Howlader, Md Mohasin, Yasmin, Shamsunnahar, Bhaskar, Ashish, & Haque, Shimul (Md. Mazharul)

(2023)

A before-after evaluation of protected right-turn signal phasings by applying Empirical Bayes and Full Bayes approaches with heterogenous count data models.

Accident Analysis and Prevention, *179*, Article number: 106882.

https://doi.org/10.1016/j.aap.2022.106882

A before-after evaluation of protected right-turn signal phasings by applying Empirical Bayes and Full Bayes approaches with heterogenous count data models

Md Mohasin Howlader
Doctoral Candidate, Queensland University of Technology,
School of Civil and Environmental Engineering,
Brisbane, Australia
E-mail: mdmohasin.howlader@hdr.qut.edu.au

Shamsunnahar Yasmin
Senior Research Fellow/Senior Lecturer, Queensland University of Technology,
Centre for Accident Research and Road Safety-Queensland (CARRS-Q),
Brisbane, Australia
E-mail: shams.yasmin@qut.edu.au

Ashish Bhaskar
Associate Professor, Queensland University of Technology,
School of Civil and Environmental Engineering,
Brisbane, Australia
E-mail: ashish.bhaskar@qut.edu.au

Md Mazharul Haque
(Corresponding author)
Professor, Queensland University of Technology,
School of Civil and Environmental Engineering,
Address: 2 George Street, GPO Box 2434, Brisbane QLD 4001, Australia
Phone: (07) 31387195
E-mail: m1.haque@qut.edu.au

ABSTRACT

Right-turn crashes (or left-turn crashes for the US or similar countries) represent over 40% of signalized intersection crashes in Queensland, Australia. Protected right-turn phasings are a widely used countermeasure for right-turn crashes, but the research findings on their effects across different crash types and intersection types are not consistent. Methodologically, the Empirical Bayes and Full Bayes techniques are generally applied for before-after evaluations, but the inclusion of heterogeneous models within these techniques has not been considered much. Addressing these research gaps, the objective of this study is to evaluate the effectiveness of protected right-turn signal phasings at signalized intersections employing heterogeneous count data models with the Empirical Bayes and Full Bayes techniques. In particular, the Empirical Bayes approach based on random parameters Poisson-Gamma models (simulation-based Empirical Bayes), and the Full Bayes approach based on random parameters Poisson-Lognormal intervention models (simulation-based Full Bayes) are applied. A total of 69 Cross intersections (with ten treated sites) and 47 T intersections (with six treated sites) from Southeast Queensland in Australia were included in the analysis to estimate the effects of protected right-turn signal phasings on various crash types. Results show that the change of signal phasing from a permissive right-turn phasing to the protected right-turn phasing at cross and T intersections reduces about 87% and 91% of right-turn crashes, respectively. In addition, the effect of protected right-turn phasings on rear-end crashes was not significant. The heterogenous count data models significantly address extra Poisson variation, leading to efficient safety estimates in both simulation-based Empirical Bayes and simulation-based Full Bayes approaches. This study demonstrates the importance of accounting for unobserved heterogeneity for the before-after evaluation of engineering countermeasures.

Keywords: Protected right-turn; Crash modification factor; Empirical Bayes; Full Bayes; Random parameters model

1. Introduction

Turn opposite direction crash (right turn crash in right-hand driving and left-turn crash in left-hand driving conditions) is one of the most serious and common crash types of signalized intersections. The right-turn crash risks at signalized intersections are associated with complex traffic maneuvers that involve opportunistic gap selection of the turning vehicles from the opposing traffic (Davis et al., 2007). Earlier studies suggest that the consequences of turn opposite-direction crashes[1] are likely to be serious because of the relatively higher travel speeds of the vehicles and their angle of impact (Wang et al., 2008). For example, in the South-East Queensland region of Australia (right-hand drive traffic), right-turn crashes at signalized intersections account for more than 40% of all intersection crashes between 2001 and 2015. More alarmingly, 35% of these crashes are fatal and major injury crashes.

The right-turn crash risks at signalized intersections can be minimized by providing protected right-turn signals, installing exclusive turning lanes, improving visibility, applying appropriate speed limits and control, and changing alignments or intersection geometry. Dedicated signal phasing (otherwise known as protected signal phasing) of turning movements can significantly improve safety among different countermeasures. Protected signal phasings remove the gap negotiation dilemma between the turning and the through traffic and thus improve safety. Signal phasing options for right-turn at signalized intersections can be permissive, protected, or partially protected (including lead protected-permissive and lag protected–permissive signal phasing) (Srinivasan et al., 2012; Islam et al., 2022). The permissive option provides no exclusive phasing for right-turning traffic but allows traffic to turn on a green signal phase after yielding to pedestrian and opposing traffic (if any). On the contrary, the protected signal design provides an exclusive window for right-turning traffic. It allows vehicles to make right turn only when a right-turn green arrow signal indication is displayed. Thus, a protected right-turn phase ensures safe passage for the turning movements by proving an exclusive right-of-way (Chen et al., 2015). The primary focus of this study is to evaluate the safety effectiveness of protected right turn phasing of signalized intersections. Specifically, we have applied Empirical Bayes and Full Bayes evaluation approaches for evaluating the effectiveness of protected right-turn signal phasing compared to permissive right-turn signal phasings at signalized intersections. Unlike existing studies, this study explicitly accounts for unobserved heterogeneity in before-after evaluations building on the estimates from random parameters

[1] In the rest of the paper, turn opposite crashes are referred to as right-turn crashes since this study employed data from Australia where driving is on the left side of the road.

safety performance functions. Further, the study demonstrates the importance of considering the effects of unobserved heterogeneity (if present) in before-after study design for computing the safety effectiveness of engineering countermeasures.

1.1. Safety effectiveness of protected right-turn signal phasing

In general, protected right-turn signal phasings are expected to have better safety effectiveness than permissible or partially-protected right-turn phasing due to the complete separation of right-turning traffic. Several studies have found such benefits of the protected right-turn phasing of signalized intersections specific to right-turn crash type (Davis et al., 2007; Harkey et al., 2008; Srinivasan et al., 2008). In contrast, a few studies reported no improvements in total injury crashes (Harwood et al., 2003; Davis et al., 2007; Srinivasan et al., 2008) and right-turn crases (Perfater, 1983) from such treatment. Hauer (2004) conducted a critical review of 14 studies from several countries and concluded that the safety effectiveness of converting protected phasing from either permissive or partially protected phasing is approximately 70% for right-turn crashes, while the safety effectiveness for other crash types is not statistically significant. With regards to rear-end crashes, several studies found positive safety effects of protected right turn phasing (Gan et al., 2005), while others found insignificant or negative safety effects of such signal phasing (Davis et al., 2007; Srinivasan et al., 2012; De Pauw et al., 2015).

Several studies suggested that permissive and partially-protected phasings are operationally more effective than protected right turn phasings (Lalani et al., 1986; Zhang et al., 2005). Chen et al. (2015) further explained that protected right-turn phasings should not be treated as a universally better solution than the permissive control, and the choice of signal phasing should be driven by potential trade-offs among safety, delay, and other factors (such as geometry, traffic flows, and operations). Apart from these issues, the safety effectiveness measure of protected right-turn phasings is also found to vary geographically for right-turn crashes representing low effects (17% ~ 25%) for studies in Canada (Lyon et al., 2005; Srinivasan et al., 2012), moderate effects (58% in Belgium, 50% in Sweden) for studies in European countries (De Pauw et al., 2015; Elvik et al., 2009), and high effects (70% ~100%) for studies in the USA (Harkey et al., 2008; Srinivasan et al., 2008).

1.2. Evaluation of engineering countermeasures

Empirical Bayes and Full Bayes methods are widely adopted evaluation techniques for before-after evaluation of engineering countermeasures. The Empirical Bayes evaluation approach

(Hauer, 1997) with negative binomial regression-based safety performance function remains the workhorse of before-after evaluation in existing literature. On the other hand, the Full Bayes evaluation approach is becoming popular over the last two decades. The Full Bayes approach has potential advantages of model specification flexibility, smaller data sample, and detailed inferencing capacity relative to the Empirical Bayes approach (Aul et al., 2006; Pawlovich et al., 2006). However, the Full Bayesian approach is computationally burdensome relative to Empirical Bayes approach.

In the Empirical Bayes approach, the crash-causality relationship is first developed through a safety performance function using reference site data. The estimates from the safety performance function are then used for predictions on the treated sites to estimate the treatment effects (Hauer, 1997). With regards to the Full Bayes approach, two different study designs are considered. In the first study design within the Full Bayes approach, the safety performance function is developed by pooling data from reference sites (both before and after period data of treatment implementation) and before period treated sites (proposed by Aul et al. (2006)). However, Lan et al. (2009) and Persaud et al. (2010) found that Empirical Bayes and such study design within the Full Bayes approach led to comparable results. On the other hand, in a different study setting for the application of the Full Bayes evaluation approach, the safety performance function is developed by pooling data for both reference and treated sites, including before and after period data of treatment implementation (proposed by Pawlovich et al. (2006)). Park et al. (2010) showed that the safety effectiveness estimates from this study design of the Full Bayes approach could significantly differ from the results obtained from the Empirical Bayes approach, specifically in the absence of a reasonably large reference group data.

It is beyond the scope of this study to present a comprehensive review of all safety studies employing Empirical Bayes and Full Bayes evaluation approaches. Please see Persaud et al. (2010) and Park et al. (2010) for detailed reviews of these approaches. Several studies have compared the performance of safety effectiveness evaluation from Empirical Bayes, and Full Bayes approaches. A summary of these studies is presented in Table 1. Table 1 shows the study design, treatment, data, safety performance functions, and a comparison between the Empirical and Full Bayes approaches. Several studies found that the safety effectiveness estimates from the Full Bayesian approach are to be comparable to those of Empirical Bayes approach (Lan et al., 2009; Persaud et al., 2010; Park et al., 2016).The mean safety estimates are mostly found to be similar, whereas the standard errors of the estimates are reported to vary, supporting the favorable outcome of the Full Bayes approach. Most of these studies argued that the sample size played a significant role towards better estimates from the Full Bayes relative to the Empirical Bayes approach. On the other hand, Islam et al. (2015) found the estimate from Empirical Bayes to be better than that of the Full Bayes approach. A number of other studies also compared the performance of the Empirical and Full Bayes approaches (Ahmed et al., 2015; Sacchi et al., 2015; D'Agostino et al., 2019). Some studies found that uncertainty estimates for the Full Bayes are larger than that of the Empirical Bayes. Park et al. (2010) argued that such larger uncertainty estimates may have resulted from the cumulative effect of incorporating parameter uncertainty into the final safety effectiveness estimates.

The prediction performance of the safety performance functions[2] also plays a significant role in safety effectiveness evaluation. For example, Park et al. (2010) found that safety effectiveness estimates of countermeasures from higher-order (multivariate model accounting for correlation among crash types) safety performance function in a Full Bayes setting results in more precise results than those from traditional count data model-based Empirical Bayes and Full Bayes approaches. Studies also found that unobserved heterogeneity consideration in safety performance functions can significantly improve the precision of parameter estimates and model predictability. In developing crash prediction models, several studies employing the frequentist approach (Anastasopoulos et al., 2009; Russo et al., 2014; El-Basyouny et al., 2014) adopted random parameters count data models and demonstrated that the random parameters model outperforms the traditional fixed parameters model. The Bayesian inference-based approach has also been employed to estimate random parameter models for developing crash prediction models (Li et al., 2008; El-Basyouny et al., 2011; Barua et al., 2014, 2015, 2016)). However, the application of this advanced safety performance functions in evaluating treatment effectiveness has received little attention despite potential benefits.

[2] Table 1 describes the safety performance functions adopted for both Empirical and Full Bayes safety evaluation methods. Poisson-gamma models are used for Empirical Bayes while variants of both poison-gamma and poison-lognormal models (linear, non-linear, multivariate, intervention) have been used for Full Bayes approaches. However, poisson-lognormal models have mostly been employed in the Full Bayes approach.

Table 1: Summary of studies comparing the performance between Empirical Bayes and Full Bayes approaches.

Study	Study design	Treatment	Data	Crash types	Safety performance functions	Outcomes
Lan et al. (2009)	Empirical Bayes Full Bayes: Hybrid Bayesian framework	Conversion from stop to signalized control	Treated sites: Dataset 1: 47 Dataset 2: 105 Dataset 3: 229 Reference/comparison sites: Dataset 1: 42 Dataset 2: 111 Dataset 3: 263	1. Total injury crash 2. Left-turn crash 3. Right-angle crash 4. Rear-end crash	Empirical Bayes: Poisson-Gamma Full Bayes: Both Poisson-Gamma and Poisson-Lognormal models with site-specific random effects	i. Safety estimates by Full Bayes are consistent with the Empirical Bayes approach. ii. Standard errors of estimates from the full Bayes method are smaller (15%~60%) across different crash types) than that of the empirical Bayes approach.
Park et al. (2010)	Empirical Bayes Full Bayes: Change-point modeling framework	Reduced posted speed limit	Treated sites: 33 Reference/comparison sites: two cases: one with 44sites and the other with 126 sites	1. Total injury crash 2. Speed violation crash 3. Fatal and major injury crash	Empirical Bayes: Poisson-Gamma Full Bayes: Multivariate Poisson-Lognormal intervention model	i. Comparable results are found for total injury crashes and fatal and injury crashes but highly different for speed violation crashes. ii. High differences in results are found for Empirical Bayes approach with a lesser number of reference sites. iii. In general, the uncertainty estimates for Full Bayes estimates are larger (33%~ 39% across different crash types) than Empirical Bayes estimates.
Persaud et al. (2010)	Empirical Bayes Full Bayes: Hybrid Bayesian framework	Conversion of the four-lane road to a three-lane cross-section with two-way left-turn lanes	Treated sites: 15 Reference/comparison sites: 1. 296 sites as used for Empirical Bayes and Full Bayes 2. 15 matched comparison sites for Full Bayes	Total injury crash	Empirical Bayes: Poisson-Gamma	i. The estimated safety effects and standard errors from the two approaches are comparable across crash types. ii. Recommends comparative study for treatments with smaller effects and larger standard errors to investigate the statistical significance of the effects.

Study	Study design	Treatment	Data	Crash types	Safety performance functions	Outcomes
Ahmed et al. (2015)	Empirical Bayes Full Bayes: Change-point modeling framework	Widening urban and rural two-lane to four-lane divided roads	Treated sites: Urban two-lane to urban four-lane: 41 Rural two-lane to urban four-lane: 43 Reference/comparison sites: Urban two-lane to urban four-lane: 381 Rural two-lane to urban four-lane: 370	1. Total injury crash 2. Fatal and major injury crash 3. Property damage only crash	Empirical Bayes: Poisson-Gamma Full Bayes: Poisson-Lognormal model	i. The estimated safety effects and standard errors are comparable between the two approaches. ii. Findings suggest intensive data requirements for the Empirical Bayes approach.
Islam et al. (2015)	Empirical Bayes Full Bayes: Hybrid Bayesian framework	Reduced posted speed limit	Treated sites: 27 Reference/comparison sites: 287	1. Fatal and major Injury crash 2. Property damage only crash	Empirical Bayes: Poisson-Gamma Full Bayes: Univriate and multivariate Poisson-Lognormal model	i. Empirical Bayes and Full Bayes approaches led to opposite conclusions when the safety effects were relatively small with high standard deviations. ii. Total injury and Property Damage Only crash reductions are statistically insignificant in the Empirical Bayes approach, while they are significant in both the univariate and multivariate Full Bayes approaches. iii. Full Bayes approach provides more precise estimates of safety effects.
Sacchi et al. (2015)	Empirical Bayes Full Bayes: Change-point modeling framework	No treatment	Treated sites: Randomly chosen Reference/comparison sites: 221	Total injury crash	Empirical Bayes: Poisson-Gamma Full Bayes:	i. Full Bayes approach provides higher consistency than naïve, comparison group, and Empirical Bayes approach among different sites.

