ability. These were: teacher shortages, discipline problems and lack of family support. Students' levels of interest increased when tasks they were asked to complete were seen to be relevant and open-ended rather than simply all textbook work. The authors also contend that it is within this framework of teaching that teachers scaffold instruction in a way that withdraws support as understanding and confidence grows.

This two-year project carried out by Koshy et al., (2009) left out a good deal of information about what actually happened in the classroom. A limitation of the study could be the lack of a measure to gauge the effectiveness of the open-ended approach that was applied. This research is relevant as it highlights that whole class instruction is not particularly useful when asking the students to complete higher order thinking tasks. Second, Koshy et al., (2009) suggested that there are different ability levels in identified gifted students and some remediation may be required. Students' interest levels also increased because the program was challenging, open-ended and relevant. The study also added that teachers disliked accelerating students through the curriculum as they believed it caused too many long-term problems for student learning.

Hidi and Renninger (2006) discussed the four-phase model of interest development where students develop in their levels of interest through four incremental phases from situational to well-developed individual interest. Abbott (2017) suggests that to gain this deepened interest; students must be engaged and active participants in the learning process that is purposeful, challenging, explorative, relevant, and includes multiple modes of learning. Abbott ascertained what students are interested in at the commencement of instruction, and works that into the classroom instruction, where possible through authentic real-world immersed enrichment challenges. By contrast, Frenzel, Pekrun and Goetz (2007) suggested that boredom is a “purely action-related emotion” (p. 482) that results from having little control over your learning. Furthermore, the researchers suggested that increased enjoyment in learning came when a feeling of competence coupled with quality instruction allowed deeper or penetrative learning to take place.

Kanevsky and Keighley (2003) in their qualitative study on ten gifted non-producing high school students discussed the complex notion of boredom. They reviewed research (Drory, 1982) that cited correlations between boredom and intelligence. They also cited other studies (Robinson, 1975) which found no correlation. The authors (Kanevsky & Keighley) suggested that there are too many inter/intra personal/environmental factors that impact on researchers that try to draw any definitive correlation between the two constructs. They suggest a range of factors which increases students’ chances of being interested in learning such as: quality instruction, competence, having a sense of control over their learning, ensuring that the learning is relevant, challenging, engaging and is multi-modal. The challenge to increase feelings of interest would be to cater for the individual as well as collective interests, while also ensuring students feel competent through mastery of the core curriculum.

Kanevsky and Keighley (2003) suggested that students should have access to choose what they learn about, the complexity of content, have control of their learning, as well as access to a caring and respectful environment and appropriate levels of challenge that takes place within the ZPD. Without control, choice, complexity, challenge and care, students can often disengage and become disinterested in learning. The authors suggested that the students may rebel in any number of ways including non-production of schoolwork, dropping out of school, and experience disenchantment with the education process or lack motivation or interest in learning. In the next section, control, choice, challenge, complexity and care are discussed within the context of Ryan and Deci’s (2017) Self Determination Theory.

3.4.4. Motivation and Self-determination Theory – General introduction.

In his lecture introducing his audience to an understanding of Ryan and Deci’s (2017) Self Determination Theory, Deci (2010) describes motivation, as the “energy for action (which) moves people to behave” in the ways they do. Rather than view motivation as a quantitative measure, Deci describes motivation that comes in two forms, intrinsic and extrinsic. According to Deci and Ryan (2002), when an individual is extrinsically motivated, the locus of control for behaviour is external to them. Regulation of motivation can then come through rewards and external control.

Conversely, when an individual is intrinsically motivated, the locus of control is within themselves. Hence regulation of motivation is driven by volitional interest. Ryan and Deci (2017), also associate volitional control and choice as core components of autonomy. This autonomy "requires integration, as experiences of full volition are characterised by lack of inner conflict and willing engagement" (p. 8). Their (Ryan and Deci's) research suggested that no-one likes to be controlled. People like to experience a sense of competence, or the "need to feel effectance and mastery", (p. 11), relatedness (having a sense of belonging and feeling cared for) as well as having this autonomy in tasks they complete, to be motivated to achieve. These, according to Ryan and Deci are considered "basic psychological needs" (p. 80) to remain motivated.

Ryan and Deci (2017) presented a self-determination continuum from non-self-determined amotivation through to self-determined intrinsic motivation. The highest external motivation comes when a person internalises a goal that has been set, rather than completing a task simply because of volitional interest or enjoyment. For Ryan and Deci when a person is intrinsically motivated, they are interested in and perform well at a task, a view supported by others (Black & Deci, 2000; Soenens & Vansteenkiste, 2005).

Soenens and Vansteenkiste shared their research from two studies on a total of 613 adolescents aged between 15 and 21, from a secondary school in Belgium. Their study examined the relative social and academic impact autonomy supported teaching and parenting had on a student's social functioning, academic engagement and feelings of competence. Results from their study supported other findings (Black & Deci, 2000) which "clearly confirmed…that more self-determined motivation to engage in scholastic activities is associated with more perceived competence, as well as with higher actual grades (academic grade point averages)" (p. 601). While the study did not report on how the students' grade point averages were calculated, it did state that they were self-reported grades.

According to Hattie (2009), students' self-reported grades are based on "estimates of their performance…(which can be) very accurate understandings of their levels of achievement" (p. 43) and chances of success. A student's self-awareness of their level of understanding, according to Hattie, is "one of the greatest influences on student achievement" (p. 31). For Hattie, this self-evaluation of ability attained by a student ranking themselves in a class's academic standing can limit their expectations for what they see as attainable. In both Hattie's and Soenens and Vansteenkiste's (2005) study, the role of the teacher in providing support with challenging goal setting and autonomy is important to overcome such limitations. By supporting autonomy and setting challenging goals, the students can internalise the process of motivating themselves to work. The use of effective feedback, autonomous and challenging goal setting through an appropriately differentiated and individualised learning program, therefore, facilitates better motivation and better academic performance. Ryan and Deci (2017) suggested that students need access to volitional choice with this positive feedback which promotes interest and enjoyment in the learning process.

Deci (2010) discusses the positive effects of choice (Zuckerman, Porac, Lathin & Deci, 1978), acknowledged feelings (Deci, 2010) and positive feedback (Ryan, 1982). Elements identified to encourage internalised intrinsic motivation (integrated regulation) include understanding the other's perspective, encouraging exploration, offering choice, challenge, providing meaningful feedback, reason, or rationale for learning a concept and minimising controlling language. These are all elements observed in the application of the Mastery Learning Model. Research presented next, explores how autonomous learning, competence and relatedness can impact on gifted students' learning.

3.4.4.1. Motivation, Self-Determination Theory and gifted students.

Research discussed in this section reveals how gifted students' motivation fluctuates along the motivation continuum. This motivation depends on the type of learning, levels of autonomy and the relevance of the challenges which are given, the kind of rewards provided, the way the teacher teaches, parental and peer influences. The scope of this study does not permit me to focus on every aspect of motivation pertaining to gifted student learning. I do, however, draw from two key theories: (a) Flow (Csikszentmihalyi, 2002) and (b) Self Determination Theory (Ryan & Deci, 2017), after first highlighting why being motivated is important within the gifted learning context and using the Mastery Learning framework.

Academic motivation is important. According to Siegle, Rubenstein and Mitchell, (2014), it is the "strongest predictor of academic achievement of gifted students" (p. 35). McCoach and Siegle (2003) go further, suggesting it to be a key difference between those who achieve and those who underachieve. According to Siegle et al. (2014), if students are to "do well in school…(they) need to believe they have the necessary skills to perform the task (self-efficacy)" (p. 35), to be involved in relevant learning, not be engaged in needless repetition, but instead, have learning paced appropriately according to their ability. Two studies on motivating gifted students in learning will now be discussed.

These two studies (Bourgeois & Boberg, 2016; Garn & Jolly, 2014) analysed the impacts two completely different programs had on gifted students' motivation to learn. First, a mixed methods study (Bourgeois & Boberg) of 680 teachers and 5392 Year-3-12 students "from a single charter school organisation in the southern United States" (p. 4) analysed the impacts leadership and teaching had on students' motivation to study math. While in the second study, Garn and Jolly completed qualitative (Interpretive Phenomenological Analysis) research on 15 high ability students "with a mean age of 9.13 (SD = 1.19)" (p. 12) from the Southeastern United States. In both studies, the authors used interview data to analyse what factors influenced students' motivation levels. In particular, Bourgeois and Boberg sought to examine reasons why "high-achieving, cognitively disengaged middle-level students experience motivation towards academic tasks in math" (p. 4). In the second study, examples are drawn from a summer learning camp for identified gifted students.

The first phase of this study by Bourgeois and Boberg (2016), "revealed that while increases in emotional engagement predicted increases in student achievement, lower levels of engagement appeared to predict higher levels of student mathematics success" (p. 4). In this study, the students identified the learning as too easy, while in the Garn and Jolly (2014) study, students were engaged and interested in learning. To delve into the reasons for these findings, Bourgeois and Boberg conducted face-to-face semi-structured interviews with eight students and their mathematics teacher and the school principal. Grades and choice were the two factors which attributed to students' fluctuating motivation levels.

Students from both studies (Bourgeois & Boberg, 2016; Garn & Jolly, 2014) were provided with a variety of rewards for getting good grades in school from both parents and the school. Students were motivated by rewards for good grades and received disappointment, punishment and more pressure for bad grades. According to Ryan and Deci (2017), externalised rewards like these diminish feelings of autonomy and this, in turn, decreases longer-term motivation levels when the rewards are withdrawn or worse when punishment is given. In the first study (Bourgeois & Boberg, 2016), students had little access to learning autonomously. These authors describe the in-class lessons as teacher-centred where all students received the same instruction at the same time. Conversely, the study by Garn and Jolly (2014) revealed that students appreciated it when the teacher allowed the students to pursue their interests in investigations and gave them choice in what and how they learned.

The students in Bourgeois and Boberg’s study had developed home study routines to achieve better grades. Whereas the students interviewed by Garn and Jolly enjoyed completing challenges that were of interest to them. These two studies reveal the benefits of having autonomous control and relevant challenge and thereby ensuring their competence is acknowledged in a meaningful sense. On the one hand, students who received mostly externalised motivation felt pressure, punishment and sometimes short-term rewards, where students who could autonomously choose what they learned and how they learned it were more intrinsically motivated. One system was seemingly built on a behaviourist model that provides external rewards and punishment, while the other seems to be based on a more constructivist approach that emphasises student-led learning.

Instead of having a focus on work, the students within the Garn and Jolly study were motivated to learn. Students particularly loved being able to investigate topics that were of interest to them which made learning “more fun and ultimately result(ed) in higher levels of engagement” (p. 16). In contrast, Bourgeoise and Boberg shared how the students would routinely pack up their books before the end of the lesson in preparation to leave. Garn and Jolly shared how the students were challenged through this access to autonomy, while Bourgeoise and Boberg reported on how the students were disengaged and found learning to be easy.

In both studies, feelings of relatedness had evidently impacted on the students’ motivation to succeed. Bourgeoise and Boberg described how the teacher was relaxed and friendly but delivered instruction from the front of the class. Conversely, Garn and Jolly described how they loved how their teachers took the time to get to know what they liked and allowed them to pursue such interests. In both studies, the students liked their teacher/s, while in the Garn and Jolly study, students' levels of motivation were positively impacted on as evidenced by their passion for, and desire to learn even when the teacher was not around. While the teacher and principal argued that their approach to teaching was successful, they measured success through grades that students achieved through hard work. Students from both studies worked hard. However, the contrast in both studies reveals how relatedness impacted on motivation levels in different ways.

Bourgeois and Boberg do not attribute the levels of disengagement to the teacher-centered instructional approaches, the rewards system or the focus on teaching to the test. They do however urge schools like this one to use more intrinsic forms of motivation, such as is discussed in the Garn and Jolly (2014) study so that students can see the value in the learning itself, rather than the grades from the test, or rewards. They also urge educators to "seek out instructional strategies and materials that are optimally challenging" (p. 14).

In a more recent study (Ben-Eliyahu, 2017) of 455 students from both secondary schools and undergraduate universities from the southeastern United States found "no difference between (self-identified) gifted and typically developing students across contexts" (para. 26) regarding their levels of self-regulation in learning. His study utilising descriptive statistical methods asserts that "learning context overrides individual differences" (para. 27). This opinion, however, is different to the findings of Vallerand, Gagné, Senécal, and Pelletier, (1994) who found that "gifted students perceived themselves as being more cognitively competent and more intrinsically motivated than regular students" (p. 174). Both arguments suggest that the learning environment and the teaching approach are important in encouraging intrinsic motivation. It would seem plausible to suggest that in classrooms where access to autonomy, relatedness and competence is limited, the classroom environment could impede gifted students' motivation levels.

The argument to remain intrinsically motivated is furthered when students are working in flow. Csikszentmihalyi (2002) defined flow as "the state in which people are so involved in an activity that nothing else seems to matter; the experience itself is so enjoyable that people will do it even at great cost, for the sheer sake of doing it" (p. 4). He identified several elements required for this flow experience, such as clear goals, immediate feedback, a balance between one's challenges and one's skills, no concern about failure, a distorted sense of time, and the activity having its reward.

Csikszentmihalyi, Montijo, and Mouton, (2018) suggest that to remain in the state of flow, “individuals must take on tougher challenges as their skills increase. They must develop new skills through deliberate practice to meet increasing challenges to remain in flow” (p. 216). By applying the Mastery Learning Model to the teaching of the same students, students were engaged in challenges at their level of ability, and that this level of challenge increased after mastery had been achieved. Without elaborating on the entire depth and breadth of Csikszentmihalyi’s Flow theory, I note how this study had the potential to observe such flow moments, once students experienced the elements as described above (Csikszentmihalyi, 2002). Next, I draw from the relevant literature to refine arguments drawing on research on mastery goals and the self-determination theory.

3.4.4.2. Mastery Learning and Self Determination Theory.

A search was conducted of EBSCOhost databases, including Academic Search Elite, American Doctoral Dissertations, Audiobook Collections, CINAHL, eBook Collection, Education Source, E-Journals, ERIC, MAS Ultra, Primary Search, Psych Articles, Psych Books, PsychEXTRA, PsychINFO) using the terms “Self-determination theory” and “Mastery Learning”. Results found were related to the use of mastery goals, rather than the explicit use of the Mastery Learning Model advocated for in this research. Some key findings, however, were able to be drawn from one article as stated here.

A study (Madjar, Nave, & Hen, 2013) of 191 Year-8 students from public junior-high schools in an urban area in the United States looked at the associations between student perceived teacher behaviours and student personal goal orientation. Their research used confirmatory factor analysis which found that the use of mastery goals were associated with autonomy supportive teaching practices ($r= .71$, $p < .001$). Their research included providing students with choice on relevant learning tasks and that such teacher practices should be embraced. These results indicated an association between the setting of mastery goals to be positively linked with the autonomy supportive practices. According to Ryan and Deci (2017), these autonomy-supported behaviours enhance feelings of intrinsic motivation, which would, therefore, suggest that the setting of mastery goals would have a positive effect on student motivation in the classroom. These findings provided support the way in which data were collected in my study. Instead of focussing on grades, Section 7.1 and 8.1 focus on students' levels of understanding, that is, what they learned.

3.4.4.3. Facilitating interest and engagement.

According to Linnenbrink and Pintrich (2003), the teacher should make a judgment on whether the student can complete the activities given to them. By doing so, the learner stays within their zone of proximal development and maintains positive levels of self-efficacy and motivation to learn. If a student's levels of self-efficacy are too high, they may over-estimate their abilities and not see the need to engage in learning. Whereas, if students' self-efficacy beliefs are low, then they may not feel confident enough to try the task. Optimal self-efficacy levels need to be slightly higher than actual ability levels so that they both engage in learning and are not feeling like it is too hard. The use of the Mastery Learning Model would, therefore, enable healthy self-efficacy beliefs, as the students can understand their ability levels through the use of formative interactives, which included prompt feedback enabling them to remain inside their zone of proximal development. The teacher could also set tasks that are within their zones of proximal development as is suggested as needed by Linnenbrink and Pintrich (2003). Table 3.3 presents the vast array of factors that can have an influence and impact on a student's ability to achieve in school. To elaborate upon all of these would be too unwieldy and beyond the scope of this thesis.

Table 3.3

Factors that can Influence Students' Achievement and Engagement

Factors	Research
Peer acceptance and other social-emotional issues	Especially prevalent in high school students (Grantham, & Ford, 1998; Guldemond, Bosker, Kuyper, and van der Werf, 2007; McCoach & Siegle, 2003; Seeley, 2004).
Family or personal circumstances	Divorce, separation, abuse, neglect, major personal or family event, Relationship break-up, Stress (McCoach & Siegle, 2003).
Cultural barriers	Students from other cultures may struggle to perform in a different cultural environment (Ford, Wright, Grantham, & Harris, 1998; Seeley, 2004).
Twice exceptionalities	Students may have other exceptionalities ADHD, autism, dyslexia, and other special needs (McCoach & Siegle, 2003; Seeley, 2004).
Organisational issues	Bad study habits (Guldemond et al., 2007; McCoach & Siegle, 2003).

Working within a social cognitive, and Vygotskian social constructivist framework, this thesis draws on research by Linnenbrink and Pintrich suggesting that "self-efficacy can lead to more engagement and, subsequently, to more learning" (p. 123) as elaborated upon in Figure 3.2. It diagrammatically represents a reciprocal causality on how students' self-efficacy levels can impact on engagement, motivation and learning. Once students realise they can achieve mastery, interest, motivational levels and self-efficacy levels can be impacted upon in a positive sense. Self-efficacy which is affected by the interplay of self, environment and behaviours can be goal-directed within the scaffolded and challenge-rich classroom to bring about deeper learning and engagement.

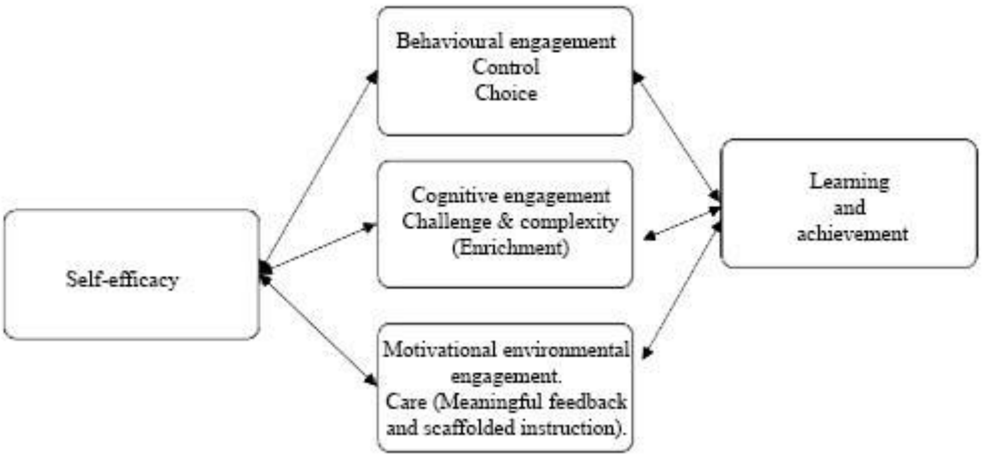


Figure 3.2. Linking self-efficacy, motivation and learning within the Mastery Learning Model (Adapted from Linnenbrink and Pintrich, 2003).

Figure 3.2 demonstrates that when a student is engaged cognitively through this appropriate and relevant challenge, their levels of self-efficacy can be impacted on in a positive sense. Similarly, when students receive support that enables autonomy and building of positive relationships, it impacts reciprocally in a positive sense on their self-efficacy, learning and achievement (Ryan & Deci, 2017).

Research (Frenzel, Goetz, Pekrun & Watt, 2010) revealed that interest in mathematics "declines from childhood through adulthood" (p. 510). The authors attribute many reasons for this decreasing interest in mathematics from increased complexity, increasingly restrictive learning environments, adolescence and social relationships. According to Eddles-Hirsch, Vialle, Rogers and McCormick (2010), "years of academic neglect may not only impinge on talent development but may also impact the social and emotional development of the gifted child" (p. 108). From this premise, therefore, any study that addresses the education of gifted children should also look at how the teacher (and learning program) maintains an appropriate level of interest in learning. In this regard, one study (Shernoff et al., 2016), revealed a strong relationship between suitable challenge levels and task engagement. This section provided examples of classroom interest and engagement from a social cognitive and Vygotskian social constructivist perspective, where students were engaged in deeper learning experiences. It has shown that possessing positive self-efficacy beliefs can potentially impact on motivation levels in classroom learning tasks given within the Mastery Learning Model context.

In the next section, I elaborate on research on how learning preferences and collaborative learning can impact on students' interest levels and engagement in mathematics.

3.5 Learning Preferences and Collaborative Learning

Studies discussed here explore the learning preferences of gifted students and some of the impacts this had on their learning. This section is not intended to analyse every aspect of this topic. However, it elaborates on some key findings discussing the importance of collaboration in learning and how this has impacted on gifted students' attitudes in studies.

An exploratory case study (Diezmann & Watters, 2001) of the collaborative learning behaviours of six gifted 11 and 12-year-old students found that only when the task complexity was high, would students choose to collaborate. They argued that while the homogeneous grouping of gifted students is important, that even if the gifted students are grouped together, the level of challenge is still "limited" (p. 27). The six students would alter the way they worked depending on the complexity of the task. Further studies (Samardzija & Peterson, 2015), discussed next, also share that students appreciated the support of their peers in the collaborative classroom.

Several important points can be derived from the phenomenological study of Samardzija and Peterson's (2015), which examined the experiences of 23 identified gifted students' learning preferences are initially discussed here. The students in their study revealed that their learning preferences were domain specific. Learners revealed that they preferred to work in groups when the division of labour was even.

Some students (N=6) preferred quiet learning environments while others (N=14) were not bothered by background noises. Three students appreciated independent self-paced work that enabled them to study topics to a greater depth. While the students in this study described independent self-paced learning as harder, they also revealed that once they have found a successful solution, you remember how you got it. Students did like being able to confirm solutions and share ideas in groups as then they did not feel “embarrassed” if they got the answer “wrong” (p. 246). Not all students preferred working alone, while some (N=5) preferred assisted individual work as they wanted to make sure they were going to “get a good grade” (p. 246).

Therefore, while students appreciated being able to share ideas in groups, the students valued good grades and fairness when making decisions about their learning preferences depending on the subject they were learning at the time. Students preferred fewer distractions in the math classroom and independent work on harder tasks. The study did not go into detail about the specifics of learning tasks or the complexity of the kind of work completed by the students and the impact this had on their learning preferences. The students enjoyed learning more complex tasks and in a way that was autonomous, self-paced and personalised, as it allowed them to discover “in-depth information” (p. 246). This finding is interesting, given the self-paced nature of my study, inviting students to explore open-ended enrichment style problems to a greater depth. Their study found that some students (N=5) liked open-ended project style of learning as it helped them to remember what they had learned, but the authors did not elaborate on what the students meant by open-ended learning tasks and whether this involved collaboration and what impacts this had, if any, on learning preferences. The students preferred discussion kinds of learning, more than worksheets and would go to the Internet, their teacher or their parents if they got stuck.

Some research (French, Walker, & Shore, 2011; Samardzija, & Peterson, 2015) supports the notion that gifted students prefer to work alone. Other research (Diezmann & Watters, 2001; Samardzija & Peterson, 2015), suggest the benefits of appropriately challenging collaborative tasks to be used with gifted students.

3.6 Summary

In this chapter, I began by arguing that the use of on-going assessments helped guide instruction in a purposeful sense. I then examined some criticisms of the

Mastery Learning Model, while also noting the positive effects it has had on both academic achievement and students' motivation levels in learning. A discussion of the theoretical underpinnings of this model followed. I discussed what engagement and interest might look like in the study, which enabled this to be monitored and recorded (to be discussed in Chapter 5). I have drawn from the research of Kanevsky and Keighley (2003), Ryan and Deci (2017) and Linnenbrink and Pintrich (2003) in discussing how the Mastery Learning Model could potentially counter feelings associated with boredom and therefore facilitate interest and engagement in deeper learning.

I provide support for the contention of Soenens and Vansteenkiste (2005) "that social (and educational) contexts that are responsive and autonomy supportive, promote the development of this volitional or self-governing" functioning" (p. 600). The implications for this research would suggest that for interest levels in the mathematics classroom to be nurtured, the teacher, the students and the teaching approach need to be supportive of student autonomy, provide students with access to social relatedness and acknowledge competence in a meaningful sense, to maintain optimal levels of motivation to learn. When a search was conducted on the application of Ryan and Deci's Self Determination Theory to a Mastery Learning framed classroom in major databases no results were found suggesting that any findings found to be novel and new. Next, I critically discuss the theoretical and methodological underpinnings that guided the collection and analysis of the collected data in Chapter 4.

Chapter 4
Methodology and Research Design

4.1 Introduction

The purpose of this study was to examine the influence the use of the Mastery Learning Model had on the achievement, interest levels and attitudes of gifted mathematics students in a Year-8 mathematics classroom. The two research questions that guided the collection and analysis of data are re-stated here:

1. In what ways does a teaching program informed by the principles of Mastery Learning influence gifted students' mathematical performances?
2. In what ways does a teaching program informed by the principles of Mastery Learning influence gifted students' attitudes, motivation and interest in learning mathematics?

An overview of the different stages of the study is presented in Figure 4.1. Section 4.2 discusses the philosophical foundations. The explanatory case study methodology and boundaries for this research are also discussed in this section. In Section 4.3, I introduce how the participants were selected and what data collection methods were employed, along with how the data were analysed. A discussion on the measures taken to ensure the quality of the research is outlined in Section 4.4. Finally, Section 4.5 discusses ethical considerations and limitations of the study.

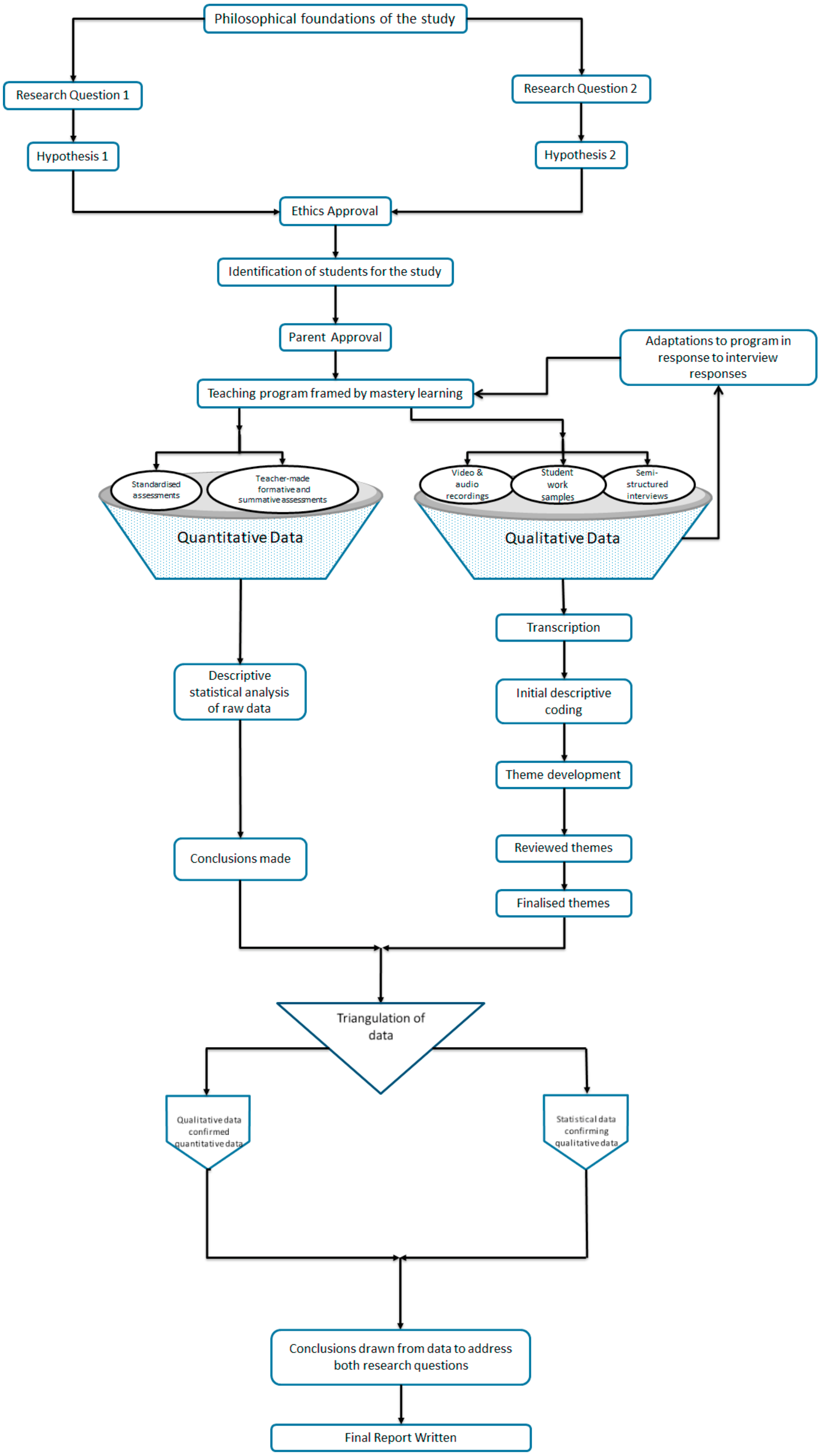


Figure 4.1. Visual representation of research design.

4.2 Ontology, Epistemology and Methodology

This section outlines the ontological and epistemological foundations that ground the methodological approach that was adopted in this study.

4.2.1. Research paradigm.

A post-positivist ontology and epistemology guided the research design, collection and analysis of data. The post-positivist worldview suggests that "no universal truth is found" (Panhwar, Ansari & Shah, 2017, p. 253). The authors suggest that we can only "explore a phenomenon as much as possible" (p. 253). From a post-positivist perspective, scientific claims can be made with both qualitative and quantitative data.

Within this worldview, participants are constructors of their social worlds (Fox, 2008). As the researcher, I would have to interpret meanings ascribed to participants' actions while understanding the "context-specificity of (their) knowledge" (Fox, 2008, p. 662) within the "real world setting" (Harrison, Birks, Franklin & Mills, 2017, p. 10) of the classroom. Similarly, I was an "insider researcher" (Greene, 2014, p. 3). As such, I gained a "rich description" (Morrow, 2005, p. 252) of the participants. I gathered both insider and outsider perspectives in order to embrace multiple standards of quality as discussed in Section 4.4. As an experienced teacher, I bring to this research my understanding of phenomena which interacted with the participants' views of phenomena and present a final interpretation of what was observed in the final results of this case study research (Yazan, 2015). For post-positivists, reality exists independent of the observer, and the closest approximation of the truth is best found when data are triangulated from multiple data sources (Sharma, 2010). According to Egbert and Sandon (2014), the researcher cannot claim complete objectivity in the analysis process. Analysis of findings are based on my subjective observations but are guided by principles of quality and rigorous research, given in Section 4.4.

This case study used a range of data collection methods to analyse the influence the Mastery Learning Model had on academic performance, attitudes and interest levels from multiple perspectives. I was then able to search for causal mechanisms from the data, as is discussed in further detail in Section 4.3.2. The next section discusses the use of this case study methodology as a framework for data collection.

4.2.2. Methodology.

This section provides a definition of and justification for the use of an explanatory case study (Yin, 2014). The use of this methodology afforded insights into a teaching and learning approach guided by the Mastery Learning Model with young gifted adolescents in a mathematics classroom.

Yin (2014) defines a case study as a "critical inquiry (which) investigates a contemporary phenomenon (the case) in depth and within its real-world context, especially when the boundaries between phenomenon and context may not be clearly evident" (p. 48). Fisher and Ziviani (2004) distinguish the explanatory case study as having how and why type research questions that "beckon a more explanatory approach" (p. 186). I analysed the complex, socially constructed interactions within their natural settings (Flyvbjerg, 2006; Gibbert, Ruigrok & Wicki, 2008; Harland, 2014; Pearson, Albon, & Hubball, 2015; Simons, 2009), a key feature of such case study methodologies. The study also sought to provide an explanation using multiple data collection methods to record these complex interactions, which consisted of students' interactions with each other, the learning challenges set within the Mastery Learning Model structure and the teacher.

In a practical sense, I sought to understand how the teaching model may have influenced students' attitudes, interest levels and academic performance. Qualitative data were "quantitised" (Kitchenham, 2010, p. 562) through student responses on academic performance. Conversely, quantitative data were "qualitised" (p. 562) to confirm any influence on student attitudes toward mathematics.

Blatter and Haverland (2012) discuss other characteristics such as participant size, and the difference in focus of the research in explanatory case studies. They revealed that the explanatory case study is usually smaller in size with fewer participants and generalizable across comparable cases. Explanatory case studies are case centred, rather than variable centred. My research was therefore bounded to only evidence from the five participants in relation to the two research questions stated. These boundaries, along with others are discussed further in the next section.

4.2.2.1. Boundaries.

Section 4.2.1 provided the theoretical foundations for the research. The stages of this explanatory case study research are presented diagrammatically in Figure 4.1. This section elaborates on the case; the boundaries, the stages of the research and how the methods (Section 4.3) were used to provide a rich explanation and note changes in attitudes, interest levels and achievement in mathematics. The boundaries of the case were the participants, the school context, the classroom, and time.

Participants. The study was bound by the participants selected (Merriam, 1998). In this case, the participants were five Year-8 students who had previously been identified within the school context as mathematically gifted (three boys and two girls). These participants are profiled in Chapter 6, where I use Neihart's (2012) profiles of the six types of giftedness to provide a richer contextualised understanding of each participant. The participants attended an urban private school on the Gold Coast in Queensland, Australia and were part of a Year-8 streamed mathematics class in this school setting. Criteria used in the selection process are discussed in detail in Section 4.3.1. Data from these five participants were used to draw conclusions about the outcomes of the study. Data from the remainder of the mathematics class was used for comparative purposes alone.

Contextual boundaries. The context or environment, domain, or the school and class also bounded this research. Swanborn (2010) spoke of the environment as the domain, suggesting that the boundaries of the domain need to be "defined in advance" (p. 47). The study in terms of context was bounded by the structure, practices and culture of an independent school that provided accelerated programs for more able students in mathematics. Based on an Australian Government measure that is used to characterise the socio-economic status of students within the school; the Index of Community Socio-Educational Advantage (ICSEA), the school contains students from economically above average families. Sheppard and Biddle (2017), suggests that students like these are from established middle-class families.

The research was conducted in a Year-8 mathematics classroom that contained some students identified as mathematically gifted. The Year-8 mathematics classroom existed within a Foundation to Year 12 private (non-government) school in Queensland, Australia. What I taught needed to conform to both local (school) and state-guided national curriculum guidelines contained in school policies, and national government syllabi requirements (Australian Curriculum Assessment and Reporting Authority, n.d.).

Time parameters. Yin (2014) recommended that the researcher bind their case with a beginning and end time for their research. According to Yin, the researcher can select all or part of the life cycle of their study. The time boundary for the entire case was 11 months (2014). This allocation of time enabled me to collect enough data, and track changes over time, to see patterns emerge.

Table 4.1

Data Collection Timeline

Dates (2014)	Data Collection Points
January	Initial data collection
February	Baseline data recorded - PAT Maths and baseline interviews.
February – June	Video and audio recordings of students working. Videos reviewed at the end of each session.
March 31 – April 3	Second round of interviews
February – June	Results from formative assessments recorded into a spreadsheet.
March & June	End of Term Summative assessment data recorded into a spreadsheet.
June 23 – 26	Third round of interviews conducted
December 2	Final interview with one student discussing his involvement in the Year-10 program.

4.2.3. Summary.

This section has discussed the theoretical foundations that framed the collection and analysis of data related to the two research questions within an explanatory case study methodology (Yin, 2014). I have presented the boundaries for the study that were framed by the research questions and aims for the study. This next section elaborates on these methods used within these research boundaries.

4.3 Methods

A range of research methods were used in this study that enabled the collection and analysis of data to provide a “richer and stronger array of evidence” (Yin, 2014, p. 109). I first provide information on how the participants were selected in Section 4.3.1. I then elaborate on and justify the use of the multiple methods in Section 4.3.2. Once the methods have been introduced, a justification is supplied for each method’s use in Sections 4.3.3 and 4.3.4. How the data were analysed is then discussed in Section 4.3.5.

4.3.1. Participant selection.

All students participating in the study were part of a mixed-gender streamed class (N=24) in Queensland Australia. A review of student records revealed that all students possessed similar Anglo-Australian and North American cultural and social attributes and spoke fluent English. These students had met minimum standards on at least two criteria (Parent/teacher nomination, standardised intelligence assessments, anecdotal evidence such as student work samples and report card results) before being placed in this class. Research conducted by Matthews & Kirsch (2011), recommended these as screening tools for such programs.

Five mathematically gifted students from this streamed class were selected using three specific criteria described here: (a) All five selected participants were averaging a minimum grade of A-minus in mathematics; (b) had met the criterion for inclusion with intelligence scores in the top five percentile ranks when compared to students their age; and (c) had completed an informal problem-solving task (Appendix G) which confirmed these students were able to reason at very high levels. The Standard Progressive Matrices (Raven, 1989) and the Slosson Intelligence Assessment-Revised (Slossan, Nicholson, & Hibpshaman, 2002) were used to identify students from the class in which this study was carried out. The identification criteria provided an overview of their content knowledge and problem-solving ability. Table 4.2 provides some basic information about each of the five participants.

During the study, Walter and Ty chose to work as partners, as did Bree and Miley. Oliver often worked independently, sometimes choosing to work on collaborative tasks with other friends from the class.

Table 4.2

Summary of Participant Backgrounds

Participant (Pseudonyms)	About the participant
Miley	A 12-Year-old girl from the United States of America who has lived in Australia for one year. Placed in the top fifth percentile of students her age according to Ravens Progressive Matrices. Ranked in the top 1% of students according to Slossan Intelligence Test. 93rd percentile Year-8 level Pat Maths Plus online. 84th percentile Year-9 level Pat Maths Plus online.
Walter	13-Year-old Caucasian Australian boy. Placed in the top fifth percentile of students his age according to Ravens Progressive Matrices. Ranked in the top 1% of students according to Slossan Intelligence Test. 89th percentile Year-8 level Pat Maths Plus online. 93rd percentile Year-9 level Pat Maths Plus online.
Oliver	12-Year-old Caucasian Australian boy. Twin to Bree. Placed in the top fifth percentile of students his age according to Ravens Progressive Matrices. Placed in the top 1% of students with Slossan Intelligence Test. 96th percentile Year-8 level Pat Maths Plus online. 99th percentile Year-9 level Pat Maths Plus online. Past teacher identified as exceptional.
Bree	12-year-old Caucasian girl. Twin to Oliver. Placed in the top fifth percentile of students her age according to Ravens Progressive Matrices. 64th percentile Year-8 level Pat Maths Plus online. 81st percentile Year-9 level Pat Maths Plus online. Past teachers identified her as exceptional and a gifted underachiever.
Ty	11-Year-old Caucasian Australian boy. Placed in the top fifth percentile of students his age according to Ravens Progressive Matrices. Placed in the top 1% of students with Slossan Intelligence Test. 85th percentile Year-8 level Pat Maths Plus online. 91st percentile Year-9 level Pat Maths Plus online.

4.3.2. Data sources.

The use of both qualitative and quantitative data sources, as discussed in this section, provided an understanding of the students' in-class behaviours and experiences (Giannakaki, 2005; Morse, 2003). The use of formative, summative and standardised assessments provided an insight into student academic performance. The quantitative data addressed Research Question 1 as stated here: In what ways does a teaching program informed by the principles of Mastery Learning influence gifted students' mathematical performances? The use of interviews, photos, in-class audio recordings and student work samples enabled a qualitative description to emerge, which addressed research question 2, stated here: In what ways does a teaching program informed by the principles of Mastery Learning influence gifted students' attitudes, motivation and interest in learning mathematics? The use of multiple data sources enabled triangulation and enhanced reliability (Yin, 2014). These data collection methods are discussed next.

4.3.3. Quantitative data.

This section discusses and justifies the use of the quantitative data sources (teacher made and standardised assessments). The participant selection process as elaborated on in Section 4.3.1 provided the baseline data necessary for creating the five students' work programs. Table 4.3 outlines how different quantitative data were collected at various points throughout the study. I used both teacher-made assessments and standardised tests, as are elaborated on in the Sections 4.3.3.1 and 4.3.3.2.

Table 4.3

Quantitative Data Informing Research Question 1

Assessment Type	Date	Data
Teacher made assessments	Beginning and end of each smaller sized unit.	Used as a baseline measure to ascertain students' understanding of specific topic given before instruction. Formative data used to track student's academic progress.
Standardised Pat Maths Plus Assessment	January 2014 July 2014	Initial pre-assessment used for baseline data Post-test data to track improvements in students' achievement.

4.3.3.1. Teacher-made assessments.

The teacher-made assessments (see sample provided in Appendix H) consisted of both formative and summative items. A combination of teacher-made online quizzes, paper-based tests and challenges were used to ascertain students' level of understanding in each unit of instruction. The results were used to guide future instruction. The students were assessed against Australian Curriculum standards (Australian Curriculum Assessment and Reporting Authority, n.d.).

4.3.3.2. Pat Maths Plus.

The Pat Maths Plus assessments (Australian Council for Educational Research, 2011) are a nationally standardised series of tests, used in Australian schools to ascertain students' general ability in mathematics (Stephanou & Lindsey, 2011). The Pat Maths Plus assessments package was developed to enable the teacher to use different tests at the start and end of the year. The norming study used to standardise the test items of the Pat Maths Plus assessments included 12996 pupils from schools across Australia from Year 3 to Year 11. This norming study revealed the reliability coefficient (0.9) of the Pat Maths Plus assessments to be "satisfactory for cognitive measures" (Stephanou & Lindsey, 2011, p. 55). Test items pertained to five mathematics strands (Number, Space, Measurement, Chance and Data and Algebra). The tests have been normed using the data collected for Pat Maths Third Edition (Stephanou, 2006). The tests consisted of a combined total of 330 questions for students in years 1-10 with each level being more challenging than the one before it. I adhered to the user guide when administering the assessment (Stephanou & Lindsey, 2011).

The Pat Maths Plus assessments allowed me to select the difficulty level at which I wanted the students to complete. Because of this variability in assessment levels, I could accurately measure a student's mathematical competence. When a student achieved mastery (85% or better) on a one-year level's assessment, I could work with the students to help them decide if they should to be working on enrichment challenges or accelerated content. It was the case for some students that they would complete a Year-8 level Pat Maths Plus assessment and then complete a Year-9 test because the Year-8 level test had been mastered. Table 4.4 gives a summary of some of the test levels.

Table 4.4

Recommended Year Levels for Pat Maths Plus Tests

Test	Number of questions	Recommended year levels
7	38	5, 6, 7
8	39	6, 7, 8
9	40	7, 8, 9

(Source: Stephanou & Lindsey, 2011, p. 6)

At the start of the school year, the students completed the Pat Maths Plus Tests online in test conditions. Every student sat at their desk with a space between their desk and the persons sitting beside them. The students first completed the 40-minute, Year-8 level test under test conditions within the user guide’s time parameters (Australian Council for Educational Research, 2011). The test was an online computer program which calculated their percentile rankings, means and stanine levels immediately following the completion of each test. The “percentile ratings and stanines provide(d) a picture of how students’ results compare with the results of students in the same year level across Australia” (Stephanou & Lindsey, 2011, p. 3).

The standardised and teacher-made tests were given before and after the study as a way of understanding if students’ achievement levels had improved or otherwise. Similarly, qualitative measures were also used to gain a richer understanding of the students’ experiences in the classroom, as are discussed in the next section.

4.3.4. Qualitative data.

The following sources of qualitative data were collected and analysed: interviews, audio recordings of classroom discussions, classroom observations, video and photograph recordings, along with student work samples, as listed in Table 4.5. Data gathered using these methods were triangulated during the analysis phase of this research, as per the recommendations of Yin (2014). These data were used to address Research Question 2 namely to gain insights into the opinions, attitudes, experiences, and behaviours of the five selected students from their unique perspectives.

Table 4.5

Qualitative Data Informing Research Question 2

Data Type	Date	Data
Semi-structured Interviews	January	Baseline data gaining students' pre-study attitudes.
	March/April	Mid-semester interviews.
	July	Final interviews.
	December	Final interview with one student on the Year-10 program.
Photographs	Whole semester	Data used to corroborate findings discussed in interviews and capture significant events.
Audio recordings	Whole semester.	Semi-structured interviews and in-class recordings.
Student work samples	Whole semester.	Data used to provide examples of student work.

The next section discusses how the use of semi-structured interviews helped me build a richer understanding of these perspectives.

