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Enhanced MMSE Channel Estimation Using Timing Error Statistics for Wireless OFDM Systems

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Abstract—Estimation and tracking of the frequency-selective time-varying channel response is a challenging task for wireless communication systems incorporating coherent OFDM. In pilot-symbol-assisted (PSA) OFDM systems, the minimum mean-square-error (MMSE) estimator provides the optimum performance based on the channel statistics (channel correlation function and SNR). In OFDM systems, FFT-block timing error introduces a linear phase rotation to data modulated on individual subcarriers. An MMSE channel estimator designed only using the wireless channel statistics performs only sub-optimally when subcarrier phase rotations due to block timing errors are present. In this paper, we show that by using the block timing error statistics of the OFDM time-synchronizer the performance of the MMSE channel estimation can be significantly improved. Numerical results show that the bit-error-probability (BEP) performance degradation due to timing errors can be almost completely recovered by the proposed technique.

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) is a promising technique for high-bit-rate wireless communications [1], [2]. Multipath immunity, bandwidth efficiency and resistance to narrow-band interference and impulse noise are the key advantages of OFDM. It has been employed for digital audio/video broadcasting (DAB/DVB) and wireless LAN (IEEE 802.11a and Hiperlan/2) standards [1]. In a wireless channel, the multipath environment causes frequency-selective fading. On the other hand, the mobility of the receiver causes time-selective fading of the OFDM subcarriers. The former is characterized by the power delay-spread of the channel, while the latter is characterized by the Doppler frequency of the channel. Estimation of this time and frequency domain fading processes at the receiver is crucial for OFDM system performance. Although differential detection could be used to detect the data from received signal in the absence of channel state information, it results in about 3 dB loss in signal-to-noise ratio (SNR), compared to coherent detection [3].

A number of channel estimation techniques are reported in literature [3]–[9]. In pilot-symbol-assisted (PSA) techniques [3], [4] channel estimation is performed by inserting pilot symbols at selected positions in the OFDM time-frequency grid (see Fig. 1). First, channel estimation is performed at the pilot locations using the known pilot data. Channel estimation at the unknown data locations are obtained via an interpolation technique. Linear, spline, and Gaussian filters have been studied as interpolation techniques [5]–[7]. For a chosen pilot pattern in the time-frequency grid, the optimal channel estimator in terms of minimum mean-square-error (MMSE) is the 2-dimensional Wiener filter [9]. Wiener filtering or MMSE channel estimation performs an optimal 2-dimensional interpolation given the channel statistics (channel correlation function and operating SNR).

In OFDM systems, FFT-block timing errors due to the time-synchronizer inaccuracy introduce phase-rotation to data carried by the individual subcarriers. This phase-rotation is linear with n-th subcarrier experiencing a rotation of $\pm2\pi nd/N$, where $N$ and $d$ are the total number of subcarriers and the block timing error (in samples), respectively [2], [13]. The block timing error $d$ can be considered as a random variable, the discrete distribution $P(d)$ of which is dependent on the particular time synchronization algorithm used and the channel conditions (strength of multipaths and SNR). The value of $d$ for consecutive blocks

![Fig. 1. Example of pilot symbol placement in the OFDM time-frequency grid. Shaded locations carry the pilot data, while unshaded locations carry the data being transmitted.](image-url)
are generally independent. Therefore, the effective channel response as seen by the OFDM receiver is the actual channel response plus the phase errors (subcarrier rotation) due to the block timing errors.

In this paper, we show the followings: (i) The time-domain MMSE interpolation becomes impossible when block timing errors are present. This is because, in general, adjacent OFDM blocks have independent (though identically distributed) timing errors, which reduces the time-domain channel correlation significantly, specifically for middle subcarriers, (ii) The frequency-domain MMSE interpolation is still possible even when the block timing errors are present. This is true for sufficiently accurate time synchronizers, i.e., when the variance $E\{d^2\}$ of $d$ is small. However, the performance degradation of the MMSE interpolation becomes significantly high as the time synchronizer accuracy worsens, i.e., when the variance $E\{d^2\}$ is large, and (iii) In frequency-domain MMSE interpolation, the performance degradation due to block timing error can be almost completely recovered by incorporating the timing error statistics into the channel correlation function.

