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Empirical evaluation of public transport travel time variability

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Abstract

Although transit travel time variability is essential for understanding the deterioration of reliability, optimising transit schedule and route choice; it has not attracted enough attention from the literature. This paper proposes public transport-oriented definitions of travel time variability and explores the distributions of public transport travel time using the Transit Signal Priority data. First, definitions of public transport travel time variability are established by extending the common definitions of variability in the literature and by using route and services data of public transport vehicles. Second, the paper explores the distribution of public transport travel time. A new approach for analysing the distributions involving all transit vehicles as well as vehicles from a specific route is proposed. The Lognormal distribution is revealed as the descriptors for public transport travel time from the same route and service. The methods described in this study could be of interest for both traffic managers and transit operators for planning and managing the transit systems.

1. Introduction

Public transport travel time reliability has been considered as a quality of service measures as the reflections of passengers' anxiety in route choice and waiting; and operators' frustrations in scheduling (Martchouk et al., 2010). Reliability has been defined by a number of authors in literature (Strathman et al., 2000, Currie et al., 2012, Abkowitz and Engelstein, 1983) as the consistency or dependability in travel times. However, the literature focusing on the variability of public transport travel time, i.e. the variance in travel time itself is relatively limited.

Understanding the travel time variability is important for transit operators. First, travel time variability could be used to explain travel time reliability. Understanding the public transport travel time variability (PTTV), one can investigate the reasons for deteriorations of travel time reliability and explain the values of reliability index. For instance, exceeding travel time spending at previous corridor causes late arriving at the later corridors and extra waiting time for passengers. Second, according to the TCQSM (TRB, 2003), recovery time is the amount of time that is added to the expected running time of a public transport service to take into account the journey time variations and allow a short break before the next departure. When the variation in running times is less due to improved reliability, it may be possible to reduce recovery time or allow longer breaks or accommodate additional services. Third, travel time variability also plays an important role in traveller trip planning and route choice (Abdel-Aty et al., 1995). It has been noted that unreliable and highly variable travel time increases in the anxiety and stress (Bates et al., 2001) and is an additional cost to the traveller (Noland and Polak, 2002). Thus ridership is lost when service is perceived to be unreliable. A study in Oregon, US found that a 10% decrease in headway delay variation could lead to an increase of 0.17 passengers per trip per timepoint (Kimpel et al., 2000).

Notwithstanding the importance of PTTV, limited exploration has been done in the literature. Although the existing general definitions of travel time variability has been proposed (Bates et al., 1987, Noland and Polak, 2002), the definitions are better suited for measuring the private than public transport travel time variability. Some confusion still arises in the practical use of them in the public transport case due to the dissimilarities between the modes of transport. First, in the case of private transport travel time variability (CTTV), the average or probe vehicles' travel time is often used since the individual car travel time is generally not available. Conversely, individual travel times of transit vehicles are often accessible due to the needs of service monitoring. The possibility of tracking each vehicle opens other means of measuring the TTV. Vehicles running on the same study site or only vehicles of a specified route, or even a particular service can be of interests. Second, while the private transport vehicles could be considered homogenous to some extent, the public transport vehicles are noticeably different. Express transit routes are significantly faster than local ones by stopping at only selected stops. Within the same route, the scheduled travel time of different services could be varied between peak and off-peak periods. Therefore, the definition of PTTV could be similar or different to the CTTV.

This paper uses Transit Signal Priority (TSP) data to establish PTTV definitions and investigate its distribution. Firstly, the paper establishes the definitions of PTTV. Secondly, the probability distribution of public transport travel time is investigated, revealing the nature and shape of travel time. The findings of this research enable transport managers and researchers to better monitoring public transport variability.

2. Travel time variability in the literature

TTV has been defined in the literature as having three main types (Bates et al., 1987, Noland and Polak, 2002). *Vehicle-to-vehicle (or Inter-vehicle)* variability is the difference between travel times experienced by different vehicles travelling similar trips within the same time period. The signal delay, driver behaviour, conflicts with pedestrians, etc. are involved in this type of TTV. *Period-to-period (Inter-period or within-day variability)* is the variability between the travel times of vehicles travelling similar trips at different times on the same day. It is mainly caused by differences in the level of demand, occurrence of accidents and incidents, weather conditions, level of daylight and so on. *Day-to-day (or inter-day variability)* is the variability between similar trips on different days within the same time period. It is attributed to the fluctuations in traffic demand, weather, driver behaviours, and incidents. This definition is independent to the congestion effects. Within the same time period, a high demand system could have little fluctuations in day-to-day travel time variability if congestions are recurrent.

A considerable quantify of studies in CTTV has been conducted in the literature using different types of data sources such as floating cars (Chien and Liu, 2012), loop detectors (Oh and Chung, 2006) or even new sources of data such as Bluetooth (Martchouk et al., 2010). Standard Deviation (SD) travel time is one of the most popular measures of travel time variability, along with Buffer time/buffer index, T90-T10, etc. Coefficient of Variations (CV) of travel time is also used by some authors (Susilawati et al., 2011).

The literature on PTTV is relatively limited. Abkowitz and Engelstein (1983) predicted the running time and running time deviation by using linear regression analysis. Their model revealed that only the link length has significant impact on the day-to-day variability of public transport travel time. Mazloumi et al. (2010) adopted the definition of variability from Noland and Polak (2002) to explore the day-to-day PTTV in Melbourne, Australia using GPS data. The nature and pattern of variability were discussed by fitting bus travel time to Normal and Log-normal distribution, followed by a linear regression analysis to investigate the different impacts to the PTTV. Schramm et al. (2010) aimed to find the features that affecting TTV on Bus Rapid Transit Systems of many cities in the US. The study explored a ratio between peak and off-peak travel times. Moghaddam et al. (2011) proposed a procedure and empirical models for predicting the SD of bus travel time based on the average bus travel time, number of signalised intersection and a ratio between volume and capacity for an origin-destination path. Currie et al. (2013) analysed PTTV when measuring the impacts of transit priority using Automatic Vehicle Location data. The SD of travel time was studied in a linear regression analysis. To the best of the authors' knowledge, there is no study aimed to properly define the TTV of public transport and examine the use of different definitions of TTV on the same data.

3. Methodology and data

3.1 Data and case study site

The Bluetooth and TSP scanners are operated at major corridors in Brisbane for monitoring and assisting traffic and transit operations, respectively. The Bluetooth scanners capture the unique MAC addresses and timestamps of all Bluetooth-enabled devices within a range of 100 m. The TSP scanners identify the unique bus vehicle identification number of all buses that passing the intersection, along with their route numbers, timestamps and service scheduled start times using RFID technology. The service scheduled start time is the departure time from the first bus stop of the service as scheduled.

The corridor between an upstream (Coronation Drive/High Street) and a downstream (Coronation Drive/Cribb Street) signalised intersection is chosen as the case study site. The corridor length is 2.29 km with speed limit of 60 km/h and 3 shared lanes of public and private transport on each side. The study site is highly congested on both morning and afternoon peak periods. In this paper, we derive the method of bus and car travel time estimation and data cleansing from a previous work by Kieu et al. (2012). The travel time is the difference between observed timestamps at upstream and downstream intersections.

