Spectrum Sensing Optimisation for Dynamic Primary User Signal

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***Abstract*—We propose a multi-layer spectrum sensing optimisation algorithm to maximise sensing efficiency by computing the optimal sensing and transmission durations for a fast changing, dynamic primary user. Dynamic primary user traffic is modelled as a random process, where the primary user changes states during both the sensing period *and* transmission period to reflect a more realistic scenario. Furthermore, we formulate joint constraints to correctly reflect interference to the primary user and lost opportunity of the secondary user during the transmission period. Finally, we implement a novel duty cycle based detector that is optimised with respect to PU traffic to accurately detect primary user activity during the sensing period. Simulation results show that unlike currently used detection models, the proposed algorithm can jointly optimise the sensing and transmission durations to simultaneously satisfy the optimisation constraints for the considered primary user traffic.**

I. INTRODUCTION

Cognitive radio is based on the concept of dynamic spectrum access, whereby non-licensed, *secondary users* (SU) are permitted to access spectrum owned by licensed, *primary users* (PU) as long as interference to PU transmission is minimal [1]–[3]. Spectrum sensing is crucial to cognitive radio, as the SU must detect the presence or absence of PU signals to decide if the SU can transmit on a given spectrum. Spectrum sensing must be performed periodically, forming a sensing cycle consisting of a *sensing period* followed by the *transmission period* [1], [3]. Greater SU throughput can be achieved by minimising the sensing duration while maximising the transmission duration. However, longer sensing period is required for greater detection performance. Thus the SU must achieve a trade-off between PU protection and SU throughput.

Spectrum sensing optimisation aims to find the optimal sensing parameters to maximise the optimisation objective while satisfying imposed constraints [2]–[4]. Analysis of PU traffic activity provides information regarding the duration and probability of observing PU states that can aid spectrum sensing. One common approach models PU traffic as a random process, which implies the PU can change between busy and idle states during a sensing cycle [2], [4]–[6]. The transmission period within a sensing cycle is typically longer than the sensing period, therefore existing studies have focused on the model of PU changing states during the transmission period [5], [7]. These studies however, assume the PU remains in a constant state during the sensing period as per conventional spectrum sensing (*static PU*) [1], [3]. This assumption is suitable for slow changing PU traffic such as TV broadcasts [14]; short sensing cycle relative to average PU traffic implies the probability of state changes during a cycle is low and occasional interference is tolerable. However, for fast changing PU traffic such as cellular traffic, public safety, WLAN and WiMAX [15]–[17], the probability of the PU changing states during the sensing period cannot be neglected. Current studies do not impose any constraints to guarantee the chosen signal model accurately reflects practical PU activity patterns. In particular, it is possible that the observed PU changes states within the chosen sensing duration (*dynamic PU*), implying that the conventional signal model of static PU is no longer valid. To the best of our knowledge, no spectrum sensing optimisation to date considers the scenario where PU changes state during *both* the transmission and sensing periods.

A dynamic PU that changes states during the sensing period will exhibit an unknown duty cycle, previously defined as the fraction of the sensing period occupied by the PU signal [8], [9]. Studies have demonstrated that performance degradations occur when a conventional detector formulated for static PU is used to detect a dynamic PU [8]–[11]. Our analysis in [9] also showed that traffic parameters and sensing durations considered in few optimisation studies ([4]–[6]) can result in the detector overestimating the probability of detection by 8% while underestimating the probability of false alarm by 26%. Inaccurate detection performance renders the integrity of sensing applications at risk. For example, sensing optimisation constraints are formulated based on calculated detection performance, implying the optimisation constraints may be violated without SU knowledge and the resulting optimisation performance is meaningless. If the PU is modelled as fast changing traffic, then sensing detector must be designed to detect a *dynamic* PU.

Limited numbers of studies have proposed to detect dynamic PU signals by modelling the PU to change states during the sensing period. For example, [12] proposed a likelihood ratio test for a dynamic PU, and [13] calculated a soft metric indicating the probability of presence of the PU. These dynamic PU detectors all have a common limitation in terms of its implication to spectrum sensing: a dynamic PU that changes states during the shorter sensing period will also change states during the relatively long transmission period. Therefore a decision on PU activity state during the sensing period cannot guarantee the state of the PU during the transmission period. Spectrum sensing for dynamic PU must actively account for state changes during both sensing and transmission period to protect PU while utilising all possible spectrum opportunities.

There is currently no common consensus on the sensing pa-

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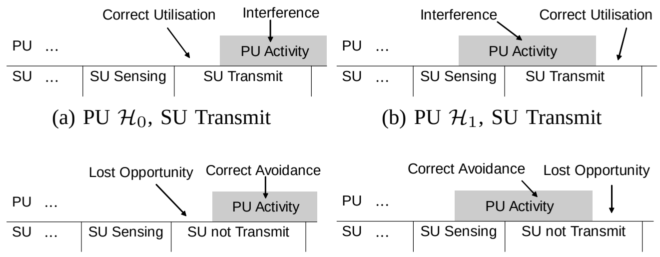
rameters, objectives and constraints that should be considered in spectrum sensing optimisation. Therefore it is difficult to compare optimisation performance across studies, as different approaches implement different system models and parameters. Some common sensing parameters include single channel sensing time [2], [4], [18], [19] and transmission duration [4], [5]. Sensing efficiency is the most common optimisation objective to describe SU throughput by comparing the durations of SU transmission period and sensing period. Greater sensing efficiency is desired, however different authors in [2], [4], [5], [18], [19] have varying definitions for sensing efficiency based on different system models and PU parameters.

There are two fundamental constraints that must be implemented regardless of the approaches to sensing optimisation, 1) interference to PU [4], [5], [7], [18], [19], and 2) lost opportunity of the SU [2], [4], [7]. Protection of PU is paramount, hence interference constraints specified by the PU must not be violated. Meanwhile, the SU must achieve greater spectral utilisation to guarantee the operation of the SU network. Interference is commonly implemented as a constraint; however lost opportunity is often simplified or unconstrained. Some approaches that simplify the constraints of interference and/or lost opportunity include assuming one metric is negligible or perfect and control the other [2], set both metrics to be equal [4], or constrain interference but allow lost opportunity to vary during optimisation [18], [19]. Optimisation through joint interference and lost opportunity constraint approach is rare [7]. In practice however, it is not sustainable for the SU to satisfy interference constraints at the expense of an inoperable SU network. Therefore *both* the interference and lost opportunity constraints must be separable but simultaneously met by the SU to guarantee the efficiency and integrity of the SU network.