4.3.4.1. Semi-structured interviews.

The use of semi-structured interviews helped me understand the learners' perspectives and opinions about mathematics from their own words (Yin, 2012). The rationale for using interviews was to understand the learning experiences of five selected gifted mathematics students. I could explore "causal inferences and explanations" (Yin, 2009, p. 102), along with "cultural meanings" (Magnusson & Marecek, 2015, p. 6) for observed attitudes and beliefs gifted students held towards their mathematical studies. I could then assess the influence the Mastery Learning Model may have had on these attitudes (in their own words).

As the interviewer, I had conversations where I "gently guide(d) a conversational partner in an extended discussion" (Rubin & Rubin, 2005, p. 4). To achieve the goal of extended discussions, I used semi-structured interviews which would allow impromptu questions, prompting for further information and deeper probing into answers, where appropriate. Following the recommendations of Rowley (2012) and Warren (2002), the interviews started with around six to twelve questions to begin the conversation (Table 4.7). They suggested the questions be asked in a flexible, attentive way so as not to ask questions they may have answered previously.

It is then possible to "generalise to broader process(es), to discover causes, and to explain or understand a phenomenon" (Rubin & Rubin, p. 7).

The original research questions listed in Table 4.6 provided me with a starting point to ensure there was a careful balance of questions pertaining to Research Question 2. I changed the direction of the interview according to the answers to the responses given by the students. This practice was in line with the recommendations of Gray (2014) and often meant I had to ask the participants to elaborate on their answers. If a response provided evidence that led to confirming the predictions made or falsifying them, then I asked the respondent to confirm or elaborate on their answer. The introductory questions were not included, as they were general conversation starters to help make the students feel relaxed and used to answering questions.

Table 4.6

Initial Interview Questions

Interview Question	Research Question #
How do you feel about mathematics?	2
How do you think you are going in mathematics?	1
What do you like about how we are learning mathematics at the moment?	1 and 2
What do you do with detailed feedback given to you after you have handed mathematics work in to be marked by the teacher?	1
Is this feedback helpful or otherwise? How?	1
How might this feedback be more helpful?	1 and 2
What aspects of this current mathematics program do you like and why?	1 and 2
What aspects of the current mathematics program do you not like and why?	1 and 2
What do you do when you come across a mathematics problem that is difficult?	1 and 2
Do you feel like this current program is challenging? If so/not how?	1
Have your attitudes towards mathematics changed in any way this year? If so, how?	1
If not, what aspects of the program would you change?	1
Have the quizzes helped you in any way with your learning in maths? If so, how?	1

According to Rowley (2012), the transcription of the data should happen as soon as is possible after the interviews. As per the recommendations of Gray (2014), the transcription process was completed as soon as possible after the interview, which helped to formulate new questions for future interviews (Gray, 2014). Once I had completed the interviews and transcription process, I could analyse the transcriptions. Photo evidence was also used to provide further evidence, is discussed next.

4.3.4.2. Photographic data

Photographs were used to corroborate findings discussed in interviews, provide further insights into students' learning experiences and capture significant events pertaining to students' interest and motivation to learn. Gersten et al. (2005), Gillham (2000), MacQuarrie (2010) and Yin (2012) suggested the many advantages of using video and photos to record participant interactions. Gillham advised that it enables the researcher to play and replay the events, to analyse individual interactions that take place in given events. A point agreed on when MacQuarrie highlighted how the use of video "adds to the depth of understanding" of what is happening (p. 2). Gillham further suggested that the video enables the researcher to watch the video recording from different places in the room, encouraging the researcher to analyse footage.

The five selected students were monitored to analyse students' interactions and what effect these had on learning. I was able to watch and often re-watch the video footage to gain further insights into significant events, such as when they solved complex problems as a group. Analysis of the sound recordings and screenshots enabled me to modify teaching methods to maximise the potential effectiveness of this study.

4.3.4.3. Audio recordings of classroom transactions.

Some (Gillham, 2000; Yin, 2014) recommended the use of audio recordings in case study research. They suggest that the researcher should consider the context of the study before using audio recording devices.

At the start of each mathematics lesson, two digital audio recorders were positioned around the classroom to record conversations that took place. These recordings helped me better grasp the students' interpretations and understanding of a lesson or key event. At first, the sound recordings impeded the students from speaking naturally. Humphrey and Lee (2004), share how the researcher should have an alternate strategy for tape-shy participants, adding that at all times the researcher should make every effort to accommodate the wishes of the participants. As time went on, however, the recording devices did not interrupt the normal flow of conversation within the classroom, which is in line with the findings of Billham (2005) suggesting this to be common.

Alternative recordings, such as diary entries, provided me with an alternative data source if something went wrong with the audio recorders. I was familiar with how to use the device well and ensured the batteries were never flat and were readily available in the classroom.

4.3.4.4. Summary.

The use of interview and direct observation in this study enabled me, the researcher, to investigate the student's interactions with each other and the environment to a greater depth. Yin (2012, 2014) shared how the case study can employ many such methods in one study. The above section has provided a caution that such research methods should focus on illuminating and providing a greater depth of understanding of the research questions. The qualitative research methods chosen enabled me access to further evidence to support claims made in Chapter 7 and Chapter 8. They also provided evidence of engagement and interest. In the next section, I discuss how these data were analysed.

4.3.5. Data analysis.

The quantitative and qualitative data were analysed soon after data collection. This process consisted of "examining, categorising, tabulating, testing, or otherwise recombining evidence" (Yin, 2014, p. 190). According to Egbert and Sandon (2014), analysis of data applying a post-positivist epistemology, cannot be completely devoid of the "researchers' absolute objectivity" (p. 21).

Section 4.3.5.1 describes the process used to analyse the quantitative data. A summary of the stages adopted from the adapted recommendations of Braun and Clarke (2006) to analyse the qualitative data are described in section 4.3.5.2.

4.3.5.1. Analysis of quantitative data.

Analysis of student results took place during and after the data collection phase. I elaborate here on the univariate analysis used to describe and "draw conclusions from the numerical data" (Riazi, 2016, p. 89). These procedures enabled the careful calculation of measures of centrality and spread of data. The boundaries of the study dictated that I would focus on the results of the five selected students, while data were drawn from the remainder of the class to provide a contextualised comparison of these results.

Students' results from online formative quizzes and Pat Maths Plus assessments (Australian Council for Educational Research, 2011) were instantaneously available to export to Microsoft Excel from the Online Learning Management System and the Pat Maths Plus website[1]. Results from paper-based formative and summative quizzes along with enrichment tasks were marked by myself and then entered manually into Microsoft Excel spreadsheets. Each smaller sized unit (Example: area of circle unit) would have its own column in the spreadsheet, while students' names would be on the side in rows. Year-8 assessment results from both teacher-made and the Pat Maths Plus assessments would be grouped on one spreadsheet, while data for Year-9 and Year-10 assessment results were kept on separate spreadsheets coupled with related results from enrichment tasks.

After the data were ordered into appropriate columns and rows, I would double check for accuracy and begin the descriptive analysis process which comprised of the following steps: (a) collating the data; (b) inserting the data into spreadsheets; (c) calculating means, standard deviations and ranges of the data; (d) checking the data; and (e) analysing these data to look for patterns.

If a student had not completed an assessment due to illness or absence, they would complete an alternate equivalent assessment, or do the formative quiz at an alternate time. If it was not practical for them to do the formative quiz due to time restraints, I would leave that spreadsheet data cell blank as per the recommendations of McBurney and White (2004) and rely on their summative results in that area for analysis. If the student did not complete an assessment because they had already mastered the pre-assessment for that particular unit, their initial pre-assessment result would be counted as the score on that particular unit of instruction. For example, if one student mastered the pre-assessment on the area of circle unit, they would receive that mark for that unit.

I then analysed for gaps or notable differences in results. If there was a significant difference noted in the results between standardised and teacher-made assessments, I investigated possible causes for the difference. For example, students

[1] https://oars.acer.edu.au/

may have got nearly every question correct on the Pat Maths Plus assessment but had more gaps in knowledge evident in the teacher made assessments. This gap suggested that the student/s did not understand a specific concept which was not tested on the more general standardised Pat Maths Plus assessment.

The standard deviation of the scores for the whole class and the five selected participants was then calculated to ascertain how spread apart the students' results were. The larger the spread indicated a greater variance in results.

Next, "the main features or properties of a distribution of scores" were summarised with graphical representations and annotations (Balnaves & Caputi, 2011, p. 26) using scatter diagrams and frequency histograms. These main features would consist of their overall percentage scores relevant to that year level of work in which each student was engaged. The baseline data included pre-assessment data collected at the start of each unit, while changes in achievement were measured by recording the students' results from either the end of unit formative or summative assessments.

I used scatter plot graphs to compare data from formative, summative and standardised tests to ascertain if the students' results were correlated. Throughout this stage of the research cycle, I aimed to "produce high-quality analyses, which require(d) attending to all the evidence collected" (Yin, 2014, p. 191). In the case of quantitative data, I checked and cross-checked data to ensure the accuracy of results, which included checking my calculations and also checking to make sure all values were inserted correctly and in the right places. I also checked if the answers on the online quizzes were correct.

Given that the sample size is small, the data were used to show growth in learning in each of the five participants. Mills, Durepos and Wiebe (2010) suggest that the use of such data in case studies is common when the researcher is seeking to explain the phenomena in context. An array of data were therefore generated on the five participants over the duration of the study, which presented evidence of students incrementally mastering concepts from higher year levels and of greater complexity. This data with the qualitative data provided evidence of student learning and development. Next I describe the processes I used to analyse the qualitative data, using an adapted framework based on the recommendations of Braun and Clarke (2006).

4.3.5.2. Analysis of qualitative data.

This section outlines the thematic analysis that was conducted with the qualitative data. I drew from the recommendations of Braun and Clark (2006), who recommended six phases to be used when conducting the thematic analysis. Table 4.7 provides a summary of these phases which show a progression of analysis from transcription through to the production of the report. The report aimed to use both interview data, video and audio recordings to tell a story. This story utilised these themes to discuss the influence the use of the Mastery Learning Model had on students' learning, attitudes and interest levels in the mathematics classroom. It involved "movement…back and forth as needed" (p. 86) between phases. Data were triangulated with support from quantitative results.

Table 4.7

Phases of Thematic Analysis

Phase	Summary of process
1. Transcription	Every interview, along with relevant in-class video and audio recordings was transcribed.
2. Initial descriptive coding	Initial coding involved annotating transcripts to summarise main points in participant responses.
3. Theme development	I grouped the themes that emerged from the interview data.
4. Reviewed themes	Discarded irrelevant themes.
5. Finalised themes	Research uncovered some unexpected findings.
6. Produced the report	The analysis was written up.

The research phases that occurred in the analysis of the qualitative data are elaborated on in this section.

In Phase 1. I would listen to the audio recordings and make notes to take into the next interview with each participant. This process would give me a general understanding of what was said. All transcription analyses from the semi-structured interviews, in-class video and audio recordings were carried out throughout the study, as close as was possible to when the event occurred. After transcription was completed, I filed the audio and video recordings into a folder in a secure location on my computer. The transcripts were prepared with Microsoft Word.

The analysis of photo footage was also conducted in the first phase. I watched the videos of the lessons and made diary notes and transcriptions to find any further evidence of any possible themes, especially from critical moments. The two-fold purpose of the photos was to add meaning to interview answers, but also capture evidence of engagement in lessons. For example, I may have used photos of the students making a discovery, concentrating, or just simply working in groups to solve enrichment challenges. If a photo potentially contained evidence of student learning or engagement, I would ask the student about the event, to ensure I attached their meanings and understandings of that event.

Phase 2. I typed notes beside the respective paragraphs, related to that paragraph. I also began highlighting possible themes as I was transcribing the recorded data.

Phase 3. I refined the data by eliminating less significant points and take notes on evidence that I felt might develop into more pertinent themes. For example, I would highlight passages where students spoke about repetition, choice and variety of learning experiences which emerged as dominant ideas during this phase.

Phase 4. The use of NVivo (2014) provided me with a tool to help with the data analysis. This program would help me identify the frequency of use of some keywords related to the research questions. If I could see that a particular word or phrase had been used by a majority of the students, I would reference it for review in this, the fourth phase of analysis. Some codes did not belong anywhere (example: parent influences) or were not pertinent across students, and the themes were not prevalent themes, so were therefore removed.

In the fifth phase, I finalised the most relevant themes relating to the research questions. An example of this process is when students spoke of how they appreciated not having to re-learn information. All responses were grouped under the theme competence. A similar process was conducted for the other themes that emerged from their interview responses and in-class recordings.

Once the final themes had been established, I was able to report on the findings from the entire study. These reports consisted mainly of drafted paragraphs under the headings of each point on sub-themes. For example, a sub-theme under autonomy may have been control or choice. Baseline data taken from the start of the study would provide background information on each participant, while also providing a baseline to track changes in attitudes or interest levels in the mathematics program.

The drafting process continued, as I searched the raw interview data for further contextualised evidence to support the themes that had been established.

The entire qualitative data analysis process saw me interacting with the data, looking for and confirming the themes that had emerged from the analysis of the raw data. Some new and unexpected findings emerged while others emerged that matched findings in previous research. A combination of an understanding of the relevant research I had conducted, coupled with the careful analysis of all the entire qualitative data set enabled me to draw conclusions as stated in Section 8.2.

4.3.6. Summary

In this section, I have provided information on how I collected, analysed and reported on qualitative and quantitative data. This analysis was done through the post-positivist theoretical lens. Data sources and collection methods were discussed, and reasons were given as to why they were used. The use of quantitative data were enhanced by qualitative findings, as presented, while qualitative data were also supported by the quantitative findings as are elaborated on in Section 8.2.

The use of multiple data sources has also been elaborated upon which enabled me to triangulate results where appropriate. Careful and rigorous protocols were adhered to in this data collection phase to ensure the quality of these results was obtained without interference or bias, as is discussed in this next section.

4.4 Quality

This research sought to embrace “multiple standards of quality” (Morrow, 2005, p. 250) known as reliability, and internal, external and construct validity. The credibility of interpretations was obtained through “prolonged engagement with the participants; persistent observation in the field; the use of peer debriefings or peer researchers” obtaining a “thick description” (p. 252) of the participants who are profiled in Chapter 6. This was obtained by being an insider researcher which in turn, carried with it the potential for bias.

Greene (2014) suggests there are both advantages and disadvantages to such insider research. Like Morrow, Greene asserts that an insider can have a more contextualised understanding of the group being studied, to having the ability to ask more meaningful questions. Greene explains that the researcher would not have to orientate themselves to the environment. He also suggests the researcher tries to adopt both an insider and outsider perspective by being immersed in the environment they are studying, but also being critically aware of potential bias and open to intensive scrutiny. To minimise bias, I adhered to Greene's processes with the use of triangulation of data, self-analysis through critically reflective diary entries, having a clear paper trail of all records and observations, along with peer debriefing such as would happen with critical discussions with my research supervisors. I took other measures to avoid bias. For example, the summative assessments were developed collaboratively by three teachers from the school. The final assessment was then checked by the mathematics head of department, while the questions on the formative quizzes were similar to those the students would encounter on the end of term exam. They were also checked by another Mathematics teacher. This process ensured a high standard of quality of both formative and summative assessments which were aligned to Australian curriculum standards and reduce the possibility of potential bias.

Gibbert, Ruigrok and Wicki (2008) revealed that "the case study method has been prone to concerns…in terms of validity and reliability" (p. 1465). These authors suggested that there should be a "clear chain of evidence" (p. 1468) that reveals how the study measured what it purported to measure and that these data were not contaminated with bias in subjective misinterpretation. While I was an active participant in the research, I also needed to remain objective in my analysis and conclusions. What follows in this section is a discussion on the steps that I took to ensure the quality of this data remained as objective as was possible.

4.4.1. Internal and construct validity.

The rigour of this research was assessed using three measures pertaining to internal validity, which is in line with recommendations by Gibbert, Ruigrok, and Wicki, (2008). Internal validity is defined as ensuring "certain conditions are believed to lead to other conditions, as distinguished from spurious relationships" (Yin, 2014, p. 85).

First, a clear framework (Figure 4.1) was established to ensure that the use of the Mastery Learning Model led to the outcomes as elaborated on in Chapter 7 and 8. A combination of teacher diary entries, unit plans along with video and audio recordings all acted to affirm the faithfulness of the implementation as described. An external auditor would be able to see and compare these pieces of evidence, confirming the research framework was adhered to faithfully.

Secondly, patterns in achievement were correlated with patterns found in previous research, while patterns in qualitative findings would be compared between students. An example might include how two or more students responded to a certain stage in the study in similar ways.

Thirdly, triangulation of the data was possible using multiple sources of data, as elaborated on in Section 4.3. Yin (2009) asserted this assessment is necessary for explanatory case studies, however, he suggested that explanation building is also important. He proposed that for multiple cases, the researcher should seek common explanations across the cases. In the case of this research, this pertained to comparing improvements in achievement along with noticing common themes observed among the five selected students.

In his 2012 work, Yin further discussed explanatory designs that can be flawed if the researcher asserts his/her own beliefs or presents flawed design or data collection procedures. In this sense, this research sought to present findings from the students' perspectives, in a manner that was in line with the research design (Figure 4.1) and identify links between the data, the hypotheses and the research questions. This process provided a chain of evidence, which should allow the reader to follow this research and get a clear sense of what had been done.

4.4.2. External validity.

Rigour of this research was also assessed by its generalizability of findings. While generalising was difficult, I drew from the recommendations of Flyvbjerg (2006) and Yin (2014) who suggested that deeper analysis and seeking out alternate explanations can aid in asserting a greater level of external validity to such findings.

Giddens (1984) criticised case study research for its lack of generalizability. However, Flyvbjerg (2006) argued that much can be learned from even single cases, drawing on single historical cases such as Aristotle's gravity experiment and other history changing discoveries by Darwin, Marx, and Freud. According to Flyvbjerg, the use of case studies is ideal for falsification. Giddens' argument ensued stating that often by trying to prove that all swans are white, that you will find black swans. The point is that the use of case studies could help you find truths that do not appear until a deeper analysis of the case/s takes place. Yin (2014) suggested that this generalizability test has historically presented a “major barrier” to doing case studies (p. 43). Yin proposes that case studies have analytical generalisability, that is the cases can inform theory. Similarly, McLeod (2010) has argued that while statistical generalisation is difficult, the generalisation of the processes and programs can be transferred from a single case study to other similar cases.

To enable this deeper analysis and to strengthen the explanations and results, a wide range of questions were asked in the interviews. Yin (2012) asserted the need for the researcher to ascertain any “substantive rivals” (p. 149), or alternate explanations. The triangulation of data sources, along with comprehensive testing and data collection, has helped minimise the impact of rival explanations for outcomes of this study. I probed further into the students’ answers when discoveries were noted that might have suggested such rival explanations. Furthermore, I asked similar questions to the other four of the five selected students, to ascertain if there was wider support, or otherwise, on the impact, this may have had on their results. Testing was also rigorous, using a combination of standardised and Australian Curriculum aligned non-standardised tests to ensure quantitative data presented a consistent picture of the results.

Towards this point, Yin (2012) suggested that “a case study can reach an acceptable degree of certainty about its conclusions, though not as airtight as in an experiment”. He argued the necessity for the case study, as much as is possible, should be able to “rule out virtually all rival hypotheses” (p. 148). Therefore, in the case of my research, statistical results were compared across students. Similarly, students’ responses to the same questions were compared to check for generalizability across each student’s responses, and rival explanations were ruled out. The research uncovered many metaphorical “black swans” as is discussed in Chapters’ Six and Seven.

4.4.3. Reliability.

The final test for the research design is to see if the findings can be replicated by a later researcher following the same or similar procedures to the original study. Riege, (2003) stipulated a set of principles that can enhance the case study’s reliability. Many of which have been adopted in this study, such as the recording of classroom interactions and audio recording interviews and transcribing verbatim what was said, using multiple data sources and recording of observations in a research diary. Some were implausible, such as the "use of multiple researchers" (p. 83). Video and audio recordings were also used to give the full account of theories and ideas with the recording of observations. These data helped me to "record observations and actions as concrete(ly) as possible" (p. 83) as I did with research diary entries. There was the provision of "meaningful parallelism of findings across multiple data sources" (p. 83). This evidence would come in the form of students' results as given in Chapter 7.1, along with transcriptions of audio recordings, research diary entries and unit and lesson plans. By adhering to such general principles, generalisation is possible by utilising the same or similar methods and procedures if the findings are similar. In every stage of the data collection and subsequent analysis, strict adherence to ethical protocols was adhered to, as is discussed next.

4.5 Ethics

This section is framed by the National Health and Medical Research Council's principles and values for ethical conduct. It has been suggested by Wagaman and Balog (2011) that when "people are mindful of their moral standards, they are more likely, to be honest" (p. 10). The guidelines as stated in the National Statement on Ethical Conduct in Human Research (NHMRC reference number: 1300000577) suggest that research conducted on young children should be to the students' best interests and that the child and their parents should give their consent to enable the researcher to research their child. The four principles to be discussed next ensure ethical principles were adhered to regarding the students' and research's beneficence, mutual respect, justice and research merit.

4.5.1. Beneficence.

The National Health and Medical Research Council speak of "beneficence" in research where the benefit of the research must justify any risks of harm or discomfort to participants" (p. 10). The maximum possible risk was that of inconvenience (participating in informal discussions and interviews) and discomfort (test-taking and student learning being recorded) and coercion (any potential teacher-student influence). Mastery Learning is a research-tested model of instruction. Modifications to the original mastery model were made for gifted students, which is in line with best practice for this population by providing the students with access to a faster paced, more challenging and complex curriculum (Rogers, 2007; Yuen et al., 2018). The experiences of students did not proceed beyond the normal experience of students in a mathematics class, and therefore any such risks, inconvenience or coercion was deemed minimal. The learning of the students was not compromised, as the students were required to meet school and curricular requirements, as per the normal expectations of the curriculum, the school and education governing bodies. Students that did not participate in this study were aware of the research taking place, and their education was not impacted in any significant way. I acted as their teacher, and results would reflect that the entire class benefited from the use of this study as is expanded on in Section 8.1.

I discussed the project (both benefits and inconveniences they would experience) with the students, and they were encouraged to talk about this with their parents. Students consent was gained before the study commenced or any data were formally collected. All parents were provided with information about the project and requested to provide consent by signing the experiment consent form. Students could have opted out of being included in the research in either one of two ways: (a) They could have simply asked to opt out; or (b) their parents could have requested that their child not be included in the research. Participants and their parents had access to their academic results. No plans were made for any involvement from outside participants or volunteers.

4.5.2. Respect.

To ensure respect, parents and students were fully informed of the purpose of the study and provided consent before commencing the research. Respect was given to each of the students by ensuring all recorded data were stored confidentially, timelines adhered to and events were carefully recorded as they happened. Students' formative and summative assessments were marked by trained teachers, and moderation of summative assessment tasks was completed, which ensured students received a fair grade when compared with their similar ability peers. If students completed a Year-9 level assessment, they would be marked according to the school's and Australian Curriculum's grading guidelines (Australian Curriculum Assessment and Reporting Authority, n.d.). Full transcripts of the interviews were given to the participants. They were welcome to change any words that had been transcribed which were different from their original sentiment. At all times, student data were kept confidential and stored in a secure online password protected environment.

During the data collection phase, students were treated respectfully through acceptable classroom and research conventions. That is, all participants were taught by an accredited teacher with formal and appropriate qualifications in teaching mathematics. In interviews, students could leave at any time and were listened to while speaking.

At the commencement of the study and research, the students agreed to participate in three interviews lasting no longer than one hour in duration each. Oliver and his mother did agree to a further interview at the end of the year so that I could gain insights into his thoughts about the Year 10 Mathematics C course that he completed in the second half of the year. This extra interview was within approved dates stipulated by the university ethics committee.

Both the parents and students gave their full consent to have their photos published and in-class conversations audio recorded as part of this research. It is quite commonplace at our school for students to be photographed, participate in in-house research, and be in videos. Parents and those in a position of leadership at the college provided their full consent to this study and were eager to have their children participate.

4.5.3. Justice.

Selection, teaching and research procedures were fair. Students were selected according to the criteria as specified on the basis of standardised assessment data, parent and teacher nominations. This selection process was fair, as all students were considered for this study based on their individual merit. No particular cultural group was favoured, and no rewards or incentives were given to any child who participated or otherwise in this study. The same guiding questions and time allocation were used for each interview. Each child was treated fairly within standard teaching procedures that might normally be seen in a Queensland classroom.

4.5.4. Research merit.

In alignment with university requirements, the proposed research was approved by a panel of appropriately qualified university academics as having met expectations of a doctoral study.

4.6 Summary

This chapter began by outlining the questions and predictions that were made. I then discussed the philosophical foundations of this research. The post-positivist ontological and epistemological foundations would provide the reader with a lens through which truth could be examined. The explanatory case study methodology was then introduced, and justification was given on how this methodology would best provide suitable evidence about the research questions given. This methodology allowed me to be an active participant in the research process in a way that would enable me to shape the direction of the research, where appropriate.

The study was conducted within the framework suggested by Yin (2014) which proposed a series of stages for such case study research. Evidence of the research phases can be seen in the research design given in Figure 4.1. The choice of the research methods enabled me to explore the effects of causes and causes of effects. The ontological and epistemological foundations, the research questions and the research design, helped shape the direction of the entire research process. A discussion then ensued by outlining how both the qualitative and quantitative data were collected and then analysed. The goal of these results which were analysed ensured that they measured what they pertained to measure, and the study methods chosen would be the best fit for finding the answer to the research questions as given. In the next Chapter, I provide a discussion that gives you, the reader, a richer understanding of the Mastery Learning Model along with the phases that were followed by the students in the learning process.

Chapter 5
The Mastery Learning Model with Adaptations for Gifted Students

This chapter describes the teaching and learning program along with adaptations made for gifted students. The six phases of the Mastery Learning Model are discussed.

The premise behind the Mastery Learning Model that differs from other models is that students in the classroom needed to achieve mastery standards (at least 85% on formative assessments) on smaller sized topic-based units of instruction before they progressed to the next unit of work or enrichment challenge. Other models advocated in gifted education such as the Schoolwide Enrichment Model (Renzulli & Reis, 1985) and Maker's (1982) model for providing for gifted and talented students incorporate various elements of the Mastery Learning model (for example: compacting, challenging enrichment tasks and acceleration). These methods do not emphasise the need for students to demonstrate mastery before they progress to the next task. The implementation of the Mastery Learning Model is presented as a series of phases as outlined in Figure 5.1, and the detailed content for an entire unit outline is presented in Appendix I. A sample online sub-unit outline is also given in Appendix J.

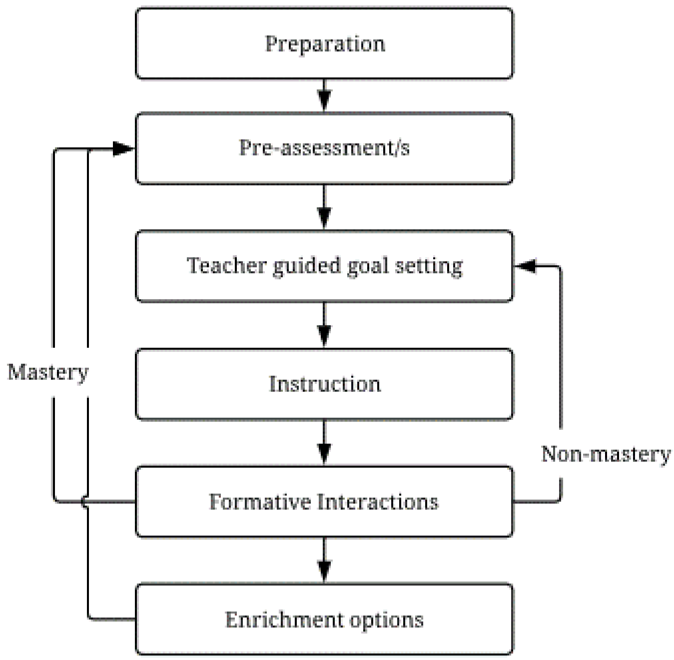


Figure 5.1. Mastery Learning Model

According to the Australian Curriculum Assessment and Reporting Authority (2016), gifted and talented students are “entitled to rigorous, relevant and engaging learning opportunities drawn from the Australian Curriculum and aligned with their individual learning needs, strengths, interests and goals” (para. 1). The program was broken into seven units taught between February and July 2014. Each unit focussed on a topic drawn from the Australian Curriculum (Australian Curriculum, Assessment and Reporting Authority, N.D.)

5.1.1. Instructional design.

The teaching program was designed according to the principles of Mastery Learning and practices appropriate for gifted education. The alignment between these frameworks and the program is outlined in Table 5.1. All teaching and learning

Table 5.1
Instructional principles adopted

Principles	Program	Reference
Social constructivism	Meaningful learning tasks Student collaboration. Student-initiated dialogue	Bandura, 1986; Vygotsky, 1978; Wood et al., 1976
Assessment	Pre-assessment (Diagnostic) Formative interactions Summative assessment	Guskey, 2010; Johnson, 2000; Lidz & Elliott, 2006; Lo and Porath, 2017; Reis, Westberg, Kulikowich, & Purcell, 1998; Sia & Lim, 2018; VanTassel-Baska & Stambaugh, 2005
Mastery	Students achieved at least 85% on core curriculum tasks. Sub-units sequentially structured	Bloom, 1968, 1971; Guskey, 2010
Self-directed learning	Student-led teacher guided scaffolded inquiry.	Betts & Kercher, 2004; Black and Wiliam, 2009; Wood et al., 1976
Advanced cognitive abilities	Opportunities for enrichment Compacting Acceleration	Bloom, 1971; Guskey, 2007; Neubauer & Fink, 2009; Rogers, 2015

resources were placed online by the teacher prior to the commencement of each teaching term. The teacher's role in each phase of instruction is introduced next and elaborated on in each related phase.

5.1.2. The teacher's role in the classroom.

Figure 5.2 reveals the teacher's role in the classroom guided by both gifted education principles and the principles of the Mastery Learning Model. It places the teacher's role in each phase as discussed in the outline of each phase below.

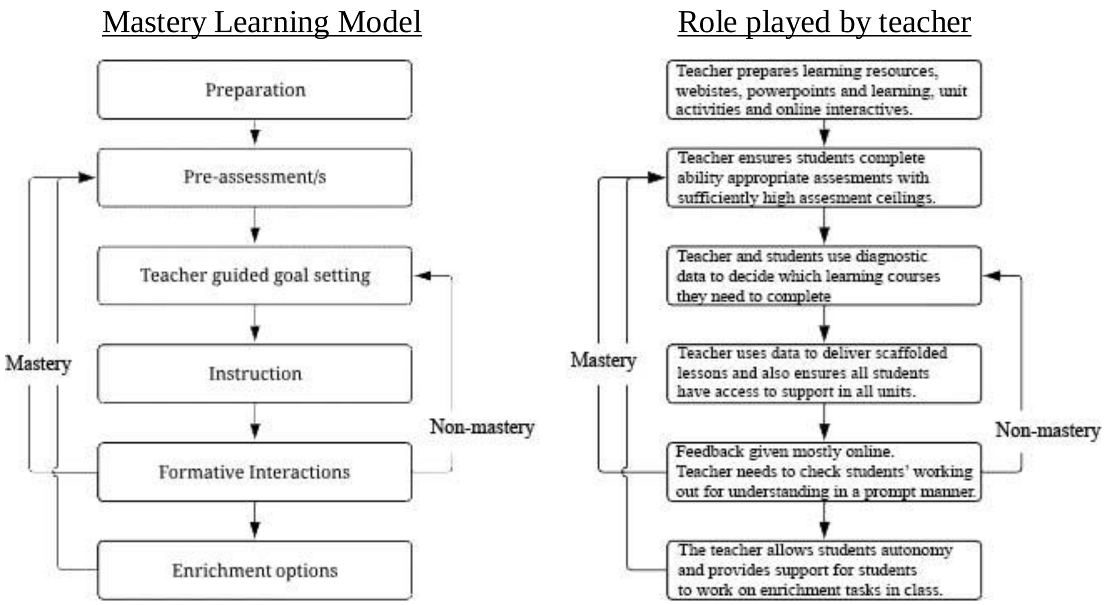


Figure 5.2 Teacher's role in the classroom.

5.1.3. Phase 1: Preparation.

Prior to the academic year commencing, I placed a series of learning units, which included pre-assessments, password protected learning activities, links to online interactives, videos, PowerPoints with scaffolded worked examples and enrichment challenges onto the Online Learning Management System. I worked collaboratively with other teachers, including the Year-9 mathematics teacher and head of mathematics department, to ensure that I capitalised on existing resources and that my assumptions and practices could be critiqued or validated. Restrictions and password protections were placed on units and investigations, particularly on units relating to higher year level concepts. Students would see only the names of the sub-units and not the contents within the units. They would receive the password to enrichment or accelerated options upon the completion of the pre-requisite course materials or assessments. An example of a unit might be the area of two-dimensional shapes. These units would then be subdivided into a series of sub-units; for example, area of the circle or areas of a sector of a circle.

A range of enrichment options were placed onto the Online Learning Management System with each unit as options for all students to complete once mastery was achieved. A lesson at the start of the year was also devoted to showing students how they could to create their own mathematical investigations. This process would see them finding a problem and devising a mathematical solution of their own.

5.1.4. Phase 2: Pre-assessment phase.

Section 3.3.1 shared research (VanTassel-Baska & Stambaugh, 2005) discussing the necessity of the effective utilisation of pre-assessments to guide instruction. In this regard, each sub-unit was designed to begin with a pre-assessment which would reveal to the teacher and the students, what elements of the upcoming unit they understood and what concepts they still needed to learn. In this phase, I needed to ensure that the questions in each pre-assessment task ranged in complexity from simple to more complex problems relevant to each intended learning outcomes.

Research cited in Section 3.3.1 revealed how pre-assessments, which included diagnostic standardised and teacher-made assessments, guided instruction. Each pre-assessment included a range of questions of different levels of complexity from across the unit. The results enabled the students (and teacher) to have confidence in their understanding or lack therein of the content and skills that were covered in the unit they were about to complete. The pre-assessment formed part of the regular instruction.

5.1.5. Phase 3: Goal setting.

Goal setting was completed informally and mostly by the students themselves with the guidance of the teacher. This guidance included me checking to ensure that each student had access to online courses responding to their pre-assessment results. Students were required to complete a minimum number of learning. The dialogical feedback comprised a conversation on the pre-assessment results which suggested a strategy for the students to complete a series of questions. Students could choose how they would learn best to understand a given concept, as is discussed next.

5.1.6. Phase 4: Instruction.

The pace, style and format of instruction varied for all students. The delivery of instruction occurred within the Gifted Instruction Model's theoretical framework which placed the gifted learner at the centre of all learning. Students were made aware of the importance of how to use results from the pre-assessments. If they did not demonstrate or feel confident in their understanding of the content contained in these pre-assessments, then they would have had the choice to either work in a group with the teacher or work independently. Students could use provided PowerPoints, textbook-based activities, learning games, interactive learning websites such as Mathletics.com.au, geogebra.com, Khan academy units or watch related video tutorials embedded in the learning management system. They had the choice to learn the concepts in a way that suited their learning preferences.

The delivery of instruction was guided by the principles of constructivism which holds that learning started with what the students already knew, and they built on this knowledge with teacher support. They mostly learnt on their own or by collaborating with their peers (Eberlein et al., 2008; Felix, 2005). If a group of students wanted help to understand a specific concept, they would be able to come and work with the teacher. During this group instruction, the teacher would engage students in scaffolded constructive dialogue. Learning would progress from the teacher initially explaining the overall concept, to students completing questions varying in complexity on their own, asking me questions as they went along.

These sessions were fluid in terms of the way students engaged with the learning activities, me as their teacher and peers. Some students, for example, would participate in the entire session, while others may only stay for the first or last half, depending on their level of understanding of the concept. These groups would typically contain at least three students, while sometimes there would be as many as 15 students in the group.

At the end of these sessions, students would then go back and work collaboratively with their peers or on their own on tasks on the Online Learning Management System. Students learning in pairs would engage in discussions, debates and complete a variety of games that helped them understand each topic. The students might also choose to use online learning materials and learn together, helping each other understand the required content. Once students felt as though they understood the key concepts, they would then move on to complete the formative quiz as is discussed next.

5.1.7. Phase 4: Formative interactives.

The principles discussed in Section 3.3.1 on formative interactives were actioned with students learning in the following ways. If students had demonstrated that they had completed all the set/required tasks, they then went on to finish the formative assessment as provided on the school's Online Learning Management System. Students were required to complete the formative assessments at school and were seated at individual desks at the time of completion of the test. Students who demonstrated 85% or higher level of mastery of the pre-assessments were not required to complete the end of the unit formative assessment. During lessons and after class I would often moderate these results by checking to make sure the students had provided sufficient reasoning to explain their understanding of each concept. This practice was important, as students may have got every question correct on an online quiz, but not shown any working out (reasoning). They would then be given some further questions, which were more complex to solve, and would be required to include their reasoning in their answers to these questions. Feedback would be attached to goal setting, as the students would be required to respond to feedback by completing extra questions or moving on to enrichment activities or another unit of instruction. According to Guskey (2007), a solid understanding of the baseline content better prepares students for work on "more advanced units" (p. 23). These more advanced options are discussed next.

5.1.8. Phase 5: Options.

Subject to performance on the formative assessment, students were provided with two options. They would work on consolidation activities or continue to work on a range of enrichment challenges.

5.1.8.1. Non-mastery results.

Failing to achieve an 85% grade on a formative quiz (Phase 5) meant that the student needed to go back and complete similar activities/tasks. This requirement could include the completion of remaining textbook-based activities contained on the online learning management system. The teacher could re-teach the concept to the student in a different way, which might include using a hands-on, kinesthetic approach, providing a different real-life illustration or getting the students to engage in a different activity on a different website. Learning was scaffolded which included the withdrawal of support as the learning progressed. It may have also been the case that there was only one component of the unit that required further consolidation. It was essential that the student/s gained an understanding of what they needed to learn to achieve mastery levels. The students would then work on enrichment or related tasks usually reserved for students from higher year levels.

5.1.8.2. Mastery results.

If time permitted and the student had demonstrated a level of mastery of the regular content, they would then work on an enrichment investigation of theirs or the teachers' choosing.

Every student was required to complete at least one enrichment task per school term. There was a choice of other tasks, which students could complete to help them receive extra credit towards their end of semester report card grade. Students had an assortment of enrichment projects (Examples given in Appendices J, K, P, Q and R) that they can work on in the classroom.

Students may have completed enrichment tasks as a way of introducing a unit of work to pique interest in the particular content area about to be covered. Alternately, the task/s may have been a way to provide students with the chance to gain a deeper level of understanding of the compacted curriculum covered. Examples of this may have included an introduction to the concept of theoretical probability, by playing a game involving a single normal dice. After the completion of a unit of work, they may have completed an enrichment task that looks at predicting weather patterns in their local area based on a study of current and past meteorological events. They could apply the knowledge learned in the probability and statistics unit to a real-world challenge and had the choice of presenting their findings in a format of their choosing. Table 5.2 presents an elaboration of phases the students may have progressed through in such enrichment tasks.

Table 5.2

Phases for Enrichment Challenges

	High levels of support	Low levels of support
Phase 1 Introductory Phase.	Students complete the enrichment challenge in pairs and groups, helping each other along the way.	Students complete enrichment challenge individually.
Phase 2 Planning phase.	A more capable other may help the student with planning and implementation of their projects helping them to set appropriate timelines, goals and objectives.	The student sets the timelines and obtains approval from the teacher for these timelines.
Phase 3 Investigation phase.	The teacher may also help by providing the students with books, websites and access to experts for the completion of set milestones.	Students find their own resources and information developing effective information retrieval skills along the way.
Phase 4 Culmination phase	Teachers guide the students and help them collate their data into tables, spreadsheets and graphs.	Students collate their data into spreadsheets and graphs obtaining help from the Internet and other more capable others when/if needed.
Phase 5 Presentation	Students present their understanding of their findings to an audience.	Students work with the teacher to select the target audience. The teacher may facilitate a meeting, or a presentation to be given to the selected audience.

The level to which each student operated differed. While one student might have been dependent on on-going support from the teacher, others might have only needed scaffolding in the form of an introductory lesson, a task sheet, and checks along the way to ensure efficient time management procedures have been incorporated. Other students, however, might require the teacher’s help or worked in collaboration with their peers to complete a task they could not have been able to do on their own.

5.1.9. Assessment and Reporting.

This case study was conducted within a school which had firm procedures to adhere to in relation to assessment and reporting. To get an A on the student's report card, the child must have demonstrated mastery on either formative or summative assessments and demonstrated a high level of proficiency on the more complex enrichment tasks. The grading system utilised by the school is given in Table 5.3, which is in accord with the Queensland Studies Authority's (2012) stipulations of a five-point scale (Sample rubric provided in Appendix K).

Table 5.3

Grading System used at School

Percentage	Grade
85-100%	A
70-84%	B
55-69%	C
30-54%	D
0-29%	E

If students did not hand in a piece of assessment by the due date, the student was required to provide the teacher with a letter, signed by their parents explaining why they are submitting it later. Extensions were given when the student provided the school with a reasonable reason (usually needed to be accompanied with a medical certificate from an accredited physician).

5.2 Summary

This chapter has sought to outline what the teaching approach looked like in the classroom. It has presented the importance of both scaffolding and support from more capable others in helping the individual work towards some level of self-efficacy. Table 3.3 in Section 3.4 highlighted a raft of possible factors that may have come to bear on the students achieving the desired goals. Placing the Mastery Learning Model inside the Social Cognitive Theory's framework, suggests such factors including the interplay between self, the environment, which included the teacher, teaching resources and online materials and personal factors exist. This Chapter has outlined a series of phases and how they looked at a classroom level, how they were taught and assessed, and how the use of scaffolding and dialogic interactives helped the students achieve a desired level of self-efficacy.

Chapter 6
Participant Profiles

This chapter profiles the five students who participated in the study. The framework proposed by Betts and Neihart (1986), subsequently revised by Neihart (2012) provides a lens to view these mathematically gifted students' behaviours, feelings and needs. Section 2.3 discussed these profiles as a way of assisting the teacher in identifying gifted students, as per their behaviours and common characteristics. The students that were selected in this study met four out of the six behaviour profiles as presented in Table 6.1.

Table 6.1

Participant Profiles (Neihart, 2012)

Type 1 – Successful – Walter/Ty	Type 2 – Creative - Miley
Well behaved, high achiever, eager for approval, positive self-concept, complacent, does not go beyond the syllabus, struggled with skills needed to learn independently.	Strongly motivated to follow inner convictions. Playful, expresses impulses, androgynous, lower levels of self-control, emotionally labile, low interest in conforming to expectations, high energy levels.
Type 6 - Autonomous Learner – Oliver and Bree	**Type 3 – Underground - Ty**
Goal setter, perseveres, high levels of self-efficacy, thrives on challenge, possesses an incremental view of ability, courageous, self-regulated and works well on own, possesses the ability to explain. Copes well with setbacks, carrying a very non-plus attitude towards learning.	Discounts their abilities, feels pressure to reject achievement behaviours, experiences dissonance about achievement goals; associates certain achievement attitudes as a betrayal of their group; withdraw from or resist talent development opportunities.

The profiles are not intended to describe any one child completely, but rather they help the reader get a better understanding of their behavioural and academic profiles. This understanding of the students extends previous ideas presented in earlier chapters highlighting the complex and multi-faceted aspects of giftedness. Information contributing to profiles discussed in this chapter came from a variety of sources including past teachers, previous year's records, report cards and more formal one-to-one interviews. I looked at these students through three different lenses: (a) a behavioural lens; (b) a social lens; and (c) an academic lens.

The information provided in Table 6.2 highlights the five selected students' results on standardised assessments. These assessments included the Slossan Intelligence Test (Slossan, Nicholson, & Hibpshaman, 2002), the Raven's Progressive Matrices (Raven, 1989) and the Pat Maths Plus Online (levels 8-10) (Australian Council for Educational Research, 2011). These results are not intended to be used separately, but rather to help build onto these profiles and help the reader understand these students by looking through the three lenses (behavioural, social and academic) adopted. Information gathered in this section has been drawn from a combination of sources, including interviews conducted before the outset of the study and test results as shown. I had also taught the students in some pull-out classes when they were in primary school. Therefore, some of the baseline data came from this teaching background with the students.

Table 6.2

Standardised Test Results

Name	Slossan Intelligence Test		Raven's Progressive Matrices	Pat Maths Plus Online Test Level 8	Pat Maths Plus Online Test Level 9	Pat Maths Plus Online Test Level 10
	IQ	%'ile	%'ile	%'ile	%'ile	%'ile
Miley	139	99th	95th+	93rd	84th	Not done
Walter	143	99th	95th+	89th	93rd	Not done
Oliver	160	99th	95th+	96th	99th	89th
Bree	Not done		95th+	81st	64th	Not done
Ty	155	99th	90th	85th	91st	72nd

6.1 Case 1 - Miley

In-school observations, interviews and referrals from other teachers identified Miley (pseudonym) as most fitting the creative profile (Table 6.1). Miley was a 12-year-old girl who came from a Caucasian established middle class (Sheppard & Biddle, 2017) family. Up until the midpoint of Year-7, Miley was home-schooled with her stay-at-home mother, as they had arrived from the USA and were travelling around Australia most of Year 7. For Miley, this meant that mathematics was mainly taught informally by watching mathematics tuition videos such as Khan Academy[2] and being asked informal questions by her mother afterwards. She reflected on her experience neutrally, suggesting "the whole focus was on understanding the concept more, instead of doing exams and stuff" (Miley, Interview, February 25, 2014).