Study	Study design	Treatment	Data	Crash types	Safety performance functions	Outcomes
					Non-linear Poisson-Lognormal intervention models	
D'Agostino et al. (2019)	Empirical Bayes Full Bayes: Change-point modeling framework	Two-lane road sections retrofitted with alternate additional overtaking lanes.	Treated sites: 16 Reference/comparison sites: 104	1. Total injury crash 2. Target crashes: multiple vehicle collisions (head-on, rear-end, and sideswipe crash).	Empirical Bayes: Poisson-Gamma Full Bayes: Poisson-Lognormal linear and non-linear models	i. Empirical Bayes approach produces lower crash modification factors and comparatively higher value (9.5%~44% across different crash types) of standard errors than the Full Bayes approach.
Jin et al. (2021)	Empirical Bayes Full Bayes: Hybrid Bayesian framework	Adaptive Signal Control Systems (ASCS)	Treated sites: Six ASCS corridors with 65 intersections Reference/comparison sites: 11 ASCS corridors with 680 observations across different signalized intersections. Intersection count is not available.	1. Total injury crash 2. Fatal and major injury crash 3. Angle crash 4. Rear-end crash	Empirical Bayes: Poisson-Gamma Full Bayes: Poisson-Lognormal spatial and non-spatial models Full Bayes; Non-hierarchical and hierarchical Poisson-Lognormal intervention models	i. Full Bayes approach with safety performance function that accounts for spatial effect and year factor shows the best performance in reducing potential bias and variance of prediction and improving the accuracy of safety effect estimation.
Park et al. (2016)	Empirical Bayes Full Bayes: Hybrid Bayesian framework	Roadside barriers	Treated sites: 147 Reference/comparison sites: 328	1. Total injury crash 2. Run-off road crash	Empirical Bayes: Poisson-Gamma Full Bayes: Poisson-Lognormal model	i. Results from both the approaches are comparable. ii. Empirical Bayes method shows more reliable estimates when (a) a sufficient sample size is obtained and (b) enough crash frequencies for both treated and reference sites are available.

1.3. Research gap

While the applications of random parameters models became popular among analysts for developing safety performance functions (crash prediction models), the application of such generalized variants of the regression models in safety effectiveness evaluation is far and few between. Most recently, Tahir et al. (2022) proposed a simulation-based framework to accommodate the effects of unobserved heterogeneity in Empirical Bayes safety effectiveness evaluation. The study concluded that the proposed simulation-based Empirical Bayes approach using panel random parameters negative binomial safety performance function resulted in more precise estimates of crash modification factors for the wide centreline treatments along two-lane two-way rural highways than those from the standard Empirical Bayes approach. To account for the unobserved heterogeneity within comparison-treatment pairs, El-Basyouny et al. (2011) applied random parameters-based Poisson-Lognormal intervention models in the Full Bayes evaluation approach. The study found that the random parameters model improves the safety estimates of a group of engineering countermeasures at signalized intersections, including improvements in signal visibility, left turn phase improvement, and left-turn lane installation. As presented in Table 1, a number of studies developed the safety effectiveness measures by employing Empirical Bayes and Full Bayes approaches with the application of traditional count regression models. It is well established that heterogenous count data model provides superior statistical fits than the traditional count model (Mannering et al., 2016; Chang et al., 2021; Bhowmik et al., 2022). However, the effects of unobserved heterogeneity in computing safety effectiveness of engineering countermeasures to mitigate right-turn crash risks have not been studied.

While many before-after evaluation studies have been conducted to evaluate the safety effectiveness of road infrastructure improvements like median treatments, reduction of the posted speed limit, overtaking lanes and roadside barriers (see Table 1), very little is known about the effects of a traffic signal phasing as a sole engineering treatment (everything else being the same). Recently the safe system approach within Australian National Road Safety Strategy has listed protected right-turn phasing as a safe system solution to improve safety of signalized intersection. However, the effectiveness of protected right-turn signals in the Australian road traffic conditions has not been rigorously examined to date.

1.4. Objective and scope

The objective of this study is to evaluate the effectiveness of protected right-turn signal phasings by applying heterogeneous count models within the Empirical Bayes and Full Bayes approaches[3]. Specifically, the safety performance function for Empirical Bayes is developed by employing random parameters Poisson-Gamma model. For the Full Bayes approach, the safety performance function is developed by employing random parameters Poisson-lognormal model.

The rest of the paper is organized in the following manner. Section 2 describes the scope of the data, samples and variables formation for the study, Section 3 explains the econometric methodologies including model specifications, estimation approaches applied to develop the heterogenous safety performance functions for the proposed Empirical Bayes and Full Bayes approaches, Section 4 presents the empirical estimates and performances of the safety performance functions, and comparison of the crash modification factors developed by the proposed approaches, Section 5 discusses the significances and justifications of the developed crash modification factors for protected right-turn signal phase implementation, and finally, Section 6 concludes with recommendations for the future direction of the study.

2. Data

2.1. Study area

The major focus of this study is to evaluate the effectiveness of protected right-turn signal phasings for Cross and T-signalized intersections located in Queensland, Australia. Specifically, ten Cross and six T signalized intersections are considered which were treated with the protected right-turn signal phasings from permissive right-turn phasings at least in one of the approaches between 2005 through 2012. For this research, a careful selection of reference sites is performed. Reference sites are selected with similar geometric and traffic characteristics to treated sites and, most importantly, with permissive right-turn signal phasings reflecting the before-condition of the treated sites. Following these criteria, a total of 59 Cross signalized intersections and 41 T signalized intersections are identified. The locations of these treated and reference intersection sites are shown in Figure 1.

---

[3] It is beyond the scope of this study to provide a detailed comparison of the performance in safety effectiveness between these Empirical and Full Bayes approaches. Our major focus is to accommodate for unobserved heterogeneity in the development of safety performance functions and accommodate the effects of such unobserved heterogeneity in computing safety effectiveness measures from both Empirical and Full Bayes approaches. However, we have provided with a brief comparison of these results across two evaluation approaches to generate insights for future research directions.

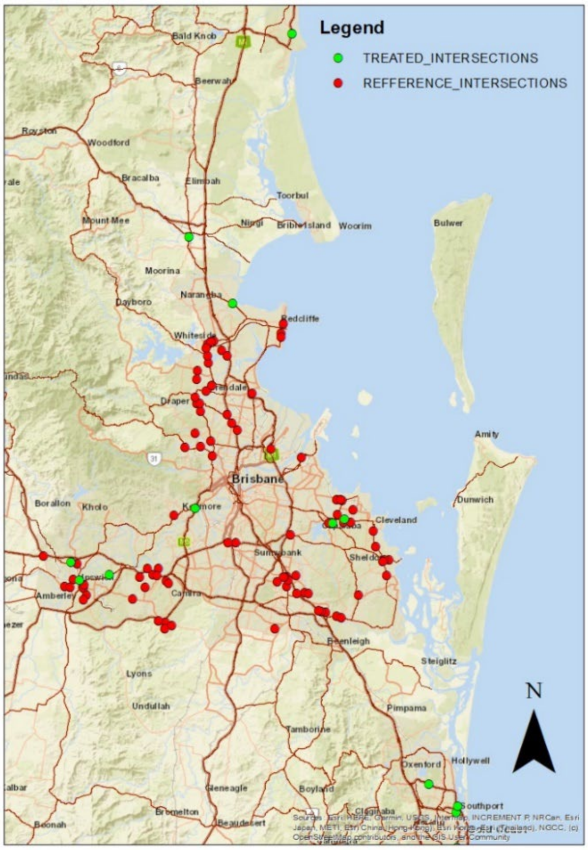

Figure 1: Treated and reference signalized intersection locations.

2.2. Sample formation

The protected right-turn signal phasing treatment was implemented between 2005-2012. However, the implementation period varied across signalized intersections. In addressing the regression-to-mean (uncertainty) effect in evaluation, this study includes at least three years of before and three years of after period data for the treated sites. Thus, the treatments implemented have different spans in the before (after) periods for both Cross and T intersections. The data for the intervention periods of the treated sites are not considered in computing the safety effectiveness of those locations.

Crash data for this study is sourced from the official crash database of Queensland for the years 2002 through 2015. The official crash database of Queensland only contains casualty crashes but does not include ‘no injury’ crashes. The injury severity outcomes are recorded as a four-point scale variable, including minor, moderate (medically treated), major (hospitalization injury), and fatal injury. Further, roadway geometry and traffic characteristics data are compiled from the ‘Australian Road Assessment Program’ and ‘A Road Management Information System’ databases for the years 2002 through 2015.

Crash prediction models for intersections are generally estimated at approach level (aggregate crash counts on individual approaches), roadway level (aggregate crash count along major and minor roads) and/or intersection level (aggregate crash counts within the influence area of intersection). Please see Wang et al. (2007) for a detailed description of these approaches. Analysis at the approach and/or roadway level may lead to excess zeros and site correlation. Moreover, assigning traffic crashes at an approach or roadway level may generate an erroneous dataset due to the uncertainty involved in crash location recording and assignment. Therefore, this study considers an intersection-level analysis for treatment evaluation. Finally, crash records, roadway geometry, and traffic characteristics are aggregated at the intersection level. Table 2 presents descriptive statistics of the crash counts and other exogenous variables in the reference sites, and Table 3 presents descriptive statistics of the crash counts and other exogenous variables in the before and after periods of the treated sites.

Table 2: Descriptive statistics for the reference sites.

Variables	Description of Variables	Cross-Intersection			T-Intersection		
CRASH COUNTS		**Mean**	**Std. Dev.**	**Total**	**Mean**	**Std.**	**Total**
Total injury crash	Total injury crashes per intersection per year	1.07	1.50	883.00	0.72	1.14	415.00
Fatal and major injury crash	Fatal and major injury crashes per intersection per year	0.39	0.76	319.00	0.22	0.53	129.00
Right-turn crash	Right-turn crashes per intersection per year	0.48	0.86	400.00	0.40	0.85	228.00
Rear-end crash	Rear-end crashes per intersection per year	0.21	0.54	175.00	0.15	0.45	86.00
CONTINUOUS VARIABLES		**Mean**	**Std. Dev.**	**Freq.**	**Mean**	**Std.**	**Freq.**
Modulus of AADT*	Natural Logarithm of module of Annual Average Daily Traffic (AADT) [Ln(Sqrt(AADT for major approach^2 + AADT on minor approach^2))]	9.10	0.68	-	9.11	0.46	-
Product AADT	Natural logarithm of product of major and minor AADT [(Ln(AADT on major approach ×AADT on minor approach)]	17.11	1.56	-	16.91	1.24	-
Number of lanes							
Major Road	Average number of lanes of major road	3.05	0.76	-	2.59	0.49	-
Minor Road	Average number of lanes of minor road	2.09	0.69	-	2.22	0.68	-
CATEGORICAL VARIABLES		**Frequency (Percentage)**			**Frequency (Percentage)**		
Median on major road	Presence of median on at least one of the approaches of major road						
Yes		742 (89.83)			546 (95.13)		
No		84 (10.17)			28 (4.87)		
Median on minor road	Presence of median on at least one of the approaches of minor road						
Yes		602 (72.88)			504 (87.8)		
No		224 (27.12)			70 (12.19)		
Exclusive right-turn lane on major road	Presence of exclusive right-turn lane on at least one of the approaches of major road						
Yes		798 (96.61)			574 (100.00)		
No		28 (3.39)			0 (0.00)		

Table 2: Descriptive statistics for the reference sites (Continued).

Variables	Description of Variables	Cross-Intersection	T-Intersection
		Frequency (Percentage)	Frequency (Percentage)
Exclusive right-turn lane on minor road	Presence of exclusive right-turn lane on at least one of the approaches of minor road		
Yes		546 (66.1)	462 (80.49)
No		280 (33.89)	112 (19.51)
Exclusive left-turn lane on major road	Presence of exclusive left-turn lane on at least one of the approaches of major road		
Yes		490 (59.32)	322 (56.1)
No		336 (40.68)	252 (43.90)
Exclusive left-turn lane on minor road	Presence of exclusive left-turn lane on at least one of the approaches of minor road		
Yes		616 (74.58)	490 (85.37)
No		210 (25.42)	84 9 (14.63)
Skewness level			
0^0 to 9^0	Indicator of maximum skewness between approaches from 0^0 to 9^0		
Yes		350 (42.37)	392 (68.29)
No		476 (57.63)	182 (31.71)
10^0 to 19^0	Indicator of maximum skewness between approaches from 10^0 to 19^0		
Yes		140 (16.95)	98 (17.07)
No		686 (83.05)	476 (82.93)
20^0 to 29^0	Indicator of maximum skewness between approaches from 20^0 to 29^0		
Yes		168 (20.34)	70 (12.2)
No		658 (79.66)	504 (87.81)
Above 29^0	Indicator of maximum skewness between approaches above 29^0		
Yes		42 (5.08)	14 (2.44)
No			
		784 (94.92)	560 (97.56)
Posted speed limit of major road			
> 60 km/hr	Indicator for major approaches with posted speed limit		
Yes		322 (38.98)	196 (34.15)
No		504 (61.02)	378 (65.85)

Table 2: Descriptive statistics for the reference sites (Continued).

Variables	**Description of Variables**	**Cross-Intersection**	**T-Intersection**
		Frequency (Percentage)	**Frequency (Percentage)**
> 70 km/hr	Indicator for major approaches with posted speed limit above 70 km/hr		
Yes		42 (5.08)	84 (14.63)
No		784 (94.92)	490 (85.36)
Posted speed limit on minor road			
> 50 km/hr	Indicator for minor approaches with posted speed limit above 50 km/hr		
Yes		350 (42.37)	266 (46.34)
No		476 (57.63)	308 (53.66)
> 60 km/hr	Indicator for minor approaches with posted speed limit above 60 km/hr		
Yes		42 (5.08)	14 (2.44)
No		784 (94.92)	560 (97.56)
Rural intersections	Intersection in rural area		
Yes		42 (7.32)	42 (7.32)
No		784 (94.92)	532 (92.68)

Abbreviations and definitions:
AADT = Average annual daily traffic; Std. Dev. = Standard deviation;
*Module of annual average daily traffic is basically the hypotenuse of a right triangle that has both major and minor road traffic counts as arms resulting in a weighted value. The property of the module of annual average daily traffic is that its magnitude is mainly determined by the larger traffic count but still reflective of the smaller one.

Table 3: Descriptive statistics for the treated sites.

Variables	Description of Variables	Cross intersections						T intersections					
		Before Period			After Period			Before Period			After Period		
CRASH COUNTS		**Mean**	**Std.**	**Total**	**Mean**	**Std.**	**Total**	**Mean**	**Std.**	**Total**	**Mean**	**Std.**	**Total**
Total injury crash	Total injury crashes per intersection per year	2.41	1.72	140.00	1.02	0.94	57.00	2.17	1.34	91.00	0.63	0.85	19.00
Fatal and major injury crash	Fatal and major injury crashes per intersection per year	0.90	0.89	52.00	0.23	0.47	13.00	0.55	0.77	23.00	0.27	0.58	8.00
Right-turn crash	Right-turn crashes per intersection per year	1.19	1.22	69.00	0.25	0.58	14.00	0.93	1.11	39.00	0.07	0.25	2.00
Rear-end crash	Rear-end crashes per intersection per year	0.67	0.89	39.00	0.68	0.66	38.00	0.90	0.82	38.00	0.43	0.72	13.00
CONTINUOUS VARIABLES		**Mean**	**Std.**	**Freq.**	**Mean**	**Std.**	**Freq.**	**Mean**	**Std.**	**Freq.**	**Mean**	**Std.**	**Freq.**
Modulus of AADT*	Natural Logarithm of module of AADT [Ln(Sqrt(AADT on major approach^2 + AADT on minor approach^2))]	9.63	0.27	-	9.72	0.23	-	9.78	0.19	- -	9.79	0.19	- -
Product of AADT	Natural logarithm of product of major and minor AADT [(Ln(AADT on major approach ×AADT on minor approach)]	18.41	0.60	-	18.62	0.46	-	18.35	0.53	- -	18.33	0.59	- -
Lane number for Major Road	Average number of lanes over the approaches of major road	3.30	0.77	-	3.38	0.73	-	2.90	0.37	- -	2.93	0.31	- -
Lane number for Minor Road	Average number of lanes over the approaches of minor road	2.87	0.79	-	2.63	0.70	-	2.19	0.40	- -	2.13	0.35	- -
CATEGORICAL VARIABLES		**Frequency (Percentage)**			**Frequency (Percentage)**			**Frequency (Percentage)**			**Frequency (Percentage)**		
Median on major road	Presence of median on at least one of the approaches of major road												
Yes		45 (77.59)			45 (80.36)			42 (100.00)			30 (100.00)		
No		13 (22.41)			11 (19.64)			0 (0.00)			0 (0.00)		
Median of minor road	Presence of median on at least one of the approaches of minor road												
Yes		45 (77.59)			45 (80.36)			30 (71.43)			18 (60.00)		
No		13 (22.41)			11 (19.64)			12 (28.57)			12 (40.00)		
Exclusive right-turn lane on major road	Presence of exclusive right-turn lane on at least one approach of major road												
Yes		58 (100.00)			56 (100.00)			42 (100.00)			30 (100.00)		
No		0 (0.00)			0 (0.00)			0 (0.00)			0 (0.00)		

Table 3: Descriptive statistics for the treated sites (Continued).

