Precision Guidance with Impact Angle Requirements

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ABSTRACT

This paper examines a weapon system precision guidance problem in which the objective is to guide a weapon onto a non-manoeuvring target so that a particular desired angle of impact is achieved using the minimum control energy. In this paper two approaches for achieving the desired interception requirements are considered.

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EXECUTIVE SUMMARY

The weapon system guidance problem involves designing control algorithms to achieve weapon trajectories with predetermined interception conditions. The need to investigate precision guidance of weapon systems results from recent developments in weapon systems and sub-systems as well as changes in the deployment and operational philosophies.

New precision guidance problems have been posed that include requirements involving impact angle criteria in addition to the requirement for zero miss-distance. These new interception requirements facilitate optimisation of the warhead/fuzing and seeker components and hence maximise weapon effectiveness. This paper proposes and analyses two solutions to this precision guidance problem.

The key contribution of this paper is a model of the engagement that highlights the structural properties of weapon trajectories that achieve the precision guidance requirements. These structural properties are used to develop an *ad hoc* controller and are further developed to proposed an optimal control solution.

The two proposed control algorithms are examined both through theory and simulation studies for a variety of engagement configurations.

An improved understanding of the precision guidance problem is developed in this paper and this understanding will enable a more thorough and efficient response to the Department of Defence's requirements for assessment, evaluation, advice and modification of weapon systems.

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Jason Ford joined the Guidance and Control group in the Weapons Systems Division in February 1998. He received B.Sc. (majoring in Mathematics) and B.E. degrees from the Australian National University in 1995. He also holds the PhD (1998) degree from the Australian National University. His thesis presents several new on-line locally and globally convergent parameter estimation algorithms for hidden Markov models (HMMs) and linear systems.

His research interests include: adaptive parameter estimation, risk-sensitive techniques, non-linear filtering, applications of HMM and Kalman filtering techniques, adaptive and non-linear control, and precision guidance.

Contents

1 Introduction

Precision guidance of weapon systems is a computationally and conceptually demanding problem [2]. Historically, due to real-time computing constraints, major approximations in the control design process have been necessary. For example, control loops have historically been represented with the "classical three-loop" topology and linear control methods applied in each loop independently [2, 5, 6]. The guidance algorithms synthesised using this linearised multi-loop methodology have been successful in meeting historic performance requirements for the target and interceptor engagements [2, 5, 6]. However, the approximations introduced using this design procedure result in both design and operational constraints on missile systems including limits to precision of delivery and limitations on operational envelopes.

Advances in computational hardware and changes in operational requirements suggest that a re-evaluation of this linearised multi-loop design methodology is timely. Recent advances in missile sub-systems mean that modern guided weapons have significantly improved computational capacity [2].

The Guidance and Control (GC) group, Weapons Systems Division (WSD) has the responsibility to initiate R&D activities relating to navigation, guidance and control of autonomous and guided weapon systems. The need for R&D in these areas is a result of the recent developments in weapon systems and sub-systems as well as changes in the deployment and operational philosophies [2]. The ability to deliver warheads with miss distance accuracy and directional precision to targets leads directly to a need for new precision guidance control laws, and new precision guidance analysis and synthesis tools.

This paper considers a new precision guidance problem that involves ensuring not only that the miss distance is minimised but also that the angle of impact is controlled to a desired impact angle [2, 3, 4]. Even though the guidance problem against partially observed manoeuvring targets is of considerable interest; in this paper the target is assumed to be fully observed and non-manoeuvring.

Inclusion of an angle of impact guidance criteria is motivated by the improved fuzing capabilities of new warheads. These warhead capabilities allow more effective fuzing and delivery of the warhead if the interceptor can be delivered to the target at specific angles.

A standard finite-time-horizon optimal control approach to the problem is given in [2].

The key contributions of this paper are two solutions to the precision guidance problem. The key advancement presented in this paper comes from the noting that there is an invariant manifold in this guidance problem. This manifold is a surface in space that has the property that once the missile is on this surface the missile performing no control action will stay on this surface and will hit the target with the requirement terminal configuration. This invariant manifold leads to an *ad hoc* solution as well as allowing the problem to be formulated as a finite time horizon optimal control problem.

The paper is organised as follows: In Section 2 the precision guidance problem is proposed. In Section 3, an invariant manifold for the problem is identified and a control solution proposed. In Section 4 an optimal controller that minimises miss-distance, collision angle and control energy is given, then the sensitivity of the optimal controller to time-to-go errors is discussed. Finally, some simulation results are given in Section 5 followed by some conclusions in Section 6.

2 The Precision Guidance Problem

Guidance is the term used to describe the process of determining the desired engagement trajectory for an intercepting missile (or interceptor) against a target. The trajectory should be designed to ensure that the interceptor collides with the target according to some predetermined requirements. In this section a state-space description of an engagement (that ignores the missile time-constant) is introduced and then the interception requirements are described.

For simplicity consider an engagement defined in continuous time and let the following definitions be in a 2-D Euclidean reference frame. Let (x_t^I, y_t^I) and (x_t^T, y_t^T) be the position of the interceptor and target respectively. Then let (u_t^I, v_t^I) and (u_t^T, v_t^T) be the velocity of the interceptor and target respectively. The lateral acceleration of the interceptor is assumed to be the control variable and the target is assumed to be non-manoeuvring.

Observations of the engagement are commonly related to the relative dynamics of the interceptor and target, so we introduce the following state variable, $X_t := [x_t, y_t, u_t, v_t]$, where $x_t := x_t^T - x_t^I$ etc.

