Investment Strategies in Retirees' Decumulation Phase

by

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ORIGINALITY STATEMENT

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

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Abstract

This thesis consists of three studies that share a common subject of investment strategies in a retiree's decumulation phase. The importance of appropriate retirement distribution plans that adequately cater for the risks in retirement is an integral component of retirement planning. The first study compares alternative withdrawal strategies in retirement to a life annuity benchmark using Australian data. By adopting the Epstein-Zin utility preferences, we are able to disentangle relative risk aversion (RRA) from elasticity of intertemporal substitution (EIS) and examine how retiree preferences are impacted by the means-tested age pension. For the second study, I derive the portfolio choice and consumption patterns for retirees with Hyperbolic Absolute Risk Aversion (HARA) utility who have bequest or capital preservation needs and access to deferred life annuities. I find that investing in deferred annuities leads to significant welfare gains, that is, there is a real option to delay annuitisation (RODA) for retirees at all risk aversion levels. Whilst risk averse retirees want to delay annuitising in retirement, they prefer a shorter delay to a longer one. Retirees derive more utility in purchasing annuities with shorter deferral periods over longer periods and benefit from purchasing deferred annuities over self-annuitisation. The final study compares the performance of the commonly nominated default retirement investment option, the lifecycle fund, to alternative investment strategies during the retirees' decumulation phase. Under different shortfall risk measures, I find that balanced portfolios with constant exposure to stocks, stock dominated portfolios as well as 'reverse lifecycle' portfolios that increase exposures to stocks over time consistently outperform the conventional lifecycle portfolio. Using a utility-of-terminal wealth approach which allows for loss aversion as in prospect theory, I find the balanced portfolio to dominate the alternative strategies at low thresholds. With increasing threshold levels, investment strategies with high constant stock allocations become dominant. The lifecycle portfolio is dominated by the 'reverse lifecycle' portfolio at all threshold levels.

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Abbreviations

ABP	**A**ccount **B**ased **P**ensions
AGA	**A**ustralian **G**overnment **A**ctuary
ALDA	**A**dvanced **L**ife **D**eferred **A**nnuity
APRA	**A**ustralian **P**rudential **R**egulation **A**uthority
ASFA	**A**ssociation of **S**uperannuation **F**unds of **A**ustralia
CAPM	**C**apital **A**sset **P**ricing **M**odel
CPI	**C**onsumer **P**rice **I**ndex
CPPI	**C**onstant **P**roportion **P**ortfolio **I**insurance
CRRA	**C**onstant **R**elative **R**isk **A**version
DB	**D**efined **B**enefit
DC	**D**efined **C**ontribution
EIS	**E**lasticity of **I**ntertemporal **S**ubstitution
E-Z	**E**pstein-**Z**in
FSI	**F**inancial **S**ervices **I**nquiry
HARA	**H**yperbolic **A**bsolute **R**isk **A**version
IMF	**I**nternational**M**onetary **F**und
IPA	**I**ndividual **P**ension **A**ccount
IRA	**I**ndividual **R**etirement **A**ccount
MEL	**M**ean **E**xcess **L**oss
MPT	**M**odern **P**ortfolio **T**heory
MTAWE	**M**ale **T**otal **A**verage **W**eekly **E**earnings
PCA	**P**ublic **C**are **A**version
RODA	**R**eal **O**ption to **D**elay **A**nnuitisation
RRA	**R**elative **R**isk **A**version
SAA	**S**trategic **A**sset **A**llocation

SE	**S**hortfall **E**xpectation
SMSF	**S**elf **M**anaged **S**uperannuation **F**unds
SP	**S**hortfall **P**robability
S&P	**S**tandard and **P**oor
SPIA	**S**ingle **P**remium **I**mmediate **A**nnuity
SPV	**S**tochastic **P**resent **V**alue
SWP	**S**ystematic **W**ithdrawal **P**lan
TAA	**T**actical **A**sset **A**llocation
TIAA-CREF	**T**eachers **I**nsurance and **A**nnuities **A**ssociation **C**ollege **R**etirement **E**quity **F**und

Dedicated to my family

Chapter 1

Introduction

1.1 Background

Human mortality has decreased significantly in the last century and today, people are living longer than ever before. Given a relatively long lifespan, appropriate investment strategies are necessary for retirees to maintain their available wealth in retirement years. Findings by Oeppen and Vaupel (2002) reveal that female life expectancy among the longest living populations in the last 160 years has been increasing steadily by almost three months every year. Australians are among the longest lived populations on the planet with longevity continuously improving. A recent Global Financial Stability Report.[1] by the International Monetary Fund (IMF) suggests that the Australian government helps to insure against longevity risks by developing their annuity markets and introducing reverse mortgages. The report explains that underestimating life expectancy could add up to about 50% to the cost of providing for retirement in Australia (Uren, 2012). The IMF says countries around the world have consistently underestimated improvements in longevity by about 3 years every 20 years. While institutional authorities expect the rate of improvement to slow down, the continuous advances in medical science suggest otherwise.

> *The more realistic predicted scenario is much more dramatic. After allowing for mortality improvements on a cohort basis it's estimated that retirees aged 65 now (i.e. in 2010) will live until 86 for men and 89 for women.*
>
> *By 2050 the average life expectancy for people aged 65 is projected to have improved to 92 for men and 93 for women. And this is an average.*

[1] This report is available online here.

> *Many will live longer than this.* (Australia's Longevity Tsunami, 2012, pg. 6)

Increasing life expectancy will thus have major implications for retirement incomes policies. With 84% of Australians over the age of 65 receiving some form of government payments, aged care is currently the eighth largest part of the government expenditure. The Government's age pension is a non-contributory payment for people satisfying age and residence requirements and whose income and assets are below certain limits. The purpose of this pension is to ensure that senior Australians have adequate means of support.

For men, the current qualifying age for Age Pension is 65 years whilst the qualifying age is gradually being increased to 65 years for women. The Intergenerational Report (2010) explored the fiscal sustainability of the age pension system and Treasury calculations forecast that age pension expenditure will rise from the current 2.7% of GDP to 3.9% in 2050. While this may not appear to be a steep increase, 1.2% of GDP represents nearly $20 billion per annum in today's terms (Australia's GDP is currently around $1.6 trillion). The debate on increasing the Age Pension qualifying age has been trending, with the Actuaries Institute, the body representing the actuarial profession in Australia, calling on government to consider an increase of the eligibility age in line with increases to life expectancy. On 13 May 2014, the Government delivered the 2014-15 Federal Budget,[2] and among other things, revealed plans to increase the Age Pension age from the current 65 to 70 in 2035. From 1 July 2025, the Age Pension qualifying age will continue to rise by six months every two years, from the qualifying age of 67 years that will apply by that time, to a qualifying age of 70 years by 1 July, 2035.

If gains in life expectancy were predictable, they would have been taken into account when planning for retirement, thereby making an insignificant effect on retirement finances. Unfortunately, mortality improvements and life expectancy are difficult to model. In this regard, longevity risk is associated with the risk that future mortality occurrences and life expectancy outcomes may differ from what is expected. Longevity risk refers to the uncertainty of the age of death giving rise to the possibility of an individual outliving her investment portfolio. As a result of this uncertainty surrounding future changes in mortality and life expectancy, retirees run the risk of outliving their available resources forcing them to reduce their standard of living in

[2] An overview of the Australia Federal Government's 2014-2015 Budget is available here.

old age.

There is the need for appropriate investment measures to be taken whilst individuals receive income and have adequate human capital to build a good portfolio to create wealth in the accumulation years. Similarly, it is important that retirees invest optimally to ensure that they do not outlive their wealth. To ensure that a decumulation strategy is appropriate and sustainable, the relevance of longevity insurance in retirement investment cannot be overlooked. With increasing life expectancies, appropriate investment products and strategies are necessary to minimise the risk of retirees outliving their wealth. Wealth decumulation strategies have become particularly important in recent years with baby boomers moving into retirement and the manifold growth in number of the Defined Contribution (DC) plans over the Defined Benefit (DB) plans.

1.2 The Shift from Defined Benefit to Defined Contribution Pension Plans

In the last three decades, we have witnessed the gradual takeover of the Defined Benefit (DB) pension plan by the Defined Contribution (DC) plan. The sponsor of the DB retirement plan bears all investment risks and ensures that the plan participant receives her benefits regardless of the market outcome. The DB plan gives the plan member certainty about her future benefit, typically by the use of a formula approach. The Australia superannuation system, like many other pension systems all over the world, is predominantly DC in nature, where the benefits of the participant depend on the accumulated contributions and investment returns. The retiree is therefore responsible for the effective management of her retirement wealth to ensure that it meets her consumption needs.

The evidence of reduction in the DB plan numbers is absolute. Globally, DC plans assets have increased from 38% in 2003 to 47% in 2013 (Towers Watson, 2014). As of June 2013, total assets (for entities with more than four members) consisted of approximately 83.6% ($891.0 million) allocated to defined contribution accounts and 16.4% ($174.6 million) allocated to defined benefits accounts in Australia. DC plans in the United States are up to 58% of total retirement plans. Under Australia's superannuation plan, employers are required to make a mandatory contribution, which is currently 9.5% into individual accounts in privately managed pension funds. Research has been extensive on the reasons for this transition from DB to DC plans and studies have attributed several factors to this shift. Milevsky and Abaimova (2005) cite the high economic cost of financing and maintaining DB pension plans as a reason for the failure of the DB to meet retirees' current demands. Their research goes further to point out the poor performance of the stock markets, low interest rates and increasing life expectancy of plan holders as other factors which are believed to compromise the effectiveness of the DB plan.

Past studies suggest that the DC plan is perceived to be a cheaper alternative to the DB plans. The DC plan also gives the participant more flexibility with investment options and is less risky compared to the DB plan, in that there is very little potential of loss to the plan participant should the corporate sponsor go insolvent (Mitchell & Schieber, 1998). DC plans also provide much if not absolute control of the plan participant's asset allocation as well as transparency in investor's investment dealings. The DC plan benefits are received by plan participants in the form of a lump-sum, instalment payments or a life annuity, providing a lot more flexibility and liquidity, unlike the DB plan which comes either as a lump sum or in the form of a life annuity. In these respects, the DC plan is applauded by many as a move in the right direction. The DC plan however has its shortcomings which cannot be overlooked. Many poorly informed DC plan holders are at risk of running a poor investment by making irrational portfolio choices. The DC plan does not provide security against a plan participant outliving her resources. In other words, the DC plan does not insure the plan holder against longevity risk. It is one major risk associated with pension investment. Other risks include inflation and consumption risk, sequencing risk of returns as well as investment and portfolio risk. These risks associated with pension investment comprise the major deficiency of the DC plan which calls for strategies to help preserve the investor's resources in retirement.

An effective decumulation plan should be able to adequately hedge against market and longevity risks. Whilst there is extensive academic research on how individuals can build a well-diversified and sustainable investment portfolio during their working lives, the same cannot be said for dissaving wealth after their retirement. This doctoral dissertation seeks to examine the diverse possibilities available in the decumulation or retirement distribution phase of retirement. I explore different portfolio withdrawal strategies to enhance a retiree's utility from her available wealth in retirement. I further explore available products such as annuities and in particular deferred annuities for the retirement phase and discuss retirement investment strategies with the view of helping retired individuals to make informed decisions in retirement.

1.3 Motivation

With the DC plan being the more popular of the two main pension plans, the relevance of asset allocation, innovative retirement products and post-retirement planning is greatly emphasised. A retiree's standard of living is dependent on her level of wealth at retirement, as this is the major determinant of the consumption levels she is able to spend from her accumulated wealth. In other words, the income annuity level used to replace the retiree's pre-retirement income is dependent on the size of her DC account at retirement. Inflation may erode the purchasing power of such retirement income and this is a major risk to the retiree. A rise in price levels increases the investor's consumption rate as she requires more money to keep up with her standard of living. The value of money in these terms is reduced and the investor is at the risk of prematurely exhausting her assets in a period of soaring inflation rates. On the effects of inflation shocks on Australia's economy, Mallick and Mohsin (2007) assert that both in the short and long run, inflation has a negative impact on both durable and non-durable consumption as well as investment.

Investment risk can be defined as the probability or likelihood of occurrence of losses relative to the expected return on any particular investment. When market volatilities cause fluctuations in the capital market early in retirement, the retiree stands the risk of a depleted portfolio which may be unable to stand the stress of regular withdrawals. These risks highlight the importance of effectively managing a portfolio's withdrawal to sustain the individual's consumption in retirement without facing portfolio ruin. Portfolio ruin occurs when the retiree exhausts her retirement wealth whilst she is still alive. To generate income in retirement, investor may choose to hold a diversified portfolio from which they make annual withdrawals to meet their consumption or decide to purchase an annuity to receive periodic income over their years. Retirees who self-annuitise their lump sum at retirement have no insurance or longevity protection in retirement and are at risk of portfolio ruin. Portfolio ruin is eliminated by holding life annuities. The cost to the annuitant may however come in the form of credit risk, which involves the possibility of loss due to the inability of the life company to fulfil its financial obligations.

The major shortfall of the DC plan is its inability to provide security against a plan participant outliving her resources. Longevity risk is covered under the DB plan where the onus lies on the plan sponsor to bear all investment risks and ensure that the plan participant receives her benefits regardless of the market outcome. The DC plan however leaves the benefits of the participant dependent on retiree's contributions' accumulated wealth level. The failure of the DC plan to replicate this important aspect of insurance makes the use of securities and investment strategies indispensable in retirement planning to help preserve the investor's resources. It is commonly believed that a life annuity is the best insurance against a retiree outliving her wealth. However, empirical studies have shown evidence of annuity aversion among retirees. In Australia, Ganegoda and Bateman (2008) argue that this is the result of retirees having a range of benefit options with their retirement fund; converting their wealth into one or more lump sums, a phased withdrawal or a life or term annuity.

With the longstanding evidence of annuity aversion among retirees, researchers have sought to find the appropriate wealth distribution programs in retirement that meet retirees' individual preferences. A primary objective of a retirement savings account is to minimise the uncertainty regarding prospective retirement spending. In retirement, the correct choice of distribution program is necessary to meet this objective. What is the appropriate drawdown strategy in retirement for rational retiree preferences with government pensions? Studies have compared the most popular alternative self-annuitisation options available for the post-retirement phase and presented findings based on various analysis in the context of the mandatory annuitisation, shortfall analysis, bequest motives and varying risk aversion levels for the Constant Relative Risk Averse (CRRA) retiree (Dus, Maurer, & Mitchell, 2005; Horneff, Maurer, Mitchell, & Dus, 2008). While the assumption of the CRRA retiree is widely accepted in pension literature, recent findings in experimental economics have shown that individual preferences may be different to what is assumed for the CRRA investor concerning consumption. Brown and Kim (2013) show that subjects prefer early resolution of uncertainty and have elasticity of intertemporal substitution (EIS) greater than the reciprocal of Relative Risk Aversion (RRA). The CRRA preferences assume EIS as being the reciprocal of RRA but current studies suggest that this is not the case. Retirees' exhibiting Epstein-Zin preferences, which allows a separation of RRA from EIS, opens up an avenue of further research and exploration of the comparison of these alternative retirement distribution programs. Furthermore, previous comparison of alternative distribution plans have failed to recognise the impact of government pensions on such outcome. These are the key motivations behind the first essay of this doctoral dissertation.

The need for innovative retirement products is a primary concern with the current ageing populations prevalent in many developed countries. There are many products available in the accumulation phase to help individuals diversify their saving through target date and balanced funds (Gomes, Kotlikoff, & Viceira, 2008). There are also lifestyle investment options for individuals to accumulate wealth to meet their lifestyle needs and wealth goals. In the post-retirement phase however, products are not as developed as those in the accumulation phase. There are distribution products such as income annuities, which pay retirees regular income for a fixed period or for life for an initial premium paid to the insurance company. This income may be fixed, indexed for inflation, or variable based on returns of an underlying asset and may have options such as deferred income and capital preservation. The existence of annuities dates back to the Middle Ages and detailed records exist for annuity pools in France as early as the 17th Century (Poterba, 1997). Annuities became very popular after the Great Depression of the 1930s which followed the stock market crash of 1929. People purchased financial products from life insurance companies because they were deemed more stable than holding shares on the financial markets. Since then annuities have expanded to many parts of the world over the years. Some insurance companies have life insurance products with withdrawal features available to retirees. There are also a number of investment products that allow individuals to invest in a variety of mutual funds mainly focussed on decumulation. These funds centre on the creation of investment products that can create a stream of income to fund retirementm (Hawkins, 2010). The option of lump sum at retirement still remains the predominant choice for retirees and retirement products see less patronage in many Anglo-American countries. [See The World Bank (1999), Financial Services Inquiry Report 2014]

There has been a long-standing interest in researching annuitisation in retirement. In his seminal study on the theory of a lifecycle consumer, Yaari (1965) asserts that in the absence of bequest motives, it is optimal for the investor to hold life annuities rather than liquid assets. Davidoff, Brown, and Diamond (2003) demonstrate theoretically the benefits and optimality of holding life annuities in retirement under less restrictive conditions than the Yaari model. One promising annuity option is the deferred annuity. The deferred annuity, also called the longevity annuity, commences periodic payments to the annuitant at some pre-specified future date conditional on survival. Deferred annuities are available in the US and the UK annuity markets but are not available in Australia. This is because pension rules prescribe minimum withdrawals during the deferred period and do not permit deferred earnings to be eligible for tax benefits. The leading provider of annuities in Australia, Challenger Limited, expects the price of a deferred annuity purchased by a 65-year-old man which begins payouts of $8,000 annually from age 80 to cost as low as $10,000 (Uren, 2012). There are plans to introduce deferred annuities on the Australian market in the near future and this is one of the motivations for this thesis. Whilst the benefits of this annuity option are widely discussed in the literature, the suitable timing of deferred annuities has not been examined. The length of the annuity's deferral period is another side of this product that is not discussed in previous literature. This leaves room for further research on the timing and length of deferred annuities and the benefits of including this product in a retirement portfolio both in the presence and absence of government means-tested age pension.

It is widely considered that asset allocation is an important driver of portfolio investment returns over the long horizon. This is sufficiently demonstrated by studies such as Brinson, Hood, and Beebower (1986) and Blake, Lehmann, and Timmermann (1999). The DC plan lasts over long horizons, typically the entire working life an individual. This makes asset allocation an important aspect of such a plan and a significant determinant of returns and final wealth. With continuously decreasing mortality, the average life expectation for a 65 year old female Australian is expected to rise to age 93 in the year 2050. Australian retirees will live to an expected average of 28 years in retirement. This represents a substantially long distribution horizon and planning in retirement as well as asset allocation in this phase of life is increasingly important. Cooley, Hubbard, and Walz (2001) propose that at least 50% of a retirement portfolio should be invested in stocks and their findings show increased sustainability of the pension fund as it tilts more towards equities. They show that the presence of bonds is mainly to restrain portfolio volatility and provide liquidity to cover an investor's living expenses. Bengen (2004) advises that if the future market follows the trends of behaviour in the past, then a retirement portfolio should contain 50-75%equity allocation.

In the last two decades, allocation to bonds in the seven largest pension markets have decreased by 12% (40% to 28%), with cash allocations reducing from 6% to 1% in the same period. Equity investments on the other hand have increased by 3% to 52% of total pension allocations, with the remaining 18% allocated to alternative assets which include real estate, private equity, hedge funds and commodities (Towers Watson, 2014). Are these changes moving in the right directions? The conventional rule of thumb to pension investment states that the investors should allocate a percentage of 100 minus their current age in risky assets (such as stocks) and the remainder in low risk assets such as bonds and cash. The lifecycle strategy is probably the most popularly used default investment plan and in Australia, lifecycle funds are increasing rapidly (QSuper, 2014). The strategy invests massively in stock at the onset when the investor is still young, because the younger investor is able to assume more risk than the older investor, according to Utkus (2005), who also believes that there is always time to regain losses if stocks should decline in the early years of investment. This investment is gradually switched to low risk assets as the individual nears retirement and the asset mix changes. Since the allocation to stocks (risky assets) and bonds and cash (non-risky assets) is unidirectional, investment in stocks reduces as that in bonds and cash increases. This has been firmly supported by works such as Malkiel (1999) who asserts that the plan participants should consider cutting back on riskier assets and increase investment in bonds and cash as they grow older.

While the lifecycle investment option is predominantly accepted in the accumulation phase, decreasing investment in risky assets with increasing age is prevalent in retirement phase as well. Retirees generally do not change their asset allocations in the retirement phase, leaving them in the default strategy, which is mostly the lifecycle plan. Retirees' reluctance to exercise their option of investment choice in retirement is due to the perceived lack of financial literacy. Biases in human behaviour is also recognised as one factor influencing this inaction, with retirees perceiving the default investment option available to them as an endorsement or recommendation by the plan provider (Beshears, Choi, Laibson, & Madrian, 2009). With studies showing that pension sustainability is immensely improved with high levels of equity in accumulation, there is growing interest in what allocations are optimal in the retirement phase (Arnott, Sherrerd, & Wu, 2013; Estrada, 2012). Investment strategies in the decumulation phase are fairly new to the retirement literature and leave a lot of room for exploration. Fundamental decisions such as holding a strategic static asset allocation (SAA) as opposed to a decreasing risky asset investment in the decumulation phase are still left unanswered. Based on historical returns, are retirees better off with the default lifecycle style of investing or exploring alternative methods? Some of these alternative methods such as increasing risky assets with age have been found to provide higher wealth levels and better downside performance in the accumulation phase. Do these alternative methods improve retirement wealth outcomes in the decumulation phase when retirees face the stress of regular income withdrawals and the imminent shocks to expected government pensions? These are some of the questions that motivate the final essay of this thesis.

1.4 Research Objective

In Australia, about half of superannuation benefits in retirement are paid as lump sums; the remaining half is paid as income streams. Individuals who wish to convert their superannuation assets into a retirement income stream may choose between Account-Based Pensions (ABP) and annuities. ABPs allow retirees to choose their investment strategy; they have the option of taking lump sums at any time whilst the remaining balance is drawn down according to the retirees' preference subject to legislative prescribed minimums. Annuities are financial products which provide the buyer with a series of regular payments in return for an upfront lump-sum investment or a series of smaller payments. Income from the annuities may be received over a specified time horizon (fixed-term annuities) or for the remainder of the individual's life (lifetime annuities). Account-based pensions account for at least 94% of current pension assets. This means retirees choose to manage their retirement benefits rather than investing in annuities that will provide them with regular periodic income. This decision entails that retirees are responsible for the size and timing of lump sums should they decide to take one as well as their choice of drawdown strategy. What constitutes an appropriate drawdown strategy is discussed in the first essay. Secondly, retirees are in charge of managing their longevity risks by choosing the ABPs. How to appropriately incorporate longevity risk management tools in the retirement plan is studied further in the second essay. Finally, retirees' asset allocation of her funds is deemed an important determinant of portfolio performance. I study different asset allocation strategies in retirement and compare the alternatives under several shortfall measures to emphasise the need for appropriate allocation strategies in a retirement account.

With the empirical evidence of annuity aversion in various countries globally, the importance of appropriate retirement distribution plans which adequately cater for the risks in retirement is increasingly becoming an integral part of retirement planning. While there are studies that compare distribution strategies under various scenarios and preferences, there are yet a number of issues that have not been addressed. Prominent among these is the assumption of Constant Relative Risk Aversion (CRRA) of retirees for the purpose of distribution choice. More recent studies have shown that retirees have RRA levels which exceed the inverse of EIS levels. This means retiree preferences in retirement are better modelled by Epstein-Zin preferences, allowing the disentangling of the two relevant preference parameters. This dissertation incorporates this knowledge in the retirement distribution plans literature, analysing a retiree's preference of distribution plans when she is not only interested in the size of her income but also smoothing her consumption over her retirement years. This study further considers how such preferences are impacted by government sponsored pensions. The presence of pension systems such as Australia's Age Pension may significantly affect a retiree's choice of drawdown plan and this is extensively investigated in the first study of this thesis. The inclusion of pensions in the drawdown plan choice analysis is a novelty in this dissertation as previous studies have generally been silent on the impact of government sponsored pensions on distribution plan choices. Other sensitivites regarding increasing life expectation and bequest motives are also discussed.

The plan participant of a DC account has a variety of options at retirement; prominent among these is the option to take a lump-sum benefit. There may be restrictions in some countries on how much of a lump sum one may access at retirement. Since the DC plan is unable to cater for longevity risks, it is important that the retiree invests in some form of insurance at retirement to cater for this risk. There are several reasons explaining retirees' aversion to life annuities, the perceived best insurance against longevity risk. An important factor accounting for this aversion is the high cost of annuities and the loss of liquidity through the purchase of this security as the decision to annuitise is generally irreversible. Pension products are generally short sold as investors are not presented clearly with the value of such products and how these products improve their financial wellbeing. A retirement product should be easy to implement and should be easily understood by customers. The deferred annuity is one innovative pension product of immense value which provides the longevity insurance retirees require at a low cost and can be purchased with a fraction of retirees' wealth leaving a large portion for the retiree to self-annuitise. Such a product should be more attractive to retirees if the value of this is apparent.

The evidence of the presence of bequest motives among the elderly is unquestionable, as has been amply demonstrated in academic literature. Marshall (1949, pg. 228) acknowledged the bequest motive when he said that family affection is the main motive for saving, and that "*a man is concerned about leaving his family to start from a higher round of the social ladder than on that which he began.*" Becker and Tomes (1976) reason that parents spend resources to improve the quality or lifetime income of their children and they derive utility from doing so irrespective of what the children may do with their boosted income. Of striking significance is the work of Kotlikoff and Summers (1980) who find that at least 80% of US capital stock is accrued through intergenerational transfers. Hurd (1989) asserts that the bequest motive is egoistic, triggered by the desire of the elderly to leave behind a positive net worth at time of death. He claims that most bequests are accidental and are a result of precautionary measures taken by individuals in the absence of complete insurance markets.

With the aftermath of the Global Financial Crisis wiping out several millions of dollars of retirement funds, retirees will look to preserve some capital should there be another market crisis again in the near future. Retirees' need for capital preservation and meeting bequest motives is incorporated in a retirement investment strategy via a portfolio insurance model and the retiree is given the option to annuitise a portion of their retirement wealth in a deferred annuity spanning different withdrawal periods. The preserved capital may serve as a retiree's bequest at the time of death and the deferred annuity caters for longevity risk. I employ the Omega Ratio as the investment performance measure to compare various available investable alternatives and this is a novelty of this study as this measure has previously not been used in the retirement literature. I use this measure for its simplicity and appropriateness in ranking preferences based on individual's preference of higher utilities over lower utilities.

There are growing concerns regarding the appropriateness of the lifecycle strategy which is widely used as a default investment strategy. The final essay looks at asset allocation strategies in the retirement decumulation phase. Whilst there are many programs and securities available for the accumulation phase, not much attention has been given to the allocation of asset in the distribution phase. The final essay of this thesis investigates the appropriateness of the lifecycle strategy in retirement, specifically comparing this widely accepted strategy to various alternative hybrids of the lifecycle model as well as varying levels of stock in SAA strategies. Further analysis using utility functions motivated by Prospect Theory helps to rank various alternative investment models for different wealth thresholds. I conduct shortfall analyses using different measures to compare the various decumation asset allocation strategies.

1.5 Thesis Structure and Research Description

The three essays in this dissertation share a common subject of investment strategies in the retiree's decumulation phase. Although I provide a general introduction and literature review, each essay has its own introduction and review of related literature. The rest of the thesis is organised as follows: Chapter 2 presents the general literature review which shapes the scope of this thesis and introduces the relevant studies in pension literature, investments and portfolio performance measures. The first essay is presented in Chapter 3; I study the effect of Means-Tested Age Pension on decumulation plan choices when a retiree has Epstein-Zin preferences. Chapter 4 presents the second essay, a study of the timing and deferral periods of deferred annuities in a retirement decumulation plan for retirees with bequest motives. The final essay is presented in Chapter 5 and I discuss the investment glidepath illusion and compare the lifecycle style investment to alternative strategies under different shortfall measures. Finally, I conclude in Chapter 6 and point out some possible avenues for future research.

Chapter 2

Literature Review

This chapter consists of six sections. In Section 2.1, I discuss the global decreasing mortality rates and the impact of this on retirement planning as increased life expectation serves as a major motivation for research in a retiree's decumulation phase. I discuss asset allocation and sustainable withdrawal strategies in Section 2.2, focussing on the different asset allocations recommended in various studies for the decumulation phase and the different withdrawal rates as suggested in previous literature. I review the history and development of the life annuity in Section 2.3. I discuss the various annuity types and demand for annuities on various markets. The section concludes with a brief discussion of the annuity puzzle and the way forward for life annuities. Section 2.4 reviews the evidence of bequest motives among individuals and the influence of bequest on consumption and asset allocation. In Section 2.5, I review the Epstein-Zin preferences and utility and examine its appropriateness in the retirement drawdown and bequest analysis. I discuss the measures of investment performance used in this thesis in Section 2.6; 2.6.1 introduces the Omega Ratio used in the second study and 2.6.2 discusses different alternative shortfall measures which are used in the third essay.

2.1 Mortality Improvements and the Need for Retirement Planning

The two major phases of peoples' financial life are their wealth accumulation and decumulation years. The importance of saving for retirement is apparent and many countries have well-structured pension plans to enable individuals to save up for their retirement. Individuals must face the task of taking appropriate measures to prepare for anticipated longer retirement lifetimes whilst governments and policy makers increasingly face concerns regarding aged-care, pension and welfare policies. Whereas there is extensive academic research to constructing well-diversified and sustainable investment portfolio during the working life of individuals, the same cannot be said for dissaving wealth after the individual's retirement. The importance of wealth dissaving strategies has become more pronounced with the increase in life expectation especially in developed countries and the growth in numbers of the DC plans over the DB plans.

The last two centuries has seen life expectancy more than doubling, from roughly 25 years to about 65 for men and 70 for women (Riley, 2005). The Organisation for Economic Co-operation and Development (OECD)[1] defines life expectation as a measure of how long on average a person of a given age can expect to live, if prevailing death rates do not change (OECD, 2013). The life expectancy for a population is directly read from mortality tables, also called a life table, which is a statistical model used to represent mortality for a given population. A recent report by the United Nations (UN)[2] reveals that the 10 countries with the highest life expectancies for the period 2010-2015 have an average of 81.7 years for both sexes combined. For the same period, 57% of the world population live in countries with life expectancy above 70 years, while 9% of the global population has a life expectancy reaching 80 or higher. Very few countries remain at life expectancy levels below age 60, with only 9% of the world population living in these countries. Between sexes, women have a life expectancy advantage of 4.5 years on average, with this advantage decreasing for less developed countries and increasing for more developed countries.

Disparity between life expectations is not only observed for the different sexes but also for geographical locations and significantly for different income levels. While these findings report the life expectancy at birth, a vital aspect of mortality expectation is the conditional survival probabilities at different ages. The conditional survival probabilities are most relevant in actuarial finance and for retirement planning. The life table provides the probability of death or survival within any one given year, thus translating an age group x into a probability of dying, q_x, during the next year. For example q_{60} is the probability of an individual dying before her 61st birthday assuming she is alive at age 60. The probability of dying by definition is $0 \leq q_x \leq 1$ and $q_N = 1$ for some large enough number say $N = 110$. The conditional probability

[1] The OECD is an international economic organisation of 34 countries founded in 1961 to stimulate economic progress and world trade.

[2] United Nations (2013)

of survival in year x is the complement of q_x, or $1 - q_x$, usually denoted by p_x. The conditional probability of survival provides an estimate of surviving n more years given that the individual has survived x years. This is denoted in actuarial notation by[3]:

$$_np_x = \prod_{i=0}^{n-1}(1 - q_{x+1}) \tag{2.1}$$

$_np_x$ is the conditional probability of survival for n years given an individual age x.

The value of $_np_x$ declines with increasing age as the probability of living for $n = 10$ years is higher when one is $x = 65$ years of age than when one is $x = 90$ years of age. Table 2.1 shows an extract of Australian Life Tables 2005-2007 (2009) which includes estimates of p_x and p_{x+10} from my calculations. The calculations of conditional probabilities help actuaries determine the expectation of life to various ages. This helps in the provision of technical and statistical knowhow for financial planners and policy makers on survival expectations of cohorts at given survival to specific ages. An example is the probability of a cohort of retirees at age 65 surviving to age 85.

Table 2.1: Extract from ALT 2005-2007 (Females)

Age(x)	l_x	d_x	p_x	q_x	$_{10}p_x$
65	92,152	626	0.9932	0.0068	0.89918
66	91,526	680	0.9926	0.0074	0.88137
67	90,846	742	0.9918	0.0082	0.86814
68	90,104	813	0.9910	0.0090	0.85335
69	89,291	894	0.9900	0.0100	0.83665
70	88,398	986	0.9888	0.0112	0.81788
80	72,308	2,647	0.9634	0.0366	0.47897
90	34,635	4,535	0.8691	0.1309	0.10492
100	3,634	1,028	0.7172	0.2828	0.0000
109	105	39	0.6320	0.3680	0.0000

l_x–*number of persons surviving to exact age* x
d_x–*number of persons dying at exact age* x
q_x– *proportion of persons dying between exact age* x *and exact age* $x+1$
p_x–*proportion of persons surviving between exact age* x *and exact age* $x+1$
$_{10}p_x$–*proportion of persons surviving between exact age* x *and exact age* $x+10$

From the extract, a 65-year-old retiree has up to 89.9% chance of surviving 10 more years in retirement while the 90-year-old has 10.5% probability of surviving to age 100. Calculations based on Australia's recent life table[4] show that 50% of Australian females aged 65 will live to age 87 as opposed to age 84, which is the life expectation at birth. One in five 65-year-old women can be expected to live to age 93 and one in

[3] Milevsky (2006)
[4] Australian Life Tables 2010-2012 (2013)

10 to at least age 96. This requires a retiree who wants to be 90% certain that her superannuation savings last as long as she lives to plan her consumption and savings for up to 31 years.