The solution to above (i) is to estimate the phase-rotation due to timing errors on block-by-block basis, and compensate for the phase error before the MMSE interpolation is performed. Methods of this kind for phase-rotation estimation are reported in [7], [8]. This is a deterministic approach to solve the problem of phase-rotation due to timing errors. However, the deterministic approach adds to the computational complexity of the OFDM receiver as the block timing error has to be estimated on block-by-block basis. Whereas, in this paper, we propose a statistical approach, which requires only the block timing error statistics, and causes no additional computational cost. The above (iii) is the major contribution of this paper, and we argue that the OFDM receiver performance can be significantly improved using the proposed technique, specially when the time synchronizer performs poorly.

This paper is organized as follows. Section II gives the details of the wireless channel model. Section III describes the details of the proposed enhanced MMSE channel estimation technique with numerical results. Section IV gives conclusions.

II. WIRELESS CHANNEL MODEL

A wireless channel model with time-varying finite impulse response (FIR) filter coefficients was incorporated. Each propagation path $i$ is characterized by a fixed delay $\tau_i$ and a time-varying amplitude $A_i(t)$. The $A_i(t)$ is a product of an amplitude $a_i$ and a Rayleigh fading process $g_i(t)$. The impulse response of the channel model can be given as

$$h(t,\tau) = \sum_{i=0}^{I-1} A_i(t) \delta(\tau - \tau_i)$$  \hspace{1cm} (1)

where $A_i(t) = a_i g_i(t)$ and $I$ is the total number of propagation paths. The fading function $g_i(t)$ is a $f_d$-limited complex Gaussian process that is independent for different paths. The term $f_d$ is the Doppler frequency, which is related to the relative speed $v$ between the transmitter and receiver and the carrier frequency $f_c$ by $f_d = v f_c/c$, where $c$ is the speed of light. The time-varying frequency response of the wireless channel can be calculated using (1) as

$$H(t, f) \triangleq \int_{-\infty}^{\infty} h(t, \tau)e^{-j2\pi f\tau} d\tau$$

$$= \sum_{i=0}^{I-1} a_i g_i(t)e^{-j2\pi f\tau_i}.$$

In the time-frequency grid of an OFDM system the channel frequency response at the $n$th tone of the $m$th OFDM block becomes

$$H[m, n] \triangleq H(m T_b, n \Delta f)$$

$$= \sum_{i=0}^{I-1} a_i g_i(m T_b)e^{-j2\pi n \Delta f \tau_i},$$  \hspace{1cm} (2)

where $T_b$ and $\Delta f$ are the block duration and the inter-subcarrier spacing of the OFDM system, respectively. The channel correlation function in the time-frequency space for different OFDM block and tone separations can be given as

$$R_{HH}[k, l] \triangleq E\{H[m + k, n + l]H^*[m, n]\}$$

$$= \sigma^2_H R_k[k] R_l[l],$$  \hspace{1cm} (3)

where $\sigma^2_H$ is the total average power of the channel response given by $\sigma^2_H = \sum_{i=0}^{I-1} \sigma_i^2$. The $\sigma_i^2$ is the average power of the $i$th propagation path. The notation $E\{\cdot\}$ denotes the expected value of a random variable. The $R_k[k]$ and $R_l[l]$ are the time-domain and frequency-domain correlation functions of the channel response, respectively. The $R_k[k]$ depends on the Doppler frequency $f_d$ of the channel and is given by [1]

$$R_k[k] = J_0(2\pi k F_d), \text{ } b \text{ } \text{w} \text{ } \text{h} \text{ } \text{a} \text{ } \text{where} \text{ } F_d = f_d T_b.$$  \hspace{1cm} (4)