Variables	Description of Variables	Cross intersections		T intersections	
		Before Period	After Period	Before Period	After Period
		Frequency (Percentage)	Frequency (Percentage)	Frequency (Percentage)	Frequency (Percentage)
Exclusive right-turn lane on minor road	Presence of exclusive right-turn lane on at least one of the approaches of minor road				
Yes		45 (77.59)	45 (80.36)	42 (100.00)	30 (100.00)
No		13 (22.41)	11 (19.64)	0 (0.00)	0 (0.00)
Exclusive left-turn lane on major road	Presence of exclusive left-turn lane on at least one of the approaches of major road				
Yes		39 (67.24)	39 (69.64)	30 (71.43)	18 (60.00)
No		19 (32.76)	17 (30.36)	12 (28.57)	12 (40.00)
Exclusive left-turn lane on minor road	Presence of exclusive left-turn lane on at least one of the approaches of minor road				
Yes		52 (89.66)	50 (89.29)	42 (100.00)	30 (100.00)
No		6 (10.34)	6 (10.71)	0 (0.00)	0 (0.00)
Skewness Level					
0^0 to 9^0	Indicator of maximum skewness between approaches from 0^0 to 9^0				
Yes		39 (67.24)	41 (73.21)	4 (9.52)	8 (26.67)
No		19 (32.76)	15 (26.79)	38 (90.48)	22 (73.33)
10^0 to 19^0	Indicator of maximum skewness between approaches from 10^0 to 19^0				
Yes		19 (32.76)	15 (27.00)	14 (33.00)	10 (33.33)
No		39 (67.24)	41 (73.00)	28 (67.00)	20 (66.67)
20^0 to 29^0	Indicator of maximum skewness between approaches from 20^0 to 29^0				
Yes		0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)
No		58 (100.00)	56 (100.00)	42 (100.00)	30 (100.00)
Above 29^0	Indicator of maximum skewness between approaches above 29^0				
Yes		0 (0.00)	0 (0.00)	24 (57.14)	12 (40.00)
No		58 (100.00)	56 (100.00)	18 (42.86)	18 (60.00)

Table 3: Descriptive statistics for the treated sites (Continued).

Variables	Description of Variables	Cross intersections		T intersections	
		Before Period	After Period	Before Period	After Period
		Frequency (Percentage)	Frequency (Percentage)	Frequency (Percentage)	Frequency (Percentage)
Posted speed limit on major road					
> 60 km/hr	Indicator for major approaches with posted speed limit above 60 km/hr				
Yes		26 (44.83)	30 (53.57)	22 (52.38)	14 (46.67)
No		32 (55.17)	26 (46.43)	20 (47.62)	16 (53.33)
> 70 km/hr	Indicator for major approaches with posted speed limit above 70 km/hr				
Yes		11 (18.97)	11 (19.64)	0 (0.00)	0 (0.00)
No		47 (81.03)	45 (80.36)	42 (100.00)	30 (100.00)
Posted speed limit on minor road					
> 50 km/hr	Indicator for major approaches with posted speed limit above 50 km/hr				
Yes		28 (48.28)	29 (51.79)	16 (38.09)	8 (26.67)
No		30 (51.72)	27 (48.21)	26 (61.91)	22 (73.33)
> 60 km/hr	Indicator for major approaches with posted speed limit above 60 km/hr				
Yes		0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)
No		58 (100.00)	56 (100.00)	42 (100.00)	30 (100.00)
Rural intersections	Intersection in rural area				
Yes		0 (0.00)	0 (0.00)	0 (0.00)	0 (0.00)
No		58 (100.00)	56 (100.00)	42 (0.00)	30 (100.00)

Abbreviations and definitions:
AADT = Average annual daily traffic; Std. Dev. = Standard deviation; DCA : Definitions for coding accidents
*Module of annual average daily traffic is basically the hypotenuse of a right triangle that has both major and minor road traffic counts as arms resulting in a weighted value. The property of the module of annual average daily traffic is that its magnitude is mainly determined by the larger traffic count but still reflective of the smaller one.

3. Methodology

In this study, the safety effectiveness of protected right-turn signal phasing has been examined by employing two different evaluation approaches (1) Empirical Bayes evaluation based on the panel random parameters Poisson-Gamma model and (2) Full Bayes evaluation based on the panel random parameters Poisson-Lognormal model. These evaluation approaches are discussed in the following sections.

3.1. Empirical Bayes evaluation with panel random parameters Poisson-Gamma model

For Empirical Bayes evaluation, The safety performance function is computed by employing the panel random effects Poisson-Gamma model (often referred to as grouped random parameters model (Mannering et al., 2016)). Separate safety performance functions are developed for each signalized intersection type (Cross and T-intersection) by different crash types (total injury, fatal and major injury, right-turn, rear-end crashes). The framework for the Empirical Bayes approach is briefly presented below.

Random parameters Poisson-Gamma model framework: Let's assume that i represents intersections and t represents years (t = 1, 2, …, T). The equation system for random parameters Poisson-Gamma model can be expressed as:

$$\log(\lambda_{it}) = \mathbf{X}_{it}^{'}\,\boldsymbol{\beta}_{it} + \varepsilon_{it}$$
$$\exp(\varepsilon_{it}) \sim \text{gamma}(\varphi, \varphi)$$
$$\boldsymbol{\beta}_{it} = \overline{\boldsymbol{\beta}}_{i} + \boldsymbol{\alpha}_{it},\ \boldsymbol{\alpha}_{it} \sim \text{Normal}(\mathbf{0}, \boldsymbol{\sigma}_{i}) \qquad (1)$$

where, $\boldsymbol{\beta}_{it}(=\beta_{0,it},.......,\beta_{k,it})^{'}$ denotes a vector of unknown regression parameters specific to intersection i and year t, $\mathbf{X}_{it}(=1, X_{1,it}\ ...., X_{k,it})^{'}$ denotes the vector of k covariates, $\overline{\boldsymbol{\beta}}_{i}\ (=\overline{\beta_{0,i}},.......,\overline{\beta_{k,i}})^{'}$ and $\boldsymbol{\sigma}_{i}\ (=\sigma_{0,i},.......,\sigma_{k,i})^{'}$ are the mean and standard deviation of parameters across intersections, $\boldsymbol{\alpha}_{it}\ (=\alpha_{0,it},.......,\alpha_{k,it})^{'}$ is the randomly distributed terms with zero mean and variance σ_{i}^{2} assumed to a realization from normal distribution. The maximum simulated log-likelihood estimation technique with Halton draws (Train, 2000; Bhat, 2003) is employed for the random parameters model estimation

Simulation-based Empirical Bayes evaluation framework: In evaluating the safety effectiveness of the protected right-turn signal phasing by employing the Empirical Bayes approach, the predicted crash counts for the before and after periods of the treated sites are generated by using the estimated parameters ($\overline{\beta_i}, \sigma_i$). In incorporating the effect of random parameters in evaluation, the predicted crash counts are computed as mean predictions generated using 1,000 random draws (generated as the realizations of normal distributions). Referring to Eq. (1), the random parameters of the safety performance function can be written as:

$$\overline{\beta_k} \sim \text{Normal}\,(\mu_k, \omega_k) \tag{2}$$

where, μ_k and ω_k are the means and standard deviations of the random regression parameters distributed across the intersections. Using the predicted crash counts, 1,000 crash modification factors are computed, and the overall crash modification factor is finally computed as an average measure of 1,000 crash modification factors.

In employing the simulation-based empirical Bayes approach, the crash modification factor for each draw is computed following Hauer (1997). Assuming a and b represent before and after periods, respectively, the before period expected number of crashes without treatment for each draw is estimated using Eq. (3) to Eq. (5).

$$E_{pb}=w_p M_{pb}+\left(1-w_p\right)N_b \tag{3}$$

$$w_p=\frac{1}{1+O\times M_{pb}} \tag{4}$$

where,

E_{pb} = expected crash counts per year in the before period of treated sites for p^{th} draw,

E_{pa} = expected crash counts per year in the after period of treated sites for p^{th} draw,

M_{pb} = predicted crash count per year in the before period of treated sites (using safety performance function) for p^{th} draw,

M_{pa} = predicted crash count per year in the after period of treated sites (using safety performance function) for p^{th} draw,

N_b = observed crash per year in the before period of treated sites,

w_p = weight factor dependent on the over-dispersion parameter and predicted crash count obtained from the p^{th} draw, and

O = overdispersion parameter.

A correction factor is multiplied with E_{pb} to generate the estimate of E_{pa} to account for differences in traffic volumes and general trend from b to a as well as to account for the length of the after period relative to the before period. Notably, annual multipliers, as the ratio of yearly observed crashes to the yearly estimated crashes from the reference group are applied to both E_{pa} and E_{pb} to account for the effect of temporal trends (weather, economy, demography) on safety. Hence, the estimate of E_{pa} is given by,

$$E_{pa}=E_{pb}\times R_p=E_{pb}\times \frac{M_{pa}}{M_{pb}} \tag{5}$$

where, R_p is the correction factor.

Finally, the crash modification factor (θ_E) and crash reduction rate (CRR_E) for simulation-based empirical Bayes approach are calculated as,

$$\text{Crash modification factor, } \theta_E=\frac{1}{p}\sum_1^p \theta_p = \frac{1}{p}\sum_1^p \frac{\frac{A_{p,sum}}{E_{p,sum}}}{1+\left[\frac{Var(E_{p,sum})}{E_{p,sum}{}^2}\right]} \tag{6}$$

$$\text{Crash reduction factors, } CRR_E = \frac{1}{p}\sum_1^p CRR_p = \frac{1}{p}\sum_1^p 100\times(1-\theta_p) \tag{7}$$

More details on the simulation-based empirical Bayes approach can be found in (Tahir et al., 2022)

3.2. Full Bayes evaluation with random parameters Poisson-Lognormal model

The safety performance function for the Full Bayes approach is developed by employing the panel random parameters Poisson-Lognormal model by pooling data for both reference and treated sites (Pawlovich et al., 2006; Li et al., 2008; Park et al., 2010). The framework for the Full Bayes approach (Change point modeling framework) is presented below, followed by the mathematical formulation of the random parameters Poisson-Lognormal model.

Random parameters Poisson-Lognormal model framework: Assuming j as intersection and t as an indicator for years (t = 1, 2, …, T), the equation system for random parameters Poisson-Lognormal change point model (Li et al., 2008) can be expressed as:

$$\log\left(\phi_{jt}\right)=\gamma_{0,jt}+\gamma_{1,jt}\tau_j+\gamma_{2,jt}t+\gamma_{3,jt}(t-t_{0j})I_{jt}+\gamma_{4,jt}\tau_j t+\gamma_{5,jt}\tau_j(t-t_0)\, I_{jt}+\mathbf{Z}'_{jt}\,\boldsymbol{\gamma}_{jt}+\kappa_{jt}$$

$$\kappa_{jt}\sim \text{Normal}(0,\sigma_\kappa^2)$$

$$\boldsymbol{\gamma}_{jt}=\overline{\boldsymbol{\gamma}_j}+\boldsymbol{\delta}_{jt}\quad,\ \boldsymbol{\delta}_{jt}\sim \text{Normal}(\mathbf{0},\boldsymbol{\psi}_j^2) \tag{8}$$

where, ϕ_{jt} is the Poisson mean, $\gamma_0 \sim \gamma_5$ are the parameters of the interaction terms accounting for the changes in the intercept and slope with respect to time (both at treated and comparison sites before and after periods of treatment implementation), $\boldsymbol{\gamma}_{\mathbf{jt}}(=\gamma_{6,jt},.......,\gamma_{l,jt})'$ denotes a vector of unknown regression parameters corresponding to $\mathbf{Z}_{\mathbf{it}}(=Z_{6,jt}\ ...., Z_{l,jt})'$ vector of explanatory variables, and κ_{jt} is the idiosyncratic error term. The extra-Poisson variation (represented by the error term κ_{jt}) is assumed to follow a normal distribution with zero mean and standard deviation $\boldsymbol{\psi}_{\mathbf{j}}\ (=\psi_{0,j},.......,\psi_{l,j})'$. The interaction terms in Eq. (8) are:

τ = Indicator for treated sites (equals 1 for treated sites, zero otherwise),
$t = t^{th}$ year in the study period ($t = 1,2, \ldots\ldots\ldots, T=14$),
$(t - t_{0j})I_{jt}$ = time trend after period for both treatment and comparison sites,
τt = time trend on treated sites for both before and after period, and
$\tau(t - t_{0j})I_{jt}$= time trend on treated sites for after period.
where,
I_{jt}= Indicator for before and after period (1 if belongs to the after period and zero otherwise), and
t_{0j} = year in which the countermeasure was installed at site j (for the comparison group, it is defined as the same year as the corresponding treatment group).

The above change point model is estimated in the Bayesian inference using the Markov chain Monte Carlo (MCMC) simulation method. The Deviance information criterion is used to determine the best set of regressors for each model.

Simulation-based Full Bayes evaluation framework: For the Full Bayesian approach, the crash modification factor is estimated as the odds ratio from the expected crashes at before and after period of both treated and comparison groups. For each group of (1) comparison-after, (2) comparison-before, (3) treated-after, and (4) treated-before, the expected crashes are estimated from the Eq. (9) to Eq. (12) transformed from the change point model considered in Eq. (8).

$$\ln(\phi_{j,t})_{Comp,B} = \gamma_{0,jt} + \gamma_{2,jt}t + \mathbf{Z}'_{jt}\,\boldsymbol{\gamma}_{\mathbf{jt}} \tag{9}$$

$$\ln(\phi_{j,t})_{Comp,A} = (\gamma_{0,jt} - \gamma_{3,jt}t_{0i}) + (\gamma_{2,jt} + \gamma_{3,jt})t + \mathbf{Z}'_{jt}\,\boldsymbol{\gamma}_{\mathbf{jt}} \tag{10}$$

$$\ln(\phi_{j,t})_{Trt,A} = (\gamma_{0,jt} + \gamma_{1,jt}) + (\gamma_{2,jt} + \gamma_{4,jt})t + \mathbf{Z}'_{jt}\,\boldsymbol{\gamma}_{\mathbf{jt}} \tag{11}$$

$$\ln(\phi_{j,t})_{Trt,B} = \{(\gamma_{0,jt} + \gamma_{1,jt}) - (\gamma_{3,jt} + \gamma_{5,jt})t\} + (\gamma_{2,jt} + \gamma_{3,jt} + \gamma_{4,jt} + \gamma_{5,jt})t + \mathbf{Z}'_{jt}\,\boldsymbol{\gamma}_{\mathbf{jt}} \tag{12}$$

Referring to Eq. (9) to (12), the posterior distributions of crash frequencies for before and after period of the treated group and before and after period of comparison groups are obtained by taking the average for appropriate sites and years first. Then the crash modification factor, θ_F, is estimated from the following equations:

$$\theta_F = \frac{\sum_g^G \phi_{TA(g)}}{\sum_g^G \{\phi_{TB(g)} R_{c(g)}\}}, \; g=1,\ldots., G \tag{13}$$

$$R_{c(g)} = \frac{\phi_{CA(g)}}{\phi_{CB(g)}} \tag{14}$$

where,

G = number of groups of treated sites,

$\phi_{CB(g)}$ = posterior distribution of expected average crash frequency at before periods for comparison groups,

$\phi_{CA(g)}$ = posterior distribution of expected average crash frequency at after periods for comparison groups,

$\phi_{TB(g)}$= posterior distribution of expected average crash frequency at before periods for treatment groups, and

$\phi_{TA(g)}$ = posterior distribution of expected average crash frequency after periods for treatment groups.