6.1.1. Social and behavioural profile.

Miley appeared to be a creative person who loved to draw and interpret mathematical patterns. She described in her original interview about how she loved drawing patterns, which she later discovered was the Hilbert Curve[3]. My original observations of her suggested she was a curious person who frequently asked questions, was highly energetic and rarely sitting still for too long unless she was trying to solve a complex problem.

When Miley was not exuding an effervescent and confident persona, it may have been because she had just been involved in and felt as though she had done poorly in a test, or she has just woken up. Miley most accurately could be described as someone who was potentially at risk of underachievement. She often expressed extremes of emotion, was highly critical, a little bit of aloof and often responded well when she was in a social environment with peers who challenged her thinking. These friends would be more likely to spend a lunch-time in the library discovery centre rather than playing sport and participating in any physical activity. Socially, Miley did tend to exhaust her friends with her high energy levels, but they have expressed in casual conversations with me how they are used to her effervescent behaviours, and they were accepting of and enjoy her high energy levels.

6.1.2. Academic Performance and Attitudes towards Mathematics

Results from interviews with Miley revealed that she was the kind of student who rated her attitudes towards mathematics as an 8/10. Why? In her words, "(she) would want time to do other things as well, but I think mathematics was a very important subject" (Miley, February 25, 2014).

[2] https://www.khanacademy.org/
[3] The Hilbert curve is a space filling curve that visits every point in a square grid with a size of 2×2, 4×4, 8×8, 16×16, or any other power of 2.

Miley appeared to like being the centre of attention in a group and possessed a rather jovial attitude on most occasions. She always did her homework, and if a topic interested her, she wanted to find out everything there was to know about that particular concept.

6.1.3. Standardised test results.

Results from the Ravens Progressive Matrices conducted at the beginning of the study (February 3, 2014) placed Miley in the top five per cent of students her age. This result was consistent wither her mathematical performance on the PAT-Maths, and previous ability assessments (Slossan Intelligence Test – R3) which placed Miley in the 99th percentile of students her age with an intelligence quotient of 139. Miley's strengths on this assessment suggested that she has a good memory and could draw and analyse similarities and differences well. The PAT Maths Plus Online assessment completed at the start of the study (January 30, 2014), placed her in the 93rd percentile when compared with other Year-8 mathematics students and even achieving as high as the 84th percentile on a Year-9 level test when compared with other Year-9 students.

6.2 Case 2 - Walter

Walter (pseudonym) was a 13-year-old boy who came from a Caucasian established middle class (Sheppard & Biddle, 2017) family. Up until the start of year seven, Walter was one of the highest achieving students in his year level, often receiving the academic dux award for his year level.

6.2.1. Walter's Social and Behavioural Profile.

Walter appeared to be a generally happy and content student who comfortably met Bett's and Neihart's profile of the successful learner, as he was a high achiever, did as he was told while sitting and listening respectfully to the teacher awaiting his/her next instruction (Table 6.1). According to Walter, he did not respond well to his main teacher in year 7. This teacher, who also taught him mathematics, seemed to teach mainly to the textbook (According to Walter) without offering him much choice in what he learned or challenge by way of access to accelerated or enriched coursework. He felt as though he learned very little and his academic achievement scores seemed to have also slid in this previous year, from being a top student at the start of the year to receiving a grade of A-minus at the end of year 7. You would often have often found Walter playing basketball or any other sport at lunchtimes with a large group of boys.

6.2.2. Academic performance and attitudes towards mathematics

As I had taught Walter before in an earlier year level, I was able to identify him as a self-regulated learner. For example, when I taught him in the earlier year, I noticed that he would often come to class and begin work straight away on the learning from the previous lesson, without being reminded to do so. He liked to see the relevance in completing a task before he decided the level of effort he would apply. In the classroom, Walter often went about the business of completing what needed to be done efficiently. In his initial interview, he shared how he preferred to learn with a variety of learning tasks. Walter was a practical, level-headed thinker who appreciated completing challenging work.

6.2.3. Standardised test data.

Walter was identified as a candidate for this class as he scored in the top 5% of students his age in the Ravens Progressive Matrices (3rd February 2014). Before this, another standardised intelligence test (Slossan Intelligence Test – R3) also placed him in the top 1% of students his age with an intelligence quotient of 143. These test results revealed strengths in the areas of social comprehension, comparative and quantitative reasoning. His previous mathematics teachers had highly recommended that I include Walter when they suggested he was a highly capable and gifted mathematics student. Interestingly, Walter performed to the 89th percentile on the Year-8 Pat Maths Plus assessment when compared with other Year-8 students. I then gave him a higher-level test where he was able to score in the 93rd percentile when compared with other Year-9 students. These results suggest that Walter was the kind of child who responded to higher levels of challenge and could be quite whimsical in responding to easier questions. According to Walter (January 30, 2014), he often struggled to get high grades on tests due to making careless errors, such as not reading the question properly or forgetting to include vital information in his response to questions.

6.3 Case 3 - Oliver

Oliver was nominated for this study by several teachers as an exceptional mathematics student and for being a consistently high achiever in class. He was jointly awarded the academic dux of the Year-7 gifted mathematics class in 2013. He turned 12 years of age at the start of this study. He comes from an established middle class (Sheppard & Biddle, 2017) background with parents who are both well-educated. A previous teacher recounted a time when Oliver was asked to write a short story utilising the fantasy genre as the framework for the story. Oliver developed a plot set in a world of peace, seclusion and quiet where he lived on a secluded and remote property.

6.3.1. Oliver's social and behavioural profile.

Oliver appeared to meet a wide range of criteria from Neihart's (2012) behaviour profiles. In the initial interview, Oliver shared how he liked to work independently and loved to set himself challenges that he often completed with his dad at home, or by himself. Initial classroom observations saw that Oliver worked mostly alone. He seemingly enjoyed the initial enrichment investigation where he used string and a pencil to find an approximation of pi (3.1415).

Oliver associated with a group of like-minded boys who loved playing computer games in their spare time and eating lollies and other sweets. Oliver and his friends were often found in the library playing on their electronic devices or talking about their latest escapades on their different gaming machines. Initial classroom observations revealed him to be socially well-adjusted and highly respected by his classmates. In the initial interview, he explained that he disliked group challenges, noisy classrooms and working with others.

6.3.2. Academic performance and attitudes towards mathematics.

Oliver expressed in his original interview his love of mathematics, mainly due to its logical nature. One interview with Oliver revealed that he preferred learning on his own. Previous intelligence test (Slossan Intelligence Test – R3) results placed Oliver's intelligence quotient at 160, putting him in the top 1% when compared with his same-aged peers. On this assessment, Oliver scored almost every quantitative reasoning question correct, again revealing an apparent strength in this area. Oliver got every single question correct scoring in the highest percentile in the Ravens Progressive Matrices. He scored in the 96th percentile when compared to other Year-8 mathematics students, the 99th percentile when compared with other Year-9 students and the 89th percentile when compared with Year-10 students on three different Pat Maths Plus assessments. The Pat Maths Plus does not offer a higher level of test online. However, this was enough to show Oliver had exceptional mathematics ability.

6.3.3. General information.

Oliver was a quiet non-assuming boy who went about the business of learning in classes. He indicated in the original interview that he hated it when he was asked to listen to the teacher teaching, as he usually knew whatever it was that the teacher was about to say.

6.4 Case 4 - Bree

One almost gets the sense that had it not been the case that if her school did not offer a program that is geared towards giftedness, then Bree (pseudonym) could have been profiled as an underground gifted girl, as many of those character traits fitted Bree's persona. That is, Bree would hide her strengths and abilities long rather than admit to being clever. Past teachers described her as a girl who was creative, quiet, reserved, but someone who did not want other people to know about her talents. At the start of the study, she did not like the subject of mathematics at all. She rated her feelings towards the subject as a "one or maybe a zero out of ten". Bree recounted that there were too many "little things to remember" and too much repetition in mathematics. When asked to clarify what she meant by "little things", she shared that there is like a formula for everything. When asked to clarify more on her comments regarding repetitiveness, she recounted how mathematics is the only subject that required you to do the same thing repeatedly.

6.4.1. Bree's social and behavioural profile.

Bree was quietly spoken and often sat in class often being respectful Her manner towards teachers and students alike. She had a small group of close friends and was seemingly socially well adjusted. When I saw her during lunch times, she was with girls who were more noisy, dominant, and opinionated.

Bree often turned to her twin brother (Oliver) for assistance, if she got stuck in class. She was very artistic and drew animals with a great deal of prowess and expertise. One example is given in Appendix L shows Bree drawing at the end of exams and leaving creative comments as well. My first encounter with Bree was when she gave me a drawing she had done of a horse. She often smiled with a resolute demeanour. According to the initial interview, she had high levels of anxiety when it came time for exams. Bree explained that she was fine with questions with only a one or two-point grade score, but she felt stressed when she came to questions that were worth more points. These and other observed traits reveal a sensitive, intelligent and thinking girl.

6.4.2. Academic performance and attitudes towards mathematics.

Bree most readily fitted under the autonomous under-achieving learner profile as given Table 6.1. Previous report cards suggested she had commendable attitudes towards learning. Grades on these report cards saw her achieving A's or an A-minus in most academic subjects. She did not like outdoor sports or health and physical education. She suggested that she enjoyed more creative subjects such as graphics, drawing and technical studies.

Bree was the kind of student who relied heavily on her excellent memory. In this respect, she possessed a more analytic and creative intelligence. Results on any standardised test should be read within the understanding that this is a girl whose results were affected by nerves and struggled with anxiety.

6.4.3. Standardised test results.

Results on the Raven's Progressive Matrices (Raven, 1989) suggested that Bree sat in the top fifth percentile of students in the visual-spatial arena when compared to students her age. She also scored higher on the Year-9 mathematics Pat Maths Plus assessment, scoring in the 81st percentile in the Year-9 level of the test and only the 64th percentile on the Year-8 test. Normally, I would not have given a student the higher-year level test. However, I gave Bree this test, making light of it, telling her to do it for a bit of fun. Due to her heightened levels of anxiety, a joint teacher/parent decision was made not to ask her to complete the Slossan Intelligence test. She was identified by her results in the Raven's progressive matrices, past teacher nomination and previous report card results. It is noted that a similar approach was taken with the whole class in the administration of the Raven's Matrices. This method was to minimise the effects of nerves and anxiety on standardised test results. Students sat the test under strict conditions, but the delivery of the test was given in a relaxed way to assure students like Bree and calm her nerves.

6.5 Case 5 - Ty

Ty was quite confident, satisfied with current achievements, did not do more than he needed to do and had shown in the initial few lessons that he lacked the skills necessary to learn independently. Ty mostly fit Neihart's (2012) successful character behaviour profile and could have been aptly grouped with underground gifted students. He was a good friend to another boy who hid his abilities in the hope to ensure maximum social acceptance. He was often quick to "discount (his) abilities...reject achievement behaviours...and would experience dissonance about achievement goals" (Neihart, 2012). He seemed to be a different person in interviews compared to the boy I often saw in the classroom (having worked with him before this study) and playground. In interviews, Ty was quiet, sometimes emotional and often introspective, while in the classroom or playground, he was a more outspoken character. An example of this was seen when he broke down in tears when speaking about his A- grade on an enrichment task. He explained to me that he tried harder on this task then he had every tried before at school, and he was upset that he still did not get an A grade. This was somewhat contrasted with the louder, more outspoken, very popular young man who championed the basketball courts at lunch times, or was the center of attention in classroom challenges. Chapter 7 describes elaborates on how his classroom behaviours changed quite noticeably throughout the study. You would have been likely to find Ty playing basketball or other sports at lunchtimes.

6.5.1. Ty's Social and behavioural profile.

Ty mixed with a large group of boys at lunchtimes and was considered to be a person that most students liked and was very popular. He was well respected and liked by his aged-peers.

6.5.2. Academic performance and attitudes towards mathematics.

Ty actively tried to blend in and did not seek out any extra challenge or enrichment in mathematics. When he was asked to complete a challenge question, he usually pondered over it for many minutes, only to arrive at a successful solution (usually an educated guess according to Ty) soon after his initial attempt. Ty often guessed and worked to solve problems with the minimal of effort. He sat up the back of the class with Walter and other peers.

Ty's results on the Slossan Intelligence Test suggested his intelligence quotient to be 155 placing him in the 99th percentile of students his age. He was also in the top 10% of students his age based on results recorded from the Raven's Progressive Matrices. The score on the Raven's and Slossan slightly disagree, while the Slossan showed quantitative reasoning to be an area of strength along with social reasoning. These results could be interpreted in any number of ways, or it could just be that Ty is not as strong when it comes to higher order thinking visual-spatial questions. Ty's results on the Slossan Intelligence test were also replicated with very high scores on the Year-8, Year-9 and Year-10 online Pat Maths Plus assessments scoring in the 85th, 91st and 72nd percentile of students respectively (when compared with results from same year level students). These results along with in-class observations, past report cards and teacher nominations confirm that Ty was a capable mathematics student who was quite happy to do what was needed.

6.6 Summary

This chapter provided an introductory insight into the school behaviour of these five case students. Interpretations of behaviours were framed by Neihart's (2012) learner profiles. My data substantiates the exceptionality of the students who had profiles like those described in Betts and Neihart's (1986) earlier research. Chapter 7 will elaborate further on this information, by highlighting how these students responded to a more individualised learning model that provided them with opportunities to extend their thinking with both enrichment and accelerated tasks.

Chapter 7
Results

In the previous chapter, the five participants were introduced and profiled. That chapter revealed how a variety of measures were used to identify each participant as mathematically gifted, and how each individual reflected different characteristics of giftedness as per Neihart's (2012) updated profiles. It drew attention to their attitudes and feelings towards mathematics at the commencement of the study. It also gave a summary of their academic backgrounds in the field of mathematics. What unfolds in this chapter are the students' responses regarding both their academic results and attitudes towards learning mathematics in a classroom framed by the use of the Mastery Learning Model. These responses are discussed considering the two key research questions which are:

1. In what ways does a teaching program informed by the principles of Mastery Learning influence gifted students' mathematical performances?
2. In what ways does a teaching program informed by the principles of Mastery Learning influence gifted students' attitudes, motivation and interest in learning mathematics?

Firstly, in Section 7.1, I will discuss how the use of the Mastery Learning Model has influenced achievement levels for the five selected students, with results for the whole class used as a point of comparison. Section 7.2 will then focus primarily on results from student interview data on how the Mastery Learning Model influenced their motivation and attitudes towards learning mathematics.

7.1 How the Mastery Learning Model influenced Achievement Levels

In this section, quantitative data are discussed. It will be argued that these results suggest that the Mastery Learning Model positively influenced students' achievement levels. The effect size of the study on the five students cannot be measured using Cohen's D, as the sample size is too small. These results do, however, provide descriptive insights into just how many concepts gifted students are often asked to "learn" even though they already understand them prior to instruction. Students were able to progress to and master more advanced content, while not sacrificing the depth that came with enrichment activities.

7.1.1. Year-8 level results.

Students completed the PAT Maths Plus Standardised achievement assessments (Australian Council for Educational Research, 2011) at the beginning and end of the semester-long teaching program. They also completed the teacher-made pre-assessments at the beginning and the end of each unit of instruction. If the student achieved a rank that placed them in the 90th percentile (or higher) on the Pat Maths Plus assessment, then they did not need to redo this level of the standardised assessment later in the year. If the student correctly answered more than 85% or more of the questions on the teacher made Australian Curriculum aligned tests, the students then did the next level's pre-assessment. Table 7.1 presents the results of these Year-8 pre-assessments that demonstrate how both Oliver and Miley scored higher than 90% on the teacher made pre-assessments, as well as on the Pat Maths Plus assessments. They then completed the Year-9 pre-assessments in the next available class timeslot. The completion of the teacher-made assessments ensured all students had understood (or otherwise) all Year-8 Australian Curriculum mathematics concepts not explicitly tested in the Pat Maths Plus assessment.

The Pat Maths Plus standardised assessment presented in Table 7.1 revealed a mean ($\bar{x} = 87.69$) and standard deviation ($\sigma = 7.14$) for the five selected students. Similar results were noted for the teacher-made assessments indicating the five selected students had mastered 86.05% of the Year-8 mathematics curriculum. The remainder of the class had mastered 78.43% (Appendix M) of the same curriculum according to the Pat Maths Plus assessment and 73.81% of this curriculum according to the teacher-made assessments. Table 7.1 also indicates that at the commencement of instruction, all five students had a satisfactory understanding of Year-8 mathematics concepts, while four had an understanding to mastery levels of these Year-8 level skills.

Table 7.1

Year-8 Level Pre-assessment Results

	Raven's Progressive Matrices	PAT Maths Results		Teacher Made Pre-assessments
	Percentile	Percentile	Per cent	Per cent score
Class mean			78.43	73.81
Oliver	95th+	96	94.87	93.7
Miley	95th+	93	92.31	90.56
Walter	95th+	89	89.74	92.86
Ty	90th	85	87.18	88.33
Bree	95th+	64	74.36	64.79

An analysis of the school's Year-8 curriculum documents revealed that students in a regular classroom would be required to understand approximately 54 Year-8 concepts (Appendix N), as prescribed by the Australian Curriculum mathematics content descriptors. As the five selected students had demonstrated mastery of a majority of these concepts (Table 7.2) in the pre-assessments, then the students were only required to cover between 10-20% of the total Year-8 mathematics program for that semester. Awareness of what students understood saved time which they could use to work on tasks of advanced complexity and challenge, such as those discussed in Section 5.1.

Table 7.2 demonstrates how a score of 85% does not always necessarily mean a student has mastered every concept or skill. Comprehensive assessments with smaller sized formative quizzes enabled students to hone in on achieving mastery of specific concepts. For example, Miley had achieved 90.56% average on her combined Year-8 level pre-quiz answers. By breaking up the teaching term's instruction into smaller sized units, it was clear that Miley had gaps in understanding some Year-8 level probability concepts, which could be taught. By setting separate quizzes, the students and the teacher could be assured that no skill gaps were present.

Table 7.2

Number of Year-8 Concepts not Understood Before the Instruction Phase

Units of Instruction	Miley	Ty	Bree	Oliver	Walter
Time		1		1	1
Congruence			1		
Perimeter			2		
Volume			2	1	1
Rates	1	1	1	1	1
Algebraic					1
Fractions					
Ratio	2	2			
Probability	3	3	3		
Circumference			1		
Circle					
Non-Mastery	6	7	10	3	4

Students demonstrated in both non-standardised and standardised assessments that they had a solid understanding of the majority of the Year-8 mathematics curriculum related to the semester one topics required by the school to be taught. From the data summarised in Table 7.1 and Table 7.2, it was evident which concepts students already understood. Pre-assessment data were able to be used, therefore, to confirm that students had mastered the set mathematics program.

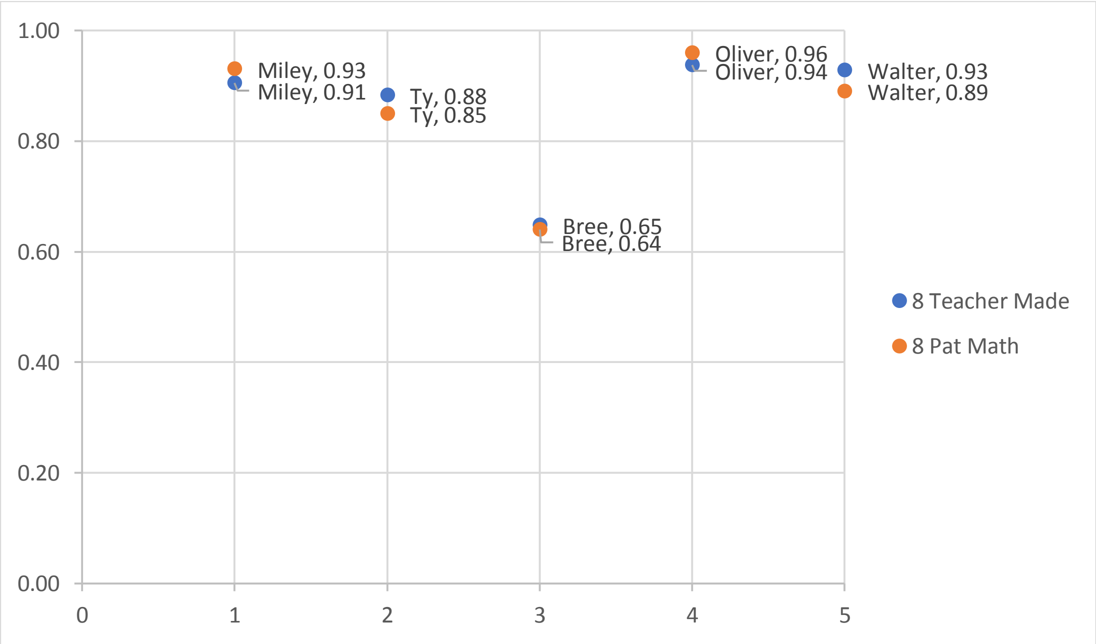


Figure 7.1. Selected students' Year-8 level pre-assessment results.

Figure 7.1 provides a scatterplot that illustrates the similarity of scores between Pat Math Plus assessments and teacher-made, curriculum aligned assessments. The scatterplot reveals how the cumulative results' from across the term, were similar. However, a closer analysis of the individual teacher-made quizzes identified some gaps in understanding some Year-8 concepts.

Students worked to fill these gaps and gains in achievement could then be noted as students were mastering content they had not previously understood. According to a cognitive/constructivist view of learning, students marks should not drop, therefore, as they are building on what they already knew. A disparity between end of semester summative (including standardised results) and formative results would be cause for further investigation. Figure 7.2 presents data comparing pre-post assessment results which demonstrate that even though the class is streamed with high ability and gifted students, that there still exists variance in ability. A mean ($\bar{x} = 86.05$) on teacher made, curriculum aligned pre-assessments and ($\bar{x} = 87.69$) on the Pat Maths Plus assessment for the five selected students reveals a high degree of mastery of Year-8 mathematical concepts that would normally be taught.

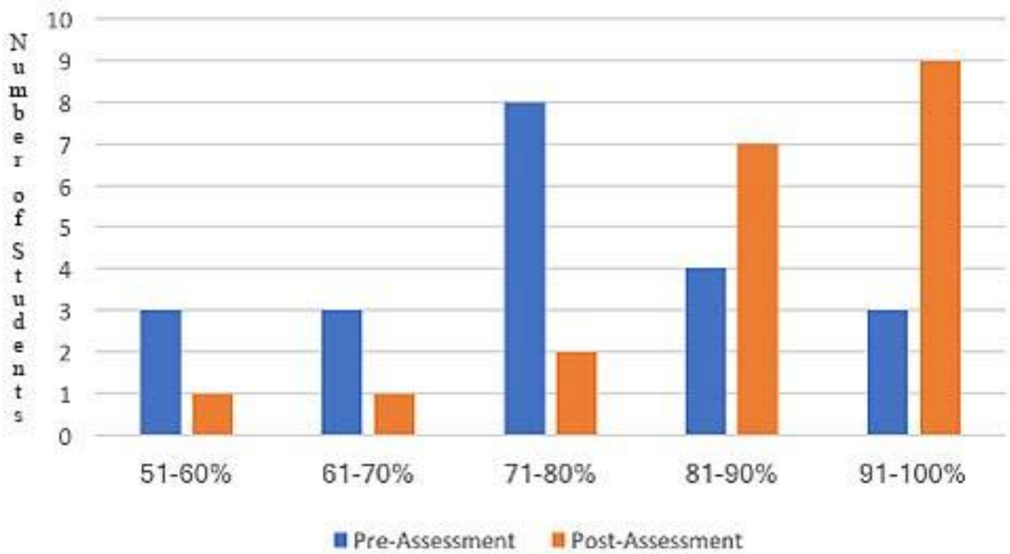


Figure 7.2. Whole class Year-8 level pre/post-assessment (teacher-made assessments) results' distribution and comparison.

A lower mean ($\bar{x} = 73.81$) for the remainder of the class on the teacher-made assessments, and ($\bar{x} = 78.43$) for the Pat Maths Plus assessment suggested that most students would need to spend more time consolidating some, but not all Year-8 level mathematics concepts. This suggested a need for these students to have access to a program differentiated in complexity and design to ensure that they could work at a level suited to their mathematical ability.

A mean growth ($\bar{x} = 7.77$) was noticed in the five selected students on the Year-8 Teacher made assessments. On this same assessment the remainder of the class noticed similar improvements in performance ($\bar{x} = 7.66$) on the Australian Curriculum aligned pre/post-assessments. A whole class mean of over 80% was also noticed on both Year-8 level assessments. Both Miley and Oliver did not complete the Post Pat Maths Plus tests as they already completed the assessment to mastery levels. This meant that the difference statistic is not applicable given they did not complete the post-test.

Table 7.3

Year-8 Level Pre-Post Test Percent Mean Comparison

	Pat Maths Plus Online			Teacher-Made Assessments		
	Pre-Tests	Post-Tests	Difference	Pre-Tests	Post-Tests	Difference
Year-8 Class (N=19)	78.43	82.91	4.48	73.81	81.47	7.66
Miley	92.31	92.31	N/A	90.56	95.73	5.17
Ty	87.18	100	12.82	88.33	92.86	4.53
Bree	74.36	76.92	2.56	64.79	85.98	21.19
Oliver	94.87	94.87	N/A	93.7	98.78	5.08
Walter	89.74	100	10.26	92.86	95.73	2.87
Selected students' mean summary	87.69	92.82	5.13	86.05	93.78	7.77
Total Class mean results	85.37	87.61	2.24	82.19	88.60	6.42

Figure 7.2 and Table 7.3 present the shift in understanding from the commencement of instruction to the completion. At the commencement, a normal skewness was evident in results, whereas, post-test results reflect that a higher percentage (79.17) of students (n=19) scored over 80% on the end of term assessments, where only 29% of students scored over 80% on the pre-assessments. These findings are similar to findings from other research (Guskey, 2010) discussed

in Chapter 2. These data reflect that improvements in results were not only limited to the five selected students. Due to the scope of this study, I will continue to focus mainly on the five selected students' results.

This section has provided insights into how the use of the Mastery Learning Model enabled the five selected students to remain challenged, working at a level appropriate to their abilities. Students were able to focus only on concepts they had not mastered, instead of completing tasks they already understood. They did not have to listen to a teacher teach concepts repeatedly that they had already mastered them. This section revealed how some students had initially demonstrated mastery, but still needed to consolidate a few skills to ensure skills gaps did not appear, while most were able to go on to higher level tasks quickly, as will be discussed next.

7.1.2. Above-year-level results.

Students worked on more complex concepts after they had mastered the related Year-8 content. I first discuss the results on formative and summative assessments related to these more complex Year-9 skills and provide examples of student responses to this higher year level work. I then provide data on how students performed on the Year-10 level content.

At the commencement of the intervention, and as depicted in Table 7.4, a spread (σ=16.89) and mean ($\bar{x} = 50$) revealed a significant difference in understanding of the higher year level concepts as given in the teacher made assessment results. The Pat Maths pre-assessment results ($\bar{x} =$ 87.5, σ = 6.12), however, suggested the five selected students could solve more complex questions correctly, while the teacher made pre-assessment showed that there were still gaps in their understanding. Numerical data provided for the remainder of the class were not reliable indicators of achievement, as not all students completed all Year-9 level assessments, since they had yet to master Year-8 level equivalent concepts.

By the end of the study, a higher mean score ($\bar{x}$ = 90.42) and smaller spread (σ=6.32) was noticed in the teacher made assessments for the same students. The slight improvement in results on the Pat Maths tests were considered inconsequential. This small change in results is attributable to the fact that only three students needed to do the Pat Maths post assessment, which noticed negligible improvements in already high scores.

Table 7.4

Comparison of Year-9 Level Pre-Post Assessment Mean Per cent Results

	Pre-Test Teacher-made tests	Pre-test Pat Maths Plus Online Results	Post-Test Teacher-made tests	Post-Test Pat Maths Plus Online Results
Year-8 Class	55.39	N/A	75.59	72.19
Miley	50	82.5	95.26	90
Ty	28.3	87.5	80.53	87.5
Bree	45.7	80	88.95	80
Oliver	80	97.5	98.95	97.50
Walter	43.8	90	88.42	95
Selected students mean scores	50	87.5	90.42	88.75

The post-assessment teacher made test results did, however, suggest gains in understanding of an average of 40% according to the teacher made assessments. An example of the kind of Year-9 level questions these students were able to master is given in Figure 7.3.

The questions are considered complex, as the students typically had not formally learned these skills. They required an understanding of Year-8 level concepts (in this case, the first four index laws) and reasoning which included the methods used to solve the Year-8 related content before they could solve the higher-level questions using other index laws. To address the problems successfully, the students would have needed to use multiple steps to demonstrate a satisfactory reasoning ability (also a higher order skill).

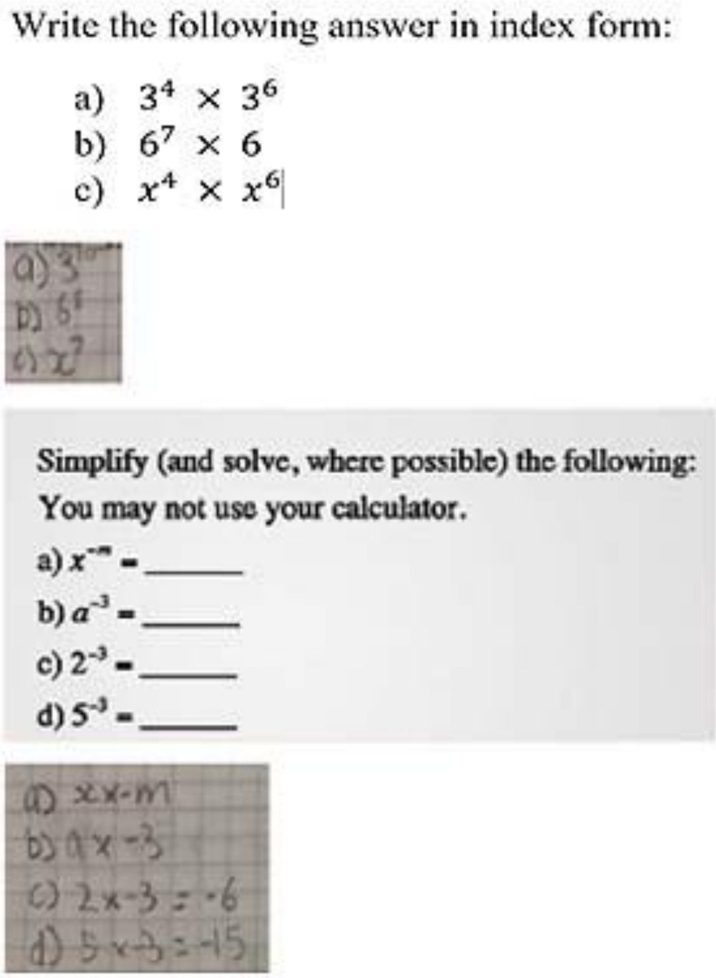


Figure 7.3. Walter's answers to a skills pre-quiz.

In the questions given in Figure 7.4, students had to understand operations to the fractional, zero and negative integer index (Year-9 level skills). The pre-assessment demonstrated that students had not learned these skills, while the formative quiz submission given in Figure 7.4 provides evidence of an improved understanding of these concepts.

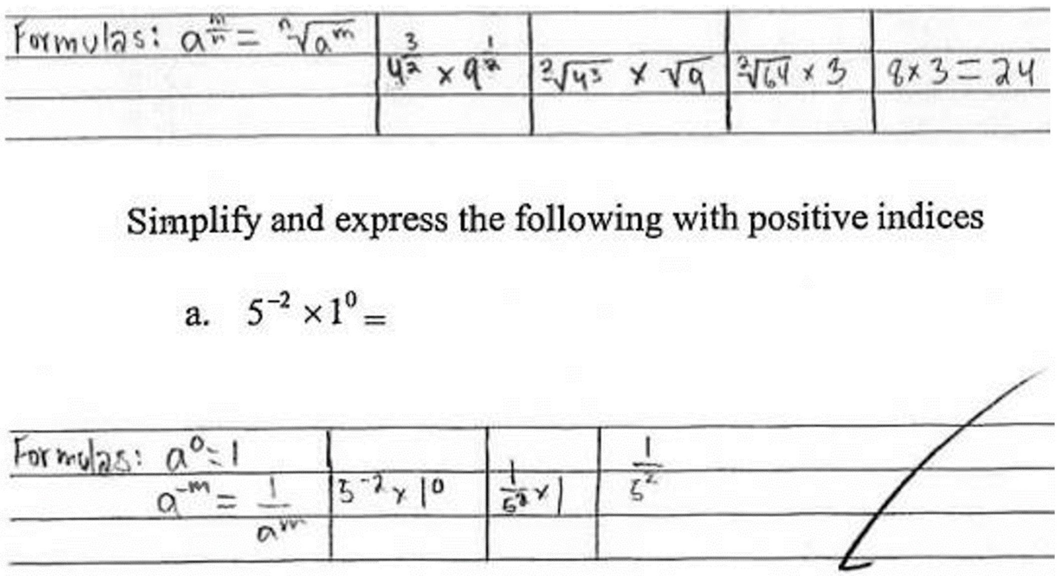


Figure 7.4. Example of answers to questions on end of term assessment.

Other questions, given in Appendix O and Figure 7.5, show similar examples of more complex learning tasks, which were completed successfully by students. The students were able to demonstrate understanding through their working out (reasoning) of such complex questions.

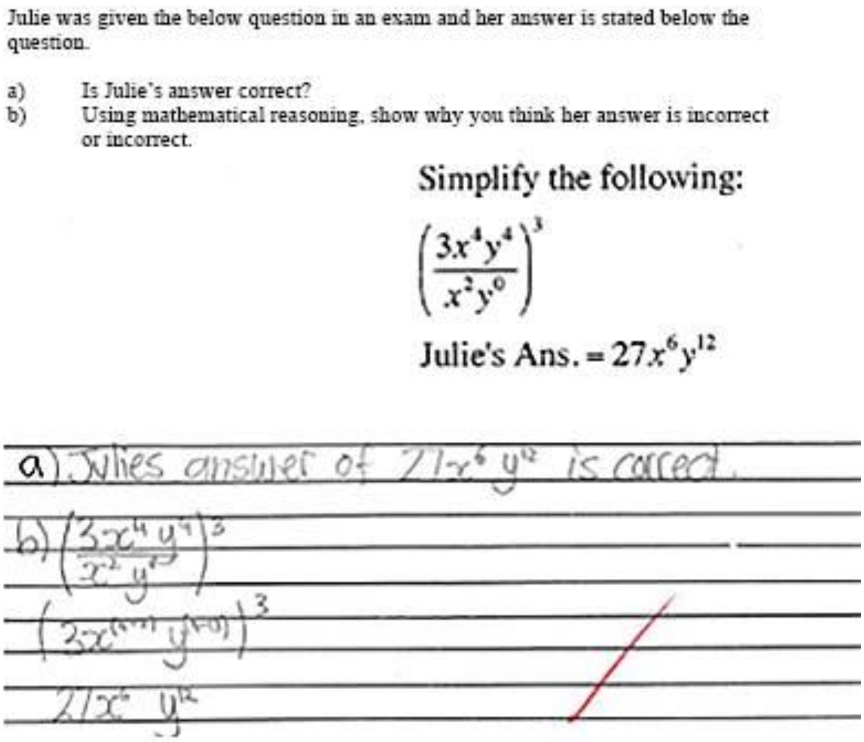


Figure 7.5. Solution to a question involving reasoning.

The gains in understanding are presented diagrammatically in Figure 7.6. This figure denotes how the students were able to master over 85% of the Year-9 mathematics curriculum, with post-assessment mean scores ($\bar{x} = 90.42$) demonstrating mastery of these skills. The smaller standard deviation ($\sigma = 6.33$) also reveals a reduced spread of data.

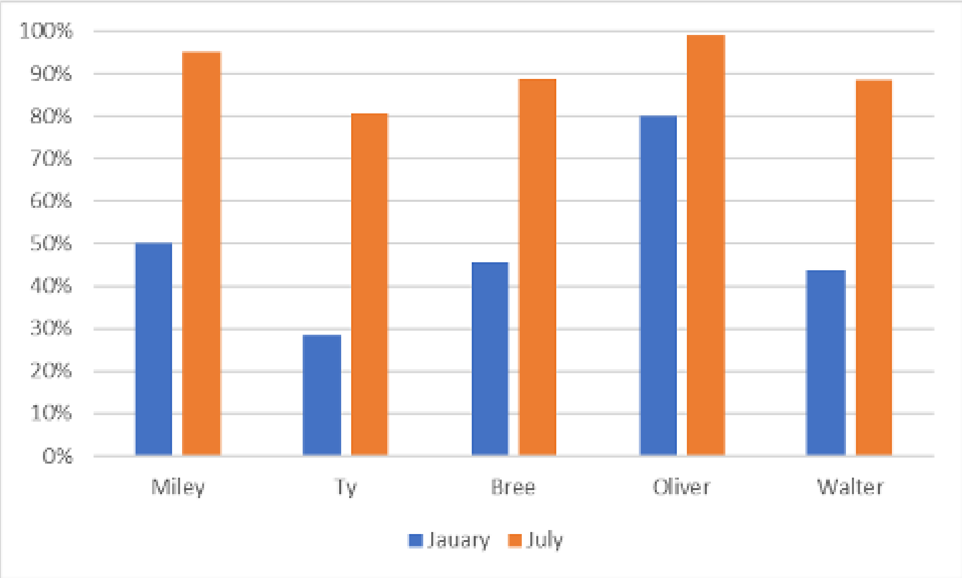


Figure 7.6. Pre-post-test comparison results on Year-9 non-standardised assessments.

Table 7.5 further reveals how these students were even able to master many Year-10 level mathematics concepts such as "expanding binomial products…factorising algebraic expressions…solving linear equations with fractions…conditional probability and constructing and comparing box plots" (Australian Curriculum Assessment and Reporting Authority, n.d.). A sample of the kind of questions included can be found in Appendix P. Section 7.2.3.1 reveals further details on Oliver's involvement in a Year-10 Mathematics C course.

Table 7.5 outlines the results for some of these students on the PAT Maths assessment when compared to other Year-10 students. Video footage revealed that Bree, Walter and Ty possessed a level of focus that was not observed in their previous tests. A mean score ($\bar{x} = 90.86$) and ($\sigma = 5.38$) demonstrated a mastery of many Year-10 concepts at the end of the study by the five selected students.

Table 7.5

Year-10 End of Semester Pat Maths Plus Results

	Mark	% Score	Percentile
Miley	34	83	94
Ty	39	95	99
Oliver	38	93	99
Walter	38	93	99

7.1.3. Summary.

A mean ($\bar{x} = 87.69$) on Year-8 assessments at the start of the term and ($\bar{x} = 90.42$) and on the end of term end of Year-9 Pat Maths Plus assessment suggest a significant improvement in achievement results. These results only tell part of the story that Section 7.2 will elaborate on further. Results reveal that students learned new skills and remained challenged. Students could not only access a higher Year-9 level work but attained mastery of these concepts, by being able to spend more time completing content pertaining to this higher level. This evidence suggests that these students were able to benefit significantly from the use of this model. Oliver was to go on later and complete a Year-10 Mathematics C course (a pre-requisite course for any students wishing to do Year-11 Mathematics C in senior school). While not part of the findings, Oliver later decided to move into the higher year level's class physically in Year-9. The above results document evidence in response to the first research question. This evidence asserts that Mastery Learning Model influenced achievement levels positively. It has shown how performance data were used to drive instruction both by students and teachers to determine next-steps in their learning. The next section discusses the raw data as presented in relation to the second research question.

7.2 Influence of the Mastery Learning Model on Motivation and Attitudes

This section addresses the second research question that asked how the Mastery Learning Model influenced students' motivation and attitudes towards mathematics. As a researcher; analysis of video, voice recordings, survey responses and interview data helped me to draw conclusions on the second research question. These foci enabled the identification of five ways that the Mastery Learning Model had influenced students' attitudes towards learning mathematics. The following themes emerged from the analysis of survey results and interviews: Autonomy, competence, challenge, relevant and complex learning tasks and social collaborative learning.

These key themes emerged from video recorded observations, survey results and interview data. A combination of the above themes enabled a sense of flow in student learning. The use of the Mastery Learning Model meant students would not be asked to complete skills they already understood. This practice, in turn, reduced the amount of repetition and associated boredom which they had experienced in past years.

7.2.1. Autonomy.

Section 7.2.1 will share how students responded to learning autonomously. According to Ryan and Deci (2017), a person can naturally internalise extrinsic motivations from significant others for autonomy, when they experience full volitional control, characterised by lack(ing) inner conflict and willing engagement" (p. 8). Section 7.2.1, therefore, analyses what volitional control and choice looked like and how students responded to it.

In responding to the question asking students to rate their attitudes towards maths on a scale of one to ten, with one representing a hatred of the subject to ten being that you wished you could do it all the time, students' average ratings shifted from 4.85 at the beginning of the study to 7.3 by the end. This section therefore elaborates on why this shift in attitude may have occurred. For example, at the commencement of the intervention students like Bree, shared how they had "no free will" in previous mathematics classes. Similarly, all five students attributed feelings of boredom to learning mathematics at the start of the intervetion. All five students described one reason for the shift in their attitudes towards mathematics was attributable to how they had more choice and control over what and how they learned, as will be elaborated on in this section.

Interview responses revealed that students experienced a sense of autonomy in their learning with self-directed, teacher, peer and parent support. This self-direction influenced students' intrinsic motivation to learn. It also encouraged a feeling of supported autonomy. The students were able to make self-directed decisions on whether to participate in a lesson, work independently of the teacher, or engage in supported learning from a fellow student. Section 7.2.1.1, will, therefore, discuss how this self-direction impacted on the selected students' motivation to learn mathematics within a classroom framed by the Mastery Learning Model. Section 7.2.1.2 discusses how the school's online learning management system provided both the scaffolding and feedback to guide this self-directed learning. The notion of autonomy implies a feeling of volitionally engaging in a behaviour (Ryan & Deci, 2017). Furthermore, this autonomy came with support (Williams, Gagné, Ryan, & Deci, 2002) that the students described as helpful. It represented a form of empowerment where individuals sensed that they had control and choice about the tasks they encountered.

The following section provides evidence of how students were able to learn in an autonomous way that avoided both gaps and repetition in learning and impacted on their attitudes towards mathematics. Interview data revealed that feedback given on formative quizzes encouraged "integrated regulation" (Ryan & Deci, 2017, p. 15) to complete mathematical tasks and learn new mathematical skills. Section 7.2.1.1 provides evidence, suggesting that all five students used feedback to help motivate them to gain an understanding of a concept to mastery levels. Interview responses revealed how other students (Oliver, Miley, Walter and Ty) were internally motivated when given access to accelerated tasks including enrichment challenges.

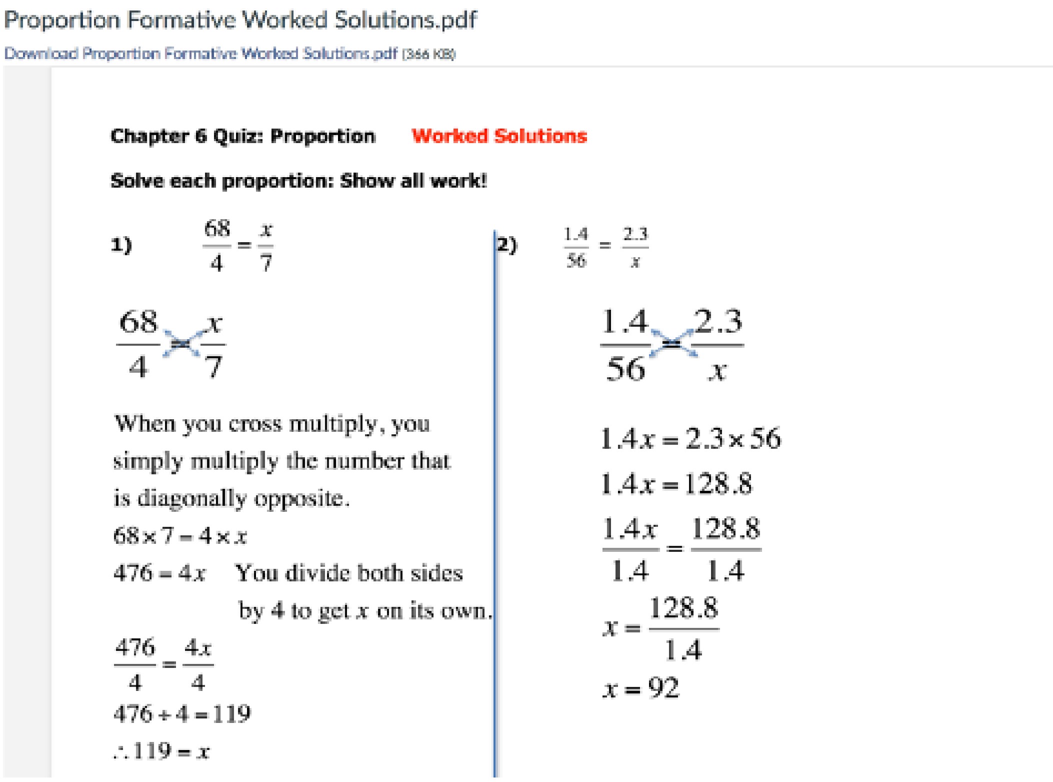
7.2.1.1. Instant feedback imbued an internalised motivation to learn.