The dynamics of $(\bar{u}_t, \bar{v}_t)$ are given in [2], hence the dynamics of the state can be expressed as follows

$$\begin{aligned} \frac{dX_t}{dt} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} X_t + \begin{bmatrix} 0 \\ 0 \\ -\sin(\theta_t^I) \\ \cos(\theta_t^I) \end{bmatrix} \bar{u}_t \\ \frac{dX_t}{dt} &= AX_t + Bu_t \end{aligned} \tag{2.1}$$

where $\theta_t^I = \tan^{-1}(v_t^I/u_t^I)$ is the interceptor heading angle, and $\bar{u}_t$ is a scalar input representing the lateral acceleration demanded in the interceptor.

Further, let $V_I := \sqrt{(u_t^I)^2 + (v_t^I)^2}$, $V_T := \sqrt{(u_t^T)^2 + (v_t^T)^2}$ and the target heading angle $\theta_0^T := \tan^{-1}(v_0^T/u_0^T)$ be known.

Remarks

1. This state-space model can be expanded to allow for target manoeuvres but this needlessly complicates the guidance problem considered in this paper. The control laws proposed here are for guidance against non-manoeuvring targets and further examination of the influence of manoeuvres is needed before applying them against such targets.

The terminal phase of an engagement is usually modelled in terms of the line-of-sight angle and rate, λ and $\dot{\lambda}$, range and range rate, R and $\dot{R}$, closing velocity, V_c, and time-to-go, t_{go}. The first five of these quantities can be determined from the engagement state as follows:

$$\begin{aligned} \lambda &= \tan^{-1}(y_t/x_t) \\ \dot{\lambda} &= \frac{1}{R^2}(x_t v_t - y_t u_t) \\ R &= \sqrt{x_t^2 + y_t^2} \\ \dot{R} &= \frac{1}{R}(x_t u_t + y_t u_t) \\ V_c &= -\dot{R}. \end{aligned}$$

If the engagement is close to a collision course engagement then the time-to-go can be approximated as follows:

$$t_{go} \approx \frac{R}{V_c}. \tag{2.2}$$

To describe the final state of the engagement we let $t_f = t + t_{go}$ denote the warhead initiation time (when the interception requirements are closest to being satisfied) and θ_D denote the desired collision angle (commonly $\theta_D = 90°$). For simplicity we assume that both the interceptor and target body axes are in the direction of their velocity vectors.

The Guidance Problem

The guidance problem is to choose control actions $\bar{u}_t$ such that the interceptor is directed, with minimum control effort, to collision with the target and that collision occurs so that the body axis of the interceptor is at a given angle to the body frame of the target.

That is, the optimal control problem is the following:

$$\min_{\{\bar{u}\}} \quad J(\{\bar{u}_t\}) \qquad \text{subject to (2.1) where}$$

$$J(\{\bar{u}_t\}) = \int_{t_0}^{t_f} \bar{u}_t dt + \gamma_R R(t_f) + (V_t - V_{ref})' \Gamma_\theta (V_t - V_{ref}) \tag{2.3}$$

where γ_R as a weighting factor, Γ_θ is a weighting matrix, $V_t = [u_t, v_t]'$ is the relative velocity, and $V_{ref} = \bar{V}_c[\cos(\bar{\alpha}), \sin(\bar{\alpha})]'$ is the desired relative velocity at interception. Here, $\bar{\alpha}$ is the desired interception vector angle determined by θ_D and θ_t^T (as shown in the next section) and $\bar{V}_c = \sqrt{V_T^2 + V_I^2 - 2V_T V_I \cos(\theta_D)}$ is the desired closing velocity at intercept.

This index contains as special cases several interesting types of guidance problems previously considered. For example, with $\Gamma_\theta = 0$ the usual proportional navigation guidance problem is recovered but with $\theta_D = 0$, $\Gamma_\theta \neq 0$, the optimal rendezvous guidance problem is recovered (see [6] for more information about these problems).

In the next section we will further examine the engagement and discover some properties that lead to an intuitive solution. Before proceeding consider the following typical engagement.

2.1 A Typical Engagement

A moderately difficult precision engagement is shown in Figure 1. In this engagement, a normal proportional navigation guidance law will not result in impact with the directional requirements satisfied. To achieve all of the engagement requirements the interceptor will have to turn away from the target for a period to ensure a precision impact is achieved. Consider an engagement that commences at a distance of 5000 m. The interceptor is traveling at a velocity of 1000 ms^{-1} in a direction 36° (measured in a counter-clockwise direction) from a line drawn between the interceptor and target. The target is travelling at a velocity of 660 ms^{-1} in a direction of 120° from the same line. Hence, the initial conditions are $(x_0^I, y_0^I, u_0^I, v_0^I) = (0, 0, 1000\cos(36°), 1000\sin(36°))$ and $(x_0^T, y_0^T, u_0^T, v_0^T) = (5000, 0, 660\cos(120°), 660\sin(120°))$ where distances are in units of m and velocities are in units of ms^{-1}.

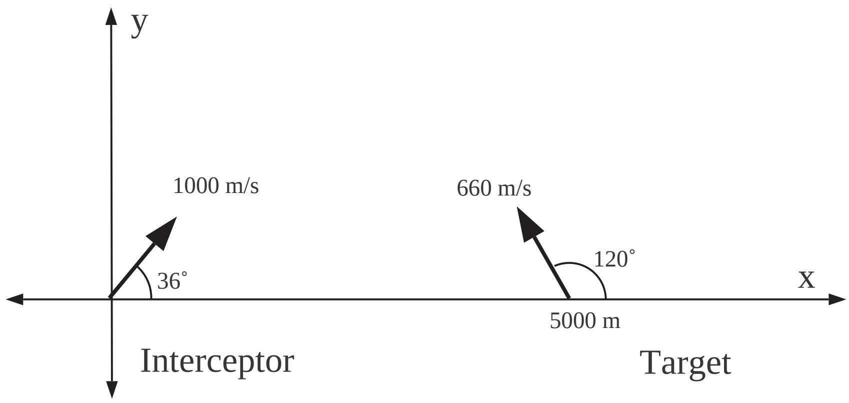

Figure 1 (U): A typical engagement. The interceptor and target are roughly heading towards the same point.