4. Empirical results

4.1. Estimation of safety performance functions

The Empirical analysis of the current study involves the estimation of four sets of safety performance functions for four types of crashes (total injury crashes, fatal and major injury crashes, right-turn crashes, and rear-end crashes). Separate safety performance functions (SPF) are developed for Cross and T intersection groups. The SPFs are developed by employing – (1) fixed parameters Poisson-Gamma model, (2) random parameters Poisson-Gamma model, (3) fixed parameters Poisson-Lognormal model, and (4) random parameters Poisson-Lognormal model. The estimation results of Poisson-Gamma models for Cross and T intersections are presented in Tables 4 and 6, respectively. Further, the estimation results for Poisson-Lognormal models are presented in Tables 5 and 7, respectively. The Tables also include the data fit measures for all crash types. From the Tables, we can observe that the random parameters variant outperforms the fixed variant of the respective model across all crash types. The results highlight the importance of accommodating unobserved heterogeneity in developing safety performance functions. The final models are developed based on the variables which are statistically significant (at 95% confidence level for the Poisson-Gamma models and within 95% credible interval for the Poisson-Lognormal models). Parameter estimates from the developed safety performance functions are briefly discussed in the following section by crash types.

4.2.1. Parameter estimates for Cross intersections

The results for total injury, fatal and major injury, right-turn, and rear-end crashes for cross intersections are presented in the second, third, fourth and fifth column panels of Tables 4 and 5, respectively. The exposure variable *product of AADT* has a positive association with the expected crash frequency for all the evaluated crash types across all models. Other explanatory variables vary for the different crash types. For the random parameters specifications of both Poisson-Gamma and Poisson-Lognormal models, the constant terms (*Intercept*) for all crash types are normally distributed, implying the existence of unobserved heterogeneity or the intersection-specific variation of common unobserved attributes for Cross intersections in the dataset.

Total injury crashes: The *presence of exclusive right-turn lane on minor road* is positively associated with total injury crash risk across all models, perhaps reflecting the compound nature of permissive signal phases (Wang and Abdel-Aty, 2008, Miller et al., 2006). The intersections in rural areas are positively associated with total injury crashes. Further, a negative association between *posted speed limit on major roads> 60 km/hr* and a positive association with the *posted speed limit on minor roads> 50 km/hr* are found. In random parameters Poisson-Gamma model, the parameters for *posted speed limit on minor roads > 50 km/hr* and *rural intersections* are found to be normally distributed, indicating the presence of unobserved heterogeneities around these variables.

Fatal and major injury crashes: The *presence of an exclusive right-turn lane on minor road* is positively associated with the expected crash frequency for fatal and major injury crashes across all models. Similarly, *rural intersections* show positive association with fatal and major injury crashes across all the models. In random parameters Poisson-Gamma model, the parameter estimates for the *product of AADT* are found to be normally distributed. Further, in the random parameters Poisson-Lognormal model, the parameter estimates for an *exclusive right-turn lane on minor road* is also found to be random.

Right-turn crashes: The *number of lanes on major roads* is positively associated with right-turn crash risk across all models. Similar to total injury and fatal and major injury crashes, the *presence of an exclusive right-turn lane on minor roads* is also positively associated with right-turn crashes. Further, a negative association between right-turn crashes and *posted speed limit on major roads > 60 km/hr* and a positive association between right-turn crashes and *posted speed limit on minor roads > 50 km/hr* are found across all model variants.

Rear-end crashes: The *number of lanes of minor roads* is positively associated with the expected crash frequency for rear-end crashes across all models. *Rural intersections* are positively associated with rear-end crashes. Further, in random parameters Poisson-Gamma model, the parameter estimate for the *product of AADT* is found to be normally distributed.

4.2.2. Parameter estimates for T intersections

The results for total injury, fatal and major injury, right-turn, and rear-end crashes for T intersections are presented in the second, third, fourth and fifth column panels of Tables 6 and 7, respectively. For the fixed and random parameters specifications of both Poisson-Gamma and Poisson-Lognormal models, the exposure variable *modulus of AADT* has a positive association with the crash risk across all crash types under consideration. It is worth mentioning that for T intersection, better model fits are found for the *modulus of AADT* over the *product of AADT*. Similar to Cross intersection models, in the random parameters specifications of both models, the constant terms (*Intercept*) for all the crash types are found to be normally distributed, implying the intersection-specific variation of common unobserved attributes for T intersections in the dataset.

Total injury crashes: Unlike Cross intersections, the presence of an *exclusive right-turn lane on minor roads* is negatively associated with the total injury crash risk across all models. *Rural intersections* are positively associated with total injury crashes. Further, a negative association between total injury crashes and *posted speed limit on major roads > 60 km/hr* is found. In random parameters Poisson-Gamma model, the parameter estimates for the *modulus of AADT* and an *exclusive right-turn lane on minor roads*, and in random parameters Poisson-Lognormal model, *posted speed limit on major roads > 60 km/hr* are found to be normally distributed.

Fatal and major injury crashes: *Rural intersections are* positively associated with fatal and major injury crash across all models. In both Poisson-Gamma and Poisson-Lognormal models, none of the variables are found to be random.

Right-turn crashes: *Posted speed limit on major roads > 60 km/hr* is negatively associated with right-turn crashes. This result could be explained by the fact that the flow of vehicles is more stratified and harmonious at higher speed limits than at lower speed limits roadways. Similar to total injury crashes, the parameter estimates for the *modulus of AADT* and *posted speed limit on major roads > 60 km/hr* are found to be normally distributed in the Poisson-Gamma model.

Rear-end crashes: The *presence of an exclusive left-turn lane on minor roads* is positively associated with rear-end crashes. *Rural intersections* are positively associated with rear-end crashes similar to total injury and fatal and major injury crashes. Further, in the random parameters Poisson-Lognormal model, the parameter estimate for an *exclusive left-turn lane on minor roads* is found to be normally distributed.

Table 4: Poisson-Gamma safety performance function estimates for Cross intersections.

Variables	Total injury				Fatal/major injury				Right-turn				Rear-end			
	Models															
	Fixed		Random		Fixed		Random		Fixed		Random		Fixed		Random	
	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value
Constant term																
Intercept [a)]	-6.772 (0.314)	<0.001	-7.087 (0.499)	<0.001	-8.094 (0.597)	<0.001	-7.576 (0.820)	<0.001	-7.92 (0.564)	<0.001	-8.832 (0.735)	<0.001	-12.863 (0.946)	<0.001	-11.413 (1.173)	<0.001
Std. Dev.	-	-	0.571 (0.043)	<0.001	-	-	0.346 (0.064)	<0.001	-	-	0.664 (0.062)	<0.001	-	-	0.319 (0.080)	<0.001
Exposure variable																
Product of AADT [a)]	0.377 (0.018)	<0.001	0.390 (0.029)	<0.001	0.386 (0.033)	<0.001	0.354 (0.047)	<0.001	0.310 (0.033)	<0.001	0.358 (0.043)	<0.001	0.593 (0.056)	<0.001	0.513 (0.069)	<0.001
Std. Dev.	-	-	-	-	-	-	0.023 (0.004)	<0.001	-	-	-	-	-	-	0.025 (0.005)	<0.001
Geometric characteristics																
Lane numbers on major road	-	-	-	-	-	-	-	-	0.410 (0.059)	<0.001	0.382 (0.087)	<0.001	-	-	-	-
Lane numbers on minor road	-	-	-	-	-	-	-	-	-	-	-	-	0.297 (0.099)	0.003	0.227 (0.129)	0.077
Exclusive right-turn lane on minor road	0.321 (0.053)	<0.001	0.245 (0.086)	0.004	0.461 (0.109)	<0.001	0.345 (0.141)	0.014	0.414 (0.089)	<0.001	0.373 (0.129)	0.004	-	-	-	-
Traffic characteristics																
Posted speed limit on major road > 60 km/hr	-0.366 (0.048)	<0.001	-0.289 (0.079)	0.000	-	-	-	-	-0.368 (0.075)	<0.001	-0.247 (0.108)	0.022	-	-	-	-
Posted speed limit on minor road > 50 km/hr [a)]	0.283 (0.052)	<0.001	0.188 (0.082)	0.028	-	-	-	-	0.452	<0.001	0.375 (0.112)	<0.001	-	-	-	-
Std. Dev.	-	-	0.221 (0.054)	<0.001	-	-	-	-	-	-	-	-	-	-	-	-
Spatial characteristics																
Rural intersections [a)]	0.473 (0.101)	<0.001	0.305 (0.155)	0.049	0.678 (0.291)	<0.001	0.689 (0.231)	0.003	-	-	-	-	1.153 (0.199)	<0.001	1.081 (0.258)	<0.001
Std. Dev.	-	-	0.487 (0.159)	0.002	-	-	-	-	-	-	-	-	-	-	-	-
Overdispersion Parameter	0.4147	<0.001	0.1107	0.011	0.619	0.004	0.2448	0.011	0.3889	<0.001	0.07	0.034	0.4023	0.077	0.0686	0.701
No of parameters	5		5		3		3		5		5		3		3	
Loglikelihood	-1098.85		-1048.93		-637.293		-624.781		-707.309		-684.182		-416.418		-408.366	
AIC	2211.7		2117.9		1284.6		1263.6		1428.6		1384.4		842.8		830.7	

Variables	Total injury				Fatal/major injury				Right-turn				Rear-end			
	Models															
	Fixed		Random		Fixed		Random		Fixed		Random		Fixed		Random	
	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value
Notes:																
Number of observations: 826; a) Random parameters																

Table 5: Poisson-Lognormal safety performance function estimates for Cross intersections.

Variables	Total injury				Fatal and major injury				Right-turn				Rear-end			
	Models															
	Fixed		Random		Fixed		Random		Fixed		Random		Fixed		Random	
	Mean (Std. err.)	95% BCI	Mean (Std. err.)	95% BCI	Mean (Std. err.)	95% BCI	Mean (Std. err.)	95% BCI	Mean (Std.	95% BCI	Mean (Std. err.)	95% BCI	Mean (Std. err.)	95% BCI	Mean (Std. err.)	95% BCI
Constant term																
Intercept a)	-7.203 (0.514)	[-8.215: -6.212]	-7.859 (1.023)	[-9.899: -5.890]	-8.494 (0.936)	[-10.37: -6.681]	-7.875 (1.274)	[-10.310: -5.263]	-8.034 (0.786)	[-9.501: -6.485]	-9.138 (1.630)	[-10.050: -6.097]	-13.100 (1.315)	[-15.650: -10.500]	-11.870 (1.662)	[-15.110: -8.552]
Std. Dev.	-	-	0.571 (0.075)	[0.437: 0.730]	-	-	0.478 (0.117)	[0.209: 0.694]	-	-	0.695 (0.105)	[0.623: 0.926]	-	-	0.482 (0.11)	[0.283: 0.714]
Exposure variable																
Product of AADT	0.388 (0.029)	[0.333: 0.446]	0.427 (0.059)	[0.315: 0.543]	0.383 (0.052)	[0.282: 0.487]	0.353 (0.072)	[0.203: 0.490]	0.320 (0.046)	[0.227: 0.407]	0.376 (0.095)	[0.314: 0.584]	0.617 (0.074)	[0.474: 0.758]	0.545 (0.094)	[0.356: 0.726]
Interaction terms																
T b)	0.822 (0.220)	[0.395: 1.250]	0.743 (0.28)	[0.195: 1.300]	0.981 (0.330)	[0.338: 1.628]	0.996 (0.366)	[0.27: 1.709]	1.098 (0.291)	[0.537: 1.684]	1.020 (0.380)	[0.767: 1.774]	0.770 (0.330)	[0.119: 1.420]	*0.718* *(0.386)*	*[-0.047:* *1.473]*
t b)	*0.027* *(0.018)*	*[-0.008:* *0.061]*	*0.018* *(0.017)*	*[-0.014:* *0.051]*	*0.044* *(0.027)*	*[-0.008:* *0.098]*	*0.035* *(0.027)*	*[-0.018:* *0.088]*	*0.018* *(0.024)*	*[-0.029:* *0.066]*	0.036 (0.025)	[0.02: 0.085]	*0.0290* *(0.032)*	*[-0.034:* *0.092]*	*0.006* *(0.033)*	*[-0.058:* *0.071]*
X4 b)	-0.068 (0.025)	[-0.116: -0.02]	-0.055 (0.024)	[-0.103: -0.007]	*-0.056* *(0.036)*	*[-0.126:* *0.014]*	*-0.04* *(0.038)*	*[-0.115:* *0.035]*	*-0.022* *(0.034)*	*[-0.087:* *0.045]*	*-0.051* *(0.035)*	*[-0.074:* *0.019]*	-0.097 (0.045)	[-0.186: -0.010]	*-0.053* *(0.049)*	*[-0.149:* *0.044]*
X5 b)	-0.113 (0.039)	[-0.189: -0.037]	-0.087 (0.035)	[-0.157: -0.019]	-0.134 (0.06)	[-0.252: -0.019]	-0.121 (0.057)	[-0.234: -0.01]	-0.175 (0.055)	[-0.287: -0.070]	-0.149 (0.052)	[-0.183: -0.049]	*-0.0530* *(0.055)*	*[-0.162:* *0.055]*	*-0.009* *(0.059)*	*[-0.125:* *0.106]*
X6 b)	*0.023* *(0.061)*	*[-0.097:* *0.143]*	*-0.02* *(0.056)*	*[-0.131:* *0.092]*	*-0.081* *(0.1)*	*[-0.279:* *0.114]*	*-0.099* *(0.101)*	*[-0.297:* *0.096]*	*-0.048* *(0.092)*	*[-0.232:* *0.131]*	*-0.097* *(0.090)*	*[-0.159:* *0.076]*	*0.100* *(0.080)*	*[-0.059:* *0.257]*	*0.021* *(0.088)*	*[-0.149:* *0.193]*
Geometric characteristics																
Lane numbers on major road	-	-	-	-	-	-	-	-	0.325 (0.086)	[0.158: 0.492]	0.325 (0.077)	[0.273: 0.474]	-	-	-	-
Lane numbers on minor road	-	-	-	-	-	-	-	-	-	-	-	-	0.216 (0.083)	[0.052: 0.379]	0.224 (0.07)	[0.085: 0.36]
Exclusive right-turn lane on minor road a)	0.381 (0.087)	[0.209: 0.555]	0.368 (0.078)	[0.217: 0.520]	0.553 (0.135)	[0.29: 0.822]	0.46 (0.192)	[0.079: 0.834]	0.488 (0.137)	[0.220: 0.757]	0.478 (0.118)	[0.397: 0.71]	-	-	-	-

Variables	Total injury				Fatal and major injury				Right-turn				Rear-end			
	Models															
	Fixed		Random		Fixed		Random		Fixed		Random		Fixed		Random	
	Mean (Std. err.)	95% BCI	Mean (Std. err.)	95% BCI	Mean (Std. err.)	95% BCI	Mean (Std. err.)	95% BCI	Mean (Std.	95% BCI	Mean (Std. err.)	95% BCI	Mean (Std. err.)	95% BCI	Mean (Std. err.)	95% BCI
Std. Dev.	-	-	-	-	-	-	0.174 (0.134)	[0.027: 0.516]	-	-	-	-	-	-	-	-
Traffic characteristics																
Posted speed limit on major road > 60 km/hr	-0.344 (0.076)	[-0.494: -0.195]	-0.339 (0.069)	[-0.475: -0.202]	-	-	-	-	-0.398 (0.106)	[-0.607: -0.19]	-0.372 (0.095)	[-0.436: -0.184]	-	-	-	-
Posted speed limit on minor road > 50 km/hr	0.230 (0.078)	[0.077: 0.382]	0.221 (0.07)	[0.083: 0.357]	-	-	-	-	0.426 (0.11)	[0.210: 0.640]	0.413 (0.097)	[0.347: 0.600]	-	-	-	-
Spatial characteristics																
Rural intersections	0.529 (0.155)	[0.224: 0.826]	0.509 (0.143)	[0.226: 0.791]	0.676 (0.223)	[0.228: 1.102]	0.671 (0.192)	[0.29: 1.05]	-	-	-	-	1.141 (0.252)	[0.624: 1.620]	1.126 (0.211)	[0.712: 1.544]
Number of parameters	5		5		3		3		5		5		3		3	
DIC	2607.01		2487.49		1536.37		1503.41		1712.87		1626.90		1144.88		1113.69	

Notes:
Number of observations: 966
a) Random parameter
b) Interaction terms : generated as, T = Indicator for treated sites, t = t^{th} year in the study period (t = 1,2, . . ., m), X4 = (t - t_{0i})I[t > t_{0i}], X5 = T*t, X6 = T*(t - t_{0i})I[t > t_{0i}]; I[t > t_{0i}] = 1 if t belongs to the after period, 0 otherwise, t_{0i} = year of countermeasure installation at site i (same imaginary construction period for the comparison group).
95% credible limits are shown for all parameters. Insignificant parameters (Interaction variables only) are shown in italic.