The $F_d$ is the normalized Doppler frequency, normalized by the OFDM block rate $1/T_b$. The $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind. For an exponentially decaying multipath power delay profile, $R_f[l]$ is given by [1]

$$R_f[l] = \frac{1}{1 + 2\pi^2 \tau_{\text{rms}}}, \text{ } \text{w} \text{ } c \text{ } \tau_{\text{rms}} = \frac{\hat{\tau}_{\text{rms}}}{T_s}.$$  \hspace{1cm} (5)

The $\tau_{\text{rms}}$ is the RMS delay-spread $\hat{\tau}_{\text{rms}}$ relative to the OFDM sampling interval $T_s$. The total number of OFDM subcarriers (tones) is denoted by $N$.

III. ENHANCED MMSE CHANNEL ESTIMATION

A. Timing Error Statistics

The discrete baseband OFDM signal consists of a sequence of OFDM blocks. The $n$th OFDM block is generated by performing the IFFT operation (multiplexing) on $N$ complex valued constellation points $\{X[n, m]\}$ (which represents the data to be transmitted), resulting in $N$
time domain samples \( \{x[m,n]\} \) \( 0 \leq n \leq N - 1 \). In OFDM systems it is essential to add last \( N_{\text{CP}} \) samples of each block as a cyclic-prefix at the front. Cyclic-prefix serves the purposes of (i) avoids Inter-Block-Interference (IBI) by absorbing the delay-spread of the frequency-selective wireless channel, and (ii) provides a safety margin against block time synchronization errors. Assuming that block timing error \( d_m \) is within the ISI free region of the cyclic-prefix (see Fig. 2), the demultiplexed (using FFT) data \( \{Y[m,n]\} \) \( 0 \leq n \leq N - 1 \) for the \( m \)th OFDM block at the receiver can be given as

\[
Y[m,n] = X[m,n]H[m,n]e^{j(2\pi/N)md_m} + W[m,n]
\]  
(5)

where \( W[m,n] \) denotes additive white Gaussian noise (AWGN), and \( H[m,n] \) denotes the channel response for the \( n \)th subcarrier at the time of \( m \)th OFDM block. The above (5) can be written as

\[
Y[m,n] = X[m,n]\tilde{H}[m,n] + W[m,n]
\]  
(6)

where \( \tilde{H}[m,n] = H[m,n]e^{j(2\pi/N)md_m} \) is the effective channel response with the effect of block timing error lumped into it. The effective channel correlation function \( \tilde{R}_H[k,l] \) in the time-frequency space can be calculated as

\[
\tilde{R}_H[k,l] = E\{\tilde{H}[m+k,n+l]\tilde{H}^*[m,n]\}
= \sigma_H^2 R_0[k]R_f[l]E\{e^{j(2\pi/N)(k+l)d_m}\}.
\]

The effective frequency-domain channel correlation function \( R_f[l] \) can be given as

\[
R_f[l] = \tilde{R}_H[0,l] = R_0[l]E\{e^{j(2\pi/N)ld_m}\} = R_0[l] \beta_f[l]
\]  
(7)

where the total average power of the channel was assumed to be \( \sigma_H^2 = 1 \). The \( \beta_f[l] \) is the contribution of the timing error statistics to the effective frequency-domain channel correlation function given by

\[
\beta_f[l] = \sum_{d=d_{\text{min}}}^{d_{\text{max}}} P(d)e^{j(2\pi/N)ld}
\]  
(8)

where \( P(d) \) is the discrete distribution (probability density function) of the block timing error \( d \) given by

\[
P(d) = \sum_{\theta=d_{\text{min}}}^{d_{\text{max}}} P(\theta)\delta(d-\theta).
\]  
(9)

Similarly, the effective time-domain channel correlation function \( \tilde{R}_T[k] \) can be given as

\[
\tilde{R}_T[k] = \tilde{R}_H[k,0] = R_T[k]E\{e^{j(2\pi/N)(d_{m+k}-d_m)}\} = R_0[k]\alpha_T[n].
\]  
(10)