In the examples discussed in this section, students would decide whether to learn in a more traditional teacher-centred approach, through self-directed online activities or via their own means of discovery with enrichment tasks. The reader will recall that all the necessary content was contained with the Online Learning Management System. The use of the Mastery Learning Model enabled students to access what they needed to learn or re-learn to attain mastery levels of understanding. Their learning was autonomous in that they identified what they did not understand, and the teacher supported them (with materials or help) to master the required skills to help with their learning.

Appendix J provides an example of how the online learning management system acted as a tool to support the Mastery Learning Model principles that contained scaffolded instruction. Students were able to proceed through a series of learning activities at their own pace, choosing which activities to complete, while some, like the pre-quiz, were compulsory. When they had mastered the pre-assessment, they were then able to proceed with the enrichment tasks. Figure 7.7 provides an example of how these quizzes came with worked solutions enabling the student to check the accuracy of their thinking. The feedback for the quizzes was instant. Students then proceeded to work on the enrichment task or the next unit of instruction. A desire to discover patterns, understand increasingly complex content that was related to the students' real lives was evident as noticed through on-going observation and interactions with each of the five students.

Ty asserted that the structure provided in the online units of instruction provided him with choice and control in his learning. He explained that he "(did not) have to wait forever in class for other students who (took) a long time to understand and could just go on and do the next task" (Ty, Interview, April 3, 2014). Oliver added to this suggesting that he did not "mind having to finish something before the end of the lesson, as long as (he got) to choose which order (he did) it in" (Interview, December 2, 2014).

Walter shared how the Online Learning Management System was "confusing at first" (Interview, April 2, 2014) but later, appreciated how various digital tools such as videos and PowerPoints helped him to learn. In response to a question asking Walter what aspects of the mastery program helped him learn best, he shared, "I collected all the PowerPoints and went through them with the skill sheet and, I would tick off all the skills once I learned them" (Interview, June 26, 2014). Walter's motivation to learn mathematics came from a desire to master skills as stated in the differentiated unit outline.



Proportion Formative Worked Solutions.pdf

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Chapter 6 Quiz: Proportion **Worked Solutions**

Solve each proportion: Show all work!

1) $\frac{68}{4} = \frac{x}{7}$

$\frac{68}{4} = \frac{x}{7}$

When you cross multiply, you simply multiply the number that is diagonally opposite.

$68 \times 7 = 4 \times x$

$476 = 4x$ You divide both sides by 4 to get x on its own.

$\frac{476}{4} = \frac{4x}{4}$

$476 \div 4 = 119$

$\therefore 119 = x$

2) $\frac{1.4}{56} = \frac{2.3}{x}$

$\frac{1.4}{56} = \frac{2.3}{x}$

$1.4x = 2.3 \times 56$

$1.4x = 128.8$

$\frac{1.4x}{1.4} = \frac{128.8}{1.4}$

$x = \frac{128.8}{1.4}$

$x = 92$

Figure 7.7. Worked solutions found inside learning module on the online learning management system.

He added how he felt “more teachers should set checkpoints of where we need to be at a particular time”. It was the use of this checklist/unit outline that he could tick off once a skill was mastered. Whether Walter completed enrichment tasks or challenging tasks online, he had control over what he learned and how he learned it. Interestingly, this replicates Williams’ et al., (2002) findings suggesting that internalisation is more likely when such a supported autonomy is evident. The scaffolding provided with the Mastery Learning Model such as Walter appreciated helped support his learning autonomously.

Similarly, Miley (Interview, February 25, 2014) agreed, stating that “As long as it is easy to follow and there’s like a pattern and you do this thing this week and another thing next week then that’s fine”. She also found the online learning platform confusing at first, as it was new. As time progressed she became more familiar with the new online interface, how to find learning units, results and other online content. Both students appreciated being able to work within a clear structure or and towards clear learning goals. A sense of supported autonomy was evident when Miley commented on how the instant feedback from the online learning management system or the teacher helped her. She shared how this feedback helped her feel confident that she understood the key concepts. She revealed how the formative quizzes "encouraged (her) at the same time as you said I was doing great on these tasks" (Miley, Interview, June 23, 2014). She added that she liked "how it (the unit of instruction) (was) divided into sections and you (could) see how you are doing…On (the learning management system) you can submit it online and get feedback straight away" (Miley, Interview, February 25, 2014). Therefore, along with a scaffolded unit, the students used feedback to gain control over what and how they learned.

Like Miley, Bree appreciated receiving feedback on how she was doing, while also remaining confident in her mathematical ability. In her final interview, she was asked if the feedback she received was helpful. She shared:

> I think they were, because you know what to do to make it better. Also, if you are slacking off, then it picks you up and lets you know that you have to pick up your act a bit more. I know that I am smart and am good at maths, but I also know that everyone makes mistakes. (Bree, Interview, June 23, 2014)

Bree used feedback, therefore to help motivate her to work hard to gain an understanding of a concept. Getting feedback did not make her feel bad when she made a mistake. Rather, it provided support to help her construct her knowledge of that concept.

All five students shared how they used the results on the formative quizzes to go back and re-learn concepts in a self-directed way, filling gaps in knowledge. Bree would rely on listening to specific parts of the lesson related to the concept when she suggests that when "(she) got it wrong (on a quiz) and (she would) specifically listen out for that concept in the lesson" (Interview, June 23, 2014). Similarly, if Oliver "(got) it wrong on the formative quiz, then (he would) either look it up on the computer or ask (the teacher)" (Interview, June 25, 2014). However, if Miley "(got) it wrong, (she would) spend ages on it until (she) figure(d) it out. Bree (would then) tell (her) the answer and tell (her) not to worry about it, but we'll keep arguing about it until one of us understands it" (Interview, June 23, 2014).

Walter also iterated that he would use the quiz results "to revise what (he) did not understand". He also shared how when he was "at home he would ask his dad for help" (Interview, June 26, 2014). In this interview, he suggested that even though he did not "enjoy doing the quizzes very much, (he saw them as) necessary…but they're good as they stop you from doing too much, doing stuff you already know”. The quizzes provided the feedback that helped the students identify what they needed to learn, as well as ensure that needless repetition did not follow. These examples of supported autonomous learning mirror Williams’ et al. (2002) findings.

Students could listen to routine teacher-directed lessons or access and learn content independently. In discussing this aspect of choice, Bree suggests that “even if (she is) always listening (to the teacher-directed lesson) anyway, (she would) still prefer that I can choose whether to listen or not, depending on what I know” (Interview, June 23, 2014). This comment revealed an assuredness that Bree would know what she needed to listen to, which would be based on what she already knew. Therefore, without using the term “mastery”, she has suggested that the structure enabled her to know whether to listen to the teacher, based on what she knew, rather than listen because she was forced to, which was the case in previous years. This acknowledged understanding enabled students to take control of their learning. The completion of quizzes was followed by helping students form connections with what they had learned with enrichment.

Students’ motivation to learn came from a sense of curiosity to explore connections with mathematics and the real world and a desire to know more. Ty shared how the enrichment challenges included “a lot more of using it (mathematics) in the real world, instead of just sitting down and just writing it out” (Interview, April 3, 2014). As will be discussed later in greater detail, Miley connected an understanding of circumference of sectors with the wiper blades on her family’s car. Oliver’s curiosity led him to relate mathematics to getting to school of a morning and experimenting with developing a working formula to help him plot a quicker route to school. Miley also shared how she simply wanted to solve a perplexing challenge and find a connection between Binomial Theorem and Pascal’s triangle (Appendix Q). Similarly, Walter shared that he loved the enrichment problems as he could see that mathematics had relevance to his life. Ty (Interview, February 25, 2014) also revealed that it was this connection to the outside world that interested him.

This section highlighted how students were supported with materials, course structure, a social learning framework and direct instruction, all critical aspects of the Mastery Learning Model. They could identify any gaps in their learning and ensure they went to the section of the course on the Online Learning Management System and learned the skills needed as per the differentiated course outline. It has also shared how students enjoyed learning skills where relevance to their lives was clear. They enjoyed the ability to explore these interests, as related to mathematics in their own way, and often volitionally in their own time. In the next section, I discuss how students' sense of autonomy was achieved in part because there was effective feedback and structure provided by both the teacher and the online learning management system and teacher. Thus, autonomy was not a laissez-faire phenomenon but an orchestrated approach that provided the necessary level of scaffolding to ensure students could progress at their own pace.

7.2.1.2. Choice and control over their learning.

When given a choice to listen to the teacher or complete the work in a self-directed way online, Walter revealed why he liked both. He shared: "I usually came up the front to re-assure what I needed to know that I knew it" (Interview, June 26, 2014). He added that while he "could learn without a teacher", this access to both types of instruction would "assure" him of a comprehensive understanding of the concepts. Walter's experience would suggest that a teaching model that provided scaffolding, but also afforded access to a more capable other (teacher, peer or adult) as is provided within the Mastery Learning Model is beneficial. Oliver also appreciated the online scaffolded help but indicated he would have liked more access to support in the form of a more capable peer or adult, as will be discussed next.

Oliver suggested that he wished he could have had more teacher-directed instruction on his Year-10 course that he completed in class. He shared: "I often learn a lot better if someone demonstrates something" (Oliver, Interview, December 2, 2014). He added:

> Sometimes you can miss out on little bits of information, so it would have been good to have been taught about that, before going into it. When you tried to find the answers online, some of them would be too advanced while others would be too basic.

Therefore, if students are going to be placed on a much higher-level program, then students like Oliver and Walter appreciated also having access to direct as well as online instruction. This discussion led to Oliver being asked what he would do in cases where he encountered a problem that was too difficult, such as is being described here. He (Oliver, Interview, December 2, 2014) shared the following in response:

> I just try and re-read the question and see what part of the problem I know and work it out. If I am not sure how to do something, I would try to figure it out on my own first and then ask my parents if I cannot figure it out and then if I still haven't arrived at a successful answer, I would ask my teacher.

Therefore, while he was placed on an advanced program with online help, further support was necessary to help him understand a concept that he could not solve on his own. This research also echoes research conducted by Diezmann and Watters, (2001) suggesting that gifted students will often seek help from other students first.

All students explained how previous teachers asked them to complete question after question, even if they already knew that particular concept. In this case, Bree's prior knowledge is being respected. She (Interview, June 23, 2014) shared,

> It was a lot better than last year's mathematics as there was no free will. It was just a matter of doing these questions, and for the rest of the lesson, we'd just work on those questions from a textbook. There's a lot more human aspect in the way that you do not have to be forced to listen…even if it is I'm always listening anyway, I'd still prefer that I can choose whether to listen or not depending on what I know.

The Mastery Learning Model enabled Bree to attain a sense of autonomy in that she had some freedom to choose her options depending on her confidence in her knowledge based on her performance on the formative quizzes and participation in the online and teacher directed lessons. This statement exemplified a significant shift in Bree's attitudes from where she hated mathematics at the start of the year to now saying it is a "subject (she) does not mind doing". She was able to choose when to listen, based on her understanding of what she already knew versus what she perceived that she needed to know to attain a mastery level of understanding of a concept.

As discussed above, students were able to use feedback from a variety of sources to help them demonstrate mastery in accordance with Mastery Learning Model's principles. In this instance, the use of online quizzes, as well as other online programs helped facilitate learning to mastery levels. For example, Oliver reported on his use of a commercial software program ostensibly designed to engage learners in mathematics, "I do not mind Mathletics[4]. The help button is useful if I do not get

[4] http://au.mathletics.com

something" (Oliver, Interview, February 25, 2014). The help button in the Mathletics program, when clicked, enabled students to see a worked solution to a similar problem providing students with the instant feedback he needed on how to solve the problem specific to what he/she was learning in class. Tools like this helped students gain an understanding in a self-directed and teacher supported way. Students learned, because they did not understand, not because they were told to complete a set number of questions. Similarly, Walter also expressed how online access to YouTube and Slideshow tutorials "helped" him in understanding unfamiliar mathematical concepts. This style of learning allowed him to work at his own pace in a self-directed manner if he, chose to.

7.2.1.3. Grades.

It was clear that some parents wanted their children to get good grades, much like the parents in the Garn and Jolly (2014) study did. Bree shared "Our parents (gave) us a small amount of money if we (her and Oliver) (got) good grades, and I also (did not) like getting bad grades" (Bree – June 23 2014). Not all students asserted this parent influence. Rather, it seemed that both Miley and Ty used grades as a form of feedback to help encourage them. Miley (Interview, June 23, 2014) shared:

> "I think that my grades encourage(d) me, because I know that I'm doing it right. Have you ever like learned something and then five minutes later, you realise that you have done it totally wrong? I feel like that when I get an A; I am doing everything correctly".

Earlier in the year, she shared that she "kinda" liked it when a teacher "explain(ed) why (she) didn't get a good grade" (Interview, February 25, 2014). After I prompted her to provide further information, she shared that she would like teachers "to focus more on the areas that (she does not) understand rather than the areas (she did) understand". Therefore, Miley, like the other students, appreciated getting good grades as a form of feedback to help them feel better about their learning. Similarly, Ty suggested a more internalised motivation to get good grades when he stated: "I'd prefer more detailed feedback to help me get a better grade" (Interview, February 25, 2014).

7.2.1.4. Summary.

Students appreciated having the autonomy to learn what they wanted to learn from a given unit outline in the way they wanted to learn these concepts. A deeper level of understanding of each of the concepts was gained through the use of enrichment challenges, which also had a positive influence on their attitudes toward mathematics. Autonomy was not learning on their own, but rather students having the volitional control over their learning and choice to learn with help from the teacher, a peer or friend or independently. Feedback also came in the form of grades. Students did not iterate any form of parental pressure; however, Bree did share of a monetary reward that came with getting good grades. Students used feedback from assessments to help them understand what they did wrong. This finding suggests a motivation to learn and understand more than getting a good grade. It was important that the feedback from the quizzes, enrichment tasks and other challenges were used to guide the learning process, as this ensured the students were completing work set at an appropriate level of complexity as will be discussed next.

7.2.2. Competence

In the previous section, various stories illustrated how the teaching model helped students complete work set at an appropriate level of complexity. This section builds on this, suggesting that a sense of self-efficacy was gained by mastery of base concepts. This positive impact allowed students to tackle problems usually reserved for pupils of up to two year levels higher in their regular instruction in class as will be discussed in Section 7.2.3. Students suggested that they did not like completing the quizzes, but they did like how this approach meant that they knew what they needed to know and could choose to work on more complex tasks.

7.2.2.1. Competence enabled students to solve more complex concepts.

After the teaching model was explained to students in the interviews, all five suggested that they liked the model as presented to them, as it avoided repetition, allowed challenge and maintained variety which encouraged genuine curiosity and interest. The mastery model acknowledged student competence which in turn allowed them to work on more challenging tasks. The five selected students all revealed how in previous years, they were often engaged in overly repetitious textbook based and decontextualised learning tasks. This repetition left them feeling as though their prior knowledge was not acknowledged in any meaningful sense. In contrast, interview data revealed that they felt this program enabled them to build on their current knowledge and engage in higher level, challenging tasks that recognised their mathematical skill levels. Miley (Interview, June 23, 2014) described how mastery affected her feelings of efficacy towards mathematics when she explained:

> I sort of know what I am doing, so when I did it (on the quiz), I got it right, so I know I know what I am doing. I'll try to explain. In piano…the more times I got something wrong, the more likely I am to do it again…if you only know how to do something the right way, then you're not going to make mistakes, as that's the only way I know how to do it…and when I get a problem, and I get it right, then that's the only way I know how to do it.

The use of quizzes enabled Miley to feel that she knew what she was doing and that it was correct. This feeling of competence then allowed her to work on more complex tasks. Miley exclaimed that she did not like continual textbook work, but rather enjoyed tasks she could "dig her teeth into" (Interview, June 23, 2014). Similarly, Ty shared how in Year 3, he was given Year-7 level work and had gaps in his learning and he was left confused as he seemed to be attempting content outside of his zone of proximal development with little to no help from the teacher or more capable other. He described learning this way as "better because you know everything you need to know" (Interview, (Ty, June 26, 2014). Therefore, students felt self-efficacious and competent in solving more difficult problems as they had learned the base skills for more advanced topics.

Miley insistently and autonomously chose to seek out a deeper understanding of more complex concepts in her own time at nights. She was driven and determined to arrive at a satisfactory solution to such higher-level challenges. When asked about her thoughts on mastery, Miley shared: "I like how it is divided into sections, and you can see how you are doing" (Interview, February 25, 2014). The sections Miley referred to, were the smaller sized units such as the examples given in Figure 7.8. These units focussed on specific tasks as shown.

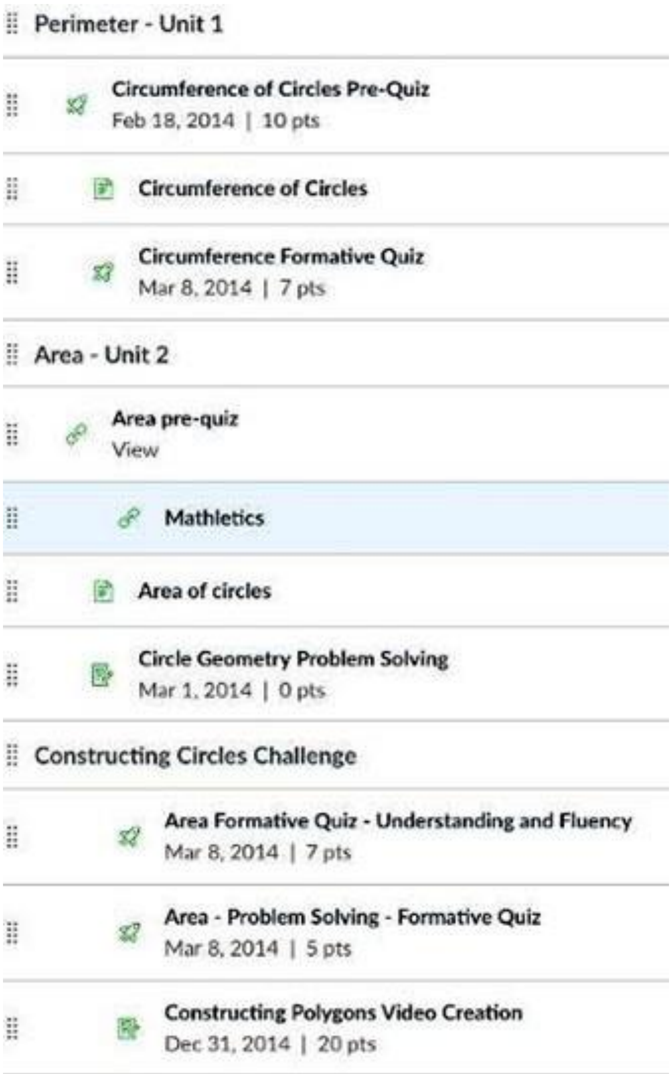

Figure 7.8. Example of online smaller sized units given to students.

In the example given in Figure 7.8, there was a section on the area of circles and before that, a small section on finding the circumference of a circle which included richer problem-solving tasks (example given below). Given that Miley understood how to find the perimeter of a sector, it meant she could successfully solve problems like the one given to her, such as that provided in Figure 7.9.

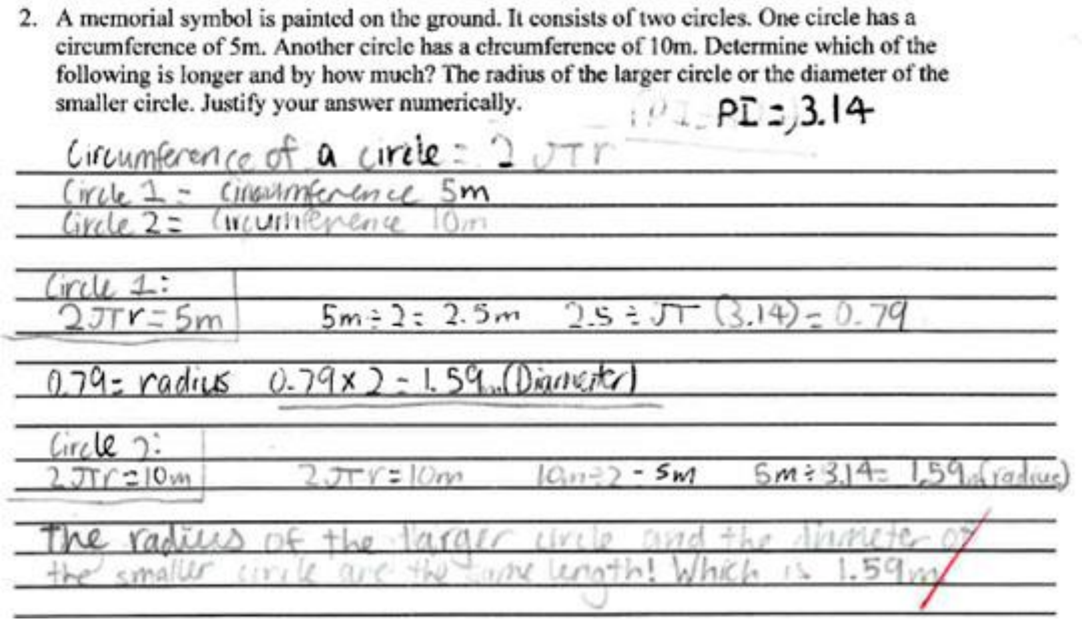

Figure 7.9. Miley's solution to this more complex problem involving circumference.

When students understood the base concepts involved, they could then solve more complex problems. These problems often involved multiple steps and often required the understanding of a range of strategies in order to solve them. Students often showed their ability to solve problems with or without the teacher's help. They could then go on and attempt the relevant end of unit quiz, which then affirmed to them that they could competently solve these harder styles of questions.

Likewise, students felt competent to help each other as will be discussed in Section 7.2.5. Oliver explained, "I get asked quite a lot of questions if they do not understand something in mathematics, but I really enjoy helping other people as it means that I can understand the concept a little better" (Interview, February 25, 2014). Through this social learning and by helping his peers, Oliver was able to gain a better understanding of a concept.

Likewise, for Bree, "It's more like when you have the understanding questions (referring to these style of questions) where there is like a story that you have to put that concept into there, but it always that interesting bit that you need logic to solve out" (Interview, February 24, 2014). All five students shared this interest and appreciated the scaffolded teacher help with "complex unfamiliar" (Queensland Curriculum and Assessment Authority, 2017, p. 2) questions. This scaffolded help came in the form of solving the question together as a whole class or with small groups after they had been asked to solve such questions in small groups or as individuals. Therefore, while the effect social learning had on students' attitudes will be discussed in Section 7.2.5, having access to peer or teacher support also had an impact on students' competence.

Students explained in interviews how they would go to the exams feeling proficient and also possessing confidence in their ability, equipped with strategies to solve such problems, even if the problems were completely different. Their prior knowledge was valued enabling them to feel competent to tackle these harder problems. Oliver surmises "In previous years, it has just been about going into the normal course, but this year, I have been able to do harder courses, which has been a lot more interesting" (Interview, June 25, 2014). Students were able to gain a sense of competence quickly, and often on their own. This efficient pacing of instruction meant students' advanced cognitive abilities were catered for.

7.2.2.2. The efficient pacing of instruction.

This section discusses how the compacted curriculum within the Mastery Learning Model framework enabled students to feel a sense of competence in light of research discussed in Section 2.3.2. Section 2.3.2 revealed how gifted students could process and understand regular classroom learning tasks at a faster rate. It also revealed how gifted students did not require as much repetition as their non-gifted peers do in their learning.

This curriculum compacting process was helpful for Oliver as he only needed to complete the pre-quiz to gain a feeling that he had mastered the content and skills given in the initial unit outline. His confidence in his ability was evident when he reported how he knew what he needed to know, and as a result, rarely listened to teacher-directed instruction. He admitted that he did not "pay that much attention (but did) listen in sometimes" (Interview, December 2, 2014). He did not need to listen as he felt already competent in this area. Achieving over 85% on the end-of-the-Year-10 course assessments confirmed his competence. When speaking with Oliver, he affirmed that he was "excited" about completing the upcoming Year-10 exam, even though he was still in Year-8. With such a high result, I was concerned, as his teacher that the work could have been even more challenging, however, he revealed that he felt the difficulty level was "about right" (Interview, December 2, 2014).

Similarly, Ty "pick(ed) stuff up really quickly. I do something once, and I remember it for a quite a while" (Ty, Interview, June 26, 2014). Oliver echoed this sentiment when he stated how in previous years, mathematics was very repetitive and even so, he would "probably do it even if (he) knew most of it already. It is often the case that we would learn one thing in the first few weeks, and they just kept revising it and revising it, and it just got really boring" (Interview, February 25, 2014). Other stories already shared, highlight how Miley and Walter also gained a sense of competence quickly. After students had completed the pre-quiz, they may have still had small gaps in their knowledge. They would then need to go and learn these skills by completing the alternate questions or activity.

Applying a scaffolded mastery approach within a social cognitive model enabled students to develop and demonstrate the sense of competence to complete more advanced tasks and in most cases, more quickly according to relevant preconditions. This proficiency was evidenced with students' helping each other, solving accelerated tasks on their own, or attaining mastery on an end of term summative assessment. No gaps were created with a compacted curriculum as students had to demonstrate mastery of all key concepts aligned with unit outcomes stated at the beginning. Interview data revealed that a teaching model framed by the Mastery Learning Model positively impacted on their feeling of competence (Ryan & Deci, 2017), as students were able to work autonomously, self-appraising their levels of mastery and determine whether further consolidation of skills was required. As Bandura (1986) suggested, it is this self-appraisal, seeing others master this work in a social learning zone (to be discussed in Section 7.2.5) that helped enable students to feel competent and then tackle more challenging tasks. All five students stated an advantage of the Mastery Learning Model was that they could feel confident that they knew what they needed to know sooner, which enabled them to spend more time on more complex enrichment tasks and challenges. There was no need for repetition.

7.2.2.3. Summary.

All students appreciated having their achievements valued and having control over what they learned. These involved students mastering core content according to a prescribed Australian Curriculum aligned unit outline. All learning was compacted, and if students did not master the compacted curriculum, they would need to complete the additional or alternate activities to ensure mastery. The process of curriculum compacting uncovered the advantages to students who learn concepts quickly. The acknowledgement of students' competence was partly the reason for students' change in attitudes towards mathematics. Instead of seeing mathematics as boring and repetitive, they saw it as challenging and relevant. Self-efficacy levels were influenced as students were seen helping each other and solving questions of greater complexity with more confidence. That is, instead of being unsure how to solve a more complex problem, they could recognise the components of the problem. An example was given showing how this recognition enabled the students to break the more challenging problems into smaller easier to solve questions in order to arrive at a successful solution. This, therefore, also had an impact on their exam results, as the students were able to solve more complex questions more efficiently. Curriculum compacting is viewed as a form of acceleration, as students are moving through the curriculum at a faster pace. Acceleration and how the students responded to a more complex program is discussed next.

7.2.3. Complexity

After students had demonstrated mastery of core concepts, they could go on to work on more complex and advanced content that are normally addressed in the Year-8 Mathematics Curriculum. This section, therefore, discusses the influence this had on their attitudes towards mathematics. Three key points are discussed:

1. Students worked on advanced content in the regular classroom;
2. Students contrasted their feelings of interest in the current more challenging rich and relevant program to their past experiences of mathematics; and
3. Three key stories are shared about how students' attitudes toward mathematics changed as a result of working on authentic enrichment challenges.

7.2.3.1. Students working on more advanced content.

Section 7.2.2 discussed how the curriculum was able to be meaningfully compacted, which is a form of acceleration (Quinlan, 2017). This section focusses on three key stories that provide evidence for how the accelerated tasks contained in the Mastery Learning Model impacted on students' attitudes towards mathematics. Oliver reflects on how he was "excited" to finally be challenged in mathematics at school. Miley described how demonstrating mastery provided her with a "stepping stone" to more complex tasks. Walter's story similarly suggests that it was these accelerated tasks that provided him with the incentive to seek a future career in mathematics.

Oliver's experiences: When asked on the positives and negatives of this current mathematics program, Oliver felt "challenged" by learning accelerated content such as "surds, completing the square, trigonometry and algorithms" and thought they were "interesting" (Interview, December 2, 2014). He retrospectively shared how he would have appreciated having being "taught about that (a Year-10 concept), before going into it". Later in that same interview, in response to a question about his thoughts in relation to what he did not like about being given choice in how he learned. He shared how he would "miss things" when learning from the textbook, as it (the textbook) did not "describe it (a given concept) too well". He would then search the Internet or use the provided video tutorials online to try and find the answers to his problems. He shared how he would still "miss out on little bits of information". Even amidst this minor confusion, and in response to a question whether his engagement levels had changed, he shared it had "changed from the early days to when (he) started the Year-10 math worked. It did change. It became more interesting" (Interview, December 2, 2014). In that interview he expressed that he wanted to continue with the accelerated course into the next year. Oliver suggested that such in-class acceleration was "great" as he could be with his aged peers and not with people who were "like fifty centimetres taller than (him)". Oliver summarised that the "Maths C (work) ha(d) been a lot more challenging than the Year-8 or Year-9 mathematics". He then continued to explain the specific concepts that he enjoyed learning about the most. While Oliver was the only student to attempt the formal, or actual Year-10 course, other students received access to Year-10 content.

Miley's experiences: Miley was given a challenge which involved her discovering the pattern that emerged when you expand the bracket $(a + b)^2$ and then expand $(a + b)^3$. She was asked to use the internet to help her explain and label the pattern that emerged and predict what $(a+b)^5$ will look like. Miley's response to this challenge, which involved understanding more advanced Year-10 level content, is discussed below in Figure 7.10.

$$(a+b)^0 = 1$$
$$(a+b)^1 = a+b$$
$$(a+b)^2 = a^2+2ab+b^2$$
$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$$
$$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$$
$$(a+b)^5 = a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$$

Figure 7.10. Miley's working on the Binomial Theorem challenge.

Miley emailed me her answer to the question shown in Figure 7.10 very late one school night. She initially shared that she "loved this" task (Personal communication, June 22, 2014). She shared how she was determined to find the pattern before going to sleep. Miley's actions in this example suggest a state of flow (Csikszentmihalyi, 2002) whereby she was not able to stop work until she figured out the relationship between her pattern of numbers and the theorem. In class the next day, I asked for her thoughts on this style of advanced questions; she used words "really cool", "interesting", "challenging" and "more engaging". This level of engagement was evidenced by her staying up into the night and not stopping until she had come to a satisfactory solution. In response to whether she enjoyed these questions, she shared that she did. She shared how these tasks were "different" and she "enjoyed this more than learning from a textbook" as with this kind of challenge "you go out and find different kind of patterns…it makes it interesting".

Walter's experiences: For Walter, completing accelerated tasks gave him more "self-confidence" in mathematics. Walter would spend many nights completing accelerated tasks involving an understanding of much more complex concepts including trigonometry, binomial theorem and the golden ratio. For example, Walter was able to work independently to find a relationship between Pascal's Triangle and Binomial theorem and submit a concise justification of this relationship. I did not ask Walter to do any of these tasks. At the time when he submitted his responses to these mathematics challenges, he shared, "I could have done a 100-page report on this, but I tried to stick to the basics" (Walter, Interview, June 25, 2014).

These experiences illustrate how students enjoyed solving more complex enrichment challenges. Students were engaged and worked in a state of flow. That is, they did not want to stop work until they had found a successful solution. Often these successful solutions came from a sense of competence and belief in their ability to solve such problems as was discussed in Section 7.2.2. This engagement is noted as an increased interest in mathematics which is contrasted with a lack of interest and challenge they had experienced in the past.

7.2.3.2. Interest in current program.

This section contrasts the types of challenge Oliver described before and after coming to this class. A discussion then follows on the effects this challenge had on his attitudes towards mathematics. Oliver's story was selected as it showcased how boredom is often a very real state for gifted students. Feelings are boredom are now contrasted with his attitudes towards mathematics when he is challenged and working in a focussed, engaged state Csikszentmihalyi (2002), would refer to as a state of "flow" (p. 71).

Prior to the study, Oliver had become resigned to the understanding that he would not be able to learn mathematics at school:

> I did find that when I was in a public school that the mathematics was too easy, so I did not get to do anything hard during school time. It was mostly boring, mostly repetitive, and mostly it clearly says how to do it. Everything was from the textbook, but there was Mathletics as well. It would not just say problem solving. It was all pretty easy. (Interview, December 2, 2014)

Oliver maintained that if he wanted to be challenged, it would only happen at home:

> I'd come home with a book and just write out random sums until I found the continuous number that it ended in. I did not get to do anything hard during school time. It was only after school that I got to do anything challenging. (Interview, February 25, 2014)

These excerpts taken from an interview with Oliver echoed the sentiments of the other four students. Adding to this, a whole class survey revealed words such as "easy" (N=23), "boring" (N=25) and "repetitive" (N=15) were words frequently used to describe their past experience with mathematics. In this example, Oliver shared his history with learning mathematics. He talked about classroom challenge in mathematics as rare, while self-set home-based challenges would be what fuelled his love of the subject. Neither affected his love for mathematics, but words used suggest negative feelings associated with classroom learning (boring, easy and repetitive) and positive feelings when speaking about what he did at home (love, fun and challenging). This is contrasted with the use of the word "challenging" (N=46) when referring to the current teaching program. This challenge is associated with both accelerated tasks and enrichment challenges given. Oliver would often give different views on which aspects of the current mathematics program he enjoyed the most, but it was clear that he enjoyed being given challenging problems set for a higher year level or, real world enrichment tasks as will be discussed in the next section.

Bree’s interest in mathematics was “a lot better than last year” (Bree, Interview, Jun 23, 2014). However, when asked to rank out of 10 how much she liked mathematics, she gave it a zero at the beginning of the term and a two out of ten at the end of the term. While she shared that she did not mind the mathematics classes, she seemingly found the challenge level set to be appropriate. She shared: “I wouldn’t say that it’s too difficult, but it’s challenging on a level that isn’t boring, but it just gives you that “oh wait, I need to do this” (Bree, Interview, February 27, 2014). Bree revealed in a later interview, that she would often get questions wrong on the formative quizzes. She would then go and seek to learn the information that she did not understand by asking her brother Oliver, or parents for help. This level was also evident in her in-class debate with Miley (Appendix E) over the correct answer to a challenging problem. She thought she was correct, but then had to seek teacher assistance as she realised she was not correct. On the one hand Bree was engaged and challenged, but her answers to questions about her interest in the subject were conflicting. She continually referred to her past experiences in mathematics that were filled with boring repetitive work. It was evident that her attitudes toward mathematics would remain mostly negative because of this on-going negative experience with the subject.

According to these findings, the students’ voices highlight how learning within the Mastery Learning Model enabled a sense of flow which impacted positively on learning mathematics. Learning mathematics in the past was classed as boring, repetitive and lacking in challenge. This style of learning is contrasted with a feeling of challenge, interest and relevance while learning within the Mastery Learning Model framework. This next section further highlights this positive impact, specifically focussing on key stories where students worked in flow on enrichment tasks.

7.2.3.3. Challenge with authentic enrichment tasks.

This section presents key stories which highlight how challenge, given in the form of enrichment tasks impacted on students' attitudes towards mathematics. Davis and Rimm (1998) defined enrichment as providing students with a "richer and more varied educational experience, a curriculum that is modified to provide greater depth and breadth than is generally provided" (p. 105). This section draws data from students' responses to interview questions about their thoughts, feelings and attitudes to mathematics as a result of completing such enrichment tasks.

Section 2.5 discussed the work of Koshy et al., (2009) who suggested that teachers often resist inviting students to engage with enrichment options because identified gifted students were not able to demonstrate an understanding of the regular curriculum for the respective year level. The use of the Mastery Learning Model framework provided these students who had demonstrated mastery the opportunity to show they understood the regular and accelerated curriculum to complete such challenges.

Three stories are discussed. Firstly, Oliver shares how he appreciates being challenged with problems with no one set answer, while Miley's excitement for learning is obvious when she is set with such challenges enabling her to work in flow. Finally, Walter describes how he now sees a purpose for learning mathematics.

Oliver revealed that one of his favourite aspects of mathematics this year was completing a traffic jam modelling assignment. He had devised a way of representing the traffic flow with a mathematical formula. This response was in answer to a challenge that asked him to formulate a plausible mathematical solution to the traffic flow problem that existed every morning on his way to school. In response to a question asking him to share his thoughts and feelings concerning enrichment tasks, Oliver shared:

> I like them; they're not just simple questions. They're not just normal problems that push you towards a certain formula. It does not give you specific information about the size of the cars how big they are it just gives you a problem and asks you how you are going to solve it...and the traffic jam was like me explaining the answers to the question…It's quite easy (continues to discuss his formula he developed for this task) … The enrichment tasks are quite fun. I have just finished or nearly finished the traffic jam I have already handed in when the perimeter and area of the same investigation. (Interview, April 1)

As previously stated, Oliver was growing accustomed to solving simple problems that involved remembering a formula, substituting into the formula and arriving at a single correct solution. In this task, he was asked to use mathematics to find the quickest route to school using actual data he had collected. Oliver would take this task a step further, by experimenting with his own formula that he subsequently tested. He later explained to his mother how long it would take her each morning to get to school.

Similarly, Miley also preferred enrichment tasks as she was able to experience a greater level of "engagement. She shared how she preferred learning through enrichment tasks because they are more "memorable" then just "looking at a bunch of words and trying to remember it" (Interview, February 25, 2014). She continued discussing a prior experience in mathematics where the teacher would continually ask her to open the textbook, and she would "look at the concepts, answer the question and then check to see if it (was) right". This comment was made in response to a query on her thoughts about an enrichment task she had just completed. I had asked her to find the relationship between the circumference of a circle and the diameter, using the Circumference formula. Once she had discovered a number that very closely resembled pi a sense of excited "engagement" ensued, as she continued to work in flow.

Earlier in the year, Walter had expressed a preference for completing higher level tasks because "it was more challenging" and gave him "a sense of accomplishment to know that you can do it well" (Walter, Interview, April 2, 2014). Later he expressed a different view sharing: "I've started to like it better this term, I guess because of enrichment tasks...I guess I feel a sense of accomplishment" (Interview, April 2, 2014). Walter recounted how the use of enrichment tasks enabled him to see a place for pursuing a career in mathematics when he shared,

> Yes, it has…I thought I would not really like to do a career with mathematics, but with the enrichment tasks, it shows me that there is more to mathematics, that there is more interesting things like the golden ratio and Pascal's triangle. It shows me that there are more interesting things to it, as it shows me there is a purpose to it.

Here, Walter attributed a change in attitude towards mathematics largely due to his work on enrichment tasks. In this case, he was referring to an investigation into the golden ratio and Pascal's triangle which revealed to him a purpose for learning mathematics. In the first example, he was involved in finding instances of the golden ratio in life and explaining his mathematical understanding. The second, involved him hypothesising why Pythagoras theorem is $a^2 + b^2 = c^2$. After being given these challenges on the school's online learning management system, he went away and worked towards a solution to both challenges at night time. In the first example he used measurements of his own body along with research in finding a close approximation to the golden ratio, whereas, in the second, he was able to use algebra with some initial assistance from the teacher to show why $a^2 + b^2 = c^2$. He shared that he "liked it (mathematics) a lot better", as the completion of these tasks left him with a sense of accomplishment.

The above examples provide evidence on how the students felt challenged or "engaged", describing how the use of such challenges made learning "memorable" (Miley, Interview, February 25, 2014). Students were "interested" (Walter, Interview, April 2, 2014), "challenged" and found learning purposeful (Walter, Interview, April 2, 2014). The word "fun" (N=12) was also a word frequently used to describe various aspects of the program that enabled the students to become engaged in the problem-solving process, rather than being passive text-book style learners.

Both examples above, present different cases for students choosing to complete enrichment tasks voluntarily and in their own time, feeling a sense of accomplishment when completed. Therefore, these examples highlight how the use of the Mastery Learning Model in the classroom enabled students to feel a heightened mathematical self-concept. These examples also highlight a significant improvement in attitude towards mathematics as a direct result of being exposed to real-world problems of greater breadth and complexity.

7.2.3.4. Summary.

A positive impact on students' attitudes towards mathematics was evidenced in a variety of different ways in the examples stated above. Examples discussed, illuminated how students felt an improved mathematical self-concept. They worked in a state of flow voluntarily at home and considered a career in mathematics, because of this access to authentic enrichment challenges.

A compacted program would enable students to work on tasks that were more challenging and investigate real life and complex problems. They were able to draw connections and see relevance to mathematics that they had not noted before, as will be discussed in further detail in Section 7.2.4. Ty commented, “last year we got tonnes of easy work which irritated me”. All five students said that one of the things they loved about this program as guided by the principles of the Mastery Learning Model, was a significant reduction in meaningless repetition. This challenge, when coupled with the ability to have control over what and how they studied, allowed students to have confidence in knowing what they needed to know, as well as identify any gaps in knowledge that they needed to “fill”.

7.2.4. Relevance of learning experiences.

Section 2.3 cited research which emphasised how students needed to learn mathematics in a way that enabled them to see the relevance in what they are learning. Norton and Reid O’Connor, (2016) in their report to the Queensland Curriculum and Assessment Authority went further, asserting the need for authentic real-world challenges in the mathematics classroom. This section argues how a program guided by the principles of the Mastery Learning Model enabled students to explore mathematics contextualised around authentic experiences. Stories of students’ learning experiences are shared to illustrate the influence this had on students’ attitudes towards mathematics. It is noted that many of the stories already shared above illustrate how this relevance was evidenced when students completed enrichment investigations exploring topics such as the Golden Ratio in real life, the area of a windscreen cleaned in the rain or how to get to school quicker each morning.

7.2.4.1. Relevance impacting on attitudes towards mathematics.

Responses from interviews suggest that while some students’ attitudes were not changed as a result of a relevant program, others were. This Section discusses three such cases. Firstly, Walter’s attitudes towards mathematics significantly shifted from one of just completing questions to considering a mathematical career. Miley began to see a purpose for learning mathematics. She was able to connect what she was learning in the classroom with what she was observing in the world outside the classroom. Ty would also note real-world connections with what he learned and the outside world.

Walter’s attitudes: Walter was asked if this teaching program changed his attitudes towards mathematics. He responded:

> Yes, it has…I thought I would not really like to do a career with mathematics, but with the enrichment tasks, it show(ed) me that there is more to mathematics, that there is more interesting things like the Golden Ratio and Pascal's triangle, it show(ed) me that there are more interesting things to it, as it shows me there is a purpose to it. (Interview, June 26, 2014)

He had found a purpose for learning mathematics and started to contemplate a future mathematical career. One task he completed required him to explain what the golden ratio is with mathematical reasoning (Appendix R). He then found as many possible examples of the golden ratio in his world. While Walter's mathematical justification was limited, he was able to get excited (as denoted by the tone of his voice in a personal conversation) about the connections he was able to form with Fibonacci's sequence. He was able to come to terms with a basic understanding of what the golden ratio is. He had done enough to make him curious to find out more, and this curiosity contributed to a significantly improved attitude towards mathematics. He had moved from hating mathematics to the point of not wanting to do it any more, to considering a future career. I asked him again about his feelings in regard to this comment a year later, and he told me how he had selected the two hardest strands of mathematics to complete in his senior years of schooling and was still hoping for a career in mathematics.

Miley's attitudes: Similarly, Miley had found relevance for mathematics which contributed to her changing attitudes towards mathematics. After learning about the origins of pi (π) and how pi came about, Miley was able to apply it to her real life on the way home from school one afternoon. When Miley (Interview, June 23, 2014) was asked why she attributed her love of mathematics to her ability to apply what she had learned to her life, she responded:

> Because you can use it for everything. Like when I was telling you about last term about circles and diameter. I could tell you about the surface area that the windscreen wipers were wiping. Like when I was walking the dog, it's like how much space do you need to keep the dog from wandering onto the street.

She had discovered this on her own where in an enrichment task and now sees a use for mathematics "for everything". She highlighted her love for mathematics in that same interview repeatedly (N=7) stating as such. Her attitudes towards mathematics had improved to a point where by the end of the study she "love(d) mathematics” when she once felt that she was “not so happy about mathematics” at the start of the year.