Table 6: Poisson-Gamma crash frequency model estimates for T intersections.

Variables	Total injury				Fatal and major injury				Right-turn				Rear-end			
	Models															
	Fixed		Random		Fixed		Random		Fixed		Random		Fixed		Random	
	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value	Mean (Std. err.)	p-value
Constant term																
Intercept a)	-8.302 (0.984)	<0.001	-8.875 (1.198)	<0.001	-7.491 (1.988)	<0.001	-7.462 (2.033)	<0.001	-5.998 (2.213)	<0.001	-5.834 (1.601)	<0.001	-12.851 (2.737)	<0.001	-12.699 (2.851)	<0.001
Std. Dev.	-	-	0.150 (0.054)	0.005	-	-	0.694 (0.103)	<0.001	-	-	0.254 (0.076)	<0.001	-	-	0.613 (0.128)	<0.001
Exposure variable																
Modulus of AADT a)	0.923 (0.106)	<0.001	0.954 (0.131)	<0.001	0.642 (0.215)	0.003	0.615 (0.219)	0.005	0.577 (0.242)	0.017	0.509 (0174)	0.003	0.997 (0.308)	0.001	0.984 (0.321)	0.002
Std. Dev.	-	-	0.058 (0.006)	<0.001	-	-	-	-	-	-	0.099 (0.008)	<0.001	-	-	-	-
Geometric characteristics																
Exclusive right-turn lane on minor road a)	-0.423 (0.079)	<0.001	-0.287 (0.147)	0.052	-	-	-	-	-	-	-	-	-	-	-	-
Std. Dev.	-	-	0.442 (0.062)	<0.001	-	-	-	-	-	-	-	-	-	-	-	-
Exclusive left-turn lane on minor road	-	-	-	-	-	-	-	-	-	-	-	-	1.678 (0.677)	0.013	1.461 (0.738)	0.048
Traffic characteristics																
Posted speed limit on major road > 60 km/hr a)	-0.923 (0.117)	<0.001	-0.957 (0.154)	<0.001	-	-	-	-	-0.743 (0.197)	<0.001	-0.814 (0.189)	<0.001	-	-	-	-
Std. Dev.	--	--	-	-	-	-	-	-	-	-	0.524 (0.163)	0.001	-	-	--	-
Spatial characteristics																
Rural intersections	1.420 (0.134)	<0.001	1.139 (0.215)	<0.001	0.969 (0.262)	<0.001	0.885 (0.250)	<0.001					1.490 (0.213)	<0.001	1.208 (0.279)	<0.001
Overdispersion Parameter	0.7303	<0.001	0.1305	0.016	0.621	0.004	0.1722	0.048	1.7724	<0.001	0.2422	0.027	0.964	0.180	0.3230	0.299
Number of parameters	4		4		2		2		3		3		3		3	
Loglikelihood	-641.72		-606.05		-325.91		-317.45		-464.09		-423.35		-235.32		-231.06	
AIC	1295.40		1230.10		659.80		644.90		936.20		860.70		480.70		474.10	

Notes:
Number of observations: 574; a) Random parameters

Table 7: Poisson-Lognormal crash frequency model estimates for T intersections.

Variables	Total injury				Fatal and major injury				Right-turn				Rear-end			
	Models															
	Fixed		Random		Fixed		Random		Fixed		Random		Fixed		Random	
	Mean (Std. err.)	**95% BCI**	**Mean (Std. err.)**	**95% BCI**	**Mean (Std. err.)**	**95% BCI**	**Mean (Std. err.)**	**95% BCI**	**Mean (Std. err.)**	**95% BCI**	**Mean (Std. err.)**	**95% BCI**	**Mean (Std. err.)**	**95% BCI**	**Mean (Std.**	**95% BCI**
Constant term																
Intercept [a)]	-8.076 (1.339)	[-10.940: -5.693]	-9.098 (2.553)	[-14.430: -4.371]	-7.47 (1.918)	[-11.410: -3.822]	-7.516 (3.29)	[-14.72: -1.595]	-5.783 (1.812)	[-9.131 :-2.28]	-6.310 (3.537)	[-12.9: 1.856]	-12.050 (2.357)	[-17.38: -7.616]	-12.230 (2.922)	[-17.96: -6.225]
Std. Dev.	-	-	0.664 (0.108)	[0.473: 0.896]	-	-	0.685 (0.156)	[0.410: 1.017]	-	-	1.066 (0.169)	[0.779: 1.444]	-	-	0.378 (0.231)	[0.036: 0.835]
Exposure variable																
Modulus AADT	0.887 (0.15)	[0.617: 1.204]	1.032 (0.284)	[0.505: 1.623]	0.622 (0.209)	[0.224: 1.048]	0.621 (0.361)	[-0.029: 1.413]	0.481 (0.198)	[0.098: 0.845]	0.551 (0.389)	[-0.348: 1.279]	0.920 (0.247)	[0.442: 1.476]	0.957 (0.319)	[0.296: 1.603]
Interaction terms																
T [b)]	1.154 (0.326)	[0.524: 1.795]	1.109 (0.432)	[0.273: 1.964]	*0.810 (0.498)*	*[-0.194: 1.767]*	*0.796 (0.612)*	*[-0.417: 1.978]*	*0.849 (0.536)*	*[-0.224: 1.892]*	*0.755 (0.67)*	*[-0.566: 2.043]*	1.377 (0.422)	[0.544: 2.215]	1.407 (0.503)	[0.432: 2.399]
t [b)]	*-0.014 (0.024)*	*[-0.061: 0.032]*	*-0.025 (0.021)*	*[-0.066: 0.017]*	*0.002 (0.037)*	*[-0.07: 0.074]*	*-0.004 (0.037)*	*[-0.076: 0.069]*	*0.037 (0.035)*	*[-0.031: 0.106]*	*0.008 (0.029)*	*[-0.048: 0.066]*	-0.091 (0.044)	[-0.177: -0.007]	*-0.083 (0.044)*	*[-0.17: 0.002]*
X4 [b)]	*-0.076 (0.045)*	*[-0.165: 0.013]*	*-0.057 (0.042)*	*[-0.138: 0.025]*	*-0.038 (0.067)*	*[-0.171: 0.093]*	*-0.028 (0.07)*	*[-0.167: 0.111]*	-0.15 (0.065)	[-0.279: -0.024]	*-0.080 (0.058)*	*[-0.196: 0.033]*	*0.092 (0.082)*	*[-0.071: 0.251]*	*0.071 (0.084)*	*[-0.095: 0.234]*
X5 [b)]	-0.113 (0.054)	[-0.22: -0.008]	-0.086 (0.044)	[-0.173: -0.001]	*-0.091 (0.082)*	*[-0.252: 0.071]*	*-0.058 (0.083)*	*[-0.219: 0.103]*	*-0.084 (0.09)*	*[-0.26: 0.094]*	*-0.063 (0.068)*	*[-0.198: 0.07]*	*-0.050 (0.072)*	*[-0.193: 0.092]*	*-0.034 (0.071)*	*[-0.174: 0.104]*
X6 [b)]	*0.045 (0.104)*	*[-0.157: 0.251]*	*0.020 (0.095)*	*[-0.166: 0.204]*	*0.115 (0.147)*	*[-0.18: 0.399]*	*0.047 (0.155)*	*[-0.263: 0.347]*	*-0.364 (0.25)*	*[-0.886: 0.09]*	*-0.403 (0.233)*	*[-0.899: 0.013]*	*0.014 (0.136)*	*[-0.255: 0.278]*	*-0.018 (0.139)*	*[-0.291: 0.254]*
Geometric characteristics																
Exclusive right-turn lane on minor road	-0.376 (0.168)	[-0.704: -0.045]	-0.843 (0.27)	[-1.384: -0.31]	-	-	-	-	-	-	-	-	-	-	-	-
Exclusive left-turn lane on minor road [a)]	-	-	-	-	-	-	-	-	-	-	-	-	1.970 (0.845)	[0.603: 3.985]	1.699 (0.847)	[0.246: 3.575]
Std. Dev.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.333 (0.228)	[0.034: 0.804]
Traffic characteristics																
Posted speed limit on major road > 60 km/hr [a)]	-0.733 (0.134)	[-0.999: -0.478]	-0.670 (0.125)	[-0.914: -0.424]	-	-	-	-	-0.628 (0.185)	[-0.999: -0.273]	-0.615 (0.167)	[-0.94: -0.289]	-	-	-	-
Std. Dev.	-	-	0.325 (0.279)	[0.03: 1.02]	-	-	-	-	-	-	-	-	-	-	-	-
Spatial characteristics																

Variables	Total injury				Fatal and major injury				Right-turn				Rear-end			
	Models															
	Fixed		Random		Fixed		Random		Fixed		Random		Fixed		Random	
	Mean (Std. err.)	95% BCI	Mean (Std. err.)	95% BCI	Mean (Std. err.)	95% BCI	Mean (Std. err.)	95% BCI	Mean (Std. err.)	95% BCI	Mean (Std. err.)	95% BCI	Mean (Std. err.)	95% BCI	Mean (Std.	95% BCI
Rural intersections	1.195 (0.23)	[0.749: 1.649]	1.171 (0.215)	[0.750: 1.587]	0.927 (0.272)	[0.366: 1.434]	0.918 (0.241)	[0.440: 1.388]					1.479 (0.303)	[0.873: 2.056]	1.261 (0.534)	[0.145 :2.248]
Number of parameters	4		4		2		2		3		3		3		3	
DIC	1512.300		1428.961		803.759		776.329		1028.050		952.735		662.241		647.709	

Notes:

Number of observations: 658

a) Random parameter

b) Interaction terms : generated as, T = Indicator for treated sites, t = t^{th} year in the study period (t = 1,2, . . ., m), X4 = (t - t_{0i})I[t > t_{0i}], X5 = T*t, X6 = T*(t - t_{0i})I[t > t_{0i}]; I[t > t_{0i}] = 1 if t belongs to the after period, 0 otherwise, t_{0i} = year of countermeasure installation at site i (same imaginary construction period for the comparison group).

95% credible limits are shown for all parameters. Insignificant parameters (Interaction variables only) are shown in italic.

4.3. Estimates of treatment effects from Empirical Bayes and Full Bayes approaches

The safety effectiveness estimates are computed based on fixed and random parameters Poisson-Gamma models in Empirical Bayes approaches and fixed and random parameters Poisson-Lognormal models in Full Bayes approaches for four crash types (total injury, fatal and major injury, right-turn and rear-end crashes) and two intersection groups (Cross and T intersections). Thus, four sets of treatment effects for four types of crashes are computed for each intersection group. The estimates of treatment effects are presented in terms of crash modification factor (CMF) or crash reduction rate (CRR) (100*(1-crash modification factors)) in percentages, uncertainty estimates (standard errors), and 95% confidence intervals (CI)/Bayesian confidence intervals (BCI) of crash modification factors. Hereinafter, the evaluation approaches based on fixed and random parameters safety performance functions will be termed as traditional and simulation-based approaches, respectively[4].

4.3.1. Treatment effects for Cross intersections

The protected right-turn phasing at cross intersections has been found to reduce about 63% of total injury crashes, as estimated by the simulation-based Empirical Bayes approach. As shown in Figure 2, the crash modification factors by both traditional and simulation-based Empirical Bayes approaches [CMF: 0.403, CI: 0.274-0.532 and CMF: 0.373, CI: 0.254-0.493] are found to be marginally lower than those of Full Bayes approaches [CMF: 0.435, BCI: 0.294-0.624 and CMF: 0.434, BCI: 0.309-0.593]. However, for both evaluation approaches, standard deviations of crash modification factors based on the simulation-based approaches [Empirical Bayes: 6.10 and Full Bayes:7.27] are smaller than those estimated from the traditional approaches [Empirical Bayes: 6.59, Full Bayes: 8.40].

The effect of protected right-turn phasings on fatal and major injury crashes at cross intersections was even higher, with a corresponding reduction of 80%. Crash modification factors are slightly lower in traditional and simulation-based Empirical Bayes approaches [CMF: 0.228, CI: 0.090-0.365 and CMF: 0.200, CI: 0.078-1.322] than those of Full Bayes approaches [CMF: 0.243, BCI: 0.120-0.430 and CMF: 0.247, BCI: 0.126-0.426]. Similar to total injury crashes, uncertainties around crash modification factors for simulation-based

[4] Full Bayes approaches based on both fixed parameters and random parameters safety performance functions are estimated through MCMC simulation. However, the fixed parameters model based Full Bayes approach is termed as traditional Full Bayes approach and the random parameters model based Full Bayes approach is termed as simulation-based Full Bayes approach in this study to distinguish them similar to the corresponding Empirical Bayes approaches and to avoid verbosity.

approaches [Empirical Bayes: 6.23 and Full Bayes: 7.74] are smaller than those of the traditional approaches [Empirical Bayes: 7.01, Full Bayes: 7.99].

The protected right-turn phasings at cross intersections are found to eliminate about 87% of right-turn crashes. Similar to total injury and fatal and major injury crashes, crash modifications for right-turn crashes are also lower in traditional and simulation-based Empirical Bayes approaches [CMF: 0.148, CI: 0.062-0.233 and 0.131, CI: 0.055-0.208] than those of the Full Bayes approaches [CMF: 0.202, BCI: 0.104-0.343 and CMF: 0.197, BCI: 0.109-0.318)]. The standard errors of crash modification factors are smaller for simulation-based approaches [Empirical Bayes: 3.89 and Full Bayes: 5.40] compared to the traditional approaches [Empirical Bayes: 4.35, Full Bayes: 6.14].

Unlike the other three crash types studied, crash modification factors for rear-end crashes are not statistically significant across all four approaches, namely, traditional Empirical Bayes, simulation-based Empirical Bayes, traditional Full Bayes and simulation-based Full Bayes approaches (Figure 2).

Overall, the results suggest that implementing a protected right-turn signal over a permissible right-turn signal is an effective treatment for reducing total injury crashes, fatal and serious injury crashes, and right-turn crashes. the treatment does not have any effect on rear-end crashes in the current study context. In general, uncertainty estimates from simulation-based evaluations are lower than traditional approaches supporting the hypothesis that random parameters model-based evaluations are likely to result in more precise estimates of countermeasure effectiveness. These precise estimates (lower standard errors) of crash modification factors in simulation-based evaluations lead to tighter confidence intervals/credible intervals than the corresponding traditional counterparts, as shown in Figure 2.

Among the variants of the Empirical Bayes and Full Bayes approaches, the simulation-based Empirical Bayes approach produces the most efficient results for total injury, fatal and major injury, right-turn, and rear-end crash types. Based on this approach, the implementation of protected right-turn signal phasings contributes to substantial (statistically significant) reduction of total injury crashes by 62.66% (95% CI = 50.70 – 74.60%), fatal and major injury crashes by 80.01% (95% CI = 67.80 – 92.20%), and right-turn crashes by 86.86% (95% CI = 79.20 – 94.50%) at Cross intersections.