In this paper we propose a multi-layer spectrum sensing optimisation algorithm for a dynamic PU signal model implementing a novel duty cycle based energy detector for spectrum sensing. We model the PU as a random process and allow the PU to change states during the sensing period *and* the transmission period to realistically reflect a fast changing PU traffic. Furthermore, we derive and impose joint constraints of interference and lost opportunity such that any changes in PU state during the sensing cycle is accounted for. The decision threshold of the proposed detector is optimised with respect to PU traffic to ensure accurate detection performance. Our optimisation algorithm allows individual control of the two constraints depending on different priority for the PU or SU and ensure that the constraints are simultaneously satisfied.

The remainder of this paper is organised as follows: Section II presents the system and optimisation model and describes the optimisation problem. Section III derives the optimisation constraints and the process of transmission period optimisation. Section IV proposes the duty cycle based detector for sensing period optimisation. Finally, Section V presents the optimisation algorithm in detail and discuss the simulation results and Section VI concludes this paper.



(a) PU $\mathcal{H}_0$, SU Transmit

(b) PU $\mathcal{H}_1$, SU Transmit

(c) PU $\mathcal{H}_0$, SU Not Transmit

(d) PU $\mathcal{H}_1$, SU Not Transmit

Fig. 1. Consequence of SU activity during transmission period. SU transmitting under $\mathcal{H}_0$ (a) and $\mathcal{H}_1$(b) leads to fractions of interference and correct utilisation. SU not transmitting under $\mathcal{H}_0$ (c) and $\mathcal{H}_1$ (d) results in fractions of lost opportunity and correct avoidance.

II. System & Optimisation Model

We consider the scenario of a single SU attempting to access a single PU channel similar to studies in [5], [6], [19]. A sensing cycle consists of a sensing period of duration τ followed by a transmission period of duration T_x. Based on the signal observed during the sensing period, SU transmits on the PU channel when the null hypothesis $\mathcal{H}_0$ is declared and does not transmits when the alternate hypothesis $\mathcal{H}_1$ is declared.

Conventional detection hypothesis assumes the PU to be static during the sensing period hence is not applicable for a dynamic PU. We redefine the detection hypothesis based on the state of the PU at the end of the sensing period as this state most closely represent the state of the PU at the start of the transmission period. $\mathcal{H}_0$ is declared when sensing ends while PU is idle, meaning PU will remain idle when transmission period starts hence SU can transmit. Vice versa, $\mathcal{H}_1$ is declared when sensing ends with PU being busy, hence SU must not transmit in the following transmission period. Our model appropriately chooses the duration of sensing and transmission periods and ensures that any changes in PU activity during the sensing cycle are accounted for.

PU traffic activity is modelled as a two-state random process where the PU alternates between ON and OFF states representing busy and idle periods. While this model is simple in nature, it is supported by experimental data such as [15], [16], [20] to represent fast changing traffic such as WLAN and cellular traffic and has been used in numerous spectrum sensing studies [2], [4], [11], [15], [16]. Similar to [2], [4]–[7], the holding times of ON and OFF states are i.i.d. exponentially distributed with departure rate r_1 and arrival rate r_0 respectively and the mean holding times of the two state are $\mu_1 = \frac{1}{r_1}$ and $\mu_0 = \frac{1}{r_0}$ respectively. The steady state probabilities of ON and OFF states are $P_1 = \frac{r_0}{r_1+r_0}$ and $P_0 = \frac{r_1}{r_1+r_0}$. As part of the spectrum occupancy analysis process prior to deploying SU network, the SU must extensively study PU traffic behaviour to identify candidate spectrum for cognitive use. Random process models and associated traffic parameter can also be established during this process.

Four examples of dynamic PU signals are shown in Fig. 1 where the PU signal contains both periods of ON states and OFF states during a sensing cycle. For this type of PU, detections performance based on the sensing period no long reflect the performance of spectrum sensing in terms of interference to the PU and lost opportunity of the SU during the transmission period. For example, if the SU transmits during the periods illustrated in Fig. 1, the SU will correctly utilise spectrum opportunity for the fraction of time where PU is OFF, but interfere with the PU for the duration that the PU is ON (Fig. 1a, 1b). On the other hand, if the SU does not transmit during these periods, the SU will correctly avoid interference when the PU is ON, but loses spectrum opportunity when the PU is OFF (Fig. 1c, 1d). Therefore we define two performance metrics as the constraints of our optimisation algorithm to ensure PU protection is correctly accounted for while achieving maximum spectrum utilisation.

Definition 1: The *interference ratio* R_I, is defined as the fraction of PU busy periods within the transmission period that is interfered by SU transmission, to be derived in (7). Interference occurs under two scenarios: 1) PU is initially OFF but changes to ON, while SU declares $\mathcal{H}_0$ and transmits (Fig. 1a), and 2) PU is initially ON but SU declares $\mathcal{H}_0$ (missed detection) and transmits (Fig. 1b). The PU specifies the maximum interference I_{max} that it can tolerate, and the SU must ensure $R_I \leq I_{max}$.

Definition 2: The *lost opportunity ratio* R_L, is defined as the fraction of PU idle periods within the transmission period that is not utilised by SU transmission, to be derived in (8). Lost opportunity occurs under two scenarios: 1) PU is initially OFF but SU declares $\mathcal{H}_1$ (false alarm) and does not transmit (Fig. 1c), and 2) PU is initially ON but changes to OFF, while SU declares $\mathcal{H}_1$ and does not transmit (Fig. 1d). The SU specifies the maximum lost opportunity L_{max} that it can sustain and the SU must constrain $R_L \leq L_{max}$ to ensure efficient operation of the SU network.