The analysis has been carried out on 4 months of Bluetooth and RFID data (March to June 2012) on inbound traffic. Only in-service buses (buses that are on operation) on working days (weekdays excluding Public Holidays and School Holidays) are considered in the study. The data of Route 411 is used on the analyses involving a single specified route. It is a timetabled local bus which connects University of Queensland and the Brisbane CBD. The frequency of Route 411 is every 15~20 minutes. It is one of the busiest routes along the study corridor. Route 411 follows the path illustrated on Figure 1.

3.2 Methodology

The variability of public transport travel time is explored in this study using two main approaches. First, the paper defines vehicle-to-vehicle, period-to-period and day-to-day TTV specifically for public transport. Each definition is illustrated and quantified in the case study using TSP data. The common knowledge of TTVs is also presented using Bluetooth data for better understanding of the proposed PTTV. The Bluetooth-enabled vehicles travel times are captured by Bluetooth scanners and their TTV is calculated. This variability might not be the true representative of all private transport vehicles' TTV because not all vehicles has enabled Bluetooth devices, but is an example of TTV from the dataset. The method of taking individual travel time from Bluetooth data has been explored by some authors in the literature (Kieu et al., 2012) and revealed as only 10% difference from the ground truth of travel time (Malinovskiy et al., 2011). Second, the distributions of bus travel time within different time windows are examined. The distribution of travel time describes the nature and shape of variability. For instance, a uniform distribution denotes no variability while a long tail skewed distribution shows that the bus could experience high and unreliable travel time.

The variability of travel time is measured using the Coefficient of Variation of travel time. CV measures the variation as a percentage of the mean, or the ratio of the SD to the mean.

$$CV = \frac{SD_{TT}}{\overline{TT}} \times 100 \qquad (1)$$

Where: CV = Coefficient of variation (%)

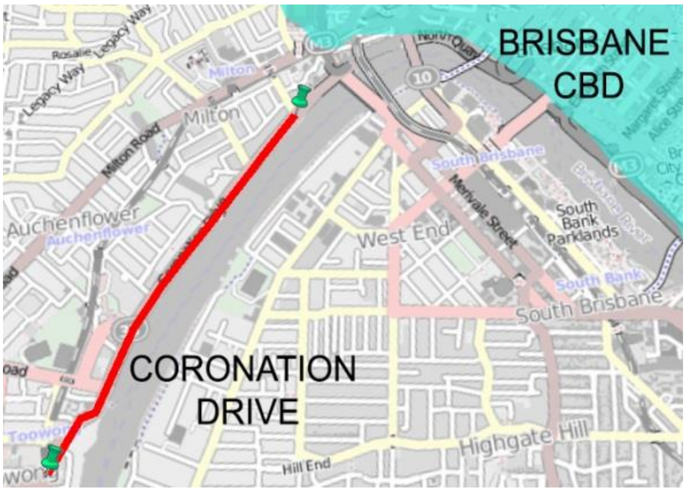
SD = Standard deviation of travel times

$\overline{TT}$ = Mean of travel times

CV is chosen as a meaningful comparison between two or more magnitude of variations. Because they are quantified in form of percentage, the variability from different sources are comparable even if they have different means or scales of measurements.

Study time period is involved in all definitions of TTV in the literature as “travelling within the same time period” (refer to section 2). If not explicitly stated otherwise, the time period for the TTV quantifications will be 30 mins. The period is short enough to be perceived by passengers as having uniform conditions and long enough to contain multiple buses in a study period.

Figure 1 Study site



4. Defining public transport travel time variability

This section aims to define the TTV of public transport based on the knowledge of TTV commonly used in existing studies (Noland and Polak, 2002, Bates et al., 1987).

4.1 Vehicle-to-vehicle travel time variability

Vehicle-to-vehicle travel time variability measures the differences in travel time experiences by multiple individual vehicles within the same period and same day that making similar journeys (Noland and Polak, 2002, Bates et al., 1987).

$$vehicle - to - vehicle\ TTV_{corridor} = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(TT_{i,p,d} - \overline{TT})^2}}{\overline{TT}} \qquad (2)$$

Here $TT_{i,p,d}$ represents the i^{th} individual travel time samples during time window *p* of day *d*; $\overline{TT}$ represents the average of *n* samples of $TT_{i,p,d}$

Because of the unavailability of individual vehicle travel times, this type of variability does not attract much attention in the literature of private transport. The Figure 2 illustrates the vehicle-to-vehicle variability of Bluetooth enabled vehicles along the study corridor on the first 3 days of May 2012.

The vehicle-to-vehicle PTTV is proposed based on the same definition of TTV. All the buses passing the study corridor within the studied day and time window (30 minutes) are considered. The PTTV between individual vehicles is showed in the Figure 3.

It could be noted from the Figure 2, Bluetooth enabled vehicles TTV are relatively stable. Some patterns can clearly be seen from the figure. During congestion build-up, the vehicle-to-vehicle TTV increases and then drops during morning peak due to jammed vehicles. In vehicle-to-vehicle PTTV case (Figure 3), although the same patterns can also be observed the fluctuations are really high among the periods. The variations of individual bus travel times could be any value between 0 to 40%. However, it might not reflect the transit performance since some routes are designed to be faster than others. Express buses only stop at a few selected bus stops along the corridor, while local routes service all the stops.

This definition of PTTV is only useful in a system where all services are relatively similar. It could be defined as ***vehicle-to-vehicle PTTV on corridor level.***

Figure 2 Vehicle-to-vehicle TTV of Bluetooth enabled devices on Coronation Drive, Brisbane.