According to the OECD (2013) report, the advances in healthcare, healthier lifestyles and improved living conditions before and after people reach age 65 significantly contributes to the observed decreasing mortality rates. With increasing life expectation in retirement, the onus lies on policy makers to ensure that welfare policies are put in place which incorporate the expected improvements in life expectancies. Individuals are responsible for planning and ensuring that their savings and accumulations during their working years can adequately sustain them in the event of long retirement horizons. Life expectancy is underestimated by many and this is evidence in a recent report by the International Monetary Fund which cautioned that the underestimation of life expectancy could add up to about 50% of the cost of providing for retirement in Australia (Uren, 2012). In the US, the American Society of Actuaries found that more than half of Americans surveyed underestimate their life expectancy, and perceive very short financial planning time horizons in retirement. The survey conducted by the Society, of the retirees and pre-retirees, found about 4 in 10 underestimated their life expectancy age by five or more years and another 2 in 10 underestimated it by 2 to 4 years. More importantly, regarding the median life expectancy, only 4 in 10 retirees correctly responded that about half of 65-year-old men and women can expect to live until median life expectancy. This report is published by Lifetime Income Risk Task Force (2012). There is a strong indication that variability of life expectancy is not well understood by the populace.

2.2 Asset Allocation and Sustainable Portfolio Withdrawals

The need for optimal asset allocation in investors' dissaving phase is essential to ensure that they do not outlive their available resources. The importance of asset allocation is highlighted in the seminal work by Brinson et al. (1986) who find that up to 94% of variation in their sample pension funds' total return is explained by the choice of asset classes and their relative weights in a portfolio. For the accumulation phase, there are several popular investment models which investors employ with the hope of saving adequately for retirement. There is the Strategic Asset Allocation strategy (SAA), a static allocation strategy which is representative of the optimal combination of different asset classes where the aggregate asset classes are assumed to be efficiently priced. (Radcliffe, 1997). The SAA is akin to the buy and hold strategy and the assets are periodically rebalanced to the target allocations as investment returns skew the original asset allocation proportions. A more active strategy is the Tactical Asset Allocation (TAA) which is not static but allows for a range of proportions for each asset class. This allows the investor to take advantage of market volatility by altering the asset proportions. This is particularly useful when an investor believes asset classes are mispriced. In terms of the length of investment horizons, while the SAA decisions relate to long investment horizons, the TAA is usually useful for shorter horizons, typically a year or less (Haugen, 1990).

Conventional wisdom suggests that individuals are more risk averse as they grow older and their investment horizons get shorter. Individuals should therefore invest more in stock when they are young with long investment horizons and switch to more conservative investments as they grow older. Bodie, Merton, and Samuelson (1992), Bodie (2003) and Samuelson (1989), among others, have agreed with such a proposition. Agnew, Balduzzi, and Sunden (2003) in their empirical study find evidence of a downward trend in stock allocation with age among US retirement accounts over the period 1995–98. However, other studies find the reduction in stock exposure with age to be insignificant (see Ameriks & Zeldes, 2004; Gomes & Michaelides, 2005; Poterba & Samwick, 2003). An investment approach that explicitly becomes more defensive with age is usually referred to as a lifecycle plan.

Lifecycle strategies periodically rebalance the investments in line with a specific asset allocation target. The strategy invests massively in stock at the onset when the investor is still young, cutting back on stocks as the investor grows older. Utkus (2005) supports this as a prudent method as the younger investor is able to assume more risk than the older investor, adding that there is time to regain losses should they occur early in retirement. Other studies suggest the lifecycle as a reasonable default option with Malkiel (1999) agreeing that the plan participants should consider cutting back on riskier assets and increase investment in bonds (conservative assets) as they grow older. The lifecycle is probably the most popularly used default investment plan.

The optimality of this strategy has been questioned in recent academic literature. Shiller (2005) believes the earnings profile of individuals is hump-shaped; young people earn relatively less income compared to their older colleagues and earnings peak at middle age. Therefore investing heavily in stock in the early years when there is very little wealth and conservatively in the later years when earnings are bigger does not help in achieving an optimal allocation. Basu, Byrne, and Drew (2011) suggest a high investment in growth assets at the beginning of the investment cycle, when the investment is able to achieve the investor's wealth accumulation target, the asset mix is adjusted to reduce allocation to stock and increase allocation to bonds and cash (conservative assets). This is continually evaluated and the asset mix is readjusted to increase growth assets allocation if at any point in time, wealth value should run down below the investor's accumulation target.

DC plans often offer a great variety of investment choices to their members, from very conservative to more aggressive investment strategies. This confronts individuals with the challenge of choosing investment strategies that most suits their individual needs and risk preferences. Even with the varied investment options available, research has shown that the majority of retirees for various reasons choose to invest in the retirement fund's default option. As most plan members have limited financial knowledge, most default strategies simplify complex savings decisions, which in turn may encourage higher participation rates. Another reason for the high participation rates in default strategies, Beshears et al. (2009) suggest, is the so called endorsement effect. Individuals tend to perceive the default investment strategy as an endorsement of a particular course of action by the plan sponsor or provider. Whilst many retirees leave their retirement fund in the default investment because of non-participation, other retirees actively choose the default option as their investment of choice. The statistics on plan members in default investment strategies are startling with over 80% of Australian superannuation plan participants actively or passively choosing the default option. Other countries such as the United States of America and the United Kingdom have up to 80% and 60-100% of plan participants in the default strategy respectively (IOPS, 2012). On the pros of choosing such a default strategy, studies by Madrian and Shea (2000) show that default funds which are specifically designed to reflect the needs of pension plan members come with numerous benefits. Since some individuals have very little financial knowledge, a well-suited default plan simplifies the savings process thereby increasing participation rates. This benefit is echoed by Byrne, Blake, Cairns, and Dowd (2007).

In the decumulation phase, asset allocation remains an important part of investment and determines the sustainability of a retiree's portfolio. Bengen (2004) advises that if future markets follow the trends of past behaviour, then a retirement portfolio should contain 50-75% equity allocation; allocating the remaining to low-risk assets. Works by Milevsky (2001) and Ameriks, Veres, and Warshawsky (2001) demonstrate through simulation, the need for holding a substantial equity allocation in retirement portfolio. Cooley et al. (2001) propose that at least 50% of a retirement portfolio should be invested in stocks and their findings show increased sustainability of the pension fund as it tilts more towards equities. They show that the presence of bonds is mainly to restrain portfolio volatility and provide liquidity to cover an investor's living expenses. The benefit of an equity-dominated portfolio in retirement is well documented in literature.

Bierwirth (1994) formulates future financial and investment strategy decisions in retirement by using historical data of past retirees' investment decisions and outcomes to forecast possible future retirees' range of outcomes. He asserts that based on actual observed experiences, one is able to predict better the ranges of future outcomes. With this assumption, a number of studies on retirement fund sustainability predict future returns based on actual historical returns. Results from asset allocation models based on historical returns have however produced mixed results for the lifecycle strategy. Milevsky and Kyrychenko (2008) demonstrate that investors with downside protection in the form of 'longevity puts' assume 5% to 30% more risk exposure than the investors without such protection. This evidence shows that in the presence of insurance for asset allocation in retirement investment, plan participants defy the conventional approach to investment, where investors' allocation to risky assets decreases with increasing age and take up more risk. The need for a more aggressive portfolio in retirement is echoed in a study by Basu et al. (2011) who suggest that investors are richer on average with a glidepath that is in reverse to the conventional lifecycle path. Arnott et al. (2013) show that the lifecycle strategy fails to meet the basic objective for a retirement solution; resulting in lower terminal values compared to the static balanced and the inverse glidepath portfolio approaches. This subsequently results in a low level of annuity income generated from the lifecycle approach in retirement. Surz (2009) strongly disagrees with the proposal of an inverse glidepath arguing that the risks associated with such a model outweigh the investment returns as proposed. In his subsequent work, Surz (2013) stresses that what matters most in retirement planning is not the direction of allocations or glide paths but saving enough before retirement. He emphasises that no glidepath is able to compensate for inadequate savings. With disagreements on the glide path paradigm, this thesis seeks to explore how the need for equity dominated portfolios yielding superior returns and terminal wealth translates to retirees' asset allocation choices and the role of glidepath in such investments in the decumulation phase.

At retirement, the retiree is left with the task of ensuring that her nest egg is able to meet her income needs as long as she is alive. Several questions arise in the retiree's quest to meet this motive. How much income is sufficient to meet her needs in retirement? How long will her portfolio last if she continues to withdrawal at her preferred rate? How long is she expected to live? Should asset allocation be changed in the retirement phase? These are a few of the important retirement decisions that need to be addressed for a comfortable retirement. At retirement, the retiree may choose to fully or partially annuitise her wealth or self-annuitise. By annuitising, a retiree pays a non-refundable lump sum to an insurance company in exchange for a guaranteed life-long consumption stream that cannot be outlived. By self-annuitising, a retiree chooses to maintain full liquidity of her wealth by investing in a diversified portfolio and withdrawing a stipulated amount annually to meet consumption needs. This way, the retiree tries to mimic the income stream received from the annuity (Milevsky, 1998).

In his pioneering work, Bengen (2004) suggests that to withdraw from a retirement portfolio without exhausting the retiree's assets, he should adopt a "safe" withdrawal rate which is annually adjusted for inflation and a corresponding asset allocation in the retiree's portfolio based on her risk tolerance. For retirees 60-65 years, he advises a withdrawal rate of 4% and a 50-75% stock allocation. This formed the basis of the popular 4% Withdrawal Rule which is well accepted in both theory and practice. In his subsequent works, he presents findings which support the 4% withdrawal rule (Bengen, 1996, 1997, 2001). Cooley et al. (2001) make analysis based on portfolios using annual overlapping historical returns data on S&P 500 and high grade corporate bonds and suggest a relevant range of withdrawal between 3 and 4% as being sustainable for fixed periods of up to 30 years in retirement. In their most recent study, Cooley, Hubbard, and Walz (2011) suggest an annual withdrawal rate of 4 to 5% which is annually adjusted for inflation. Hubbard (2006) considers different withdrawal rates and various portfolio allocation strategies using S&P 500 and high grade US corporate bonds. He analyses the sustainability of withdrawals over fixed periods of up to 30 years using overlapping monthly asset returns and emphasises the benefit of holding an equity dominated portfolio.

Some influential studies use the Monte Carlo simulation to explore thousands of possible future scenarios of returns on assets. Ameriks et al. (2001) demonstrate through Monte Carlo simulation and by using historical returns, the need for holding a substantial equity allocation in retirement portfolio. Using a portfolio of returns on S&P 500, bonds and cash, they conclude, based on both approaches, that increasing the stock market exposure of portfolios increases the portfolios probability of sustaining systematic withdrawals. Their analysis suggests an annual withdrawal rate of 4.5% that is adjusted for inflation annually and different fixed horizons of up to 40 years. Pye (2000) uses the Monte Carlo simulation of returns to analyse the sustainability of various withdrawal rates in real terms, that is, after these rates have been reduced for tax and other expenses. He suggests up to 3.5% annual withdrawal over a 35-year pension horizon.

Works by Spitzer, Strieter, and Singh (2007)) and Spitzer (2008) have questioned the widely accepted 4% rule as being an 'oversimplification of a complex process'. They suggest the distribution process is a complex decision involving asset allocation, risk tolerance, withdrawal size and expected returns and a fixed 4% rule adjusted for inflation is not always a safe rate but a dynamic approach that performs better in the context of these complexities. More recent works by Zolt (2013) support the idea of a dynamic withdrawal approach to withdrawal, suggesting that retirees forego the proposed annual inflation increases when portfolio performance is less than expected. While Harris (2009) agrees that the distribution process is a complex one, he finds that sustainability is primarily determined by sequencing risk. He suggests withdrawal rates of 2 to 4% as being safe. For different underlying assumptions regarding the distribution of portfolio returns, Athavale and Goebel (2011) find that a withdrawal rate of 2.5% is sustainable for a 35-year retirement horizon.

With the 4% rule remaining debatable, researchers believe its underlying philosophy is useful in outlining the liability aspect of retirement planning and helps in retirees forming realistic expectations (Drew & Walk, 2014). There are several possible withdrawal strategies and it is important that the retiree makes a choice that optimises her utility based on her preferences and remaining life expectation. Popular distribution rules include the fixed withdrawal, fixed percentage of wealth, the '1/T' rule and the 1/E [T] rule, with studies comparing these strategies to life annuities (see Dus et al., 2005; Horneff, Maurer, Mitchell, & Dus, 2008). The fixed withdrawal strategy involves the retiree withdrawing a fixed dollar amount of her wealth every year. This amount may or may not be adjusted for inflation. The fixed percentage withdrawal is a strategy in which the individual makes a withdrawal of a defined percentage of her remaining wealth year after year. Although, the fixed percentage may fluctuate excessively with market volatility, the retiree never runs into retirement ruin. The '1/T' rule is a strategy in which one divides her wealth into a number of parts, the parts representing the maximum possible duration of her payout plan. The 'T' is usually set to the limiting age according to a mortality table and therefore diminishes with age, increasing her withdrawal as she consumes a fraction of her wealth each year. The third strategy, the 1/E[T] rule is similar to the second but the investor adjusts her consumption level annually based on her expected remaining lifetime rather than keeping a fixed lifetime expectation across the whole investment horizon. The optimality of these strategies and what influences retirees' choice of withdrawal strategy is an open discussion among researchers. The selection of withdrawal strategy that is best for retirees based on their preferences is also an important area for policy makers in post-retirement planning.

I believe modelling a withdrawal plan over a fixed horizon is an unsuitable method of analysis since there is no guarantee for life expectation over such fixed periods. It becomes an increasingly ineffective planning strategy when we put this in the perspective of decreasing mortality which is evident globally. The use of rolling estimates in financial planning, which assume that the returns on assets in the future will be in the exact sequence as experienced in the past, is another area of concern. This assumption is a weak one since there is a very small probability of this occurrence in the same sequence. The recent Global Financial Crisis (GFC) also gives a different view to the subject of investment sustainability. Although the withdrawal strategies discussed may suit the pre-GFC period, they may not necessarily be a perfect selection for planning since the possibility of another shock to the financial markets cannot be discounted and future planning needs to account for this possibility. To account for these shortfalls, the investment models proposed in this doctoral dissertation depend on stochastic lifetimes that are based on life expectation experiences from the appropriate life tables. I use block bootstrap resampling methodology that requires less restrictive assumptions about data distribution and characteristics. I also use a rich dataset spanning 85 years of daily returns on US stocks, bonds and bills (1928-2013). I discuss these models in detail in Chapters 3, 4, and 5.

2.3 Life Annuities

A life annuity is a series of payments made continuously or in equal intervals, such as monthly, quarterly or annually while a given life survives (Bowers & Hickman, 1997). Annuities may be limited to a given term of years or payable for the whole of life with payments beginning immediately or deferred over an agreed period. Annuities-due refers to annuity payments due at the beginning of the payments intervals while annuities-immediate commences payment at the end of the payment interval. For an agreed initial premium, a life company agrees to make regular payments to an annuitant over the agreed period. The existence of single premium life annuities dates back to the Middle Ages and detailed records exist for annuity pools in France as early as the 17th Century. Although annuities were available at different times, they only became commercially available to individuals in 1812, when a life insurance company in Pennsylvania began marketing contracts to individuals. The post stock market crash of 1929 marked the beginning of tremendous growth in the sale of annuities. People purchased annuities because they were deemed more stable than holding shares on the financial markets. The first variable annuity was sold in 1953 by the Teachers Insurance and Annuities Association College Retirement Equity Fund (TIAA-CREF) and since then many features have been added to the traditional fixed income annuity. Annuities have expanded to many parts of the world over the years (Poterba, 1997).

The United States of America (US) and the United Kingdom (UK) have very well-developed annuity markets. Though the US has the largest pension fund industry in the world, the UK has the biggest annuity market, setting up about 500,000 annuities each year. The Australian market is not as well developed as the US and UK. This may be due to the various options available to retirees in Australia such as the government sponsored means-tested pension scheme (Evans & Sherris, 2009). The demand for annuities is however growing; annuity sales have grown from $514 million in 2007 to $2 billion in 2011 and are expecting to further increase by 25% in 2012 (Uren, 2012). Life annuity sales by Challenger Ltd, Australia's biggest annuity provider, for the March 2014 quarter were $496 million, an increase of 29% on the prior corresponding period (Challenger Limited, 2014). Annuities are sold only by life insurance companies. In Australia, life insurance companies are controlled by the Life insurance Act 1995 and regulated by the Australian Prudential Regulation Authority (APRA).

There are many annuity types, with the primary ones being the term, life and deferred annuities. The term annuity pays the annuitant or beneficiary an agreed payout income over a fixed period whether or not the annuitant survives. The life annuity pays the annuitant regular income contingent on the annuitant being alive and ceases upon death. Finally, the deferred annuity offers payments to the plan participant only after a specified age determined at the time of purchase. This is referred to as the deferred period, and payments are made on condition that the annuitant survives the specified age. A fixed payout annuity provides periodic fixed dollar amounts to the plan participant, and the level of payment is usually agreed on at the time of purchase. Annuities can include a variety of features such as escalation guarantees in the form of inflation indexation using the Consumer Price Index (CPI) or variable payouts when the annuity is dependent on the performance of the underlying investment fund. There are also annuities which provide the option of a return of capital if the annuitant rescinds her decision within a limited period.

There is the Joint and Survivor (J&S) annuity, where the annuitant specifies that upon death, a surviving spouse will continue to receive income as long as the spouse lives. There is the period certain annuity which guarantees that if the annuitant dies early, a beneficiary will receive income payments until an agreed future period. Annuities can be purchased at any age, and at all ages females receive less payment per period than males. This is because females live longer on average and the life company compensates for making more payments by making lower payments (Milevsky, 2006). There are also enhanced annuities, providing income to people in poor health and terminal conditions such as cancer patients. These incomes are usually higher than incomes received by healthy annuitants.

In discrete time, the actuarial present value paying a unit amount at the end of each year the annuitant x survives. Whole life annuity immediate is denoted by:

$$a_x = (1 + L_x) \sum_{t=0}^{w-x} \frac{{}_tp_x}{(1 + r_t)^t} \tag{2.2}$$

a_x is the actuarial present value of the life annuity at age x
r_t denotes the yield on a zero coupon bond maturing at time t
${}_tp_x$ is the conditional probability of an individual age x surviving to age $x + t$
L_x is the Insurance Loading Factor charged at issue age x.

The Insurance Loading Factor covers the commissions, all expenses and taxes. This amount is multiplied by the pure annuity premium to arrive at the annuity's market price (Milevsky, 1998). The work of Mitchell, Poterba, and Warshawsky (2000) asserts that the loading factor increases with increasing issue age and this reasoning is applied to the annuity pricing in this thesis.

In his seminal work on the theory of a lifecycle consumer, Yaari (1965) asserts that in the absence of bequest motives, it is optimal for the investor to hold life annuities rather than liquid assets. Since the work by Makeham Yaari, there have been several studies emphasising the benefits of individuals holding life annuities in retirement. Duff (2001) suggests that annuities are the only products that offer the guarantee of a lifetime contingency that cannot be duplicated by any other financial strategy in investment. Ameriks et al. (2001) suggest that in the same logic of diversifying a portfolio to avert financial ruin, it is prudent to annuitise an individual's wealth to hedge longevity risk. Panis (2003) shows that retirees who purchase life annuities maintain their level of satisfaction during retirement, whilst retirees who do not purchase annuities become less satisfied over time. Davidoff et al. (2003) demonstrate theoretically the benefits and optimality of holding life annuities in retirement under less restrictive conditions of the Yaari model. More recently, Bateman and Kingston (2007) describe annuities as the only form of retirement benefit that insures against longevity risk while Blake, Wright, and Zhang (2011) suggest that a life annuity is a critically important component of a well-designed retirement portfolio since it absolutely caters for longevity.

With knowledge of the numerous benefits of annuitising, governments in some countries institute policies, which ensure mandatory annuitisation at certain ages after a period of phased withdrawals. In Chile for example, James, Martinez, and Iglesias (2006) find that up to two-thirds of all retirees have annuitized. They argue that this high rate of annuitisation is explained by the country's guarantees and regulations that place restrictions on payout choices, and insurance they receive through minimum pension guarantee as well as elimination of DB plan components, giving insurance companies selling annuities a competitive advantage. In Switzerland, the inclusion of annuitisation as a default choice in retirement, a legal guarantee of benefits and favourable tax structure among others are believed to explain the high rates of annuitisation according to Avanzi (2010). The Reister Personal Pension Account plan in Germany provides some tax inducements for participants who voluntarily save in Individual Pension Accounts (IPA) during the accumulation phase. This accentuates the government's interest in enhancing the asset accumulation of its citizens in Germany's ageing population. At retirement, a lump sum amount of 20% of the individual's accumulated assets can be withdrawn from the IPA and the remainder is drawn out in the form of a life annuity or a phased withdrawal[5] (Börsch-Supan, Coppola, & Reil-Held, 2012). The phased withdrawal amounts are either constant or increasing and relapse into an immediate annuity at age 85. In the UK, a portion of the accumulated funds can be withdrawn as a lump sum at retirement. Until April 2006, the plan participant was legally obligated to purchase an annuity at age 75 with the remainder of her accumulated assets Blake (2006). In Canada, the retiree must purchase a life annuity with her tax-sheltered savings or create a managed withdrawal plan at age 69 (Milevsky & Robinson, 2000).

[5]This was increased to 30% in the Riester's 2005 reforms.

In the United States on the other hand, annuitisation is not compulsory for 401(K) plan participants. The retirees thus roll them into Individual Retirement Accounts (IRA) and manage these accounts themselves. Tax laws require that minimum distributions begin at age 70.5. Similarly, there is no obligation to annuitise retirement wealth in Australia. Retirees have a range of benefit options with the retirement fund; they can convert their wealth into one or more lump sums, a phased withdrawal or a life or term annuity (Ganegoda & Bateman, 2008). Annuity sales in Australia have been steadily growing since the last quarter of 2011, an indication that the life annuity is gradually gaining some popularity in the market. Research predicts that 2013 should record the highest annuity purchases in Australia since 2004 (Plan for Life, 2013).

Despite the impact of government support and theoretical benefits of annuitisation well documented in literature, there is considerable empirical evidence of voluntary annuitisation aversion among investors (Blake, Cairns, & Dowd, 2003). This has led to what economists call the annuity puzzle. There are some conflicting yet interesting interpretations of the annuity puzzle. Firstly, people believe in receiving support from families and the ability to pool longevity risk within families and this deters them from annuitizing their wealth at retirement (Kotlikoff & Spivak, 1981). Friedman and Warshawsky (1990) suggest the 'actuarially unfair' pricing of annuities accounts for its low voluntary patronage under plausible assumptions. Their suggestion is based on findings that the annuity yield is emphatically depressed due to the loading factor imposed by the life office to cater for transaction costs and adverse selection among annuitants. Brown (2001) suggests this aversion could be attributed to investors' knowledge of their subjective survival probabilities. Therefore, investors in poor health will avoid purchasing annuities since the decision to purchase annuities is irreversible. Another reason for the annuity aversion, according to Munnell, Sundén, Soto, and Taylor (2002) is the presence of other annuitised resources such as Social Security and employer-sponsored defined benefits plans. In Australia, this could be explained by the availability of the means-tested public age pension.

The evidence of retirees aversion to annuitising based on bequest motives is however mixed. Hurd (1987) suggests that the bequest motive has no influence in the decumulation arrangements, and hence the annuitisation decision of the elderly. Bernheim (1992) refutes the no bequest argument and suggests that the decision not to annuitise wealth is due to strong bequest desire of individuals; he adds that many investors would not convert all their assets to annuities even in a perfect market. Brown (2001) finds that the bequest motive does not have a significant effect on the annuitisation decision and suggests that people in poor health are less likely to annuitise their wealth. More recently however, Lockwood (2012) finds that people with plausible bequest motives are likely to be better off not annuitising any wealth at available rates.

According to Ganegoda and Bateman (2008), the demand for life annuities in Australia is very low relative to other developed countries such as the UK, the US and Switzerland. This is unfortunate as empirical and theoretical utility maximising studies by Poterba and Wise (1998) and Mitchell et al. (2000) show that even with high insurance loading factors, the individual with rational utility function parameters is better off annuitising. The latter study concludes that the life annuity is attractive because of the protection it provides individuals against the risk of outliving their own assets, given uncertainty of remaining lifetime.

With increasing life expectancy and uncertain financial market returns, the annuitisation topic remains a field of great interest. Life companies are constantly finding ways to make annuities attractive to retirees and their efforts need to be complemented by policy makers to help reduce the cost of age care, especially in developed countries with ageing populations. A government-supported annuity scheme will help ease the burden on the private sector and reduce adverse selection. This will encourage competitive products and reduce the loading factor on annuities making them more attractive. Governments can also provide viable markets for hedging longevity risks facing the life annuity providers by offering securities such as longevity bonds, which pay future returns based on an index of future mortality, or other reinsurance based contracts (Blake, Cairns, Dowd, & MacMinn, 2006). Some researchers believe that since the government has the strongest credit rating, it is able to provide the assurance of contract performance that many private sectors cannot (Evans & Sherris, 2009). I strongly believe that annuitising should be an important part of a retirement plan and this doctoral thesis seeks to find innovative ways of including annuities in a retirement plan and making the annuity attractive to the ordinary retiree.

2.4 The Evidence and Influence of Bequest Motives

The evidence of the presence of bequest motives among the elderly is undeniable; however, the reasons for bequests and its influence on individual decision making remain unresolved in academic literature. Early works by Marshall (1949) acknowledge that the elderly indeed have bequest motives and believes that family affection is the driving factor for saving. Becker and Tomes (1976) suggest that parents spend their resources to improve the quality or lifetime income of their children and they derive utility in doing so irrespective of what the children may do with their boosted income. Findings by Masson (1988) also suggest that households derive some utility from bequests. Of striking significance is the work of Kotlikoff and Spivak (1981) who find that at least 80% of US capital stock is accrued through intergenerational transfers[6]. Laitner and Juster (1996) [hereafter referred to as L&J] find that about half the respondents in their dataset of TIAA-CREF households have significant bequest motives. Their work draws conclusions based on data from both households with and without children. More recent works on the presence of bequest motives by Kopczuk and Lupton (2007) find that households with bequest motives spend about 25% less on consumption expenses compared to households without bequest motives. They also find that both the presence and magnitude of bequest motives are statistically and economically significant. They conclude that among elderly single households, about 80% of their net wealth will be bequeathed with approximately half this amount attributed to bequest motives. More recent studies by Ameriks, Caplin, Laufer, and Van Nieuwerburgh (2011), who investigate Public Care Aversion (PCA) and bequest motives among individuals, find that the desire to leave a bequest is more prevalent than previously believed.

There are others who believe that intergenerational transfers should not be necessarily concluded as individuals meaning to meet bequest motives. According to Modigliani (1988), the pure bequest motive, the accumulation of wealth solely for the purpose of being distributed to heirs and not for an individual's own consumption, is a characteristic of a rather small number of households. These are mostly located in the highest income and wealth brackets. Among the lower income brackets, intergenerational transfers are mostly a precautionary motive reflecting uncertainty about the length of life and not due to a pure bequest motive. He asserts that bequests originating from the precautionary motive are different by nature from those dictated by the bequest motive, with the former belonging to the pure life cycle accumulation.

[6]Based on his empirical work, Modigliani strongly disagrees with this finding and claims that life-cycle saving was the primary source of capital accumulation and share of wealth received through transfer does not exceed one-fourth (Modigliani, 1988). Ando and Modigliani (1963) proposed the life-cycle hypothesis of saving under the key assumption that individuals neither expect to receive nor desire to leave inheritance.

Hurd (1987) hypothesised that if the pure bequest motive is an important source of terminal wealth, then retired households with living children should have more wealth and consequently save more than households without children. He however finds that households with children have similar decumulation patterns as households without children and concludes that the bequest motive is not important for a broad cross section of households. To this, I believe the work of L&J employs a more generalised and acceptable dataset unlike Hurd, who assumes the set of households with children to be the set of households with bequest motives. In his subsequent work, Hurd (1989) asserts that the bequest motive is egoistic, triggered by the desire of the elderly to leave behind a positive net worth at time of death. He claims that most bequests are accidental and are a result of precautionary measures taken by individuals in the absence of complete insurance markets.

Whether bequests are a result of pure bequest motives or precautionary savings, their presence cannot be ignored. Distinguishing between pure bequest and precautionary motives is extremely difficult as precautionary savings can also serve the bequest motive (Dynan, Skinner, & Zeldes, 2002, 2004). The relevance of incorporating the bequest desires of retirees into their portfolio build-up and withdrawal rate selection is important. Jousten (2001) studies the consumption and wealth profiles of retirees in the presence of bequest motives. He introduces incorporating the bequest motive into the standard life-cycle model and finds that consumption is monotone non-increasing in the linear bequest parameter for the simplest certainty case; however this result does not hold when life-span uncertainty is incorporated in the model except in the case of a strictly interior solution. The impact of bequests on consumption and saving decisions still remains an open discussion. This is important as assumptions regarding the desire to leave bequests are a central element to policy recommendation related to the distribution of wealth, taxation, and charitable contributions in many countries. This thesis aims to address the gap by providing insight into this aspect of pension decumulation and seeks to identify suitable consumption patterns as well as investment strategies for individuals with substantial bequest motives.

2.5 Epstein-Zin Preferences and Utility

In the intertemporal additive expected utility standard model, the intertemporal EIS is confined to the inverse of the coefficient of RRA. As a result, RRA and EIS cannot be distinguished(Attanasio & Weber, 1989; Hall, 1985; Zin, 1987). Previous attempts to remedy this issue have led to intertemporal inconsistencies. As Blake et al. (2011) explain, risk aversion measures the desire of individuals to stabilise their consumption across different states of nature in a given time period. An individual with a high degree of risk aversion will want to avoid uncertainties in her consumption in a given time period, or a reduction in consumption in a bad state of nature such as a fall in equities. EIS measures the desire of the individual to smooth consumption across time. An individual with low EIS will avoid consumption volatility over time and more so will seek to avoid a reduction in her consumption level relative to previous period consumption level.

Following the work of Kreps and Porteus (1978), Epstein and Zin (1989) introduce a class of utility preferences which enable disentangling relative risk aversion from the elasticity of intertemporal substitution. Using the Epstein-Zin utilities,[hereafter referred to as E-Z], investor's preferences are defined recursively over current consumptions and the certainty equivalent of next period utility. Empirical observations support the disentangling of relative risk aversion from elasticity of intertemporal substitution. Using the term structure of asset returns, Schwartz and Torous (1999) disentangle the EIS from the RRA based on Breeden's (1979) capital asset pricing model (CAPM). They show that while a high RRA is associated with a low EIS, the inverse relations as assumed in the power utility are not an exact estimate. A more recent empirical study by Blackburn (2008) found that RRA over the period 1996-2003 changed dramatically whilst the EIS remained fairly constant. His conclusion is based on observed S&P 500 option prices with a range of expiry dates over the period and rejects the assumption of a reciprocal relationship between the two concepts.

The work by E-Z has become a standard tool in intertemporal models. Previous use of this model in pension plans includes works by Horneff, Maurer, Mitchell, and Dus (2008) and Blake et al. (2011). Horneff, Maurer, Mitchell, and Dus (2008) analyse the utility cost of selecting different restrictive annuitisation strategies over the gradual annuitisation using the E-Z approach. They find changing EIS to have a substantial influence on asset allocation decision since life annuities offer a constant consumption level that may be preferred by investors with low EIS. Blake et al. (2011) analyse optimal funding and investment strategies of a rational lifecycle planner under E-Z preferences and find EIS to have a marginal impact on optimal asset allocation both before and after retirement. They assert that when EIS is greater than the inverse of RRA, it is optimal after retirement for the individual to spend less than the pension received and invest the saving in annuities.

The use of E-Z preferences is relevant in this study as it opens up an insightful assessment of retiree preferences and allows for an analysis of varying degrees of RRA and EIS on consumption decision choice. Long term investors focus on the consumption stream that a level of wealth is able to provide instead of the wealth level itself as Campbell and Viceira (2002) note that they consume out of wealth and derive their utility from consumption rather than from wealth. As a result, saving and investment decisions in retirement are primarily driven by retirees' preferences between current and future consumption. As Chew and Epstein (1989) explain, it is perfectly rational for an individual to care about the way consumption uncertainty resolves over time.