It should be noted that \( \alpha_T[n] \) is a only function of \( n \) (absolute subcarrier index), but not \( m \) or \( k \). This is because \( d_{m+k} \) and \( d_m \) should be treated as independent random variables with the identical distribution of \( P(d) \), i.e., i.i.d random variables. Therefore, \( \alpha_T[n] \) when \( k \neq 0 \), can be expressed as

\[
\alpha_T[n] = E\{e^{j(2\pi/N)nd_{m+k}}\} = \sum_{d} P(d)e^{-j(2\pi/N)nd}.
\]

In order to illustrate the influence of the timing error statistics on the effective channel correlation function, we selected two typical OFDM time-synchronizer performances as given in Fig. 3. Note that Fig. 3(b) depicts a relatively poor time-synchronizer performance (with a larger error spread or variance) compared to that of Fig. 3(a). The difference in performance depicts the time synchronization accuracy of two different block timing algorithms. The plots of \( \beta_f[l] \) for the two time-synchronizer performances, calculated according to (8), are given in Fig. 4. For an OFDM system with \( N = 512 \) subcarriers and RMS power-delay spread of \( \tau_{\text{rms}} = 5 \) samples, the effective frequency-domain channel correlation function \( R_f[l] \) (for the two time-synchronizer performances) is plotted in Fig. 5. For the time-domain case, we demonstrate that the effective channel correlation function can reduce to very-low values even for close by blocks in time. For an OFDM system with \( N = 512 \) subcarriers and normalized Doppler frequency of \( F_d = 0.01 \), the effective time-domain channel correlation function \( R_T[k] \) (for the two time-synchronizer performances) is plotted in Fig. 6.
B. Analysis of MMSE Channel Estimation

As described in Section I, MMSE channel estimation utilizes pilot symbols inserted at selected positions in the OFDM time-frequency grid. Channel estimations are first obtained for the pilot locations using the known pilot data. The same for the unknown data locations are obtained using MMSE interpolation (2D Wiener filtering) which utilizes frequency and time domain channel correlation functions. In Section III-A, we have shown that in the presence of block timing error, to assure optimum MMSE channel estimation, it is necessary to absorb the timing error statistics into the channel correlation function. The modified channel correlation functions were termed effective channel correlation functions ($\tilde{R}_{f}[l]$ and $\tilde{R}_{f}[l]$) in contrast to the standard channel correlation functions ($R_{f}[l]$ and $R_{f}[l]$).

In this section, we derive expressions for the MMSE channel estimation error (mean-square-error) for the following cases.

Case-I: With the standard channel correlation function used, when timing errors are not present,

Case-II: With the standard

(see Fig. 4) specifically for middle subcarriers (indices around $N/2$).

Fig. 3. Examples of typical OFDM block time synchronizer performances: (a) for synchronizer-(a) relatively good synchronizer performance with $d = 0$ for 70% of the time, and (b) for synchronizer-(b) relatively poor synchronizer performance with $d = 0$ for only 40% of the time.

Fig. 4. Plot of $\beta_f[l]$ for the two time-synchronizer performances ($N = 512$). Note that $\beta_f[l]$ has the property of even-symmetry across the total subcarrier range, and also for symmetric $P(d)$, $\beta_f[l]$ is real.

Fig. 5. Plot of $R_f[l] = R_f[l] \beta_f[l]$ for the two time-synchronizer performances ($N = 512$ and $\tau_{\text{max}} = 5$ samples). For reference, original $R_f[l]$ is also shown.