Ty’s attitude: Ty had already iterated how in previous years the repetitive nature of learning mathematics had affected his motivation to want to learn it. Ty responded to a question asking him about how his attitudes towards mathematics had changed by answering,

> A lot more of using it in the real world, instead of just sitting down and just writing it out….and like if you’ve already learned something, you do not have to go back and learn it again and again and again. (Ty, February 25, 2014)

He continued when he suggested that his attitudes toward mathematics had shifted as he was able to find this relevance. Furthermore, he was able to have an element of control over what and how he learned. In an interview on April 3, 2014, Ty revealed how he enjoyed the use of the digital learning platform as he was able to choose between these real-world tasks and regular mathematics learning, but at a more complex level. He contrasts previous attitudes associating the learning of mathematics as “really boring” to that fact that it “has got a lot better this year” because he “always got a choice of different things to do”. In this sense, he liked the “real world” tasks as it added variety to what he was learning.

7.2.4.2. Summary

While these stories have been introduced in other sections, they were discussed in greater detail here to elaborate on how the provision of authentic real-world challenges impacted on the students’ attitudes towards mathematics. As a result of these three students finding relevance in their learning, attitudes shifted from a state of boredom and giving up hope, to one of interested engagement. The students also enjoyed learning independently of the teacher outside the classroom. All five students expressed how they liked that this program was different from previous year’s mathematics learning as they could see a contextualised purposeful use for it. While this included Bree, she still did “not like mathematics”, even though she had continually received the highest grade in the subject.

7.2.5. Social learning.

This section discusses findings which suggest social learning had an impact on students’ attitudes towards mathematics. Bree, Miley, Walter and Ty enjoyed the social aspect of the classroom. Oliver, however, indicated that he preferred the quieter classroom learning environment, allowing him more time to work individually. He also enjoyed helping others too. Section 7.2.5.1 provides an excerpt from two separate in-class discussions between Miley and Bree and Walter and Ty which reveal how students responded to challenging questions in a collaborative sense. Section 7.2.5.2 elaborates on an example of social learning where other students depended on Oliver's help, while he tried to complete his own learning where he was continually interrupted with questions for assistance by these students.

In a question asking her about her feelings regarding the social nature of the classroom, Miley explained, "I love when I get something wrong, that I can explain it with my friends so that they can help me understand. When I talk to my friends, they often tell me of a different way of solving a problem, and it helps me" (Interview, June 23, 2014). In the examples discussed below, the students started at a point where they did not know what to do, or how to solve the question, to the point of arriving at a successful solution. Both examples show how they were working together to solve complex, unfamiliar mathematics problems usually reserved for students in higher year levels. The transcript of the in-class conversation also presents the excited tones of their voices as they figure out how to solve the second problem.

7.2.5.1. Debating the answers.

There are repeated examples taken from class discussions which show the students debating the correctness of their answers. It was these debates that helped both Miley and Bree understand the concepts better. In-class audio and video recordings chronicled how students tutored each other and debated the correctness of their answers.

Increased excitement levels were also evident by the tone and volume of their voices. In this example, Miley recounted a debate she had with Bree over whose answer was correct.

> Yes, but then when I get it wrong, I spend ages on it until I figure it out, and Bree will tell me the answer and tell me not to worry about, but we'll keep arguing about it until one of us understands it. (Interview, June 23, 2014)

In this example, Miley recounted arguing with Bree over the correct answer with the end result being an improved understanding of the concepts in question. The students were participating in an in-class activity that required them to pair off and solve a range of more challenging problems together. Bree and Miley turned this into a competition to see who could solve the question correctly first. An argument ensued over who got the first question correct, until (after a few attempts) a consensus could not be reached, so the girls asked the teacher for the correct answer. Once Miley realised that she was correct, there was an instant display of excitement when she exclaimed, “Yes, I won!” This debating over the correct answer happened many times, as it was often the case that the students would engage in the kind of learning that required them to work together. They were also encouraged to find creative ways to make this learning fun. For Bree and Miley, this meant turning the challenge questions into a game.

Bree:	I know how to do this with a calculator…
Miley:	Miley bangs the desk in excited tones…..signifying to Bree while giggling that she has the answer
Bree:	You got it wrong….
Miley:	Hang on…I still think it’s 540
Bree:	(Bree works out question again verbalising her thinking) Hang on. I’ll put it into the calculator.
Miley	See, that’s what I got.
Bree:	Wait a second. Wait a second. Nah, I got it. This will take me ages…hang on…)Asks the teacher for help).

An extract was also taken from a conversation between Ty and Walter, which replicated this previous example discussed here. Walter ant Ty’s response when they were asked to simplify the following ratio:

$$\frac{5}{8}:1\frac{3}{4}$$

Teacher:	Is this question hard?
Walter:	No, you just simplify it.
Teacher:	So how do you simplify it?
Ty:	You just turn it into the same um…what do you call it…the same denominator…
Teacher:	Yes…good…
Walter	They both go into 8.

Ty: (Talking about the second portion of the ratio, Ty proposes to turn the mixed number to an improper fraction by saying). You turn that into seven.

Walter: Yes.

They then both seem to add the two fractions together to get 14 somehow. In both examples, the students could not agree on a correct answer. The students demonstrated a sense of competitiveness when the one who got the answer correctly celebrated. In both examples, the students asked the teacher to adjudicate, edifying who was correct. As the teacher, I would not give the students the solution, but rather provide them with guiding questions to help them understand the processes involved in solving the question. Students thought out loud expressing a dogged (ascertained by the heightened and determined tone and volume of their voices) determination to prove the other person wrong. While one student was correct, the other student who was not correct was able to learn from their partner who would explain how they arrived at the (correct) answer.

The discussions referred to above provides examples of how students co-constructed their understanding of different and complex mathematical problems. The audio recordings had many (N=25) occasions of one student talking over the top of the other in rushed and excited tones. The students spoke in a rushed way because their co-construction of knowledge was drawing closer to a successful solution. Video footage revealed this active engagement and excitement in the learning process. Four out of the five students suggested that they liked to learn both with their friends and independently as well. Bree (Interview, February 24, 2014) revealed the advantages to her for learning in pairs when she shared:

> The best way I learn is when I interact with the people around me. I like it better when you can work in pairs and ask each other when you are not sure rather than just being told to sit down for a period of time and just do something on your own.

A reference to learning in previous years where the "classroom was quiet". Oliver revealed that while he liked helping his friends, he also liked working alone. He shared how he sometimes found it hard to concentrate if the classroom was too noisy. These examples also further revealed how students were able to receive specific help from the teacher, as well as from their peers. The next section presents the outworkings of a social cognitive model where students tutored each other and worked collaboratively to help each other in a less competitive sense.

7.2.5.2. The social construction of learning and peer tutoring.

The previous section reported on in-class recordings of students helping each other, all sharing that they liked the aspect of the program that enabled them to work together. This section shares students' thoughts on helping each other. Walter, Ty and Bree all shared that this helped them learn. In contrast, Oliver shared a preference for individual work but did not mind helping his friends on occasion.

Oliver was used to teaching himself, so he did not need others around him to learn. He revealed in every interview how he preferred to work on his own. He shared,

> I do prefer working on my own, but I always do like being near someone if I do not understand something or if I am reading a textbook and I do not know what it means. I do not really enjoy being in large classrooms, as it usually means doing group work…I do not really like group work. (Interview, February 25, 2014)

Oliver would describe how he, "love(d) being able to work things out logically, even if (he did) not know the concepts" (Interview, February 25, 2014). In one interview, he shared how he liked to help other people but was often frustrated when he tried to help his sister. The following sample discussion with two students provides an insight into why Oliver prefers independent work.

Student 1:	"Oliver: You do the hard questions, and we'll do the easy ones".
Oliver:	"No worries (starts on the hardest problem as identified at the outset by the teacher)".
Student 1:	"This one is hard!"
Oliver:	"It's obvious. Even if they want the per cent of that whole circle, it'd still be equivalent to that. Hey, do we have to do working out?"
Student 1:	"I hate these questions (referring to the fact that he doesn't like problem-Solving)".
Oliver:	"I know how to figure that question out (Continues to help the student and then goes back to his own work momentarily)".
Student 2:	"Oliver! What is cross multiplication?" (Personal communications, February 25, 2014)

This discussion continued for the next 45 minutes. Oliver’s frustration was becoming clear in the tone of his voice. He continued, however, to help the two boys he was working with. At first, he was happy to help, but as time went on, he seemed to show increasing levels of frustration. The students continued to ask Oliver for help at very regular time intervals, and yet he was still able to complete the more complex questions that the rest of the class struggled with, on his own.

In this case, the group assigned questions at the outset ensuring a fair division of the workload. While the other students in the group tried hard to complete the “easier” questions, it was seen to be an easier route to the correct solution just to let Oliver provide the group with the “correct” solutions for most of the questions. Oliver managed to finish all of the questions on the challenge sheet. I soon realised Oliver needed to be explicitly told that he had the choice to work on his own, on a separate task, or work in the group. From that point on, he decided to work on his own on different tasks.

Figure 7.11 taken from this lesson shows Oliver doing the work, while the other two students watched on. In a question asking him to explain how I can tell if he was engaged and challenged, he shared that “If (he was) looking down intently at (his) work and constantly doing something” then he was engaged. He confirmed this to be the case in Figure 7.11.

Figure 7.11. Oliver is working on this task, while peers watch (Used with permission).

Social learning was observed in many ways providing evidence of students co-constructing new knowledge together, defending their answers to each other and experiencing frustration. It further suggests how choice impacted on their attitudes to learning mathematics. That is, some students chose to work individually and others, in groups. This section has revealed how by working in pairs, errors could be identified by the partner and the students could work in a social construction zone to find a satisfactory solution together. It has also revealed one reason why gifted students prefer to work alone, as they are often left with the lion's share of thinking and responsibility for completing the work. However, for some, a sense of relatedness impacted on students' to learn, as students encouraged each other construct new knowledge. Students were attentively listening in some points, while trying to learn from each other, and talking over the top of each other at other times doggedly trying to get their point across. These social discussions revealed a heightened sense of interest by the excited tones in the students' voices and the cheers when they were able to solve challenging problems together, that they would have struggled to solve on their own. Conversely, it also highlighted the case for allowing students to choose their own learning environments (within reason). While some students preferred to work in quieter isolation, this was not the case for all gifted students.

7.3 Summary

Section 7.2 has provided a wide variety of student testimonies that suggested their attitudes towards learning mathematics had improved when learning inside a classroom framed by the Mastery Learning Model. A sense of autonomy in learning enabled students to be challenged and stay motivated to achieve goals as stated in a unit outline. Students felt a sense of competence by achieving mastery on formative quizzes and being able to progress to solve more complex problems. This competence was also reflected at the end of term summative exams. Students' perceived self-efficacy was also seemingly influenced, as they were able to work on and demonstrate mastery of more complex tasks. Students remained challenged and were able to find connections with what they learned and what they saw outside the school, making their mathematics learning more relevant and interesting for them. Students were also able to co-construct their understanding of complex problems with their self-selected peers, on their own and with their teacher. They had control over what they learned and appreciated not having to re-learn concepts they already understood.

Chapter 7.1 provided evidence that students could learn and potentially master Year-8, Year-9 and Year-10 level mathematical concepts. Their understanding of key skills was demonstrated by their performance on enrichment activities, in-class learning tasks, along with achievement on standardised and non-standardised tests. The scaffolded online instruction structure was kept strictly within the Mastery Learning Model framework. The study did identify problems with the model that need to be addressed if the model were to be used by future educators, as will be discussed in Section 9.3.

Chapter 8
Discussion

The purpose of this study was to examine the learning experiences of five mathematically gifted students in Year-8 (10 – 11 years of age) extension mathematics class. The rationale and selection of these students was discussed in Chapter 4. They participated in the class conducted in a naturalistic format in which the teaching approach was informed by principles of mastery learning and gifted education (Chapters 2 and 3). Data were gathered from these five participants in the form of interviews, formative and summative tests, and personal communications. In this chapter, these data will be discussed to address the following research questions.

1. In what ways does a teaching program informed by the principles of Mastery Learning influence gifted students' mathematical performances?
2. In what ways does a teaching program informed by the principles of Mastery Learning influence gifted students' attitudes, motivation and interest in learning mathematics?

First, I will examine in Section 8.1, the mathematics achievement of the five selected students. Second, in Section 8.2 I will discuss findings from interviews, personal communication and surveys that elaborate on how students' interest, attitude and motivation levels in mathematics were impacted on in light of the literature reviewed and the second hypothesis. In Section 8.3, I present the revised Mastery Learning Model that is recommended for use with mathematically gifted students.

8.1 Mathematics Achievement

The five selected students, along with most other students in the class were able to individually master the Year-8 curriculum by completing the required and compacted learning activities at a faster rate. This section argues that when students are engaged in mathematical learning in a teaching program grounded in a mastery learning approach, they engage in deep learning, collaborative problem-solving and solving above grade level mathematical challenges in a way that meets meet both syllabus mandated requirements and the need for gifted learners to remain challenged.

First, in Section 8.1.1 academic performance as measured by both teacher-made curriculum aligned tasks and standardised assessments is examined. Section 8.1.2 reports on how students were able to move through the curriculum at a rapid rate that did not sacrifice understanding. Finally, Section 8.1.3 reveals how opportunities for deeper learning and problem solving were provided.

8.1.1. Academic performance.

Results discussed here and in Section 7.1 confirm previous research (Guskey, 2007; Kulik, Kulik & Bangert-Drowns, 1990; Wambugu and Changeiywo, 2008) reviewed in Section 3.2, suggesting that the use of the Mastery Learning model can "reduce variation in student learning outcomes" (Guskey, 2007, p. 9) ensuring more students achieve understanding to mastery levels than they do with more traditional teaching methods. However, the research reported in this thesis extends previous research by providing a teaching model and in-depth analysis of the students' experiences during the implementation of the model.

Previous research has indicated positive outcomes but has lacked the detail of context. For example, Section 3.2.2 discussed a study carried out by Shafie, Shahdan and Liew (2010) where 70%

of university-level students (N=30) achieved an A grade in a mathematics subject as part of a Bachelor of Education degree. Their study did not elaborate on what the students had to do to get an A in a Bachelor of Education course. However, the authors did share that the students achieved Mastery when they achieved a grade of 80% or better. These results were also similar to those reported in a study by Mitee and Obaitan (2015) testing the effectiveness of the Mastery Learning approach in 401 secondary school chemistry students in Nigeria. They found that 69% of their students achieved a grade of 80% or better. These studies did not elaborate on the detail of their teaching methods.

Findings from Section 7.1 revealed that 70% of the whole class in my study achieved a score of 80% or better on the teacher-made and Australian Curriculum aligned tests. Similarly, when using the Year-8 Pat Maths Plus assessment (Australian Council for Educational Research, 2011), 68% of students from my research achieved mastery (using the 80% mastery level). Students felt as though they "(knew) everything (they) need(ed) to know" (Ty, Interview, June 26, 2014), by effectively using assessment data and feedback in the smaller sized units to guide their learning. Studies cited had the common element of allowing students to progress in their learning once mastery had been demonstrated. Like my study, they also had a high percentage of students receiving mastery. The novelty of my findings lies in the revelation of the nature of the learning, types of learning tasks and the students' responses to these tasks and the learning environment.

Key findings included how students used the smaller sized units and accompanying formative quizzes to help them understand what they needed to learn. In response to a question asking for his thoughts about needing to master concepts before moving on to the next topic, Walter (Interview, February 26, 2014) shared that it was "really helpful as it really shows if you know it and if you don't". Ty (Interview, February 25, 2014) shared how he liked how he could "move onto the next thing" after he had understood a concept.

The achievement of academic results noted here, drawn from Section 7.1, are discussed in light of selected studies given in Section 3.2. Results discussed in Section 7.1.1 revealed a pre/post-test improvement in the whole class mean scores of 5.02% according to Pat Maths Plus assessments, and 7.75% in teacher-made assessments. Over 94% of students from the remainder of the class had scored over 70% on end of semester teacher-made Year-8 assessments. Figure 7.2 and Figure 8.1 diagrammatically reveal a shift from an evenly distributed curve at the commencement of instruction, to a negatively skewed bell curve, with most of the class scoring above 80% on final teacher-made assessments. Guskey used this figure to visually represent a reduction in the "variation in students' achievement levels…yielding a distribution of achievement more like that shown in the figure" (p. 80). He did not attribute numerical values to the scores, whereas the figure depicting the change in my results is drawn from student data. Therefore, while Guskey's figure represents what might be expected, Figure 8.1 shows what actually happened. These results are similar to Guskey's (2005a) findings, presented diagrammatically in Figure 8.1, but different from Ihendinihu's (2013) results. Ihendinihu's research, discussed in greater detail in Section 3.2.2, recorded a pre-post-test mean improvement of 25.66% in high ability students. My results differ significantly from Ihendinihu's study that labels students as "high ability" (p. 851) based on the results of a pre-test. The author's use of the term "high ability" is seemingly misleading, as these students performed well on a single test, and were not identified as gifted using multiple measures as some (McClain & Pfeiffer, 2012; Pfeiffer, 2002) would recommend. Bautista (2012) also discussed an intervention with high and low ability students but did not provide details on whether these students are gifted or just performed well on a pre-test.

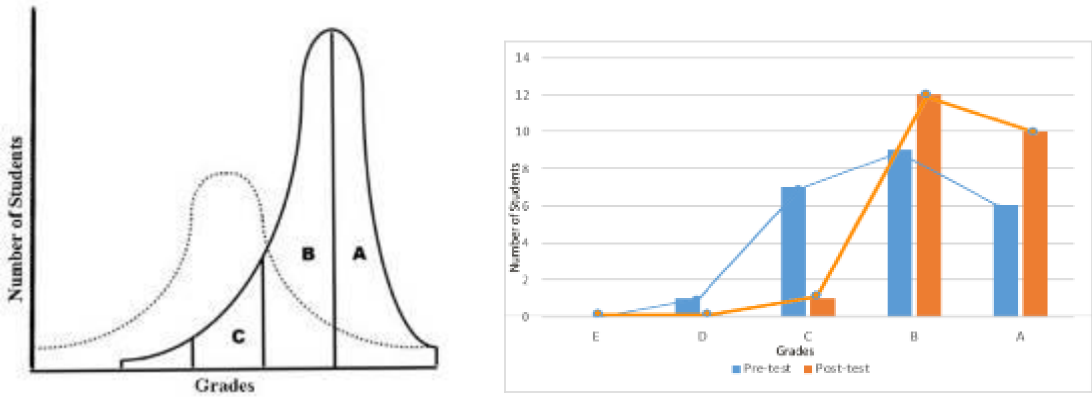

Figure 8.1. A comparison between Guskey's (2005a) findings and the current study.

Just as Guskey (2005a) observed more students achieving B and A grades, so too did these results show more students (N=22) achieving above 70%, scoring a B grade or higher, out of the entire class (N=23) according to the Year-8 level teacher-made assessments. Ten students were able to demonstrate above 85% level of understanding at the end of this study according to the Year 8 Australian Curriculum (Australian Curriculum Assessment and Reporting Authority, n.d.) aligned teacher-made Maths assessments. The program was able to be differentiated not only in what was taught but how it was taught. Some students were able to work on Year-9 level content and even to mastery levels.

During the implementation of the Mastery Learning Model, all five students were able to attempt on Year 9 level content, with some mastering many of these concepts. Data revealed how Oliver mastered all Year-9 concepts and spent much of his time successfully completing many Year-10 tasks. Figure 7.6 shows the improvement in students' understanding of Year-9 level concepts. The novelty of these findings is discussed next in Section 8.1.2.

8.1.2. Extended learning outcomes through accelerative practices.

According to Pat Maths Plus assessment (Australian Council for Educational Research, 2011) results, ten students from the entire class were able to achieve a grade of 80% or better on the Year-9 test, while nine were able to achieve this feat on teacher-made assessments that were aligned with the Year-9 Australian Curriculum (Australian Curriculum Assessment and Reporting Authority, n.d.). Results on the Year-9 level Pat Maths Plus assessment ($\bar{x} = 76.21$) and ($\bar{x} = 73$) on teacher made Year-9 level assessments were observed across the whole class. No studies were able to be found which reported similar research after searching *EBSCO, PsychInfo, CINAHL, PsychARTICLES, PsycEXTRA, PsycINFO, Eric Plus* and *Emerald* databases using the terms "Acceleration" and "Mastery Learning".

Significant gains in achievement on higher year level assessments are discussed here before pertinent conclusions are drawn. The results summarised in Table 7.2 revealed that while some students in the target group still needed to master some specific Year-8 concepts, four were able to progress to Year-9 level, while one was ready to begin working on Year-10 level content. The large standard deviation ($\sigma = 16.89$) on the teacher made pre-assessments along with the large spread (51.7) of scores suggest each student had a different level of understanding of the Year-9 mathematics concepts. Numerical data provided for the remainder of the class were not reliable indicators of achievement, as not all students completed every Year-9 level assessment, because they had yet to master Year-8 level equivalent concepts. Table 7.4 revealed a pre-test mean score for the four of the five target students ($\overline{x} = 49.43$) and standard deviation ($\sigma = 21.82$) for non-standardised Year-9 level concepts and pre-test mean score ($\overline{x} = 87.5$) and standard deviation ($\sigma = 6.84$) for the standardised Pat Maths Plus assessment. The PAT Maths Plus assessment was more generalised and contained only 4-5 questions per mathematics strand, when compared with up to 20 questions per mathematical strand in the teacher made tests. These findings are pertinent, as they highlight the importance of a comprehensive assessment regime that would identify specific areas of strength and potential weaknesses in every student. Many of the tests were digitally delivered, results were quickly collated and the resultant tailoring of the program able to be compiled promptly by the students. That is, they were able to identify gaps in learning from their printed results and ensure they went and learned the skills necessary to fill such gaps. Four of the five students then went on and completed a Year-10 level standardised pre-test achieving a mean score ($\overline{x} = 92$) and standard deviation ($\sigma = 3.46$). These results reveal significant gains in achievement and also reveal how students were able to progress on from Year-8 level content that they had already mastered most Year-9 level concepts (Example in Appendix N) and many Year-10

(Example in Appendix O) level concepts. Given these findings, two pertinent conclusions are made.

First, a series of smaller sized standardised tests for gifted mathematics students would make the process of identifying gaps in learning simpler. Second, these findings highlight that prompt feedback enabled instruction to be tailored to the individual's learning achievements.

These findings add to research already provided (Section 2.5.1) on the effectiveness of curriculum compacting, but add a further dimension, by asserting that compacting of smaller sized units is important. Furthermore, the findings confirm that students learn in different ways and at different paces. In order for teachers to cater for the individual differences of students in their classrooms in a meaningful way, a well-planned flexible learning environment is necessary, which provides students with the opportunity for autonomy in goal setting and effective feedback.

Interview data support these findings, suggesting students remained challenged by avoiding needless repetition. The work was not too difficult or overwhelming, so was within the students' ZPD. The use of a range of assessments enabled a more individualised program to be tailored for each student. Students were able to use both feedback and individualised goal setting to notice the above gains when they completed work with the respective formative assessments set at appropriate levels of complexity. Hattie and Jaeger (1998) suggest that goal setting when accompanied with effective feedback to be the "most powerful single moderator that enhances achievement" (p. 114). The authors suggest that teachers utilising mastery goals will also notice improvements (Effect size=0.50) in educational performance. While they did not point specifically to the Mastery Learning Model, the findings of this thesis add support to their suggestion. That is, according to Hattie and Jaeger (1998), you would expect improvements in results when you use a teaching model that incorporates both feedback and challenging tasks. Results shared above reveal that students were challenged within this model where improvements in both attitudes (Section 7.2) and achievement (Section 7.1) were noted.

Hattie and Jaeger (1998), suggest that the "incidence of feedback in the typical classroom is very low" (p. 114). In contrast in this study, students received prompt, and sometimes instant automated, peer, self and teacher feedback. According to Box, Skoog and Dabbs (2015); Donnelly (2010), and Tomlinson (2005) we now know that one reason this does not happen more often is because teachers feel constrained to cover a crowded curriculum (McGraw, 2018) coupled with the pressure to have students perform well in high stakes testing (Johnsen, 2017). The findings from my research have shown how teachers can be assured that their students have not only learned the required skills contained in the curriculum but have been challenged via enrichment and more complex content. While I cannot assert these findings would be replicated in a non-gifted classroom, others (e.g., Guskey, 2015) suggest there is "extensive research evidence (showing) the use of mastery learning can have exceptionally positive effects on student learning. My results with this target group of five gifted students, therefore, add to these research findings in asserting the effectiveness of this teaching model to improve student learning and provide gifted students with opportunities for deeper learning and problem-solving.

8.1.3. Opportunities for deeper learning and problem solving.

The findings suggest that students became engaged in deeper learning and problem solving because of a compacted curriculum that provided them with more time to complete authentic enrichment tasks. This section substantiates this claim by identifying components of the model that provided opportunities for deeper learning. These included the elimination of repetition, enrichment challenges and choice in selection pathways.

The elimination of repetition and provision of challenging activities were cited by four of the five students as contributing factors to students liking instruction framed in the way it was. In Section 7.1.2, Miley and Bree both related to past experiences when mathematics instruction involved a heavy reliance on the use of what they described as repetitive textbooks. According to Bree (Interview, April 1, 2014), this was one reason she did not like maths. In discussing his use of textbooks, Ty asserted that they would make learning harder to remember as he had no real-life association to draw from when it came time for exams (Interview, April 3, 2014). Students' prior learning involved no authentic enrichment tasks such as they experienced in a classroom framed by the Mastery Learning Model. Norton and Reid O'Connor (2016) in their report to the Queensland Curriculum and Assessment Authority spoke of such textbook-centred instruction as common in Australia and suggested the need for authentic real-world problems in mathematics classrooms.

The use of the Mastery Learning Model enabled deeper learning through enrichment challenges. Research by Neubauer and Fink (2009) suggested that cognitive growth happens as a result of challenge, such as was used and discussed here. In relation to his thoughts on enrichment challenges, Oliver (Interview, April 1, 2014) shared:

> I like them they're not just simple questions. They're not just normal problems that push you towards a certain formula. It does not give you specific information about the size of the cars how big they are it just gives you a problem and asks you how you going to solve it.

Oliver had repeatedly shared in interviews how he enjoyed completing the Traffic Jam enrichment task (Appendix S). He also enjoyed the challenge of being involved in accelerated work his older brother in Year-10 could not do.

Oliver expanded on his thinking in justifying his solution to this task as provided in Appendix S. In it, you can see how he not only experiments with developing his own formula but tests the formula in a variety of situations. Oliver engaged in deeper learning by experimenting with different formulas to solve a complex, real-life, relevant and unfamiliar problem. He provided sound mathematical reasoning to the problem, while also noting exceptions to when this will/not work. When asked which enrichment tasks he found most challenging, Oliver (Interview, December 2, 2014) explicates:

> Especially the traffic jam one. To find how long it would take, you needed to make a formula. You couldn't just punch the numbers in straight away. You needed to find something that worked, no matter what the numbers would be. So sometimes, you could try replacing it with 2's and 3's to see if you get a constant result and make sure it made sense. I liked it because there was no definite or one correct answer.

Stories in Section 7.2 shared how the five selected students felt challenged when compared to previous years' experiences when learning mathematics. Oliver pursued an understanding of the relationship of mathematics to traffic. Miley used mathematics when walking the dog, or when the windscreen wipers went on. Walter found relevance for mathematics to the point of wanting to pursue a mathematics career. The completion of these tasks was not mandatory. Bree and Miley revealed in interviews how they found tasks to be "more engaging" and "more memorable", while Walter suggested they were "more relevant" than regular mathematics tasks.

Similar to Matsko and Thomas's (2014) study, the effects were not only measured by improved grades. Rather, these students were engaged, interested and found more relevance to such tasks. The impact, therefore, was able to be measured by students pursuing these learning experiences (often at night) of their own volition.

Just as Matsko and Thomas's (2014) research had students choosing their own enrichment tasks, this study also noted the importance of allowing students autonomy to explore enrichment tasks to gain a greater depth and relevance to the topics covered. Once the students had mastered the required curriculum at a faster pace, they were able to remain challenged through the completion of enrichment tasks. The effectiveness and impact this had on mathematics achievement is measured by responses from students suggesting they felt challenged, felt the tasks were relevant, and the students were engaged in learning to a point where they were voluntarily working on these tasks at night time. These results support the findings in the literature and suggest the imperative for all teachers to compact the curriculum. This compacting meant that gifted students had more time to delve into solving authentic tasks through such enrichment challenges which is in line with the recommendations of Norton and Reid O'Connor (2016). The assuredness of understanding to mastery not only enabled students to participate in varied learning experiences with such enrichment challenges but also helped ease tensions associated with exams which is now discussed.

8.1.4. Opportunities for Students who Struggle with Exams.

The use of formative quizzes that accompanied the smaller sized units, provided students who once struggled with exams due to anxiety or perfectionistic tendencies with more chances to demonstrate their understanding.

Test anxiety, according to Vogelaar, Bakker, Elliott, and Resing (2017), often impinges on gifted students' ability to be able to demonstrate their potential as was seemingly the case with Bree. Their study suggested that the use of dynamic assessments, such as pre-post testing, can help lessen the negative impacts anxiety can have on gifted students in assessment. Section 6.4 shared how Bree attributed her low scores on tests to anxiety in testing situations. She later shared how she often struggled with longer questions worth more marks (Bree, Interview, April 1, 2014). These longer questions caused her considerable stress and anxiety, while Walter struggled with time.

Walter wanted every answer to be perfect and struggled to complete exams on time. Walter revealed in interviews and through observations that he would always over-analyse a question and tended to think of it as a lot harder than it actually was. According to Fletcher and Speirs Neumeister (2012), perfectionistic tendencies can negatively impact on students' achievement and motivation to achieve. The authors posit that the setting of mastery goals can be beneficial for such students. Walter later shared how the use of the skills sheet and the formative quizzes would help assure him that he knew what he needed to know (Interview, June 26, 2014).

Therefore, for both Bree and Walter, formative quizzes helped them to demonstrate their potential and understanding to mastery of the prescribed and often more complex content, as they completed these quizzes with no pressure or time restrictions (yet under test conditions where students sat with no-one beside them as they did the test). These students did not need to do formative tests if they had already mastered the pre-quiz.

8.1.5. Summary.

This section has discussed research findings that support the literature on the effectiveness of the Mastery Learning Model to positively impact on students' achievement levels. It has furthered research on the effectiveness of the Mastery Learning model that included accelerative and enrichment options for gifted students. These accelerative options allowed students to work on authentic tasks related to their interests and ability, which is in accord with recent recommendations (Norton & O'Connor, 2016) for mastery and authentic real-world learning opportunities to make mathematics learning relevant.

The novelty of my findings and the influence the use of the Mastery Learning Model had on students' academic achievement were noted, as follows: (1) No study was found which tested the influence the use of the Mastery Learning Model had on identified gifted students; (2) This study provides insights into the learning experiences of gifted students that are provided with accelerative options which allowed them to explore and understand to mastery levels content to a deeper level; (3) I have posited how the use of the Mastery Learning Model enabled students, who normally struggle in exams due to stress and perfectionistic tendencies, to master complex and often accelerated content; (4) The success of the program is not merely dependent on improved test scores, but also on students' attitudes towards mathematics being impacted on positively to the point of students completing authentic learning tasks in their own time at night. (5) Students were able to master content up to two-year levels beyond the current age-based year level; (6) The use of the Mastery Learning Model enabled data to be used by both students and teachers to drive challenge appropriate learning that some teachers report as difficult to achieve; and (7) a combination of a comprehensive assessment regime and a students' ability to master higher year level content provided extensive evidence to support a subject acceleration to an advanced Year 10 mathematics class.

These findings support that the use of the Mastery Learning Model had a positive influence on students' achievement levels as stated. The ability for students to be engaged in higher level and more challenging work had impacted on their attitudes towards mathematics as will be discussed further in Section 8.2.

8.2 Mastery Learning Model's Influence on Attitudes Towards Mathematics

This section explores the affective experiences of students during the program with particular attention to the selected students' attitudes towards mathematics. This analysis addresses the second research question. The findings support the assertion that students' interest levels in mathematics generally improve when they are given access to a curriculum that provides them with access to control, choice, challenge, complexity, and care in the learning process. The analysis draws on data from interviews, personal communications, surveys and student work samples. Four important findings emerge related to students' sense of autonomy, engagement with complex but authentic tasks, response to feedback and a sense of relatedness. The section concludes with a discussion of how students appreciated learning collaboratively with similar aged peers, but also with students of a similar ability level.

Students had the freedom to learn in a way that suited them on challenging content. Enrichment tasks provided students with a deeper understanding of the mathematics content and increased interest in learning mathematics as a result. The discussion that unfolds in this section, therefore, elaborates on the key themes which emerged from my research in light of literature reviewed in Section 3.4.

8.2.1. Autonomy.

This section examines the collected data from Section 7.2.1, which focussed on how students experienced and appreciated a sense of autonomy in their learning, in the light of research reviewed in Section 3.4. In Section 7.2.1, I described how the students had volitional control and choice over what and how they learned and that this impacted positively on their motivation to learn in mathematics, often independently of the teacher. According to Ryan and Deci (2017), having volitional control and choice is important if students are to feel motivated to learn, along with feeling a sense of relatedness and competence. Williams, Wallace and Sung (2016) suggest, however, that the effectiveness of giving choice is dependent on "student characteristics, teachers' self-efficacy beliefs, and teachers' classroom management practices" (p. 529). The giving of control and choice is countered by an "overcrowded curriculum" (McGraw, 2018, p. 157), high stakes testing (Johnsen, 2017) which often causes teachers to teach to the test (Suprayogi, Valcke, & Godwin, 2017) traditional means rather than allowing students the chance to investigate topics of interest to them (Hagay & Baram-Tsabari, 2015). This thesis suggests that when teachers allow students to work in a self-paced and guided way to achieve mastery, they encourage integrated regulation of behaviours that motivate students to want to learn. Students were interested in and found relevance in the learning material through varied learning experiences, enrichment investigations and having autonomy in the learning process.

Garn and Jolly (2014) examined the motivational experiences of 15 high ability mathematics students from the south Southeastern United States and found that students appreciated having choice, which helped them feel a sense of control over their learning. That study, like mine, noticed students gaining increased levels of intrinsic motivation that came through being engaged in learning experiences related to the students' interests. Conversely, the authors discussed the pressures these students faced from parents to get good grades and with being gifted.

Section 7.2.1 discussed how grades were utilised by students as a form of feedback to ascertain their level of understanding of what they had studied. According to Garn and Jolly, grades are seen as a controlling form of introjected motivation on student learning. This form of external motivation is described by Ryan and Deci (2017) as "adopting a regulation or value yet doing so in a way that is only a partial and incomplete transformation or assimilation" (p. 185). That is, students would partially regulate their behaviours on the basis of their grades. In another study, Ryan and Weinstein (2009) suggested that grades are perceived as controlling by students and autonomy-thwarting. This argument makes sense, as formative feedback will be almost instant and enable students to re-learn content or move on to the next concept, whereas students have to wait until the end of the unit to receive a grade. My findings, discussed in Section 7.2.1.3, were that the use of quizzes guided instruction and put the focus of grades on supporting understanding. My findings do not support the conclusions of Ryan and Weinstein's research. My analysis, however, of the impact of grades had on students' motivation levels was not a key focus of this thesis. Therefore, this finding is taken within the context of the impact the Mastery Learning Model had on students' motivation to learn mathematics and grades could be viewed as a contributor to their motivation to understand key concepts. That is, it was a regulator that provided students with the impetus to learn a concept if a grade revealed they had not understood the related content. The intrinsic motivation for students to learn came from an interest in what they were learning and understanding this content, as is discussed next.

In-class video recordings revealed how students were consistently focussed on and had control over their learning. Like the students in the Garn and Jolly (2014) study, Miley, Oliver, Walter and Ty all disclosed how they tried a lot harder because they were interested in the learning content and the way it was delivered. Students were working at night of their own volition, when the tasks, in and of themselves were non-compulsory learning challenges. Therefore, having a choice in what students learned impacted on students' intrinsic motivation levels to try harder.

Within a classroom framed by the use of the Mastery Learning Model, students learned according to a self-ascertained and teacher guided readiness. This study adds to the research on the effectiveness of self-paced instruction as some (e.g., Balentyne, Varga, Cooper, Edelman, & Huett, 2016) suggest is needed. In their research, Balentyne et al. found that gifted students' attitudes towards mathematics improved when they were given access to self-paced instruction. Results from my study revealed how students were interested in the learning process, and the learning was challenging. All five students completed more than was expected of them, with students completing non-compulsory enrichment tasks. Section 7.2.1 noted the importance of effective and timely feedback to guide student learning as is discussed next.

The self-directed learning process saw students work with the teacher to ensure the learning tasks were sufficiently compacted in a way that focussed on demonstrating understanding before progressing to the next concept/s. That is, the students determined if they needed to participate in more learning, based on results from formative quizzes. Ty shared how he mastered content quickly and enjoyed having the chance to work on harder content in a way that suited his learning style (Interview, April 3, 2014).

This process counteracts the problems identified by teachers in Box et al.'s, (2015) study. According to Box et al., teachers reverted to "low level drill and recite teaching method(s)" (p. 974) to ensure they "cover the curriculum". If I, as the teacher, asked all students to complete set tasks, then this could perceivably take a good deal longer as well. The Mastery Learning Model enabled students to use pre-assessment results from smaller units to decide what they had to learn. They did not complete a compacted chapter of the textbook when they had already mastered the content contained in the textbook. Added to this, the results in Section 7.1 showed they were able to master all curriculum materials and higher-level materials as well.

Goal setting was based on timely and individualised feedback, which helped students learn independently of a teacher. Once mastery was accomplished, the teacher would provide students with an array of options to work on. Students, therefore, had control over what and how they learned as they could, for example choose to work on a related enrichment task, work on a collaborative mathematics challenge such as those given in Appendix T or continue to work on more difficult accelerated content. Ryan and Deci (2017) propose that people possess a basic psychological need for autonomy (control over their goals), competence (mastery) and relatedness (closeness). According to Hattie and Jaeger (1998), feedback, challenging tasks, and specific goal setting are important influences on student achievement. Therefore, the link would suggest that if students know they have understood concepts to mastery, a feeling of competence would be generated through the teacher feedback. Students would be able to set meaningful goals with their teacher that included enrichment challenges and other more complex work. If Hattie, Deci and Ryan are correct, then this could explain Balentyne et al.'s lack of educational gains in achievement. Section 7.2.1.1 revealed that students appreciated having clear goals, timelines and a clear structure to work within. Given Balentyne et al.'s (2015) paper did not mention goal setting, and Hattie and Jaeger (1998) assert this to be an important factor in student learning, my research would suggest this to be necessary. This goal setting, however, would need to be completed by students as part of the differentiation process.

The use of curriculum compacting enabled students to master regular content faster and spend most of their time on complex problem solving, enrichment or higher year level content. Studies (Box et al., 2015; Donnelly, 2010; Smith & Southerland, 2007; Tomlinson, 2005, 2016) note the many problems teachers face preventing them from implementing known strategies for differentiating instruction. One such impediment is an over-crowded curriculum placing perceived time constraints on teaching (Jones, 1997; Lave, 1991, 1993; McGraw, 2018; Smith & Southerland, 2007) and teachers' beliefs about how students learn (Box et al., 2015; Smith & Southerland). According to this research, time restraints and the knowledge that gifted students have the neurological capacity to understand regular mathematics tasks at a faster rate can place a limitation on students' learning. Findings discussed in Section 7.1, however, suggested that a compacted curriculum which was responsive to students' levels of academic ability enabled the five students to spend most of their time learning higher year level concepts, complex problem solving or authentic enrichment investigations. Students' attitudes are also impacted positively when they are provided with this freedom with their learning. Findings from this study confirm those discussed by Guskey (2015) and Bloom (1987) which suggest that the use of Mastery Learning allows students who master content early opportunities to work on enrichment or extension work. In contrast to their studies, however, the use of a case study approach enabled an understanding from the gifted students' perspectives of their learning experiences.

This section has highlighted how students appreciated the autonomy to work at their own pace and have control over what and how they learned this content suggested that time restrictions often prevent teachers from differentiating instruction. It posited that this style of self-paced, mastery guided autonomous learning provided teachers with the confidence that the students understood to mastery the key content needed for state testing and agreed with research that showed how the use of the Mastery Learning Model actually found more time for differentiation in the form of enrichment and higher-level tasks. While Section 7.1.2 elaborated on the impact of the use of the Mastery Learning Model on academic performance, this next section will discuss students' responses to interview questions in relation to how this access to extended learning opportunities impacted on their attitudes towards mathematics.

8.2.2. Feedback Affect.

Studies (Hattie & Jaeger, 1998; Ryan & Deci, 2017) discuss the importance of feedback, goal setting and competence. However, they do not outline how these goals are realistically achievable within the classroom setting. Other research (Pryor, 2015) examined the potential of formative assessments to be somewhat illusory. However, in this program, three strategies were employed to promote engagement. These were: (a) teacher set performance goals; (b) enabling guided and self-paced student goals; and (c) providing on-going corrective feedback promptly. Prior to this study, two out of the five selected students had grown to dislike mathematics; one had decided to do the “bare minimum to get an A-minus grade”. While the other student found a revitalised energy through this teaching program which was devoid of repetition, rich in challenge and gave him access to learning from higher year levels. Students were able to feel competent by achieving mastery levels and were able to set meaningful goals based on topics they had not mastered, or on feedback given.

8.2.3. The complexity of content motivating learning.

An argument is presented in this section that students developed positive attitudes towards learning mathematics by being provided with access to a challenging learning program that they could master. I point out how this research is new and adds to findings shared in Section 2.3 on differentiating instruction for gifted students. In particular, Oliver’s story, discussed in Section 7.2.3, provided a good case for the use of the Mastery Learning Model as an alternative acceleration option as it does not force students into classes with older and physically larger students.

After mastery was achieved, students could work on content usually reserved for students in older year levels. Students saw the value in completing a balance of different types of complex tasks. These included accelerated content, enrichment tasks and collaborative challenges. A variety of findings discussed in Section 7.2.2 and Chapter 7.1 revealed that not only were students able to master at-year level concepts at a faster pace but were also able to move on to higher level learning. This in-class and ability-appropriate acceleration did not automatically mean that every student would be challenged, as was the case with Oliver, who was also able to demonstrate a mastery of Year-9 level concepts and many Year-10 level concepts as per the results given in Section 7.1.2 from the Pat Maths Plus standardised assessments and teacher-made assessments. While Oliver enjoyed working on these higher-level questions, he repeatedly (N=8 times) shared over three different interviews that he loved working on the traffic jam enrichment task. Students in Samardzija and Peterson's (2015) study, also found it harder to solve complex tasks but noted how these enrichment tasks were more memorable. This finding suggests the importance of providing gifted students with both enrichment challenges and higher-level challenge tasks to enable them to remain engaged and ensure the learning experiences are memorable. It is also important to note that the students were given the time and support to solve these problems successfully. They also had access to a knowledgeable teacher who could help them on these more complex tasks. If teachers focus on providing instruction where students move from one topic to the next without allowing them with such opportunities to succeed at solving more complex, deeper and authentic problems, then it would seem likely that more of the same types of work would be boring, as the students revealed in this study as discussed in Section 7.2.

Students received access to and were able to master more advanced work. A variety of accelerative practices currently used in schools was discussed in Section 2.5. I drew from Lubinski and Benbow's (2000) work, which defined acceleration as "appropriate developmental placement" (p. 138). Quinlan (2017) discusses an array of accelerative options available for teachers to use. She lists one such strategy, "flexible pacing" (p. 25) that can allow "students to work at their own pace" (p. 25). Like Carroll (1963), however, I would think that all students "should be allowed to proceed at their own rate" (p. 20). According to neurological research, this would mean that gifted students can master regular content at faster rates (Neubauer & Fink, 2009). However, this task of providing for these gifted or advanced learners is met with challenges (Dixon, Yssel, McConnell, & Hardin, 2014) as already stated. According to Berman, Schultz and Weber (2012), many teachers view gifted children in their classrooms "as nothing more than peer-tutoring candidates who are ahead of the game". They go on, suggesting that such practices take away "time that should be used for their own academic development" (p. 19). The use of the Mastery Learning

Model enabled students to work at their own pace with their age and ability peers. The collaborative nature of the classroom also meant that students like Oliver had the choice to also act as a peer tutor. As his teacher, however, I made sure this did not come at the expense of his own development. That is, it was not used as a strategy to meet his advanced needs, such as is suggested is often the case by Berman, Schultz and Weber. While I was able to help Oliver with his Year 10 level work, Section 7.2.1.2 did reveal that Oliver still wished he could receive more specific help in class. The provision of only one teacher meant that I was often unable to help Oliver when he needed it. This caused some frustration for Oliver, as there were times when he struggled to find the answers to very particular questions. One strategy that could have been used to alleviate this would have been for students to have access to a specialist teacher as was noticed in Stamps (2004) study. The use of the Mastery Learning Model did provide students with work from higher year levels, and results (Section 7.1.2) would suggest that the strategies employed did help them achieve mastery of this advanced content.