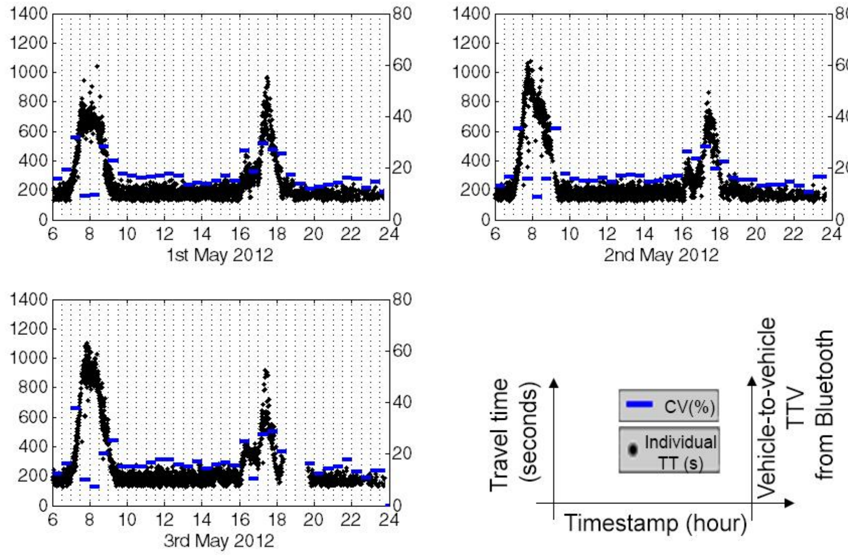
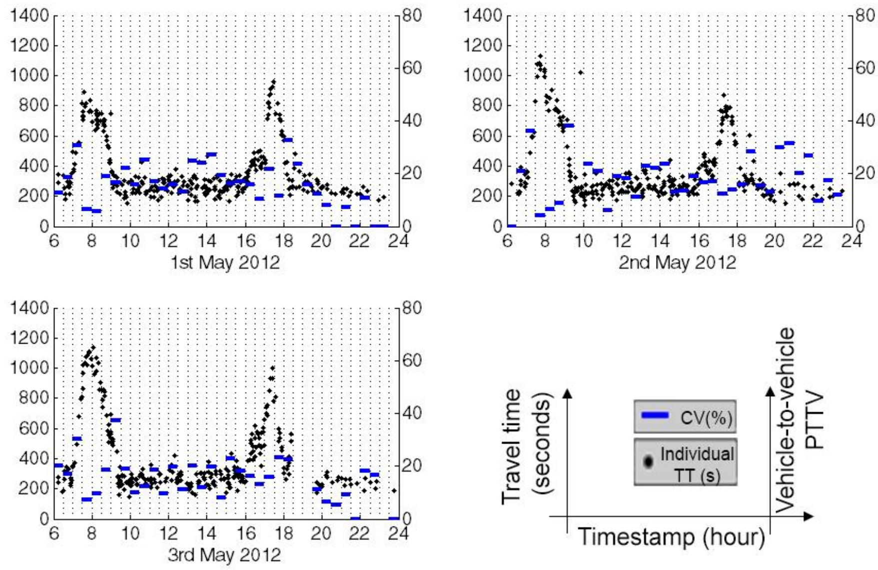


Figure 3 Vehicle-to-vehicle PTTV from multiple routes on Coronation Drive, Brisbane.



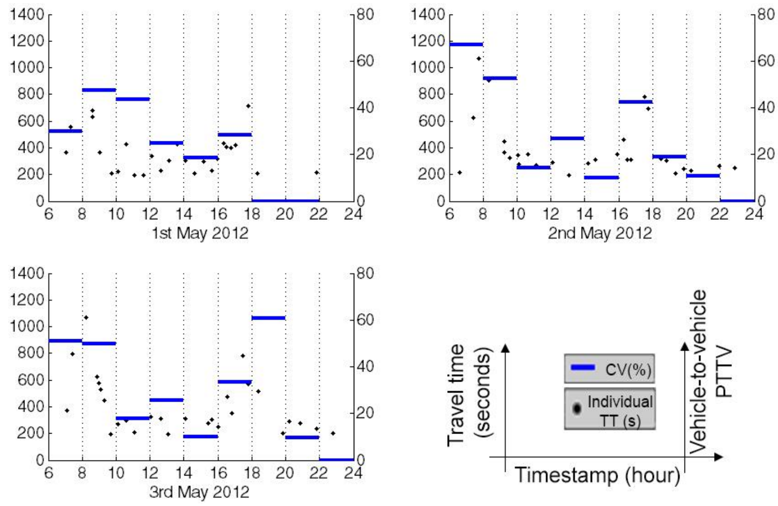
From the transit operators' point of view, there could be another definition of vehicle-to-vehicle PTTV that reflects the performance of a specific bus route. Only vehicles of the same route are considered. Here the meaning of "similar journeys" in the basic definition of vehicle-to-vehicle TTV is vehicles of the same route travelling on the same corridor at the same time of day. Short time periods would be inadequate in this case due to insufficient number of vehicles of the same route within a small amount of time.

$$vehicle - to - vehicle\ TTV_{route} = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(TT_{i,r,p,d} - \overline{TT})^2}}{\overline{TT}} \qquad (3)$$

Here $TT_{i,r,p,d}$ represents the i^{th} individual travel time samples of route *r* during time window *p* of day *d*; $\overline{TT}$ represents the average of *n* samples of $TT_{i,r,p,d}$

The Figure 4 illustrates an example of the definition on the first 3 days of May 2012. As an example, the chosen time window is 2 hours in order to accommodate at least 5 buses of the chosen bus route (route 411).
As expected when only route 411 is considered, the vehicle-to-vehicle PTTV is generally stable during off-peak period, and increases up to around 70% during peak periods. This definition of PTTV could be useful as a performance measure for transit operators. It helps find the variance between bus-to-bus travel time between intersections or stops. However, insufficient or highly varied number of bus in time windows could negatively affect the robustness of this definition, especially in the comparison between multiple periods. The definition could be named as ***vehicle-to-vehicle PTTV on route level***.

Figure 4 Vehicle-to-vehicle PTTV from route 411 on Coronation Drive, Brisbane.



4.2 Period-to-period travel time variability

Period-to-period travel time variability measures the differences in travel time of vehicles making similar trips at different times on the same day (Noland and Polak, 2002, Bates et al., 1987). CTTV is usually measured using average travel time values within a certain time window, or using the travel time of a floating car (which also represents the average travel time of the traffic flow) on the same study sites (Chien and Liu, 2012, Oh and Chung, 2006).

$$period - to - period\ TTV_{corridor} = \frac{\sqrt{\frac{1}{t}\sum_{p=1}^{t}(TT_{p,d} - \overline{TT})^2}}{\overline{TT}} \qquad (4)$$

Here $TT_{p,d}$ represents the p^{th} averaged travel time samples of all the vehicles during time window *p* of day *d*; $\overline{TT}$ represents the average of $TT_{p,d}$ on each of *t* time windows. The Figure 5 shows the period-to-period TTV of Bluetooth-enabled devices for the first 3 days of May 2012.
A similar definition of period-to-period PTTV could be derived by taking the average travel time of all vehicles within a time window. The average value of travel time acts as a bus probe to represent the bus travel time in general. One could argue that by taking the average, we also normalise some sources of variability such as signal delay, driver behaviour, etc. However, taking the average is necessary to avoid biased period-to-period PTTV. We consider an example of *n* individual buses with different travel times TT_1 to TT_n at period p_1. At period p_2, if *n* buses with exactly the same travel times TT_1 to TT_n are observed, in the definition of period-to-period TTV, the value of CV should be zero. If individual bus travel time samples are used for calculating period-to-period TTV, CV would be larger than zero. It is merely the vehicle-to-vehicle PTTV of two periods p_1 and p_2. If the average value $\overline{TT}$ of TT_1 to TT_n is used to calculate the PTTV, $\overline{TT}_{p1}$ would be equal to $\overline{TT}_{p2}$ and CV would

be zero. The Figure 6 illustrates the variability of bus travel time on the first 3 days of May 2012.