The precision guidance problem is to determine the sequence of lateral acceleration commands required to control the interceptor to collision with the target at a desired impact angle.

3 The Invariant Manifold Solution

In this section we examine the engagement and propose a solution motivated by an invariant manifold. Figure 2 shows an engagement with the target heading parallel to the y-axis direction ($\theta_0^I = 90°$) and the interceptor heading parallel to the x-axis direction and located on a collision course with impact angle 90°. In this engagement, no control action is required to ensure that collision with impact angle 90° occurs and hence $\bar{u}_t = 0$ is the optimal control action if $\theta_D = 90°$.

All collision course trajectories that require no control action to achieve collision with a

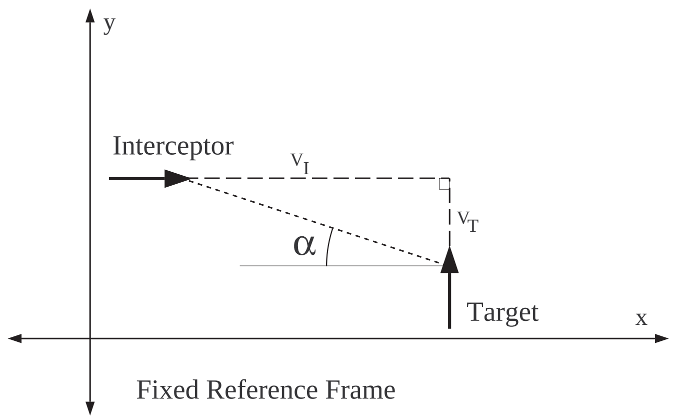

Figure 2 (U): Desired engagement configuration when $\theta_D = 90°$. The angle α (the desired line-of-sight) can be determined from the relative speeds of the platforms.

impact angle of 90° can be found by scaling, rotation and reflection of the engagement shown in Figure 2. It then follows that collision courses that achieve an impact angle of 90° are defined by two features: a line-of-sight angle of $\alpha = -\tan^{-1}(V_T/V_I)$ and a line-of-sight rate of zero, ie. $\dot{\lambda} = 0$.

Collision course engagements that achieve collision with a general impact angle θ_D and general target orientation θ_0^T can similarly be drawn. When the target orientation is $\theta_0^T = 90°$, the desired line-of-sight to achieve a general impact angle, θ_D, is $-(\sin^{-1}\left((V_T/\bar{V}_c)\sin(\theta_D)\right) + \theta_D - 90°)$ where $\bar{V}_c = \sqrt{V_T^2 + V_I^2 - 2V_T V_I \cos(\theta_D)}$. Hence, for a general target orientation (θ_0^T) and desired impact angle (θ_D), an interceptor is on a collision course satisfying the impact angle requirements if and only if the line-of-sight angle is constant and has the value $\bar{\alpha} := -(\sin^{-1}\left((V_T/\bar{V}_c)\sin(\theta_D)\right) - \theta_0^T + \theta_D)$.

To further understand the properties of the required collision consider the same engagement (with $\theta_D = 90°$) shown in the relative reference frame in Figure 3. This figure demonstrates that an interceptor on a collision course with the desired impact angle has a constant relative heading (measured clockwise from the negative x-axis) of α (or $\bar{\alpha}$). Hence, if both the line-of-sight angle to the target and the relative heading are α then $\bar{u}_t = 0$ is the optimal control strategy.

The two conditions shown in Figures 2 and 3 define a manifold in space described by the line-of-sight angle and relative heading of the interceptor. When the interceptor is in this manifold no control action is required to ensure the collision requirements are satisfied.

Further, this manifold is invariant because once on the manifold the interceptor remains in the manifold unless a control action is performed. It also follows that engagements which satisfy the collision requirements must involve trajectories that enter this manifold.

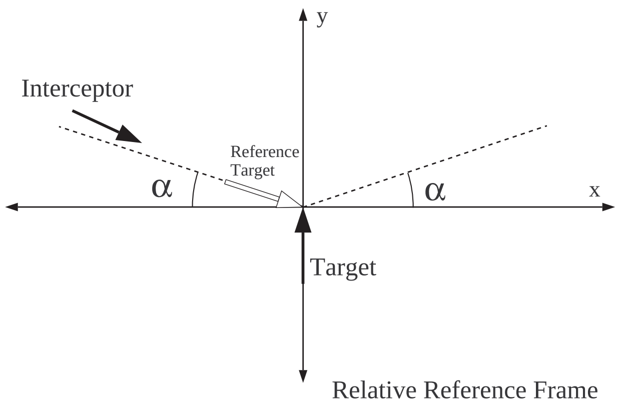

Figure 3 (U): The engagement in the relative reference frame. The angle α describes an invariant manifold. Once on this line (in a location and velocity sense) the interceptor will satisfy the engagement requirements without performing any control action. A reference target is also shown on the diagram.

This control problem can be now considered in a sliding model control or model reference control context (see [8, 9]). The intuitive idea would be to design a controller that drives both the line-of-sight angle and the relative heading of the interceptor to $\bar{\alpha}$. If these quantities can be driven to $\bar{\alpha}$ before the collision occurs then all the precision guidance requirements are achieved.

Consider the following controller:

$$\bar{u}_t = K_1(\lambda - \bar{\alpha}) + K_2(\theta_R - \bar{\alpha}) \tag{3.1}$$

where $\theta_R = \tan^1(v_t/u_t)$ is the relative heading angle, and K_1, K_2 are non-linear range-dependent gain terms. A controller of this structure may be able to drive the interceptor onto the invariant manifold. However, the performance of this type of *ad hoc* controller is very dependent on the choice of K_1 and K_2.