The proposed optimisation algorithm allows the interference and lost opportunity constraints to be individually specified and jointly constrained, unlike other methods that impose assumptions to simplify the constraints hence cannot ensure both metrics satisfy the PU and SU [2], [4], [18], [19]. Furthermore, our approach allows different priority to be applied depending on the interest of the PU or SU. If a PU demands greater interference protection then I_{max} can be set lower without affecting the constraint of lost opportunity. On the other hand if the SU requires better QoS for its operation then L_{max} can be set lower while still satisfying the interference constraint.

The signal model and constraints considered are more generic and suitable for the traffic patterns of fast changing PU. It is possible to consider such formulation in future regulations for this type of traffic pattern. According to current regulations [21], the scenario illustrated in Fig. 1d is not a lost opportunity as the SU is not permitted to access the spectrum when the PU is detected. Based on our model however, this scenario remains feasible as long as average interference to PU is constrained. Nevertheless, if this scenario remains infeasible in future regulation, the proposed system model and optimisation algorithm are still valid with only slight changes in the lost opportunity ratio.

Similar to [4], [5], sensing efficiency η is the optimisation objective defined as the duration of transmission period over the duration of the entire sensing cycle (1). The optimal sensing parameters, i.e. the optimal sensing duration $\hat{\tau}$ and optimal transmission duration $\hat{T}_x$, are obtained by solving the following optimisation problem,

$$\text{Find: } \hat{T}_x, \hat{\tau}$$

$$\text{Maximise: } \eta = \frac{T_x}{\tau + T_x} \tag{1}$$

$$\text{Subject to: } R_I \leq I_{max},\, R_L \leq L_{max}\,. \tag{2}$$

For the purpose of this study, we calculate the optimal sensing parameters while demonstrating the relation between sensing efficiency, transmission duration and sensing duration. Therefore η is calculated over the entire operating range of T_x and the optimisation problem is solved through two layers,

1) For values of transmission duration within the operating range, find the associated detection performance required to satisfy the optimisation constraints.
2) Find the minimum sensing duration that can satisfy the calculated detection requirements.

The optimal sensing parameters are thus calculated by finding the pair of sensing and transmission duration that result in maximum sensing efficiency. The operating range of T_x is defined by an upper maximum T_{xu} and lower maximum T_{xl}. T_{xu} is based on PU traffic parameters and derived in Section III, T_{xl} is derived based on detection parameters and explained in Section IV.

The variables and their probability distributions involved in the optimisation algorithm are numerically derived and evaluated, hence an explicit optimisation solution is not available. The proposed algorithm implements an iterative approach by computing the sensing period for all values of transmission durations within the operating range to find the maximum sensing efficiency. Minimum sensing duration is also iteratively calculated by the duty cycle detector. Furthermore, many expressions involve the integration operation, which are numerically evaluated. Therefore the computation cost depends on iteration resolution, where greater resolution increases optimisation accuracy but longer convergence time to find the maximum.

For fixed integration resolutions, increasing sensing duration resolution by a factor of N_1 will increase computation time by factor of N_1. Similarly, a N_2^{th} fold increase in transmission duration resolution also results in a N_2^{th} fold increase in computation time. The effect on resolution between the two sensing parameters is multiplicative, implying a simultaneous resolution increase by N_1 and N_2 results in total computation increase of $N_1 \times N_2$.

This study aims to demonstrate accurate optimisation performance by implementing high resolution hence may not be computationally efficient. As topics of future research, we aim to find the optimal resolution to minimise computation load while ensuring computation accuracy and also derive more efficient optimisation solutions.

III. Transmission Period Optimisation & Optimisation Constraints

A. Expected Duration of PU States in Transmission Period

The interval availability of the PU can be interpreted as the fraction of the transmission period that the PU is in either ON or OFF states [22]. The expected interval availability accounts for arbitrary number of PU state changes and is sufficient to describe the duration of ON and OFF states, depending on the rate parameters r_1, r_0 and transmission duration T_x. There are four combinations of interval availability of interest, A_{00}, A_{01}, A_{11} and A_{10}, where A_{ij} denotes the expected interval availability of state j given initial state i, with subscript 0 for state OFF and subscript 1 for state ON. The expression for each term is given as [22],

$$A_{00} = \frac{r_1}{r_1+r_0} + \frac{r_0}{(r_1+r_0)^2 T_x}\left(1-e^{-(r_1+r_0)T_x}\right) \tag{3}$$

$$A_{01} = 1 - A_{00} \tag{4}$$

$$A_{11} = \frac{r_0}{r_1+r_0} + \frac{r_1}{(r_1+r_0)^2 T_x}\left(1-e^{-(r_1+r_0)T_x}\right) \tag{5}$$

$$A_{10} = 1 - A_{11} \tag{6}$$

As the transmission period decreases and $T_x \to 0$, the probability of PU changing states also decreases, hence the interval availability converges to $A_{00} \to 1$, $A_{01} \to 0$, $A_{11} \to 1$ and $A_{10} \to 0$. However, longer transmission period increases the probability of PU state changes and for long transmission period such as $T_x \to \infty$, A_{00} and A_{11} decreases while A_{01} and A_{10} increases and the interval availability converges to the steady state probabilities of $A_{00} \to P_0$, $A_{01} \to P_1$, $A_{11} \to P_1$ and $A_{10} \to P_0$.

B. Optimisation Constraints

R_I and R_L are derived to satisfy the optimisation constraints of $R_I \le I_{max}$ and $R_L \le L_{max}$ based on PU interval availability (3)-(6) and detection performance of the spectrum sensing detector. Detection performance are measured by P_F and P_D; the probabilities of SU deciding PU is present under $\mathcal{H}_0$ and $\mathcal{H}_1$ respectively. The expressions for R_I and R_L are

$$R_I = \frac{P_1 A_{11}(1-P_D) + P_0 A_{01}(1-P_F)}{P_1 A_{11} + P_0 A_{01}}\,, \tag{7}$$

$$R_L = \frac{P_1 A_{10} P_D + P_0 A_{00} P_F}{P_1 A_{10} + P_0 A_{00}}\,. \tag{8}$$

For a given value of T_x (fixed A_{ij}), over-satisfying the constraints of I_{max} and L_{max} with reduced R_I and R_L requires smaller P_F and larger P_D, which further requires longer sensing period hence decreases sensing efficiency. Since maximum sensing efficiency is desired, the optimisation constraints are set to achieve $R_I = I_{max}$ and $R_L = L_{max}$, allowing for shortest sensing duration.