A variety of stories on how a challenging program enhanced students' attitudes towards learning mathematics were shared in Section 7.2.3.2. These research findings contrast those of Box et al., (2015) who shared the stories of teachers who stopped using formative assessment to guide instruction. The teachers in their study decided to revert to low-level drill and recite methods to cover the curriculum and prepare the students for a high stakes test. Teachers in the Box et al.'s, study, along with the myriad of research given already, show the nexus between what teachers know that they should do, yet do not do because of factors already stated. Teachers, according to Smith and Southerland (2007), revert to strategies they believe will work. Results from my study highlight how different students had gaps in their learning as a result of a teacher-centred approach. If teachers are feeling the pressure to fill such gaps, and see an "overcrowded curriculum" (Rubin, Abrego, & Sutterby, 2014, p. 4), then they revert to teacher-centred instruction methods. These methods, which often lack the use of differentiation of the way content is delivered are common, according to Bondie (2018), Smith and Southerland, Jones (1997) and Lave (1993). Students from this study who had experienced this kind of learning felt confused, bored and unchallenged. They thought mathematics was irrelevant, repetitive and full of questions which only had one correct answer.

The nature of these results is limited, and further studies would need to be conducted in larger numbers to confirm all the reasons for gaps in student learning. It does, however, provide new information on a method for ensuring gifted students can receive access to an appropriate level of challenge. These findings further reveal how challenge impacted on academic results as discussed in Section 7.1.2. Students were able to remain challenged, were engaged, maintained a positive mathematical self-concept through achieving mastery goals, and engaged in high-level tasks in a collaborative manner. The next section, therefore, draws on results shared in Section 7.2.4 and research cited in Section 3.4.

8.2.4. Relevance and variety in learning.

Maker (1982) argued that the curriculum needs to be abstract, complex, more varied and organised differently for gifted learners. In this study, both Ty (Interview, June 26, 2014) and Oliver (Interview, June 25, 2014) shared in separate interviews that they liked being given access to new challenges where they could discover interesting mathematical facts and use logic to solve more challenging problems. Due to the nature of the investigations utilising the Mastery Learning Model, their studies in mathematics had connections to their lives outside the classroom (Section 7.2.4.1). They attributed engaging with relevant investigations as the reason for their "love" of mathematics and why mathematics was "more interesting". Students would complete these enrichment tasks, often of their own volition. Walter even indicated at the start how he preferred to work on higher-level challenges, whereas later in the study, he said that "he liked it (studying mathematics) a lot better this term because of the enrichment tasks". Miley was able to find a relationship with mathematics and her love of creative and endless patterns and with the windscreen wipers on the car (Interview, June 23, 2014). These results add to the findings discussed in Section 2.3.3 from Kim's (2016) meta-analytic review of enrichment programs. In this study, Kim pointed to both academic gains and socioemotional benefits (career goal setting) of such enrichment tasks.

The Mastery Learning Model enabled diversity in their learning, as students were able to work on enrichment challenges and a variety of activities and types of activities they completed. According to data gathered in Section 7.2.4, this added interest to the mathematics program. This experience is contrasted with the repetitive and monotonous nature of previous years' exposure to mathematics that had them listening to the teacher endlessly and complete excessive numbers of simpler textbook questions as discussed in Section 7.1.

8.2.5. Social learning and collaboration in the mathematics classroom

The theoretical underpinnings of the Mastery Learning Model are re-presented in Figure 8.2, as discussed in Section 3.4 which places the gifted learner at the centre of a social constructivist classroom, where students were engaged in challenging and collaborative tasks. This section considers the influence social learning had on the selected gifted students. There is substantial research on the role of collaboration in learning by gifted students. According to Rogers (2002), gifted students need opportunities to learn and socialise with like-ability peers. Other research (e.g., Mersino, 2010) advocates the benefits of shared learning with gifted students. A study by Diezmann and Watters (2001) of six gifted 11-12-year-old students, suggested that as the challenge level of mathematical problems increased, the students tended to collaborate to solve tasks. Samardzija and Peterson's (2015) qualitative research further encourages teachers to consider individual learning preferences in highlighting reasons why some students may prefer a more social setting where others favour a quieter learning experience. This research adds to this issue in revealing the impact a flexible and social learning environment had on students' attitudes towards mathematics.

Considering this research, I revealed in Section 7.2.1 how Oliver sometimes liked to help his friends, but he also "like(d) working alone", where he would "use logic and the computer to help him solve unfamiliar questions. If he still got stuck, he would then go to the teacher for help. While results discussed in Chapter 7 revealed how Bree, Miley, Ty and Walter enjoyed discussing and debating answers with their friends, they also enjoyed the autonomy of learning to mastery. Section 7.2.3 shared Ty's example of "listening and listening" to the teacher talk about concepts "he already knew". This teacher-centred practice of teaching all students the same content from the front exemplifies the need for a more student-centred approach to learning, where debating and collaborating over answers to complex problems were commonplace.

In one story discussed in Section 7.2.4, Miley shared that she "learned best" when she was working with her friends on enrichment tasks as they were more memorable. A study by French, Walker, and Shore (2011) suggested that gifted students enjoyed collaborative learning if done in a non-threatening and fair way. In the example discussed, both students (Miley and Bree) would almost talk over the top of one another in excited tones to find a solution together. Both girls enjoyed social interaction in lessons, as it helped them solve complex questions that they had spent a long time trying to solve on their own. This suggests that gifted students will collaborate if the problem is complex and engaging. If the problems are routine, there is no incentive or need to collaborate, echoing the sentiment of the research of Diezmann and Watters (2001).

Social learning may have some disadvantages. An interesting story was revealed in Section 7.2.3 highlighting how less able students rested on Oliver's mathematical giftedness by asking him to answer the more challenging questions, while they struggled to answer other less complex questions. This story illustrates how students like Oliver can have their learning hampered by continually helping others. Oliver shared how he enjoyed these activities, even though he was called upon by his group members to do most of the work. This story supports the findings of French, Walker, and Shore (2011) and Samardzija and Peterson (2015) who suggest this uneven distribution of labour to be one of the reasons why gifted students often prefer to work alone, as is the case here with Oliver.

While not all students enjoyed learning in a social setting, all shared that they benefited from having access to variety in learning. All five students were asked if they preferred working alone or with their peers. All, except Oliver, shared how they preferred a flexible classroom learning environment that allowed the open discussion of ideas in learning. This section adds to research in suggesting that when gifted students work with like-minded peers, they prefer to be able to communicate ideas, but also like the option to work in a quieter learning zone as well, depending on the type of challenge being set. It notes the GIM given in Figure 7.2 which suggests that gifted students exist with a social zone of challenge and learning where students can access each other, technology tools, parents and the teacher to assist them with understanding more complex tasks.

8.3 A Revised Mastery Learning Model

Having considered the findings of this study, a revised Mastery Learning Model is proposed and presented in Figure 8.2. This revised model accommodates findings not predicted by the original conceptualisation. For example, it was not possible to use individualised learning contracts on my own and maintain the differentiation contained within each smaller sized unit. Instead of writing up learning contracts for each student, I had to ensure the students were given units of instruction set appropriate complexity levels and depth. Research cited in Section 2.3 highlighted the need for greater depth in students' learning. It was important that I monitor students' progress online via the school's learning management system. I also had to collect the students' books regularly and encourage some students to go back and complete more work on tasks they had not mastered. I did not force students to do the enrichment tasks. I made sure the enrichment tasks were relevant and interesting enough so that they wanted to do them, as discussed in Walter's example in Section 7.2. By not creating learning contracts, I inadvertently exhibited to the students that I trusted them to do the right thing. Interestingly, they saw the importance of learning for mastery, and therefore, this sense of autonomous learning eventuated, as discussed in Section 7.2.1. The six phases of implementing the revised Mastery Learning Model are discussed next.

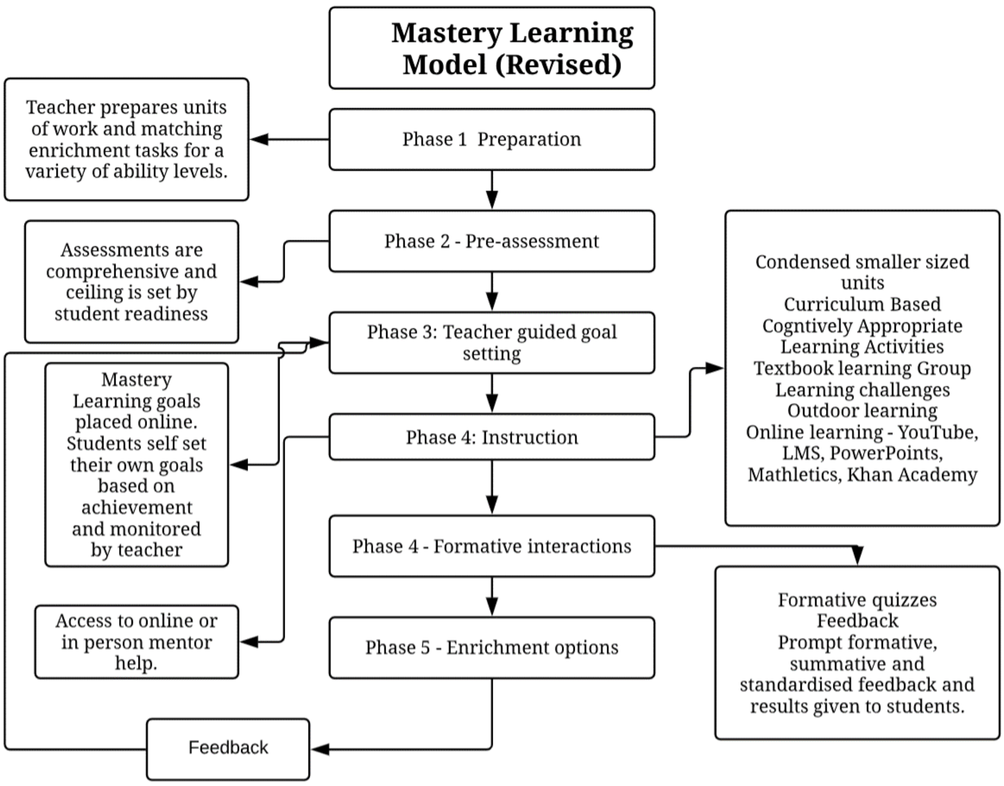

Figure 8.2. Mastery Learning Model (revised).

The role of the teacher remains consistent with the original model as given in Figure 5.2. This model given in Figure 8.2 notes the importance of having access to an expert teacher who can mentor students who are working on content that is significantly different to what the rest of the class is doing. This is in response to one criticism given by Oliver suggesting that he found it difficult to learn on his own, and would have really appreciated specific modelling of questions that he was being asked to complete that were set at an advanced Year-10 level. This tutor would ideally be available for students like Oliver during the week to provide them with focussed support on questions that may not be covered in the online advanced course. This modification to the original Mastery Learning Model is minor and in line with studies conducted by Stamps (2004).

Phase 1: The teacher ensures all unit modules, YouTube videos, PowerPoints, online interactives, group and individual activities are placed online in a format that enables students to work on appropriately challenging activities at their own pace, but under the guidance of a teacher. This phase is important and may require teachers to work collaboratively with other teachers utilising their expertise such as discussed with the involvement of the librarian in Chessman's (2002) study, or the gifted support teacher in Stamps (2004) study.

Phase 2: The use of online pre-assessments matched to the Australian Curriculum is helpful for teachers who have little time. The use of standardised and non-standardised assessment instruments is needed. Research already shared in Section 7.1, and Section 3.3 discussed the importance of utilising both standardised and non-standardised tests, as the teacher needs a detailed understanding of the students' strengths and possible weaknesses before the commencement of instruction. It was suggested that smaller sized topic specific standardised tests would be more useful in ascertaining students' understanding.

Phase 3: When the teacher uses online assessments, often the feedback is instant. Hattie and Timperley (2007) noted the positive effects computer-assisted instruction coupled with prompt feedback had on student achievement. Students use the immediate feedback in a teacher-guided session to help them set individual learning goals. For example, if the student did not master all concepts on a Year-8 level assessment, the teacher would assign the student to just a Year-8 level online course. Once all content has been mastered, students are encouraged to complete relevant real-world, authentic enrichment tasks.

Phase 4: Students participate in a range of learning activities as set out in the online learning management system. It is important that the activities are varied in both type and complexity. Some students may be enrolled in multiple levels of learning.

Phase 5: Guskey (2015) points out how at the early stages of the use of the Mastery Learning model, some teachers felt they were being reduced to “record keepers of student progress” (p. 756). The use of online quizzes, which provide instant feedback, was helpful in ensuring that the job of the teacher is one of a monitor of progress, rather than a record keeper.

Phase 6: Upon completion of relevant summative or formative assessments, students would need to re-evaluate and reset learning goals, upon noting the feedback given by the teacher.

The original Mastery Learning Model proposed Phase six of the study to consist of enrichment challenges. As students had been used to mainly textbook learning, there was a lack of depth, challenge and relevance. As the teacher, I would explain the benefits of completing enrichment challenges. Many students chose not to do them at first, however as they saw other students complete them, they started to see the relevance of such tasks and decided to try some themselves. In this sense, the learning was autonomous, as the students had volitional control over their learning during this phase of instruction. This control could be after the initial pre-assessment or during their learning, to enrich their learning experience, making it more relevant to their lives, as per Bloom’s (1974) original intent with his Mastery Learning Model. He elaborates on this intent:

> I find great emphasis on problem-solving, applications of principles, analytical skills, and creativity. Such higher mental processes are emphasised because this type of learning enables the individual to relate his or her learning to the many problems he or she encounters in day-to-day living. (p. 578)

By including enrichment as part of the learning activities (phase four instead of phase six), students had this opportunity to be involved in the kind of learning Bloom intended in a classroom, guided by the principles of the Mastery Learning Model. Rather, and according to Guskey (2015), the misunderstanding was that Mastery Learning was about learning basic lower level skills. This revised model suggests that rather than the teacher telling the students what to do; he or she would guide the students through possible learning pathways and also ensure students complete the required learning, when necessary.

8.4 Summary

Chapter 8 presented findings related to the two research questions as stated. It noted key findings that both added to already existing and related research, and also posited new research. Students' test results asserted a positive impact in that the learning program allowed students to work on and master content usually reserved for students in higher year levels. Graphs were given, presenting pre-post test data which interestingly mirrored other findings noticed with non-gifted students. The smaller sized units enabled the teacher and the students to hone in on specific gaps in understanding. It was noted that even though a child might score 90% on a test, they may still have gaps that they can master in a time efficient manner with the use of student led compacting of these units of learning.

The level of challenge also impacted positively on the students' attitudes towards learning mathematics. By having their understanding confirmed, students were able to work on memorable and engaging enrichment tasks. The adherence to the Mastery Learning Model protocols meant that students would be able to learn in a way that suited them. Students' interview responses and test results revealed a contrast in the way learning was structured with the Mastery Learning Model and how they learned mathematics in previous years. Rather than learning and relearning content, students would only need to learn a concept or skill once. The application of this learning to authentic and even student-centred real-world contexts meant the tasks would be more memorable. That is, students saw examples the mathematics they were learning in their everyday lives and wanted to know more through a natural sense of curiosity. There were many examples cited, where my results mirrored those of other research, as discussed. A strong theme to emerge from this research was that choice and level of autonomy played a significant part in remaining challenged and engaged in the learning tasks. These learning tasks were able to be set at appropriate challenge levels, enabling students to remain in their zones of socially constructed cognitive development, and therefore remain engaged.

Another important finding of this study was that the learning experiences of the students were varied. Students engaged in real-world individual and small group challenges as well as more traditional, compacted knowledge and skill-based learning. The nature of the compacted units enabled students to work confidently on such tasks. The use of the Mastery Learning Model confirmed the findings of Guskey (2010) which asserted that students would have more time at the end to work on enrichment tasks but added new qualitative findings which revealed students' thoughts on such a study.

Finally, a revision of the original Mastery Learning Model has been presented. The main difference in this model is the removal of learning contracts from the original Mastery Learning Model. This change was made very early on in the study as it would have been an unwieldy process for any teacher to scour through all the students' results and make necessary amendments to their learning goals. By setting their own goals, students had more control over their learning, which data suggested, they appreciated.

Impressive achievement results were noted as were improvements in attitudes towards learning mathematics. The improvement in attitudes was evidenced by interview responses, considerable engagement in relevant real-world tasks, which students would complete into the nights, of their own volition. This improvement was also ascertained by students contrasting their feelings associated with the given mathematics program and those the students had completed in prior years.

Chapter 9
Conclusions

9.1 Introduction

In an article summarising the non-negotiables of gifted education, VanTassel-Baska, (2005) noted that students require access to a complex accelerated curriculum, flexible grouping, access to online resources, a differentiated curriculum with access to real-world problems. Norton and Reid O'Connor's (2016) findings stand in agreement, suggesting the need for all students to have access to this challenge and authentic learning. This study tested the effectiveness of a model conceptualised as a Mastery Learning Model, which incorporated these elements VanTassel-Baska suggested into a single intervention. The Mastery Learning Model required the teacher to ensure students received meaningful feedback which enabled students to feel a sense of competence that helped them to complete harder challenges successfully. This model needed to be revised further to ensure students could take more responsibility for setting teacher guided learning goals. Students were compelled to seek out the assistance of teachers, parents, computer aides and their peers, due to the complexity of the tasks set. The students had time to attempt to solve questions on their own first and only seek help when they had exhausted other options. Students were able to construct both meaning and understanding through a variety of learning styles and modes of delivery. These students' answers revealed that even though they were in a class with mostly high ability students, the content still was not differentiated. They explained how they all listened to the same instructions, delivered in the same way and then completed the same textbook questions in spite of their levels of understanding.

A comprehensive body of research was reviewed revealing that teachers often know how to differentiate for gifted students, but do not do so. This is due to various pressures such as time restraints, state/national testing agendas and an over-crowded curriculum they face in their day to day jobs (Brimijoin, 2005; Johnsen, 2017; Tomlinson, 2016; VanTassel-Baska & Stambaugh, 2005).

Research reviewed in Chapter 3 cited a large range of studies that have tested the potential effects on students' academic achievement when teachers apply Bloom's Mastery Learning model (Bautista, 2012; Corbett & Anderson, 1994;

Guskey, 2010; Guskey & Pigott, 1988; Idendinihu, 2013; Shafie, Shahdan, & Liew, 2010). This body of research reveals it as a strategy whose success has been widely documented. A review of all the peer-reviewed education and psychology databases was unable to find any specific studies that researched the impacts the Mastery Learning model on gifted mathematics students. Chapter 3 discussed how Bloom's intent for student learning within a Mastery Learning model should be similar to learning experiences students would have in a one to one learning environment. Bloom hypothesised that all students could achieve mastery in the classroom if given the opportunities to do so.

This original study undertook to examine two research questions through the use of an explanatory case study methodology. The Mastery Learning Model guided the teaching of mathematics in a Year-8 class over 24 weeks. Five students identified as gifted mathematics students, their learning, attitudes towards learning, were monitored, through the collection of data from interviews, direct observation, formative and summative (including standardised) tests. The research proposed to investigate in what ways a teaching approach guided by the principles of the Mastery Learning Model impacted on students' achievement, interest and motivation levels in mathematics.

Results from this study revealed how mathematical achievement levels did improve as shown in pre-post test data. Students engaged in deep learning, problem-solving and extended learning outcomes through group problem-solving challenges and various enrichment tasks. Results were able to justify in-class acceleration options. Pre-test data revealed that students understood to mastery levels, most of the Year-8 mathematics curriculum, but little of the Year-9 curriculum. Some students, however, did understand many Year-9 concepts.The measure of the influence the Mastery Learning Model had on students' academic achievement, therefore, was their ability to not only master Year-8 level concepts but also move on to Year-9 and Year-10 level content. Many students, including the five in this study, were able to master a range of these more advanced concepts. Students were able to gain a deeper and more contextualised understanding of why they learned mathematics, to a point where students were completing non-compulsory learning tasks of their own volition at night and in their own time. One student even suggested that these enrichment tasks encouraged him to seek out a career in mathematics later in life.

Additionally, according to interview data and personal communications, student interest levels in mathematics improved. An explanation for this increased interest was that they were able to learn autonomously in a way that avoided needless repetition and acknowledged their achievements in a meaningful sense. The students' attitudes shifted from one of feeling bored and frustrated with an overly repetitive subject to one that saw relevance and provided challenge. Four of the five students enjoyed learning in a social setting, and this impacted positively on their attitudes towards mathematics. They were able to discuss and debate ideas, as well as complete group challenges together in a cooperative sense while learning from each other.

9.2 Theoretical Implications of Findings

A range of literature was reviewed on ways to cater for gifted students (Dai & Renzulli, 2008; Gagné, 1985; 2013; McAllister and Plourde, 2008; Rogers, 2007; VanTassel-Baska, 2004). Many such methods involve out of school workshops and tutoring. Many highlight the benefits of individual components (e.g., autonomous learning, curriculum compacting, ability grouping and acceleration practices). No studies were able to document the students' responses to a teaching and learning model that combined best practices in gifted education into one model such as some (Bain, Bourgeois, & Pappas, 2003; Ambrose, Van Tassel-Baska, Coleman, & Cross, 2010) suggest is needed. The application of the Gifted Instruction Model (Figure 9.1) presented students with the ability to learn in a preferred non-repetitive way on appropriate and challenging content.

My research also furthered the work of Ryan and Deci (2017) with qualitative findings on gifted students' perspectives on an education program that provided autonomous learning opportunities, acknowledged student competence through mastery goal setting with relatedness support which encouraged open collaboration and social dialogue in an accepting environment. Students' competence was acknowledged meaningfully both in the pace, depth and breadth of the curriculum. The use of curriculum compacting was integral as it allowed for the gifted student to master regular content at a faster pace. Therefore, the evidence presented, suggests that the use of this model catered for their advanced cognitive abilities. This research has shown that gifted students can achieve to their potential when they are given access to relevant and meaningful challenge and autonomy while having their voice valued by their peers and teachers.

Students' strengths and gaps in learning were identified, and students were able to develop a preferred learning path that enabled access to a challenging and self-paced learning program. Their attitudes towards textbook style learning revealed how they responded better to a more flexible and multi-layered program that utilised a variety of online and interactive activities, that aimed to share with students a real-world relevance of what and why they are learning in mathematics. Research uncovered why teachers overly relied on textbook-based instruction and the detrimental effects this can have on students' attitudes towards learning that subject. This research discussed their responses to a more varied and flexible program that encouraged higher level thinking and challenges in a variety of settings.

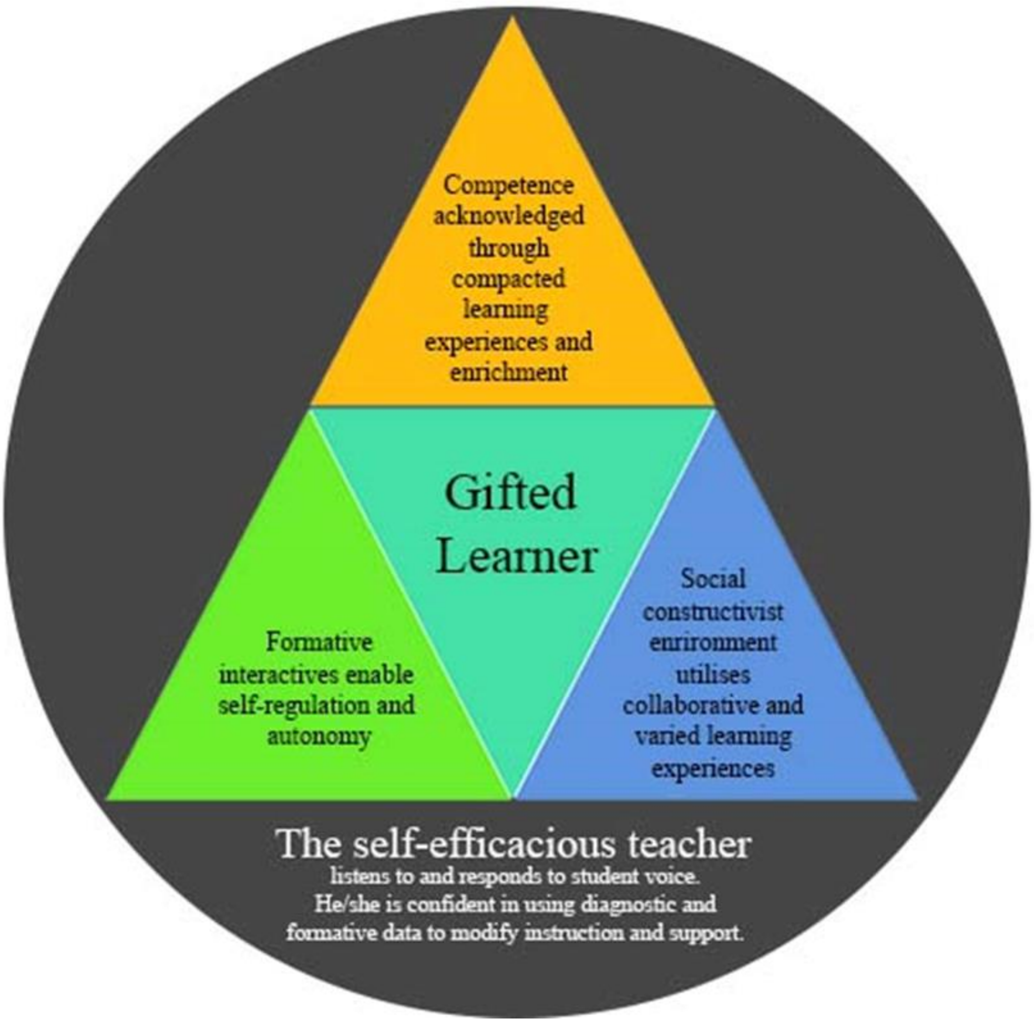

Figure 9.1. Gifted instruction model (GIM)

The Gifted Instruction Model acknowledges the influence of a highly efficacious teacher who sees the importance of responding to students' needs in a meaningful sense. While I taught mathematics, most statistical analysis used to guide instruction responsively was done by the online learning managements system, Microsoft Excel or the online standardised assessment device. I simply had to collate these data and further calculate measures of central tendency and spread for the purpose of reporting on the results in this thesis.

Another important finding from this study highlighted how I used data from a comprehensive assessment for learning regime to tailor instruction to specific students' individual needs a purposeful sense. A vast array of research reviewed revealed the potential for the Mastery Learning Model to improve educational outcomes for all students, including the gifted. My findings mirrored those of Guskey's (2005a, 2010) showing the normal distribution of results at the beginning of the study shifting to a negatively skewed curve by the end. My research furthered Guskey's findings in that students were able to master higher year level content using curriculum compacting within the mastery learning framework. While some (Ritchotte, Rubenstein, & Murry, 2015) would assert there is no program that will "ameliorate underachievement for all gifted students" (p. 103), studies like this one carry merit for government level and school curriculum writers to consider, as discussed further in Section 9.6. The limitations of such findings are discussed next.

9.3 Limitations

This study explored a teaching program that promised the potential of a more individualised level of instruction in an age where a national testing agenda places heightened pressures on teachers to get good results from students. This pressure was real for me, as the teacher, as this was a lead in year for the Year-9 level National tests. If these students performed poorly in the following year, then my principal would have asked for a plausible explanation.

This research explored a broad range of themes. All five students were from middle-class Caucasian families. Therefore, cultural factors that might impact on results from other studies were not noted here. The impact of the teaching approach on academic performance was measured by showing how the students were able to demonstrate mastery of higher year level concepts as well as same year level content and skills. This high level of mastery was confirmed by students maintaining mastery level grades into the next year. One study (Samardzija, & Peterson, 2015) noted the impact “the classroom teacher’s personality, competence, accessibility, and concern for students” can have on results. This furthers Hattie’s (2012) claims, suggesting that impassioned and efficacious teachers can influence student results. As the teacher/researcher, I was not able to comment on my performance, although I did use video footage and diary entries to analyse my teaching methods along with student responses to make amendments, as noted.

A brief note was made regarding students’ over-excitabilities and how this can potentially have an impact on student attitudes towards their schooling experience. Given that I was the students’ teacher, there was an element of familiarity with me as their teacher. Even though I asked for the students to give honest and raw answers to my questions, it is possible that they were nice or kind in their responses. I felt as though their answers were honest, as there was a consistency across student responses that duplicated perspectives from different students.

The students were recorded with both video and voice recorders. It is also possible that the Hawthorn effect (Henry et al., 2015) may have also impacted on behaviours. I noticed that at the start of the research, the students were grabbing the voice recorder and saying hello to their teacher on it. Towards the end of the study, the students were very used to the presence of the recording devices, and the classroom behaviours and conversations were what one would reasonably expect to observe when watching lessons of students engaged in these kinds of learning tasks.

9.4 Rival Explanations

Yin (2014) shared how the researcher should consider rival explanations in any case study research. This section therefore examines a range of rival explanations that may either have impacted on these results or could impact on results if the study were to be replicated by another teacher.

I have cited research (Hattie, 2012; Samardzija & Peterson, 2015) which suggests that effective teachers can have an impact on student outcomes. Hattie (2012) revealed that teachers are “among the most powerful influences on learning” (p. 18). He argues how the teacher’s experience, their passion levels about the subject and their education can impact on student learning. Within this context, it is possible that my passionate teaching, many years of experience and schooling background could have contributed to the successful learning outcomes. If someone were to try to replicate this study, these factors should be noted. As discussed in Section 4.4 a range of strategies were used to ensure credible interpretation of events.

The motivation to get good grades may have impacted on students' motivation to do well mathematics. In her initial interview, Bree revealed how she was driven to get good grades. Students' interview responses indicated that grades were important to them. However, they only suggested this as a factor when explicitly asked about the impact of grades on their attitudes towards learning mathematics. This finding would imply that grades may impact on students' motivation to get good results, if this study were to be replicated by another teacher. The influence was not seen as significant; however it is noted here. This finding is in line with claims by Ryan and Deci (2017) who identified grades as extrinsic motivators and controlling measures. Their influence was minor.

Further to grades, other factors could have potentially impacted on students' attitudes, achievement and interest levels such as parental involvement. Four out of the five students, (not Ty), shared their parental involvement at home helping them or providing incentives for them to do well in mathematics. Levpuscček, Zupančič, and Sočan (2013) suggested that parental pressure had a negative effect on students' self-efficacy levels, while support had the reverse effect. Therefore, these interview responses concur with the findings of Levpuščěk, Zupančič, and Sočan suggesting the parental involvement added a mostly positive impact on students' attitude and achievement (understanding) levels towards mathematics. Should a replication of this study be attempted, a researcher could also include parents in the interview process to better understand the impacts they have in a classroom framed by the Mastery Learning approach.

While this section has discussed factors that potentially impact on the results as presented in this study, their impact could not be measured, while also staying within the bounds of this study. Further research could be conducted to explore such implications.

9.5 Future Research

There were a range of questions this research did not cover. This section, therefore, proposes future research that could be conducted in relation to the results from this study.

Firstly, this research uncovered interesting findings in relation to autonomy, competence and relatedness in the mathematics classroom. A search of peer-reviewed databases using the terms "Mastery Learning" and "Self Determination Theory" revealed no research. While this research has uncovered new findings in this area, it has also provided potential future larger-scale studies that could be conducted that focus on the impacts a Mastery Learning has on students' motivation to learn mathematics over a longer duration of time.

Research (Brown & Group of Eight Universities, 2009; Larkin & Jorgensen, 2016) cited, revealed that mathematics is often associated as a less interesting, difficult, and boring subject. Do students find mathematics uninteresting because of gaps in their learning? Larkin and Jorgensen's (2016) study revealed that even younger students as young as Year-3 are turning away from mathematics. Why are students disengaging in mathematics at a younger age, and can the use of the mastery learning impact positively on these attitudes? Further research has shown that the teacher has a significant impact on outcomes. One article (Perschbacher, 2016) suggested that students do not understand the more difficult mathematics tasks because they did not fully understand the basics. I propose a longitudinal study, noting changes in both academic achievement and attitudes toward mathematics. Bree came into this classroom hating mathematics. She thought that it was confusing, there were too many facts to remember, and no matter what the teacher did, she would not like it. At what point do students like Bree write off mathematics as too confusing, and why? The current research uncovered that a significant amount of repetition in mathematics as a reason for student disengagement from it as a subject. Other students in the study suggested it was because teachers just taught to an irrelevant textbook. Therefore, conducting research with younger students to ascertain if students at a younger age are more engaged in mathematics when it is more concrete, and they are more easily able to see the relevance in the real world would be beneficial. A follow-up study would, therefore, seek to ascertain whether the patterns of underachievement and lack of interest in mathematics can be transformed if given access to a program that is autonomy, relatedness and competency supportive.

Section 3.3.1 examined debates on the use of standardised and non-standardised tests to examine a program's effectiveness. The standardised test results used in this study were helpful. However, it would have been better to have a series of smaller concept based standardised tests which measured students understanding of specific concepts. Such results would provide the teacher with meaningful data for students' academic performance when compared with their age or ability peers across a larger sample of students. I also noted the impacts high stakes testing has on teachers willingness to use research-backed strategies for differentiation in their teaching. The use of such smaller sized standardised and less formally conducted assessments could provide state and national government authorities with more meaningful data on students' developmental achievement. This research has uncovered that students struggle in larger exams to demonstrate their understanding because of anxiety and other issues commonly faced by gifted students. Therefore, this suggested follow up work could ensure such data provides more accurate and meaningful data for teachers and curriculum developers alike. The data could be used to track student achievement across the country and show potential strengths and weaknesses that may be in common in classrooms today.

The understanding of such results could then also be traced by the use of eye trackers (Cohors-Fresenborg, Kramer, Pundsack, Sjuts, & Sommer, 2010) and pulse rate oximetry (Amat et al., 2016). This kind of deeper scientific analysis could help researchers understand engagement in the classroom through different scientific lenses. Follow-up studies to track student engagement using these medical technologies along with video and voice recordings would be useful. This research would intend to observe differences in engagement when students' complete quizzes and when they do exams. How do students' responses compare when they are in a less formal setting (but still working on their own) and a more formal exam setting? The research would focus on accurately measuring student performance and finding the best way to do so.

The current research has also uncovered how students liked a mixture of both traditional learning with enrichment investigations. Follow-up research could track the pulse rates and eye engagement between two groups of students within two different settings. One would be within a traditional learning environment, and the other, learning within a classroom environment shaped by the use of the Mastery Learning Model. My research noticed students engaging in learning with little to no interference or behaviour correction from the teacher. Video footage and audio analysis revealed some off -behaviours, but the footage gathered showed that students were mostly engaged and interested in learning. The second research question analysed changes in students' attitudes and interest levels in mathematics. The results reveal promising findings. However, it would be good to see the differences in engagement in a classroom guided by the principles of the Mastery Learning Model and a regular classroom where the teacher acts as the main facilitator and deliverer of knowledge.

It is recommended to conduct follow up research that investigates these findings to a greater extent in classrooms where the researcher is not also the teacher. There is a large amount of data (Gunderson, Ramirez, Levine, & Beilock, 2012; Hattie, 2012) which suggests the teacher has a significant impact on mathematics achievement.

Further studies would also be beneficial that incorporate the findings of Diezmann and Watters (2001) and Daniels and Piechowski, (2009) which consider the impact gifted students' heightened sensitivities may have on their learning. Studies noted that gifted students often prefer to work alone. It would be good to narrow this down and explore exactly why some do and if their sensitive natures are partly the reason for this.

It is conceivable that if this study was considered for trial for a whole school, that students could be working on curriculum content at different levels despite their ages. This proposed follow up study would require careful planning, as the teachers that are to be working with students should be familiar with the subject area and be able to provide accelerated support advanced students, like Oliver needed. Oliver shared how he would have appreciated having more support and access to lessons at the start of the advanced Year-10 mathematics course he completed. I found that I could not be everywhere. It may well have been advantageous if Oliver could have participated in such lessons with extra support from a suitably qualified support teacher.

One of my principle interests in conducting this study was to track students' attitudes and achievements within an autonomous supportive, social learning environment. Future research, therefore, would continue to explore this area in an attempt to track student performance and attitudes across genders, class, and differing settings. This section has proposed a variety of follow-up studies that examine these questions from different perspectives with different foci. The end goal would be to reverse the evident pattern of underachievement of gifted students in the area of mathematics.

9.6 Implications of this Study

The findings from this research has implications for students, their teachers and teacher practice, students' parents, school leaders and policy decision-makers alike. These implications are discussed in this section.

9.6.1. Implications for Students

Many gifted students underachieve and often drop out of school (Ritchotte & Graefe, 2017). They become sick and tired of what Hill-Wilkinson (2016) referred to as "menial" repetitive tasks (p. 73). Hill-Wilkinson's research suggests that gifted students are underachieving because they are not being challenged. Similar to her study, students discussed in this thesis were used to completing overly repetitive irrelevant learning activities, which, according to Hill-Wilkinson often results in underachievement. A range of research has been discussed which states the importance of students' competence being acknowledged in a meaningful sense. As a result, students will come into mathematics lessons and be challenged.

Research has revealed how students' interest in mathematics declines as they progress through school (Frenzel, Goetz, Pekrun & Watt, 2010). Students like Bree and Walter perceived past mathematics learning as irrelevant and overly repetitive. As a result, they disengage from learning mathematics. This thesis has discussed gifted students' reactions to a program that enabled them to learn autonomously, and have their competence acknowledged within a social constructivist learning environment. This research shows potential as it provides the impetus for further exploration in ways to improve students' motivation to want to learn mathematics.

Students can also complete tasks that relate to their interests. Miley loved drawing patterns and investigating patterns in nature. She could relate this interest to mathematics, not because the teacher planned a unit on patterns, but simply because she started to see mathematics in the world she lived in through the enrichment investigations given.

Similarly, students can begin to investigate varying career options as a result of studies they conducted in class. Walter started the year simply completing the required tasks. As the year progressed he completed a range of enrichment tasks that made him re-think a career in mathematics. Students could potentially develop their passions that could lead to a future career.

If this approach was used by teachers of gifted classes, students could spend more time pursuing their passions in other STEM (science, technology, engineering and mathematics) fields. Instead of spending time re-learning content, they could spend more time exploring passions similar to what Walter, Oliver, Miley and Ty did. These students had mastered the curriculum and had the option of completing enrichment tasks or accelerated content. The more Walter accessed these types of challenges, the more he wanted to do them. This may also in turn lessen the dependence on students completing accelerated content in classes such as Ty experienced in Year-3.

A study by Blaas (2014) revealed how gifted students often feel isolated and excluded and this can have negative implications on their academic performance. This contrasts with students from this study who did not feel isolated and were included in active and often heated academic debates and ability appropriate learning. Students actively challenged each other, and their intellectual abilities were valued by the teacher and their peers in a variety of collaborative learning challenges. This was evidenced by interview answers from students revealing how they enjoyed learning which allowed them to work collaboratively with their friends. The students had autonomous control over their learning and chose how they would learn each concept. Ryan and Deci (2017) speak of relatedness as not only a sense of belonging but having a voice that is heard which then in turn impacts on the person's motivation to achieve. Learning happened socially, and students were engaged and interested in learning in varied ways with their peers. This social learning sits in contrast to the isolation many gifted students feel in classrooms where they sit at their desk and complete endless activities and often ostracised as being different (Blaas, 2014).

With the use of the Mastery Learning Model, students can be challenged, engaged, accepted and see a potential future involving concepts they are learning in school. These implications can not only improve their love of learning, but also have a positive impact on their academic performance as well. The use of the Mastery Learning Model can also impact on teaching decisions and teachers in general.

9.6.2. Implications for Teachers

Research (Leikin, Leikin, Paz-Baruch, Waisman, & Lev, 2017; Shaw, 2007) revealed that mathematically gifted students can complete regular mathematics tasks at faster rates. This same research argued that gifted students exert more mental effort on complex tasks. This thesis provided evidence of this ability for identified gifted students to master regular mathematics concepts at a faster pace. Evidence from in-class recordings also suggested that students needed to exert more mental effort as they leant on the support of their peers in order to solve such complex tasks. While I did not have the ability to track neural effort with medical imaging technologies, this research does provide evidence that could be investigated further. These findings also provide support for the argument which urges teachers to provide students with access to compacted units of instruction for learning the regular coursework.

Further research discussed in Chapter 2 also highlighted how gifted students are often hard to identify. By using the Mastery Learning Model, the teacher was able to meaningfully track student learning. This tracking process involved the collation of computer-generated data that was also confirmed by the teacher examining the students' reasoning in their responses, such as those provided in the examples given. The use of the Mastery Learning Model, therefore, has the potential to simplify the complex task of identifying giftedness, ensuring that such mathematics gifts and talents are not wasted.

Students from my study revealed how past teachers had given them a textbook containing content from three or more-year levels higher and asked to sit quietly and work through it on their own. Other students' mathematics learning only happened at home, where their parents would give them more challenging mathematics tasks. It is clear from research that gifted students are often given busy work. Teachers will therefore be able to use specific data on each child to provide an education tailored to each individual child's ability levels.

One study (Ritzema et al., 2016) revealed how teachers today are spending most of their time helping struggling learners. However, researchers (Herman, Osmundson, Dai, Ringstaff, & Timms, 2015), have found that teachers do not use formative assessments and formative results, for a variety of reasons. These included teachers' self-concept, time, and high stakes testing. Teachers can use this data to alter both what they are teaching, whom they are teaching it to and how key concepts are being taught. For example, if a teacher notices that a student has not demonstrated mastery of a given concept after a prolonged period of time, they can make adjustments to their teaching to ensure this student receives the support necessary to master this concept. They can also feel assured their students have learned the relevant concepts in the curriculum to prepare them for high-stakes testing.

At the end of every year, the data for each child should go with them to the child's next teacher. Using the Mastery Learning approach, teachers could be able to plan their units of instruction with a very clear understanding of what their students understand. The teacher should also come to the classroom with a clear knowledge of a range of investigations that students can engage in that will make learning meaningful and engaging. Some teachers may need to do as Stamps (2004) did in obtaining outside expertise to help the more advanced students. This outside support may come from other teachers or parents who may be experts in a given field related to the student's interests.

9.6.3. Implications for Parents

When parents have a detailed understanding of what their child understands, they can work with the teacher to help their child learn skills they may be struggling with. Parents can also use the rich data the use of the Mastery Learning Model offers by providing their children with further opportunities outside of school to apply their learning to the world they live in. Parents can also feel assured that their child is being challenged and is socially happy at school.

9.6.4. Implications for Schools

Key decision makers in the schools will be empowered by the data generated from a school-wide application of Bloom's Mastery Learning Model. First, they can track students' progress in a meaningful sense, by generating specific reports on what students have mastered and how they have mastered these concepts. Given the amount of research which cites teacher's lack of time, it would seem logical for teachers to receive support to free up such time to ensure records are updated and accurate. This support would enable the school to use data to drive decisions that could benefit student learning from across the school.

Second, student support decisions should also be informed by these data with the purpose in mind to ensure that students with all levels of academic needs and interests are catered for. This might involve, for example, timetabling a teacher in the senior school to work with an advanced child or children from younger year levels once a week, and therefore allowing that student/s to remain with his same age peers.

Third, schools should also be providing support for teachers by allowing them time and training to develop enrichment investigations that will both challenge their students and maximise their interest in what they are learning.

9.6.5. Implications for Education Policy Makers

I have presented evidence that reveals how nationwide standardised testing is circumventing good teaching practice to a point where teachers succumb to the pressure of teaching to these tests. Results from my study suggest the positive benefits a comprehensive assessment for learning regime can have when coupled with a meaningfully differentiated program. An alternative for such a national testing regime is suggested, where one major test is divided up into many smaller moderated online quizzes that all students from around the country complete.

This proposed shift in how we understand students' academic understanding and progress could be carefully monitored and performance measured on standardised and teacher-made assessments. This shift could potentially focus on students demonstrating mastery, instead of having a focus on the giving of grades on a test. Studies cited replicate my findings suggesting that grades are an extrinsic motivator for student achievement. This is contrasted with an education that measures success when students achieve mastery, rather than by placing a graded judgement that can negatively impact on their motivation to learn that subject.

If instituted on a national level, government bodies, along with schools would have more meaningful data on their students. This purposeful use of data can be used to shape future education programs and agendas, look at areas of deficiency and provide students with access to support, resources and funding that delivers better outcomes for individuals.

Results from this study add to the findings discussed in Chapter 3 which reveal how the Mastery Learning Model has the potential to impact positively on achievement and attitudes towards learning for all students, including gifted students. This study revealed how students pursued mathematical investigations that impacted on their thoughts about a potential career. These investigations had a positive impact on students' attitudes towards mathematics. Research cited how students' interest in mathematics is falling. Therefore, this model if applied to a broader context could reverse such trends.