Simulation studies illustrate that it is possible to choose these gains so that a stablising controller is achieved (stablising for a large part but not all of the state space). The following controller has been demonstrated in simulation studies to work for a large variety of initialisation and engagement configurations (but it is not a unique choice).

$$\bar{u}_t = -(100\, rad \times ms^{-2}) \left(\frac{200}{D_h}(\lambda - \bar{\alpha}) + \frac{R_u}{D_\lambda}(\theta_R - \bar{\alpha}) \right) \tag{3.2}$$

where $D_\lambda = (\lambda - \bar{\alpha})^2$, $D_h = (\theta_R - \bar{\alpha})^2$ and $R_u = 10 + (1/20m)\min((R_t - 500m), 0)$. Note that the angles are measured in radians and the control $\bar{u}_t$ is in the units of ms^{-2}.

The intuitive idea in this controller is that for large ranges and when close to the correct line-of-sight angle the controller drives to minimise the relative heading error, but if close to the correct relative heading then the controller drives to minimise the line-of-sight error.

In the simulation section performance studies of this controller are presented. The key disadvantage of this *ad hoc* approach is that it can demand very large control inputs. In the next section an optimal control solution to this problem is presented.

4 An Optimal Control Solution

In this section an optimal control solution to the guidance problem specified in Section 2 is given. The development presented here follows the approach taken in [6] for solving the optimal control problem in both the proportional navigation and optimal rendezvous guidance problems. The control solution achieved is based on a linearisation about near collision course engagement geometries. The optimal controller that results is hence limited in application to situations where the terminal guidance problem starts with the interceptor pointing approximately at the collision point (say within $\pm$ 30°).

To develop a 2 state model of the engagement, assume that the interceptor and the target are heading towards a nominal collision point as shown in Figure 4. In this linearised model, the engagement is defined in terms of distances measured perpendicular to the nominal line-of-sight and angles are measured counter clockwise from the nominal line-of-sight.

Let x_1 be the target distance minus the interceptor distance (distances are measured perpendicular to nominal line-of-sight) and the actual line-of-sight angle (for small angles) is $\lambda_t = x_1/R$ where R is the range. Let $x = [x_1, x_2]'$ where $x_2 = \dot{x_1}$, then a 2 state linearised model can be written as

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \bar{u}_t^0 \tag{4.1}$$

where $\bar{u}_t^0$ is a control action perpendicular to the line-of-sight.

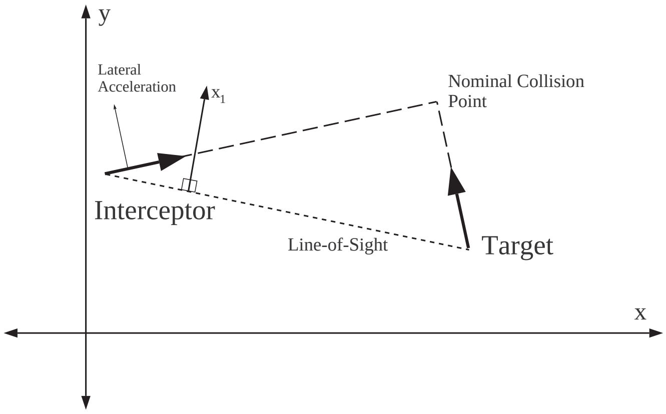

Figure 4 (U): Two-dimensional interceptor-target engagement geometry

The optimal control problem from Section 2 can be expressed as

$$\min_{\{\bar{u}\}} \quad J(\{\bar{u}_t\}) \quad \text{subject to (4.1) where}$$

$$J(\{\bar{u}_t\}) = \int_{t_0}^{t_f} \bar{u}_t^0 dt + \gamma_R x_1^2 + \gamma_\theta (x_2 - x_d)^2. \tag{4.2}$$

Here $x_d = V_I \sin(\theta_D)$ is the desired terminal lateral velocity. Although there is an ambiguity in this terminal velocity description, this ambiguity can be removed in implementation.

Theorem 1 *The solution to the optimal control problem defined by (4.2) is*

$$\bar{u}_t^0 = -\left[g_1 x_1(t) + g_2(x_2(t) - x_d)\right] \tag{4.3}$$

where

$$g_1 = \frac{\gamma_R t_{go}(\frac{1}{2} t_{go}\gamma_\theta + 1)}{t_{go}^4 \frac{\gamma_R \gamma_\theta}{12} + t_{go}\gamma_\theta + t_{go}^3 \frac{\gamma_R}{3} + 1}$$

$$g_2 = \frac{\frac{1}{3} t_{go}^3 \gamma_R \gamma_\theta + t_{go}^2 \gamma_R + \gamma_\theta}{t_{go}^4 \frac{\gamma_R \gamma_\theta}{12} + t_{go}\gamma_\theta + t_{go}^3 \frac{\gamma_R}{3} + 1}.$$

Proof: This is a standard optimal control problem and the proof presented here follows development given in [6].

The first-order necessary conditions for optimality are the following adjunct equations.

$$\dot{\lambda}_1 = 0, \quad \dot{\lambda}_2 = -\lambda_1, \quad \text{and} \quad \bar{u}_t^0 = -\lambda_2 \tag{4.4}$$

with the terminal conditions

$$\lambda_1(t_f) = \gamma_R x_1(t_f), \quad \text{and} \quad \lambda_2(t_f) = \gamma_\theta(x_2(t_f) - x_d). \tag{4.5}$$

Thus solutions for the adjunct variables are

$$\lambda_1(t) = \gamma_R x_1(t_f) \quad \text{and} \quad \lambda_2(t) = \gamma_R x_1(t_f)(t_f - t) + \gamma_\theta(x_2(t_f) - x_d). \tag{4.6}$$