Solving (7) and (8) for P_F and P_D using $R_I = I_{max}$ and $R_L = L_{max}$ gives the detection requirements P_{Fopt} and P_{Dopt} that must be simultaneously achieved by the sensing detector to satisfy the optimisation constraints,

$$\begin{aligned} P_{Fopt} =& \frac{A_{10}(I_{max}-1)(A_{01}P_0 + A_{11}P_1)}{P_0(A_{11}A_{00} - A_{01}A_{10})} \\ &+ \frac{A_{11}L_{max}(A_{00}P_0 + A_{10}P_1)}{P_0(A_{11}A_{00} - A_{01}A_{10})}\,, \end{aligned} \tag{9}$$

$$\begin{aligned} P_{Dopt} =& \frac{A_{00}(1-I_{max})(A_{01}P_0 + A_{11}P_1)}{P_1(A_{11}A_{00} - A_{01}A_{10})} \\ &- \frac{A_{01}L_{max}(A_{00}P_0 + A_{10}P_1)}{P_1(A_{11}A_{00} - A_{01}A_{10})}\,. \end{aligned} \tag{10}$$

P_{Fopt} and P_{Dopt} are both functions of T_x, implying that to satisfy constant I_{max} and L_{max} at increasing T_x requires more stringent detection performance of smaller P_{Fopt} and larger P_{Dopt}. Based on this trend, there comes a point where $P_{Fopt}(T_x = T_{xF}) = 0$ and $P_{Dopt}(T_x = T_{xD}) = 1$. When $T_x > T_{xF}$ or $T_x > T_{xD}$, the calculated P_{Fopt} and P_{Dopt} become invalid. Therefore the upper maximum $T_{xu} = \min(T_{xF}, T_{xD})$ defines the maximum duration of T_x where the calculated detection performance remains valid. For the purpose of this study, we numerically calculate $P_{Fopt}(T_x)$ and $P_{Dopt}(T_x)$ until T_{xu} is found.

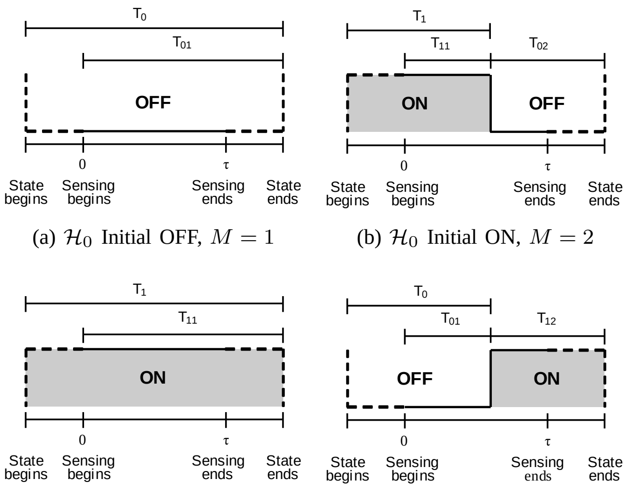
IV. Sensing Period Optimisation & Duty Cycle Based Detector

A duty cycle based energy detector is proposed for sensing period optimisation to calculate the minimum sensing period required to achieve the detection requirements calculated in Section III. Our analysis in [9] showed that longer sensing duration increases the probability of observing more state changes which degrades detection performance. On the other hand, enforcing the constraint of static PU model requires a very short sensing period, which may be insufficient to achieve the detection requirements. Therefore we impose a constraint on the sensing duration τ_{2max} such that the PU changes states *at most* once (maximum two observed states) during the sensing period.

Detectors proposed in the literature aiming to detect dynamic PU derive signal models where the PU changes states during the sensing period. However, these proposals do not specify the conditions where the PU traffic behaviour can be accurately represented by the proposed signal model. For example, without constraining the sensing duration with respect to PU traffic, it is possible that the number of state changes exhibited by the PU differ from what is considered by the system model. This results in the same hidden problem as conventional detectors, whereby practical activity patterns of a dynamic PU differs from the assumed static PU signal model, hence the system assumption is invalid. Therefore our imposed constraint on sensing period further ensures the proposed signal model will always accurately reflect PU behaviour as long as $\tau \le \tau_{2max}$.

A. Detection Model

The proposed signal model suggests that there can be four possible combinations of PU state transition depending on the initial PU state when sensing begins and the number of



(a) $\mathcal{H}_0$ Initial OFF, $M = 1$ (b) $\mathcal{H}_0$ Initial ON, $M = 2$

(c) $\mathcal{H}_1$ Initial ON, $M = 1$ (d) $\mathcal{H}_1$ Initial OFF, $M = 2$

Fig. 2. PU activity for different initial state and up to two observed states.

observed states M, as illustrated in Fig. 2. Using I_i to denote the event the PU is initially in state OFF ($i = 0$) and ON ($i = 1$) when a sensing cycle begins, $\mathcal{H}_0$ is declared when sensing ends with PU in OFF state and consists of scenarios for Fig. 2a (I_0, $M = 1$) and Fig. 2b (I_1, $M = 2$). $\mathcal{H}_1$ is declared when sensing ends in PU state ON and consists of scenarios for Fig. 2c (I_1, $M = 1$) and Fig. 2d (I_0, $M = 2$).