Fig. 6. Plot of $R_f[k] = R_f[k] \alpha_1[n]$ for the two time-synchronizer performances ($N = 512$ and $F_d = 0.01$). For reference, original $R_f[k]$ is also shown.
channel correlation function used, when timing errors are present, and **Case-III**: With the effective channel correlation function used, when timing errors are present. These three cases are summarized in Table I. We consider the frequency-domain MMSE interpolation with a comb-type pilot arrangement [7] as shown in Fig. 7. Assume an OFDM system with $N$ number of subcarriers with $N_p$ pilot subcarriers uniformly inserted in the frequency-domain. The pilot subcarrier spacing $L = N/N_p$ is selected to be an integer. The pilot subcarrier indices are given by $n = rL$, where $r = 0, 1, 2, \ldots, N_p - 1$. The estimated channel response at the pilot subcarrier locations for a given OFDM block (we drop the block index $m$ for notational simplicity) is given by

$$\hat{p} = \left[ \hat{H}[rL] \right]_{0 \leq r \leq N_p - 1}^T$$

where

$$\hat{H}[rL] = H[rL] + W[rL] X[rL]$$

where $X[rL]$ and $W[rL]$ are the known pilot data and AWGN noise, respectively. Estimated channel response at all subcarrier locations $\hat{h} = [\hat{H}[n]]_{0 \leq n \leq N - 1}^T$, using MMSE interpolation is given by [1]

$$\hat{h} = \hat{R}_{hp} R_{pp}^{-1} \hat{p}$$

where $\hat{R}_{hp}$ is the cross-covariance matrix between $\hat{h}$ and $\hat{p}$, with $h = [H[n]]_{0 \leq n \leq N - 1}^T$. The $R_{pp}$ is the auto-covariance matrix of $\hat{p}$. Also,

$$R_{pp} = E\{p p^H\} = R_{pp} + \frac{1}{SNR} I$$

where $R_{hp}$ is the auto-covariance matrix of $p = [H[rL]]_{0 \leq r \leq N_p - 1}^T$ and $I$ is the unit matrix. It is assumed that the pilot data represents the average signal constellation power, i.e., no boosted pilots. Also, $R_{hp} = R_{hp}$ as $H[n]$ and $W[rL]$ are statistically independent for all $n$ and $r$. Therefore, the optimum MMSE channel estimator can be given as

$$\hat{h} = G_{opt} \hat{p}$$

where $G_{opt} = R_{hp} \left( R_{pp} + \frac{1}{SNR} I \right)^{-1}$. (15)

Let $e = h - \hat{h}$ be the corresponding channel estimation error vector. The auto-covariance matrix of $e$ is given by [10], [11]

$$R_{ee} = R_{hh} - R_{hp} G^H - G R_{hp}^H + G R_{pp} G^H.$$  

The above (16) is true for any estimator (interpolator) matrix $G$. For the optimum MMSE estimator $G = G_{opt}$ given in (15), $R_{ee}$ becomes

$$R_{ee} = R_{hh} - G_{opt} R_{hp}^H.$$  

The mean-square-error (MSE) of the channel estimation can be given as

$$MSE = \frac{1}{N} \sum_{n=0}^{N-1} R_{ee}(n, n) = \frac{1}{N} \text{tr}(R_{ee})$$  

where $\text{tr}(\cdot)$ denotes the trace of a matrix. Let $R_{hh}, R_{hp}$, and $R_{pp}$ denote the covariance matrices calculated using the standard channel correlation function $R_{L}[l]$, and $\hat{R}_{hh}, \hat{R}_{hp},$ and $\hat{R}_{pp}$ denote the covariance matrices calculated using the effective channel correlation function $\hat{R}_{L}[l]$. Using above (16)–(18), the channel estimation error (MSE) for the three cases I, II, and III can be given as follows:

1) **Case-I**:

$$MSE_1 = \frac{1}{N} \text{tr}(R_{ee1})$$  

where

$$R_{ee1} = R_{hh} - R_{hp} \left( R_{pp} + \frac{1}{SNR} I \right)^{-1} R_{hp}^H.$$  

2) **Case-II**:

$$MSE_2 = \frac{1}{N} \text{tr}(R_{ee2})$$  

where

$$R_{ee2} = \hat{R}_{hh} - \hat{R}_{hp} G^H - GR_{hp}^H + GR_{pp} G^H$$  

$$\hat{R}_{pp} = \hat{R}_{pp} + \frac{1}{SNR} I$$  

and $G = R_{hp} \left( R_{pp} + \frac{1}{SNR} I \right)^{-1}$.  