Now substitution of $\bar{u}_t^0 = -[\gamma_R x_1(t_f)(t_f - t) + \gamma_\theta(x_2(t_f) - x_d)]$ into (4.1) and integrating from t_0 to t using appropriate initial condition information gives

$$\begin{aligned} x_1(t) &= \left[\frac{1}{2}\gamma_R(\frac{t}{3} - \frac{t_0^3}{3}) - \frac{1}{2}\gamma_R t_0^2(t - t_0)\right] x_1(t_f) - \frac{1}{2}\gamma_R(t - t_0)^2 t_f x_1(t_f) \\ &\quad -\frac{1}{2}\gamma_\theta(t - t_0)^2(x_2(t_f) - x_d) + x_1(t_0) + x_2(t_0)(t - t_0) & (4.7) \\ x_2(t) &= \frac{1}{2}\gamma_R x_1(t_f)\left((t^2 - t_0^2) - (t - t_0)\right) - \gamma_\theta(t - t_0)(x_2(t_f) - x_d) + x_2(t_0). & (4.8) \end{aligned}$$

Equations (4.7) and (4.8) can be evaluated at $t = t_f$ and then solved for $x_1(t_f)$ and $x_2(t_f)$. Then $\bar{u}^0$ can be written as affine in $x_1(t)$ and $x_2(t)$. This leads to the Theorem statement.

□

An interesting special case of controller (4.4) is the control problem where $\gamma_R \to \infty$ and $\gamma_\theta \to \infty$ and hence the gain terms become $g_1 = 6/t_{go}^2$ and $g_2 = 4/t_{go}$. The controller then achieves the terminal conditions $x_1(t_f) = 0$ and $x_2(t_f) = x_d$ (when used against non-manoeuvring targets). To write the controller in terms of the usual observables we note that $\lambda = x_1/(V_c t_{go})$ and $\dot{\lambda} = (x_1 + x_2 t_{go})/V_c t_{go}^2$ and the controller can be written as

$$\bar{u}_t^0 = -V_c(4\dot{\lambda} + 2(\lambda - \bar{\alpha})/t_{go}). \tag{4.9}$$

Although the optimal control problem in this section has been defined on the linearised system and the control action has been defined as perpendicular to the line-of-sight, the control solution is commonly used for the system defined by (2.1) by projecting the control action $\bar{u}^0$ into the lateral missile axis as follows $\bar{u}_t = \bar{u}_t^0/\cos(\theta_t^I - \lambda)$.

Even with this projection, $\bar{u}_t = \bar{u}_t^0/(\cos(\theta_t^I - \lambda))$ is the optimal control for the guidance problem given in Section 2 only when the interceptor is both on a collision course and its velocity is parallel to the line-of-sight. This is a fairly restrictive requirement but simulation studies demonstrate that the controller $\bar{u}_t^0/(\cos(\theta_t^I - \lambda))$ is stabilizing under moderate relaxation of these collision course and line-of-sight requirements.

Comparing the optimal control with the *ad hoc* controller used previously demonstrates two separate strategies for precision guidance. Both controllers involved terms that drive λ to $\bar{\alpha}$ but the controllers have different strategies for achieving the correct heading angle.

Although driving the line-of-sight rate to zero achieves the required collision if the interceptor is already close to a collision course, the strategy of driving the heading error directly achieves the required collision performance for a larger variety of engagements. In particular, in some engagements it is necessary to increase the line-of-sight rate so that the interceptor can manoeuvre before driving the line-of-sight rate to zero. In these types of engagements driving the relative heading directly achieves the desired collision conditions while driving the line-of-sight rate drives the interceptor away from the engagement.

Remarks

2. The guidance law with impact angle constraints presented in [3] is not optimal in general; however, it does includes the optimal law presented here in certain engagements with certain parameter choices. In addition, [3] considers non-linear pursuit kinematics and establishes initial heading error requirements. The advantage of the approach presented here is that parameter values used are optimal on linearised system and no parameter tuning is required.

3. This paper has not considered the effect of unmodelled dynamics, missile time-constants nor hard control constraints on the precision guidance problem. All of these issues may have a significant impact on the performance of the proposed guidance law and these effects need to be examined in detail before applying the proposed law in a realistic setting.

4. Implementations of this guidance law need to resolve the ambiguity in angular descriptions and the angular difference $\lambda - \bar{\alpha}$.

5. A wider-aspect guidance law based on the optimal controller is as follows

$$\bar{u}_t = \begin{cases} sign\left(\frac{\bar{u}_t^0}{\cos(\theta_t^I - \lambda)}\right) \min\left(\left|\frac{\bar{u}_t^0}{\cos(\theta_t^I - \lambda)}\right|, U_{max}\right) & \text{for } |\theta_t^I - \lambda| < 90^\circ \\ sign\left(\frac{\bar{u}_t^0}{\cos(\theta_t^I - \lambda)}\right) U_{max} & \text{for } |\theta_t^I - \lambda| \geq 90^\circ \end{cases}$$

   where $sign(.)$ returns the +1 when the argument is positive and -1 when the argument is negative, and U_{max} is the maximum desired lateral acceleration. This controller is not optimal but simulation studies demonstrate that it is stabilizing for many initial heading configurations.

6. Precision guidance against manoeuvring targets is not considered in this paper but mild manoeuvres should not greatly effect the closed-loop performance of the controllers (assuming that the target manoeuvres are measured). In general terms, the performance of the proposed guidance laws against manoeuvring targets should be worse than proportional navigation laws but more investigation is require to determine the actual degradation in performance.

4.1 Interpretation as an Optimal Rendezvous Problem

Typically, an optimal rendezvous trajectory shows the interceptor manoeuvring in behind the target and a tail-chase occurring. This is because the optimal rendezvous guidance problem is defined as finding a controller that minimises both the distance and lateral velocity between the interceptor and the target at the collision point (see [6]).