We briefly define the variables and expressions previously derived in [9] required for the proposed detector. Analytical expression are not available due to the complexity of the variables involved. Therefore the distribution of variables and associated expressions are computed and evaluated numerically. Variables illustrated in Fig. 2 are as follows:

- T_i: Duration of PU state when sensing begins, measured from last state transition to next state transition. Distribution given as $T_i \sim \exp(\mu_i)$.
- T_{i1}: Forward recurrence time of first observed PU state, measured from start of sensing to next PU state transition. Density function given as

$$P(T_{i1}) = \int_{T_{i1}}^{\infty} \frac{1}{T_i} P(T_i)\, dT_i\,. \tag{11}$$

- T_{i2}: Duration of second observed state, measure from end of first observed state to next state transition. Distribution given as $T_{i2} \sim \exp(\mu_i)$.

The probability of observing M PU states are derived as,

- $P_{M1}(\tau)$: Probability of observing strictly one state for given τ.

$$P_{M1}(\tau) = P(T_{01} > \tau)P_0 + P(T_{11} > \tau)P_1 \tag{12}$$

- $P_{M2}(\tau)|I_i$: Conditional probability of strictly two state for given τ and initial state i.

$$P_{M2}|I_0 = \int_0^{\tau} P(T_{12} > \tau - T_{01})P(T_{01})\, dT_{01}\,, \tag{13}$$

$$P_{M2}|I_1 = \int_0^{\tau} P(T_{02} > \tau - T_{11})P(T_{11})\, dT_{11}\,. \tag{14}$$

- $P_{M2}(\tau)$: Average probability of observing strictly two state for given τ.

$$P_{M2}(\tau) = (P_{M2}(\tau)|I_0)P_0 + (P_{M2}(\tau)|I_1)P_1 \tag{15}$$

Duty cycle D is defined as the fraction of the sensing period occupied by PU signal, i.e.

$$D_i = \frac{C_i}{\tau} \tag{16}$$

where C_i is the *cumulative duration* of ON states for hypothesis $\mathcal{H}_i$. The distribution of C_i cannot be accurately described by expected interval availability using (3)-(6) as only one or two PU states are observed during the sensing period and for a more realistic scenario, we model sensing to start randomly during PU's first state. C_{im} denotes the cumulative duration of ON states with initial state i and m number of observed states. The distribution of C_0 and C_1 can then be expressed as,

$$P(C_0 \le x) = \frac{P(C_{01} \le x)(P_{M1}|I_0)P_0}{(P_{M1}|I_0)P_0 + (P_{M2}|I_1)P_1} + \frac{P(C_{12} \le x)(P_{M2}|I_1)P_1}{(P_{M1}|I_0)P_0 + (P_{M2}|I_1)P_1}\,, \tag{17}$$

$$P(C_1 \le x) = \frac{P(C_{11} \le x)(P_{M1}|I_1)P_1}{(P_{M1}|I_1)P_1 + (P_{M2}|I_0)P_0} + \frac{P(C_{02} \le x)(P_{M2}|I_0)P_0}{(P_{M1}|I_1)P_1 + (P_{M2}|I_0)P_0}\,. \tag{18}$$

The distribution for C_{im} for each scenario in Fig. 2 is summarised below. Sensing period has duration τ, hence $0 \le C_{im} \le \tau$.

$$P(C_{01} \le x) = 1\,, \tag{19}$$

$$P(C_{02} \le x) = \int_{\tau - x}^{\tau} P(T_{12} \ge \tau - T_{01})P(T_{01})dT_{01}\,, \tag{20}$$

$$P(C_{11} \le x) = u(\tau)\,, \tag{21}$$

$$P(C_{12} \le x) = \int_0^{x} P(T_{02} \ge \tau - T_{11})P(T_{11})\, dT_{11}\,. \tag{22}$$

B. Duty Cycle Based Energy Detector

The constraint on sensing duration is imposed by ensuring the probability of observing more than two states at τ_{2max} is negligible. For a significance level of $p = 0.999$, we use (12) and (15) to numerically compute the maximum sensing period τ_{2max} to achieve

$$P_{M1}(\tau_{2max}) + P_{M2}(\tau_{2max}) = p \approx 1\,. \tag{23}$$

The proposed duty cycle detector integrates the distribution of duty cycle into the test statistic of the energy detector Y_D to compare with the decision threshold λ. For simplicity and by convention [2], [4]–[7], the effect of fading is ignored and both noise and PU signal are assumed to be zero mean, Gaussian distributed with variance σ_n^2 and $\sigma_s^2 = \gamma\sigma_n^2$ respectively, where γ is the SNR. Therefore the test statistic is χ^2 distributed with L degrees of freedom conditioned to the observed duty cycle D such that $Y_D|D \sim \chi_L^2 \sigma_n^2 (1 + \gamma D)$. $L = \tau W$ represents the sample size and is given as the time bandwidth product of the sensing period τ and channel bandwidth W. The

TABLE I
OPERATING PARAMETERS FOR DUTY CYCLE BASED ENERGY DETECTOR DEMONSTRATION.

T_x (ms)	P_{Fopt}	P_{Dopt}	P_{Dmax}	τ_{min} (ms)
41.2	0.0831	0.9508	0.9667	5.3
53.0	0.0781	0.9658	0.9658	9.5
59.5	0.0752	0.9743	0.9651	-

conditional density of Y_D depends on D and varies between observations, while the averaged density of Y_{D_i} for hypothesis $\mathcal{H}_i$ is numerically calculated by integrating $Y_{D_i}|D_i$ over the probability of D_i to get,

$$P(Y_{D_i}) = \int_0^1 P(Y_{D_i}|D_i)P(D_i)dD_i \,. \tag{24}$$

Detection performance of the detector, P_F and P_D, are the probabilities that $Y_D > \lambda$ for each of the detection hypotheses,

$$P_F = 1 - F_{Y_{D_0}}(\lambda) \,, \tag{25}$$

$$P_D = 1 - F_{Y_{D_1}}(\lambda) \,. \tag{26}$$

where $F_{Y_{D_0}}(\lambda) = P(Y_{D_0} \le \lambda)$ and $F_{Y_{D_1}}(\lambda) = P(Y_{D_1} \le \lambda)$ are the distribution functions of Y_{D_0} and Y_{D_1} respectively.