3) **Case-III**:

$$MSE_3 = \frac{1}{N} \text{tr}(R_{ee3})$$  

where

$$R_{ee3} = \hat{R}_{hh} - \hat{R}_{hp} \left( \hat{R}_{pp} + \frac{1}{SNR} I \right)^{-1} \hat{R}_{hp}^H.$$
C. Analysis of Bit Error Probability

In this section, the benefit of the proposed enhanced MMSE channel estimation technique is evaluated in terms of bit error probability (BEP) in a Rayleigh fading channel. The channel estimation error analyzed in Section III-B is analytically translated into BEP by providing approximate close-form BEP expressions for QPSK and M-QAM modulation modes.

The post-equalized (zero-forcing) data symbol \( \hat{X}[n] \) for the \( n \)th subcarrier of the OFDM system becomes

\[
\hat{X}[n] = \frac{H[n]X[n] + W[n]}{H[n]} = X[n] + \frac{\delta H[n]}{H[n]}X[n] + \frac{W[n]}{H[n]} \tag{22}
\]

where the second term presents the self-interference due to the nonzero channel estimation error \( \delta H[n] = H[n] - \hat{H}[n] \). It is reasonable to assume the estimated channel response \( \hat{H}[n] \) and the channel estimation error \( \delta H[n] \) as independent complex Gaussian random variables [14]. Thus, the instantaneous effective SNR for the \( n \)th subcarrier is obtained as

\[
\gamma_{\text{eff}}[n] = \frac{|\hat{H}[n]|^2\sigma_x^2}{p_{H}[n]^2\sigma_x^2 + \sigma_w^2}
= \frac{\alpha_n \gamma}{\beta_n \gamma + 1} \tag{23}
\]

where \( \alpha_n = |\hat{H}[n]|^2 \) and \( \beta_n = |\delta H[n]|^2 \) and \( \gamma = \sigma_x^2/\sigma_w^2 \). The average BEP can be obtained for different modulation schemes (M-PSK and M-QAM) by integrating the Gaussian Q-function related expressions given in [14] with the instantaneous SNR in (23) over the distributions of \( \alpha_n \) and \( \beta_n \). Such BEP expressions are derived in [14] for QPSK and M-QAM and we utilize them in this BEP analysis. The average BEP for QPSK mode can be approximated as follows [14], [15]:

\[
P_b[n] \approx \frac{1}{2} \left( 1 - \frac{\pi \alpha_n}{2 \beta_n} \exp(-\xi_n) \text{erfc} \left( \sqrt{\xi_n} \right) \right) \tag{24}
\]

where \( \text{erfc}(\cdot) \) is the complementary error function, \( \alpha_n = \mathbb{E}\{\alpha_n\}, \beta_n = \mathbb{E}\{\beta_n\} \) and

\[\xi_n = \frac{\alpha_n \gamma + 2}{2 \beta_n \gamma}.\]

The average BEP of square M-QAM with Gray bit mapping is [14], [15]

\[
P_b[n] \approx \frac{2}{\log_2 M} \left( 1 - \frac{1}{\sqrt{M}} \right) \times \left( 1 - \frac{3\pi \alpha_n}{2(M - 1)\beta_n} \exp(-\xi_n) \text{erfc} \left( \sqrt{\xi_n} \right) \right) \tag{25}
\]

where

\[\xi_n = \frac{3\alpha_n \gamma + 2(M - 1)}{2(M - 1)\beta_n \gamma}.\]