The definition of PTTV denotes the variance in average bus travel time on different periods of a day. While the average travel time within off-peak periods are relatively stable, the average travel time at peak periods are not the same every day. Therefore, the use of this definition is two-fold. First, period-to-period PTTV shows how much is the variability in public transport travel time between peak and off-peak periods of the studied day. Second, when comparing the variability of different day, a high CV denotes a congested traffic due to higher average travel time during peak periods. For instance, Figure 5 and Figure 6 show higher congestion in 2nd and 3rd of May 2012 (especially during morning peak) compared to 1st of May. The CVs of these two days are also noticeably larger than the one of the first day. This definition could benefit traffic managers in monitoring the variance of bus travel time between different periods in general. It could be defined as ***period-to-period PTTV on corridor level.***

Figure 5 Period-to-period TTV of Bluetooth enabled devices on Coronation Drive, Brisbane

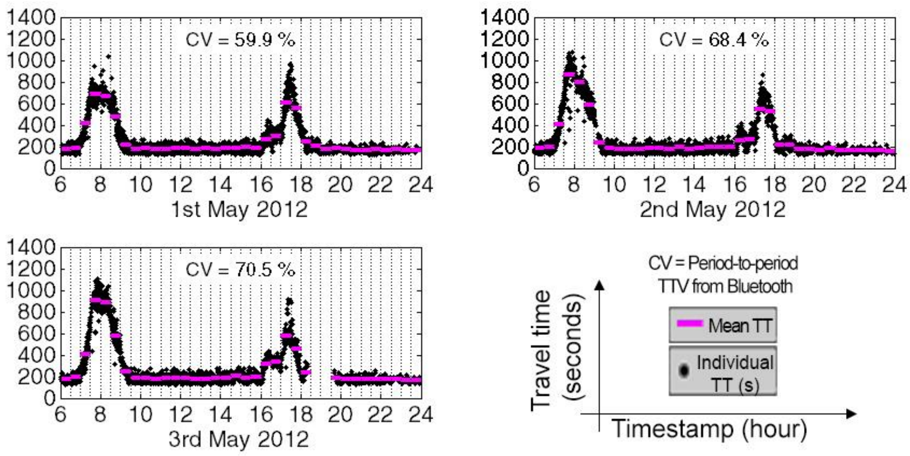
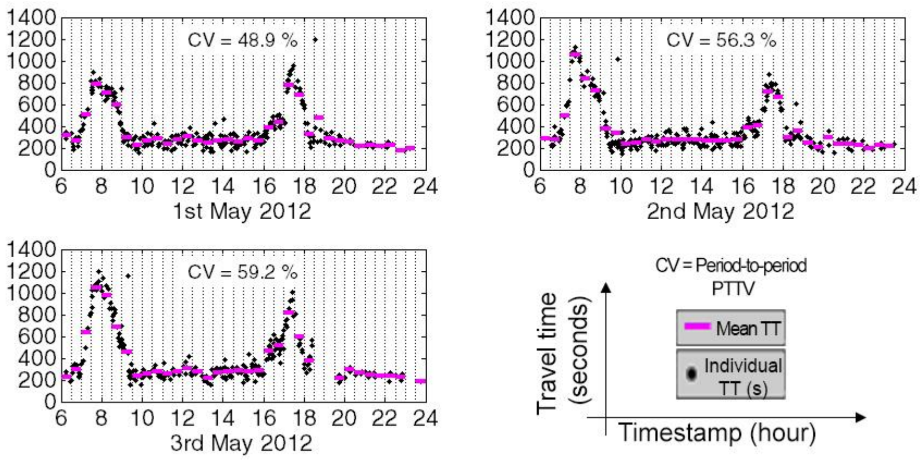


Figure 6 Period-to-period PTTV on Coronation Drive, Brisbane



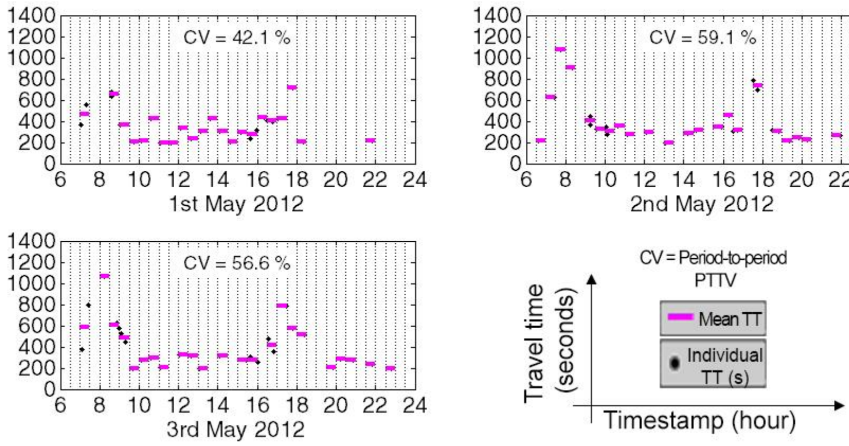
Another definition which benefits transit operators could be proposed if the data of only 1 specific bus route is used.

$$period-to-period\ TTV_{route} = \frac{\sqrt{\frac{1}{t}\sum_{p=1}^{t}(TT_{p,r,d}-\overline{TT})^2}}{\overline{TT}} \qquad (5)$$

Here $TT_{p,r,d}$ represents the p^{th} averaged travel time samples of vehicles from route *r* during time window *p* of day *d*; $\overline{TT}$ represents the average of $TT_{r,p,d}$ on each of *t* time windows.

The Figure 7 presents the route-level definition of period-to-period PTTV. This definition of period-to-period PTTV might be more suitable on the route level, where transit operators are interested in how stable is their service. This definition could be called ***period-to-period PTTV on route level.*** It shows the variations in travel time of buses from a specific route on different periods. Therefore, it could be used as a within-day stability indicator for the travel time of a route. The information might be of interest for transit operators. However, similar to the vehicle-to-vehicle case for route level, the accuracy suffers from the variations in number of buses in each time window.

Figure 7 Period-to-period PTTV of route 411 on Coronation Drive, Brisbane



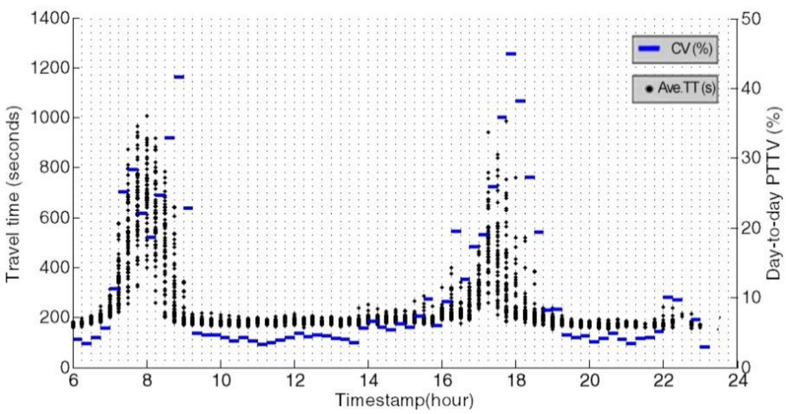
4.3 Day-to-day travel time variability

Day-to-day TTV measures the variability between travel times of vehicles on similar trips on different days within the same time period (Noland and Polak, 2002, Bates et al., 1987).
Day-to-day CTTV is often calculated in the literature by multiple day average travel time values within a certain time window, or using the travel time of floating cars on the same study sites (Chien and Liu, 2012, Oh and Chung, 2006).

$$day - to - day\ TTV_{corridor} = \frac{\sqrt{\frac{1}{D}\sum_{d=1}^{D}(TT_{d,p} - \overline{TT})^2}}{\overline{TT}} \qquad (6)$$