A *constant false alarm rate* (CFAR) approach is used to achieve a required probability of false alarm of P_{Fr}. Threshold λ and associated P_D are calculated from (25) and (26),

$$\lambda = F^{-1}_{Y_{D_0}}(1 - P_{Fr}) \,, \tag{27}$$

$$P_D = 1 - F_{Y_{D1}}\left(F^{-1}_{Y_{D0}}(1 - P_{Fr})\right) \,. \tag{28}$$

C. Minimum Sensing Period

A transmission period T_x requires the detector to achieve detection performance P_{Fopt} and P_{Dopt} to satisfy the constraint of I_{max} and L_{max}. To implement the CFAR approach we apply the required false alarm rate P_{Fopt} as P_{Fr} into (28) and calculate the associated P_D. For constant P_{Fr}, P_D is then dependent on τ. For the purpose of maximising sensing efficiency, $P_D(\tau)$ is numerically computed for $0 < \tau \le \tau_{2max}$ using (28) until the minimum sensing period τ_{min} that can achieve $P_D(\tau_{min}) = P_{Dopt}$ is found.

Conventional sensing detectors can achieve arbitrary detection requirements by increasing L. In the case of constant W, performance is improved by increasing τ as long as the PU remains static. However, for dynamic PU, increased τ will have two distinct effects: 1) Longer τ increases L, which improves detection performance, but 2) longer τ increases the probability of observing less favourable D, which degrades detection performance. To demonstrate this effect, operating parameters outline in Table I are used to calculate detection performance for $r_1 = 3$, $r_0 = 1$, $W = 10$ kHz, $\gamma = 0$ dB and presented in Fig. 3. For this PU traffic, $\tau_{2max} = 43$ ms.

As seen in Fig. 3, τ_{min} is the minimum sensing duration that achieves $P_D(\tau_{min}) = P_{Dopt}$. For example, $P_D(5.3\text{ms})$ at point A in the figure achieves $P_{Dopt} = 0.951$. We also see that for a given set of operating parameters, detection performance is limited by a maximum achievable P_{Dmax} as P_D decreases for longer τ. This implies that it may be possible that two

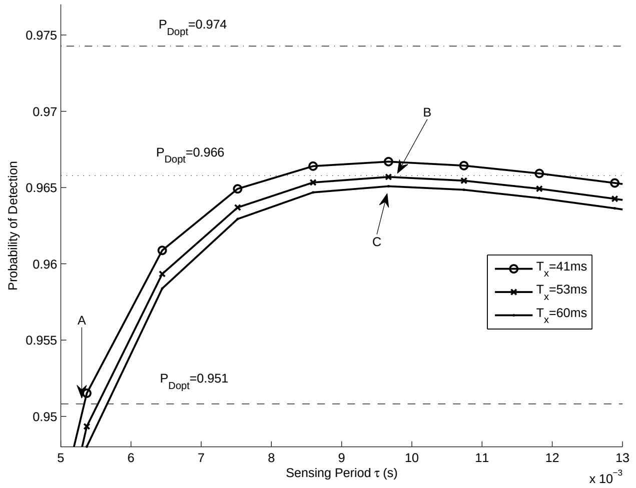


Fig. 3. τ_{min} computed as $P_D(\tau_{min}) = P_{Dopt}$ for each T_x. A: $\tau_{min} = 5.3$ ms, B: $\tau_{min} = 9.5$ ms, C: no valid τ_{min}. Detection performance limited to maximum achievable P_{Dmax}.

values of τ can satisfy P_{Dopt}, however the shorter τ_{min} is desired for greater sensing efficiency.

Longer T_x requires smaller P_{Fopt} and larger P_{Dopt} to satisfy the detection requirements while the achievable P_{Dmax} decreases. P_{Dmax} is dependent of P_{Fopt}, and together with P_{Dopt}, all are dictated by T_x. The lower maximum T_{xl} thus defines the maximum T_x where the calculated detection requirements can be practically achieved by the sensing detector such that $P_{Dopt}(T_x = T_{xl}) = P_{Dmax}(T_x = T_{xl})$. As indicated by point B in Fig. 3, $\tau_{min} = 9.5$ ms achieves $P_{Dopt} = P_{Dmax} = 0.966$. For $T_x > T_{xl}$, $P_{Dopt} > P_{Dmax}$ and no valid τ_{min} is possible as indicated by point C. The proposed optimisation algorithm stops once $T_x = T_{xl}$ ensuring that a valid τ_{min} is always computed.

The effect of different PU SNR is presented in Fig. 4 using $P_{Fopt} = 0.08$, $P_{Dopt} = 0.95$. Higher SNR results in stronger signal energy hence better detection performance. Therefore the required τ_{min} to achieve P_{Dopt} is reduced. For example, $\tau_{min} = 6.2$ ms at point A with $\gamma = -0.25$ dB, while $\gamma = -1.3$ dB requires $\tau_{min} = 13$ ms. In this demonstration, $\gamma = -1.75$ dB cannot achieve the detection requirements, as indicated by point C.

D. Optimisation Implementing Conventional Detector

For comparison, we also investigate the resulting R_I and R_L when the conventional detection model is implemented instead of the duty cycle based model. The conventional detector is invalid when detecting a dynamic PU during the sensing period, hence the error in detection will be propagated through the optimisation process.

The conventional detector can always achieve the detection requirements calculated in Section III by arbitrarily increasing the sensing duration. However, since PU exhibits duty cycle, the true detection performance achieved at this sensing duration will differ from the calculated detection requirements and can be found following the analysis in [9]. We then

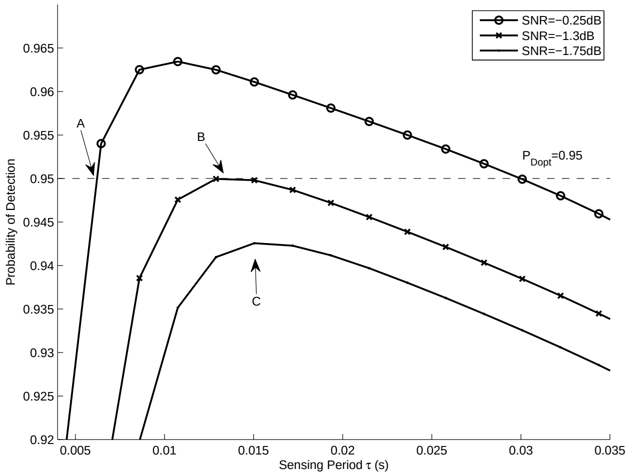


Fig. 4. Higher PU SNR (γ) provides stronger PU signal energy hence smaller τ_{min} can achieve the desired detection performance. A: $\tau_{min} = 6.2$ ms, B: $\tau_{min} = 13$ ms, C: no valid τ_{min}.