On the other hand, to achieve collision perpendicular to the target's velocity vector the interceptor may have to turn away before turning back towards the target and will have a very large relative velocity at the collision point. Hence, even though both the precision guidance problem and the optimal rendezvous guidance problem are both concerned with the final distance and lateral velocities, the trajectories that occur will look significantly different.

However, the differences in expected interceptor trajectories tend to hide the basic structural similarity between the two guidance problems. This basic structural similarity can be seen in the similarity of the control solutions and can also be understood by considering Figure 3.

Figure 3 shows that the desired collision angle, the relative velocities, and the target orientation define an invariant manifold. To connect our guidance problem with the optimal rendezvous problem we introduce a reference target defined in the relative reference frame. This reference target is defined as being located at the origin but lying in the invariant manifold. Now consider an optimal rendezvous guidance problem to this reference target in the relative reference frame.

The solution of the optimal rendezvous guidance problem to this reference target is the same solution as the precision guidance solution presented above. From this similarity it is possible to show that, in the relative reference frame, the trajectories of the interceptor look the same as trajectories for an optimal rendezvous problem in a fixed reference frame.

4.2 Sensitivity to Time-to-go

In this section we analyse the sensitivity of the control-loop performance to t_{go} errors by considering the closed-loop equations of motion. The closed-loop motion of the engagement, when under control by (4.9), is described by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{-6}{t_{go}^2} x_1 + \frac{-1}{t_{go}} x_2. \end{aligned} \tag{4.10}$$

By substituting in (4.10) it can be shown that the solution is

$$\begin{aligned} x_1 &= c_1 t_{go}^2 + c_2 t_{go}^3 - x_d t_{go} \\ x_2 &= -2c_1 t_{go} - 3c_2 t_{go}^2 + x_d \end{aligned} \tag{4.11}$$

with the following constants of integration

$$\begin{aligned} c_1 &= \frac{x_2(t_0)}{t_{go}} + \frac{3x_1(t_0)}{t_{go}^2} + \frac{2x_d}{t_{go}} \\ c_2 &= \frac{-x_2(t_0)}{t_{go}^2} - \frac{2x_1(t_0)}{t_{go}^3} - \frac{x_d}{t_{go}^2}. \end{aligned} \tag{4.12}$$

The closed-loop equations of motion (4.11) provide the trajectories of a controlled engagement from any initial condition $x(t_0)$. Hence the sensitivity of the controller's performance to t_{go} (or t_f) errors can be examined by considering the effect of these errors on the closed-loop trajectories.

Consider the derivative of $x_1(t)$ with respect to t_f

$$\left.\frac{dx_1(t)}{dt_f}\right|_{t=t_f} = x_d. \tag{4.13}$$

Hence the sensitivity of miss-distance (ie. $x_1(t_f)$) to time-to-go errors is proportional to x_d. In the special case of $\theta_D = 0°$ (ie. optimal rendezvous), the sensitivity goes to zero demonstrating that optimal rendezvous guidance is relatively insensitive to time-to-go errors. However, when $\theta_D = 90°$ the sensitivity of miss-distance to time-to-go errors is at its maximum. Further, in between the angle of impact extremes, this analysis demonstrates that the influence of time-to-go errors on miss distance increases as the desired impact angle approaches perpendicular angles.

5 Simulations

To illustrate the performance of the two proposed control algorithms computer simulations were performed using Matlab^{TM}. The simulations were implemented using discrete-time versions of the state space description given in Section 2 with a sampling period of $h = 0.01\ s$. At this sampling rate collision distances can be resolved to within 0.83 m. Sample and hold versions of the two controllers were used with acceleration limits of 50 ms^{-2} applied to threshold the control actions. Trajectory plots are presented that show the interceptor and target from t_0 until a short time after t_f. No collision effects are modelled.

In these simulations, the target and interceptor have velocities of 66 ms^{-1} and 100 ms^{-1} respectively and the desired impact angle was $\theta_D = 90°$. Low velocity engagements are used to provide exaggerated interceptor trajectories but the reader should note that these trajectories are not representative of engagements at higher velocities.

The controllers are examined in two types of engagements: near collision course engagements and engagements that require large manoeuvres.

5.1 Near Collision Course Engagements

For the purpose of this simulation study, the engagement presented in Figure 1 is considered a near collision course engagement. To exaggerate the effect of the precision guidance objective we simulated the performance of both controllers in a low velocity engagement commencing at 1000 m with $\theta_0^T = 120°$ and $\theta_0^I = 5°$. The closed-loop trajectories of both controllers are shown in Figure 5. Both controllers achieved miss-distances less than 1 m and achieved impact angles within 2° of the desired angle.

To achieve the desired interception, the missile first manoeuvres to the left before turning right to achieve collision. Both trajectories show this manoeuvre sequence but the trajectories are not the same. The *ad hoc* controller achieves interception in a shorter time than the optimal controller but uses more control energy.