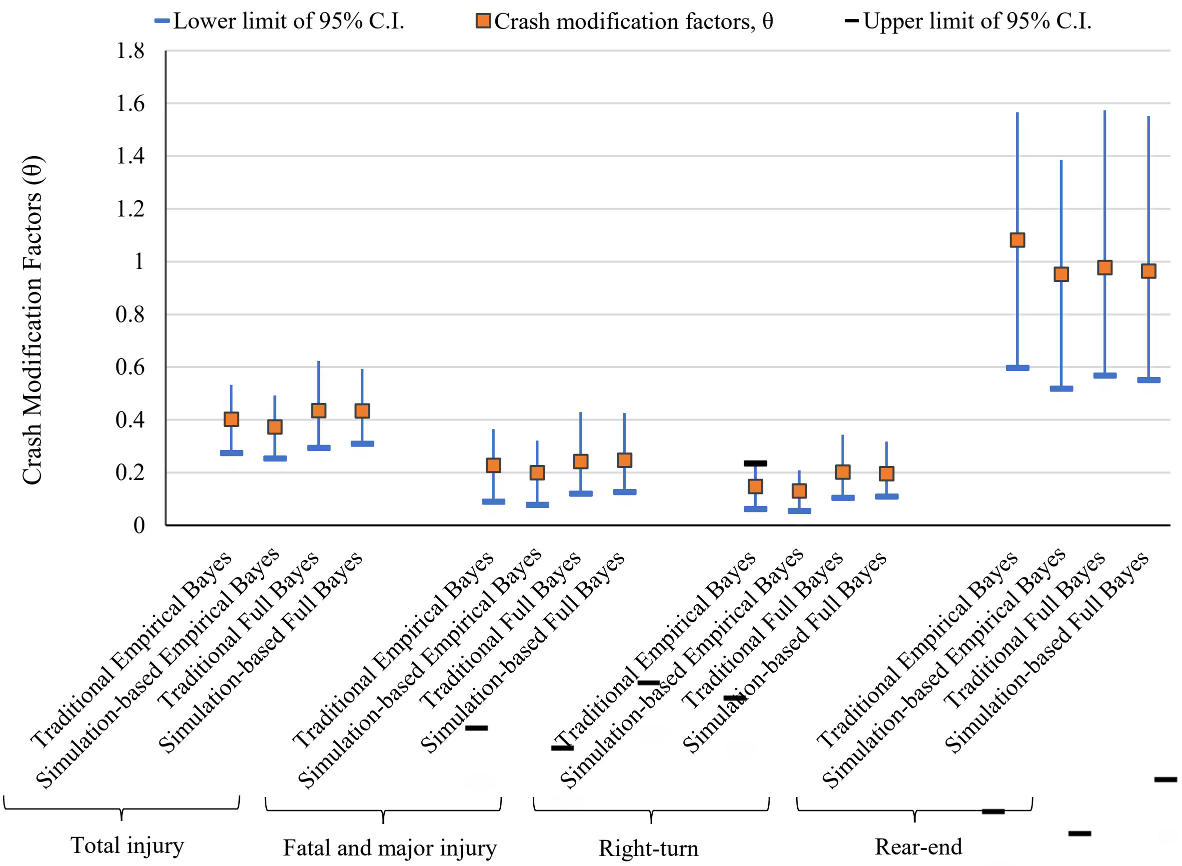

Figure 2: Crash modification factors for different crash types at Cross intersections.

4.3.2. Treatment effects for T intersections

The protected right-turn signal at T-intersections has been found to reduce about 59% of total injury crashes. As shown in Figure 3, the crash modification factors based on traditional and simulation-based Empirical Bayes approaches [CMF: 0.436, CI: 0.216-0.655 and CMF: 0.406, CI: 0.206-0.607)] are comparable to those of the Full Bayes approaches [CMF: 0.386, BCI: 0.206-0.659 and CMF: 0.430, BCI: 0.253-0.675]. Moreover, the estimates of the standard deviations of crash modification factors from simulation-based approaches [Empirical Bayes: 10.26 and Full Bayes:10.86] are smaller than those of traditional approaches [Empirical Bayes: 11.20, Full Bayes:11.73].

For fatal and major injury crashes, crash modifications factors are significant in both the Empirical Bayes approaches [0.557 (95% CI: 0.120-0.994) and 0.477 (95% CI: 0.169-0.783)] whereas not significant in the Full Bayes approaches [0.619 (95% BCI: 0.239-1.301) and 0.592 (95% BCI : 0.243-1.196)]. Similar to total injury crashes, uncertainties around crash modification factors are lower for simulation-based approaches [Empirical Bayes: 6.23 and Full Bayes: 7.74] than those of traditional approaches [Empirical Bayes: 7.64, Full Bayes: 9.02].

The protected right-turn signal at T intersections has been found to eliminate 91% of right-turn crashes. For right-turn crashes, crash modifications are lower for both Empirical Bayes approaches [CMF: 0.108 (95% CI: 0.000-0.258) and 0.093 (95% CI: 0.000-0.221)] compared to those of Full Bayes approaches [0.139 (95% BCI: 0.030-0.370) and 0.137 (95% BCI: 0.033-0.343)]. Like other crash types, the standard errors of crash modification factors are smaller for simulation-based evaluations [Empirical Bayes: 6.54 and Full Bayes: 8.15] than that of traditional evaluation approaches [EB: 6.49, FB: 9.00].

For rear-end crashes, mixed results on crash modification factors are observed for different approaches. Crash modifications factor is found significant from simulation-based Empirical Bayes approach [CMF: 0.607, CI: 0.236-0.978] whereas not significant for traditional counterpart [CMF: 0.759, CI: 0.296-1.222]. However, crash modification factors are not significant for both traditional and simulation-based Full Bayes approaches [CMF: 0.575, BCI: 0.246-1.150 and CMF: 0.560, BCI: 0.332-1.479].

Similar to Cross intersections, the simulation-based Empirical Bayes approach produces the most efficient estimates of crash modification factors. Following the approach, implementation of protected right-turn signal phasings at T intersections demonstrates substantial (statistically significant) reduction of total injury crashes by 59.37% (95% CI = 39.30 – 79.40%), fatal and major injury crashes by 52.40% (95% CI = 21.70 – 83.10%), and right-turn crashes by 90.75% (95% CI = 77.90 – 100.00%), whereas a comparatively low reduction of rear-end crashes by 39.31% (95% CI = 2.20 – 76.40%). The spread of the crash modification factors at T intersections from the simulation-based Empirical Bayes approach compared to the rest of the approaches are shown in Figure 3.

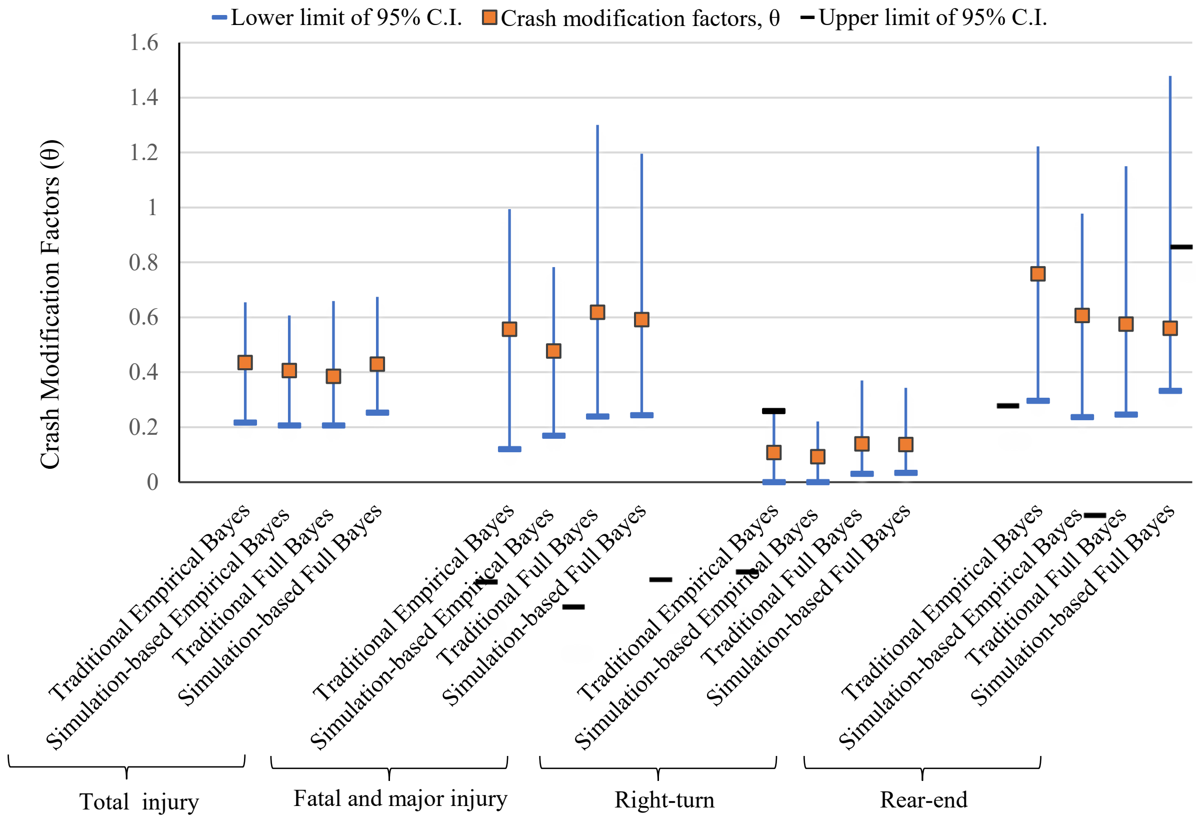

Figure 3: Crash modification factors for different crash types at T intersections.

4.4. Comparative distribution of treatment effects

The comparative distributions of the crash reduction rates from simulation-based Empirical Bayes and simulation-based Full Bayes approaches for both Cross and T intersections are presented in Figure 4 and Figure 5, respectively. Simulation-based Empirical Bayes approach can demonstrate the distribution of safety effects instead of a point estimate by the traditional Empirical Bayes approach. On the contrary, the distribution of safety effects is inherently produced in the Full Bayes approach. As such, the distributions from these two different approaches now can be compared on a common platform that can generate additional information on the overall effectiveness of treatments.

For the Empirical Bayes approach, 1,000 simulations are created from a random combination of parameters (section 3.1), whereas 1,000 simulations are systematically recorded from Markov chain Monte Carlo simulations of the Full Bayes approach. The comparative distributions depict the relative locations of mean, minimum, and maximum safety effects of the protected right-turn signal treatment by the two approaches.

From Figures 4 and 5, it is observed that despite the lower standard errors, the maximum treatment effect from the simulation-based Empirical Bayes is higher than the respective simulation-based Full Bayes maximum in multiple instances (e.g., total injury crashes and right-turn crashes at Cross intersections). In contrast, the minimum treatment effects are smaller in the simulation-based Full Bayes approach for all crash types in both Cross and T intersections. The higher spread of distribution is expected due to higher standard error estimates in the Full Bayes approach considered in the study. However, the comparative distributions show that the relative locations of mean, minimum, and maximum safety effects can randomly vary. Notably, for this study, though the mean crash reduction rates are predominantly lower in the simulation-based Full Bayes approach than in the simulation-based Empirical Bayes approach, it is not necessarily the same for minimum and maximum crash reduction rates.

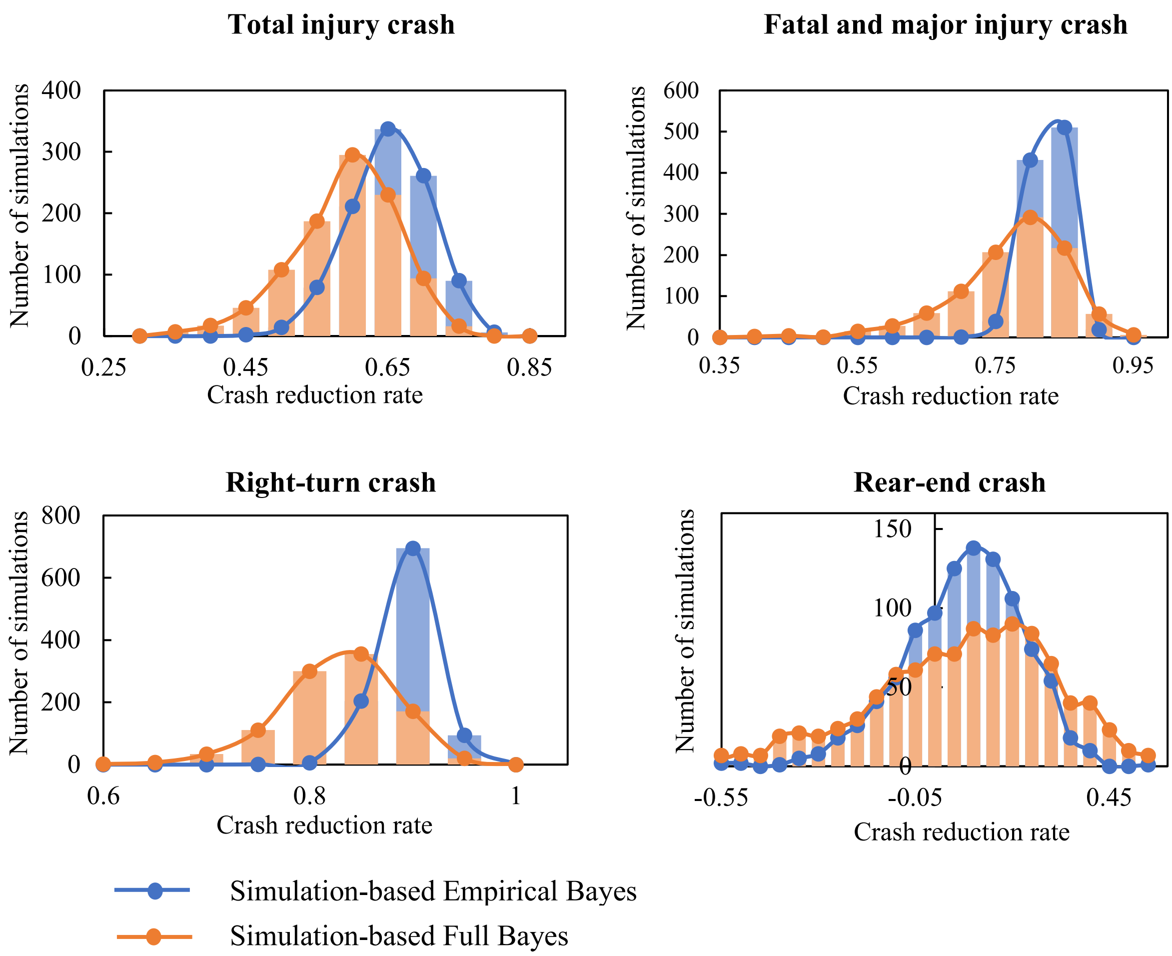

Figure 4: Distribution of crash reduction rates for Cross intersection.