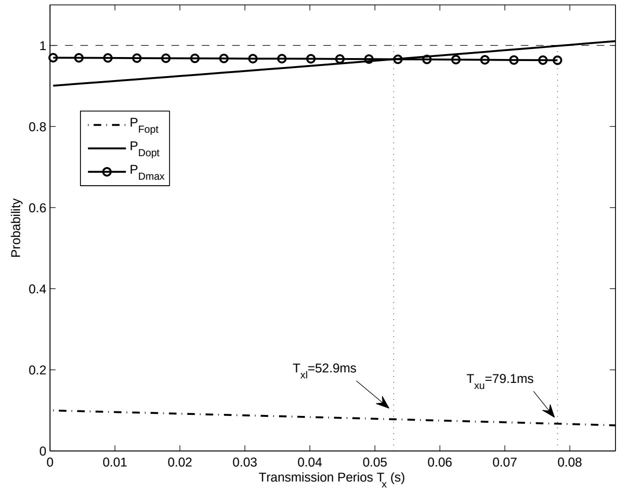


Fig. 5. Operating range of transmission duration showing detection requirements and detection performance.

TABLE II

OPERATING RANGE AND OPTIMAL SENSING PARAMETERS FOR VARIOUS PU TRAFFIC.

PU Traffic (r_1, r_0)	T_{xu} (ms)	T_{xl} (ms)	τ_{2max} (ms)	η_{max}	$\hat{T}_x$ (ms)	$\hat{\tau}$ (ms)
(1, 1)	232	166	76.2	0.9570	128	5.7
(1, 2)	118	74.7	53.7	0.9237	65.0	5.4
(1, 3)	79.1	42.0	43.0	0.8760	37.1	5.3
(2, 1)	118	86.7	53.7	0.9258	69.9	5.6
(3, 1)	79.1	51.7	43.0	0.8860	42.0	5.4

calculate the resulting interference ratio and lost opportunity ratio. Results in [9] showed that using the conventional model to detect a PU exhibiting a duty cycle results in reduced probability of detection and increased probability of false alarm. Therefore a spectrum sensing detector implementing the conventional detection model will violate the optimisation constraints.

V. OPTIMISATION & SIMULATION RESULTS

In this section we discuss the parameters and procedure of the proposed optimisation algorithm and simulation results. As mentioned earlier, this algorithm is designed to calculate optimal sensing parameters by presenting the trend of η across the entire range of T_x. There are potential to improve the efficiency of computation and are topics for future research.

When considering a practical implementation scenario, sensing optimisation does not need to be computed in real time. The optimal sensing parameters for each PU traffic model can be computed during the spectrum occupancy study and stored in a database. Depending on the active PU profile, the SU network can look up the database for associated sensing parameters.

A. Proposed Spectrum Sensing Optimisation

The upper and lower maximums of T_x (T_{xu} and T_{xl}) defines the operating range of the optimisation algorithm. An example of the relationship between the operating range, detection requirements and detection performance is presented in Fig. 5 using $r_0 = 3$, $r_1 = 1$. Increasing T_x requires more stringent P_{Fopt} and P_{Dopt} which decreases P_{Dmax}. $T_{xl} = 52.9$ ms defines the transmission duration where $P_{Dopt}(T_{xl}) = P_{Dmax}(T_{xl})$ and the sensing detector cannot practically achieve the detection requirements beyond T_{xl}. $T_{xu} = 79.1$ ms is the maximum limit of T_x where $P_{Dopt}(T_{xu}) = 1$. Therefore $T_{xl} < T_{xu}$ and the algorithm stops at $T_x = T_{xl}$.

The algorithm to solve the optimisation problem in (2) is described as follows:

1) Define operating range of T_x: $0 < T_x \leq T_{xu}$ as described in Section III-B
2) For a value of T_x, calculate P_{Fopt} and P_{Dopt} using (9), (10)
3) Find P_{Dmax} as described in Section IV-C
4) If $P_{Dopt} \leq P_{Dmax}$, find τ_{min} such that $P_D(\tau_{min}) = P_{Dopt}$
5) Calculate η using (1)
6) Repeat for all values of T_x
7) If $P_{Dopt} > P_{Dmax}$, stop iteration and find η_{max}

B. Simulation Results & Discussion

PU channel has constant bandwidth $W = 10$ kHz and the PU signal SNR is $\gamma = 0$ dB. We perform optimisation on PU with different traffic parameters as outlined in Table II. The optimisation constraints are chosen to be $I_{max} = L_{max} = 0.1$, which correspond to the common spectrum sensing benchmark of $P_D = 0.9$ and $P_F = 0.1$.

A summary of optimisation parameters and performance are presented in Table II. The optimal sensing parameters $\hat{T}_x$ and $\hat{\tau}$ have been computed to achieve maximum sensing efficiency η_{max}. $\hat{T}_x < T_{xl}$ since T_{xl} is the maximum T_x where the sensing detector can satisfy the optimisation constraints.