Also, \( \alpha_n + \beta_n = \mathbb{E}\{|H[n]|^2\} \) and assuming the channel power to be unity, i.e., \( \mathbb{E}\{|H[n]|^2\} = 1 \), we have \( \alpha_n + \beta_n = 1 \). The expressions in (24) and (25) can be used to calculate the average PEB for the \( n \)th subcarrier with the variance of the channel estimation error substituted from (17) (Section III-B) as \( \beta_n = \mathbb{E}\{\delta H[n]|^2\} = \mathbf{R}_{\text{corr}}(n,n) \). The average BEP for the OFDM system (averaged across all the data subcarriers) can be given as

\[
P_{\text{avg}} = \frac{1}{N_d} \sum_{n \in D} P_b[n] \tag{26}
\]

where \( D \) represents the index set consisting of all data subcarriers and \( N_d = N - N_p \) is the total number of data subcarriers.

D. Numerical Results

An OFDM system with \( N = 512 \) number of subcarriers with frequency-domain pilot subcarrier spacing of \( L = 8 \) was assumed. The RMS power delay-spread of the multipath wireless channel was taken to be \( \tau_{\text{rms}} = 8 \) samples. The cyclic-prefix length was assumed to be longer than the channel delay-spread, leaving an ISI free region to absorb the block timing errors. An average channel SNR range of 0–50 dB, at steps of 5 dB was considered. The channel estimation performance was evaluated for the three cases I, II, and III, using (19)–(21), respectively. Fig. 8 shows the results for the time-synchronizer-(a). The same for the time-synchronizer-(b) is shown in Fig. 9. As can be seen, with the effective channel correlation function used (Case-III), the channel estimation performance can be re-established almost to the original performance (Case-I). It should be noted that the channel estimation performance degradation due to the time synchronization error is more for synchronizer-(b) (Fig. 9: Case-II) than that for synchronizer-(a) (Fig. 8: Case-II). This is an expected result as the synchronizer-(b) has a larger error variance (poor performance) compared to that of the synchronizer-(a).

However, the proposed technique (Case-III) brings the MMSE
The average bit error performance was evaluated for the three cases I, II, and III, using the BEP expressions (uncoded) provided in Section III-C. Fig. 10 shows average BEP for the time-synchronizer-(a) for QPSK and 16-QAM modulation modes. The same for the time-synchronizer-(b) is shown in Fig. 11. As can be seen, there is a significant BEP degradation due to time synchronization error in both cases (more severe for the poor synchronizer-(b)). However, for both synchronizers, the proposed technique (Case-III) re-establishes the BEP performance to the original performance (Case-I). The BEP error floor observed in all the cases is mainly due to the residual MMSE channel estimation error [10] and can be improved by increasing the number of pilot subcarriers or by incorporating forward-error-correction (FEC).

These results demonstrate the potential of the proposed statistical approach to significantly reduce the degradation of both the channel estimation and the bit error performances caused by poor time synchronization. This proves that the performance degradation of OFDM systems (bit-error-rate) due to time synchronization error [12], [13] can be effectively mitigated using the proposed technique.

IV. CONCLUSIONS

In this paper, we proposed a technique of improving the performance of MMSE channel estimation under poor time synchronization conditions for wireless OFDM systems. The key features of the technique are: (i) it incorporates the statistics of the OFDM block timing error (time synchronizer performance) into the MMSE channel interpolation matrix, and (ii) it causes no additional computational burden compared to deterministic block-by-block estimation of the time synchronization error. We introduced the effective channel correlation function that takes into account both the channel statistics and the time-synchronizer statistics. Performance evaluation of the proposed technique was carried out in terms of the channel estimation error and the bit error probability using two typical time-synchronizer error distributions. Numerical results showed that the performance degradation of the standard MMSE channel estimator due to block timing errors can be almost completely recovered using the proposed technique. Specifically, when the time-synchronizer performs poorly (which is normally the case when channel SNR is low or the multipath dispersion is large), the proposed statistical strategy of improving the performance of MMSE channel estimation proves valuable.

REFERENCES


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