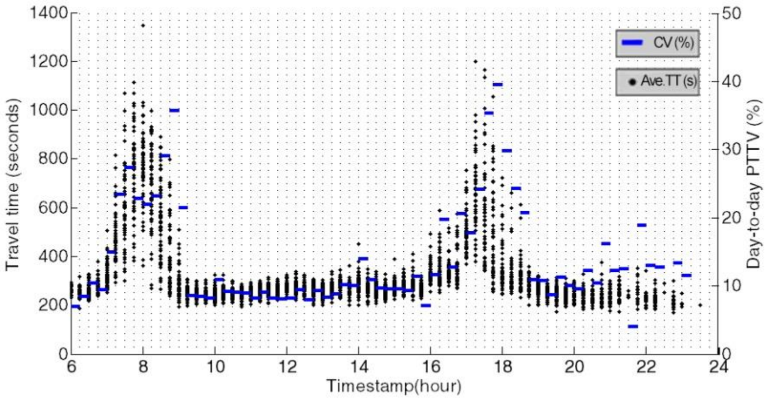
Here $TT_{d,p}$ represents the d^{th} averaged travel time samples of all the vehicles of day *d* during time window *p*; $\overline{TT}$ represents the average of $TT_{d,p}$ on each of *D* days. The Figure 8 illustrates the definition of day-to-day TTV through Bluetooth-enabled devices on all working days from March to June 2012 (in total 72 days).

Figure 8 Day-to-day TTV of Bluetooth enabled vehicles on Coronation Drive, Brisbane



Derived from the same definition, the day-to-day PTTV could also be measured using TSP data. An average of all buses within a study time window (30 mins) on a working day is calculated to represent the average bus travel time on day d_i, time window p_i. The average values are used by the same reason as discussed in the previous section. The Figure 9 shows the measurement of day-to-day PTTV on all working days from March to June 2012.
Day-to-day TTV from Bluetooth-enabled devices and buses share similar pattern. The CV is relatively low and stable during off-peak periods at around 5% for Bluetooth-enabled devices and 10% for bus. They are both sharply increasing and reducing during congestion built-up and dissipation, in which the CV could be up to 40% in the Bluetooth case and 35% in the bus case. It denotes that the average bus travel time could vary by up to 35% during these periods. The difference between traffic demands, weather condition, and incidents could be the main contributions to this variation. The TTV drops during morning peak period due to highly congestion. This definition of PTTV could be useful for traffic managers in monitoring the inter-day variability of bus travel time in general. However, the travel time is averaged, which could normalise some variability between individual buses such as driver behaviour and signal delay. This definition of variability could be named **day-to-day PTTV on corridor level.**

Figure 9 Day-to-day PTTV on Coronation Drive, Brisbane



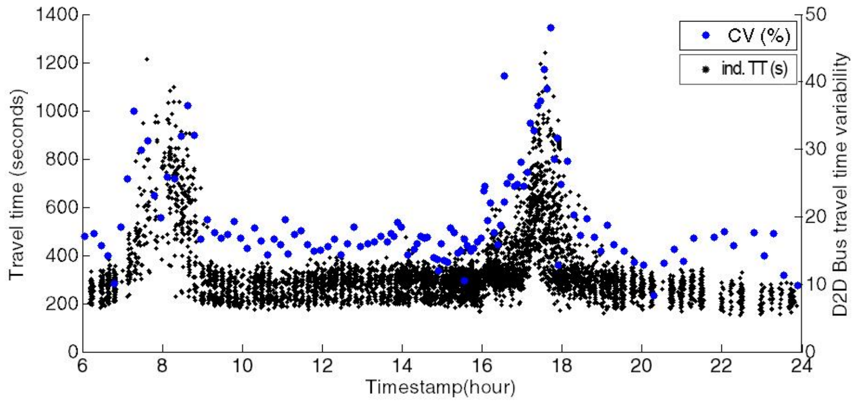
Utilise the availability of bus route number and service scheduled start time; another way to measure the day-to-day PTTV could be defined. This definition focuses on the variability of a specific bus route rather than all the buses. The buses of the same route and service scheduled start time are considered, because these buses are scheduled to travel time similarly.

$$day - to - day\ TTV_{service} = \frac{\sqrt{\frac{1}{D}\sum_{d=1}^{D}(TT_{d,r,s} - \overline{TT})^2}}{\overline{TT}} \quad (7)$$

Here $TT_{d,r,s}$ represents the d^{th} individual travel time sample of the bus of route *r* which is scheduled to start at time *s* of day *d*; $\overline{TT}$ represents the average of $TT_{d,r,s}$ on each of *D* days. The definition of PTTV is illustrated in the Figure 10. While the black points represent the individual bus travel time over 4 months data of working days, the blue points show the service scheduled start time and variability of the service. The service scheduled start time is earlier than the real observed time at the upstream intersection. This gap is approximately 3-5 minutes vary from case to case.
A similar pattern to the previous case could be observed. The day-to-day PTTV of services starting during off-peak periods is relatively low. It is the evidence that these services are reliable. However, as the congestion increasing, the variability is raising up. At a certain level of congestion, the day-to-day variability is getting lower because the vehicles are jammed with each other. This information is useful for transit operators in scheduling, particularly in deciding the timetable and recovery time. The reliability performance of each service on

multiple days could also be discovered by this definition. It could be defined as ***day-to-day PTTV on route level.***

Figure 10 Day-to-day PTTV of route 411 on Coronation Drive, Brisbane



4.4 Discussion of the public transport travel time variability definitions

For each type of TTV, two definitions have been proposed in this section. The first definition is similar to the private transport case, where the bus travel time in general is analysed and all buses are considered in the calculation. They facilitate the traffic managers to investigate the variability of bus travel time in general by considering all passing buses. The PTTV is explored in the comparison with other modes of transport. For instance, PTTV provides some insights on how much the attractiveness of public transport modes is compared to the private counterpart. Therefore, it is useful for macroscopic strategic planning purpose. The second definition utilises the additional information in public transport data: the route number and the service scheduled start time. The PTTV of a specified bus route or a service scheduled start time is proposed. The second definition is useful for transit operators to measure the performance, explore the sources of unreliability and optimise timetables. Hence, the application of the second definition is on a smaller scale compared to the first one, as a specific local or express transit route/service is of interest. The Table 1 summaries the proposed definitions.