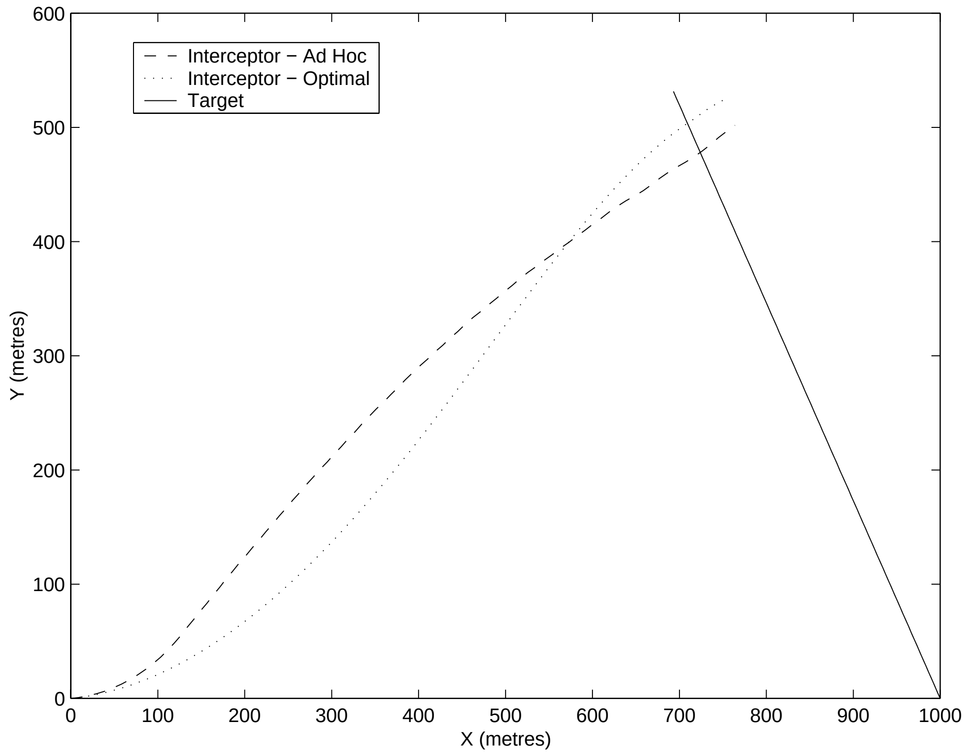

Figure 5 (U): A near collision course engagement.

A comparison of control actions is provided in Figure 6. This figure demonstrates that the optimal controller uses less control energy to achieve similar terminal conditions. The optimal controller also spreads the control actions over more time, while the *ad hoc* controller exhibits switching behaviour when near the invariant manifold. The gain of the *ad hoc* controller (ie. scale of both K_1 and K_2) can be further tuned to reduce the control energy used (and switching).

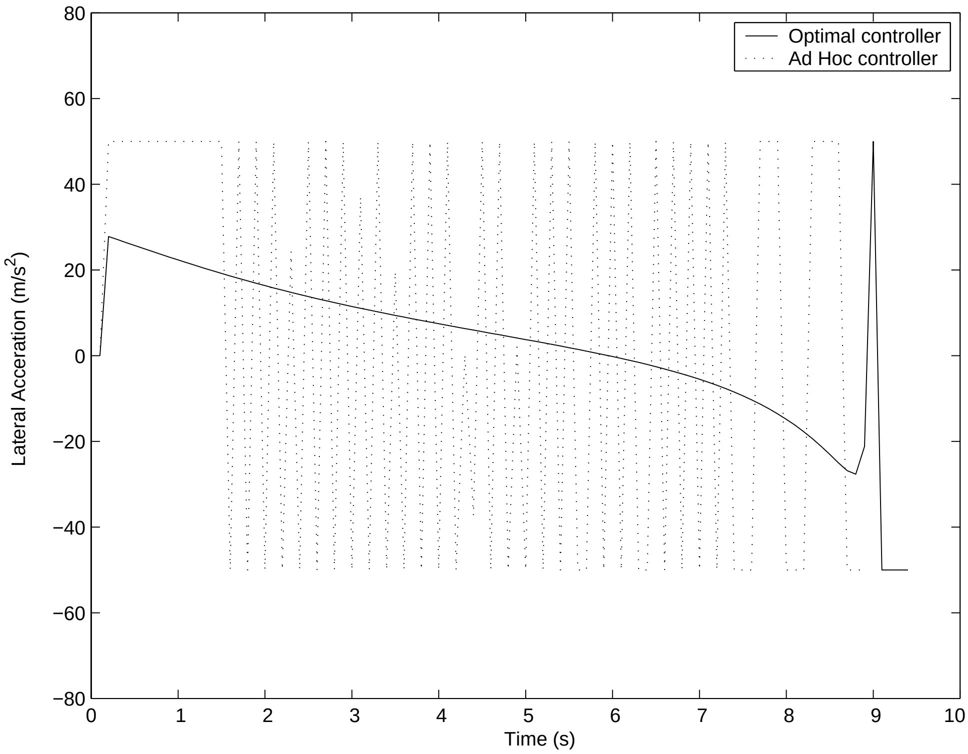

Figure 6 (U): The lateral acceleration commands demanded expressed in ms^{-2}.

5.2 Large Manoeuvre Engagements

For the purposes of this paper, large manoeuvre engagements are defined as engagements where the interceptor needs to perform a turn greater than 50°. In extreme engagements, the interceptor may have to turn 180°, or a full 360°. Consider the same engagement as before except with the interceptor initial heading $\theta_0^I = 140°$. The *ad hoc* controller is used to control the interceptor. In this engagement, the interceptor is initially heading away from the target.

Figure 7 shows the closed-loop trajectory of the engagement using the *ad hoc* controller. A miss-distance less than 0.1 m was achieved with an impact angle within 1° of the desired impact angle. Although the interceptor performs only one manoeuvre, for much of the engagement the interceptor violates the conditions used to generate the optimal control (ie. the engagement is not near a collision course geometry). The optimal controller also achieves impact, but the resulting trajectories are unsatisfactory because the interceptor loops several times before impacting with the target.