Following the work of Horneff, Maurer, Mitchell, and Dus (2008) and Blake et al. (2011), the E-Z utility preference in this thesis is adapted to include the mortality risk as observed from the Australia life table, Australian Life Tables 2005-2007 (2009). The utility function is defined over a single consumption pattern that is selected at retirement and remains the effective wealth decumulation strategy through retirement. A retiree aged,x, has preferences defined by a discrete-time recursive utility function denoted by:

$$U_x = \left\{ (1-\beta) \times (C_x)^{1-\frac{1}{\phi}} + \beta p_x \times \left(E_x[(U_{x+1})^{1-\gamma}]\right)^{\frac{1-\frac{1}{\phi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\phi}}} \tag{2.3}$$

U_x is the level of utility at age x
C_x is the consumption level at age x
p_x is the one year survival probability at age x
β is the individual's personal one-year discount factor
γ is the level of Relative Risk Aversion (RRA)
ϕ is the level of Elasticity of Intertemporal Substitution (EIS).

The last surviving age in this analysis is age 100 when we assume $p_{101} = 0$ and with individuals having no bequest motives, the recursive equation for the terminal condition is given as:

$$U_{100} = \left\{ (1-\beta) \times (C_{100})^{1-\frac{1}{\phi}} \right\}^{\frac{1}{1-\frac{1}{\phi}}} \tag{2.4}$$

I include the impact of bequest motives on the utilities retirees derive from their consumption and wealth[7] assuming retirees derive their total utility from consumption as well as the possibility of meeting their bequest motives. Motivation for the modelling of bequest motives is discussed in works of Laitner (2002) and Gomes and Michaelides (2005). To include the impact of bequest motive on the utility analysis, I follow the study by Gomes and Michaelides (2005) and use a slightly different form of the E-Z utility function given by:

$$U_x = \left\{ (1-\beta p_x) \times (C_x)^{1-\frac{1}{\phi}} + \beta E_x \left[(U_{x+1})^{1-\gamma} + (1-p_x) b \frac{(W_{x+1}/b)^{1-\gamma}}{1-\gamma} \right]^{\frac{1-\frac{1}{\phi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\phi}}} \tag{2.5}$$

with the terminal condition being given as:

$$U_{100} = b \frac{(W_{101}/b)^{1-\gamma}}{1-\gamma} \tag{2.6}$$

where b determines the strength of the bequest motive and W_x is the remaining wealth level at age x and all remaining parameters as defined earlier.

2.6 Measures of Investment Performance

Measuring investment performance is of significance to this thesis as I evaluate different investment strategies and portfolio distribution rules as well as asset allocation strategies. For evaluating investment strategies, the most widely used model in finance is the classic mean-variance analysis of Modern Portfolio Theory (MPT) as developed by Markowitz (1952). Markowitz uses expected return of individual assets and their variance-covariance to derive an efficient frontier such that all portfolios lying on this frontier have maximised their expected returns for their levels of variance. This gave investors a formal mathematical approach to asset selection and portfolio management. The main shortcomings of Markowitz risk-return approach is the assumption of variance giving a complete assessment of risk for portfolios and the assumption of returns following a normal distribution. The assumption of normal returns is rejected by many analysts as well as practitioners (Michaud, 2008). It is generally accepted that portfolio selection based on the mean return and variance of return is a simplification relative to including additional moments that might

[7] The presence of bequest motives is one important explanation given to explain the annuity puzzle. It is believed that retirees with bequest motives are reluctant to annuitise since they lose their liquidity by purchasing annuities with their retirement wealth.

more completely describe the distribution of returns of the portfolio. Studies by Lee (1977) and Kraus and Litzenberger (1976) suggested different portfolio measures that included higher moments such as skewness while other studies such as Fama (1965) and Elton and Gruber (1995) provide more realistic descriptions of the distribution of returns of portfolios. More recently, theories such as the Omega Ratio by Keating and Shadwick (2002) enable the incorporation of all portfolio return moments in the calculation and ranking of alternative portfolios. I use the Omega Ratio approach to evaluate alternative investment strategy performances discussed in the second essay of this thesis.

2.6.1 The Omega Ratio

The Omega Ratio proposed by Keating and Shadwick (2002) is a preference-free performance measurement that is used to rank and evaluate portfolios. It is able to capture higher moment effects beyond the traditional mean-variance for a financial returns distribution and requires no strict assumptions regarding the distribution of returns as they are all encoded in the Omega Ratio. The Omega is especially suited for financial performance where investors are interested in the risk and reward characteristics of various investment options. It involves partitioning returns into losses and gains relative to a return threshold. For each return level, Omega Ratio calculates a probability weighted ratio of returns above and below the partitioning. It operates under the simple rule of investors preferring more to less. Given the choice between two portfolios with the same predicted return, investors should prefer the portfolio with the highest Omega Ratio. This maximises the potential for making the desired level of return, and minimises the probability of extreme losses.

Keating and Shadwick (2002) define the Omega as:

$$\Omega(u) = \frac{\int_u^b [1 - F(x)]dx}{\int_a^u [F(x)]dx} \tag{2.7}$$

Where (a, b) is the interval of returns
F is the cumulative distribution of returns
$\Omega(r)$ is the probability weighted ratio of gains to losses relative to the threshold r

The Omega Ratio requires no estimates, unlike standard statistical estimators, and as such faces no sampling uncertainties as inference are made directly from the observed distribution. Keating and Shadwick assert that the different ranking of funds and portfolios derived from alternative measures such as Sharp ratios, Alphas or Value at Risk is due to the additional information incorporated in Omega. When higher moments are of little significance however, we find the Omega to agree with the traditional measures of portfolio performance. For a range of thresholds covering a distribution, the different Omega Ratios for the range of thresholds form an Omega Function.

2.6.2 Alternative Shortfall Analysis

Different investment and income shortfall measures are used in the third essay of this study. To compare asset allocation strategies quantitatively, we usually require a risk-return framework for decision making under uncertainty. One of the approaches taken by financial economists is to maximise expected discounted value of a time-separable utility function for uncertain future benefits, which may include expected bequest in some cases. The constant relative risk aversion utility is commonly used and I refer to studies by Blake et al. (2003) and Milevsky and Young (2007) who use such an approach. A limitation of this approach as Pye (2000) points out is the lack of an explicit measure of retiree's risk aversion. I refer to risk-value approach models which use explicit measures of risk and value with functions reflecting the associated trade off. I compare strategies based on their actual observed value levels and accompanying standard deviations. I measure return as being the level of income generated from retiree's wealth as well as the expected possibility of bequest, and risk as the possibility of not reaching a desired level of consumption. Previous studies define the operative risk measure as the probability of ruin; which occurs when the retiree runs out of money before her uncertain date of death after regular periodic withdrawals to meet consumption. Ruin occurs whenever the retiree's desired consumption exceeds her available wealth level at the time of consumption. This is demonstrated in previous studies (see Albrecht & Maurer, 2002; Ho, Milevsky, & Robinson, 1994; Milevsky, 1998; Pye, 2000). Milevsky and Robinson (2000) explain that the probability of ruin is the probability that the Stochastic Present Value (SPV) is greater than the initial wealth to support the consumption.

Expressed in a mathematical framework, an individual is currently age x with an initial wealth of $W_0 = w$ and a desired lifelong consumption stream of k real dollars per annum. For a fixed time of death, T, usually represented by the terminal age on the life table used and a fixed real interest rate r, the present value of the individual's desired consumption is given by:

$$k\bar{a}_{T\rceil r} = k\int_0^T e^{-rt}dt = \frac{k(1-e^{-rt})}{r} \tag{2.8}$$

If Equation 2.8 is greater than the initial wealth, w, the individual does not have enough to meet her consumption stream and ruin occurs with certainty.

Further to shortfall analysis based on explicit terminal wealth levels, as Campbell and Viceira (2002) suggest that long term they consume out of wealth and derive their utility from consumption rather than from wealth, I consider other shortfall measures based on consumption levels. These are income levels that the retiree is able to generate from her level of wealth. Arnott et al. (2013) further echo that for retirement purposes, it makes sense to gauge the success of a portfolio in terms of annuitised income rather than notional portfolio gains or losses. I follow the work of Dus et al. (2005) and incorporate into the shortfall calculations the size of shortfall when it occurs. This measure incorporates the possible size of shortfall should it occur. Based on the resulting wealth levels of wealth from alternative asset allocations in retirement and the resulting income levels, the size of shortfall is measured relative to benchmark such as income generated by a life annuity purchased at retirement. Shortfall Probability (SP) relative to an income benchmark is represented as:

$$SP(B_t) = P(B_t < z) \tag{2.9}$$

This is a measure of the probability that an income or withdrawal level B_t, derived from the individual's current wealth is less than the chosen benchmark z, a life annuity income level. A further shortfall measure to incorporate the severity of shortfall referred to as the Shortfall Expectation (SE) by Dus et al. (2005) and denoted by:

$$SE(B_t) = E[max(z - B_t, 0)] = MEL(B_t) \cdot SP(B_t) \tag{2.10}$$

Shortfall Expectation represents the sum of losses weighted by their probabilities, the unconditional average loss. MEL is the Mean Excess Loss and measures the conditional expected shortfall given that shortfall occurs, measuring how badly on average a strategy performs (Artzner, Delbaen, Eber, & Heath, 1999). This measure is expressed as:

$$MEL(B_t) = E[z - B_t | B_t < z] \tag{2.11}$$

This thesis bases on these shortfall measures to analyse income levels resulting from wealth accumulations from alternative asset allocation strategies in retirement.

Chapter 3

Comparative Study of Post-Retirement Withdrawal Plans

3.1 Introduction

Wealth decumulation choices have become particularly important in recent years with baby boomers moving into retirement and the manifold growth in number of Defined Contribution (DC) plans over Defined Benefit (DB) plans around the world. In a typical DC plan, the benefits depend on the accumulated value of the participant's contributions and the investment returns. The participant has a choice to receive the benefits as a lumpsum at retirement or annuitise. Empirical evidence, both in the US and other countries, suggests that the former is overwhelmingly preferred to the latter. In doing so, most participants effectively assume the responsibility of managing their own retirement assets to meet consumption needs over their remaining lifetime. This is a significant responsibility as their self-managed decumulation plans need to bear the associated investment and longevity risks.

It is commonly believed that the life annuity is the best insurance against a retiree outliving her wealth. Yet the decision of whether to annuitise one's retirement savings is a complex one. Apart from the prevailing annuity prices, several other factors including availability of government pensions, family support, life expectancy, and bequest motives need to be considered. There is much empirical evidence demonstrating retirees' aversion to voluntary annuitisation. Whilst many researchers attribute this to the high cost of annuities, the impact of other factors is also highlighted in the extant literature. For example, Brown (2001) suggests this aversion could be attributed to investors' knowledge of their subjective survival probabilities where investors in poor health will tend to avoid purchasing annuities since the decision to purchase annuities is irreversible. The presence of bequest motives can also discourage annuitisation (Bernheim, 1992).

In this study, I investigate the alternative decumulation choices for Australian retirees. Given the annuity aversion observed among Australian retirees, I explore the desirability of following alternative decumulation plans, akin to self-annuitisation by the retirees, vis-à-vis purchasing a life-annuity at retirement. In each of these plans, the retiree invests in a pool of assets and withdraws a fixed or variable amount periodically to support consumption. These decumulation plans have advantages in terms of meeting liquidity requirements, offering the possibility of greater consumption over a lifetime, and bequeathing unspent assets in the event of early death. However, they can also expose the retiree to investment and longevity risk. Uncertainty in investment returns means that the value of pension assets would be volatile and the corresponding withdrawal amounts can rise above or fall below the level of benefits offered by a life annuity. Also, the uncertainty about remaining lifespan means the retiree would always run the risk of outliving her assets irrespective of whether they make fixed or variable withdrawals from their pool of assets.

The Australian case is of particular interest due to its unique institutional and policy settings. Australia has had a compulsory superannuation system since 1992 which makes it mandatory for all employers to make contributions towards their employees' superannuation to a fund of their choice. The minimum contribution rate is currently set at 9.5% of an employee's gross earnings but will increase to 12% by 2022. The system is dominated by DC plans which hold 83.6% of the $1.6 trillion total superannuation assets as at 30 June 2013 (APRA, 2014). Despite putting in place a regime backed by strong public policy to ensure retirement savings while in employment, Australia has no defined policy for decumulation as workers move into retirement. Once retired, workers have the option to withdraw their accumulated savings either as a lumpsum or as a series of regular payments over an identifiable period of time. If a retiree opts for the former, she can either self-manage her assets (and spending withdrawals) or purchase a life annuity from the market providers at the prevailing price. If the latter option is taken, the retirement savings either continue to be invested by the fund in nominated assets from where the retiree can make withdrawals as required[1], or exchanged for a regular income usually guaranteed for life or for a fixed term. Therefore, irrespective of whether the retiree opts for lumpsum or phased withdrawal, a choice has to be made by the retiree between self-managing their decumulation and annuitisation.

The uniqueness of the Australian system is also characterised by a universal, means-tested age pension. Unlike the pay-as-you-go Social Security system in United States, the age pension in Australia is paid from general tax revenues and provides all citizens above 65 years of age with an income stream the rate of which is determined by their income and asset level relative to certain thresholds [2]. Thus the age pension provides a basic level of sustenance for all retirees. However, even in the presence of this safety net, retirees require supplementary income to maintain a comfortable lifestyle in retirement. The Association of Superannuation Funds of Australia (ASFA) Retirement Standard estimates this amount to be $41,190 per year an individual retiree and $56,236 per year for a retired couple (ASFA, 2013)[3]. The full rate of government age pension as of September 2013 provides an annual income of $21,913 per retiree and $33,036 per couple. The full rate pension serves about 41% of the 76% of Australians over 65 years of age who are on some form of government pension. The remainder of retirees depend on the part rate age pension and pension income through the Department of Veteran Affairs (CRIR, 2013). Therefore, even in the presence of government support, it is essential for the retirees to manage their decumulation efficiently in order to live a comfortable retired life.

The uniqueness of this study is the inclusion of a pension model in the retirement decision making. I use the Means-Tested Age Pension, which is characteristic of the Australian pension system. A previous study by Butt and Deng (2012) includes the presence of the means-tested government pension in their asset allocation study. They find that a 100% allocation to growth assets is optimal for retirees who desire large expenditure relative to their initial balance. Other asset allocations are sensitive to changes in initial balance and desired expenditure levels, as well as interactions with the Age Pension. The influence of government sponsored pensions has hitherto, not been considered in decumulation drawdown choices by retirees. I investigate decumulation drawdown plan choices for retirees in the presence and absence of these pensions. I also control for bequest motives and allow for mortality improvements.

[1] Payments in this case need not be made at regular intervals and the payment amount can also vary. However, there is a minimum withdrawal amount specified under current legislation.

[2] I elaborate on the means-tested age pension in the next section.

[3] These estimates are based on the assumption that the retiree owns her residential dwelling.

Whereas previous studies addressing the issue of withdrawal plan choice adopt the CRRA utility, recent studies have shown that individuals prefer early resolution of uncertainty and have RRA levels, which are greater than the reciprocal of the EIS. This finding is consistent with the predictions by recursive preferences such as the Epstein-Zin (Brown & Kim, 2013). I incorporate this finding into this study, with retirees desiring smooth consumption over a variable consumption and deriving utility from consumption smoothing. The model retiree has Epstein-Zin preferences, which enables a separation between RRA and the EIS. Whilst risk aversion relates to the retiree's desire to stabilise consumption across different states of nature in a given time period, the EIS measures the desire of the individual to smooth consumption over time. In the real world, since returns on investments are volatile, variable withdrawal strategies return varying consumption levels that may not be utility enhancing for a retiree with the desire to smooth her consumption across time and different market states. For this reason, the Epstein-Zin preferences provide a very useful basis for making comparisons between different forms of self-annuitisation and annuitisation. This is another innovation this study introduces to the literature.

My results are generally consistent with the existing literature on asset allocation for sustainability of self-managed portfolios. I find that an increase in the allocation to equities in the retiree's portfolio reduces the shortfall probabilities. However, higher allocation to equities also increases the volatility in returns and withdrawal levels for variable withdrawal strategies. A retiree with Epstein-Zin preferences who may want to reduce the variability in her level of income will moderate the equity allocation of her retirement fund to maintain a reasonably smooth consumption. I find the ranking of alternative strategies to be different from those observed under the CRRA utility assumptions. Pensions and bequest motives also play significant roles in retirees' choice of withdrawal. Annuities dominate phased withdrawals in the absence of bequest motives and are the least preferred strategy otherwise. The presence or absence of pension income determines which of the alternate withdrawal strategies dominates in my set up. The impact of pension is thus seen as a significant condition for retiree's choice of drawdown program.

The remainder of this study proceeds as follows. Section 3.2 reviews the literature on sustainable portfolio withdrawals, pension income, life annuities, and post-retirement distribution choices. The data and methodology is described in Section 3.3. Section 3.4 presents the analysis of my results and Section 3.5 concludes.

3.2 Literature Review

3.2.1 Sustainable Withdrawal Strategies

Bengen (2001) suggests that if the future market conditions follow historical trends, a retirement portfolio should contain 50-75% equity allocation. For retirees aged between 60 and 65 years, he suggests an annual inflation-adjusted withdrawal rate of 4.3%, and the corresponding asset allocation to be based on investors risk tolerance. In his subsequent study, Bengen (2004) proposes a 'safe' withdrawal rate of 4.58% when the withdrawal rates are allowed to vary over the retirement years based on investment performance. Other studies by Cooley et al. (2001) and Hubbard (2006) are in agreement that a retiree's pension fund should be composed of a significant amount of equities to be sustainable. The former argues that the presence of bonds is mainly to restrain portfolio volatility and provide liquidity to cover an investor's living expenses. They suggest withdrawal rates between 6-8% as being sustainable up to 30 years in retirement. They use annual overlapping historical returns data in their analysis. The latter study analyses the sustainability of different withdrawals rates over fixed periods of up to 30 years by using overlapping monthly asset returns.

Other important studies by Ameriks et al. (2001), Pye (2000), Stout and Mitchell (2006) and Mitchell and Kovar (2011) use the Monte Carlo simulation method to explore a wide range of possible future scenarios under different asset returns. Their conclusions are in unison emphasising the need for an equity dominated retirement portfolio. However, these studies are not in agreement about sustainability of withdrawal rates, with prescription ranging from 3.5-8% of retiree's remaining wealth. Although the Monte Carlo simulation remains an acceptable approach to portfolio analysis, I believe the general acceptance of a single withdrawal rate for annual wealth decumulation is an oversimplification for countries like Australia that have regulations regarding minimum drawdown rates for tax purposes. Rather than a fixed percentage of wealth used in previous literature, my study uses withdrawal rates based on the Australian Government's minimum drawdown rules. I elaborate on this withdrawal strategy in Section 3.3.

3.2.2 The Age Pension

The age pension referred to in this study is the Australian means-tested age pension. Australians who reach the required age are eligible for this age pension. The current required age for a male is 65 and 64.5 for female, increasing to 67 by July 2023. The single rate age pension is set to 27.7% of Male Total Average Weekly Earnings (MTAWE). The pension payment is indexed to the CPI. The amount of pension a person receives depends on several factors such as the individual being single or with a partner, and owning a house or not. Most importantly, the pension amount is directly linked to the lower of two assessments; an income and asset test.

Income levels lower than $152 per fortnight and assets less than $196,750 for a homeowner make her eligible for full rate pension. The maximum fortnightly income for a single female homeowner as at March 2013 is $808.40. This rate decreases with increasing level of assets and income, providing the retiree with a part-rate pension. Beyond an upper threshold, which is $1,768.80 income per fortnight and $735,750 in assets, the individual is disqualified from receiving any pension income. The individual's income is based on the lower of the two tests and is disqualified if she fails either of the tests. The assets and income tests applied are based on reported figures as at March 2013.[4]

3.2.3 Life Annuities

The annuity is the major type of longevity risk management security available on the market. Many investors believe the best way of insuring against longevity risk is by purchasing a life annuity, a financial product that allows the purchaser to receive periodic payments, fixed or variable, as long as they are alive, in exchange for an initial premium (Brown, 2001). There has been a long-standing interest in researching life annuities. In his seminal study on the theory of a lifecycle consumer, Yaari (1965) asserts that in the absence of bequest motives, it is optimal for the investor to hold life annuities rather than liquid assets.[5] Davidoff et al. (2003) demonstrate theoretically the benefits and optimality of holding life annuities in retirement under less restrictive conditions of the Yaari model.

[4]See http://www.humanservices.gov.au/customer/services/centrelink/age-pension for further details on Australia's means-tested age pension.

[5]The existence of annuities date back to the Middle Ages and detailed records exists for annuity pools in France as early as the 17th Century (Poterba, 1997). Annuities became very popular after the Great depression of the 1930s, which followed the stock market crash of 1929. People purchased financial products from life insurance companies because they were deemed more stable than holding shares on the financial markets. Since then annuities have expanded to many parts of the world over the years.

There are many annuity types; this research however focuses on the level Single Premium Immediate Annuity (SPIA). The annuity provides fixed level periodic payouts that commences one period after purchase. Annuities can include a variety of features such as escalation guarantees in the form of inflation indexation and return of capital. In discrete time, the fixed immediate annuity paying one dollar per year contingent on the survival of the annuitant is priced as:

$$a_x = (1 + L_x) \sum_{t=0}^{w-x} \frac{{}_tp_x}{(1+r)^t} \tag{3.1}$$

Where a_x is the cost of a \$1 income paid annually for life for an annuitant age x, ${}_tp_x$ denotes the conditional probability of an individual age x surviving to age $x+t$, the discount function which is expressed by $1/(1+r)^t$ with r being the discount rate or the interest rate at which the annuity is compounded annually. I include in the annuity pricing, an Insurance Loading Factor, L, charged at issue age x. The Insurance Loading Factor covers the commissions, all expenses and taxes. This amount is multiplied by the pure annuity premium to obtain the annuity's market price (Milevsky, 1998). The work of Mitchell et al. (2000) asserts that the loading factor increases with increasing issue age and this reasoning is applied to the annuity pricing in this study.

3.2.4 The Distribution Plan Choice

The ostensibly poor value of life annuities has driven the search for new retirement-income drawdown programs. Retirees seek to match the benefit of life annuities through these programs whilst still maintaining their liquidity and bequest potential. The natural alternative to annuitisation is investment among the various asset classes such as equity, fixed income and real estate with periodic withdrawals, which creates an income stream equivalent to that generated by the annuity to support consumption and bequest, in some cases, during retirement. The choice of drawdown plan, commonly referred to as self-annuitisation or phased withdrawal plan depends on several factors such as adequacy, reliability and the possibility of leaving a bequest for heirs. Pension programs such as the Riester plan in Germany provide tax inducement and encourage life-long pension income for individuals in an annuity fashion. Until April 2006, plan participants in the UK were legally obligated to purchase an annuity at age 75 with the remainder of their accumulated assets. In Canada, a retiree must purchase a life annuity with her tax-sheltered savings or create a managed withdrawal plan at age 69 (Milevsky & Robinson, 2005).

Some important studies on the decumulation distribution plan choice include works by Blake et al. (2003) who compare the life annuity to different distribution programs with differing equity exposures assuming mandatory annuitisation at 75. They find the best distribution program does not usually involve a bequest, the most important decisions depend on equity distribution, the optimal program also depends on relative risk aversion and the program choice fairly insensitive to bequest. Dus et al. (2005) quantify the risk and return profiles for fixed and variable withdrawal strategies to a life annuity benchmark using a shortfall framework. Using German data, they compute the expected present value of benefit payments and bequest potential, analysing four phased withdrawal programs; a Fixed, Fixed Percentage, 1/T rule and 1/E[T]. 'T' refers to the remaining lifetime according to the chosen life table and 'E[T]' refers to the individual's expected lifetime. They find 1/E[T] is attractive as a standalone strategy but not attractive with mandatory annuitisation. Another important study by Horneff, Maurer, Mitchell, and Dus (2008) compares life annuity to phased withdrawal strategies and a combination of both. They adopt a Constant Relative Risk Aversion utility framework and work with differing levels of risk aversion. The study concludes that the Fixed Percentage is preferred across a broad range of risk preferences while the 1/E[T] is preferred at low and moderate risk aversion but not for the very risk averse. Fixed strategy is not appealing to retirees because of the chance of ruin, and they find the 1/T strategy to be the worst performer over the long term. The annuity is attractive for high risk averse retirees with no bequest motive.

This study compares the level income SPIA to three withdrawal strategies, the Fixed, the Fixed percentage, hereafter referred to as the Percentage of Wealth strategy and the 1/E[T] which I refer to as the Life Expectancy withdrawal strategy. I omit the fourth strategy, the 1/T rule as previous studies find this to be the worst performing strategy. I use Australian returns data and mortality data and find the dominant strategies at various conditional survival ages for retirees with Epstein-Zin preferences. I allow for mortality improvements and further analyses incorporate the impact of bequest motives. A novelty of this study is the inclusion of pension income and an investigation of its impact on retirees' choice of drawdown strategy when they have Epstein-Zin preferences.

3.3 Data and Methodology

3.3.1 Data

An important factor in any robust analysis on retirement planning is the inclusion of mortality. Many earlier pension models that disregarded the stochastic nature of life expectation overstated or understated the chances of portfolio sustainability and ruin. Recent studies however, make use of the stochastic life expectations to model the pension plans. Milevsky and Robinson (2005) introduce the concept of stochastic present value in the retirement withdrawal modelling whilst Stout and Mitchell (2006) perform sustainable spending analysis under stochastic life expectation.

The model used in this study depends on stochastic lifetimes based on life expectation experiences from the Australian life tables 2005-2007 provided by the Australian Government Actuary (AGA). This life table provides mortality experiences and captures the mortality observations among Australians over the three-year period centred on the 2006 Population and Housing Census. The price of annuity and annual annuity payments are obtained from the life table with discrete observations between the initial age, typically the retirement age (65 years), and the terminal age (100 years). The life table estimates the male and female life expectancies at birth as 79 and 84 years respectively.

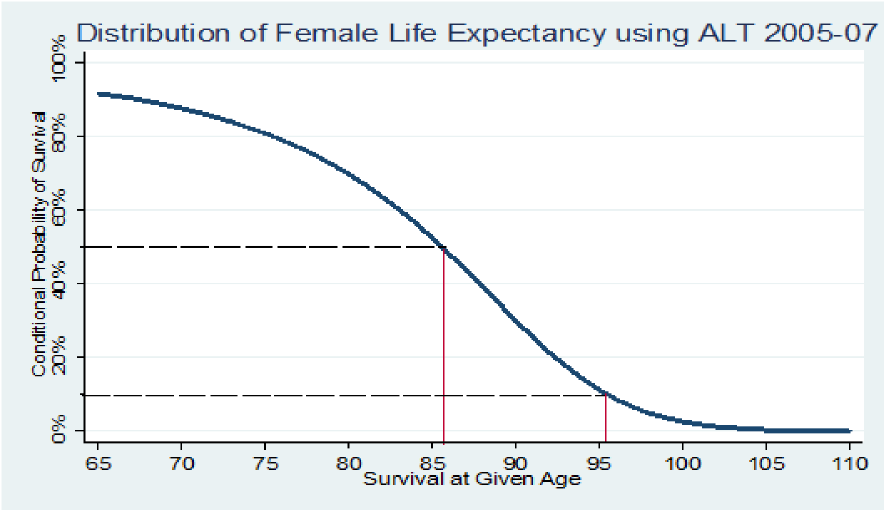
All simulated returns are based on annual returns data from Dimson-Marsh-Staunton Global Returns Data (DMS Global)[6] spanning 111 years over the period 1900 through 2011. The simulated returns, which are descriptive of historical asset class returns, are assumed to represent future asset class returns.
I make the government-sponsored pension income calculations based on the Australian System. I refer to pension rates provided by Department of Human Services for a single female homeowner. At smaller retirement balances, the retiree receives a higher pension income and vice versa. This complements the retiree's drawdown and helps to avoid retirement ruin which occurs in the absence of pensions.

An analysis of the life table in Figure 3.1 shows that the conditional survival probabilities for retirees are indeed higher than the life expectancy at birth values. Whilst life expectancy at birth for females is estimated at age 84, we expect one in every two females age 65 will live to age 86, the median life expectation in retirement. Another

[6] (Dimson, Marsh, & Staunton, 2012)

Figure 3.1: Retirees Conditional Survival Probabilities (Females)

This figure presents the conditional survival probabilities of a female Australian aged 65 surviving to given ages using the probability distributions from the ALT 2005-2007. It shows the median life expectation in retirement as well as the 10th percentile.



significant observation is that with respect to the life tables, one in every 10 females in retirement is expected to live until age 96, requiring a retirement planning horizon for up to 31 years. This implies that a retiree who wants to be 90% certain that her superannuation savings last as long as she lives is required to plan her consumption and savings for up to 31 years. The conditionality of these calculated survival rates makes them vital tools in retirement investment planning.

The annuity analysed in this study is the level Single Premium Immediate Annuity (SPIA). The annuity provides fixed level periodic payouts that commences one period after purchase. I embed the annuity prices with a loading factor that increases with age at a rate of $[0.2 + 0.005(x)]$. The loading factor, otherwise called the insurance load refers to the increment in the price of annuity due to taxes, commissions and other expenses charged by the life office. The annuity's market price is obtained from the pure actuarial premium after weighting the premium by the loading factor. The choice of the loading factor and its rate of increase are informed by literature on annuity pricing in earlier works. (see Ganegoda & Bateman, 2008; Milevsky, 1998; Mitchell et al., 2000).

3.3.2 The Epstein-Zin Preferences and Consumption

Epstein and Zin (1989), hereafter referred to as E-Z, introduce a class of utility preferences which enable disentangling RRA from the EIS. Using the E-Z utilities, investor's preferences are defined recursively over current consumptions and the certainty equivalent of the next period's utility. Previous use of this model in pension plans includes works by Horneff, Maurer, Mitchell, and Dus (2008) and Blake et al. (2011). Horneff, Maurer, Mitchell, and Dus (2008) analyse the utility cost of selecting different restrictive annuitisation strategies over the gradual annuitisation using the E-Z approach whilst Blake et al. (2011) analyse optimal funding and investment strategies of a rational lifecycle planner under E-Z preferences.

The use of E-Z preferences is relevant in this study as it opens up an insightful assessment of retiree preferences and allows for analysis of varying degrees of RRA and EIS. The utility function is defined over a single consumption pattern that is selected at retirement and remains the effective wealth decumulation strategy through retirement. A retiree aged x, has preferences defined by:

$$U_x = \left\{ (1-\beta) \times (C_x)^{1-\frac{1}{\phi}} + \beta p_x \times \left(E_x[(U_{x+1})^{1-\gamma}]\right)^{\frac{1-\frac{1}{\phi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\phi}}} \tag{3.2}$$

U_x is the level of utility at age x
C_x is the consumption level at age x
p_x is the one year survival probability at age x
β is the individual's one-year discount factor
γ is the level of RRA
ϕ is the level of EIS.

Following previous studies, the E-Z utility preferences in this study are adapted to include the mortality risk as observed from the Australia life table, ABS ALT 2005-2007 [7] that I truncate at age 100. The last surviving age in this analysis is 100 when we assume $p_{101} = 0$ and with individuals having no bequest motives, I obtain a recursive equation for the terminal condition as:

$$U_{100} = \left\{ (1-\beta) \times (C_{100})^{1-\frac{1}{\phi}} \right\}^{\frac{1}{1-\frac{1}{\phi}}} \tag{3.3}$$

Retirees who employ the phased withdrawal strategy derive their consumption from the annual withdrawals from their investment portfolios. Retirees who choose to annuitise at retirement derive their annual consumption from their annuity income. I assume retirees derive their entire utility from the level of consumption at any age x. I also include the impact of bequest motives on the utilities retiree derive from their

[7] Australia Bureau of Statistics, Australian Life Tables (2005-2007)

consumption and wealth.[8] I assume retirees derive their total utility from consumption as well as the possibility of meeting their bequest motives. Motivation for the modelling of bequest motives is discussed in works by Laitner (2002) and Gomes and Michaelides (2005). To include the impact of bequest motive on the utility analysis, I follow the study by Gomes and Michaelides (2005) and use a slightly different form of the E-Z utility function given by:

$$U_x = \left\{(1-\beta p_x)\times(C_x)^{1-\frac{1}{\phi}} + \beta E_x\left[(U_{x+1})^{1-\gamma} + (1-p_x)b\frac{(W_{x+1}/b)^{1-\gamma}}{1-\gamma}\right]^{\frac{1-\frac{1}{\phi}}{1-\gamma}}\right\}^{\frac{1}{1-\frac{1}{\phi}}} \tag{3.4}$$

with the terminal condition being given as

$$U_{100} = b\frac{(W_{101}/b)^{1-\gamma}}{1-\gamma} \tag{3.5}$$

Where b determines the strength of the bequest motive and W_x is the remaining wealth level at age x. All remaining parameters are as defined earlier.

3.3.3 Parameter Calibration

I derive the lifetime consumption utilities from the different withdrawal paths for the investor and make comparison with the utility derived from annuity payouts for a retiree with similar characteristics. I assume the hypothetical retiree to be a 65-year-old female with a retirement balance accumulated from her defined contribution superannuation plan during her working life. The retiree has no other source of income besides the government-sponsored Age Pension. I assume the retirees have a risk aversion range of γ=2–5, an intertemporal substitution coefficient of ϕ=0.1–0.5. I choose these baseline preference parameters in line with lifecycle literature such as Cocco, Gomes, and Maenhout (2005), Horneff, Maurer, and Stamos (2008) and Blake et al. (2011). In the attempt to estimate a value of the discount factor from household consumption patterns, empirical studies find values close to one (Laibson, Repetto, & Tobacman, 2007). In line with this and previous pension literature, I set the hypothetical retiree's discount factor to β=0.96 (Blake et al., 2011; Cocco et al.,

[8]The presence of bequest motives is one important explanation given to explain the annuity puzzle. It is believed that retirees with bequest motives are reluctant to annuitise since they lose their liquidity by purchasing annuities with their retirement wealth.