Table 1 Summary of proposed definitions

Type of TTV	PTTV definition	Data used	Utilities	Weaknesses
Vehicle to vehicle	Corridor-level	Time window, day of study, individual travel time	Variation between individual public transport vehicles in general	The difference between express and local services might introduce some inaccuracy
	Route-level	Time window, day of study, specified route, individual travel time	Performance measures of a specific route	Results could be affected by the difference in number of bus on each time window
Period to period	Corridor-level	Day of study, average travel time within a time window	Variability between peak and off-peak public transport travel time in general, and daily congestion indicator	It only gives one value for a day, thus has limited information on the pattern of PTTV
	Route-level	Day of study, specified route, average travel time within a time window	Stability indicator for travel time of a specific route	Number of buses on each time window should be similar

Day To day	Corridor-level	Time window, days of study, average travel time within a time window on a day	Variations of public transport travel time in multiple days in general	The travel time is averaged, which could normalise some variability between individual buses
	Route-level	Days of study, specified route, specified service start time, individual travel time	Insights of the pattern of PTTV, a method for monitoring performance and optimising the time tables	Service number for each vehicle is required

The *vehicle-to-vehicle PTTV on corridor level* is useful if the bus services are relatively similar. If distinctive services such as local and express bus routes are all considered in the measuring the PTTV, the value of CV could increase from the fact that these routes are designed to operate differently. The *vehicle-to-vehicle PTTV on route level* is useful in uniform headway-based frequent system, where the buses are distributed equally and within short time gap. The *period-to-period PTTV on corridor level* and *on route level* is beneficial to investigate the variance between peak and off-peak travel time of public transport. The higher period-to-period variability also denotes a more congested day. However, the vehicle-to-vehicle and especially the period-to-period definitions of PTTV have limited information on the pattern of public transport TTV as the data of only a single day is explored.

The definition of *day-to-day PTTV on corridor level* reflects the variability of bus travel time in general on multiple days. All the buses within the same time window are considered by averaging their travel times. By examining the CV of the average travel times, the pattern of the daily public transport variability can be discovered. The journeys within off-peak periods are relatively reliable while the average travel time during peak periods are fluctuated, especially during congestion built-up and dissipation. However, by averaging the travel time of individual buses, we also normalise some sources of variability such as driver behaviours, signal delay, etc. Conversely, the definition of *day-to-day PTTV on route level* measures the variability in travel time of a specified service on a specified route. It does not only reveal the pattern of TTV, but also provide a method for monitoring performance and optimising the time table of transit services.

In comparison between the three types of travel variability for public transport, the definition of *day-to-day PTTV on route level* is the most intuitive in representing the pattern of variability, because the individual inter-day travel time samples are used. Moreover, from transit passengers point of view, the variability of travel time of the same service or route on multiple days would be more important than the variability among different vehicles (vehicle-to-vehicle PTTV) or among different periods of a day (period-to-period PTTV). Many transit commuters travel daily by a specific route/service at around a specific time of the day. Therefore, day-to-day PTTV (especially the route-level one) is more advisable for exploring the TTV of public transport than other two types of variability.

5. Day-to-day public transport travel time distribution analysis

Probability distribution of travel time shows the shape and nature of travel time variability. A uniform distribution denotes no variability, while a long tail skewed distribution shows unreliable travel time. It is useful to assume a parametric family to the public transport travel time because of 3 reasons.

First, travel time distribution is also essential in public transport planning. Resource allocation such as timetable is not often planned on the basis of average travel time, but on minimizing the opportunity that any journey would exceed the scheduled time (Moghaddam et al., 2011). Probability density function of public transport vehicle travel facilitates calculation of the probability that travel time would be higher than a predefined threshold. Given a network design and desired frequency setting, transit operators is interested in

minimizing the probability that their transit vehicles would experience the travel time higher than the predefined running time.
Second, travel time distribution is the essential part of any simulation study on public transport. In traffic simulation, one would be interested in reproducing number of vehicles that have the travel time follows a parametric probability distribution similar to the reality.
Third, the knowledge of public transport travel time distribution can also benefit various of statistical studies such as travel time prediction (Chien and Kuchipudi, 2003), route choice (Liu et al., 2004), dynamic scheduling {Hickman, 2001 #518} where transit vehicle travel time is modelled using the assumed family of distribution.
However, the literature on the distribution of public transport travel time is still limited and inconsistent. Existing studies fitted the distribution to both symmetric, i.e. Normal distribution (Taylor, 1982), and skewed distribution, i.e. Lognormal distribution (Andersson et al., 1979) or both of them (Mazloumi et al., 2010). Most of the transit travel time distribution analyses in the literature have just only explored common distributions at limited time periods.
A comprehensive seven-step approach is applied in this section. Based on the previous discussion, we focus on the probability distribution of bus travel from the *same route* and *service*, i.e. on **route level**. Most continuous distribution types test whether the distribution is symmetrical (e.g. Normal, t-location scale, Error) or skewed (e.g. Lognormal, Burr, Weibull) as the nature of bus travel time is continuous. The analysis neglects only the discrete types of distribution (e.g. Binominal, Negative binominal, Poisson) as well as Uniform and limited samples distributions (Triangular, Rectangular).
The list of 23 fitted distribution types includes: Beta, Birnbaum-Saunders, Burr, Chi-Squared, Dagum, Erlang, Error, Exponential, Frechet, Gamma, Generalized Pareto, Inverse Gaussian, Levy, Logistic, Log-logistic, Lognormal, Nakagami, Normal, Rayleigh, Rician, Pareto, t location-scale and Weibull.

5.1 Seven-step approach for public transport travel time distribution analysis

Travel time samples of each service are fitted by the Maximum Likelihood Estimation (MLE) method to estimate the parameters of each distribution. Most existing studies of travel time distribution analysis performed one of the three common goodness-of-fit tests named Chi-Squared; Kolmogorov-Smirnov (KS); and Anderson-Darling to find whether the data follows the specified distribution (hypothesis H0). Any p-value larger than the significance level (α) fails to reject H0 and the distribution is considered as significantly fitted with the data. However, this method has two key drawbacks (Durbin, 1973). Chi-squared requires large sample size, while the other two test goodness-of-fit of distribution with predefined parameters, if estimated from the data, then original critical values of the test are not valid.
Literature offers other approaches which solve the aforementioned problems, but they also have their own disadvantages. First, the information creation technique such as Bayesian Information Creation (BIC) (Schwarz, 1978) measures the relative quality of a statistical model by trading off the complexity (by considering the number of parameters) and goodness-of-fit of the fitted distribution (by considering the maximized value of the log-Likelihood). However, the BIC statistic is difficult to interpret. The fitted distribution with the lowest BIC is the “best” descriptor of the data, without a hypothesis testing to validate the goodness-of-fit. Second, the best fitted distribution could be examined graphically by using the probability plot, histogram, stem & leaf plots, scatter plot, or box & whisker plots. This graphical approach does not provide a reference point so that multiple distributions can be compared within multiple time periods. Third, recent goodness-of-fit tests such as Lilliefors test (Lilliefors, 1967) extends the KS test by determining the critical value by a Monte Carlo simulation, which enables estimating the distribution parameters from the data. However, the critical values table supports only a few limited types of distributions, restricting the study to a few selected distributions.
To overcome the limitation of the existing approach in travel time distribution analysis, this paper extends the Liliefors test to support all types of distribution by calculating critical

values using parametric bootstrap (D'Agostino and Stephens, 1986, Babu and Rao, 2004). The analysis follows the following steps.

Step 1: Consider each type of distribution. MLE method is employed to estimate distribution parameter(s) from bus travel time data.

Step 2: Generates random numbers from the studied distribution using the parameter(s) from Step 1.