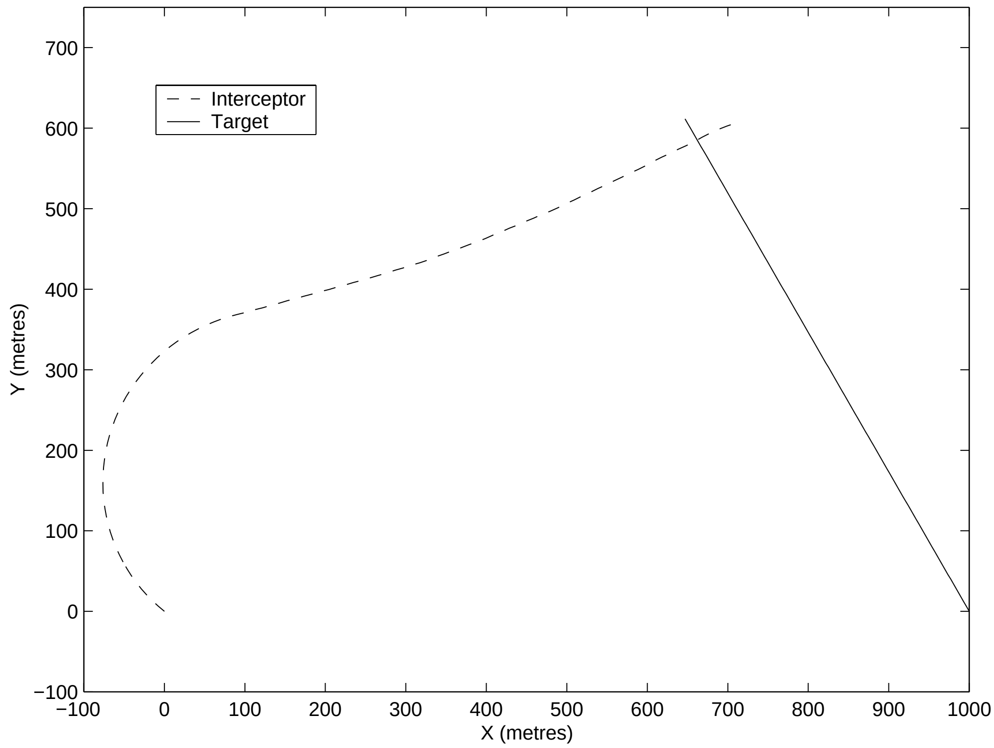

Figure 7 (U): A large manoeuvre engagement with interceptor controlled by the ad hoc controller.

Now consider an engagement where the interceptor and target are heading directly towards each other. Figure 8 shows the closed-loop trajectory of the engagement for the *ad hoc* controller (the trajectory for the optimal controller is very similar). The *ad hoc* controller achieves a miss-distance less than 0.4 m with an impact angle within 2° of the desired impact angle, while the optimal controller achieves a miss-distance less than 0.6 m with an impact angle within 0.1° of the desired impact angle. Notice that the interceptor needs to head away from the target (and off the line-of-sight) before turning and achieving the required collision course.

It should be noted that for some regions of the state space, particularly those in which the interceptor has to move from one side of the target to the other, the *ad hoc* controller described in this paper does not achieve intercept. However, simple modification of the controller to avoid these situations should ensure collision if the interceptor has the kinematic ability.

Finally, note that large manoeuvre engagements at higher velocities require very large acceleration efforts to ensure interception. The acceleration requirements may be unrealistic

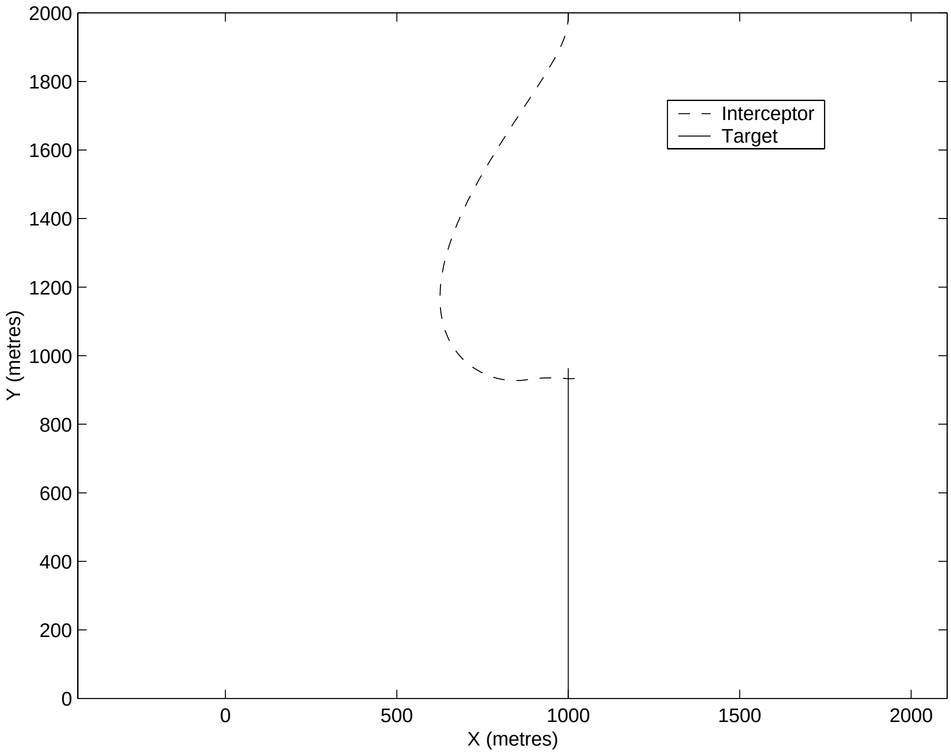

Figure 8 (U): A head-on engagement with the interceptor controlled by the ad hoc controller.

unless the engagement is at the low velocities considered in these simulations.

6 Conclusions

This paper presented two solutions to a precision guidance problem for missiles in which the control objective is to guide a missile on to a non-manoeuvring target so that a particular desired angle of impact is achieved using the minimum control energy. The key contribution of this paper is the presentation of an invariant manifold approach to the guidance problem that leads to both an *ad hoc* controller and an optimal control for a linearised form of the guidance problem. Simulations of typical engagements are presented that demonstrate the performance characteristics of the controllers.

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19. ABSTRACT

This paper examines a weapon system precision guidance problem in which the objective is to guide a weapon onto a non-manoeuvring target so that a particular desired angle of impact is achieved using the minimum control energy. In this paper two approaches for achieving the desired interception requirements are considered.