2005; Horneff, Maurer, & Stamos, 2008).

By setting the baseline parameter values as above, the E-Z utility function collapses to the CRRA utility, where ϕ=1/γ. I use this reference point to analyse the robustness of the results with different parameter values. I begin with a bequest motive b=0 and the subjective survival probabilities are obtained from the Australian Life Tables. When bequest is present, b=2.5, in line with previous literature (see Gomes & Michaelides, 2005; Laitner, 2002). The annuity is priced using the life tables with a loading factor of L=0.205. I base the choice of the loading factor on a recent annuity quote by Challenger Limited reported in *The Australian* (Bullock, 2012).

3.3.4 Sensitivity Analysis

I find the changes in the utility levels for the annuity and different withdrawal strategies using varying preference parameter levels. The EIS coefficient is varied between ϕ=0.1 and 0.5 to ascertain the effects it has on retirees' utility of consumption. I maintain the discount parameter and bequest level while keeping the risk aversion level fixed. Next, I hold the EIS coefficient constant and for varying levels of risk aversion between γ= 2 and 10, I investigate the changes to utility for retirees on condition that they live to 100 years. I extend this analysis by investigating the consumption utility paths for a retiree with bequest motives. This is done for the baseline preferences. By varying the levels of risk aversion and EIS, I find the utility levels to change in trends similar to that observed for the case without bequest motives. Also, I analyse the utility paths for different asset allocations and analyse the impact on the consumption and hence the utilities of consumption. Finally, I analyse the retiree's drawdown choices in the presence and absence of means-tested pensions.[9]

3.3.5 Portfolio Simulations

I create an investor's portfolio that consists of equities, bonds and bills from Australia. I assume different allocations of these assets and observe their growth over the retirement horizon in the simulations. I use bootstrap resampling of historical observed returns and 10,000 iterations to generate future returns. The assumption is that the individual annual historical returns contained in the total set of historical

[9] Graphs of different asset allocations are not displayed in this study due to space limitation but are available upon request

returns have an equal probability of occurring in the future, but without regard to any sequencing of returns contained in the historic data. This methodology overcomes the statistical problems of using overlapping multi-period returns.

I set retirees' initial retirement account balances to $500,000. I choose this account balances since the average aggregate personal balance for Australian account based pensions and Self-Managed Superannuation Funds (SMSF) are $275,000 and $466,909 respectively according to Australian Tax Office (ATO) and Australian Prudential Regulation Authority estimates (CRIR, 2013). SMSFs are used by over 1 million Australians and makes up close to a third of total superannuation wealth with current median SMSF account balance exceeding $500,000 (ATO, 2014). This informs my choice of starting wealth. The bootstrap simulations I employ are performed over three different portfolio allocation strategies. Previous literature on asset allocation in retirement recommends a 50-75% equity allocation (see Bengen, 2001, 2004; Cooley et al., 2001). I set the baseline asset allocation to a balanced portfolio allocation of 50/50% equity-bonds/bills proportions. The other asset allocations used in this analysis are 30-70% equity-bonds/bills, and 70-30% equity-bonds/bills allocations.

3.3.6 Withdrawal Strategies

I analyse four different withdrawal strategies. The choices of these withdrawal strategies are based on prior literature and the objective of making a meaningful comparison with annuity payouts. The strategies are described below.

◦ A Fixed withdrawal strategy in which the retiree withdraws a fixed amount from her portfolio each year. This is done till the retiree's death or the complete depletion of her wealth, whichever happens earlier. This strategy is made to mimic the returns from a fixed annuity purchased with the same amount of wealth, that is, I set the size of the withdrawal to equal the life annuity payout divided by the initial wealth.[10]

◦ A variable withdrawal strategy, which I call the Percentage of Wealth strategy,[11] where the retiree receives a specified portion of her wealth every year. This withdrawal has the advantage of not running into ruin no matter how long the retiree survives. The disadvantage is that the withdrawal is very volatile and increases

[10]This is akin to the $1/\ddot{a}_y$ rule used in Blake et al. (2003) and the 'fixed benefit approach' by Horneff, Maurer, Mitchell, and Dus (2008)

[11]I refer to this strategy as Percent of Wealth in Tables and Figures for paucity of space

or decreases depending on how well the portfolio performs. I link the Percentage of Wealth withdrawal to the minimum withdrawal payments for Australia's current account based pensions as regulated by the Superannuation Industry Supervision Regulations 1994 and the Retirement Savings Accounts Regulations 1997. Hence, unlike past studies, the Percentage of Wealth withdrawn from the retiree's pension fund is not fixed, but adjusted after every five years or decade to meet the required minimum withdrawal quota as shown in Table 3.1.

◦ The final phased withdrawal strategy is the $1/E(T)$ rule (I refer to as Life Expectancy strategy). $E(T)$ refers to the retiree's expected remaining lifetime. The level of withdrawal relates to the investor's remaining life expectation. The investor consumes a larger fraction of her wealth as her remaining lifetime $E[T(x+t)]$ decreases.[12] This is similar to the 401(k) drawdown rule employed in the US to ensure that retirement savings are not left to bequest.[13] The Australian Life Tables 2005-2007 (2009) for Females are used to provide the expected remaining lifetimes.

The Percentage of Wealth withdrawal strategy used in this study follows the current minimum drawdown payments as shown in Table 3.1. I assume withdrawals are made at the beginning of the year and the portfolios are annually rebalanced after withdrawal to assume the set allocations. This is repeated annually until the earlier of the demise of retiree or depletion of retiree's wealth.

TABLE 3.1: Minimum Annual Pension Payments
(For Account-Based Pensions)

	Regular Percentage factors	**Temporary relief 2012/13 year**
Age		
65-74	5%	3.75%
75-79	6%	4.50%
80-84	7%	5.25%
85-89	9%	6.75%
90-94	11%	8.25%
95 or older	14%	10.50%

Source: *Adapted from Schedule 7, Superannuation Industry (Supervision) Regulations 1994 and Federal Government news releases dated 18 February 2009, and 12 May 2009, and 30 June 2010, and 29 November 2011.*

[12]The complete expectation of life is calculated as $E[T(x+t)]=\sum_{t=0}^{w-x} {}_tp_x$. These values are given in the Australian Life Tables 2005-2007 (2009). x refers to the age of the retiree and t to the retiree's expected remaining lifetime.

[13](see Dus et al., 2005; Horneff, Maurer, Mitchell, & Dus, 2008).

In addition to the income retirees receive from their invested assets, they are also entitled to a government-sponsored pension. The amount of pension income they receive depends on the amount of assets they own and income they earn from their assets. For this study, I assume a 65-year-old female homeowner whose retirement balance and her income drawdown respectively define her assets and income. Her pension income is based on the level of her assets and is adjusted annually by the CPI. For income and asset levels above an upper threshold, retirees are disqualified from receiving government pension. Retirees qualify for full government pension when their income is below the lower threshold and part rate pension if they fall within the set thresholds.

3.4 Results

3.4.1 Baseline Asset Allocation and Preferences

Asset allocation is a major determinant of investment performance in explaining variation in investment performance (see Brinson et al., 1986; Brinson, Singer, & Beebower, 1991; Ibbotson & Kaplan, 2000). In Australia, there exist some fundamental differences between the asset allocations of SMSFs and other types of superannuation funds. One difference is the high allocation to cash, property, shares and trusts by SMSFs (Valentine, 2011). To examine the variation in asset allocation, consistent with Ellis, Tobin, and Tracey (2008), I analyse the samples based on the proportion invested in growth and defensive assets. With uncertainty in the literature relating to the definition of asset allocation categories such as cautious, conservative, balanced, growth and aggressive, I set up two broad categories for the analysis. I define a conservative asset allocation to a 30/70 growth to conservative assets proportion and a growth asset allocation as a 70/30 growth to conservative asset allocated proportions. Hence the baseline asset allocation is a balanced portfolio allocation of 50/50 growth to defensive asset proportions.

TABLE 3.2: Summary of Portfolio Values for Simulated Returns in the presence of Pension Income (A)

Simulated returns on a 50/50 growth/defensive asset allocated investment portfolio with $500,000 starting value showing probability of ruin, average ending portfolio and the distribution of the ending portfolio values. 'Fixed' represents a fixed real withdrawal amount of $30,000 maintained throughout the retirement horizon. The 'Percent of Wealth' takes a 4% withdrawal of remaining wealth annually. The 'Life Expectancy' pattern depends on withdrawing a fraction of wealth relative to retirees' remaining life expectation. SPIA refers to a Single Premium Immediate Annuity paying level income to the retiree for life. Probability of ruin represents the probability of the portfolio balance going negative whilst retiree is still alive. Means-tested age pension income is included in the withdrawal amounts. The simulations are capped at age 100.

Withdrawal Type	Average withdrawal (Standard deviation)	Probability of ruin	Average Ending Portfolio	Percentiles of distribution of ending portfolio			
				5	25	50	75
Fixed	35,530(5,846)	3.03%	1,221,074	71,498	491,363	705,649	1,347,865
Percent of Wealth	49,218(29,086)	0%	838,528	340,863	522,524	698,826	997,529
Life Expectancy	67,545(33,686)	0%	473,780	55,152	234,710	491,596	640,093
SPIA	40,026(0)	0%	0	0	0	0	0

The results of portfolio simulations for different withdrawal strategies are reported in Table 3.2. The Fixed withdrawal strategy has a ruin probability of 3.03% and a high average ending portfolio value that is over 200% the size of the starting value. This represents the only phased withdrawal strategy with a ruin probability. The high ending portfolio values at all percentiles could explain retirees' high utility from bequest demonstrated by their dependence on a level of consumption independent of their wealth. Alternatively, these ending portfolio values show large consumption foregone due to the limiting nature of the withdrawal strategy.

The Percentage of Wealth withdrawal has a high average withdrawal of $49,218 per annum with a standard deviation of $29,086. It has an average ending wealth of 1.3 times the starting value. The Percentage of Wealth average ending portfolio and distribution of portfolio ending values are fairly balanced, with the 25th percentile being higher than the starting value and the 5th percentile below the starting value but never running into ruin. The ending portfolio value percentiles of the Percentage of Wealth withdrawal are lower than that of the Fixed strategy at higher percentiles but higher at lower percentiles. This means for those who end up with terminal wealth values below the 50th percentile, the Percentage of Wealth withdrawal provides a higher average consumption as well as higher average ending portfolio needed to meet bequest motives. On the upside, the Percentage of Wealth provides lower terminal values than the Fixed withdrawal strategy, showing less allocation to bequest but less consumption forgone as the average consumption level is higher for the former.

The Life Expectancy withdrawal has the highest withdrawal rate, averaging $67,545 with a standard deviation of $33,686. Since this strategy does not run into ruin and its terminal values are the lowest among the strategies under study up to the 50th percentile, this portrays an efficient spending by the retiree. The SPIA has an average withdrawal of $40,026 and zero standard deviation. The annuity is irreversible and hence there is no return of capital or chance of bequest.

The differences between the alternative withdrawal strategies may not be striking but I am able to rank the strategies based on their probabilities of ruin, and their average withdrawal and standard deviations. The Fixed withdrawal strategy has the lowest average withdrawal and a deviation of 16% about the mean. This strategy may not be attractive for its low withdrawal levels as well as its chance of ruin, although the ruin probability is low. The Percentage of Wealth and Life Expectancy withdrawal strategies do not run into ruin and are attractive for retirees as they give them a guarantee of income as long as they are alive. These two strategies however have the highest deviations about the mean consumption level, with deviations of up to 60% and 50% about the mean consumption level for the Percentage of Wealth and the Life Expectancy withdrawals respectively.

Although retirees with bequest motives may avoid full annuitisation, annuities are attractive in this study for two reasons. Firstly, annuities provide a high average withdrawal with minimal standard deviation; the level SPIA has zero standard deviation. Although the level of the annuity payments depend on the initial premium and a number of factors at purchase, the regular income received from annuities is unaffected by market returns and hence wealth changes. Secondly, annuities provide income with lifetime certainty and hence retirees never run into ruin. With less variation and no chance of ruin, annuities serve as an appropriate benchmark for retirees in the absence of bequest motives. The impact of bequest motives is explored later in my analysis.

FIGURE 3.2: Withdrawal-to-Wealth patterns for Alternative Withdrawal Strategies

I show the withdrawal-to-wealth ratios for the three different withdrawal strategies. I denote the Fixed withdrawal strategy path by a line with stars; the Percent of Wealth withdrawal path by a line with circles and the Life Expectancy withdrawal path by a line with triangles. The plot represents the average of 10,000 simulated withdrawal paths for a $500,000 initial investment.

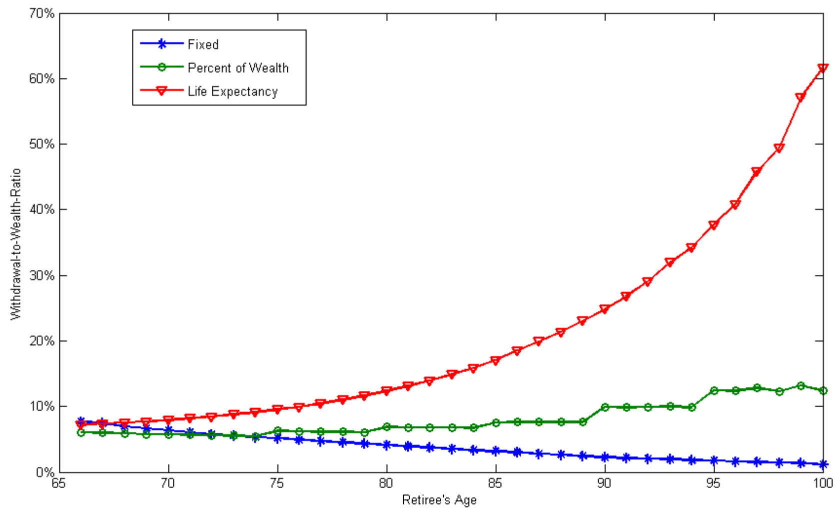


Figure 3.2 shows the withdrawal-to-wealth patterns of the various withdrawal strategies conditional to the retiree surviving to the age of 100 years. The Fixed withdrawal strategy shows a decreasing profile with increasing age, making the Fixed withdrawal

a smaller portion of investor's total wealth. This is expected as I find terminal wealth to increase on average with increasing age. The Percentage of Wealth increases in a stepwise manner as the retiree ages. This increase is attributed to the increase in the minimum drawdown ratios applied to ensure that retiree meets the tax-exempt rules. After every few years, a larger proportion of wealth is withdrawn, increasing the withdrawal to wealth ratio. The Life Expectancy withdrawal strategy ends with the retiree depleting her wealth at the final surviving age. As the retiree's withdrawal amount is a ratio of her expected remaining lifetime, she withdraws higher proportions of her wealth as she grows older and has lower remaining life expectation. Her wealth is depleted rapidly in the latter years, withdrawing over 60% of remaining wealth at age 99 and withdrawing the remainder of her wealth in the subsequent year as she does not expect to live beyond age 100.

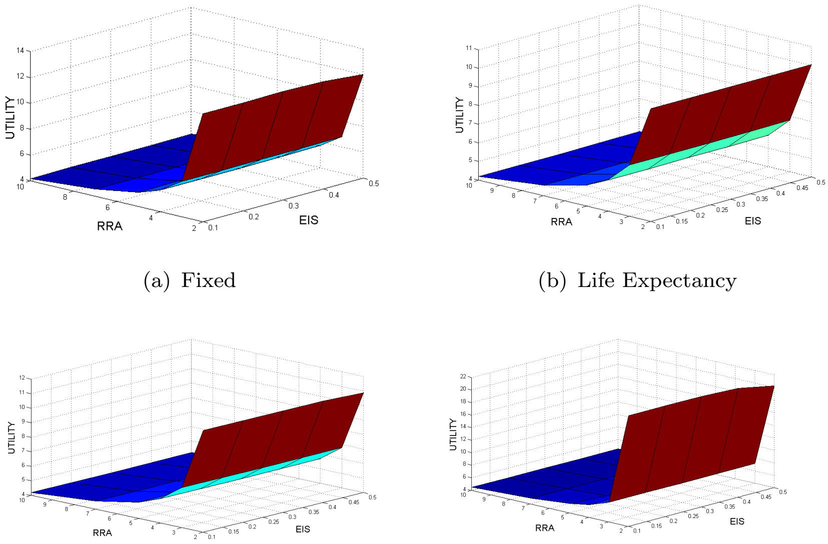
3.4.2 Utilities from Alternative Withdrawal Strategies

I calculate the expected utilities retirees derive from their consumption and bequest from choosing any of the three alternative withdrawal strategies by using the Epstein-Zin utility function. I also calculate the expected utility a retiree derives from consumption by choosing to fully annuitise her wealth at retirement age 65. I set retiree's baseline preference to a moderate RRA level of γ=5, an EIS level of ϕ=1/5. I set the hypothetical retiree's discount factor to β=0.96. I choose these baseline preference parameters in line with lifecycle literature (see Blake et al., 2011; Cocco et al., 2005; Horneff, Maurer, & Stamos, 2008).

I measure the impact on expected utility of varying RRA levels for a given level of EIS for all the alternative withdrawal strategies. I also investigate the response of expected utility to different EIS levels for set RRA. The EIS coefficient is varied between ϕ=0.1 and ϕ=0.5 to ascertain the effects it has on retirees' utility of consumption. I study a range of RRA between γ=2 and gamma=10 and investigate the changes to expected utility for retirees between retirement age 65 and age 100. I show the expected utility plots of varying EIS and RRA for various withdrawal strategies from the baseline asset allocation in Figure 3.3.

For the lowest level of relative risk aversion of 2, I find expected utility to be significantly high and decrease with increasing levels of risk aversion. Increasing levels of risk aversion sees the expected utility to decrease for a given level of EIS. I make a

Figure 3.3: Expected Utilities with varying RRA and EIS



(a) Fixed

(b) Life Expectancy

(c) Percent of Wealth

(d) SPIA

number of significant findings from calculating the expected utilities for the alternative withdrawal strategies' consumption streams and terminal wealth levels. Firstly, I find the decrease in utility for increasing RRA to be steep for the Annuity, decreasing in steepness for the Fixed, Percentage of Wealth through to the Life Expectancy strategy in that order. Taking the inverse of RRA as the estimate of EIS, as in the case of a CRRA investor as a baseline preference estimates, I compare the impact of having RRA levels that are increasing or decreasing relative to the baseline level. Whenever RRA is less than the inverse of EIS, the individual derives higher utility from her consumption and bequest levels and will thus find the strategies whose utility increase steeply with increasing RRA appealing. Hence, when retirees have lower risk aversion than dictated by the inverse of EIS, they will prefer the Annuity over the phased withdrawal strategies. Among the phased strategies, the Fixed strategy will be the dominant strategy while the Percentage of Wealth is preferred over the Life Expectation strategy. However, when RRA is greater than the inverse of EIS, I find the reverse preference ranking to be true. The high risk averse retiree will want to avoid the possibility of no consumption or bequest in the event of portfolio ruin, and hence will prefer a phased withdrawal strategy that neither ends in negative balance or loss of consumption. The utility levels decreases smoothly with increasing RRA for a given level of EIS and retirees' preferences respond to this. The phased withdrawal strategies become more utility enhancing than the annuity; with retirees preferring the Life Expectancy drawdown to the two alternative phased strategies.

The Percentage of Wealth ranks higher than the Fixed withdrawal and the annuity is the least preferred. The preference of phased withdrawals over annuities and RRA levels being higher than the inverse of EIS is consistent with prediction by recursive preferences.

Compared to the CRRA investor, an investor who is willing to accept consumption volatility over time or late resolution to uncertainty has increased expected utility only if he is less risk averse (has RRA lower) than predicted by the inverse function for the EIS coefficient. Retirees' time to resolution of uncertainty determines how much utility they derive from their wealth levels and consumption streams. This is an important finding as previous studies have overlooked the influence of EIS in retirees' withdrawal strategy selection. The impact of the EIS is significant and the timing of resolution informs the drawdown plan choice for retirees of various risk aversion levels.

I rank the different withdrawal strategies for various RRA and EIS coefficients and find the dominating strategies at different combinations of the preference parameters. The results of the dominant strategies are shown in Table 3.3. I show the dominant strategies based on conditional survival to age 86 and again unconditional to survival capped at age 100. Panel A ranks the alternative strategies based on retiree's surviving to age 86. I find that for a low risk averse retiree, that is γ=2 and γ=3, the Life Expectancy withdrawal strategy dominates other withdrawal strategies at low EIS levels and gives way to dominance by the Percentage of Wealth strategy and Annuities at moderate to increasing EIS levels. As RRA increases beyond γ=3, the Life Expectancy withdrawal dominates the other withdrawals and Annuities, the Percentage of Wealth is the second preferred strategy and is closely trailed by the Fixed withdrawal strategy.

The Annuity remains the least preferred strategy. At age 86, the Life Expectancy strategy is reaching its peak withdrawal levels and it is preferred over the other strategies for the high withdrawal levels and high bequest potential. The Percentage of Wealth withdrawal, which is next in preference is also attractive for its high average withdrawal and high portfolio ending values. The Fixed withdrawal strategy, although with a lower average withdrawal level has good bequest potential and therefore is preferred to the Annuity. Annuities in this case become unattractive as retirees receive utility not only from their level of withdrawals but also from the possibility of meeting bequest motives.

Table 3.3: Dominating Withdrawal Strategies (A)

I rank the alternative withdrawal strategies according to their expected utilities. I denote the Percentage of Wealth withdrawal by P, the Fixed withdrawal by F, the Life Expectation by L and Annuitisation by A. I present Panels A and B showing the dominant strategies conditional on retiree surviving to age 86 and unconditional on retiree's survival capped at age 100 respectively.

Panel A Dominating Withdrawal Strategies at age 86

RRA\EIS	0.1	0.2	0.3	0.4	0.5
2	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A
3	P,F,A,L	L,P,F,A	P,F,A,L	P,F,A,L	P,F,A,L
4	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A
5	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A
7	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A
10	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A

Panel B Dominating Withdrawal Strategies at age 100

RRA\EIS	0.1	0.2	0.3	0.4	0.5
2	P,L,F,A	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A
3	P,L,F,A	L,P,F,A	P,F,A,L	P,F,A,L	P,F,A,L
4	L,P,F,A	P,L,F,A	P,L,F,A	P,L,F,A	P,L,F,A
5	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A
7	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A
10	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A

Panel B shows the dominating strategies at age 100. I find the rankings of the withdrawal strategies to follow a similar trend in the long term. A major difference is the increase in dominance of the Percentage of Wealth withdrawal to γ=4. There is now a fair balance between the Life Expectancy and the Percentage withdrawals making these two the most preferred strategies for retirees with bequest motives. The dominance of these two strategies is consistent with findings of previous literature (Dus et al., 2005).

3.4.3 Changing Asset Allocations

I vary the retirement portfolios' asset allocations from the baseline proportions to see the impact on the average ending portfolios, probability of ruin and the average withdrawals for the different withdrawal strategies. I do this for the conservative and growth portfolios and analyse the summary statistics of the simulations in Table 3.4.

TABLE 3.4: Summary of Portfolio Values for Simulated Returns in the presence of Pension Income (B)

Panel A shows simulated returns on a 30/70 growth/defensive asset allocated investment portfolio with $500,000 starting value. Panel B shows simulated returns on a 70/30 growth/defensive asset allocated investment portfolio of same starting value. I show the probability of ruin, average ending portfolio and the distribution of the ending portfolio values. ‘Fixed’ represents a fixed real withdrawal amount of $30,000 maintained throughout the retirement horizon. The ‘Percent of Wealth’ takes a 4% withdrawal of remaining wealth annually. The ‘Life Expectancy’ pattern depends on withdrawing a fraction of wealth relative to retirees’ remaining life expectation. SPIA refers to a Single Premium Immediate Annuity paying level income to the retiree for life. Probability of ruin represents the probability of the portfolio balance going negative whilst retiree is still alive. Means-tested age pension income is included in the withdrawal amounts. The simulations are capped at age 100.

Panel A 30/70 Growth/ Defensive Assets

Withdrawal Type	Average Withdrawal (Standard deviation)	Probability of ruin	Average Ending Portfolio	Percentiles of distribution of ending portfolio			
				5	25	50	75
Fixed	36,948(5,807)	3.93%	624,553	209,416	451,244	537,504	696,864
Percent of Wealth	42,826(18,182)	0%	647,078	380,862	510,026	584,527	733,709
Life Expectancy	59,822(23,391)	0%	538,740	238,120	451,506	526,486	624,136
SPIA	40,026(0)	0%	0	0	0	0	0

Panel B 70/30 Growth/ Defensive Assets

Withdrawal Type	Average Withdrawal (Standard deviation)	Probability of ruin	Average Ending Portfolio	Percentiles of distribution of ending portfolio			
				5	25	50	75
Fixed	34,787(5,964)	3.45%	1,000,823	236,156	511,641	652,238	1,056,773
Percent of Wealth	56,712(45,537)	0%	869,968	380,526	523,389	693,098	1,009,182
Life Expectancy	75,640(47,411)	0%	667,558	278,139	494,230	598,492	788,219
SPIA	40,026(0)	0%	0	0	0	0	0

Comparing the summary statistics from the conservative and growth portfolios, I make three main observations. Firstly, with the exception of the Fixed withdrawal strategy, the growth portfolio provides higher average consumption in other withdrawal strategies than the conservative portfolio. The higher average withdrawals however come with higher deviation about the mean, and this is especially significant for the two variable strategies, the Life Expectancy and the Percent of Wealth. Secondly, the growth portfolio has lower ruin probabilities compared to the conservative portfolio. Although the difference in ruin probability for the Fixed withdrawal strategies between the two asset allocation categories looks marginal, the probability is reduced by about 13.9% in response to asset allocation changes. Finally, I find the ending portfolio values to be higher for the growth portfolio than the conservative portfolio. This means for the purpose of meeting bequest motives, the growth portfolio provides higher terminal values and is more attractive than a conservative portfolio.

I find increasing the allocation of growth assets of a retirement portfolio to come with several advantages. Varying asset allocation from a conservative, through a balanced to a growth portfolio increases the average ending portfolios significantly for all the withdrawal strategies. Since the average withdrawal levels for the variable strategies, the Percent of Wealth and the Life Expectancy, respond to changes in wealth levels, I observe significant increases in their withdrawals. The remaining withdrawal strategy, the Fixed withdrawal, which is wealth independent, does not make any significant increases to the average withdrawals but shows increased ending portfolio values for increasing growth assets allocations. The Fixed withdrawal strategy actually decreases in average withdrawal size with increasing growth asset proportions as the higher wealth levels will account for low pension income, gradually eroding the average withdrawal size. The increased terminal wealth reduces the ruin probabilities, making portfolios with higher growth allocations less likely to be depleted while the retiree is still alive compared to portfolios with less growth asset allocation.

3.4.4 Decreasing Mortality and Alternative Asset Allocations

I run further comparative withdrawal strategy simulations based on the survival patterns from the Australian Life Tables. I begin with conditional survival to ages 86, 92 and age 100 for the baseline asset allocation strategy. Whilst life expectancy at birth for females in Australia is estimated at age 84, we expect one in every two females aged 65 will live to age 86, the median life expectation in retirement. The

Australian Life Tables 2008-2010 (2011) provide mortality experiences and capture the mortality improvements observations among Australians for the past 25 and 100 years as well as projected mortality improvements over 25 and 100 years. With the incorporation of the 25-year mortality improvements, the median life expectation in retirement is expected to increase to age 92. I simulate the withdrawal patterns and wealth dynamics for retirees who live until age 86, and the results are based on the conditional probability of retirees surviving to this age. Further to that, I construct summary tables based on the expectation of improved mortality experiences, conditional on retiree's survival to age 92. Also, I show tables for conditional survival to age 100, which is the final age for the life tables, incorporating all withdrawal and portfolio dynamics based on the retiree surviving 35 years in retirement. Finally, I show the wealth patterns for retirees which are unconditional on survival, and I cap this at age 100.

I present the summary statistics in Tables 3.5 and 3.6.

TABLE 3.5: Summary of Portfolio Values for Simulated Returns in the presence of Pension Income (C1)

Panels A and B show simulated returns on the baseline asset allocation investment portfolio with $500,000 starting value based on retiree surviving to ages 86 and 92 respectively. ‘Fixed’ represents a fixed real withdrawal amount of $30,000 maintained throughout the retirement horizon. The ‘Percent of Wealth’ takes a 4% withdrawal of remaining wealth annually. The ‘Life Expectancy’ pattern depends on withdrawing a fraction of wealth relative to retirees’ remaining life expectation. SPIA refers to a Single Premium Immediate Annuity paying level income to the retiree for life.

Panel A. Conditional on surviving to age 86

Withdrawal Type	Average Withdrawal (Standard deviation)	Probability of ruin	Average Ending Portfolio	Percentiles of distribution of ending portfolio			
				5	25	50	75
Fixed	35,597(5,796)	3.39%	1,186,549	55,919	491,605	921,584	1,595,870
Percent of Wealth	48,111(23,434)	0%	997,515	370,572	619,610	863,524	1,231,211
Life Expectancy	70,318(34,147)	0%	531,089	242,111	357,099	474,857	639,850
SPIA	40026(0)	0%	0	0	0	0	0

Panel B. Conditional on surviving to age 92

Withdrawal Type	Average Withdrawal (Standard deviation)	Probability of ruin	Average Ending Portfolio	Percentiles of distribution of ending portfolio			
				5	25	50	75
Fixed	34,976(6,098)	7.96%	1,632,323	-129,537	554,993	1,190,887	2,244,888
Percent of Wealth	56,659(34,686)	0%	986,619	330,822	569,805	838,679	1,234,505
Life Expectancy	76,181(37,426)	0%	256,363	117,111	171,673	227,766	306,569
SPIA	40,026(0)	0%	0	0	0	0	0

Table 3.6: Summary of Portfolio Values for Simulated Returns in the presence of Pension Income (C2)

Panel C show simulated returns on the baseline asset allocation investment portfolio with \$500,000 starting value based on retiree surviving to ages 100. Panel D shows simulated returns on the baseline asset allocation unconditional on survival and capped at age 100. 'Fixed' represents a fixed real withdrawal amount of \$30,000 maintained throughout the retirement horizon. The 'Percent of Wealth' takes a 4% withdrawal of remaining wealth annually. The 'Life Expectancy' pattern depends on withdrawing a fraction of wealth relative to retirees' remaining life expectation. SPIA refers to a Single Premium Immediate Annuity paying level income to the retiree for life.

Panel C. Conditional on surviving to age 100

Withdrawal Type	Average Withdrawal (Standard deviation)	Probability of ruin	Average Ending Portfolio	Percentiles of distribution of ending portfolio			
				5	25	50	75
Fixed	34,011(6,279)	11.35%	2,673,966	-451,611	777,781	1,890,336	3,868,489
Percent of Wealth	68,504(44,149)	0%	792,014	234,352	425,062	687,542	1,133,668
Life Expectancy	71,866(33,666)	0%	45,978	29,239	36,490	43,051	54,589
SPIA	40,026(0)	0%	0	0	0	0	0

Panel D. Unconditional on surviving capped at age 100

Withdrawal Type	Average withdrawal (Standard deviation)	Probability of ruin	Average Ending Portfolio	Percentiles of distribution of ending portfolio			
				5	25	50	75
Fixed	35,530(5,846)	3.03%	1,221,074	71,498	491,363	705,649	1,347,865
Percent of Wealth	49,218(29,086)	0%	838,528	340,863	522,524	698,826	997,529
Life Expectancy	67,545(33,686)	0%	473,780	55,152	234,710	491,596	640,093
SPIA	40,026(0)	0%	0	0	0	0	0

From Table 3.5, I find the average withdrawal levels for the Fixed withdrawal strategy to decrease with increasing surviving age. This is explained by examining the increasing trend in ending portfolios for the strategy. The increasing average ending portfolios result in lower allocated pension which declines the average withdrawal size as retiree ages. The two remaining withdrawal strategies, the Percentage of Wealth and Life Expectancy withdrawals also show increments with increasing survival for the baseline asset allocation strategy. The Percentage of Wealth withdrawal increases in a stepwise manner as the retiree ages. This increase is attributed to the minimum drawdown ratios applied to ensure that retiree's earnings are tax-exempt. The Life Expectancy strategy's withdrawal increases gradually with age and peaks at the age of 88 years and thereafter depletes rapidly, creating a hump shape pattern for its withdrawal stream.

Probability of ruin, as expected, is a direct function of age. Retirees are more likely to deplete their assets the longer they live. The increasing probability of ruin in the setting is able to partly explain the decrease in the average withdrawal levels for the Fixed withdrawal strategy and increasing standard deviation of this withdrawal plan. With the availability of pension income, retirees who deplete their retirement portfolios are entitled to the full-rate pension, which is about a half of their income for a retiree under the Fixed withdrawal strategy. The significant drop in consumption decreases the retiree's mean consumption and increases the standard deviation of her withdrawal.