Step 3: Use MLE to re-estimate distribution parameter(s) from the generated data. The parameter(s) is used to build theoretical cumulative distribution function (c.d.f) F(x) at each value of the generated data

Step 4: Calculate the KS statistics D_N^*, i.e., maximum difference between the empirical distribution function (e.d.f.) $S_N(x)$ from the generated data and the theoretical c.d.f. F(x) at each value of the generated data.

$$D_N^* = \max |S_N(x) - F(x)| \quad (8)$$

Where the e.d.f. SN(x) of N samples is calculated as in Equation (9).

$$S_N(x) = \begin{cases} 0 & x < x_1 \\ \frac{i}{N} & x_i < x < x_{i+1}, \quad i = 1, \ldots., N-1 \\ 1 & x_N \leq x \end{cases} \quad (9)$$

Step 5: Repeat Step 2 to Step 4 a large number of time (say 10000) to gather the set of D_N^*. Since significance level (α) equals 0.05, the 95th percentile of the set is chosen as the critical value DC.

Step 6: Compute the observed KS statistic DN between the e.d.f. from the bus travel time data and the c.d.f. at each sample of the bus travel time, and compare it to the simulated critical value. If DN < DC, the test fails to reject the null hypothesis that the distribution could describe bus travel time data.

For each service, the list of accepted distribution types can be found. However, the KS test with parametric bootstrap does not provide a measure to compare the goodness-of-fit at each service if multiple distributions are accepted. A hybrid approach was then used, in which the top five distribution types in the number of passed KS test were chosen as the five candidates for the descriptor of bus travel time. The BIC statistic test was then conducted to find the goodness-of-fit of each of 5 candidates to the bus travel time.

Step 7: BIC statistics are calculated for each candidate distribution from Step 6. The distribution type with lowest BIC is best fitted to the bus travel time data.

The BIC can be formulated as follows (Schwarz, 1978)

$$BIC = k \ln n - 2 \ln L_{max} \quad (10)$$

Where:

n = number of observations

k = number of parameters to be estimated

Lmax = maximized value of the likelihood function of the estimated distribution

This seven-step approach investigates the best descriptor of public transport travel time.

5.2 Analysis results and discussion

The Step 6 of the seven-step approach reveals five candidates of bus travel time distribution: Burr, Gamma, Lognormal, Normal and Weibull. While Normal and Lognormal are commonly used in public transport studies (Taylor, 1982, Andersson et al., 1979, Mazloumi et al., 2010), the other three are relatively new in the area. The KS test results and histogram of each distribution type, along with the lowest 2 distribution types in BIC statistics are presented in Figure 11. The following presents each aforementioned candidate to justify its overall goodness-of-fit to the bus travel time data.

The Burr distribution has been recently used in traffic engineering and believed as superior to Lognormal, Normal, Weibull and Gamma in modeling urban road travel time (Susilawati et al., 2011). Burr distribution is described as a heavy-tailed, highly-skewed distribution. Figure 11 shows that while the Burr distribution only passed the KS test at 18/37 services, it is the best fitted distribution where bus travel time is high left skewed and long tailed, especially with a range of travel time with very high occurrences. However, this travel time pattern appears in only a few services. The 5 services where Burr has the lowest BIC and the 4 services where it comes second are all in congestion build-up and dissipation periods.

The Weibull distribution has been widely used to represent travel time on arterial roads (Al-Deek and Emam, 2006) and especially on duration-related studies such as traffic delay durations (Mannering et al., 1994) and waiting time at unsignalized intersections (Hamed et al., 1997). Weibull distribution has been described as flexible representing right-skew, left-skew and also symmetric data. The BIC results show that Weibull is almost always within the top 2 in negative skewed travel time patterns. The histogram identifies that this pattern appears the at morning congestion period (because most vehicles experience a high travel time) and mid-peak period (when bus travel times are stable at around 300 seconds, but some faster buses form the “left tail” of the distribution). As the services with negatively skewed distribution are few in the dataset, Weibull distribution has the lowest BIC in only 3 services.

The Normal distribution has been suggested as the descriptor of bus travel time in a number of studies (Taylor, 1982, Mazloumi et al., 2010). It has a symmetric shape and its characteristics are thoroughly studied in statistics, which facilitates theoretical research. Figure 11 shows that Normal distribution is still a strong candidate as the descriptor of bus travel time in this study by passing the KS test in 20/37 services and having the lowest BIC statistics in 8 services, most of which are in mid-peak period.

The tests results indicate the Gamma and Lognormal distributions to be superior. The Gamma distribution has been long considered one of the first candidates for distribution of travel time. Polus (1979) believed that travel time on arterial road would “closely follow” a Gamma distribution, and for this reason Dandy and McBean (1984) suggested Gamma distribution as the descriptor for in-vehicle travel time. Lognormal distribution is conversely used to represent bus travel time (Andersson et al., 1979, Mazloumi et al., 2010) due to the flexibility and ability to accommodate skewed data.

While the Gamma distribution passes the KS test in 30/37 service, the Lognormal distribution passes in only one less services (29/37 services). Both of them are the optimal descriptors of bus travel time with moderate skewness and kurtosis (i.e. absolute value of skewness smaller than 1 and kurtosis smaller than 3). This type of travel time pattern is dominant in the dataset, which is why Gamma and Lognormal passed most KS tests.

Again both Lognormal and Gamma distribution have the capability to model both heavy and light tailed data, but the Lognormal is capable of representing higher skewed and longer tailed data, as it came with the Burr distribution in the top 2 lowest BIC statistic in several services. The BIC statistics also indicate that Lognormal is the best fitted distribution in more services than any other distribution types (14/37 services).

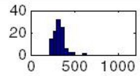
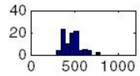
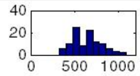
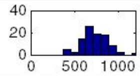
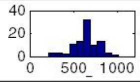
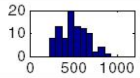
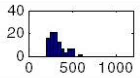
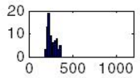
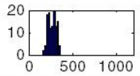
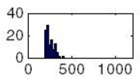
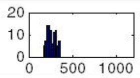
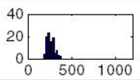
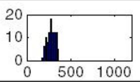
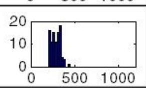
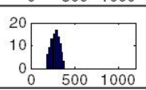
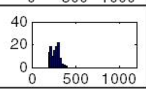
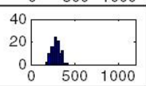
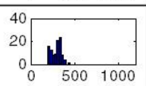
5.3 Hartigan Dip test for examining the bimodality

The histograms on Figure 11 show some signs of bimodality on two services before and after the morning peak period. Testing the bimodality is best conducted with the Hartigan Dip test. Dip statistics express the largest difference between the empirical distribution function and a unimodal distribution function that minimizes that maximum gap (Hartigan and Hartigan, 1985). If the p-value of the test is more than the significance value (chosen as 0.05), the data is concluded as unimodal distributed.