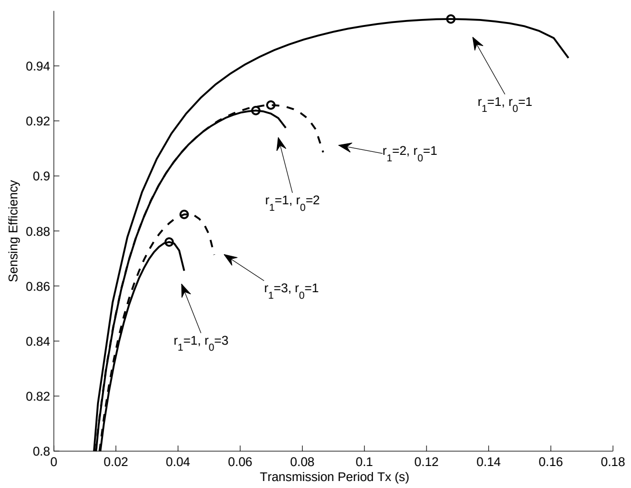


Fig. 6. Plotting sensing efficiency against transmission period for various PU traffic. Circle indicates maximum sensing efficiency with optimal transmission period.

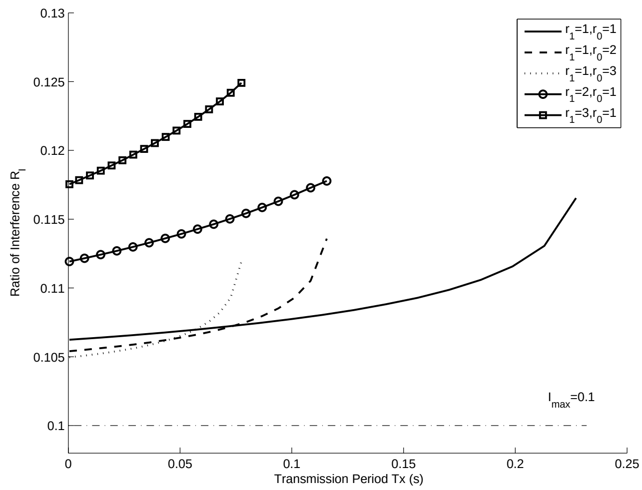


Fig. 8. Implementing conventional detection model calculates sensing parameters that violates the interference constraints.

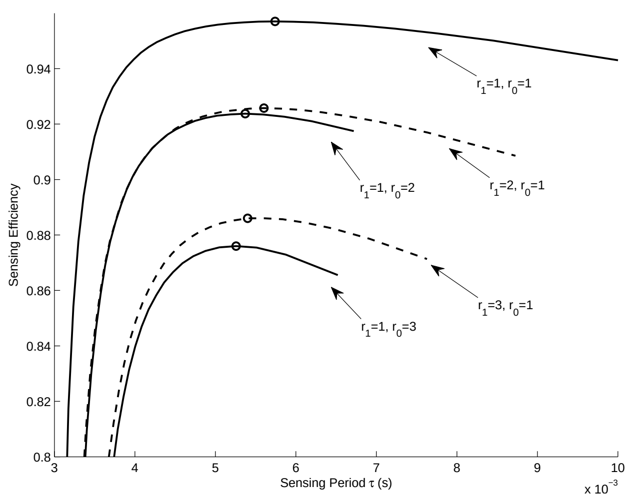


Fig. 7. Plotting sensing efficiency against sensing period for various PU traffic. Circle indicates maximum sensing efficiency with optimal sensing period.

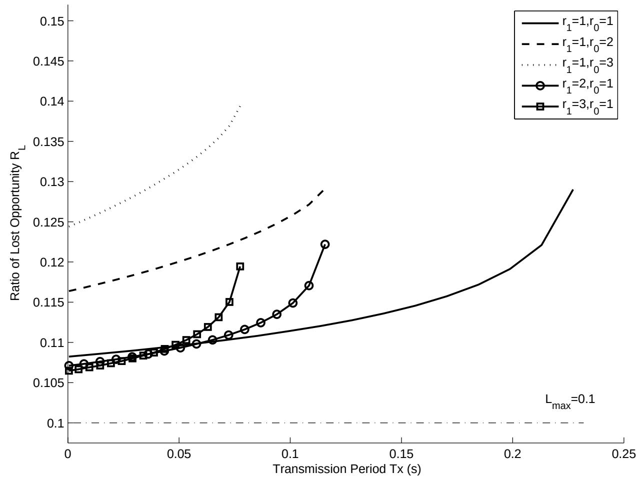


Fig. 9. Implementing conventional detection model calculates sensing parameters that violates the lost opportunity constraints.

Also, $\hat{\tau} < \tau_{2max}$ because the constraint on sensing duration is imposed for the accuracy of the signal model.

Fig. 6 and 7 presents the set of transmission period and sensing period that produces maximum sensing efficiency, indicated by circle markers. From Fig. 6, we see that longer T_x initially increases η until the maximum. Beyond this point, further increasing T_x requires longer τ which then decreases η. This phenomenon is also reflected by τ in Fig. 7. Fast changing PU traffic (represented by larger r_1 and r_0) leads to more prominent duty cycle effect. Therefore as r_1 and r_0 increases, the transmission period must be reduced to satisfy the optimisation constraints, which then reduces sensing efficiency.

The effect of implementing the conventional detection model is presented in Fig. 8 and 9. The calculated performance of the conventional detector differ from the achieved performance. Therefore the resulting optimisation leads to sensing parameters that violate the optimisation constraints as $R_I > I_{max}$ (Fig. 8) and $R_L > L_{max}$ (Fig. 9), recalling that the constraints were designed to be $I_{max} = L_{max} = 0.1$. This implies that optimisation using the conventional detector is meaningless as the constraints are not satisfied. We also see that R_I is more sensitive to changes in r_1, while R_L is more sensitive to changes in r_0.

VI. Conclusion

In conclusion, this paper presents a spectrum sensing optimisation to maximise secondary user sensing efficiency applicable to a more realistic scenario where primary user traffic is modelled to change state during both the sensing period and transmission period. Primary user changing states during the sensing period exhibits a random duty cycle, hence we implement a duty cycle based energy detector to accurately compute detection performance. Simulation results show that the proposed algorithm does indeed jointly optimise sensing duration and transmission duration to simultaneously satisfy both constraints of interference to the primary user and lost opportunity of the secondary user. We also proved that implementing a conventional detector which assumes primary user activity is constant during the sensing period will in fact violate the optimisation constraints under such realistic model.

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