The terminal portfolio values for the Fixed withdrawal strategy increases with increasing survival age. There is however, a wide range of wealth levels as some retirees run into ruin and others have large portfolio values. This shows the diverse extremes possible for those who live longer, the wealthy get wealthier, while the numbers of retirees who experience portfolio ruin also increase among surviving population. The average ending portfolio values for the Percentage of Wealth withdrawal decreases gradually, whilst the Life Expectancy terminal values depletes significantly after the withdrawal levels peak in the late eighties. Of all the strategies analysed, the Life Expectancy strategy has the lowest payouts and terminal values in the latter years of retirement.

With increasing life expectation in the population, the Life Expectancy and to some extent, the Percentage of Wealth strategies are attractive to retirees due to the increasing withdrawal levels with increasing age. These two strategies do not run into ruin and vary withdrawals in response to changes in wealth levels. The major disadvantage to this strategy is the large and increasing standard deviation with age, making the retiree uncertain of meeting her consumption needs for any given year. The Fixed withdrawals, although having lower average withdrawals and increasing ruin probabilities with increasing surviving age also have some attractive features. Retirees who desire a stable income will opt for the Fixed withdrawal strategy, which is less volatile, although that comes at a cost, that is, a chance of ruin but is able to maintain retirees' lifestyle over a long period. The large ending portfolio values for this strategy also make it attractive to retirees with bequest motives as the ending portfolio values increase with increasing surviving age.

3.4.5 The Impact of Pensions on Baseline Asset Allocation

I further investigate the influence of the pension income in the analysis. I do this for the baseline asset allocation as well as the conservative and growth portfolios. The Australian Means-Tested Aged Pension scheme is a unique characteristic of Australia's pension system. The pension income received by retirees under this scheme is dependent on the result of an asset and income levels test. For the purpose of this study, I base the pension income level on retiree's remaining wealth level at all times in retirement prior to her death. The decreasing pension income with increasing wealth levels in retirement and vice-versa has the tendency to smooth consumption and possibly bias the results. This creates difficulty in comparing the result of this study to others such as Dus et al. (2005) and Horneff, Maurer, Mitchell, and Dus (2008) that are based in countries with no government-sponsored pension income.

To create a comparative analysis, I omit pension income for the various withdrawal plans, repeat the simulations, and calculate the expected utilities for a retiree who chooses any of the strategies or a level income annuity. I show a summary of withdrawal and wealth levels in the absence of pension income in Table 3.7.

TABLE 3.7: Summary of Portfolio Values for Simulated Returns in the Absence of Pension Income (D)

Simulated returns on a 50/50 growth/defensive asset allocated investment portfolio with $500,000 starting value showing probability of ruin, average ending portfolio and the distribution of the ending portfolio values. 'Fixed' represents a fixed real withdrawal amount of $30,000 that is maintained throughout the retirement horizon. The Percent of Wealth withdrawal takes a variable percentage of remaining wealth based on retiree's age. The Life Expectancy withdrawal refers to a consumption pattern that depends on withdrawal of a fraction of wealth relative to retirees' remaining life expectation. SPIA refers to a Single Premium Immediate Annuity paying level income to the retiree for life. Probability of ruin represents the probability of the portfolio balance going negative whilst retiree is still alive. The average terminal portfolio values show the average ending balance of the portfolios after the various withdrawals are made.

Panel A. Conditional on surviving to age 86

Withdrawal Type	Average Withdrawal (Standard deviation)	Probability of ruin	Average Ending Portfolio	Percentiles of distribution of ending portfolio			
				5	25	50	75
Fixed	30,000(0)	2.03%	681,206	- 10,194	291,852	566,422	930,433
Percent of Wealth	32,196(16,256)	0%	751,640	332,767	521,964	688,370	904,228
Life Expectancy	49,755(27,079)	0%	300,131	132,928	208,450	275,085	361,537
SPIA	30,000(0)	0%	0	0	0	0	0

Panel B. Unconditional on surviving capped at age 100

Withdrawal Type	Average Withdrawal (Standard deviation)	Probability of ruin	Average Ending Portfolio	Percentiles of distribution of ending portfolio			
				5	25	50	75
Fixed	30,000(0)	4.99%	578,214	178,719	420,352	506,329	646,262
Percent of Wealth	32,646(19,032)	0%	637,758	380,568	500,000	577,357	721,564
Life Expectancy	47,292(26,595)	0%	460,857	147,773	367,248	484,530	549,754
SPIA	30,000(0)	0%	0	0	0	0	0

Table 3.7 summarises the statistics for the alternative phased withdrawal plans and the annuity in the absence of pension income. The Fixed withdrawal strategy remains at a constant level at all ages and this level is equal to what is received as annuity income should the retiree decide to annuitise. With no pension income, retirees who deplete their retirement portfolios may borrow to meet their consumption needs. The ruin probability, although low at the median life expectation in retirement, increases with increasing age and this depresses the average ending portfolio for the Fixed withdrawal plan. Whilst 2% of the surviving population run into ruin by age 86, further analysis show 26.95% of surviving population at age 100 run into ruin. Overall the cohort of 100,000 retirees show a 5% ruin probability accompanied by a reduction in average ending portfolio values at age 100. Even with the reduction in terminal wealth over the long horizon, the 5th percentile for the overall cohort shows higher ending wealth levels compared to the 5th percentile of retirees who survive to age 86. There is a marginal increase in the average withdrawal level of the Percentage of Wealth withdrawal plan over the long horizon but with a significant change in the standard deviation. The average withdrawal level of the Life Expectancy is lower in the long term compared to the short term as the average at age 86 is close to the withdrawal's highest peak after which withdrawal levels decrease for the remaining years in retirement. With the decrease in withdrawal levels is an accompanying marginal decrease in the standard deviation of withdrawal for this strategy.The average ending portfolio values for the Percentage of Wealth withdrawal decreases gradually, whilst the Life Expectancy terminal values are higher on average in the long term as they vary over wide ranges.

The Life Expectancy and the Percentage of Wealth strategies remain attractive to retirees due to the increasing withdrawal levels with increasing age. They are also attractive because, unlike the Fixed withdrawal strategy, they do not run into ruin. The major disadvantage to these strategies is the large standard deviations about their average withdrawal levels, which creates difficulty for future financial planning. The Fixed withdrawal strategy, although with lower average withdrawals and increasing ruin probabilities with increasing surviving age, provides a stable income. The Annuity provides similar benefit and has the advantage of not running into ruin. I compare and rank the various strategies based on their expected utilities in Table 3.8.

In the short term, as shown in Panel A of Table 3.8, I find a dominating preference for the Life Expectancy withdrawal strategy. Second in the preference is the Percentage of Wealth withdrawal strategy followed by the Fixed withdrawal strategy and the Annuity. Whilst the Life Expectancy strategy dominates for most combinations of

Table 3.8: Dominating Withdrawal Strategies (B)

I rank the alternative withdrawal strategies according to their expected utilities. I denote the Percentage of Wealth withdrawal by P, the Fixed withdrawal by F, the Life Expectation by L and Annuitisation by A. I present Panels A and B showing the dominant strategies conditional on retiree surviving to age 86 and unconditional on retiree's survival capped at age 100 respectively.

Panel A Dominating Withdrawal Strategies at age 86

RRA\EIS	0.1	0.2	0.3	0.4	0.5
2	L,P,F,A	L,P,F,A	P,F,L,A	P,F,L,A	P,F,L,A
3	P,F,A,L	L,P,F,A	P,F,A,L	P,F,A,L	P,F,A,L
4	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A
5	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A	P,L,F,A
7	L,P,F,A	L,P,F,A	L,P,F,A	,LP,F,A	L,P,F,A
10	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A	L,P,F,A

Panel B Dominating Withdrawal Strategies at age 100

RRA\EIS	0.1	0.2	0.3	0.4	0.5
2	F,L,P,A	L,F,P,A	P,L,F,A	P,L,F,A	P,L,F,A
3	F,P,A,L	L,F,P,A	F,P,A,L	F,P,A,L	F,P,A,L
4	L,F,P,A	F,L,P,A	L,F,P,A	L,F,P,A	L,F,P,A
5	L,F,P,A	L,F,P,A	L,F,P,A	L,F,P,A	P,L,F,A
7	L,F,P,A	L,F,P,A	L,F,P,A	L,F,P,A	L,F,P,A
10	L,F,P,A	L,F,P,A	L,F,P,A	L,F,P,A	L,F,P,A

the preference parameters, I find the Percentage of Wealth strategy to be the most attractive withdrawal strategy at low levels of risk aversion and high EIS. The Fixed withdrawal strategy is also more attractive at low levels of risk aversion and high EIS than at high risk aversion levels. Increasing risk aversion makes the chance of ruin a significant concern and makes the Fixed strategy unattractive for retirees with a low risk threshold. Whilst the Annuity has an advantage in this respect, the importance attached to the presence of bequest motives underscores the benefit of annuitising.

In Panel B of Table 3.8, up to age 35 years in retirement, I find the Fixed and the Percentage of Wealth strategies to dominate the other strategies at low levels of risk aversion of $\gamma = 2$ to 3. The Life Expectancy and Annuity follow in rank respectively. Although the Fixed withdrawal runs to ruin 4.99% of the time, this is significantly low and overshadowed by the many who receive stable incomes throughout their lifetimes. The strategy remains attractive for low levels of risk aversion with ending portfolio levels reasonably high, only second to the Percentage of Wealth strategy giving a rich bequest potential. For the moderate to high risk averse retiree, the Life Expectancy withdrawal dominates the other withdrawal strategies. Next in preference is the Fixed withdrawal, the Percentage of Wealth being the least preferred phased withdrawal strategy is preferred to the Annuity. Life Expectancy withdrawal is preferred for its high levels of withdrawal and its ability to maintain bequest potential. The Fixed withdrawal is preferred for its stability but largely attractive for its large bequest potential. The Percentage of Wealth withdrawal provides an average terminal portfolio that is higher than that provided by the Fixed withdrawal plan. A significant downside to this strategy is it's moderate average withdrawal which is just marginally higher than the Fixed withdrawal, albeit a rather large standard deviation. This makes the Percentage of Wealth the least preferred among the alternative withdrawal plans when retirees have Epstein-Zin preferences in the absence of pension income. The bequest motives of retirees reduce the importance of including Annuities in the retirement plan.

In the presence of pensions, I find the Life Expectancy withdrawal to dominate the other withdrawal strategies, with the Percentage of Wealth withdrawal being preferred at low levels of risk aversion. Next to the dominating Life Expectancy withdrawal for a wide range of RRA and EIS levels is the Percentage of Wealth and finally, the Fixed withdrawal strategy, with the Annuity being least preferred. In the presence of pension income however, the Life Expectancy withdrawal continues to dominate the other withdrawal strategies with the Fixed withdrawal strategy preferred at low risk averse levels. The Fixed withdrawal also becomes the second preferred strategy after the Life Expectancy for a broad range of RRA and EIS. Of significance is the dominance of the Fixed withdrawal over the Percentage of Wealth withdrawal in the absence of pension income. The reason for this observation is that the means-tested age pension serves as a buffer for retirees' withdrawals, increasing with less disposable wealth and vice versa. This makes high volatile wealth dependent strategies such as the Percentage of Wealth attractive. The absence of pensions reveal the extent of volatility and in many cases, the low levels of withdrawals for retirees on this strategy when returns are low. On the contrary, the absence of pensions show the stability from the Fixed withdrawal, which is useful for retirement planning, making it more attractive for retirees compared to the Percentage of Wealth strategy.

3.4.6 The Influence of Bequest on Withdrawal Plan Choice

The impact of bequest is a significant part of any pension study. I investigate the influence of bequest on the results of this study. Our baseline analysis included pension income and bequest motive, Section 3.4.5 investigates the results in the absence of pension income. In this section, I analyse how retirees' preference for the alternative withdrawal strategies change when they have no bequest motives and their expected utilities are entirely based on withdrawals from their wealth in retirement. I do this in two different scenarios; firstly, when retiree has access to pension income such as the Means-Tested Aged Pension in Australia and secondly, in the absence of pension income. I compare the former to the results of the baseline case and the latter to the results from Section 3.4.2.

3.4.7 Pensions Present With No Bequest Motives

TABLE 3.9: Dominating Withdrawal Strategies (C)

I rank the alternative withdrawal strategies according to their expected utilities received from their annual withdrawals. I denote the Percentage of Wealth withdrawal by P, the Fixed withdrawal by F, the Life Expectation by L and Annuitisation by A. I present Panels A and B showing the dominant strategies conditional on retiree surviving to age 86 and unconditional on retiree's survival capped at age 100 respectively.

Panel A Dominating Withdrawal Strategies at age 86

RRA\EIS	0.1	0.2	0.3	0.4	0.5
2	P,L,F,A	P,L,F,A	P,L,F,A	P,L,F,A	P,L,F,A
3	P,L,F,A	P,L,F,A	P,L,F,A	P,L,F,A	P,L,F,A
4	A,P,F,L	A,P,L,F	A,P,L,F	A,P,F,L	A,P,F,L
5	A,F,P,L	A,F,P,L	A,P,F,L	A,P,F,L	A,P,F,L
7	A,F,P,L	A,F,P,L	A,F,P,L	A,F,P,L	A,F,P,L
10	A,F,P,L	A,F,P,L	A,F,P,L	A,F,P,L	A,F,P,L

Panel B Dominating Withdrawal Strategies at age 100

RRA\EIS	0.1	0.2	0.3	0.4	0.5
2	P,A,F,L	P,A,F,L	P,A,F,L	P,A,F,L	P,A,F,L
3	P,A,F,L	P,A,F,L	P,A,F,L	P,A,F,L	P,A,F,L
4	A,L,F,P	A,L,F,P	A,L,F,P	A,F,L,P	A,F,P,L
5	A,L,F,P	A,L,F,P	A,L,F,P	A,F,L,P	A,F,L,P
7	A,L,F,P	A,L,F,P	A,L,F,P	A,F,L,P	A,F,L,P
10	A,L,F,P	A,L,F,P	A,L,F,P	A,F,L,P	A,F,L,P

At age 86, shown in Panel A of Table 3.9, I find the Percentage of Wealth withdrawal to dominate other phased withdrawals and the Annuity at low levels of RRA, that is, γ=2 to γ=3 and all levels of EIS. Next in preference is the Life Expectancy withdrawal, the Fixed and finally the Annuity. The Percentage of Wealth strategy is preferred for its high income and lower standard deviation compared to the Life Expectation withdrawal, which has a high income as well but a greater level of dispersion about the mean income level coming in as the second in preference. At age 86, the ruin probability for the Fixed Strategy is low and highly insignificant as retirees choose this strategy over the Annuity at low risk aversion levels. As risk aversion increases, the moderate and high risk averse retirees prefer to annuitise, with Annuities dominating the phased withdrawal strategies. The Percentage of Wealth withdrawal shifts to be the second preferred strategy after the Annuity at moderate RRA level of γ=4, and further dominated by the Fixed withdrawal plan at increasing RRA levels.

When retirees have no bequest motives and derive all their utility from their withdrawals, they prefer stability in consumption over varying and volatile consumption. The presence of pensions make the Fixed withdrawal more attractive for moderate to high risk averse retirees as it increases the average withdrawal level with minimum change in standard deviation. The two least preferred strategies at high risk averse levels, the Life Expectancy and Percentage withdrawals, have high deviations about the mean consumption level even in the early years in retirement. This amount of volatility early in retirement is undesirable and reduces utility among retirees.

In the long term, as shown in Panel B of Table 3.9, the Annuity dominates phased withdrawal plans for retirees with no bequest motives, crowding out the Percentage of Wealth withdrawal dominance for moderate to high risk averse retirees. The Percentage of Wealth strategy is still preferred at low risk averse levels of γ=2 and 3, while retirees who are more risk averse will prefer to annuitise. With annuities dominating the phased withdrawal plans, the Life Expectancy is preferred among the three alternative phased withdrawal strategies. The next preferred is the Fixed withdrawal, with the Percentage of Wealth strategy being the least preferred. After the peak of withdrawal income in the late eighties, the Life Expectancy withdrawal on average provides higher utilities than any other phased withdrawal strategy. At low levels of EIS, the Life Expectancy withdrawal is only second to annuities for moderate to high risk averse retirees. At high EIS levels however, the Fixed withdrawal strategy is preferred to the Life Expectancy withdrawal as it has a reasonable average withdrawal level as well as a lower dispersion about the mean withdrawal. The Percentage of Wealth is the least preferred for retirees with no bequest motives.The Fixed strategy is preferred to the other phased withdrawals at high RRA and high EIS for the stability in income it provides, it doesn't run to ruin because of the presence of pensions but decreases in level. Even with the decrement for retirees who run out of wealth, the dispersion is minimal compared to that observed for the remaining phased withdrawal strategies, with extremely high standard deviations.

Compared to the baseline result with pensions and bequest motives present, I find that retirees' choice of alternative withdrawal plan depends on their decision to bequeath or not. Retirees choose annuities over phased withdrawals only if they have no bequest motives. In the presence of bequest motives, the Life Expectancy and the

Percentage of Wealth withdrawals are the dominant phased withdrawal strategies in the long term. Annuities and the Life Expectancy withdrawal dominate when there is no bequest motive.

3.4.8 No Pensions Present and No Bequest Motives

TABLE 3.10: Dominating Withdrawal Strategies (D)

I rank the alternative withdrawal strategies according to their expected utilities. I denote the Percentage of Wealth withdrawal by P, the Fixed withdrawal by F, the Life Expectation by L and Annuitisation by A. I present Panels A and B showing the dominant strategies conditional on retiree surviving to age 86 and unconditional on retiree's survival capped at age 100 respectively.

Panel A Dominating Withdrawal Strategies at age 86

RRA\EIS	0.1	0.2	0.3	0.4	0.5
2	P,L,F,A	P,L,F,A	P,L,F,A	P,L,F,A	P,L,F,A
3	L,P,F,A	F,L,P,A	L,P,F,A	L,P,F,A	P,L,F,A
4	F,A,P,L	F,A,P,L	F,A,P,L	F,A,P,L	F,A,P,L
5	F,A,P,L	F,A,P,L	F,A,P,L	F,A,P,L	F,A,P,L
7	F,A,P,L	F,A,P,L	F,A,P,L	F,A,P,L	F,A,P,L
10	F,A,P,L	F,A,P,L	F,A,P,L	F,A,P,L	F,A,P,L

Panel B Dominating Withdrawal Strategies at age 100

RRA\EIS	0.1	0.2	0.3	0.4	0.5
2	L,P,A,F	L,P,A,F	L,P,A,F	L,P,A,F	L,P,A,F
3	A,L,P,F	A,L,P,F	A,L,P,F	A,L,P,F	A,L,P,F
4	A,F,L,P	A,F,L,P	A,F,L,P	A,F,L,P	A,F,L,P
5	A,F,L,P	A,F,L,P	A,F,L,P	A,F,L,P	A,F,L,P
7	A,F,L,P	A,F,L,P	A,F,L,P	A,F,L,P	A,F,P,L
10	A,F,L,P	A,F,L,P	A,F,L,P	A,F,L,P	A,F,P,L

In the short term, as shown in Panel A of Table 3.10, analysing the ranking of phased alternative withdrawal plans and the Annuity at age 86, I find the Percentage of Wealth and Life Expectancy withdrawals to dominate the Fixed strategy and the Annuity at low levels of RRA, th is, at γ=2 and 3 respectively at all levels of EIS. The Fixed withdrawal is preferred to the Annuity at low levels of low risk aversion, making the latter the least preferred strategy at low risk averse levels. As risk aversion increases to γ=4, the dominating strategy is the Fixed withdrawal strategy, next in preference is the Annuity, followed by the Percentage of Wealth and the Life Expectation strategies. As risk aversion increases, the moderate and high risk averse retirees prefer the Fixed withdrawal and Annuities to the other alternative withdrawal plans, with the Percentage of Wealth preferred to the Life Expectancy withdrawal. When retirees have neither pension income nor bequest motives and derive all their utility from their withdrawals, they prefer stability in consumption over varying and volatile consumption. The Fixed withdrawal is more attractive in the early retirement years, in this case at age 86, with its moderate average withdrawal level and minimum standard deviation. It also has a low chance of ruin and this makes a fair comparison with the Annuity. The Life Expectancy and Percentage withdrawals have high deviations about the mean withdrawal levels, with the former beginning at very low levels and increasing later in retirement.

In the long term, the Life Expectancy withdrawal dominates at low risk aversion level of γ=2 and is closely trailed by the Percentage of Wealth withdrawal. The annuity is preferred to the Fixed withdrawal plan. The more risk averse retirees prefer to annuitise, with annuities dominating all levels of EIS for RRA levels greater than 2. With annuities dominating the phased withdrawal plans, the Fixed withdrawal is preferred among the three alternative strategies. The next preferred is the Life Expectancy, with the Percentage of Wealth being the least preferred. With neither pensions nor bequest motives, annuities dominate for moderate to high risk averse retirees as they provide a steady income which cannot be outlived. The preferred strategy among the phased withdrawal plans, the Fixed withdrawal, performs a similar role as the annuity albeit with a chance of ruin. Among the variable withdrawal strategies, the Life Expectancy withdrawal is preferred to the Percentage of Wealth withdrawal for its higher average withdrawal over the entire retirement horizon.

Compared to the results from Section 3.4.6 with pensions present but no bequest motives, I find that retirees' choice of alternative withdrawal plan depends to some extent on the availability of pension income. While the annuity dominates in both scenarios, there is a significant change in the ordering of the phased withdrawal strategies. At low risk aversion levels, the Percentage of Wealth withdrawal plan dominates when pension income is present whilst the Life Expectancy withdrawal dominates in the absence of pension income. Overall, I find the Life Expectancy and Percentage of Wealth withdrawal plans to be more attractive when there are pensions than when there is no pension income. The Life Expectation dominates the Percentage of Wealth strategy in both cases as it has a higher withdrawal level with a lower standard deviation to withdrawal proportion. The means-tested-aged pension acts as a reserve income that makes these aformentioned strategies appear smoother, and the retiree does not experience the severity of volatility from these strategies. When this becomes more apparent in the absence of pensions, retirees are better off with Fixed withdrawal strategy than the two variable strategies for long-term planning and consumption purposes.

3.5 Conclusion and Discussion

Just like a well-defined superannuation contribution scheme, it is equally important to have well-structured drawdown rules in the decumulation phase. Investment strategies are also critical to ensure that individuals' accumulated wealth at retirement is sustainable. Inappropriate decumulation strategies can cause serious repercussions for the post retirement lifestyle. Using the institutional characteristics of annuity markets in Australia, I show which drawdown strategies dominate the annuity and vice-versa and retirees' preferences under varying conditions. For retirees who are willing to accept some financial risk in exchange for retaining the benefits of liquidity and bequest, they choose one of three different withdrawal strategies which they default into at retirement: the Fixed, the Percent of Wealth and the Life Expectation withdrawals. I analyse how these strategies compare to the life annuity in the presence and absence of pension income, bequest motives and improving mortality under Epstein-Zin preferences.

By using the Epstein-Zin utilities, we are able to disentangle RRA from the EIS. An individual with a high degree of risk aversion will want to avoid uncertainties in her consumption in a given time period, or a reduction in consumption in a state such as a fall in equities. An individual with low EIS will avoid consumption volatility over time and more so will seek to avoid a reduction in her consumption level relative to previous period consumption level. By varying the degrees of EIS and RRA, I observe changes to the retirees' choice of drawdown strategy based on the expected utilities from the strategies. Most importantly, I find that for various levels of RRA and a given level of EIS, whenever the inverse of EIS is greater than the RRA level, there is increased utility from the same wealth and withdrawal profile and vice-versa. Compared to the CRRA investor, an investor who is willing to accept consumption volatility over time or has late resolution to uncertainty has a high expected utility only if the investor is less risk averse than predicted by the inverse EIS for the CRRA utility function. I find retirees choose to self-annuitise via phased withdrawal strategies over buying annuities whenever risk aversion is higher than predicted by the CRRA utility. High risk averse retirees look to avoid the possibility of portfolio shortfall; resulting in having zero income and not meeting their bequest needs. These risks are eliminated in two of the phased strategies considered in this study, the Life Expectancy and Percent of Wealth withdrawal strategies, with retirees choosing these strategies over the Fixed strategy and annuity option. This is a core reason for the preference of such phased withdrawal strategies over annuitisation in retirement. This is an important finding as previous studies have overlooked the influence of EIS in retirees' withdrawal strategy selection. This result is consistent with previous studies that show that subjects prefer early resolution of uncertainty and have RRA greater than the reciprocal of the EIS.

For different equity allocation levels in the retirement portfolio, I find increased sustainability with increasing equity levels. Chances of ruin for the Fixed withdrawal strategy is significantly minimised when the equity levels are increased from conservative through balanced to a growth portfolio. The average withdrawals and terminal values for the remaining phased withdrawal strategies increase with increasing equity allocation. This finding is in agreement with works such as Cooley et al. (2001) who assert that a retirement fund should have at least 50% allocation to equity to be sustainable. Asset allocation does not change the order of preference for the alternative strategies. The Life Expectancy strategy ranks top among the alternative strategies, with the Percent of Wealth strategy being the second preferred strategy and the Fixed strategy being the least preferred phased strategy; preferred only to the life annuity.

When government sponsored pensions are present, I find the Life Expectancy withdrawal to dominate the other withdrawal strategies, with the Percentage of Wealth withdrawal being preferred at low RRA and high EIS. Life Expectancy withdrawal dominates for a wide range of RRA and EIS levels and the second preferred strategy is the Percentage of Wealth. The Fixed withdrawal strategy is next in the ranks, with the Annuity being the least preferred strategy. In the absence of pension income however, the Life Expectancy withdrawal continues to dominate the other withdrawal strategies with the Fixed withdrawal strategy preferred at low risk averse levels. The Fixed withdrawal also becomes the second preferred strategy after the Life Expectancy for a broad range of RRA and EIS. The absence of pensions shifts plan dominance to the Fixed Withdrawal over the Percentage of Wealth withdrawal. I find the inclusion of a pensions model such as the means-tested Age Pension included in this study to serve as a buffer for retirees' withdrawals, increasing with less disposable wealth and vice versa. This makes high volatile wealth dependent strategies such as the Percentage of Wealth attractive. The absence of pensions reveal the extent of volatility and in many cases, the low levels of withdrawals for retirees on this strategy when returns are low. On the other hand, the absence of pensions show the stability from the Fixed withdrawal, which is useful for retirement planning, making it more attractive for retirees compared to the Percentage of Wealth strategy. This finding makes the presence of pensions a significant determinant of retiree's choice of drawdown strategy in retirement.

Finally, I find the presence of bequest motives among retirees has influence on their distribution plan choice. Retirees choose annuities over phased withdrawals only if they have no bequest motives. When retirees have no bequest motives and derive all their utility from their withdrawals, they prefer stability in consumption over varying and volatile consumption. Annuities dominate the phased withdrawal strategies, with the Fixed withdrawal being the most attractive phased withdrawal strategy. The presence of pensions makes the Fixed withdrawal more attractive for moderate to high risk averse retirees as it increases the average withdrawal level with minimum change in standard deviation and the chance of ruin is eliminated by the pensions. The two least preferred strategies at high risk averse levels, the Life Expectancy and Percentage withdrawals, have high deviations about the mean consumption level even in the early years of retirement. In the absence of bequest motives however, the Life Expectancy and the Percentage of Wealth withdrawals are the dominant phased withdrawal strategies for the long term.

The results should be interpreted with caution, as one has to be mindful about the study's limitations. I assume the annuitisation decision is made at retirement and study the life utility paths until age of death to compare its performance with other withdrawal strategies. Future studies could price annuities at all ages in retirement, giving the retiree the choice of annuitising at any given age. The retiree may however need the forward-looking approach I use in this study to determine when there are significant utility gains from annuitising to switch to annuities. Also, this study assumes an incomplete annuity market where the level income life annuity, is the only available annuity. A fruitful line of research could be to explore the inclusion of other annuity types such as deferred annuities in my settings as this may yield different and interesting results.

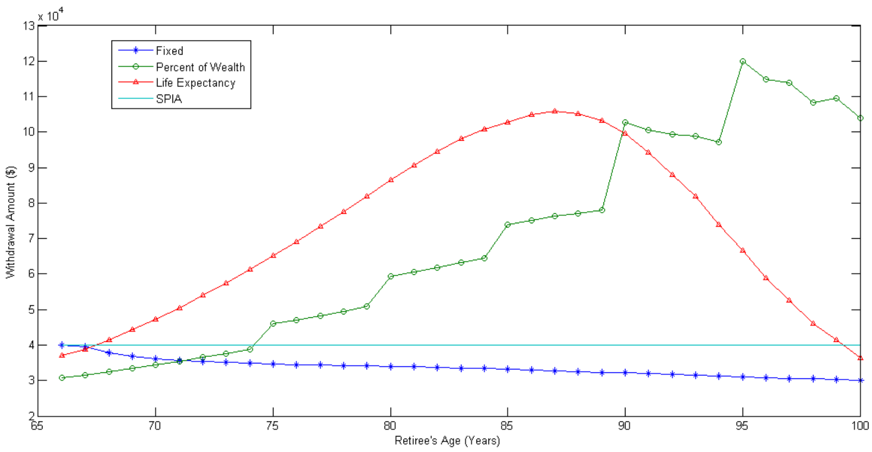
Appendix

3.A Withdrawal Strategies

The four different withdrawal strategies used in this study are carefully chosen with varying properties to meet the needs of different retirees. They present an assortment of choices for retirees of different risk preferences. The dynamics of the various strategies over time are presented in Figure 3.A.1.

FIGURE 3.A.1: Withdrawal Strategy Patterns

This figure shows the withdrawal patterns of the four different strategies; the Fixed withdrawal path is shown with a line with stars; Percentage of Wealth withdrawal path by a line with circles; the Life Expectancy withdrawal path by a line with triangles and the SPIA payouts with a solid line. The plot represents the average of 10,000 simulated withdrawal paths for a $500,000 initial investment.



The Fixed withdrawal strategy, whereby the retiree receives a fixed cash amount annually, replicates the SPIA. There are two sides to holding a fixed withdrawal strategy; there is always a chance of financial ruin, which will occur when returns on the retirement wealth are very low or negative over long periods as the level of income does not change to reflect changing wealth levels. Nevertheless, when returns are high and the portfolio grows steadily, the retiree continues to maintain her level of withdrawal, foregoing a large amount of potential consumption and leaving large amounts of wealth at her death. This should be appealing to retirees with substantial bequest potential. The decreasing levels observed over time are a reflection of the accumulating wealth at older ages leading to a lower pension income, thereby decreasing the level of income on average.

The Percent of Wealth withdrawal does not run to ruin even in times of poor returns. The level of consumption however increases or decreases with respect to the growth of the fund size. It is attractive to retirees who are not willing to take the chance of possible financial ruin. The major disadvantage with this strategy is that extreme volatility in returns causes equal variation in the level of consumption. The stepwise increasing in the level of income is commensurate with the minimum withdrawal amounts specified under Australia's current legislation. The Life Expectancy withdrawal begins with a lower annual payout, which rises above the other withdrawal strategies in the early years of retirement. It peaks in the mid-eighties and declines thereafter, although the withdrawal fraction increases with age. The decreasing income levels are due to less remaining wealth in the retiree's account after the consistently large withdrawals. It has low terminal values, ensuring an efficient consumption based on the individual's remaining life expectation. Finally, the SPIA provides a fixed income amount over the lifetime of the individual. The individual will only fully annuitise if she has no bequest motives.

Table 3.A.1: Dominating Withdrawal Strategies

I rank the alternative withdrawal strategies according to their expected utilities. I denote the Percentage of Wealth withdrawal by P, the Fixed withdrawal by F, the Life Expectation by L and Annuitisation by A. I present Panels A and B showing the dominant strategies conditional on retiree surviving to age 92 and unconditional on retiree's survival capped at age 100 respectively for the 50-50 Growth/Defensive strategy.