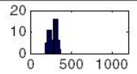
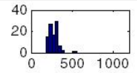
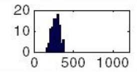
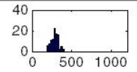
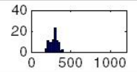
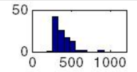
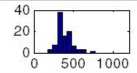
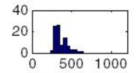
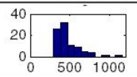
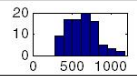
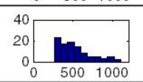
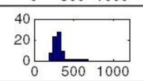
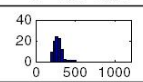
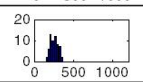
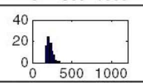
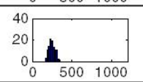
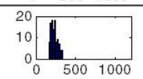
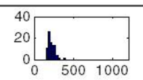
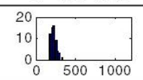
The results from Figure 11 show that although the bimodality is significant in only two services, the distributions of travel time in many services before and after the morning peak period are also nearly bimodal (p-value slightly larger than 0.05). The bimodality of travel time is mainly caused by a mixture of congested and uncongested population of traffic. Earliness or excessive congestion on some days, or generally the spread of congestions could be the main reason. These services are within the congestion build-up and dissipation periods, where speed could be free flow or congested depending on a day-to-day basis. The study was conducted on inbound traffic only, which means the pattern is not repeated for the afternoon.

Figure 11 Descriptive statistic of analysis results

Burr | Lognormal | Normal | Gamma | Weibull

Service	Histogram	# samples	Skew-ness	Kurto-sis	KS test with bootstrap resampling (1 for Accepted, 0 for rejected)					Hartigan Dip test		Lowest BIC	2nd Lowest BIC
					Burr	Normal	Log normal	Weibull	Gamma	Dip statistic	p-value		
6:51		103	1.22	7.05	0	1	1	0	1	0.04	0.38	Lognormal	Gamma
7:08		93	0.83	3.81	1	1	1	1	1	0.06	**0.02**	Lognormal	Gamma
7:26		104	0.42	2.68	1	0	1	0	1	0.05	0.06	Gamma	Lognormal
7:46		95	0.19	4.03	0	0	0	0	0	0.03	0.97	Normal	Burr
8:05		91	-0.56	3.84	1	1	1	1	0	0.03	0.77	Weibull	Normal
8:25		91	0.20	2.57	0	1	0	1	1	0.04	0.21	Weibull	Gamma
8:45		92	1.00	3.39	1	0	1	1	1	0.03	0.7	Lognormal	Burr
9:05		79	0.58	2.25	1	1	1	1	1	0.02	0.99	Lognormal	Gamma
9:25		120	-0.01	1.94	0	1	1	1	1	0.05	**0.04**	Normal	Gamma
9:55		109	0.82	3.11	1	1	1	1	1	0.03	0.55	Burr	Lognormal
10:25		93	0.18	2.03	1	1	1	1	1	0.04	0.28	Gamma	Lognormal
10:55		109	0.53	2.56	1	0	1	1	1	0.04	0.14	Lognormal	Gamma
11:25		96	-0.23	2.18	0	0	0	0	0	0.04	0.44	Weibull	Normal
11:55		94	0.24	2.55	0	1	1	0	1	0.04	0.55	Gamma	Lognormal
12:25		108	-0.02	2.23	0	0	0	0	0	0.03	0.82	Normal	Gamma
12:55		110	0.21	2.48	1	0	1	0	1	0.03	0.46	Gamma	Lognormal
13:25		113	0.12	2.66	0	0	0	0	0	0.04	0.31	Normal	Gamma
13:55		102	-0.05	2.26	1	1	0	1	1	0.04	0.13	Normal	Gamma

Burr | Lognormal | Normal | Gamma | Weibull

Service	Histogram	# samples	Skew-ness	Kurto-sis	KS test with bootstrap resampling (1 for Accepted, 0 for rejected)					Hartigan Dip test		Lowest BIC	2nd Lowest BIC
					Burr	Normal	Log normal	Weibull	Gamma	Dip statistic	p-value		
14:25		99	-0.22	1.98	0	1	1	1	1	0.04	0.31	Weibull	Normal
14:55		97	1.37	8.40	1	0	1	0	1	0.05	0.06	Lognormal	Gamma
15:10		102	-0.06	2.50	0	1	0	1	0	0.02	0.97	Normal	Weibull
15:25		100	0.00	2.70	0	1	0	0	1	0.03	0.82	Normal	Gamma
15:46		90	-0.06	2.61	1	1	1	1	0	0.03	0.97	Normal	Weibull
16:05		105	2.12	9.68	1	1	1	1	1	0.02	0.96	Burr	Lognormal
16:20		97	1.11	4.99	1	1	1	1	1	0.04	0.38	Burr	Lognormal
16:35		88	1.10	3.88	1	1	1	1	1	0.04	0.49	Burr	Lognormal
16:51		89	1.75	7.76	1	0	1	1	1	0.04	0.39	Lognormal	Burr
17:13		91	0.98	4.62	0	0	1	0	1	0.03	0.71	Lognormal	Gamma
17:33		101	0.99	3.21	0	0	1	1	1	0.03	0.8	Lognormal	Gamma
18:07		72	2.23	9.54	1	1	1	1	1	0.03	0.86	Burr	Lognormal
18:37		93	1.27	6.30	1	0	1	0	1	0.03	0.61	Lognormal	Burr
19:10		77	0.16	2.21	0	0	1	0	1	0.04	0.51	Gamma	Lognormal
19:40		98	0.97	4.44	0	0	1	0	1	0.03	0.95	Lognormal	Gamma
20:05		106	0.24	2.56	0	0	1	0	1	0.03	0.78	Gamma	Lognormal
20:40		105	0.55	2.65	0	0	1	0	1	0.03	0.84	Lognormal	Gamma
21:40		85	1.17	5.53	0	1	1	1	1	0.02	0.99	Lognormal	Gamma
22:40		79	0.71	3.37	0	1	1	0	1	0.04	0.72	Lognormal	Gamma

6. Conclusion

Literature on public transport is limited on reliability studies, notwithstanding the importance of travel time variability in explaining reliability, performance measurement, scheduling and route choice. This paper proposes three definitions of public transport travel time variability and a new approach for studying the distributions of day-to-day travel time variability.

The definitions of PTTV are based on the common definitions of TTV proposed by Bates et al. (1987). The three definitions are vehicle-to-vehicle, period-to-period and day-to-day TTV. Utilise the extra information of individual bus (route number and scheduled service start time), we proposed the definitions for corridor and route level. The new definitions suit better to public transport, and should be of interest to traffic managers and transit operators. The corridor-level variability considers all buses that passing by an urban intersection. It provides the information of variability of buses in general. The corridor-level PTTV could be useful for strategic traffic planning, especially in comparison between multiple modes of transport. The route-level variability considers the travel time from a specific bus route, or a specific scheduled service start time. It can be used for performance measurement, optimising recovery time and scheduling. Among the definitions, the day-to-day definition of route-level travel time variability is the most intuitive.

The distribution of public transport travel time reveals the pattern of variability within the buses of the same route and service. The comprehensive seven-step approach allows fitting most of the continuous types of probability distribution to all services. Each type of distribution is tested by both KS test with parametric bootstrapping and BIC method, identifying Lognormal distribution as the descriptor of day-to-day public transport travel time.

The definitions and modelling methods presented in this paper established a strong basis for future researches on travel time variability and distribution. The factors that causing the long tail of the public transport travel time distribution or high probability of travel time delay can also be explored in future studies. In the meantime, the findings of this paper are best suited for PTTV and service performance monitoring.

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