9.7 Concluding Remarks

Throughout my 20-year teaching career, I have seen teachers continually teach in isolation, teach the way they think is best, and teach to tests. Students share stories of classrooms where teachers commonly teach all children the same content at the same time. Every time I walk past a classroom today and see a teacher up the front of the classroom, I wonder if students are truly constructing their learning, and being challenged, or are they listening to more facts that they quite possibly already know. I once so carelessly believed that I taught the students so many valuable lessons, and maybe I did. Oliver's words echo back when he shared "I do not really pay that much attention. I do listen in sometimes" (Interview, December 2, 2014). How many students simply switch off to boring and irrelevant textbook based instruction? This study has shown me what research says about good teaching practice. There is an abundance of literature cited in this thesis that has shown how the use of the Mastery Learning approach works within regular classrooms.

Many parents of gifted students turn to home-schooling, as they have lost hope in the schooling system for their child who is different. There are those that work hard and achieve success despite their education program in schools. I have seen so many children with untapped potential who decided at some point in their schooling lives to give up. For many, schooling is irrelevant, boring and unnecessarily repetitive. That is, students do the same things, learn the same things over and over again. This should not be allowed to continue, and programs such as the one discussed in this research deserve merit.

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Appendices

Appendix A Typical characteristics of gifted students
(Adapted from Reznicek, 2006)

Type	Description of Characteristic
Communication skills	Highly expressive Has an unusual ability to communicate verbally, nonverbally, physically, artistically or symbolically. uses particularly apt examples, illustrations or elaborations.
Inquiry	Questions Experiments and explores. Asks unusual questions for his or her age. Plays around with ideas; Possesses extensive exploratory behaviours directed toward eliciting information about materials, devices or situations.
Insight	Learns new concepts quickly Senses deeper meanings Has an high ability to draw inferences. Appears to be a good guesser. Is keenly observant. Has a heightened capacity for seeing unusual and diverse relationships, integration of ideas and disciplines
Reasoning	Uses logical approaches to figure out solutions. Can have highly conscious, directed, controlled, active, intentional Can possess forward-looking and goal-oriented thought. Has the ability to make generalizations and use metaphors and analogies. Can be a critical thinker
Imagination-creativity	Produces many ideas; Can be highly original. Can solve problems through non-traditional patterns of thinking. Can show exceptional ingenuity in using everyday materials. Is keenly observant. Can have wild, seemingly silly ideas. Can be fluent, flexible producer of ideas. Is highly curious.

Appendix B
Sample screenshots from video recordings

Permission given to display faces.

Figure B 1*Students working collaboratively on online tasks*

Figure B 2*Bree and Miley working enrichment challenge*

Figure B 3*Bree and Miley working collaboratively on online tasks.*

Figure B 4*Ty working on enrichment task with partner*

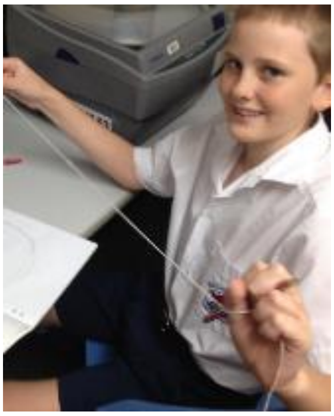
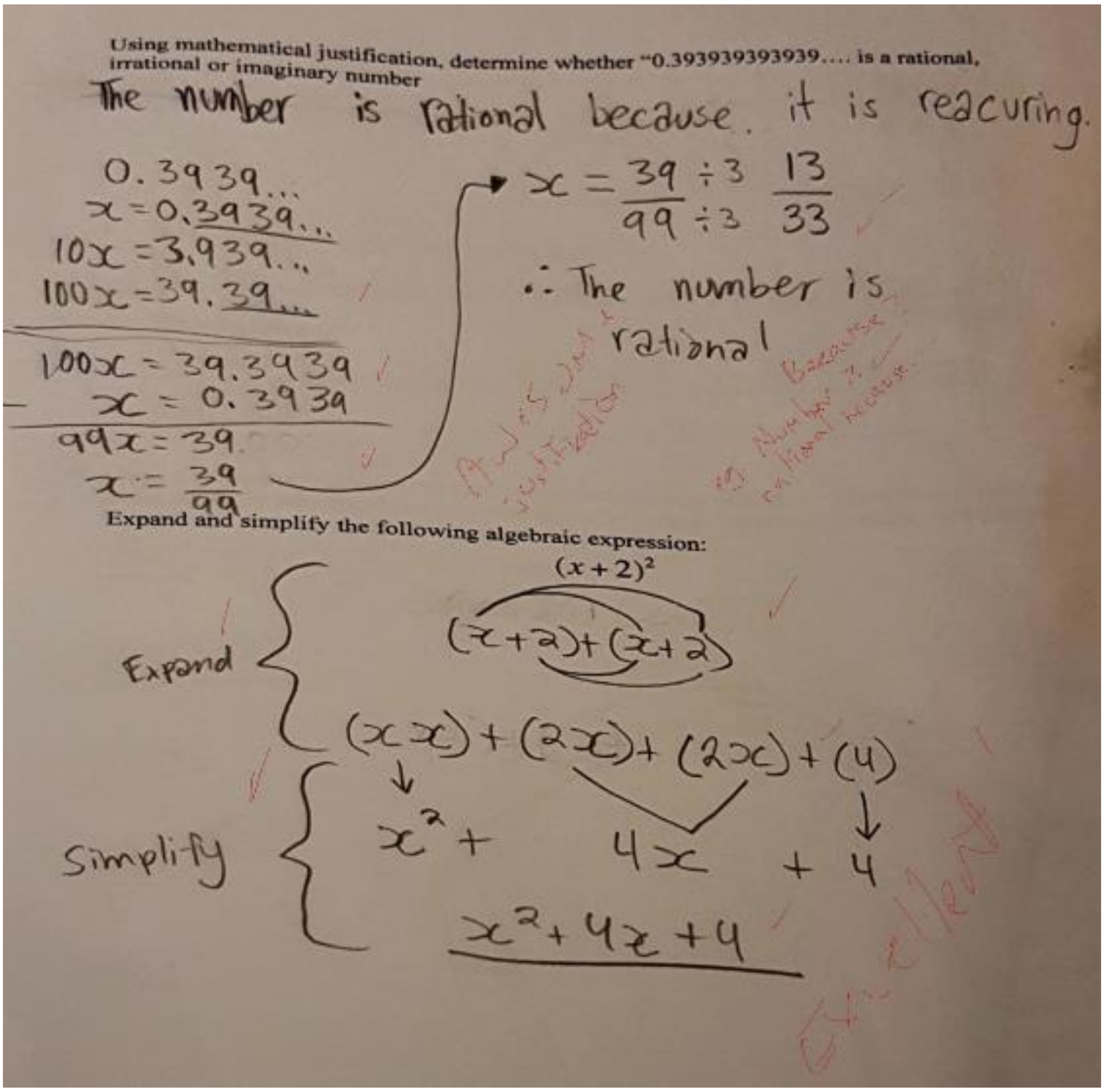
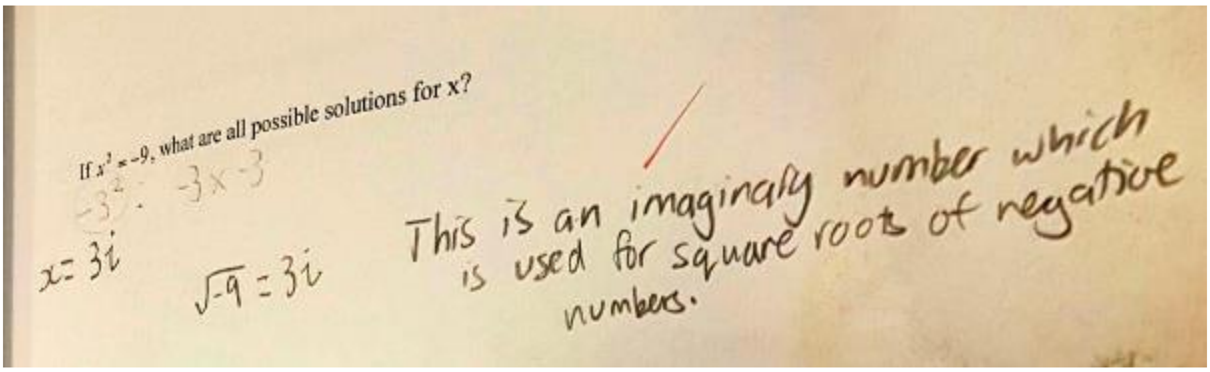


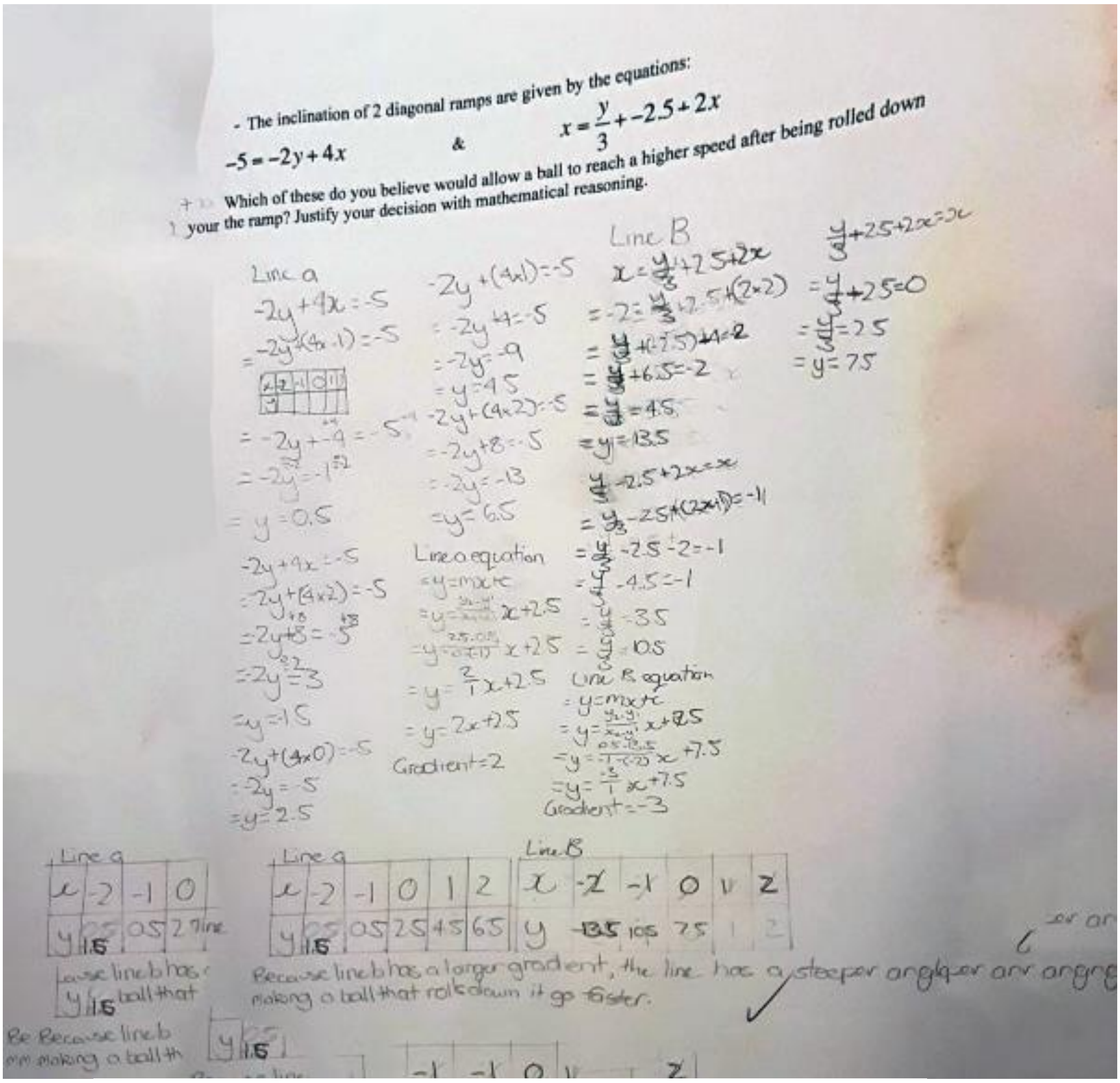
Figure B 5*Oliver working on opening enrichment investigation*

Figure B 6*Bree and Miley working on collaborative enrichment challenge*

Appendix C
Year-9 level work samples







- The inclination of 2 diagonal ramps are given by the equations:

$-5 = -2y + 4x$ & $x = \frac{y}{3} + -2.5 + 2x$

Which of these do you believe would allow a ball to reach a higher speed after being rolled down your the ramp? Justify your decision with mathematical reasoning.

Simplify and express the following with positive indices

a. $5^{-2} \times 1^{0} =$

Formulas: $a^0 = 1$			
$a^{-m} = \frac{1}{a^m}$	$5^{-2} \times 1^0$	$\frac{1}{5^2} \times 1$	$\frac{1}{5^2}$

Appendix D
Sample interview transcript

Ty: I got an A- every term last year as well.

Int: Do you like the way maths is structured?

Ty: Yeah, its different to previous years. Because if you finish something really fast, you can go on and do the next thing whereas last year, we had to keep listening and listening to the teacher, when we already have got and you'd have to listen to the teacher teach something you already knew for the rest of the week.

Int: You got to the exam and you seemed to understand everything you needed to know. Can you explain to me how you got to that stage?

Ty: I did all the formative quizzes. I pick stuff up really quick. I do something once and I remember it for a quite a while.

Int: What aspects did you not like about the maths program this semester?

Ty: With computers I am really slow, so I get kinda frustrated using computers, and if you use them all the time, it gets kind of annoying.

Int: How have your attitudes towards maths changed?

Ty: Last year, I found it really boring, so I'd probably give it like a 3/10, whereas this year I am doing stuff that I didn't really know before, so it's probably like a 7.

Int: What would make it a 10?

Ty: A bit more outdoorsy stuff.

Int: What motivates you to do better at maths? Grades or learning new things?

Ty: A bit of both, because you always want to get good grades, because you will fall behind otherwise.

Int: What are your thoughts regarding the enrichment tasks?

Ty: They were interesting. One week you got to do something different, which made it a bit more interesting.

Int: How do you feel about the fact that you have to master a skill before going onto the next level?

Ty: It helps because it makes sure you don't miss something, because like in grade 3, I was doing like grade 7 maths and I missed something along the way, and then I would get stuck because I didn't know something. This is better because you know that you know everything you need to know.

Int: What about feedback?

Ty: Yeah, because it tells you what you haven't learnt yet.

Int: How do you feel that you could be further along the path with maths then you are..for example, another student is going to be doing year 10 maths c work, and you are just as good as he is in maths, but you're not. How does that make you feel?

Ty: I don't really like doing outside of class work. If I can do all the I need to do in class, then that's fine, but I don't want extra work to do outside of class. Like, I know I would like grade 10 work, but yeah…

Int: How did you prepare for your exam?

Ty: I listened in class..I don't prepare for exams as it stresses me out when I go into an exam.

Int: How do you feel about getting an A- when you are capable of getting an A or better?

Appendix E
Sample in-class audio recordings

25th Feb 2014
M – What is a composite shape?
Int – What do you think it is? When you have got a composite of materials, what have you got?
M – There composite.
Int – There's two things combined. So looking at those questions, if you had this shape joined with this shape, how would you find the area of both of them joined together?
M – You'd add that one to that one.
Int. – When you have a triangle and a rectangle and they are joined together like that one, what will you do?
M – Find the area of the triangle and then the area of the rectangle and then add them together.
Int. – Remember that you only do the questions you haven't understood and you don't have to do all of them.
M – Okay.

Thursday March 13th

Figure E.1. Walter and Ty working quietly together while the teacher teaches a new concept on the whiteboard.

It was often the case that while the teacher was teaching the class students would work collaboratively and quietly in a way that is not disruptive of other students' learning. The video footage of this and other lessons show students around these students listening to the lesson attentively, while these students go on learning in a way that suits them. In this particular lesson both students at random time intervals looked up to the teacher to make sure they understood what they needed to know from that lesson.
Example of students working collaboratively helping each other while the teacher is teaching.
Interesting in a group bingo game activity these three boys were solving this question. In their heads they came to the answer of 13 without working anything out on paper.

> "Arthur is typing a paper that is 390 words long. He can type 30 words in a minute. How long will it take for him to type the paper?

You need to use the cross-multiplication method to solve this question".

Instead of using cross multiplication as requested, they told me that they simply entered into the calculator 390 divided by 30 to get 13. The purpose of this question was to expose students to a word problem situation which would give them practice at using the cross multiplication method. For these boys, they chose the "easy" route to solve the question.

James mows 4 times as much grass as does Samantha, as he is stronger. For every 1 metre of grass that James mows, Jade will mow the same. If there is a total of 100 square metres of grass to mow, then how much grass will each of them mow?

Interesting that when both Wim and Ty couldn't solve the above question, they chose to guess the answer rather than try to solve it.
Students were asked to simplify the following:

$$\frac{5}{8}:1\frac{3}{4}$$

Wim, Oliver and S….Students
Teacher: Is this question hard?
Wim: No, you just simplify it.
Teacher: So how do you simplify it?
Ty: You just turn it into the same um…what do you call it…the same denominator…
Teacher: Yes…good…
Wim: They both go into 8.
Ty: Talking about the second portion of the ratio, Ty proposes to turn the mixed number to a improper fraction by saying "You turn that into 7"
Wim: Yes.
They then both seem to add the two fractions together to somehow get 14.

What I noticed about this lesson was a sense of jealousy when it was announced who was leading the maths race because the group leading had Oliver in it. Comments made like "common, common" by Ty suggesting to S to move to the next question as they had already solved the question on the screen showed a sense of urgency about solving the question. The photo below depicts three boys working together. This picture displays a sense of engagement in focus. This section of the video showed Oliver remain focused scratching his head, moving through the questions at a fast a pace as he could. This picture shows a level of focus with him intent on solving the problem in question. It also shows that he was intent on solving the questions in his head. It was also interesting that as the teacher was so busy walking around the class helping students and so caught up in the excitement of the lesson, that he didn't pick up on the fact that these students weren't showing their working out on paper. This is something that I will have to watch out for incoming lessons. It will also be interesting to see what kind of reaction takes place.

Figure E.2. Picture that is representative of the type of activities that were commonplace in this classroom. Students not involved in study have had faces blurred for the sake of anonymity.

Rather than putting any working out on paper, these boys would try to solve the problem completely in their heads. This picture also shows that Oliver was the main person completing the questions and doing the lion's share of thinking. At the end of the lesson when the teacher was explaining the answers to the whole class you could hear the students debating the answers to questions. The actual audio footage wasn't clear, but intermittent words did make it clear that the students were still trying to solve a question they didn't get time to solve during the challenge. It was also interesting to note that while I was working through the worked solutions to these problems that these students were not listening to what I was saying. Rather they were intently focused on trying to work through to a successful answer of the mowing question (given above) that they still hadn't finished. This showed that the students were determined to find a solution to all the problems and get as many of the questions correct as is possible. In an informal survey of interest in the activity all students were asked if they preferred this style of learning from each other. Every student in the class barring this group of boys had their hands up. The reason these boys didn't have their hand up was that they were still focused on solving this perplexing lawn mowing question.

Excerpt from a Transcript of an In-Class Recording of Conversation between Bree and Miley (March 20, 2015)
In Class recording…

Sounds of students trying to work out the answer…
Bree: I know how to do this with a calculator…

Miley: Miley bangs the desk in excited tones…..signifying to Bree while giggling that she has the answer
Bree: You got it wrong….
Miley: Hang on…I still think it's 540
Bree: (Bree works out question again verbalising her thinking) Hang on. I'll put it into the calculator.
Miley: See…that's what I got…
Bree: Wait a second. Wait a second. Nah…I got it…This will take me ages…hang on…Asks teacher for help…
Teacher: Yes, that answer is 540.
Miley: (in an excited voice) Yes, I won!

Separate Discussion on the Same day with Walter and Ty
Walter: What are you doing?
Ty: Wait, that's not right.
Walter: It's 13.
Ty: What did you get?
Walter: I got 13.
Ty: Ah what's the length of the top one.
Walter: We did not get the top one.
Ty: What if we got the answer wrong because we did not read the question properly?
Walter: That's why I took so long, because I did read it properly.

Appendix F
Sample teacher diary entry

Key Question	Data to be collected	Examples & why this data is important
How does the use of the adapted Mastery Learning model influence gifted mathematics students' motivation and Attitudes towards learning mathematics?	Screenshots of significant events. These photos provide a further source of evidence and understanding of the classroom environment.	 *Figure F.1.* Students collaborating in a regular lesson 19th March, 2014 Group problem solving challenge which required students to solve challenging problems together in a group. This is a significant event for 3 reasons. 1. Students were noticeably engaged and interested in task. 2. This particular shot shows the three girls actually debating their answer justifying why they thought their answers were correct. 3. These activities were commonplace activities and took place regularly throughout the term.  *Figure F.2.* Students working on opening enrichment challenge 12th Feb 2014 Students using different materials to find the relationship between the circumference and the diameter of a circle. Event is significant as students were engaged, interested and on-task. Students worked together to problem solve.

Appendix G
Opening enrichment task

Students break in to small groups of 2-3 students per group.

Students are given a ball of string, a pair of scissors and two pencils. They are asked to find as many discoveries in relation to the circle as they can. They first formulate questions and then provide steps to solve their problems that they have created.

After approximately 10 minutes, the teacher provides further scaffolding by offering hints to individual student groups.

Hints included:

1. How can you find the radius/diameter?
2. How is the area of the circle effected by the length of the string?
3. How can you find the length of the circumference?
4. How can you find pi (if you did not already know it)?
5. How could you find the perimeter/area of a segment?
6. What other problems could you create with the string, that is not related to a piece of string?

Students share their findings with each other and collaborate with other teams. Discoveries and questions are place on the whiteboard at the front of the room.

Appendix H
Sample formative assessments

The timetable shows the arrival and departure times for trains in 24-hour time.

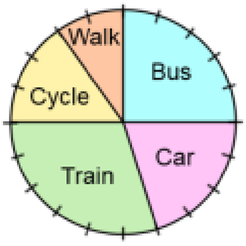
Amanda arrives at Black Hill station at 1:45 p.m. and wants to catch the first train to Cloverton.

Station						
Dairyvale						
Arrives	10:57	11:25	12:00	12:33	13:04	13:50
Departs	11:00	11:28	12:03	12:36	13:07	13:53
Black Hill						
Arrives	11:47	12:15	12:50	13:23	13:54	14:40
Departs	11:52	12:20	12:55	13:28	13:59	14:45
Cloverton						
Arrives	12:18	12:46	13:21	13:54	14:25	15:11
Departs	12:22	12:50	13:25	13:58	14:29	15:15

How long will she wait for a train to arrive at Black Hill?

- ○ 2 minutes
- ○ 5 minutes
- ○ 9 minutes
- ○ 14 minutes
- ○ 22 minutes

Ways of getting to school



What percentage of students travel to school by car?

- ○ 4%
- ○ 5%
- ○ 15%
- ○ 20%
- ○ 25%

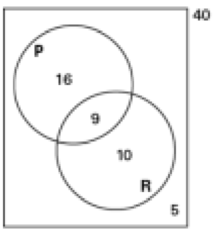
Sara surveyed 40 people.

She asked them:

'Do you like pop music?' and

'Do you like rock music?'

Sara drew this diagram to show her results.



P: like pop music
R: like rock music

Sara chose one person to interview.

She chose at random from the people that liked pop music.

What is the probability that this person will **not** like rock music?

- ○ $\frac{9}{25}$
- ○ $\frac{16}{25}$
- ○ $\frac{16}{40}$
- ○ $\frac{25}{40}$

Appendix I
Sample unit outline

Table I 1

Sample unit outline

Timeline	Australian Curriculum (Column 1)	Regular Instruction (Column 2)	Year-9 Content (Column 3)	Enrichment (Column 4)
March	Y8: Solving rates and ratio problems (ACMNA188) Y9: Explore proportion and the relationship between graphs and equations corresponding to simple rate problems (ACMNA208).	Unit pre-quiz. Students complete a range of problems involving rates. Formative Quiz.	Students chart distance/ displacement time and speed / velocity time graphs. Examples: Students might construct and use existing graphs to find and record rates of change.	Students investigate a topic of their choosing in relation to change per Appendix K.
March April	Y8. Collecting data. (ACMSP284) Construct & analyse stem-and-leaf plots and histograms (ACMSP282) Mean, median, mode and spread (Affect outliers have on data) (ACMSP207). Y9. Construct & describe data in histograms; stem & leaf plots using terms including ‘skewed’, ‘symmetric’ and ‘bi modal’ (ACMSP282).	Pre-Quiz – Game Organise, display and interpret data in frequency tables, histograms and frequency polygons. Calculate the measures of centre for a given set of data including range, mean and median. Formative quiz.	Distinguish between the measures of central tendency (mean, median, mode) and measures of spread (range & interquartile range). Interpret already existing (Construct their own) box and whisker plots.	Students continue to work on investigation from previous unit using skills learned in this unit to help them complete their investigations.

Table I 1 *Sample unit outline (continued)*

Timeline	Australian Curriculum (Column 1)	Regular Instruction (Column 2)	Year-9 Content (Column 3)	Enrichment (Column 4)
Unit 3: Aug 8, 9 & 13.	Y8. Investigate the effect of individual data values, including outliers, on the mean and median (ACMSP207). Y9. Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread (ACMSP283).	Students examine outliers in data and explore real world contextual use of this information.	Students investigate and Distinguish between the measures of central tendency (mean, median, mode) and measures of spread (range & interquartile range).	Students use the information learned in this lesson to help them work on their Change investigation (Appendix K)
Unit 4: Aug 15, 16, 20, 22, 23, 27, 29 & Sept. 3, 5 & 6.	Y8. Plot linear relationships on the Cartesian plane (ACMNA193). Solve and verify linear equations (ACMNA194). Y9. Find the distance between 2 points, the midpoint and gradient of a line segment (interval) on the Cartesian plane (ACMNA214), (ACMNA294). Y10. Explore graphical representations of algebraic equations (including quadratics and other exponentials) (ACMNA239).	Students translate tabulated coordinates into graphical form and apply this to linear functions. Then provide link to the 'Gradient and Y-Intercept Method' for graphing linear equations in the form y = mx+c.	Scaffold information to help students synthesise two alternate methods for graphing linear equations: the double intercept method and also manipulating the linear equation until it IS in the form y = mx + c	Students analyse the differences in graphical representations of equations using www.fooplot.com Students use this site and their calculations to create a stained-glass window as per Appendix T.

Table I 1 *Sample unit outline (continued)*

Timeline	Australian Curriculum (Column 1)	Regular Instruction (Column 2)	Year-9 Content (Column 3)	Enrichment (Column 4)
Unit 5: October 8, 10, 11, 15, 17, 18 & 22.	Y8. Students develop volume formulas and solve problems pertaining to volume. (ACMMG198) Y9. Calculate volumes and surface areas of various prisms and cylinders (ACMMG217), (ACMMG218) and (Y10. ACMMG242).	Explore the difference between s/area and volume? How do s/area and volume change as you manipulate the dimensions of the objects? Are there any similarities in the formulas for differing objects?	Students justify why different everyday items are packaged the way they are by exploring the aspects of surface area & volume of these solids.	Students complete straw volume investigation.
Unit 6: October 24, 25 & 29	Y8. Solve problems involving 12-24 hour time and time zones. (ACMMG199).	Convert between 12/24 hr time and solve problems involving time. Explore problems involving Australian and Global time zones.	Calculate local departure and arrival times of international flights using accessible time zone information online. Students investigate biorhythms, and effects international flights have on your body.	

Table I 1 *Sample unit outline (continued)*

Timeline	Australian Curriculum (Column 1)	Regular Instruction (Column 2)	Year-9 Content (Column 3)	Enrichment (Column 4)
Unit 7: October 31, November 1, 5, 7 & 8.	Y8. Complimentary events, two-way tables and Venn diagrams (ACMSP292), (ACMSP204). Describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and'. (ACMSP205). Y9. Use data from online media sources and elsewhere to calculate predictions with/out replacement (ACMSP225), (ACMSP226) and (ACMSP227).	Pre-quiz game Students complete a range of experiments and use results and data to predict possible future occurrence.	Students complete investigations on a choice of topics including: The chance of winning in The Lotto. The chance of having a serious accident if driving over the speed limit. The chance of developing serious health problems due to poor diet. Negotiated topic of choice.	

Appendix J
Sample sub-unit with learning activities and online interactives

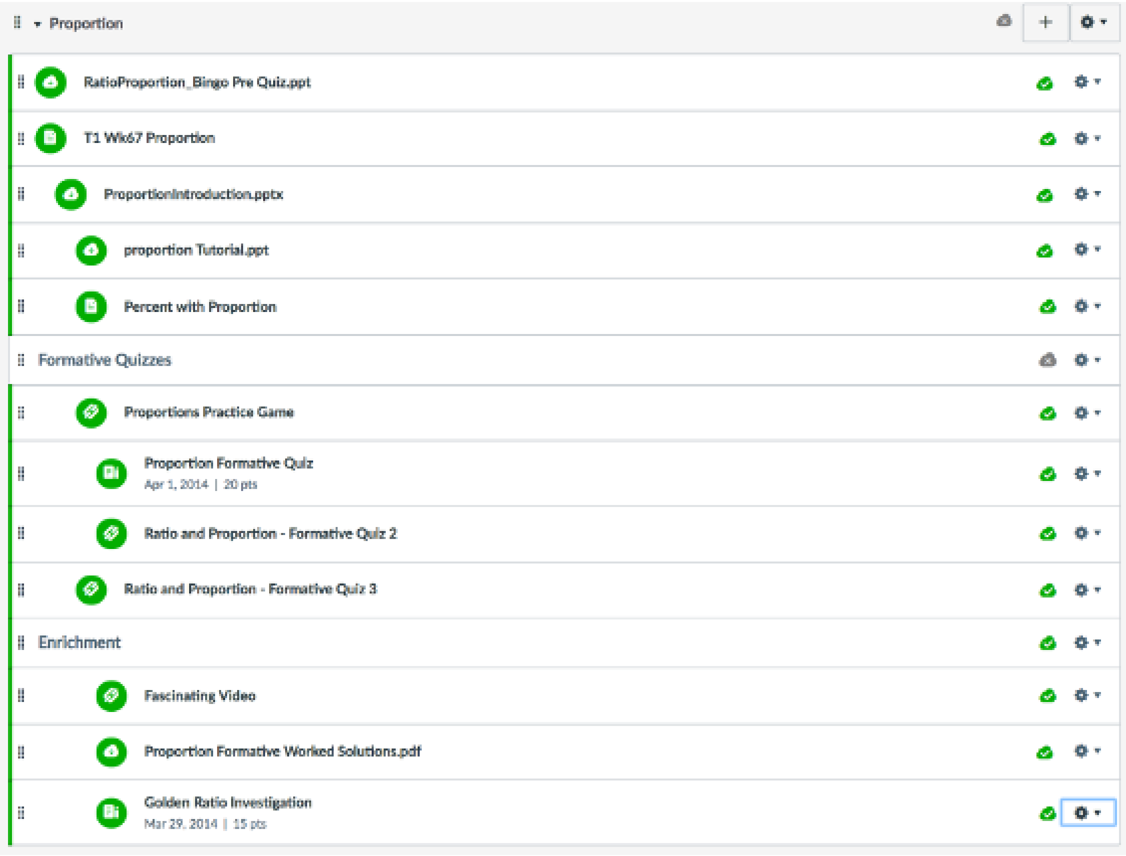
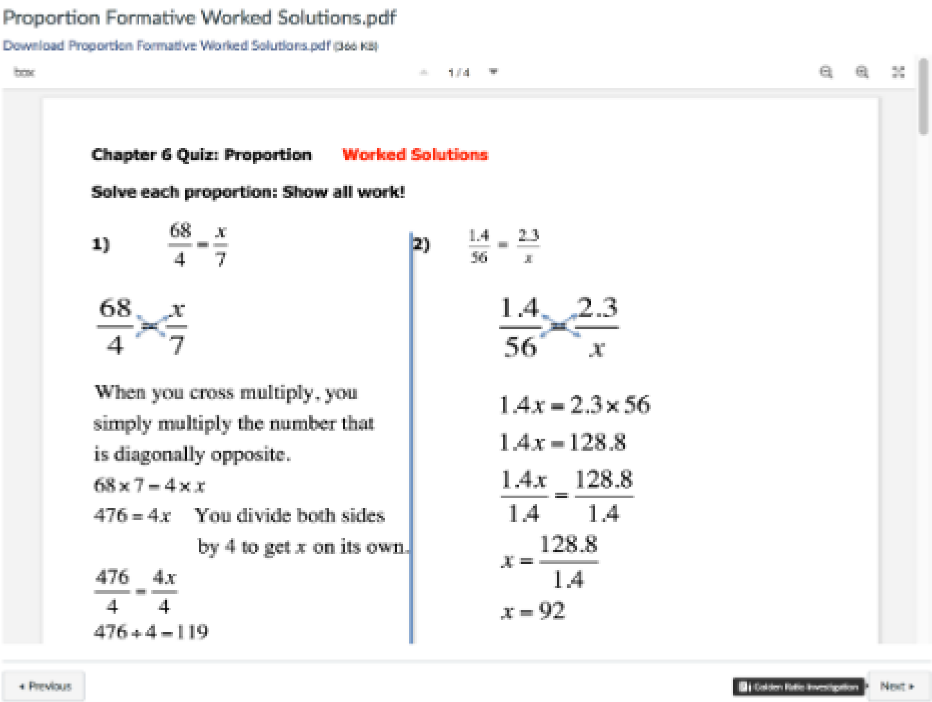


Figure J 1*Sample sub-unit on a Year-9 concept - Proportion*



Proportion Formative Worked Solutions.pdf

Download Proportion Formative Worked Solutions.pdf (366 KB)

Chapter 6 Quiz: Proportion **Worked Solutions**

Solve each proportion: Show all work!

1) $\frac{68}{4}=\frac{x}{7}$

$\frac{68}{4}=\frac{x}{7}$

When you cross multiply, you simply multiply the number that is diagonally opposite.

$68\times 7=4\times x$

$476=4x$ You divide both sides by 4 to get x on its own.

$\frac{476}{4}=\frac{4x}{4}$

$476\div 4=119$

2) $\frac{1.4}{56}=\frac{2.3}{x}$

$\frac{1.4}{56}=\frac{2.3}{x}$

$1.4x=2.3\times 56$

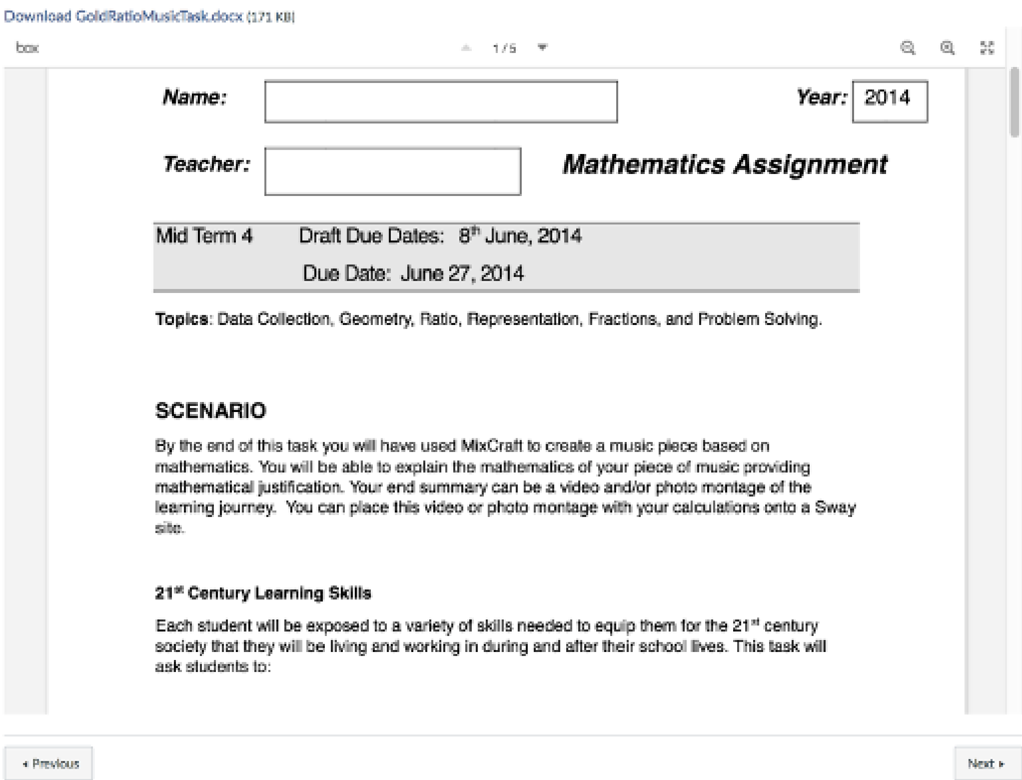
$1.4x=128.8$

$\frac{1.4x}{1.4}=\frac{128.8}{1.4}$

$x=\frac{128.8}{1.4}$

$x=92$

Figure J 2*Sample worked solution taken from PowerPoint*



Download GoldRatioMusicTask.docx (171 KB)

Name:		***Year:***	2014
Teacher:		***Mathematics Assignment***	

Mid Term 4 Draft Due Dates: 8th June, 2014

Due Date: June 27, 2014

Topics: Data Collection, Geometry, Ratio, Representation, Fractions, and Problem Solving.

SCENARIO

By the end of this task you will have used MixCraft to create a music piece based on mathematics. You will be able to explain the mathematics of your piece of music providing mathematical justification. Your end summary can be a video and/or photo montage of the learning journey. You can place this video or photo montage with your calculations onto a Sway site.

21st Century Learning Skills

Each student will be exposed to a variety of skills needed to equip them for the 21st century society that they will be living and working in during and after their school lives. This task will ask students to:

Figure J 3*Golden ratio proportion enrichment task*

Appendix K
Sample five-point marking rubric

Understanding and Skills dimensions		A	B	C	D	E
		The folio of student work has the following characteristics:				
	Understanding & Fluency	Connection and description of mathematical understandings in a range of situations, including some that are complex unfamiliar	Connection and description of mathematical understandings in complex familiar or simple unfamiliar situations	Recognition and identification of mathematical understandings in simple familiar situations	Identification of simple mathematical understandings in rehearsed situations	Statements about obvious mathematical understandings
		Accurate and efficient recall and use of facts, definitions, technologies and procedures to solve problems	Accurate recall and use of facts, definitions, technologies and procedures to solve problems	Recall and use of facts, definitions, technologies and procedures to solve problems	Some recall and use of facts, definitions, technologies and simple procedures	Attempted recall and use of facts, definitions, technologies and given procedures
		Consistent use of appropriate mathematical language, conventions and symbols	Use of appropriate mathematical language, conventions and symbols	Use of mathematical language, conventions and symbols	Use of aspects of mathematical language, conventions and symbols	Use of everyday language
	Problem solving & Reasoning	Systematic application of problem-solving strategies to investigate a range of situations, including some that are complex unfamiliar	Application of problem-solving strategies to investigate complex familiar or simple unfamiliar situations	Application of problem-solving strategies to investigate simple familiar situations	Identification of problem-solving strategies in rehearsed situations	Attempted use of given problem-solving strategies
		Mathematical modelling and representation of a range of situations, including some that are complex unfamiliar	Mathematical modelling and representation of complex familiar or simple unfamiliar situations	Mathematical modelling and representation of simple familiar situations	Statements about simple mathematical models and representations in rehearsed situations	Isolated statements about given mathematical models and representations
		Evaluation and interpretation of the results of, and conclusions reached in, investigations and inquiries	Analysis of the results of, and conclusions reached in, investigations and inquiries	Description and comparison of the results of, and conclusions reached in, investigations and inquiries	Statements about the results of, and conclusions reached in, investigations and inquiries	Isolated statements about investigations and inquiries
		Clear explanation of mathematical thinking, including justification of choices made and strategies used	Explanation of mathematical thinking, including reasons for choices made and strategies used	Description of mathematical thinking, including discussion of choices made and strategies used	Statements about choices made and strategies used	Isolated statements about given strategies
		Justification of the reasonableness of answers	Discussion of the reasonableness of answers	Checks of answers	Use of given strategies to check answers	Attempted use of given strategies to check answers

| Year 6 standard elaborations Australian Curriculum: Mathematics

Figure K 1*Assessment Rubric (Queensland Studies Authority, 2012)*

Appendix L
Sample of Bree's artistic ability in a response on a mathematics exam.