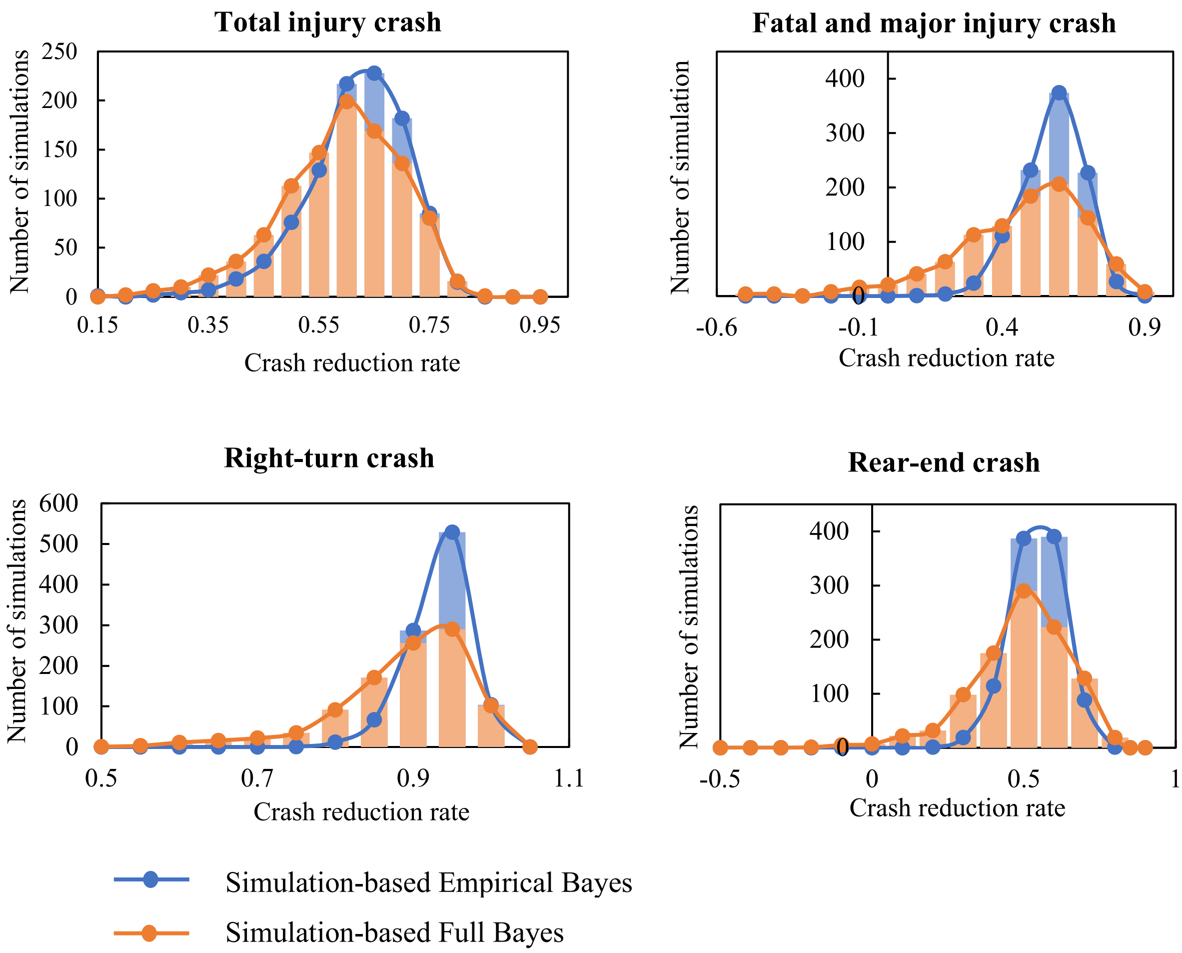

Figure 5: Distribution of crash reduction rates for T intersections.

5. Discussion

This study evaluates the safety effectiveness of protected right-turn signal phasings for Cross and T intersections employing random parameters-based safety performance functions for both Empirical Bayes and Full Bayes approaches. In line with the prior studies (Barua et al., 2014; Rusli et al., 2017), the present study shows that all the random parameters-based safety performance functions outperform the fixed parameters-based safety performance functions based on improved model fit statistics (loglikelihood, AIC, and DIC). It is worth mentioning that the primary intention while predicting crashes in both Empirical Bayes and Full Bayes approaches is to take the best-fitted models irrespective of the model specifications. The underlying hypothesis is that the models with improved fit would also lead to improved safety estimates. The findings of this study support this hypothesis and show lower uncertainty estimates of crash modification factors leading to tighter confidence intervals for both Empirical Bayes and Full Bayes approaches with heterogenous count models (Figures 2 and

3). Thus, the simulation-based Empirical Bayes approach and simulation-based Full Bayes approach outperform their traditional counterparts for all crash types at both signalized Cross and T-intersections. Notably, in the current study context, the most efficient safety evaluation is demonstrated by the simulation-based Empirical Bayes approach in terms of the lowest standard error estimates.

Protected right-turn signal phasing at intersections effectively reduce total injury crashes with a crash modification factor of 0.373 (95% CI: 0.254-0.493) and 0.406 (95% CI: 0.206-0.607) for Cross and T intersections, respectively. Protected right-turn phasing treatment is widely implemented for improving intersection safety among different possible variants of right-turn signal phasings, including permissive, permissive-protected, and protected phasing (refer to Islam et al. (2022) for a comprehensive list of right-turn phasings). Existing studies reported that protected right-turn signal phasings have greater safety benefits than the other variants. Davis et al. (2007) reported a 41.9% reduction in all injury crashes for the change of permissive to protected right-turn signal phasings in comparison to a 14.7% reduction for the change of permissive to permissive-protected right-turn signal phasings. Gan et al. (2005) also reported higher effectiveness of protected right-turn phasing (27% reduction) than protected-permissive phasing (10%). The elimination of the gap acceptance maneuver for right-turning traffic by the protected right-turn signal phasings can be attributed to this safety improvement. As reflected through the crash modification factors, protected right-turn signal phasings are more effective for Cross intersections than T-Intersections. It is expected because traffic movements at Cross intersections are generally more complex.

The crash modification factors for right-turn crashes are 0.131 (95% CI: 0.055-0.208) and 0.093 (95% CI: 0.000-0.221) at Cross and T intersections, respectively. The crash modification factors suggest that right-turn crashes are reduced by 86.86% at Cross intersections and 90.75% at T intersections, which are the highest among other crash types considered in the study. It is not surprising that protected right-turn signals almost eliminate all right-turn crashes by separating the right-turning and through movements in different phases of a signal cycle.

The protected right-turn signal implementation also results in a high reduction of fatal and major injury crashes. The crash modification factors for fatal and major injury crashes are 0.200 (95% CI: 0.078-0.322) and 0.476 (95% CI: 0.169-0.783), representing 80.01% and 52.40% crash reduction for Cross and T intersections, respectively. In general, the severity of right-turn crashes tends to be high because of the high speeds of through vehicles (Wang et al.

(2008)). Moreover, the crash victims are more prone to severe injury due to the angle-type collision involving right-turning vehicles. As mentioned elsewhere, protected right-turn phasing exclusively eliminates such conflicts between through and right-turning vehicles. Therefore, the risk of serious injuries or fatalities is likely to be lowered in a signalized intersection with protected right turn signal phasings.

The safety effectiveness of protected right turn signal phasings for rear-end crash types varies across intersection types. A significant crash modification factor is found for T intersections [0.607 (95% CI: 0.236-0.978)] but not for Cross intersections [0.952 (95% CI: 0.518-1.386)]. The effects of protected right-turn signal phasings on rear-end crash types are mixed. Gan et al. (2005) reported a significant reduction in rear-end crashes by 31%. On the other hand, Srinivasan et al. (2012) reported an increase in rear-end crashes by 9.1%. Similarly, Davis et al. (2007) reported an increase of 30.77% in rear-end crashes. Some other studies (Bahar et al., 2007; Shahdah et al., 2014; De Pauw et al., 2015) reported that the effects of protected right-turn signal on rear-end crash types are not statistically significant. It looks like a detailed investigation is needed on how protected right-turn signal influence the car-following or queuing behavior and the resultant crash risk. Right-turn and rear-end crash types are highly likely to be correlated as an additional right-turning phase leads to higher phase transitions over time and a shorter through phase that may increase the risk of rear-end crashes.

While the protected right-turn signal decreases crash risks in general, the safety effectiveness of protected right-turn signal phasing varies across geographical contexts. The crash reduction rate for right-turn crashes for the current study in Australia is as high as 86%~90%. In comparison, a before-after study by Lyon et al. (2005) on 35 treated intersections in Toronto, Canada, showed only a 17% reduction in right-turn crashes. Another study by Srinivasan et al. (2012) on 59 treated junctions in Canada revealed a 23.8% decrease in right-turn crashes. A 50% decrease in right-turn crashes was observed in a related study by De Pauw et al. (2015), utilizing data from 103 modified junctions in Belgium. In contrast, based on 10 treated intersections in Minnesota, United States, Davis et al. (2007) found a right-turn crash reduction of 99.99% by using a full Bayesian evaluation approach. Similar to the findings by Davis et al. (2007), other studies (Harkey et al., 2008; Srinivasan et al., 2008) also showed right-turn crash reduction as high as 98%~99% based on data from the United States. These differences in the safety effectiveness at different geographical locations might be attributed to the confounding factors related to the driving environments, safety standards and local driver behaviour. As such, the safety effectiveness of protected right-turn signal phasings may not be directly transferrable and requires careful consideration for adopting the corresponding crash modification factors for a particular geographical context.

Overall, the present study shows higher estimates of standard errors in the Full Bayes approaches than those of the Empirical Bayes approaches. The uncertainty estimates of safety performance function parameters simultaneously propagate into the crash modification factor estimates of the Full Bayes approach (Park et al., 2010). On the contrary, the Empirical Bayes approach does not directly transfer the parameter uncertainties to the estimates of crash modification factors. It estimates the parameters of the safety performance functions in one step and the crash modification factors in the next step, considering the safety performance function parameters as true estimates. However, in the change point model within the Poisson-log normal approach, five interaction terms in the safety performance functions are retained irrespective of their insignificance. Though these interaction terms provide additional information on the intervention, these terms generated additional uncertainty along with the other explanatory variables in the safety performance functions. As such, the additional uncertainty induced by these interaction terms in the Full Bayes approach may have resulted in the higher uncertainty of the crash modification factors.

Although simulation-based Empirical Bayes approaches show the lowest uncertainty estimates, some uncertainty estimates within this approach are still quite large (rear-end crash types in both intersections and fatal and major injury crashes in T intersections). Several studies have compared the performances of different distributional assumptions to deal with excess zeros in crash data. It has been found that different variants of Poisson-Gamma models (Negative Binomial Lindley, Negative Binomial-Generalized Exponential) can provide superior statistical fits (Lord et al., 2011; Rusli et al., 2018). It is also worth mentioning that the Empirical Bayes method is reported to produce reliable estimates when (a) a sufficient sample size is available and (b) enough crash frequencies for both treated and reference sites are available (Park et al., 2016). In this study, the observed crash counts for the aforementioned crash types with high uncertainty estimates are too low compared to the remaining cases (Tables 2 and 3). Due to this low crash counts, uncertainties around parameter estimates in the Empirical Bayes approaches may be large. A worthwhile research direction would be examining the performance of Lindley or Generalized Exponential models for before-after evaluation of low count crash types.

In addition to the mean safety estimate, the minimum and maximum safety effects can be critical to the selection of countermeasures. The distributions of crash modification factors provide better insights into safety assessment in addition to point estimates (mean) only. The simulation-based evaluation approaches allow the testing of statistical significance of safety effectiveness at any quantile within the simulation ranges. The statistical significance of the maximum or minimum crash reductions can be tested using the corresponding standard error from simulations. Figures 6 and 7 depict the 95% CI of all the simulated safety estimates for total injury crashes at Cross intersections by the simulation-based Empirical Bayes and simulation-based Full Bayes approaches, respectively. For example, the confidence intervals for maximum crash reduction rate of total crashes by protected right-turn signal at Cross intersections are 0.707-0.855 and 0.605-0.889 from simulation-based Empirical Bayes and simulation-based Full Bayes distributions, respectively. This test on the maximum crash reduction rates reveals that protected right-turn signal phasing is an effective treatment even at its maximum crash reduction limit. Following this process, an informed decision can be made by testing the treatment effectiveness for various percentiles of crash modification factors or crash reduction rates. Similarly, the statistical significance of minimum and maximum treatment effectiveness can be vital for the decision-making on engineering countermeasures.

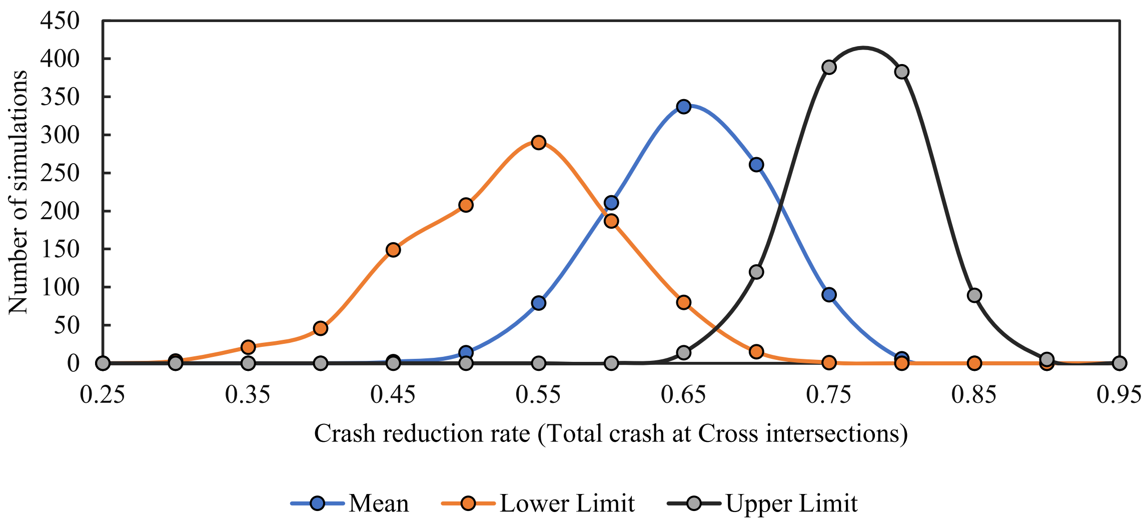

Figure 6: Significance of minimum and maximum crash reduction rates from simulation-based Empirical Bayes distribution.

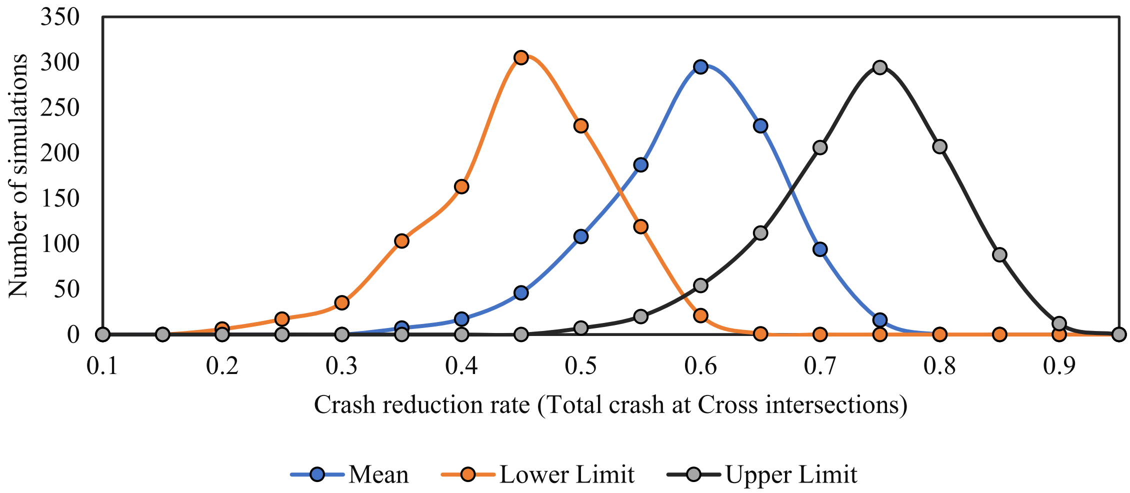

Figure 7: Significance of minimum and maximum crash reduction rates from simulation-based Full Bayes distribution.