Panel A Dominating Withdrawal Strategies at age 92

RRA\EIS	0.1	0.2	0.3	0.4	0.5
2	P,L,F,A	P,L,F,A	P,L,F,A	P,L,F,A	P,L,F,A
3	P,L,F,A	P,L,F,A	P,L,F,A	P,L,F,A	P,L,F,A
4	A,P,F,L	A,P,L,F	A,P,L,F	A,P,F,L	A,P,F,L
5	A,F,L,P	A,F,L,P	A,F,L,P	A,F,P,L	A,F,P,L
7	A,F,L,P	A,F,L,P	A,F,L,P	A,F,L,P	A,F,P,L
10	A,F,L,P	A,F,L,P	A,F,L,P	A,F,L,P	A,F,P,L

Panel B Dominating Withdrawal Strategies at age 100

RRA\EIS	0.1	0.2	0.3	0.4	0.5
2	L,P,F,A	L,P,F,A	P,L,F,A	P,L,F,A	P,L,F,A
3	L,P,A,F	L,P,A,F	L,P,A,F	L,P,A,F	L,P,A,F
4	L,A,P,F	L,A,P,F	L,A,P,F	L,A,P,F	L,A,P,F
5	L,A,P,F	L,A,P,F	L,A,P,F	L,A,P,F	L,A,P,F
7	L,A,P,F	L,A,P,F	L,A,P,F	LA,P,F	L,A,F,P
10	L,A,P,F	L,A,P,F	L,A,P,F	L,A,P,F	L,A,P,F

3.B Summary of Portfolio Values for Alternative Asset allocations given Improving Mortality Expectations

TABLE 3.B.1: Summary of Portfolio Values for Simulated Returns for 30/70 Growth/Defensive assets for Decreasing Mortality Expectations

Panel A Conditional on surviving to age 86

Withdrawal Type	Average withdrawal (Standard deviation)	Probability of ruin Ruin	Average Ending Portfolio	Percentiles of distribution of ending portfolio 5	25	50	75
Fixed	37,015(5,637)	3.89%	768,564	26,105	340,143	643,107	1,050,080
Percent of Wealth	42,440(15,495)	0%	760,431	336,450	520,185	697,137	922,760
Life Expectancy	62,296(24,222)	0%	424,363	222,428	314,144	396,516	502,621
SPIA	40,026(0)	0%	0	0	0	0	0

Panel B. Conditional on surviving to age 92

Withdrawal Type	Average withdrawal (Standard deviation)	Probability of ruin Ruin	Average Ending Portfolio	Percentiles of distribution of ending portfolio 5	25	50	75
Fixed	36,668(6,272)	9.96%	896,091	- 145,647	242,405	679,275	1,298,920
Percent of Wealth	47,631(21,643)	0%	695,140	266,298	440,635	621,130	861,856
Life Expectancy	65,758(25,665)	0%	197,623	104,276	147,457	183,370	235,432
SPIA	40,026(0)	0%	0	0	0	0	0

Panel C. Conditional on surviving to age 100

Withdrawal Type	Average withdrawal (Standard deviation)	Probability of ruin Ruin	Average Ending Portfolio	Percentiles of distribution of ending portfolio 5	25	50	75
Fixed	35,996(7,098)	19.91%	1,103,513	- 593,960	21,007	767,352	1,733,990
Percent of Wealth	52,887(27,045)	0%	492,684	149,144	303,293	418,689	586,621
Life Expectancy	60,459(24,003)	0%	37,856	26,601	31,891	36,546	41,226
SPIA	40,026(0)	0%	0	0	0	0	0

TABLE 3.B.2: Summary of Portfolio Values for Simulated Returns for 70/30 Growth/Defensive assets for Decreasing Mortality Expectations

Panel A Conditional on surviving to age 86							
Withdrawal Type	Average withdrawal (Standard deviation)	Probability of ruin Ruin	Average Ending Portfolio	Percentiles of distribution of ending portfolio 5	25	50	75
Fixed	34,786(5,965)	4.60%	1,693,330	11,288	617,000	1,241,867	2,307,375
Percent of Wealth	54,502(34,702)	0%	1,275,772	360,597	697,337	1,059,324	1,605,226
Life Expectancy	78,607(47,736)	0%	652,375	233,772	395,814	559,737	805,421
SPIA	40,026(0)	0%	0	0	0	0	0

Panel B Conditional on surviving to age 92							
Withdrawal Type	Average withdrawal (Standard deviation)	Probability of ruin Ruin	Average Ending Portfolio	Percentiles of distribution of ending portfolio 5	25	50	75
Fixed	34,113(6,189)	8.40%	2,646,862	- 199,605	723,344	1,786,978	3,638,099
Percent of Wealth	67,591(53,581)	0%	1,370,463	311,999	664,758	1,071,946	1,788,442
Life Expectancy	87,644(54,015)	0%	330,741	113,441	191,134	275,790	415,259
SPIA	40,026(0)	0%	0	0	0	0	0

Panel C Conditional on surviving to age 100							
Withdrawal Type	Average withdrawal (Standard deviation)	Probability of ruin Ruin	Average Ending Portfolio	Percentiles of distribution of ending portfolio 5	25	50	75
Fixed	33,069(5,870)	9.93%	6,034,238	- 526,262	1,015,898	3,747,046	7,977,668
Percent of Wealth	93,694(94,649)	0%	1,343,653	276,396	508,464	1,008,363	1,672,713
Life Expectancy	88,216(59,131)	0%	58,769	30,357	38,672	49,804	69,127
SPIA	40,026(0)	0%	0	0	0	0	0

Chapter 4

Portfolio Choice for Retirees with Bequest Motive: The Case for Deferred Life Annuities

4.1 Introduction

Lifetime annuities provide longevity insurance in retirement, ensuring that retirees do not outlive their wealth. Despite the impact of government support and the benefits of annuitisation that are well documented in literature, there is considerable evidence of annuity aversion among investors (Blake et al., 2003). This has led to what economists call the 'annuity puzzle'. The actuarially unfair pricing of annuities is one of the important reasons attributed to this aversion of annuities by retirees according to Friedman and Warshawsky (1990). They find that the annuity yield is depressed due to the loading factor imposed by the life companies to cater for transaction costs and adverse selection among annuitants. Brown (2001) argues that the aversion to annuities could be attributed to investors' knowledge of their subjective survival probabilities. Therefore, investors in poor health will avoid purchasing annuities since the decision to purchase annuities is irreversible. According to Bernheim (1992), the presence of bequest motives can also discourage annuitisation. Individuals will avoid locking in wealth through annuitisation but rather hold liquidity to meet their bequest needs.

In order to mitigate some of the above problems with traditional annuities, annuity providers in recent years have introduced innovative products such as deferred annuities. Unlike immediate annuities, which begin paying income to the annuitant almost immediately after purchase, the deferred annuities allow the buyers to shift the commencement of the payment to an agreed future date. Whilst no money is returned to heirs if the annuitant dies before the payout begins, the upside to surviving the deferring period is that the retiree receive higher mortality credits (additional return above the risk-free rate of return on the annuity income) from the redistribution of annuitised wealth among surviving participants from annuitants who died during the deferring period. In effect, purchasing a deferred annuity is a far less expensive proposition relative to an immediate annuity with identical payouts.

The inclusion of immediate annuities in retirement planning is deliberated extensively in previous studies (Ameriks et al., 2001; Blake et al., 2003; Horneff, Maurer, Mitchell, & Dus, 2008; Horneff, Maurer, & Stamos, 2008). Deferred annuities in retirement, although available in some annuity markets, are less discussed in retirement literature. Important studies have found the deferred annuity to be preferred to immediate full annuitisation in retirement (Horneff, Maurer, & Rogalla, 2010; Milevsky, 2005; Scott, 2008; Scott, Watson, & Hu, 2007, 2011). Milevsky (2005) introduces a concept product he called the Advanced Life Deferred Annuity (ALDA), a variant of the pure deferred annuity contract. This product is indexed for inflation, acquired in installments and pays off towards the end of the retiree's lifecycle. Gong and Webb (2010) show that households would prefer the ALDA to either immediate, postponed, or self-annuitisation. They also show the deferred annuity to provide the very necessary longevity insurance for retirees at a lower cost. Scott (2008) and Scott et al. (2011) show that purchasing longevity annuities substantially increases spending in retirement, it addresses longevity risk and the problem of deciding what portion of an individual's assets to annuitise as it usually does not require full annuitisation. Horneff et al. (2010) derive an optimal lifecycle portfolio choice and investor's consumption pattern when they are faced with uncertain labor income, a risky capital market, and mortality risk and have access to deferred annuities prior to retirement. They introduce gradual annuitisation with deferred annuities until retirement age. They also use time-varying mortality that exhibits both expected improvements in future life expectancy as well as stochastic variations around this trend.

In this essay, I incorporate a single premium pure deferred annuity in retirement portfolio for a Hyperbolic Absolute Risk Aversion (HARA) utility maximising investor who faces expected mortality improvements, risky capital markets and an incomplete annuity market at retirement. The retiree has access to a government-sponsored pension and has bequest and capital preservation needs. She has the option to self-annuitise or purchase deferred annuities with varying deferred periods with a portion of her wealth at retirement age 65 or at some later age. I evaluate and rank the various investment choices available to the retiree based on the distribution of terminal utilities she derives from making her investment choice.

This essay makes two important contributions to the literature. Firstly, whilst past studies discuss the benefits of deferred annuity, they are silent on the suitable length of the annuity's deferral period and what influences retirees' choice of deferred annuities. In this essay, I examine appropriate deferral periods based on retirees' risk preferences and desire to increase consumption and terminal utility. Secondly, I consider the appropriate timing of deferred annuities both in the presence and absence of government means-tested age pension and how the timing of the deferred annuity purchase affects the length of the annuity deferral period.

Deferred annuities are sold in a number countries including the United States of America (US), the United Kingdom (UK) and Chile (Ganegoda & Bateman, 2008) . However, in this essay, I examine them in an Australian setting. Although Australia has a small annuity market compared to countries such as the US and the UK, the market has been showing signs of growth in recent times.[1] Until recently, Australian pension rules prescribing minimum withdrawals did not permit deferred earnings to be eligible for tax benefits. However, the Australian government announced plans for concessional tax treatment for deferred lifetime annuities commencing from July 2014. This makes it an opportune time to study the efficacy of including deferred annuities in a retirement portfolio for the Australian retiree. The leading provider of annuities in Australia, Challenger Life Company, which sells over 95% of all life annuities in Australia, expects the price of a deferred annuity purchased by a 65-year-old man which begins payouts of $8,000 annually from age 80 to cost as little as $10,000 (Uren, 2012). This is about 10% of the price of an immediate life annuity purchased at the same age and up to 50% less than purchasing an immediate annuity at age 80 for a similar payout. Purchasing a deferred annuity within a pension portfolio leaves a larger portfolio size, due to its lower price at purchase, along with an increased possibility of higher investment returns and the opportunity for higher consumption before the annuity payments begin. Whilst the model calibration in this essay rests on assumptions that are realistic in an Australian setting, my results are generally applicable in other markets.

[1] Annuity sales since the last quarter of 2011 indicate that the lifetime annuity is gaining popularity in the market. If the trend continues, 2013 should record the highest annuity purchases in Australia since 2004 (Plan for Life, 2013).

Like this study, Horneff et al. (2010) derive portfolio choice and consumption patterns over life-cycle for households with access to deferred annuities. This essay differs from theirs in two ways. Firstly, I examine the inclusion of deferred annuities in a more realistic 'post retirement' setting as I believe that purchasing annuities before retirement comes at substantial costs to the investor. The latter constrains the upside potential of the retiree's portfolio by forfeiting possible returns from investable assets, decreasing her level of wealth at retirement and exposing her to a lower standard of living.[2] Moreover, the option of gradual annuitisation is unavailable in many markets, including Australia, which renders purchase of deferred annuities prior to retirement impractical. This strategy also does not take into account the tax benefits that may be available to retirees for purchasing annuities with their retirement lumpsum after they retire. Instead I propose a single premium deferred annuity that is purchased whilst an individual is in retirement. The timing and purchase amount of retiree annuitisation are based on her consumption preferences and her desire to maximise lifetime utility.

Secondly, Horneff et al. (2010) do not consider the utility from bequest.[3] In contrast, I allow the retiree to use a Constant Proportion Portfolio Insurance (CPPI) strategy to invest the non-annuitised portion of the retiree's assets that is able to cater for the bequest motive of retirees. A CPPI strategy sets a wealth floor level, which denotes the minimum acceptable level of wealth prior to death. The retiree's allocations to risky and risk free assets are based on her portfolio value at all times: increasing (decreasing) her risky investments with increasing (decreasing) portfolio value to ensure that the wealth floor is preserved at all times. This answers the aspect of the annuity puzzle arising from the presence of retirees having bequest motives. Moreover, the CPPI model serves as an efficient means of capital preservation which is essential for retirees in the aftermath of the recent Global Financial Crisis and the resulting decline in the value of many investment funds. To my knowledge, this is the first study to incorporate the CPPI model in retirement planning.[45].

[2]Surely at the age of say 40, one would be aiming to grow their wealth rather than decumulate with annual annuity purchases. The fact that there is almost zero uptake of annuities among investors prior to retirement proves the point.

[3]Although they assume that the household spends only part of their wealth in buying annuities leaving the remainder for bequest, they do not ensure that the bequest is not depleted below a certain minimum acceptable level.

[4]This model was proposed by Perold (1986) and Black and Jones (1987) as an alternative to complex option replication approaches to portfolio insurance.

[5]The complexity of this strategy along with the periodic balancing may make implementation of this strategy difficult

In what follows, I introduce the CPPI model which is used to derive the optimal consumption and portfolio allocation of risky and risk-free assets in Section 4.2. I discuss the wealth dynamics in 4.2.1, the setup of the CPPI model in 4.2.2 and deferred annuities in 4.2.3. I also elaborate on consumption and risk aversion in 4.2.4, government pensions in 4.2.5, mortality dynamics in 4.2.6 and the model calibration in 4.2.7. The Omega Ratio is discussed in Section 4.2.8. Section 4.3 presents my findings and Section 4.4 concludes the essay.

4.2 The Model

4.2.1 Wealth Dynamics

For this study, I employ the Constant Proportion Portfolio Insurance (CPPI) Model, which is augmented to include parameters to meet the decumulation planning. The retiree with HARA preferences has access to three post-retirement financial assets: a risky equity fund, a risk-free bond fund and deferred annuities. She has the option of purchasing deferred annuities on or after her retirement age. The retiree does not fully annuitise her wealth for two reasons. Firstly, she requires consumption in the time preceding the annuity payouts and secondly, because she has an expected bequest level that cannot be violated under any circumstances.

At retirement, the retiree holds an initial wealth, W_{65} of which she has the option to purchase a single premium '$n - year$' deferred annuity worth, W_a. This option is exercised at any age between 65 and 75 years. Her wealth after the annuity purchase, the remaining wealth, is represented by W_0 . The retiree receives an annual pension income of P_t, which is indexed annually for inflation. For the analysis with wealth dynamics, I may often refer to remaining wealth, W_0 as the retiree's initial or starting wealth and this is used henceforth. The retiree's initial wealth is determined by whether or not she purchases annuities at age 65. That is, initial wealth takes the form of Equation 4.1 when the retiree annuitises at age 65 and Equation 4.2 otherwise.

$$W_0 = W_{65} - W_a + P_{65} \tag{4.1}$$

$$W_0 = W_{65} + P_{65} \tag{4.2}$$

The retiree decides the proportion of her wealth to allocate to the risky and risk-free assets at retirement and is free to choose her proportion to invest in annuities later in retirement. The retiree invests her remaining wealth and pension income in the bond and equity funds and makes annual withdrawals of size, C_t for consumption. After purchasing deferred annuities, the retiree's wealth level at hand at all times prior to receiving the annuity payments, is a function of remaining wealth and pension income. For simplicity, I assume the pension income is adjusted annually for inflation, π, rather than semiannual indexation to the Consumer Price Index (CPI).

Before the annuity payments begin, the retiree's portfolio changes as:

$$W_{t+1} = P_t(1+\pi) + [P_{t+1} + (1-\alpha_t)(1+r_f)W_t + \alpha_t(1+s_t)W_t] - C_t \tag{4.3}$$

Where r_f is the bond return and s_t is the stock return, α is the proportion of wealth invested in the risky asset fund.

With the addition of the annuity payments, A_t after the deferred period, wealth changes are recorded as:

$$W_{t+1} = P_t(1+\pi) + [A_t + P_{t+1} + (1-\alpha_t)(1+r_f)W_t + \alpha_t(1+s_t)W_t] - C_t \tag{4.4}$$

Where r_f is the bond return and s_t is the stock return.

When annuity payments begin, the retiree's wealth at hand is a function of her current wealth, the annual pension income and the annual annuity payment received. Since I consider a deferred annuity with fixed payments, the increment in terms of retiree's wealth level is constant with an annual fixed upward adjustment. For the two financial assets available to the retiree, the risk free asset evolves deterministically with an exponential growth at a constant rate of r. For a unit of the risk free asset valued at X_t,

$$\frac{dX_t}{X_t} = rdt \tag{4.5}$$

I denote the value of a unit of the risky asset by S_t and the asset follows a Geometric Brownian Motion with drift μ and volatility σ.

$$dS(t) = \mu S(t)dt + \sigma S(t)dZ_t \tag{4.6}$$

Where dZ_t is a standard Wiener process.

4.2.2 The CPPI Model

The retiree's bequest motive informs her choice of wealth "Floor" level, F_0 for the CPPI model. The Floor represents the present value of the retiree's expected level of bequest, B_T. It is also the minimum amount of remaining wealth a retiree is willing to accept in her worst-case scenario prior to death. This level of wealth is determined at the beginning of the portfolio cycle and is less than the initial wealth level, satisfying the budget constraint,

$$F_0 \leq W_0 \tag{4.7}$$

The initial Floor level selected grows to the guaranteed bequest level at the end of the investment horizon such that;

$$F_t \equiv B_T e^{-r(T-t)}, t \in [0, T] \tag{4.8}$$

At every time t, for various levels of retiree wealth equal or exceeding the wealth Floor level, I define a Cushion, K_0, as the difference between the retiree's available portfolio wealth, W_t and the wealth Floor. This equals the surplus of the retiree's available wealth.

The initial cushion represents how much the CPPI investor is prepared to lose, and will vary with the investors' initial wealth level.

$$K_t \equiv max(0, W_t - F_t) \tag{4.9}$$

The retiree's portfolio value is therefore, represented at all times by the sum of the portfolio's floor level and the portfolio cushion.

$$W_t = K_t + F_t \tag{4.10}$$

According to the CPPI model, the retiree holds a constant multiple, m of the cushion with leverage greater than 1. This constant multiple is invested in the risky asset. The value of the multiple determines the type of portfolio insurance.

- For $m = 1$, the payoff is linear and represents the buy and hold strategy
- For $m < 1$, the payoff is convex and represents the constant-mix strategy
- For $m > 1$, the payoff is concave and the CPPI holds.

With the choice of a multiplier parameter, the weight of portfolio invested in the risky asset is obtained as:

$$\alpha_t \equiv \frac{mK_t}{W_t} \tag{4.11}$$

α is the proportion of the cushion invested in risky assets. The portfolio value is thus defined by:

$$\frac{dW_t}{W_t} = \frac{(W_t - mK_t)}{W_t}\frac{dX_t}{X_t} + \frac{mK_t}{W_t}\frac{dS_t}{S_t} \tag{4.12}$$
$$= (1-\alpha_t)rdt + \alpha_t\frac{dS_t}{S_t} \tag{4.13}$$

As the portfolio value increases, the investor increases her share in the risky assets. The investor becomes more conservative as the portfolio value declines towards the set floor and increases her share in the riskless asset whilst decreasing her share of the risky asset. The portfolio value is kept above the floor at all times, ensuring that $F_t \leq W_t$. At the horizon, the floor is obtained with certainty, and this value may be equal or greater than the bequest value.

$$W_T \geq F_T \geq B_T \tag{4.14}$$

4.2.3 Deferred Annuities

The deferred annuities used in this essay provide no death or surrender benefits. Upon surviving the deferred period, the retiree receives fixed annual payouts whose level is specified at the time of annuitisation. For an '$n - year$' deferred whole life annuity due payable of \$1 at the beginning of every year while the retiree age x, survives the deferred period $x + n$, onwards, the present value random variable is represented as:[6]

[6] Bowers and Hickman (1997)

$$Y = \begin{cases} 0 & \text{if } 0 \leq T < n, \\ {}_{n|}\ddot{a}_{\overline{T+1-n}|} & \text{if } T \geq n \end{cases} \tag{4.15}$$

In addition, its actuarial present value is:

$$E[Y] = {}_{n|}\ddot{a}_x = {}_nE_x\ddot{a}_{x+n} \tag{4.16}$$

$$= \ddot{a}_x - \ddot{a}_{x:\overline{n}|} \tag{4.17}$$

$$= \sum_{t=n}^{\infty} v^t {}_tp_x; \tag{4.18}$$

${}_tp_x$ denotes the conditional probability of an individual age x surviving to age $x+t$, v^t is the discount function which is expressed by $1/(1+r)^{-t}$ with r being the discount rate or the interest rate at which the annuity is compounded annually. T denotes the age of death and n, the length of deferral. I include in the annuity pricing, an insurance loading factor, L, which increases the actuarial present value of the deferred annuity. The insurance loading factor covers the insurance company's commissions, all expenses and taxes. This amount is multiplied by the pure annuity premium to arrive at the annuity's market price (Milevsky, 1998). The work of Mitchell et al. (2000) assert that the loading factor increases with increasing issue age and this reasoning is applied to the annuity pricing in this essay. Hence, the market price of the deferred annuity is given by:

$${}_{n|}\ddot{a}_x = (1+L)\sum_{t=n}^{\infty} v^t {}_tp_x \tag{4.19}$$

The illiquid nature of annuities makes investors unable to respond to significant market declines or changes to subjective mortality beliefs. Nevertheless, the retiree has the benefit of full longevity insurance as well as being able to meet bequest motives.

4.2.4 Consumption and Risk Aversion

The CPPI investor has consumption preferences assumed to belong to the HARA type of utility functions. The use of HARA utility enables the setting of a minimum consumption level that the retiree chooses. This is a non-zero consumption floor for the retiree whereby her utility increases only when her consumption rises above this minimum level. This way, I am able to ensure that consumption does not fall below some reasonable subsistence level in retirement. The inclusion of a consumption floor rather than the absolute consumption levels as assumed in CRRA models is motivated by earlier studies such as Litzenberger and Rubinstein (1976). They suggest that utility is better measured relative to a reference level and increases only when consumption increases above this reference level. Habit formation literature supports the notion that the investor's consumption is not determined by her absolute consumption level but rather by the relative position of her consumption to some 'stock of habit'. This stock of habit could be represented by her consumption history or the history of aggregate consumption, (Campbell & Cochrane, 1995). This consumption reference could also be set at a level that the investor deems to be a 'fair' consumption.

The model is based on the selection of a fair consumption reference point. This reference is equal to the annual subsistence estimate for retirees as calculated by the Association of Superannuation Funds of Australia (ASFA) Retirement Standard. ASFA estimates that a retiree will require $41,190 per year whilst a couple will require an estimated $56,236 per year for a comfortable retirement. These figures are effective as at the first quarter of 2013 (ASFA, 2013). A comfortable retirement is defined as one that enables an older, healthy retiree to be involved in a wide range of leisure and recreational activities and to have a good standard of living through the purchase of such things as household goods, private insurance, a decent car and the like. The estimates are based on the assumption that the retiree or retired couple owns their own home. I assume that retirees plan to consume a minimum of the ASFA estimate and derive an increase in utility when annual consumption exceeds the minimum. This makes the HARA utility an appropriate utility function for my analysis.

I represent the annual subsistence estimate for the retiree for every year in retirement by $\tilde{C}$. C_t denotes the actual annual consumption level for the retiree, which is equal to the higher of 5% of her wealth level and $\tilde{C}$. γ represents the retiree's relative risk aversion level or the coefficient of relative risk aversion. Following earlier studies by Milevsky and Young (2007) and Kingston and Thorp (2005), I represent the annual discount rate by r , which I equate to the annual risk-free rate of interest. Instantaneous force of mortality is denoted by λ_t for any given age t. The discounted direct utility at time t which is n years before payouts from the deferred annuity begin is given by:

$$U(C,t;n) = \frac{(C_t - \tilde{C})^{1-\gamma}}{1-\gamma} e^{-(rt + \int_{x+n}^{x+n+t} \lambda_s ds)} \quad (4.20)$$

$$= \frac{(C_t - \tilde{C})^{1-\gamma}}{1-\gamma} e^{-rt} \times e^{-\int_{x+n}^{x+n+t} \lambda_s ds} \quad (4.21)$$

As long as ${}_tp_x$, the conditional survival probability of an individual age x surviving t more years, is constant or decreasing with respect to t, the instantaneous force of mortality is represented in actuarial notation by:

$${}_tp_x = e^{-\int_x^{x+t} \lambda(s)ds} \quad (4.22)$$

Resulting in the utility of consumption being given by:

$$U(C,t) = \frac{(C_t - \tilde{C})^{1-\gamma}}{1-\gamma} e^r t \times {}_tp_x \quad (4.23)$$

4.2.5 Pension Income

For the government sponsored pension income analysis used in this study, I refer to the Australian means-tested Age Pension. Australians who reach the required age are eligible for this age pension. The current required age for a male is 65 and 64.5 for a female, increasing to 67 by July 2023. The single rate age pension is set to 27.7% of Male Total Average Weekly Earnings (MTAWE). An individual's pension income allocation depends on several factors such as the individual being single or with a partner, and owning a house or not. Most importantly, the pension income amount is directly linked to the lower of two formal assessments; an income and asset test. Income levels lower than \$152 per fortnight and assets less than \$196,750 for a homeowner makes an individual eligible for full rate pension. The maximum fortnightly income for a single female homeowner as at March 2013 is \$808.40. This rate decreases with increasing level of assets and income, providing the retiree with a part-rate pension. Beyond an upper threshold, which is \$1,768.80 income per fortnight and \$735,750 in assets, the individual is disqualified from receiving any pension income. The lower threshold for the income test, that entitles an individual to Full Age Pension, is adjusted in line with the Consumer Price Index on 1 July of each year. Additional adjustments to the thresholds in March and September affect individuals on a Part Rate Pension. The individual's income is based on the lower of the two tests and is disqualified if she fails either of the tests. The assets and income tests applied are based on reported figures as at March 2013.[7]

[7] For details on current rates, see http://www.humanservices.gov.au/customer/services/centrelink/age-pension

4.2.6 Mortality Dynamics

The life table used for the analysis is the Australian Life Tables 2010-2012 (2013). This life table provides mortality experiences and captures the mortality improvements observations among Australians for the past 25 and 100 years as well as projected mortality improvements over 25 and 100 years. The table is provided by the Australia Bureau of Statistics and is extracted from the Human Mortality Database. Human mortality has decreased significantly in the last century. Findings by Oeppen and Vaupel (2002) reveal that female life expectancy in the last 160 years has been increasing steadily by almost three months every year. According to the United Nations,[8] the current population ageing is unparalleled in the history of humanity. The population of older people, 60 years and over, at the world level is expected to surpass the population of children, below 15 years, for the first time in the year 2045. In the more developed parts of the world, the population of older people exceeded the population of children in 1998. The United Nations also report that life expectancy at birth increased by 40% over the 20th Century.

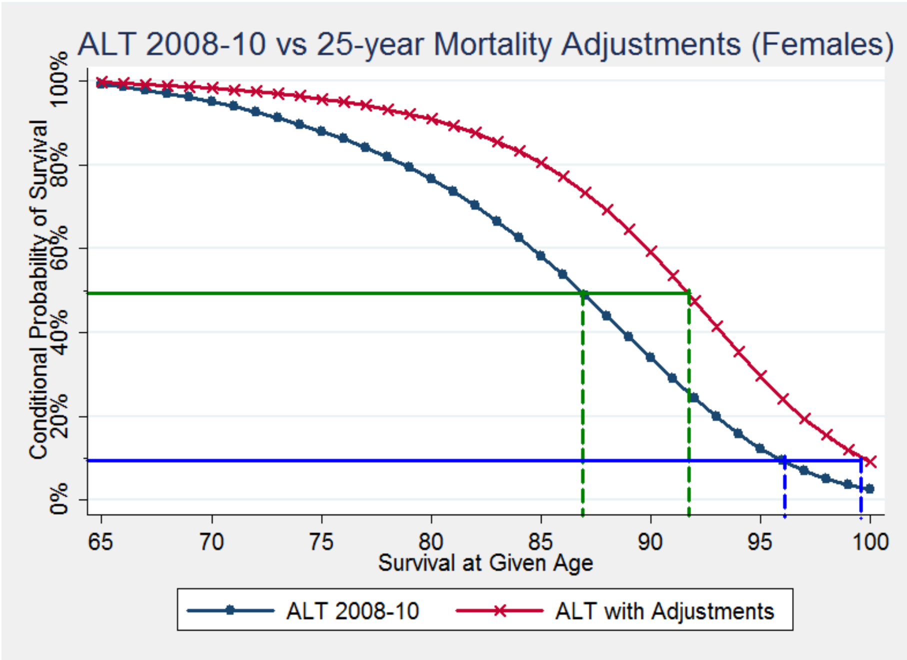
Population ageing is not an Australian issue alone. According to the American Mortality Table, RP–2000 (2000), the probability that a female age 65 will live to age 80 is 70.9%; that of a male is 62.7%. Given a couple who are both 65 years, the probability of at least one spouse attaining 80 years is as high as 89.1%. With these high and increasing life expectancies, it has become expedient that we include some mortality adjustments in retirement planning. Some academics believe that mortality rates over time follow stochastic processes (Cairns, Blake, & Dowd, 2006; Milevsky & David Promislow, 2001). To account for mortality improvements, I use the Australia mortality improvement rates as provided by the government actuary in the Australian Life Tables 2008-2010 (2011). I adjust the survival probability rates for 25 year-improvements with reference to the percentage rates.

An analysis of the Life Table used in this study as shown in Figure 4.1 shows that whilst life expectancy at birth for females is estimated at age 84, we expect one in every two females age 65 will live to age 87, the median life expectation in retirement. With the incorporation of the 25-year mortality improvements, we expect the median life expectation in retirement to increase to age 92. Another significant observation is that with respect to the life tables, one in every ten females in retirement is expected to live until age 96. With the improvements used in this study, there is a 10% chance of a retiree living past age 99. This means we expect one in every ten retirees to live

[8] World Population Ageing: 1950-2050 (2002).

Figure 4.1: Survival Probabilities showing 25-year Improvements

Figure 1 compares the survival probability $({}_tp_{65})$ under the ALT 2008-2010 with the survival probabilities adjusted for 25-year improvements.



to age 99, requiring a retirement planning horizon for up to 34 years. This requires a retiree who wants to be 90% certain that her superannuation savings last as long as she lives to plan her consumption and savings for up to 34 years. All these estimates are higher than the estimates quoted at birth; they are based on the retiree living to retirement age and are helpful for retirement planning. The conditionality of these calculated survival rates makes them an appropriate assessment based on survived ages in retirement. This should serve as a motivation for individuals to plan for longer retirement horizons.

The impact of improving mortality has two significant effects on the purchase of deferred annuities. Firstly, with the expectation of decreasing mortality rates, insurance companies will adjust the prices of annuities accordingly; this means purchasing annuities later will be more expensive because the annuity-loading factor, which caters for commissions, taxes, adverse selection etc. will be higher. This is an incentive for early annuitisation and great saving for investors who purchase deferred annuities. Secondly, decreasing mortality rates means the risk averse investor will derive higher utilities from future consumption. This makes the purchase of deferred annuities with payouts in advanced ages an attractive retirement strategy.

4.2.7 Model Calibration

I begin my model with a set of baseline parameter values as shown in Table 4.1.[9] I use the Australian Life Tables 2010-2012 (2013) as the standard female mortality table. At an interest rate of 2% per annum, the prices of an 10, 15 and 20-year pure deferred whole life annuity paying one dollar per annum at the start of each year from age 75, 80, 85 for an annuity purchased at age 65 are ${}_{10|}\ddot{a}_{65}$=10.80,${}_{15|}\ddot{a}_{65}$=7.112 and ${}_{20|}\ddot{a}_{65}$=4.05 respectively.

TABLE 4.1: Baseline Parameters

This table presents the baseline parameters used in the analysis. The wealth levels, consumption levels, wealth floor and annual pension income are in real dollar terms. The risk free rate, equity premium and volatility are in percentages per annum. Interest and loading factor for annuity calculation is also stated as a percentage per annum and the loading increases with age in 0.05 percentage increment. Starting age and time horizon are stated in years.

Asset Returns	
Real risk-free rate, r	0.02
Equity Premium,μ	0.06
Volatility of annual equity return,σ	0.18
Preference parameter	
Relative risk aversion,γ	2,5,10
CPPI Model	
Starting age,	65
Starting wealth, W_{65}	500,000
Annual Pension Income, P_t	19,000
Annuity Purchase Amount, W_a	50,000
Floor	200,000
Minimum Annual Consumption, $\tilde{C}$	41,160
Time horizon, t	40
Multiplier, m	1.25
Deferred Annuity	
Interest rate, i	0.02
Loading factor, L	0.15

I use the baseline parameters for a retiree who annuitises a tenth of her wealth at retirement. She purchases a 10, 15 or 20-year deferred annuity that begins payouts when she is 75, 80 or 85 years respectively. She receives annual pension income which is indexed for inflation, and this income is used for consumption and the remainder is invested with her retirement portfolio of equities and bonds. She makes monthly withdrawals at the beginning of the month for consumption.

[9] Australia's equity premium for the period 1883 to 2011 has averaged 6% at a 5% significance level (Handley, 2012). Other baseline parameters are in line with recent literature such as Blake et al. (2011) and Gomes and Michaelides (2005). The choice of m is discretionary and the CPPI holds as long as $m > 1$. I make use of monthly rates derived from the annual statistics to equal the frequency of retiree's monthly consumption to avoid bias.

4.2.8 The Omega Ratio

I measure the performance of the alternative investment options available to the retiree by the use of the Omega Ratio. The Omega Ratio developed by Keating and Shadwick (2002) is a preference free performance measurement that ranks and evaluates portfolios. The Omega is especially suited for financial performance where investors are interested in the risk and reward characteristics of various investment options. The entire returns distribution including all higher moments is encoded in the Omega Ratio. For each return level, Omega provides a probability adjusted ratio of gains to losses relative to the referenced return. It operates under the simple rule of investors preferring more to less. Given the choice between two portfolios with the same predicted return, investors should prefer the portfolio with the highest Omega Ratio. This maximises the potential for making the desired level of return, and minimises the probability of extreme losses.
Keating and Shadwick (2002) define the Omega as:

$$\Omega(u) = \frac{\int_u^b [1 - F(x)]dx}{\int_a^u [F(x)]dx} \tag{4.24}$$

Where (a, b) represents the upper and lower bounds of the range of return distribution and u is the threshold return. The numerator is one minus the cumulative distribution function higher than the threshold, u, up to the upper bound of the returns range and the denominator, $F(x)$ is the cumulative distribution function from the lower bound of the returns range to the threshold, u.