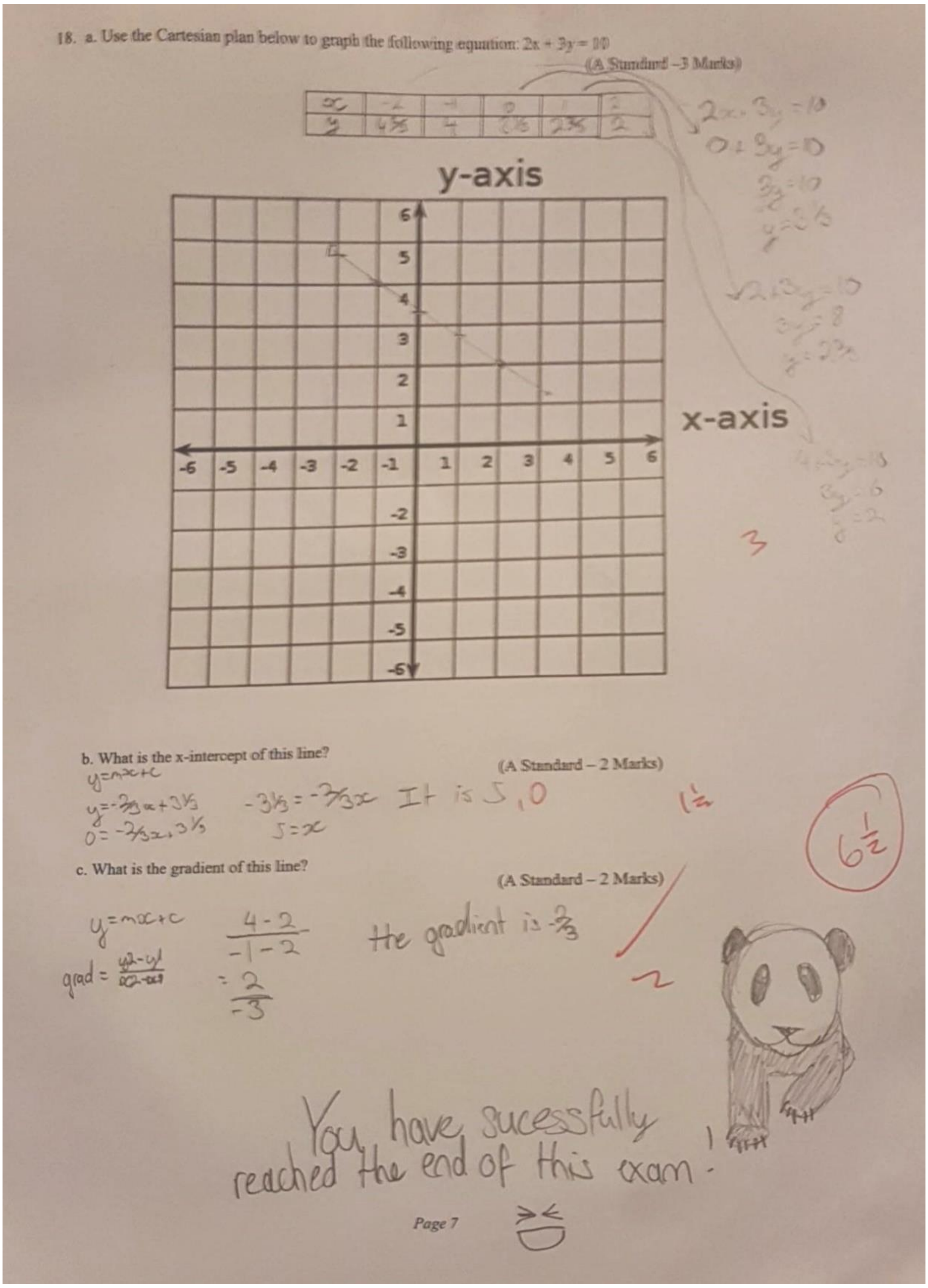


Figure L1*Sample of Bree's artistic ability in a response on a mathematics exam.*

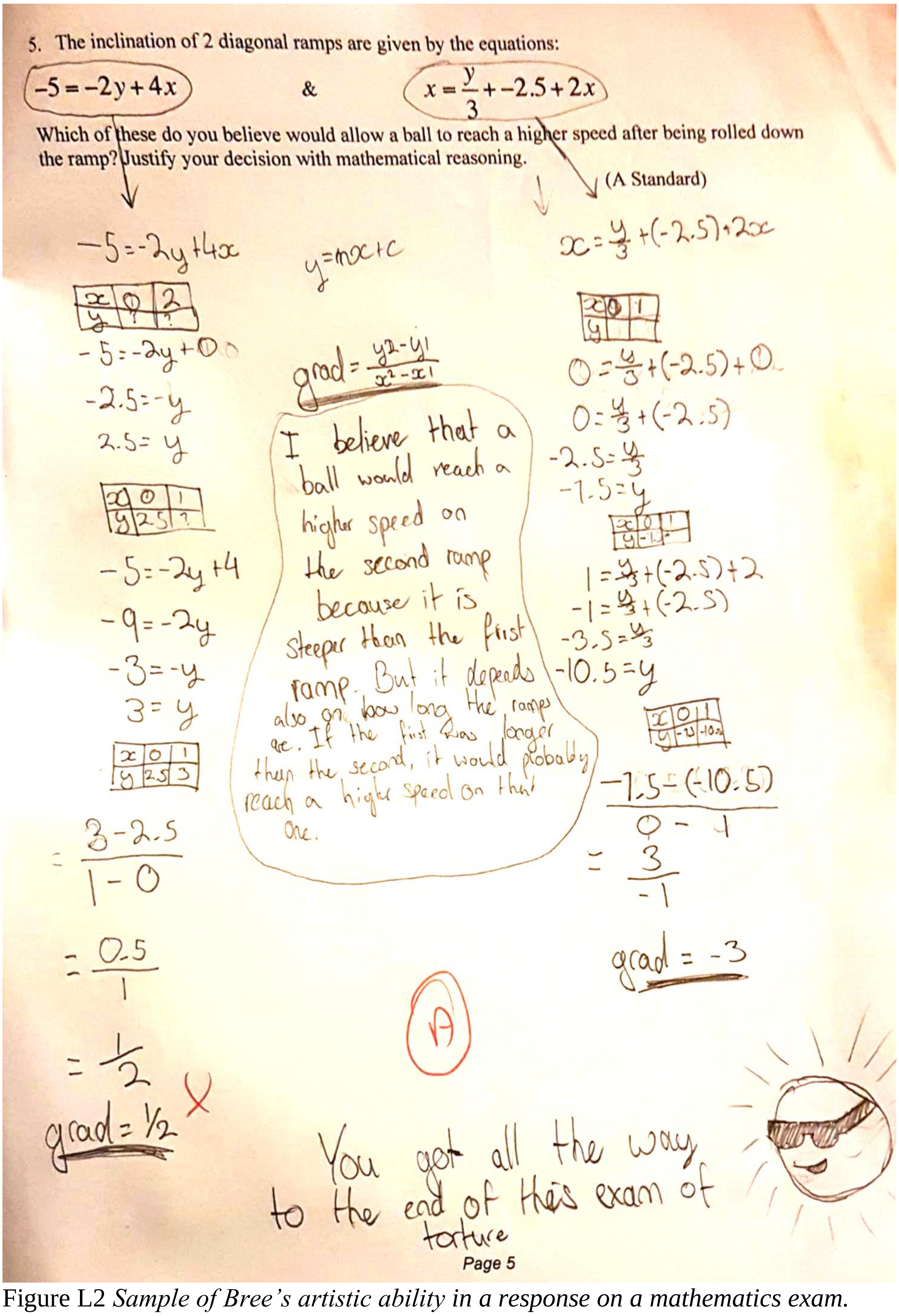


Figure L2 *Sample of Bree's artistic ability in a response on a mathematics exam.*

Appendix M
Remainder of Class Assessment Results

	Y8 Pre-Quiz Results	Y8 Formative Results	Y9 Pre-Quiz Results	Y9 Formative Results
Student 1	87.78%	86.19%	72.73%	91.48%
Student 2	56.89%	77.43%	52.63%	52.13%
Student 3	79.37%	80.60%	51.61%	79.68%
Student 4	79.23%	86.29%	52.00%	79.36%
Student 5	74.24%	79.00%	37.50%	83.46%
Student 6	73.33%	90.43%	42.86%	70.31%
Student 7	58.93%	73.93%	44.00%	96.46%
Student 8	81.43%	74.46%	64.29%	72.90%
Student 9	76.33%	92.06%	62.50%	73.79%
Student 10	69.66%	81.88%	64.71%	76.57%
Student 11	72.70%	80.44%	80.95%	81.99%
Student 12	71.43%	92.02%	35.71%	73.69%
Student 13	67.33%	82.32%	45.45%	63.71%
Student 14	64.56%	79.14%	46.15%	74.08%
Student 15	80.56%	88.66%	58.82%	76.77%
Student 16	69.70%	65.71%	48.00%	60.65%
Student 17	71.36%	73.29%	83.33%	76.56%
Student 18	93.78%	82.55%	53.85%	77.05%
Average	73.81%	81.47%	55.39%	75.59%
S.Deviation	0.089647432	0.06887114	0.1339883	0.09961299

Figure L.1. *Remainder of Class Results*

Appendix N
Concepts Covered in Year-8 Australian Mathematics Curriculum

Concepts	Approx. Number of Concepts taught
Review of Number concept, Review integers	
+, - of integers	2
x, / of integers	2
Combined operations of integers	1
Index Notation	1
Index Laws	
• Xing numbers in index form with same base	1
• /ing numbers in index form with the same base	1
• Power of Zero	1
Indices	
a) Raising a Power to Another power	1
b) Intro to real numbers	1
c) Operation using fractions with like and unlike denominators (bridging)	2
Ratio and space	
• Simplify and comparing ratios, include using fractions	2
• Properties of quadrilaterals and triangles	2
Geometry	
• Perimeters and areas of triangles and quadrilaterals – Parallelograms, rhombuses, kites	5
Number – Rational, irrational and real numbers	
• Revise of terminating, recurring and non-terminating decimals	3
• Define rational and irrational (incl. pi), real and imaginary numbers including integers	3
• Bridging Content – using variables, substitution, working with brackets, substituting positive and negative numbers	3
Algebraic Expressions	1
	1
	3

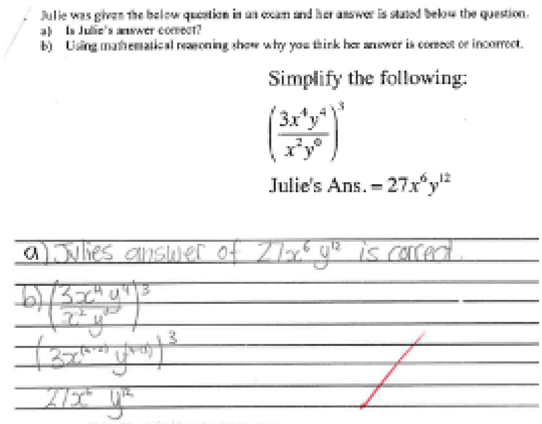
- Use the four operations to simplify algebraic expressions
- Apply BODMAS rule to simplify algebraic expressions by also grouping like terms
- Bridging Content – using variables, substitution, working with brackets, substituting positive and negative numbers, associative law, number laws and variables

Appendix O
Year-9 Sample questions given in formative quizzes

Express numbers in scientific notation (ACMNA210)
Example: Write the number 0.0065 in scientific notation.
1.32×10^{-4} as an ordinary number.

Apply index laws to numerical expressions with integer indices (ACMNA209)
Eg.

Simplify and express with positive index
$$\frac{x^{-4}}{4} \times \frac{20x^{9}}{x^{3}}$$


Julie was given the below question in an exam and her answer is stated below the question.
a) Is Julie's answer correct?
b) Using mathematical reasoning show why you think her answer is correct or incorrect.

Simplify the following:

$$\left(\frac{3x^4y^4}{x^2y^0}\right)^3$$

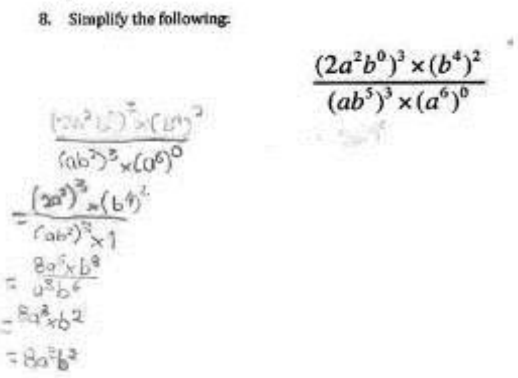
Julie's Ans. $= 27x^6y^{12}$

a) Julies answer of $27x^6y^{12}$ is correct.

b) $\left(\frac{3x^4y^4}{x^2y^0}\right)^3$

$(3x^{4-2}y^{4-0})^3$

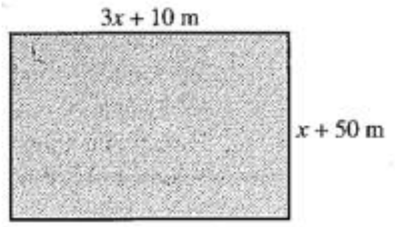
$27x^6y^{12}$

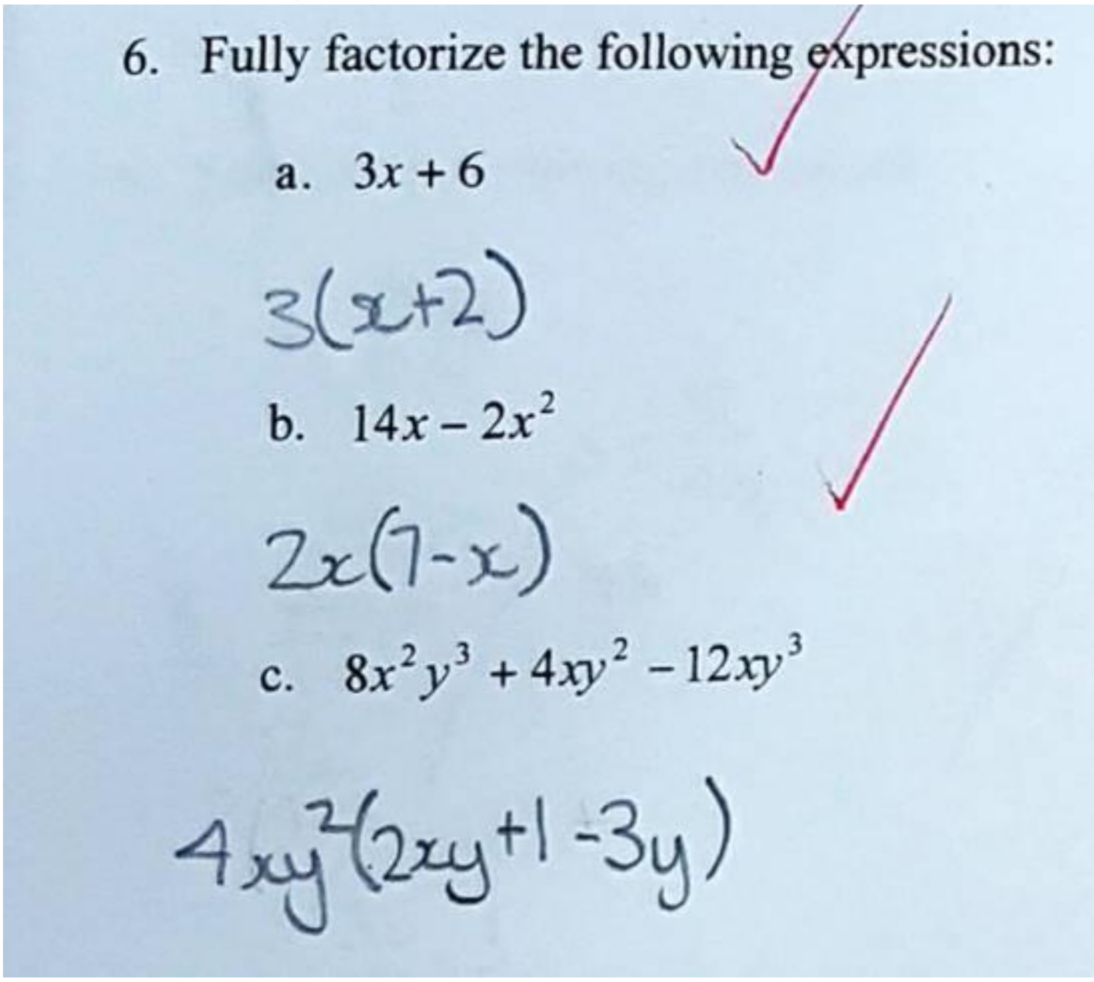

8. Simplify the following:

$$\frac{(2a^2b^0)^3 \times (b^4)^2}{(ab^5)^3 \times (a^6)^0}$$

Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate (ACMNA213)
Example: Expand and remove the bracket $(3c^2d)^3$

Example: The total distance (perimeter) around the following rectangular field is 480m. Find the value of x and use this result to find the length and width of the field.





6. Fully factorize the following expressions:

a. $3x + 6$

$3(x+2)$

b. $14x - 2x^2$

$2x(7-x)$

c. $8x^2y^3 + 4xy^2 - 12xy^3$

$4xy^2(2xy+1-3y)$

Sketch linear graphs using the coordinates of two points and solve linear equations (ACMNA215)

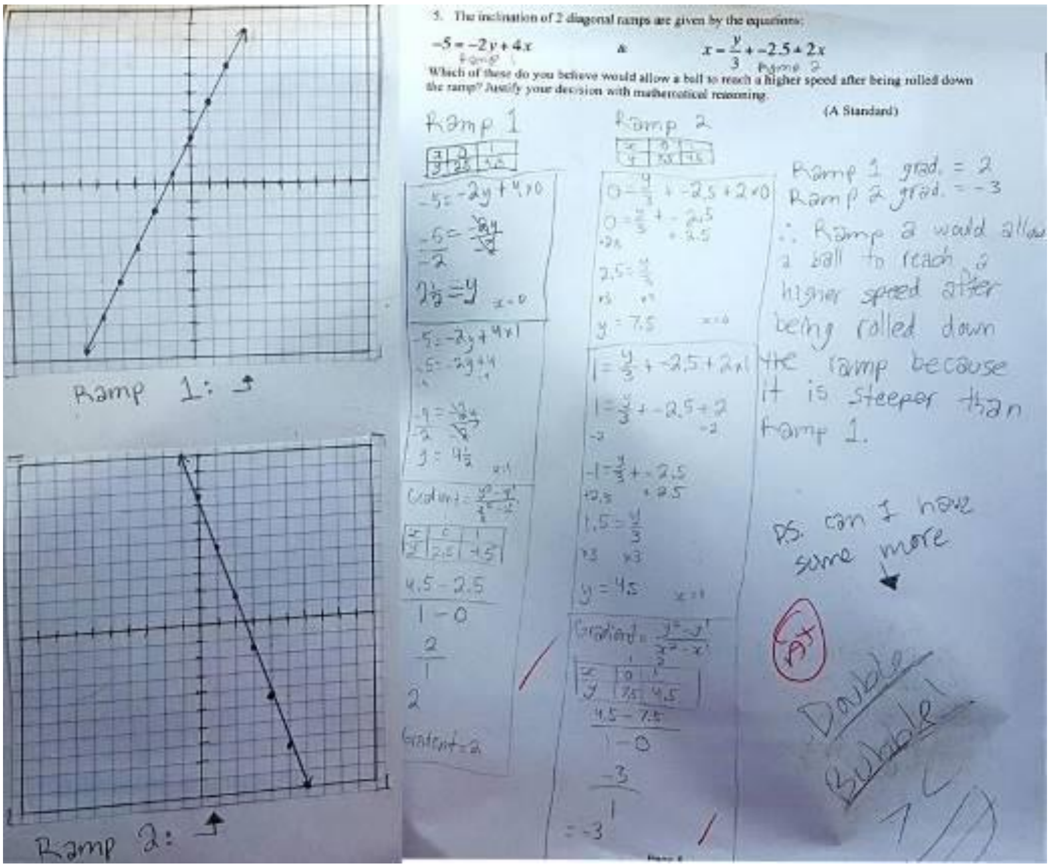


Figure O.1Miley's solution to a complex unfamiliar question with Linear Equations

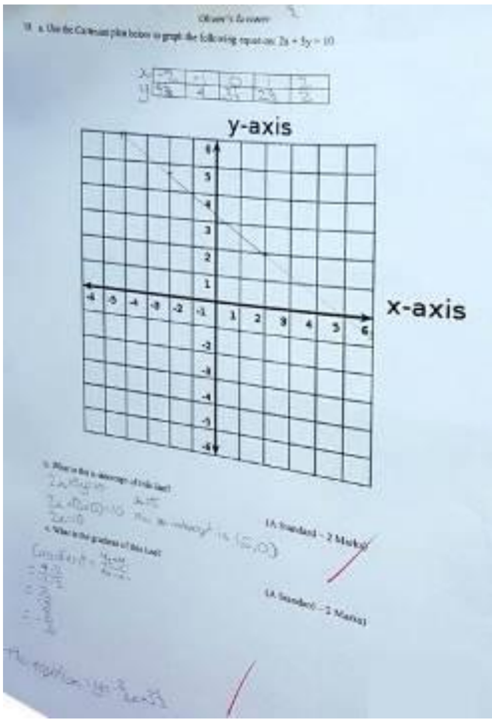
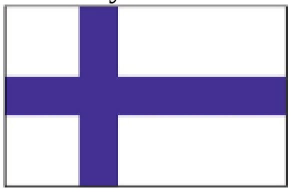


Figure O.2. Oliver's solution to a complex linear equation question on a formative quiz

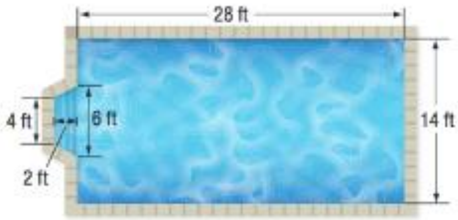
Calculate areas of composite shapes (ACMMG216)

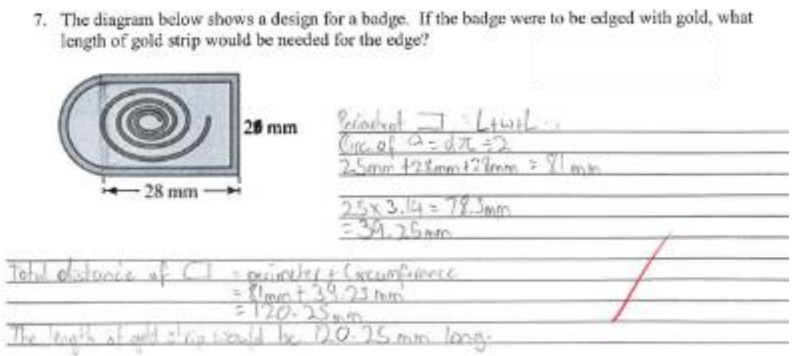
Examples:

The diagram at right shows the flag of Finland, which consists of a blue cross, whose width is a uniform 9 cms, against a solid white background. The flag measures 46cms by 60cms. What is the area of the white part of the flag?



What is the area of the pool depicted below?

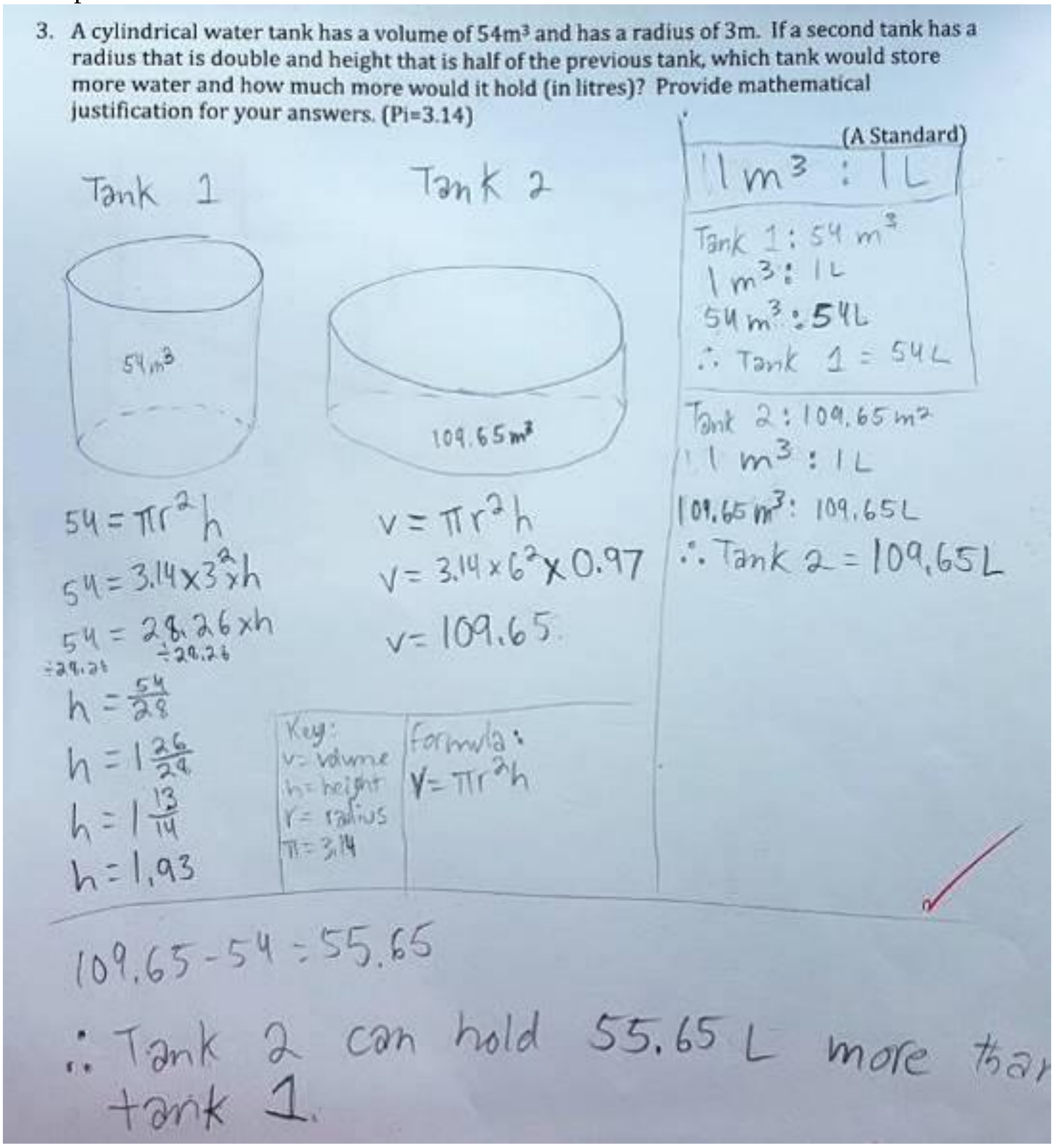




7. The diagram below shows a design for a badge. If the badge were to be edged with gold, what length of gold strip would be needed for the edge?

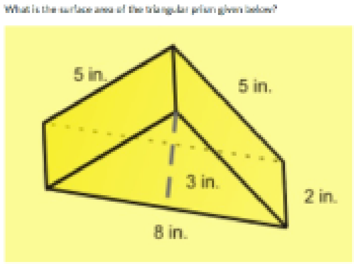
Calculate the surface area and volume of cylinders and solve related problems (ACMMG217)

Example:



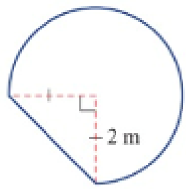
Solve problems involving the surface area and volume of right prisms (ACMMG218)

Example:



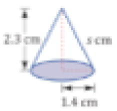
Investigate very small and very large time scales and intervals (ACMMG219)

Investigate Pythagoras' Theorem and its application to solving simple problems involving right angled triangles (ACMMG222)
Example:
Use your knowledge learned so far to find the perimeter of the shape given below (Pi.=3.14):

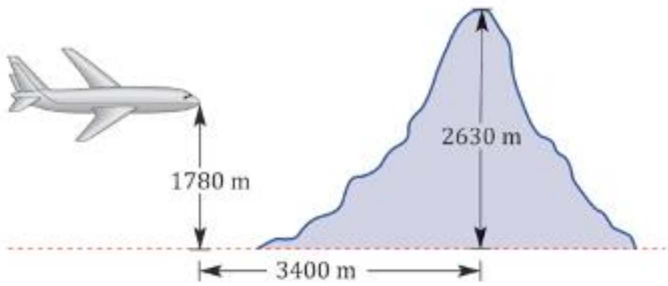


Apply trigonometry to solve right-angled triangle problems (ACMMG224)
Example:

The slant height, s cm, of the following cone, correct to one decimal place, is:

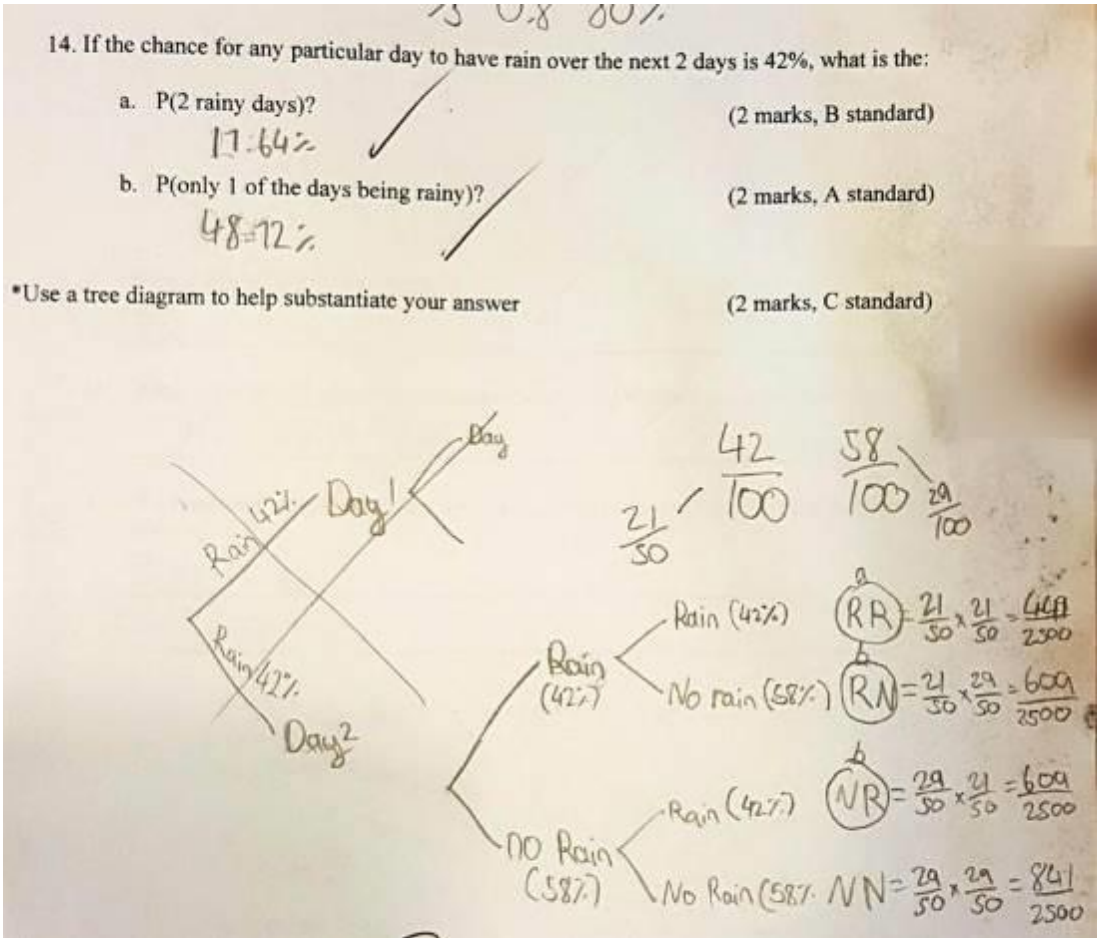


A pilot flying a plane at an altitude of 1780 m sees a mountain peak 2630 m high, 3400 m away from him in the horizontal direction.



Find the pilot's angle of elevation to the mountain peak to the nearest degree.

List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays. Assign probabilities to outcomes and determine probabilities for events (ACMSP225)
Example:



14. If the chance for any particular day to have rain over the next 2 days is 42%, what is the:

a. P(2 rainy days)? (2 marks, B standard)

17.64%

b. P(only 1 of the days being rainy)? (2 marks, A standard)

48.72%

*Use a tree diagram to help substantiate your answer (2 marks, C standard)

Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including ‘skewed’, ‘symmetric’ and ‘bi modal’ (ACMSP282)

Appendix P
Year-10 Work Example – Oliver

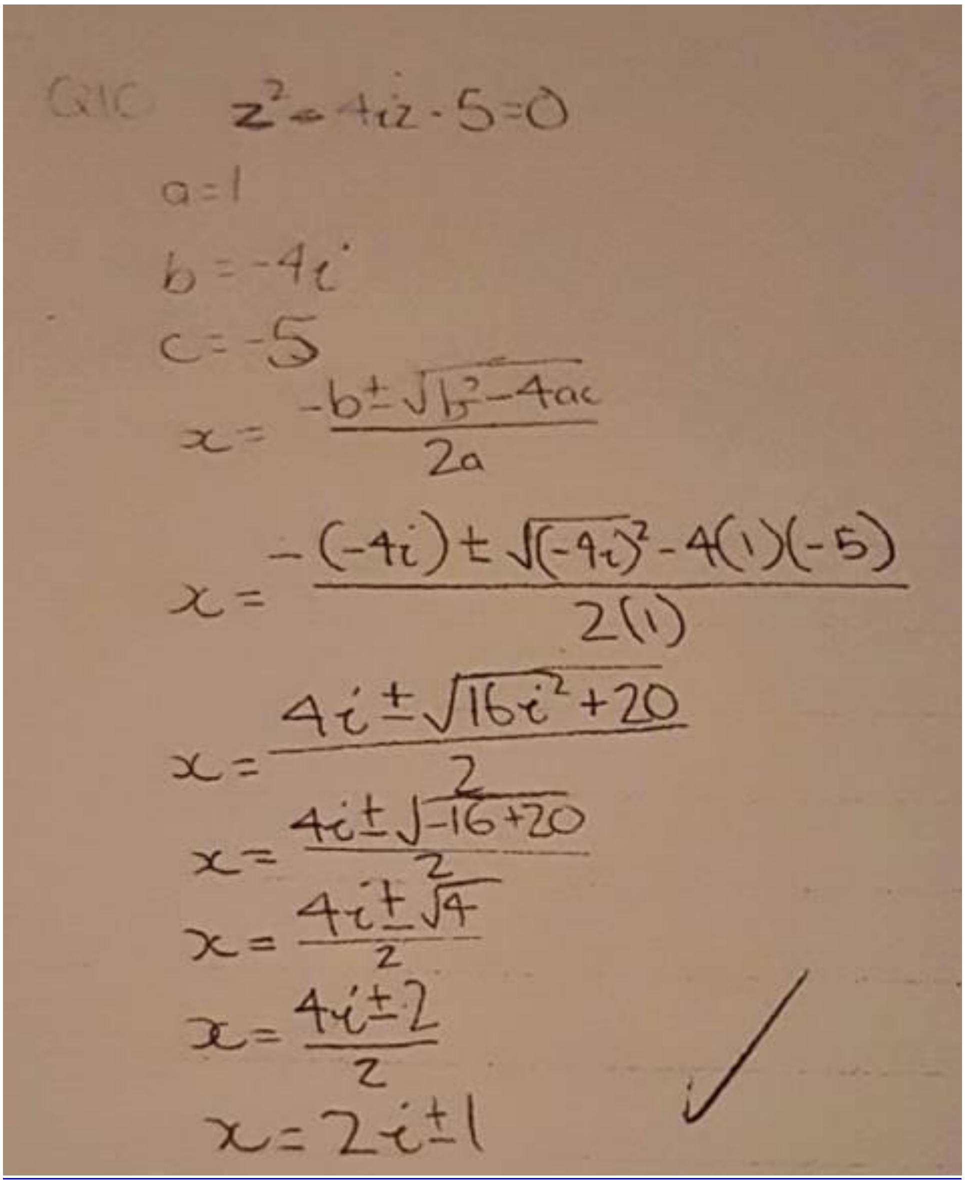


Figure O 2*Sample Year-10 level formative quiz question and Oliver's correct solution*

Appendix Q
Miley's solution to binomial theorem challenge

Miley's experiences: Miley was given a challenge which involved her discovering the pattern that emerged when you expand the bracket $(a + b)^2$ and then expand $(a + b)^3$. She was asked to use the Internet to help her explain and label the pattern that emerged and predict what $(a+b)^5$ will look like. Miley's response to this challenge, which involved understanding more advanced Year-10 level content, is discussed below.

$$
\begin{aligned}
(a+b)^0 &= 1 \\
(a+b)^1 &= a + b \\
(a+b)^2 &= a^2 + 2ab + b^2 \\
(a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\
(a+b)^4 &= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 \\
(a+b)^5 &= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5
\end{aligned}
$$

Figure P 1*Miley's working on the Binomial Theorem challenge.*

Appendix R
Golden Ratio Investigation

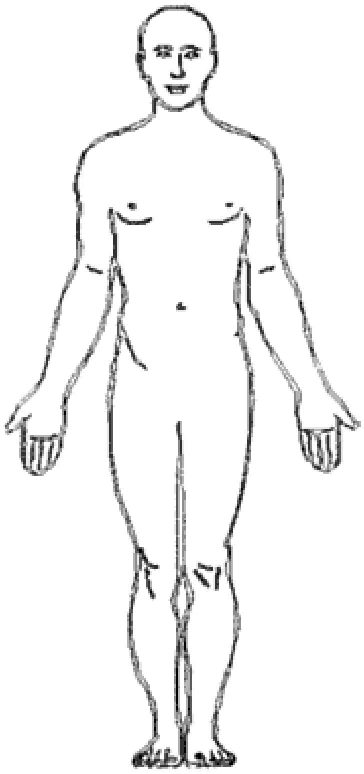
Mid Term 4 Draft Due Dates: 8th June, 2014
Due Date: June 27, 2014
Topics: Data Collection, Geometry, Ratio, Representation, Fractions, and Problem Solving.

SCENARIO
By the end of this task you will have used MixCraft to create a music piece based on mathematics. You will be able to explain the mathematics of your piece of music providing mathematical justification. Your end summary can be a video and/or photo montage of the learning journey. You can place this video or photo montage with your calculations onto a Sway site.

21st Century Learning Skills
Each student will be exposed to a variety of skills needed to equip them for the 21st century society that they will be living and working in during and after their school lives. This task will ask students to:
Collaborate: That is: They will work together in pairs, have a shared responsibility for a joint outcome. They will make substantive decisions together about goals, content, the process and finished product. Each learner will be interdependent on the other group members to complete the task successfully. They will learn that if one team member does less than the others, that the whole groups performance will be affected (as is the case in life outside of school).
ICT for Learning: Learners' use of ICT is required to construct knowledge in ways that add value to learning? i.e. They will be Movie Maker, MixCraft, AutoCollage, Sway, MovieMaker, Mix, Camera, OneNote, Microsoft Excel, Sound Spectrum, to model a solution to a mathematical challenge. Learners use ICT to design and create new ideas, products and solutions. These ideas will be presented to the principal and other invited parties for their consideration.
Self Regulation: Learning activities provide substantive time and opportunity for learners to develop self-regulation skills. Students will know the learning intentions and associated success criteria in advance of the learning work and learners do have the opportunity to plan their own work. Completed drafts will receive peer and teacher feedback enabling learners to use feedback to improve their learning.
Real World Learning: Learners DO work with real-world issues, opportunities, challenges and problems for authentic audiences and real-life benefits. They actively inquire and pose questions to identify authentic needs, opportunities and define problems AND they DO generate possibilities, design and test out ideas and solutions when designing the perfect school campus.
Skilful Communication: Learners are required to produce coherent communication using a range of communication modes. They ARE required to design their communication for a particular audience and are required to produce substantive, multi-modal communication. That is: Their video or photo collage will be presented at assembly and to invited guests to view what they have been learning about. Their presentations will be video recorded and with theirs and their parent's permission published in an appropriate online place.

Task One:
Some website and video tutorial links are provided below. You should provide a mathematical understanding of the Golden Ratio (Phi) and how it relates to both Fibonacci's sequence and then music.
Where do we see the golden ratio in life and history?
Provide at least three examples of where we see the golden ratio. In providing the examples you will explain the concept of the golden ratio (Phi) which is also known as the golden ratio as seen with these examples. You are encouraged to use photos you have taken yourself with measurements using OneNote.
Video Links:
Golden Ratio According to Donald Duck
Mathematics in Nature:
https://www.bing.com/videos/search?q=Donald+Duck+Golden+Ratio&Form=VQFRVP#view=detail&mid=4EF5F5693AE99CF9DD9A4EF5F5693AE99CF9DD9A

Watch this video and answer and explain what the golden ratio is:
https://youtu.be/5uQiEIWGG9I
https://youtu.be/3u7SXLf9t1A
Now that you have an understanding of the Golden Ratio, explain what you have learned in your OneNote document.
What is the relationship of the Golden Ratio to Fibonacci? Watch the following video and write down a summary of what you have learned, including mathematical justification.
http://bit.ly/zOn1Ws
Using a tape measure and a partner, see if you can find the golden ratio with various parts of your body.
Record your mathematical observations and measurements using attached worksheet.
Record your measurements using the excel spreadsheet given here (http://1drv.ms/1f9BB6o) with your measurements on OneNote enabling you to submit your task online.
An example is given here:
https://pistrucciartworks.files.wordpress.com/2011/02/proportion.jpg



Task Two: Math and Music

Golden Ratio and Music: http://magicsongs.net/made-in-your-image/the-golden-ratio-in-music/
Golden Ratio and Music Video:
https://www.bing.com/videos/search?q=golden%20ratio%20basics&qs=n&form=QBVR&pq=golden%20ratio%20basic&sc=0-13&sp=-1&sk=#view=detail&mid=6282D3E800B76847E3A06282D3E800B76847E3A0
Watch the videos:
Fibonacci and Music: http://www.youtube.com/watch?v=acTrvMlpuxA&t=10m40s
Pi and Music: http://www.youtube.com/watch?v=acTrvMlpuxA&t=28m50s
Take notes on what you learn.
Go to the following websites and answer the question given below.
http://mathcentral.uregina.ca/beyond/articles/Music/music1.html
http://mathandmusic.tripod.com/rhythm.htm
Answer the question: How are music and mathematics related?

Task Three:
A study carried out by Bristol University (http://www.bris.ac.uk/news/2011/8116.html) where they were able to come up with a computerized algorithm to mathematically predict the next number 1 music hit with a 60% chance of success. Links to this study are placed on Canvas for your interest. You are not expected to understand this algorithm. Your teacher has provided you with a number of number one songs. Use your understanding of songs from the websites given below to find and explain any mathematical patterns found in this song.
http://mathcentral.uregina.ca/beyond/articles/Music/music1.html
http://mathandmusic.tripod.com/rhythm.htm
The Math of Music: https://www.youtube.com/watch?v=NKZrhPytdo4

https://www.youtube.com/watch?v=zAxT0mRGuoY
http://www.youtube.com/watch?v=acTrvMlpuxA&t=8m46s

Music Analysis: http://www.clivestockerweb.co.uk/pop_resources/Analysis.html
Describe the mathematical patterns that you find and justify why they are patterns.

Task Four:

Your job is to now use your understanding of all of the above to create your own mathematical song. You can either:

1. Create a song based on a famous existing mathematical pattern.

e.g. Such as the one provided above and again here:
http://www.youtube.com/watch?v=acTrvMlpuxA&t=8m46s

2. Create a song with a similar mathematical pattern to the one you discovered with the songs provided to you.

Task Five:

Provide mathematical justification for your song. i.e. Explain how your song that you have created reflects the pattern you have decided on. You will need to provide evidence that you understand the pattern and describe your pattern mathematically. You should be encouraged to use both musical notes and algebra in your answer.

Appendix S
Traffic jam enrichment investigation – Oliver's solution

Traffic Jam Enrichment Submission completed by Oliver

The Traffic Jam (Voluntary enrichment task completed by Oliver in term 1 2014).
Last Sunday an accident caused a traffic jam 12kms long on a two way motorway.
How many cars do you think are in the traffic jam?

My Thinking:
If a traffic jam is 12kms long, the amount of cars would be the length of the jam divided by the length of the cars. Let's substitute the length of the cars with c and the length of the jam with the value 12,000m. Our substituted formula is 12,000/c. However, we need to take into account the distance between each car. This means we need to make a new formula with the distance between each car as g. Our formula is now 12,000/c - [(12,000/c x g)/c]. This new formula allows us to see the amount of cars in the traffic jam without gaps (12,000/c), add the gap in (12,000/c x g) and see how many car lengths are lost (/c).

Estimations
The following estimation will need to be made in order for us to get the accurate amount of cars in the traffic jam.
How long each car is.
The gap between each car.
Let's substitute these values. We will change how long the cars are with 4.5m and the gap between the cars with 2m. The formula substituted is now 12,000/4.5 - [(12,000/4.5 x 2)/4.5]. Because the sum now has values, we can find out the answer.

Working Out
12,000/4.5 - [(12,000/4.5 x 2)/4.5]
= 2666 - [(2666 x 2)/4.5]
= 2666 - [5332/4.5]
= 2666 - 1184
= 1482 cars in the traffic jam

Clearing the Traffic Jam
The accident cleared and the cars drove away one every two seconds. It is very simple and easy to find the time it took for the last car to move by multiplying the amount of cars by two. The answer is: 1482 x 2
= 2964 or 49 minutes and 24 seconds

The Road to School
To prevent yourself from being late to school, there will be some measurements you need to take. These measurements are;
The time the lights are green before going red.
The time the lights are red before going green.
The distance of road between you and the school.
The time school starts.

All of these measurements will be used during the formula, so make sure you have not missed one. Don't worry if we use other measurements because these can be estimated. You will need to divide the length from your house to the school by the average length of a car, so if the car length is c and the distance between you and the school is d, the formula is currently d/c. This is how many cars are between you and the school.

Now we need the seconds is takes a car to move and how long the lights are green. If we substitute the time it takes a car to move with s and the lights time as l, we can make a different formula to discover how many cars move per traffic light cycle with l/s. Using the time the lights are red before going green, we can find the rate the cars move. So, if we substitute the time the lights are red with r, we get the ratio l/s : l + r. This ratio tells us that the time per traffic light cycle (l + r) and the amount of cars that pass each time (l/s).

We also need to add in the gap like last time and the gap will still be g. The time school starts is f.

Now that we know how many cars pass each traffic light cycle, we can figure out the equation with substitution. We will need to substitute values into f - ((((d/c - (d/c x g))/c) / (l/s)) x (l + r)). This gives us our complete equation because we have the amount of cars (d/c) divided by the amount of cars that get through each cycle (l/s) to give us the amount of cycles we need multiplied by the time it takes the traffic lights to change (l + r). As explained before, we have the gap taking away from the amount of cars.

Final Answer

For an example, we will make up a traffic jam situation. The car length will be 4.5m and the gap will be 2m, like last time. The distance is 3,000m and the lights go on for 10 seconds and then off for 30 seconds. School will start at 9:00:00am or at 32,400 past midnight and it takes 1.5 seconds for a car to move. The formula will only work if there is one set of lights, no roundabout and the lights are near the end of the journey. The formula is as follows;

f - (((d/c - ((d/c x g)/c)) / (l/s)) x (l + r))
= 32,400 - (((3,000/4.5 - ((3,000/4.5 x 2)/4.5)) / (10/1.5)) x (10 + 30))
= 32,400 - (((666 - ((666 x 2)/4.5))/6) x 40)
= 32,400 - (((666 - (1332 / 4.5))/6) x 40)
= 32,400 - (((666 - 296)/6) x 40)
= 32,400 - ((370 / 6) x 40)
= 32,400 - (62 x 40)
= 32,400 - 2480
= 29,920 or 8:18:40am

Changing the Formula

To add in more lights all you have to do is repeat the bracketed part of the formula with the different measurements. For example, two sets of lights would be;

f - ((((d/c - ((d/c x g)/c)) / (l/s)) x (l + r)) + (((d/c - ((d/c x g)/c)) / (l/s)) x (l + r)))

This time you get from completing the formula might not be completely accurate even if the cars queue up from your destination to your start. This is because this formula is if no cars turn out of lane or go any way other than the way you are going.

Appendix T
Sample of Enrichment Tasks

Enrichment task	Explanation	Thinking involved
Students created mathematics music rap.	Students had to create a musical rap that would help students understand how to find the area, surface area or volume.	Meta-cognitive thinking
Area scavenger hunt	Students would have to solve a variety of clues and in the process find the area, surface area and volume of various shapes outside the classroom.	Problem solving Using measurement in the real world.
Algebraic stained glass window	Students had to create a stained glass window with graphed algebraic equations. Once the lines and curves were drawn, students would also find line intercepts and equations.	Finding mathematical patterns. Justification of mathematical reasoning.
Golden ratio in music	Students created a simple song using mathematical patterns and in the process explore where the golden ratio can be seen in life.	Mathematical reasoning and analysing patterns.
Traffic jam task.	Students had to use mathematics to find a faster route to school.	Creating formula and justifying reasoning.
Binomial theorem	Find the relationship between pascals triangle and expanding algebraic brackets.	Finding patterns. Discovering new information.
Probability pick a box	Students had to analyse the probability of quiz show games and provide mathematical reasoning for stating whether they thought the games were fair, or otherwise.	Application Reasoning