This study also underscores the effect of parameters' heterogeneity on overdispersion of observed crash data while applying the simulation-based Empirical Bayes approach. The overdispersion parameter estimates in random parameters Poisson-Gamma models are exceptionally low compared to that of fixed parameters Poisson-Gamma models. This property is very typical of a random parameters model as some unobserved heterogeneities are now captured by the random parameters. On the other hand, the traditional Empirical Bayes approach requires crash data to be overdispersed since the overdispersion parameter is needed for estimating expected number of crashes. In this study, the overdispersion parameters of rear-end crashes are not significant from the fixed parameters-based Poisson-Gamma models at both types of intersections (Cross intersections: 0.4023, p-value: 0.077, T intersections: 0.964, p-value: 0.180). Despite this insignificance, the rear-end crash reduction rates (CRR) are estimated using the traditional Empirical Bayes approach to compare with the other approaches. Subsequently, the crash reduction rates for rear-end crashes by the traditional Empirical Bayes approach are found substantially low (Cross intersections: CRR = -8.18%, T intersection: CRR = 24.10%) compared to the simulation-based counterpart (Cross intersections: CRR = 4.82%, T intersection: CRR = 39.31%). More interestingly, highly comparable crash reduction rates are observed from simulation-based Empirical Bayes, traditional Full Bayes and simulation-based Full Bayes approaches (Cross intersections: CRR = 4.82%, CRR = 2.04%, CRR = 3.56% and T intersection: CRR = 39.31%, CRR = 42.52%, CRR = 43.98%). Several studies claim that the Full Bayes approach can produce reliable estimates even in the absence of overdispersion in the crash data (El-Basyouny et al., 2010).

Therefore, the similarity in results from simulation-based Empirical Bayes and Full Bayes approaches in the study suggest that the heterogenous count data model in the simulation-based Empirical Bayes approach can contribute in two ways: i) mitigating the overdispersion in crashes originating from the heterogeneity in the crash contributing factors and therefore increasing the reliance on other parameter estimates while predicting crashes, and ii) leading to reliable safety estimates even in the absence of overdispersed crash data.

6. Conclusions

This study focuses on investigating the safety effectiveness of protected right-turn signal phasing at signalized intersections in a before-after safety evaluation framework. The study applied random parameters Poisson-Gamma in the Empirical Bayes approach and random parameters Poisson-Lognormal models in the Full Bayes approach to examine the effects of protected right-turn signal phasing compared to permissive right-turn signal.

Findings suggest that a protected right-turn signal at both cross and T intersections is an effective treatment in reducing right-turn crashes and fatal and major injury crashes, but it has no detrimental effects on rear-end crashes. A protected right-turn phasing treatment is primarily implemented to reduce right-turn crashes. The estimates of crash reduction rates for the right-turn crashes were found to be 87% for Cross intersections and 91% for T intersections, respectively. The Protected right-turn signal was also found to reduce 63% and 59% of total injury crashes and 80% and 53% of fatal and major injury crashes for Cross and T intersections, respectively.

Overall, uncertainty estimates from the simulation-based evaluation approaches were more precise than traditional approaches. These findings support the hypothesis that random parameters models result in a more precise estimate of safety due to the consideration of unobserved heterogeneity. Among different evaluation approaches, the estimates from the simulation-based Empirical Bayes approach were found to be the most efficient.

The scope of this study is limited to evaluating the effectiveness of protected right-turn phasings in a before-after setting in which traffic signal phasings have been changed from a permissible right-turn signal to a protective right-turn signal. A worthwhile research direction would be studying the effects of different variants of right-turn signal phasing strategies like permissive-protected, protected-permissive, lag-protected, lead-protected, partially protected right-turn, split approach, and diamond overlap turn. It is quite challenging to identify treated sites where only right-turn traffic signal plans have been updated. In this research, extensive efforts have been made with consultation with the corresponding road authority to identify the treatment sites. It would be worth investigating the sensitivity of crash modification factors with respect to the sample sizes. In addition, the count models accounting for excess zeros (e.g., Lindley model, generalized exponential model) could be adopted with the simulation-based Empirical Bayes and Full Bayes approaches.

References

Ahmed, M.M., Abdel-Aty, M. and Park, J. 2015. "Evaluation of the safety effectiveness of the conversion of two-lane roadways to four-lane divided roadways: Bayesian versus empirical Bayes." *Transportation research record* 2515 (1): 41-49. https://doi.org/10.3141/2515-06.

Anastasopoulos, P.C. and Mannering, F.L. 2009. "A note on modeling vehicle accident frequencies with random-parameters count models." *Accident Analysis & Prevention* 41 (1): 153-159. https://doi.org/10.1016/j.aap.2008.10.005.

Aul, N. and Davis, G. 2006. "Use of propensity score matching method and hybrid Bayesian method to estimate crash modification factors of signal installation." *Transportation research record* 1950 (1): 17-23. https://doi.org/10.1177/0361198106195000103.

Bahar, G.B., Masliah, M., Wolff, R. and Park, P. 2007. *Desktop reference for crash reduction factors [Report no. FHWA-SA-07-015]*. United States: Federal Highway Administration. https://rosap.ntl.bts.gov/view/dot/16438.

Barua, S., El-Basyouny, K. and Islam, M.T. 2014. "A full Bayesian multivariate count data model of collision severity with spatial correlation." *Analytic Methods in Accident Research* 3: 28-43. https://doi.org/10.1016/j.amar.2014.09.001.

Barua, S., El-Basyouny, K. and Islam, M.T. 2015. "Effects of spatial correlation in random parameters collision count-data models." *Analytic Methods in Accident Research* 5: 28-42. https://doi.org/10.1016/j.amar.2015.02.001.

Barua, S., El-Basyouny, K. and Islam, M.T. 2016. "Multivariate random parameters collision count data models with spatial heterogeneity." *Analytic methods in accident research* 9: 1-15. https://doi.org/10.1016/j.amar.2015.11.002.

Bhowmik, T., Yasmin, S. and Eluru, N. 2022. "Accommodating for systematic and unobserved heterogeneity in panel data: Application to macro-level crash modeling." *Analytic methods in accident research* 33: 100202. https://doi.org/10.1016/j.amar.2021.100202.

Chang, F., Yasmin, S., Huang, H., Chan, A.H. and Haque, M.M. 2021. "Injury severity analysis of motorcycle crashes: A comparison of latent class clustering and latent segmentation based models with unobserved heterogeneity." *Analytic methods in accident research* 32: 100188. doi: https://doi.org/10.1016/j.amar.2021.100188.

Chen, L., Chen, C. and Ewing, R. 2015. "Left-turn phase: Permissive, protected, or both? A quasi-experimental design in New York City." *Accident Analysis & Prevention* 76: 102-109. https://doi.org/10.1016/j.aap.2014.12.019.

D'Agostino, C., Cafiso, S. and Kiec, M. 2019. "Comparison of Bayesian techniques for the before–after evaluation of the safety effectiveness of short 2+ 1 road sections." *Accident Analysis & Prevention* 127: 163-171. https://doi.org/10.1016/j.aap.2019.02.009.

Davis, G.A. and Aul, N. 2007. *Safety effects of left-turn phasing schemes at high-speed intersections [Report no. MN/RC-2007-03]*. Minnesota, United States: Minnesota Department of Transportation. https://conservancy.umn.edu/bitstream/handle/11299/5565/200703.pdf.

De Pauw, E., Daniels, S., Van Herck, S. and Wets, G. 2015. "Safety Effects of Protected Left-Turn Phasing at Signalized Intersections: An Empirical Analysis." *Safety* 1 (1): 94-102. https://doi.org/10.3390/safety1010094.

El-Basyouny, K., Barua, S. and Islam, M.T. 2014. "Investigation of time and weather effects on crash types using full Bayesian multivariate Poisson lognormal models." *Accident Analysis & Prevention* 73: 91-99. https://doi.org/10.1016/j.aap.2014.08.014.

El-Basyouny, K. and Sayed, T. 2010. "Full Bayes approach to before-and-after safety evaluation with matched comparisons: case study of stop-sign in-fill program." *Transportation research record* 2148 (1): 1-8. https://doi.org/10.3141/2148-01.

El-Basyouny, K. and Sayed, T. 2011. "A full Bayes multivariate intervention model with random parameters among matched pairs for before–after safety evaluation." *Accident Analysis & Prevention* 43 (1): 87-94. https://doi.org/10.1016/j.aap.2010.07.015.

Elvik, R., Vaa, T., Hoye, A. and Sorensen, M. 2009. *The handbook of road safety measures*. 2nd ed. Bingley, UK: Emerald Group Publishing.

Gan, A., Shen, J. and Rodriguez, A. 2005. *Update of Florida crash reduction factors and countermeasures to improve the development of district safety improvement projects [Report no. BD015-04]*. Tallahassee, Florida, United States: Florida Department of Transportation. http://ftp.fdot.gov/file/d/FTP/FDOT%20LTS/CO/research/Completed_Proj/Summary_SF/FDOT_BD015_04_rpt.pdf.

Harkey, D., Raghavan, S., Jongdea, B., Council, F., Eccles, K., Lefler, N., Gross, F., Persaud, B., Lyon, C. and Hauer, E. 2008. *National Cooperative Highway Research Program Report 617: Accident Reduction Factors for Traffic Engineering and ITS Improvement*. Washington, DC, Unitaed States:Transportation Research Board. http://doi.org/10.17226/13899.

Harwood, D.W., Bauer, K.M., Potts, I.B., Torbic, D.J., Richard, K.R., Rabbani, E.R., Hauer, E., Elefteriadou, L. and Griffith, M.S. 2003. "Safety effectiveness of intersection left-and right-turn lanes." *Transportation Research Record* 1840 (1): 131-139. https://doi.org/10.3141/1840-15.

Hauer, E. 1997. *Observational before/after studies in road safety. estimating the effect of highway and traffic engineering measures on road safety*. UK: Emerald Group Publishing.

Hauer, E. 2004. "Left-Turn Protection, Safety, Delay, and Guidelines: A Literature Review." *Draft. Available at: http://members.rogers.com/hauer*.

Islam, M.T. and El-Basyouny, K. 2015. "Full Bayesian evaluation of the safety effects of reducing the posted speed limit in urban residential area." *Accident Analysis & Prevention* 80: 18-25. https://doi.org/10.1016/j.aap.2015.02.026.

Islam, S.M., Washington, S., Kim, J. and Haque, M. 2022. "A comprehensive analysis on the effects of signal strategies, intersection geometry, and traffic operation factors on right-turn crashes at signalised intersections: An application of hierarchical crash frequency model." *Accident Analysis & Prevention* 171: 106663. https://doi.org/10.1016/j.aap.2022.106663.

Jin, W., Chowdhury, M., Khan, S.M. and Gerard, P. 2021. "Investigating the impacts of crash prediction models on quantifying safety effectiveness of Adaptive Signal Control Systems." *Journal of safety research* 76: 301-313. https://doi.org/10.1016/j.jsr.2020.11.003.

Lalani, N., Cronin, D., Hattan, D. and Scads, T. 1986. "A Summary of the Use of Warrantsfor the Installation of Left-TurnPhasing at Signalized Intersections." *ITE journal*: 57-59. https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.417.5617&rep=rep1&type=pdf.

Lan, B., Persaud, B., Lyon, C. and Bhim, R. 2009. "Validation of a full Bayes methodology for observational before–after road safety studies and application to evaluation of rural signal conversions." *Accident Analysis & Prevention* 41 (3): 574-580. https://doi.org/10.1016/j.aap.2009.02.010.

Li, W., Carriquiry, A., Pawlovich, M. and Welch, T. 2008. "The choice of statistical models in road safety countermeasure effectiveness studies in Iowa." *Accident Analysis & Prevention* 40 (4): 1531-1542. https://doi.org/10.1016/j.aap.2008.03.015.

Lord, D. and Geedipally, S.R. 2011. "The negative binomial–Lindley distribution as a tool for analyzing crash data characterized by a large amount of zeros." *Accident Analysis & Prevention* 43 (5): 1738-1742. https://doi.org/10.1016/j.aap.2011.04.004.

Lyon, C., Haq, A., Persaud, B. and Kodama, S.T. 2005. "Safety performance functions for signalized intersections in large urban areas: Development and application to evaluation of left-turn priority treatment." *Transportation research record* 1908 (1): 165-171. https://doi.org/10.1177/0361198105190800120.

Mannering, F.L., Shankar, V. and Bhat, C.R. 2016. "Unobserved heterogeneity and the statistical analysis of highway accident data." *Analytic methods in accident research* 11: 1-16. https://doi.org/10.1016/j.amar.2016.04.001.

Park, E.S., Park, J. and Lomax, T.J. 2010. "A fully Bayesian multivariate approach to before–after safety evaluation." *Accident Analysis & Prevention* 42 (4): 1118-1127. https://doi.org/10.1016/j.aap.2009.12.026.

Park, J., Abdel-Aty, M. and Lee, J. 2016. "Use of empirical and full Bayes before–after approaches to estimate the safety effects of roadside barriers with different crash conditions." *Journal of safety research* 58: 31-40. https://doi.org/10.1016/j.jsr.2016.06.002.

Pawlovich, M.D., Li, W., Carriquiry, A. and Welch, T. 2006. "Iowa's experience with road diet measures: Use of Bayesian approach to assess impacts on crash frequencies and crash rates." *Transportation research record* 1953 (1): 163-171. https://doi.org/10.1177/0361198106195300119.

Perfater, M.A. 1983. "Motorists'reaction to exclusive-permissive left-turn signal phasing." *Transportation Research Record* (926). http://onlinepubs.trb.org/Onlinepubs/trr/1983/926/926-002.pdf.

Persaud, B., Lan, B., Lyon, C. and Bhim, R. 2010. "Comparison of empirical Bayes and full Bayes approaches for before–after road safety evaluations." *Accident Analysis & Prevention* 42 (1): 38-43.

Rusli, R., Haque, M.M., Afghari, A.P. and King, M. 2018. "Applying a random parameters Negative Binomial Lindley model to examine multi-vehicle crashes along rural mountainous highways in Malaysia." *Accident Analysis & Prevention* 119: 80-90. https://doi.org/10.1016/j.aap.2018.07.006.

Rusli, R., Haque, M.M., King, M. and Voon, W.S. 2017. "Single-vehicle crashes along rural mountainous highways in Malaysia: an application of random parameters negative binomial model." *Accident Analysis & Prevention* 102: 153-164. https://doi.org/10.1016/j.aap.2017.03.002.

Russo, B.J., Savolainen, P.T., Schneider IV, W.H. and Anastasopoulos, P.C. 2014. "Comparison of factors affecting injury severity in angle collisions by fault status using a random parameters bivariate ordered probit model." *Analytic methods in accident research* 2: 21-29. https://doi.org/10.1016/j.amar.2014.03.001.

Sacchi, E. and Sayed, T. 2015. "Investigating the accuracy of Bayesian techniques for before–after safety studies: The case of a “no treatment” evaluation." *Accident Analysis & Prevention* 78: 138-145. https://doi.org/10.1016/j.aap.2015.03.006.

Shahdah, U., Saccomanno, F. and Persaud, B. 2014. "Integrated traffic conflict model for estimating crash modification factors." *Accident Analysis & Prevention* 71: 228-235. https://doi.org/10.1016/j.aap.2014.05.019.

Srinivasan, R., Council, F., Lyon, C., Gross, F., Lefler, N. and Persaud, B. 2008. "Safety effectiveness of selected treatments at urban signalized intersections." *Transportation Research Record* 2056 (1): 70-76. https://doi.org/10.3141/2056-09.

Srinivasan, R., Lyon, C., Persaud, B., Baek, J., Gross, F., Smith, S. and Sundstrom, C. 2012. "Crash modification factors for changes to left-turn phasing." *Transportation research record* 2279 (1): 108-117. https://doi.org/10.3141/2279-13.

Tahir, H.B., Haque, M.M., Yasmin, S. and King, M. 2022. "A simulation-based empirical bayes approach: Incorporating unobserved heterogeneity in the before-after evaluation of engineering treatments." *Accident Analysis & Prevention* 165: 106527. https://doi.org/10.1016/j.aap.2021.106527.

Wang, X. and Abdel-Aty, M. 2007. "Right-angle crash occurrence at signalized intersections." *Transportation Research Record* 2019 (1): 156-168. https://doi.org/10.3141/2019-19.

Wang, X. and Abdel-Aty, M. 2008. "Modeling left-turn crash occurrence at signalized intersections by conflicting patterns." *Accident Analysis & Prevention* 40 (1): 76-88. https://doi.org/10.1016/j.aap.2007.04.006.

Zhang, L., Prevedouros, P.D. and Li, H. 2005. "Warrants for protected left-turn phasing." *Transport Res Rec* 1073 (1): 28-37. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.489.29&rep=rep1&type=pdf.