A high Omega Ratio means that there is more density return on the right of the threshold level than on the left. I compare the different investment options available to the retiree by choosing a common threshold as reference and construct the Omega functions taking the range of terminal utilities as thresholds. The Omega function is a construction of Omega Ratios taking all points in the terminal utilities range as thresholds.

I define the Omega function in an equivalent form given by:

$$\Omega(u) = \frac{E[max(R - u, 0)]}{E[max(u - R, 0)]} \tag{4.25}$$

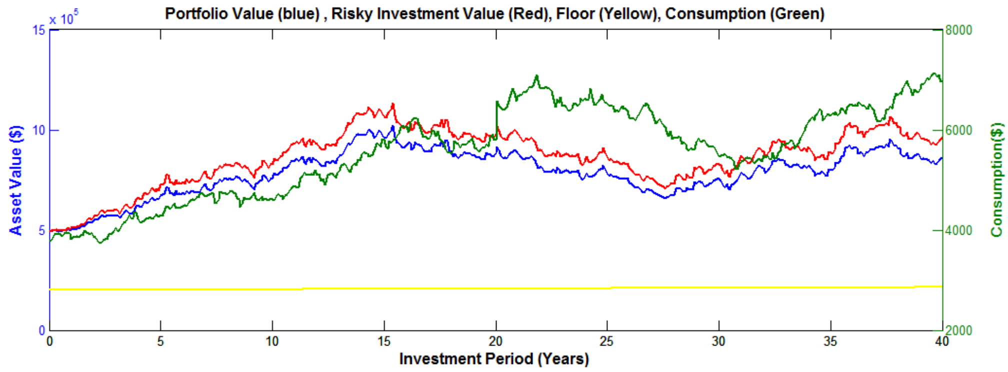
For utility level R and benchmark utility level u. F is the cumulative probability distribution function of the utilities, with $F(x) = P[R \leq x]$.

4.3 Results

Retirees choose between purchasing deferred annuities and self-managing their retirement portfolio via the CPPI strategy. They have the option to purchase deferred annuities for deferred periods of 10, 15 and 20 years. The annuity payout phase should begin before or by age 85. This means the option of the 20-year deferred annuity can only be chosen at age 65, whilst the other annuities can be purchased after some years in retirement. The retirees' choice of deferral length is primarily dependent on their belief in their subjective survival probabilities. To model the survival patterns for the annuitants, I use the 25-year mortality improvements with respect to the Australian Life Tables 2008-2010 (2011). A retiree who believes to possess higher than average survival probabilities will prefer a longer deferred period. Such people believe that based on their health status, age of death of parents, the continuous advancement in medical care, improvement in nutrition and others, they may live to very old ages.

Figure 4.1: Portfolio Dynamics

I show the portfolio dynamics for the retiree who does not invest in annuities. I show retiree's beginning wealth and its changes over time as well as the percentage of wealth invested in equities and its value over time. I also show the wealth Floor as well as consumption values for the retirement plan horizon.



The wealth dynamics shown in Figure 4.1 are a record of one path of the 100,000 simulations used in this study. The blue line represents the value of retiree's portfolio. This is a two-asset portfolio of risky and risk-free assets. The red line represents the path of the risky asset fund; I find that the portfolio value tracks the risky asset fund at all times. Between 20 to 25 years in retirement when the value of the risky investment decreases, the allocation to risky assets is reduced and the portfolio value is sustained by its allocation to the risk-free asset, keeping it above the decreasing risky asset fund value. The yellow line represents the wealth floor, the level of starting wealth that the retiree hopes to preserve at all times irrespective of market outcomes.

The portfolio value never gets below the wealth floor level in this recorded path and all others and the retiree is able to hold the accumulated floor value as a bequest level or preserved capital. The green line represents the retirees' monthly consumption, which remains positive throughout the horizon.

FIGURE 4.2: Terminal Portfolio Values

I show the distribution of the terminal portfolio values (in $) for retirees who purchase 10-year deferred annuities at age 65 and invest the remainder of their assets according to the CPPI strategy. The distribution is based on 100,000 simulations. Nsim represents the number of simulations.

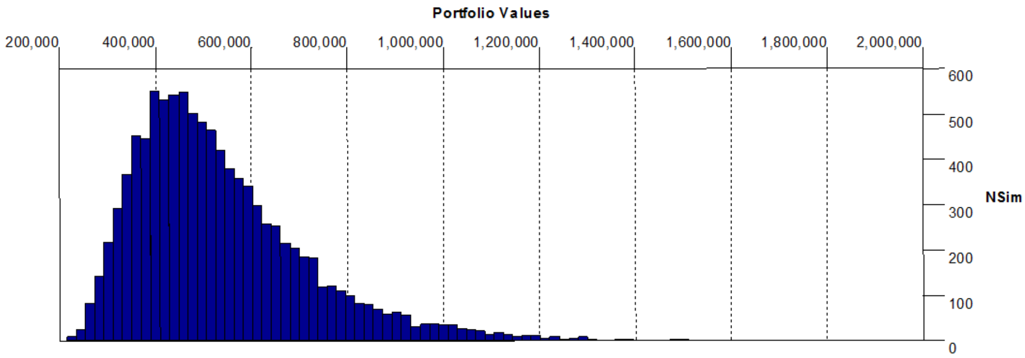


Figure 4.2 shows the ending portfolio value distributions for the 10-year deferred annuity purchased at age 65. Of all the simulations, I find that the lowest portfolio value occurs at $200,000, which is the wealth floor that the retiree sets at the initiation of the CPPI strategy. The portfolio ending values are log-normally distributed with a mean around 80% of the starting wealth level. The ending value increases to as much as $1,500,000 in some cases. The portfolio ending values show a convex profile, which emerges from the dynamic risk management method employed by the CPPI strategy. That is, the investor avoids portfolio risks when risky assets are in decline and increases her share of the risky assets when the trend is positive, thus riding the bull markets and minimising loss in bear markets.

4.3.1 Varying Annuity Deferring Periods at Retirement

I begin my analysis for a retiree who makes her annuitisation decision at age 65. The retiree decides whether to buy a deferred annuity or not to annuitise. In case of the former, the retiree is allowed three choices of deferring periods of 10, 15, and 20 years and makes a one-off decision. If the retiree decides not to invest in annuities but self-annuitises, she may choose to withdraw a fixed percentage of her wealth every month. Again, in this case, I allow two variations based on whether the retiree has a lower floor in her consumption plan or not.

Table 4.1: Annuitising at Age 65: Summary Statistics

I present the summary statistics for the simulated monthly consumption levels. No Constraint represents a strategy of a fixed percentage of wealth withdrawal without regard for a subsistence level, Lower Constraint represents a similar percentage strategy with a lower consumption constraint level; both are self-annuitisation strategies. Annuitants may choose between 10, 15 and 20-year deferred annuities and invest 10% of their starting wealth in annuities.

	No Annuitisation		**Annuitise 10% of wealth at age 65**		
	No Constraint	**Lower Constraint**	**10years**	**15years**	**20 years**
Mean	4584.1	4602.2	4675.5	4826.3	5096.2
Standard Deviation	1070	1032	927.5	1073.7	1387.7
Minimum	2148.6	3430.8	3430.8	3430.8	3430.8
Maximum	20456	24801	19005	22335	21539
5th Percentile	3303.2	3430.8	3578.3	3448.1	3430.8
50th Percentile	4354.3	4352.3	4424	4583.4	5276.4
95th Percentile	6568.4	6548.8	6469.9	6811.1	7554.7

Table 4.1 reports the summary statistics for the distribution of simulated monthly consumption levels. I find that retirees' average consumption level increases by including a lower constraint. The lower constraint refers to the monthly subsistence estimated income required by retirees for a comfortable retirement. I also find that the standard deviation of consumption income decreases whilst the minimum and maximum values for the self-annuitised investor with a lower constraint are higher compared to the simple fixed percentage of wealth withdrawal strategy.

For retirees who choose to invest a part of their starting wealth in deferred annuities, I find that the mean consumption rates increase with longer deferred periods. This is due to the larger payouts for longer deferred annuities in later ages increasing the consumption levels and hence the mean consumption. As expected, the standard deviation also increases with increasing annuity deferral periods. The various percentiles also show increasing consumption rates at all levels with increasing annuity deferral periods. This means in terms of cash values, on average, longer deferral periods should be attractive compared to shorter deferral periods for retirees if they seek higher incomes late in life.

I compare the performance of the alternative investment options available to the retiree by the use of the Omega Ratio. I analyse the options available to the retiree at age 65. These include self-annuitisation with or without a lower constraint as well as the option is to annuitise using deferred annuities for deferred lengths of 10, 15 and 20 years.

FIGURE 4.3: Omega function for Alternative Investment Choices (A)

I show the Omega functions[10] for alternative investment choices based on a retiree option to self annuitise or purchase the deferred annuity with a portion of her starting wealth at retirement age 65 years. The various deferral periods are 10, 15 and 20 years. I measure the performance of the alternative investments relative to different utility thresholds of all values within the utility range based on a relative risk aversion level of 2.

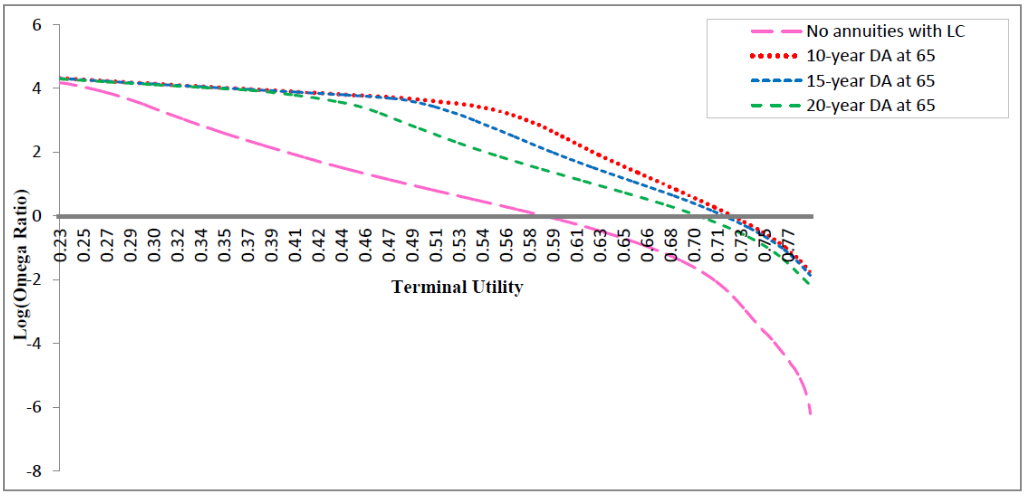


Figure 4.3 compares the performance of the alternative investment choices at retirement age 65 by using the Omega function. The Omega function is calculated from Omega ratios with reference to the expected terminal utilities from the 100,000 simulations for alternative investment choices. I use the full range of the terminal utilities to calculate the Omega Ratios that form the Omega function in Figure 4.3. The expected utilities are calculated under HARA preferences with a relative risk aversion of 2 and a minimum consumption defined by the ASFA retirement estimate.[11] Retirees have the option of choosing 10, 15 or 20-year deferred annuities. They may choose not to annuitise their retirement wealth and hold a lower constraint on their annual consumption withdrawals, withdrawing the maximum of 5% of their remaining wealth or the ASFA estimate, which acts as the lower constraint.

The Omega Ratios at the various thresholds provide a measure of the probability weighted comparison of potential gains to losses relative to the thresholds. I am able to rank the investment options based on the magnitude of their Omegas. For the simple rule of investors preferring more to less, an investment with higher Omega is preferred to a comparable investment with lower Omega. For a loss threshold of 0.7 for comparative analysis, the choice of which is arbitrary and exogenously specified,

[10] I use the natural logarithm of the Omega Ratios due to the large variation in the Omega values over the range of terminal utilities.

[11] I make analysis for other levels of relative risk aversion and calculate the Omega Ratios for the corresponding terminal utilities. I omit those figures from this presentation.

I find Omega ratios of 3.35, 2.27 and 1.03 for the 10, 15 and 20-year deferred annuity investment options respectively. I find an Omega ratio of 0.021 for self-annuitising strategy with lower constraint. I omit the option of self-annuitisation without lower constraint as consumption values lower than the subsistence level results in negative infinity utility which the risk averse individual will be unwilling to accept.

The Omega function analysis for the full range of utility observations show the 10-year deferred annuities strategy is preferred to the other investment strategies. Next in preference, is the 15-year deferred annuity purchased at age 65 and followed by the 20-year deferred annuity strategy. All three annuitisation strategies are preferred to self-annuitisation. Risk averse retirees will prefer to annuitise and derive high utility from doing so. They will however prefer a shorter deferral horizon to a longer one. For the different levels of risk aversion used in this essay, I find that by reducing the risk aversion levels, retirees tend to lean more towards self-annuitisation while increasing risk aversion increases the Omega values for the alternative annuitisation options. A significant observation I make in the evaluation and ranking of the alternate strategies is that retirees do not prefer high consumption with high standard deviation late in retirement. They choose a short deferral period with reasonably high consumption with moderate dispersion about the mean consumption.

4.3.2 Varying Annuitisation Delay Times with Equal Deferral Periods

I continue my analysis for a retiree who makes her annuitisation decision at age 65 or later in retirement at age 70 or 75. The retiree may choose to self-annuitise until later in retirement and decide at which point to buy a deferred annuity. Retirees may purchase 10-year deferred annuities at 65, 70 or 75 with payments beginning at age 75, 80 and 85 respectively. Retirees who choose not to immediately invest in deferred annuities may participate in the financial market with their starting wealth for the first five years in retirement and annuitise at age 70. Retirees may delay annuitising for 10 years after retirement before purchasing annuities at age 75. Retirees may also purchase 15-year deferred annuities at 65 or 70 with payments beginning at age 80 and 85 respectively. Retirees who are confident of their health status may choose longer deferral periods to provide enhanced lifetime income later in retirement.

This delay to annuitising increases the possibility of substantial equity return from market participation, increasing consumption later in retirement as consumption level is a direct proportion of available wealth. On the downside, this exposes their investment to market volatility. The annuity premium paid in all cases is a fraction of retirees' starting wealth rather than current wealth to encourage comparison of annuitisation at retirement age with other times in retirement.

Table 4.2: Annuitising at Different Ages: Summary Statistics (A)

I present the summary statistics for the simulated monthly consumption levels. Annuitants may choose between 10-year deferred annuities at age 65, 70 or 75 or 15-year deferred annuities at age 65 or 70. Annuitants invest 10% of their starting wealth in annuities.

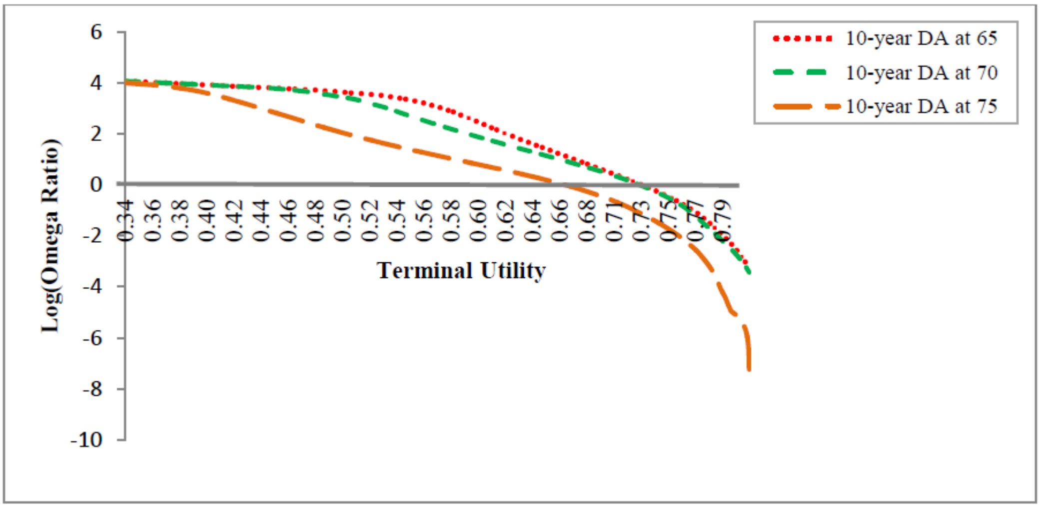
	10-year Deferred Annuities at Different Ages			**15-year Deferred Annuities at Different Ages**	
	Age 65	**Age 70**	**Age 75**	**Age 65**	**Age 70**
	10years	**10years**	**10years**	**15years**	**15years**
Mean	4676	5118	5406	4826	5546
Standard Deviation	928	1250	1434	1074	1532
Minimum	3430	3430	3430	3430	3430
Maximum	19005	22144	21547	22335	21468
5th Percentile	3578	3662	3659	3448	3662
50th Pecentile	4424	4790	5101	4583	5341
95th Percentile	6470	7499	8074	6811	8333

From Table 4.2, I find that annuitising at age 75 provides significantly higher mean consumption values than annuitising at age 70 or 65. As expected, dispersion about the mean payout increases with increasing delay period. A five-year delay to annuitising increases the standard deviation of monthly consumption relative to immediate annuitising at age 65, whilst a 10-year delay gives an even higher standard deviation. Comparing percentiles, the 10-year deferred annuity at age 75 provides higher consumption levels at the 50th and 95th percentiles. This is closely matched by the 10-year deferred annuities at age 70 strategy, which has a higher value at the 5th percentile. Percentile values for the annuity purchase strategies at age 70 are also considerably higher than that of a similar strategy at age 65.

For the 15-year annuity deferral strategies, I find annuitising at age 70 to provide a higher mean consumption than annuitising at age 65. The five-year delay before annuitising enhances the portfolio value when market returns are high and increases retirees consumption payouts in latter years. Standard deviation for consumption is higher for the strategy at age 70 relative to similar strategy at 65. Also, the 15-year deferred annuity at age 70 provides higher consumption levels at all percentiles.

Figure 4.4: Omega function for Alternative Investment Choices (B)

I show the Omega function for alternative investment choices for retirees who purchase a 10-year deferred annuity at age 65, 70 or 75. I measure the performance of the alternative investments relative to different threshold of all values within the utility range based on a relative risk aversion level of 2.

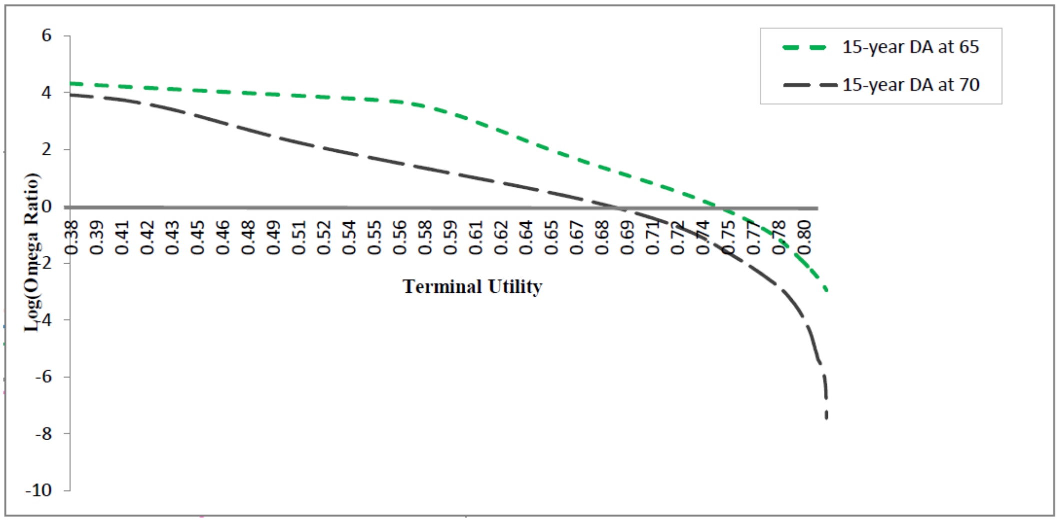


I compare the terminal utilities derived from purchasing 10-year deferred annuities at ages 65, 70 or age 75 in Figure 4.4. The retiree in this case has the option to choose the length of delay to annuitisation in order to participate longer in the money markets and maximise her utility. She invests the same proportion of starting wealth in annuities when it is purchased at a later age.

I find that at the risk aversion level of 2, retirees prefer annuitising at age 65 or age 70 to annuitising at age 75. The retiree is indifferent between purchasing the 10-year deferred annuity at age 65 and 70 for low thresholds of the terminal utility. Annuitising at age 65 is preferred to annuitising at age 70 as the terminal threshold increases with higher Omega values for the former. For different risk aversion levels, I find annuitising at age 70 becomes less attractive as risk aversion increases. It is however preferred to annuitising at age 75 at all risk aversion levels. The difference in Omega Ratios is significant for terminal utilities at all levels of the distribution. For the loss threshold of 0.7, I find an Omega Ratio of 3.33, 2.80 and 0.16 for alternate investments with 10-year deferred annuities purchased at age 65, 70 and 75 respectively. For the same deferral periods on annuities, risk averse retirees will prefer early annuitisation to late annuitisation.

Figure 4.5: Omega function for Alternative Investment Choices (C)

I show Omega function for alternative investment choices when retirees purchase a 15-year deferred annuity at age 65 or 70. I measure the performance of the alternative investments relative to different utility threshold of all values within the utility range based on a relative risk aversion level of 2.



I compare the two strategies available to the retiree which include a 15-year deferred annuity in Figure 4.5. This option is available at age 65 and age 70, the latter after a five year delay. The retiree may participate longer in the money markets and invest the same proportion of starting wealth in annuities at age 70. I find that at a risk aversion level of 2, retirees prefer annuitising at age 65 to annuitising at age 70. For the loss threshold of 0.7, I find an Omega Ratio of 2.08 for the 15-year deferred annuity at age 65 and a low Omega ratio 0.50 for annuitising at age 70 for the same deferral length.

4.3.3 Varying Deferring Periods and Timing of Annuitisation

I make further analysis for a retiree who makes her annuitisation decision regarding both the deferral period for annuitisation and the timing of the annuity purchase. The retiree may choose to self-annuitise until later in retirement and decide to annuitise for the available deferral periods. Retirees may purchase 10, 15 and 20-year deferred annuities at 65 or 10 and 15-year deferred annuities at age 70. Alternatively, retirees may choose to continue self-annuitising or purchase 10-year deferred annuities at age 75. Retirees who choose not to immediately invest in deferred annuities may participate in the money market with their starting wealth for five or ten years in retirement before annuitising. The annuity premium paid in all cases is a tenth of retirees' starting wealth.

TABLE 4.3: Annuitising at Different Ages: Summary Statistics (B)

I present the summary statistics for the simulated monthly consumption levels for annuitants who choose between 10-year deferred annuities at age 65, 70 or 75 or 15-year deferred annuities at age 65 or 70. Alternatively, they may choose to purchase 20-year deferred annuities at age 65 or self-annuitise and hold a lower constraint. Annuitants invest 10% of their starting wealth in annuities.

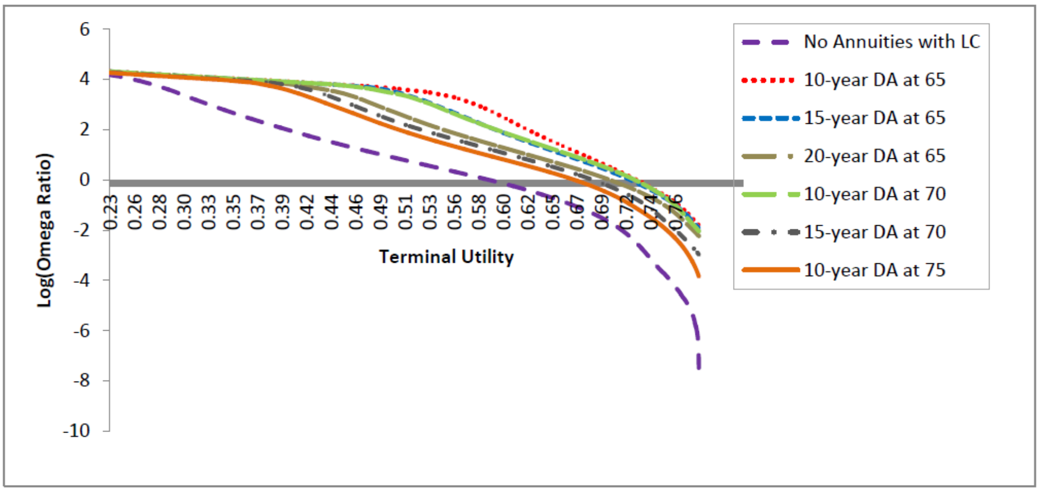
	No Annuities	Annuitise at age 65			Annuitise at Age 70		Age 75
	L.C.	**10years**	**15years**	**20 years**	**10years**	**15years**	**10years**
Mean	4602	4676	4826	5096	5118	5546	5406
Std Dev	1032	928	1074	1388	1250	1532	1434
Minimum	3430	3430	3430	3430	3430	3430	3430
Maximum	24801	19005	22335	21539	22144	21486	21547
5th Percentile	3430	3578	3448	3430	3662	3662	3659
50th Pecentile	4352	4424	4583	5276	4790	5341	5101
95th Percentile	6549	6470	6811	7555	7499	8333	8074

I compare my results from Sections 4.3.1 and 4.3.2 in Table 4.3 and observe some trends in the consumption values for the different strategies. Firstly, self-annuitisation provides the lowest mean consumption among the various investment alternatives. It has the highest maximum consumption, which shows the high amount of dispersion about the mean. The standard deviation of the self-annuitised strategy only exceeds that of the 10-year deferred annuity at age 65. It is the lowest at the 5th and 50th percentiles among the alternate strategies. Secondly, there is a significant increase in mean consumption with increasing annuity deferral periods across all ages. A 20-year deferred annuity at age 65 provides a higher mean consumption than a 15-year deferred annuity at the same age. The 15-year deferred annuity strategy provides higher mean consumption compared to the 10-year deferred annuity at the same age. A similar relationship is observed between the investment options at age 70. Also, the 15-year deferred annuity purchased at age 65 beginning payments at age 80 provides consumption levels which are lower than that provided by a 10-year deferred annuity at age 70 also beginning at age 80. The 20-year deferred annuity at age 65 is dominated in terms of consumption value by the 15-year deferred annuity purchased at age 70. They both begin annuity payments at age 85. This means retirees who are interested in high levels of consumptions later in retirement may choose to delay annuitisation to later dates and purchase deferred annuities with short deferral periods than purchase deferred annuities with longer deferral periods early in retirement.

Figure 4.6 shows Omega functions derived from the monthly consumption of retirees who choose to annuitise a part of their starting wealth at age 65, 70 or 75. I also show the Omega function for the self-annuitisation strategy. At age 65, the retiree has the options of 10, 15 and 20-year deferred annuities whilst the retiree at age 70 only has the option of 10 and 15-year deferred annuities. The retiree at age 75 makes a choice between purchasing a 10-year deferred annuity or self-annuitisation. I find that for a risk aversion level of 2, self-annuitisation is clearly dominated by deferred

FIGURE 4.6: Omega function for Alternative Investment Choices (D)

I show the Omega function for alternative investment choices in retirement for annuitants who choose between 10-year deferred annuities at age 65, 70 or 75 or 15-year deferred annuities at age 65 or 70. Alternatively, they may choose to purchase 20-year deferred annuities at age 65 or self-annuitise and hold a lower constraint. Annuitants invest 10% of their starting wealth in annuities. I measure the performance of the alternative investments relative to different thresholds of all values within the utility range based on a relative risk aversion level of 2.



annuitisation of different lengths and purchased at different times. The retiree will choose a 10-year deferred annuity purchase at age 65 over 15 and 20-year deferred annuity at same age. Comparing this to the retirees' choice at retirement age 70, I find the 10-year deferred annuity at 65 to dominate annuitising at age 70. The retiree is mostly indifferent in the choice between a 10-year deferred annuity at age 70 and a 15-year deferred annuity at age 65 with both beginning annuity payments at age 80 for a wide range of terminal utilities. At the higher end of the terminal value distribution, I find the annuitisation strategy at age 70 to dominate that at age 65. The 20-year deferred annuity at age 65 is preferred to the 15-year deferred annuity at age 70 whilst annuitising at age 75 is the least in preference. The difference in Omega Ratios is more significant for terminal utilities in the centre of the distribution and less significant at the lower and upper tails.

For the loss threshold of 0.7, I find an Omega Ratio of 4.35 for a 10-year deferred annuity strategy at age 75. The Omega Ratios for investment alternatives at age 70 yield 4.72 and 4.46 for the 10 and 15-year deferred annuities respectively. Both strategies are dominated by the 10-year deferred annuity purchased at 65, with superior Omega Ratios of 4.76 and 4.71 respectively for the 10 and 15-year deferred annuities. The 20-year deferred annuity strategy has an Omega ratio of 4.57. The self-annuitisation strategy is dominated by all annuitisation strategies, giving an Omega Ratio of 3.66 for the same threshold level.

By varying the risk aversion levels, I find the difference in Omega Ratios to decrease with increasing levels of risk aversion and vice versa. This is expected as retirees with low levels of risk aversion will want to participate longer in the money market with the hope of increasing equity premium and increasing the value of their fund than lose some liquidity through annuity purchase early in retirement. At risk aversion levels lower than 2, the option of 10-year deferred annuity at age 70 becomes more attractive than annuitisation at age 65. This finding could be explained by Milevsky and Young (2007) as Real Option to Delay Annuitisation (RODA). They assert that the uncertainty in future interest rates, mortality rates, insurance loading and annuity design increase the value of delaying annuitisation. With an expectation of increased wealth, individuals will accept the increase in wealth as a substitute to annuitising at retirement age 65. At risk aversion levels of 2 and higher, the utility derived from waiting to annuitise at age 70 decreases, and the high risk averse retiree is happy to purchase annuities at retirement age 65 rather than take a bet on receiving equity premium. The retiree does not make significant gains in annuitising at a later age and will choose a shorter waiting period over a long one. The ranking of the annuitisation options based on their deferral lengths at specific ages however do not change for different levels of risk aversion.

4.3.4 Deferred Annuitisation in Absence of Age Pension

Finally, Horneff et al. (2010) incorporate state-organised social security and occupational pension plans which run as a form of defined benefit scheme. The pension income of that directly links to the amount of contribution an individual makes. They begin with a baseline replacement ratio of 68% of final salary and make further analysis with medium and low replacement rates of 50% and 30% respectively. They find that for low (high) replacement rates for the pension income, annuity purchases commence earlier (later) and the fraction of total financial wealth invested in annuities increases (decreases). All the previous results are obtained on the premise of the availability of pension income. The Australia pension system is fully government sponsored and not dependent on individual contributions. The level of pension received, full or part rate is not fixed but changes in response to retiree's asset and income levels from semiannual reassessments.

To see the impact on a retiree's choice of investment strategy when they have no pensions, I perform the analysis in the absence of means-tested pension income. This new result should be representative of other countries without welfare schemes such as Australia's means-tested pension.

Table 4.4: Annuitising at Different Ages with No Pension: Summary Statistics

I present the summary statistics for the simulated monthly consumption levels. Annuitants may choose between 10-year deferred annuities at age 65, 70 or 75 or 15-year deferred annuities at age 65 or 70. Alternatively, they may choose to purchase 20-year deferred annuities at age 65 or self-annuitise and hold a lower constraint. Annuitants invest 10% of their starting wealth in annuities.

	No Annuities	**Annuitise at age 65**			**Annuitise at Age 70**		**Age 75**
	L.C.	**10years**	**15years**	**20 years**	**10years**	**15years**	**10years**
Mean	3535	3763	3869	4033	4164	3971	3903
Std Dev	359	353	449	659	637	604	544
Minimum	3430	3430	3430	3430	3430	3430	3430
Maximum	10321	12306	13167	15370	12623	13336	12775
5th Percentile	3430	3430	3430	3430	3430	3430	3430
50th Percentile	3430	3777	4012	4446	4482	4363	4226
95th Percentile	4176	4261	4476	4894	4924	4774	4641

Table 4.4 shows the summary statistics derived from simulating the various retirement decumulation strategies in the absence of pension income. Again, I find deferred annuitisation to provide higher consumption levels relative to self-annuitisation in retirement. The self-annuitisation strategy has lower mean consumption as well as lower consumption levels across the various percentiles. The 50th percentile of consumption for the self-annuitising strategy is at the subsistence level, which is also the level of lower constraint. Similar to the observations in the presence of pension income, I find the mean consumption levels to increase with increasing annuitisation deferral periods at age 65. The various percentiles also show increasing consumption levels as I increase the deferral periods for annuities. Annuitising at age 70 shows a decreased mean consumption for increasing deferred periods.

For annuitisation strategies beginning payments at age 80, that is, the 15-year deferred annuity at age 65 and 10-year deferred annuity at age 70, I find that annuitising later provides higher consumption levels than early annuitisation. Delaying annuitisation for five years increases the mean consumption levels significantly. The 20-year deferred annuity at age 65, 15-year deferred annuity at age 70 and 10-year deferred annuity begin payments at age 85. Comparing these annuitisation strategies, I find annuitising at age 65 to provide higher consumption income relative to annuitising later at ages 70 and 75. Unlike previously seen in the presence of pension income, early annuitisation with longer deferral periods clearly provides higher consumption than delayed annuitisation. Without the benefit of pension income, retirees seeking a higher level of consumption in retirement may choose early annuitisation with longer deferred periods to late annuitisation or late annuitisation. While annuitising with 10-year deferred annuities at age 70 yields high average consumption, retirees are better with longer deferred period annuities at age 65 than holding annuities with deferred periods exceeding 10 years at age 70. Annuitising at age 70 however yields higher consumption levels than annuitising at age 75.

FIGURE 4.7: Omega function for Alternative Investment Choices in the Absence of Pension Income

I show the Omega function for alternative investment choices in retirement in the absence of pension income. Annuitants may choose between 10-year deferred annuities at age 65, 70 or 75 or 15-year deferred annuities at age 65 or 70. Alternatively, they may choose to purchase 20-year deferred annuities at age 65 or self-annuitise and hold a lower constraint. Annuitants invest 10% of their starting wealth in annuities. I measure the performance of the alternative investments relative to different thresholds of all values within the utility range based on a relative risk aversion level of 2.

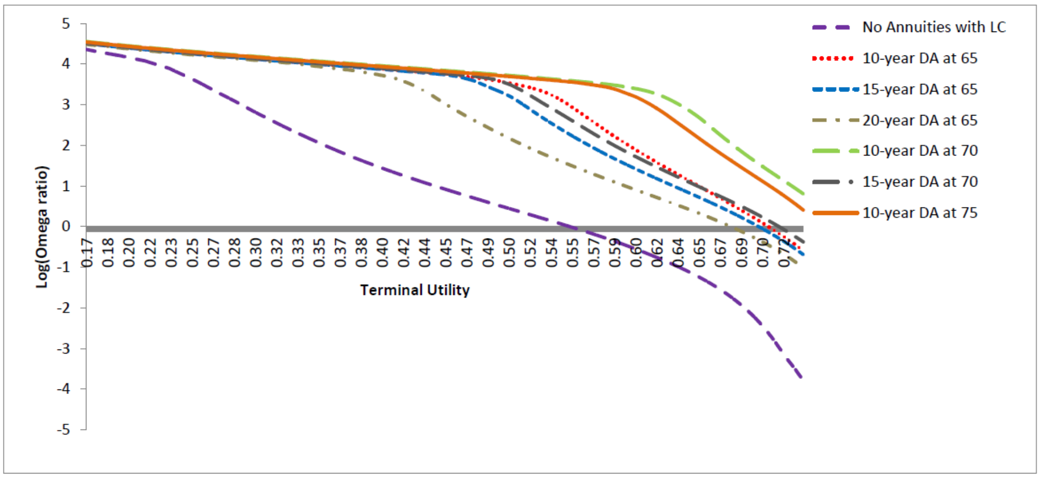


Figure 4.7 shows the Omega function for alternative investment strategies available to the retiree when she has no pension income. Retirees prefer to self-annuitise longer and prefer purchasing deferred annuities at age 70 and 75 over annuitising at 65. Whilst deferred annuitisation continues to dominate self-annuitisation, I find retirees prefer to delay annuitisation when they have no pension income. Annuitisation with 10-year deferred annuities at age 70 is the preferred strategy among the alternatives; annuitising at age 75 is preferred to annuitising at age 65. The 15-year deferred annuity purchased at age 70 in this case is preferred to an equivalent length annuity purchased at age 65 and the 20-year deferred annuity purchased at age 65 is the least preferred strategy. For different levels of risk aversion, I repeat the simulations and find the value of delaying annuitisation to be higher for low to moderate risk averse retirees. The difference becomes less significant for high risk averse retirees as they tend to prefer early annuitisation to later. The absence of pensions does not

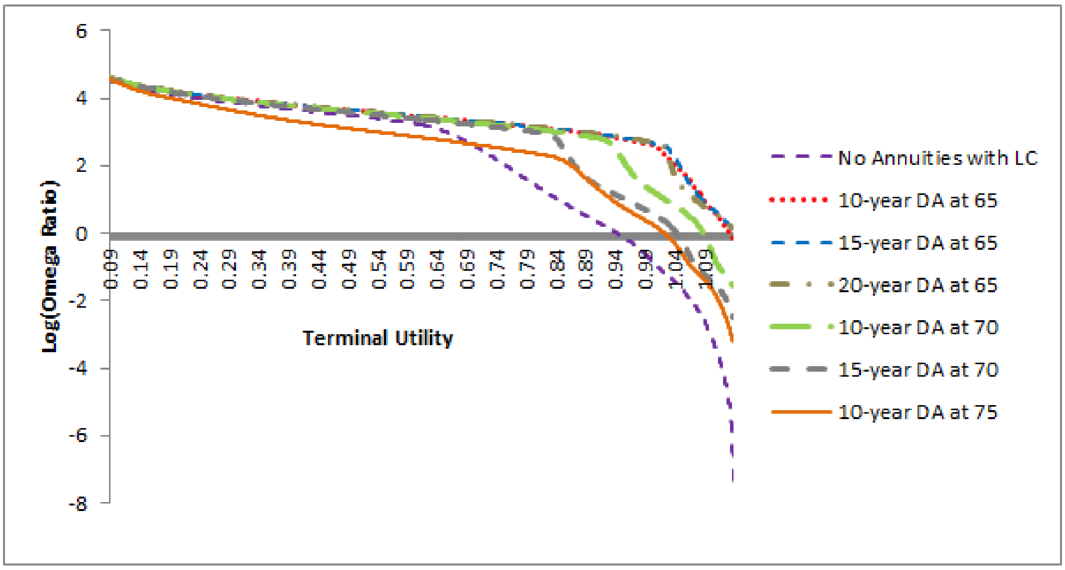
alter the annuitisation preferences of the high risk averse retirees as they prefer early annuitising at retirement age 65 to annuitising later in retirement.

4.3.5 Deferred Annuities in the Absence of Bequest Motives

I have previously assumed retirees have bequest or capital preservation needs and invest via the CPPI strategy. I analyse retiree investment choice when they have no bequest motives. I set the asset allocation to a balanced strategy with 50% investment in the risky asset and the remainder in the risk-free asset. I allow the risk-free asset to evolve deterministically with exponential growth and the risky asset to follow a geometric Brownian motion as discussed in Section 4.2.2. Assuming retirees have no capital preservation or bequest motives, they derive utility only from the consumption they make from their retirement portfolios, preferring more consumption to less. Government sponsored means-tested age pension is available to all retirees and the level of pension income received depends on the size of a retiree's portfolio balance.

FIGURE 4.8: Omega function for Alternative Investment Choices in the Absence of Bequest Motives

I show the Omega function for alternative investment choices in retirement when retirees have no bequest motives. Annuitants may choose between 10-year deferred annuities at age 65, 70 or 75 or 15-year deferred annuities at age 65 or 70. Alternatively, they may choose to purchase 20-year deferred annuities at age 65 or self-annuitise and hold a lower constraint. Annuitants invest 10 percent of their starting wealth in annuities. I measure the performance of the alternative investments relative to different thresholds of all values within the utility range based on a relative risk aversion level of 2.



At retirement age 65, I find retirees to be indifferent between the choice of purchasing deferred annuities at ages 65 or 70 and self-annuitisation over low thresholds of the terminal utility. Purchasing annuities at age 75 is the least preferred strategy given low threshold levels, but performs better than self-annuitisation at higher thresholds. The safety of the government means-tested aged pension is an incentive for retirees to self-annuitise in retirement when they have no bequest motives. Annuitisation at age 65 is preferred to later annuitisation at age 70 and 75. Retirees choose to purchase deferred annuities of different deferral lengths at age 65 to annuitising later. The 10-year deferred annuity at age 70 is preferred to 15-year deferred annuity at the same age. Both strategies are preferred to the 10-year deferred annuity at age 75 for higher thresholds of terminal utilities. The differences between the alternative strategies are significant for higher terminal utility thresholds but less significant otherwise.

4.4 Conclusion and Discussion

The analysis of deferred annuities in retirement planning presents a new explanation for the well-documented aversion to life annuities by retirees. With the irreversibility of annuity purchase and the liquidity lost by retirees who annuitise, there continues to be an incentive to delay annuitisation. Milevsky and Young (2007) quantify this option for retirees with Constant Relative Risk Aversion (CRRA) preferences, referring to this as the Real Option to Delay Annuitisation (RODA). The inclusion of deferred annuities rather than immediate annuities provides some benefit to retirees in terms of lower purchasing premiums in addition to the absolute cover when retirees survive the deferred period. This study presents a habit persistence model in terms of investor preferences, and assumes investors have Hyperbolic Absolute Risk Aversion (HARA) preferences. I also model a decumulation strategy using the Constant Proportion Portfolio Insurance (CPPI), which ensures that retirees are able to meet their bequest or capital preservation needs. I incorporate mortality improvements allowing for the constantly improving mortality experiences. Finally, I evaluate and rank the performance of the different investment options by the use of the Omega function with reference to the expected terminal utilities for the various strategies.

Low risk averse retirees will choose self-annuitisation strategy with downside constraints over a simple percentage of wealth withdrawal; allowing consumption to vary over a broad range of values yet ensuring that consumption remains above a sustenance level. Compared to the benefits of annuitising, I find retirees to prefer purchasing deferred annuities at retirement age to provide higher and more utility-enhancing consumption to any of the two self-annuitising strategies discussed in this essay. I also find that even at high levels of risk aversion, the retiree is better off purchasing deferred annuities than self-annuitisation if she is interested in the possibility of obtaining superior consumption levels leading to superior terminal utilities.

For the length of annuity deferral periods, I find that risk averse retirees prefer a shorter timeframe to a longer one. At age 65, a retiree prefers a 10-year deferred annuity to a 15-year deferred annuity, and a 15-year deferred annuity to a 20-year deferred. At age 70, the 10-year deferred annuity is preferred to the 15-year deferred annuity purchased at the same age. Retirees prefer to purchase deferred annuities with longer deferral periods only at retirement age 65 and derive less utility when they purchase annuities with deferral periods exceeding 10 years later in retirement. For two strategies that both begin payments at age 85, a 20-year deferred annuity purchased at age 65 is preferred to a 15-year deferred annuity purchased at age 70. Since there is no benefit should the retiree die during the deferral period, retirees are reluctant to purchase annuities with lengthy deferred periods late in retirement. Instead, retirees choose a short deferral period where they believe their conditional survival probabilities are still high enough to survive and receive the annuity payouts.

On the timing of annuity purchase, I find that the option to delay annuitisation exists for retirees of all relative risk aversion levels within the range used in this essay. My analysis shows that while risk averse retirees choose to annuitise in retirement, the timing of annuitisation depends on their level of risk aversion. The low risk averse retirees choose to annuitise later than the retirement age of 65. They gain utility in the expectation of stochastic improvements in their wealth levels through investing assets in the financial market for the first few years in retirement. Since their consumption in the years before the annuity payments begin is a direct function of wealth level, the low risk averse retiree will bet on increasing consumption through the financial market in the early years of retirement while they still have high survival probabilities. There is less option to delay annuitisation, however, for more risk averse retirees. They prefer early annuitisation to guarantee their income instead of betting on the financial market in anticipation of equity premium.

The option to delay annuitisation is more valuable in the case of no exogenous pension income than when government pensions are available. Low to medium risk averse retirees will prefer to delay annuitising until age 70 or age 75 to bet on the financial markets with the possibility of increasing their wealth level before eventual annuitisation later in retirement when there is no pension income. I find the presence of pension income to be an incentive for retirees at all risk levels to transfer risk to life companies and benefit from the longevity protection provided by the pension income and the increased consumption from annuitisation. Deferred annuitisation is more attractive to retirees who have access to government-sponsored pension than retirees who do not.

Finally, when retirees have no bequest motives, they prefer annuitising earlier in retirement to later. Annuities of varying deferral lengths at retirement age 65 are preferred to annuitising at age 70, while the latter is preferred to annuitising at age 75. At low thresholds of terminal utility, the self-annuitisation strategy performs better than annuitisation at age 75 although this is reversed for high terminal wealth thresholds. The certainty of the government pensions makes self-annuitisation attractive as retirees are able to increase their consumption without losing utility from lowering bequest levels.

By purchasing a deferred annuity with a fixed proportion of initial wealth, I am able to make comparisons among annuity purchases at different ages in retirement. This is however, a limitation on the amount of premium invested in annuities as retirees may choose to annuitise a proportion of their current wealth rather than a proportion of starting wealth at different ages. Basing their annuity purchase on the wealth at hand rather than a fixed value determined at retirement age could lead to different annuity premiums and hence different payout values. Secondly, the estimates in modelling mortality improvements may serve to limit other possible future mortality scenarios. Although this serves as an adequate model for future mortality, mortality is not predictable and the model only tells part of the story. Different mortality experiences may prompt improved annuity designs, necessitate increasing or decreasing loading factors, making annuities less or more expensive respectively hence giving incentive to early or late annuity purchases. Lastly, this study allows annuity payouts to begin at age 85 at the latest. Although there is no theory for this choice, I believe it serves as a reasonable age considering that one in two Australian retirees is expected to live past 85. Future studies may allow annuity payouts to begin at later ages based on the extent of improvement in mortality rates.

Appendix

4.A Expected Mortality Improvement Rates

TABLE 4.A.1: Mortality Improvements

Future Percentage Mortality Improvement Factors Females. Life Tables 2008-2010 provided by the Australian Bureau of Statistics (November 2011).

Age	25 Year	Age	25 Year
65	-2.5195	89	-1.148
66	-2.5118	90	-1.0346
67	-2.5014	91	-0.9324
68	-2.4881	92	-0.845
69	-2.4718	93	-0.7725
70	-2.4524	94	-0.7159
71	-2.4297	95	-0.6747
72	-2.4035	96	-0.6335
73	-2.3736	97	-0.5923
74	-2.3398	98	-0.5511
75	-2.3018	99	-0.5099
76	-2.2593	100	-0.4687
77	-2.212	101	-0.4275
78	-2.1808	102	-0.3863
79	-2.132	103	-0.3451
80	-2.0719	104	-0.3039
81	-1.9981	105	-0.2627
82	-1.915	106	-0.2215
83	-1.824	107	-0.1803
84	-1.7247	108	-0.1391
85	-1.6177	109	-0.0979
86	-1.5057	110	-0.0567
87	-1.3887	111	0
88	-1.2687		

The mortality improvement calculations are expressed mathematically as:

$$q_t(t) = q_x \times [1 + \frac{I_x}{100}]^{(t-2009)} \tag{4.26}$$

Where $q_t(t)$ is the mortality rate at age in year t

q_t is the mortality rate reported for age x in the 2008-2010 Life Table

I_x is the rate of improvement at age x shown in Table 4.A.1

4.B Life Annuities - A Review

4.B.1 The Whole Life Annuity-Due

The Whole Life Annuity-due pays a benefit of a unit \$1 at the beginning of each period (year) that the annuitant (x) survives.

The Present Random Variable is denoted as:

$$Y = \ddot{a}_{\overline{T+1}|} \tag{4.27}$$

Where T represents T_x, the curtate future lifetime of x

The Actuarial Present Value (APV) of the Whole Life Annuity-due is denoted as:

$$\ddot{a}_x = E[Y] = E[\ddot{a}_{\overline{t+1}|}] = \sum_{t=0}^{\infty} \ddot{a}_{\overline{t+1}|} Pr[T = t] \tag{4.28}$$

$$= \sum_{t=0}^{\infty} \ddot{a}_{\overline{t+1}|} \cdot {}_{t|}q_x = \sum_{t=0}^{\infty} \ddot{a}_{\overline{t+1}|} \cdot {}_t p_x q_{x+t} \tag{4.29}$$

$$= \sum_{t=0}^{\infty} v^t {}_t p_x \tag{4.30}$$

4.B.2 Temporary Life Annuity-Due

The Temporary Life Annuity-due or the '$N - Term'$ Annuity pays a benefit of a unit \$1 at the beginning of each year as long as the annuitant (x) survives, for up to a total of n years, or n payments.

This annuity has a Present Value random variable expressed as:

$$Y = \begin{cases} \ddot{a}_{\overline{T+1}|}, & \text{if } T < n, \\ \ddot{a}_{\overline{n}|} & \text{if } T \geq n \end{cases} = \ddot{a}_{\overline{min(T+1,n)}|} \tag{4.31}$$

The Actuarial Present Value (APV) of the Temporary Annuity is denoted as:

$$\ddot{a}_{x:\overline{n}|} = E[Y] = \sum_{t=0}^{n-1} \ddot{a}_{\overline{t+1}|}\, {}_tp_x q_{x+t} + \ddot{a}_{\overline{n}|}\, {}_np_x \tag{4.32}$$

$$= \sum_{t=0}^{n-1} v^t\, {}_tp_x \tag{4.33}$$

4.B.3 The Deferred Whole Life Annuity-Due

The Deferred Whole Life Annuity due pays a benefit of a unit \$1 at the beginning of each period (year) while the annuitant (x) survives from $x+n$ onward.

The Present Value random variable can be expressed in a number of ways:

$$Y = \begin{cases} 0 & \text{if } 0 \leq T < n, \\ {}_{n|}\ddot{a}_{\overline{T+1-n}|} = v^n \ddot{a}_{\overline{T+1-n}|} = \ddot{a}_{\overline{T+1}|} - \ddot{a}_{\overline{n}|} & \text{if } T \geq n \end{cases} \tag{4.34}$$

The Actuarial Present Value (APV) of the '$N - Year'$ Deferred Whole Life Annuity is denoted as:

$${}_{n|}\ddot{a}_x = E[Y] = \sum_{t=n}^{\infty} v^t\, {}_tp_x \tag{4.35}$$

$$= {}_nE_x \ddot{a}_{x+n} = \ddot{a}_x - \ddot{a}_{x:\overline{n}|} \tag{4.36}$$

4.B.4 The Recursive Relationship between Annuities

$$\ddot{a}_x = 1 + vp_x \ddot{a}_{x+1} = 1 + {}_1E_x \ddot{a}_{x+1} \tag{4.37}$$

$$= 1 + vp_x + v^2\, {}_2p_x \ddot{a}_{x+2} = 1 + {}_1E_x + {}_2E_x \ddot{a}_{x+2} \tag{4.38}$$

The $E's$ are multiplicative, therefore generalising the recursions to:

$$\ddot{a}_x = \sum_{t=0}^{\infty} {}_tE_x = \sum_{t=0}^{n-1} {}_tE_x = \sum_{t=n}^{\infty} {}_tE_x \tag{4.39}$$

apply change of variable $t^* = t - n$

$$= \ddot{a}_{x:\overline{n}|} + \sum_{t^*=0}^{\infty} {}_nE_x \cdot {}_{t^*}E_{x+n} = \ddot{a}_{x:\overline{n}|} + {}_nE_x \sum_{t^*=0}^{\infty} {}_{t^*}E_{x+n} \tag{4.40}$$

$$= \ddot{a}_{x:\overline{n}|} + {}_nE_x \ddot{a}_{x+n} = \ddot{a}_{x:\overline{n}|} + {}_{n|}\ddot{a}_x \tag{4.41}$$

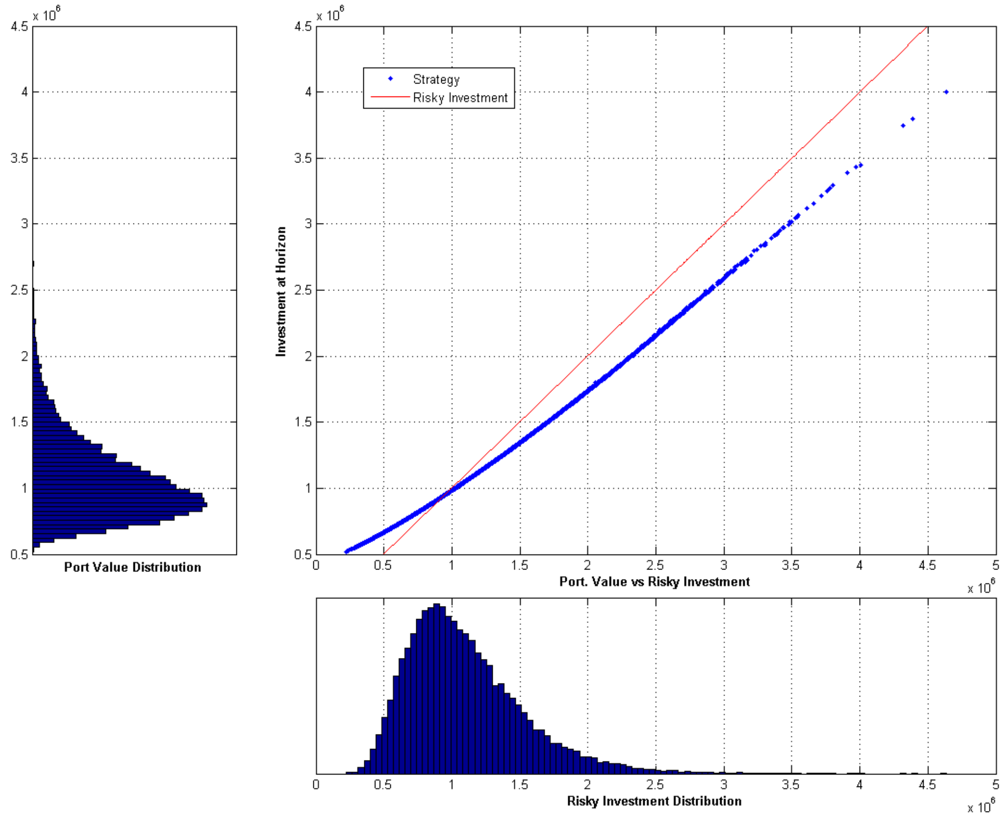
The Whole Life Annuity is the sum of an '$N-Year'$ Term Annuity and an '$N-Year'$ Deferred Life Annuity.

4.C The CPPI Strategy

The CPPI exhibits a convex strategy profile payoff. A convex strategy protects the investor when the risky asset underperforms, although they fail to capture all the upside when the risky asset rallies. Protecting the downside makes convex strategies suitable to provide portfolio insurance. I show the evolution of the risky asset and the total portfolio value for the 10-year deferred annuity purchased at age 65 strategy. The convexity of the CPPI strategy is discussed in greater depth in Meucci (2010).

FIGURE 4.C.1: Portfolio Dynamics

I show the Portfolio Value and Risky Investment distributions, and a scatter-plot of the final payoff of the CPPI strategy over the payoff of the Risky Investment, with the strategy profile showing a convex profile.



Chapter 5

Lifecycling Strategies in the Decumulation Phase

5.1 Introduction

There is the need for appropriate investment measures to be taken whilst individuals receive income and have adequate human capital to build a good portfolio to create wealth in the accumulation years. Similarly, it is important in retirement that retirees invest optimally to ensure that they do not outlive their available wealth. Whereas there is extensive academic research to building a well-diversified and sustainable investment portfolio during the working life of individuals, the same cannot be said for dissaving wealth after the individual's retirement. Increasing life expectancy is a strong incentive to save and manage adequately one's retirement wealth. The importance of wealth dissaving strategies after retirement has become more pronounced with the growth in numbers of the Defined Contribution (DC) plan over the Defined Benefit (DB) plan.

To sustain retirees' income in retirement, retirees may purchase life annuities, which provide periodic income for life. Alternatively, retirees could invest their retirement wealth in a range of stocks (risky assets) and bonds and cash (non-risky assets) and make periodic withdrawals to meet their income needs. Whilst the life annuity is believed to be the only form of retirement benefit that absolutely insures against longevity risk (Bateman & Kingston, 2007), there remains strong empirical evidence of annuity aversion among retirees. Retirees choose self-annuitisation for flexibility, liquidity and control over their retirement wealth. What are the best investment strategies to help retirees meet their income needs as well as lower the chances of portfolio ruin in retirement?

Retirees' age, health status, risk tolerance levels, as well as the total value of their assets and their desire to leave bequests may influence their choice of investment strategy. Self-annuitisation decisions and corresponding investment decisions provide no investment or longevity protection for retirees. Whatever the overall investment strategy, diversification of underlying investments in any asset class is essential in the reduction of investor's overall risk. Conventional wisdom suggests that investors ought to be less risk averse at younger ages and invest a high proportion of wealth in growth assets rather than defensive assets, while investment in growth assets should decrease as an individual ages and their investment horizon reduces. Studies by Bodie et al. (1992), Bodie (2003) and Samuelson (1989), among others, support this proposition.

There is the conventional rule of thumb approach to pension investment that states that the investors should allocate a percentage of 100 minus their current age in stocks (risky asset) and the remainder in low risk assets such as bonds and cash. There is also the static Strategic Asset Allocation (SAA) where investors choose to invest either conservatively, balanced or aggressively in stocks and invest their remaining wealth in bonds and cash. Their wealth levels are rebalanced frequently to assume the desired risky, non-risky asset proportions. The lifecycle strategy is probably the most popularly used investment plan QSuper (2014). The lifecycle approach explicitly becomes more defensive with increasing age. Plan participants invest massively in stock at the onset when the investor is still young, because the younger investor is able to assume more risk than the older investor, as explained by Utkus (2005), with the belief that if stocks should decline in the early years of investment, there is ample time for a retiree to regain losses. This investment is gradually switched to non-risky assets as the individual nears retirement and the asset mix changes. The allocation to stocks and bonds and cash is unidirectional, hence investment in stocks reduces as that in bonds and cash increases.

Empirical studies by Agnew et al. (2003) find US retirement accounts over the period 1995-98 to follow this logic, observing a downward trend in allocation to stocks with increasing age among investors. Other studies find this reduction in stock exposure with increasing age to be insignificant (see Ameriks & Zeldes, 2004; Gomes & Michaelides, 2005; Poterba & Samwick, 2003). Another school of thought introduces us to dynamic asset allocation as an alternative to the conventional strategy of pension allocation. These examine the shortfalls of the traditional method and propose that the switch of investment from growth to conservative assets as the plan holder nears retirement should not happen as a predetermined arrangement by the pension plan provider but rather take into account several factors, such as the retiree achieving a specific wealth target to make this all-important switch in asset mix (Basu et al., 2011).

The optimality of these strategies is questioned in academic literature. Shiller (2005) suggests the earnings profile of individuals is hump-shaped; young people relatively earn less income compared to their older colleagues and earnings peak at middle age. Therefore investing heavily in stock in the early years when there is very little wealth and conservatively in the later years when earnings are higher does not help in achieving an optimal allocation. Yet others have argued the optimality of a 100% stock investment throughout the investment horizon for risk-neutral investors and the conventional lifecycle investment for risk-averse investors (Haberman & Vigna, 2002).

Blanchett (2007) used Monte Carlo simulations to compare fixed asset allocations to a wide range of investment paths that reduce the stock allocation during retirement. He assesses the results using different outcome measures and concludes that fixed asset allocations provided superior results to allocations which reduce the stock allocation later in retirement. His study however does not consider increasing allocation to stocks. More recent studies by Arnott et al. (2013) argue that a reverse approach to the lifecycle/target date fund glidepath with increasing stocks delivers greater terminal wealth levels for investors. They yield higher wealth levels than the traditional lifecycle approach even at the lower tail of the wealth distribution. Estrada (2012) comprehensively studies the lifecycle investor glidepath in 19 countries, finding that the alternative strategies provide higher upside potential, and more limited downside potential, although with higher uncertainty. Surz (2013) asserts that what matters most in retirement planning is not the direction of allocations or glidepaths but saving enough before retirement. He emphasises that no glidepath is able to compensate for inadequate savings. These studies derive their conclusions based on the results from the accumulation phase of retirement planning.

Although the focus of this study is on the asset allocation in the retirement phase, the relevance of the accumulation phase cannot be underscored. Effective asset allocation decisions and changes begin before retirement. The shift in stocks to bonds for target date funds or lifecycle participants begins several years before retirement and allocation changes for static strategy holders typically begin in the periods commonly referred to as the transition period. The transition period is often the decade prior to retirement when an investor begins to assess her ability to meet the basic objectives of her retirement savings; which is firstly, to maximise the real value of her retirement nest egg, subject to reasonable risk controls and secondly to minimise uncertainty around her prospective retirement income. The retirement conversion plans are also decided in the transition period.

This study contributes to this important debate on investments and asset allocation in the retirement distribution phase in several dimensions. While previous propositions of a reverse target date fund style have been limited to the benefits of wealth outcomes in the accumulation phase, this study proposes a similar approach to investment in the post-retirement phase. The input of withdrawals for income in retirement provides a different look at this approach as these approaches not only help measure the terminal wealth after the investment horizon but also the ability of the approaches to sustain retirement income to meet retiree's needs. This further enables a comparison of approaches on a broader base such as portfolio ruin and income shortfall in retirement.

Secondly, unlike the straight-line linear increase approach considered in previous studies, I take a step further to include different hybrids of the lifecycle approach besides the reverse lifecycle. Specifically, I consider two Partial Lifecycle approaches as well as the Reverse Lifecycle approach. The partial approaches are a hybrid of the static allocation and the lifecycle approach while the Reverse Lifecycle acts in full reverse to the typical lifecycle strategy in accumulation phase, increasing assets allocated to stocks with increasing age and decreasing investment horizon.

Finally, I combine two of the conversion phase approaches as discussed by Schaus (2010). I examine how much of guaranteed income (annuity income) a retiree is able to purchase with her terminal wealth resulting from engaging in one of the asset allocation approaches over the investment horizon with a systematic withdrawal plan. I exclude the impact of government pensions, making retirees' income in retirement entirely dependent on their accumulated wealth at retirement and returns on this wealth based on the chosen line of investment.

5.2 Background and Assumptions

5.2.1 Retirement Wealth

A retirees' accumulated wealth at retirement age is a result of savings and growth of savings achieved through periodic contributions throughout the accumulation period. I do not analyse the lead up to retirement age but assume a retiree has a nest egg at retirement and this forms the basis of our asset allocation study. Investors do not purchase annuities immediately at retirement but choose to make systematic periodic withdrawals from their retirement wealth to meet their consumption needs. They invest the remainder of their wealth according to a strategy of their choice and earn returns on their investments. This investment has no longevity or investment guarantees and may deplete whilst the retiree is still alive.

I begin our analysis with different levels of initial retirement starting with a balance of \$500,000 and adjust upwards and downwards in further analysis to investigate the impact of differing wealth levels.

5.2.2 Retirement Income

Schaus (2010) provides a comprehensive approach to the retirement conversion phase for retirees. He discusses three main approaches: the income-only plan, the Systematic/Partial Withdrawal Plan (SWP) and the Guaranteed Income plan. The first is usually undertaken by the very wealthy who are able to live off the income they earn on their savings without the need to spend the principal. The second, which most retirees face, plan a drawdown plan that consists of income from both the principal and interest earned on it. The final plan is for the lower risk tolerant or investors in anticipation of high longevity; they convert all or part of their assets into an immediate or deferred income producing annuity.

I consider the second and third conversion approaches for retirees as the majority of retirees need to actively manage their retirement wealth and make systematic withdrawals or purchase guarantees to meet their income needs in retirement. I acknowledge the diverse and varied conclusions on the sustainability of the 4% 'Golden rule' as proposed by Bengen in 1994. Whilst earlier studies firmly supported this withdrawal rate (see Guyton, 2004; Guyton & Klinger, 2006; Pye, 2000), recent literature questions the sustainability of the 4% 'safe withdrawal rate' and its ability to sustain retirement portfolios. Spitzer, Strieter, and Singh (2007) and Spitzer (2008) suggest that the 4% rule may be an oversimplification while studies by Sharpe, Scott, and Watson (2007) believe the rule is inefficient. Other studies which oppose the 4% rule include Harris (2009), Pfau (2011), and Drew and Walk (2014). Based on a comprehensive analysis across 19 countries, Drew and Walk (2014) argue that the 4% rule does present us with an opportunity to form a baseline which can dramatically improve the expectations of what is possible in retirement but not a silver bullet approach to retirement withdrawals.

For our intended investment horizon, I consider a short horizon approach in which a 4% is sufficiently sustainable. I consider a 20-year investment horizon which is shorter than what is used in the aforementioned studies that draw conclusions on the 4% withdrawal rule. Using the 4% rule as the baseline, I investigate the sustenance of other withdrawal sizes ranging between 3%-8% of initial wealth level. At the end of the investment horizon, the retiree converts her terminal wealth into a guaranteed income product, specifically a whole life immediate annuity. With the increasing life expectation, guaranteed income products are increasingly becoming attractive and I consider how much income a retiree is able to generate from her terminal wealth based on the investment strategy she chooses at retirement.

Using a fixed income level in retirement may be an over simplification of the patterns of expenditure that might be desired in retirement. Ding (2012) finds that for wealthy households, expenditure clearly decreases with age. Other studies by Higgins and Roberts (2011) and Yogo (2009) find utility from consumption to decrease with increasing age due to health declines. Chen, Scott, and Chen (2007) assert that older retirees spend less than younger retirees on all items except medical care. Studies by Rice and Higgins (2009) after controlling for net income, wealth, and other factors, find the decline in spending with age to be significant. Hatcher (2007) shows increasing age has a negative impact on consumption. In reality however, the decreasing expenditure may not necessarily be desired, but may be a function of declining wealth levels and thus ability to consume.

In addition to analysing fixed desired expenditure in retirement, I discuss an alternative variable scenario. I therefore investigate a variable withdrawal plan, with the initial income level set to a percentage of starting wealth, with subsequent income levels at 4% of remaining wealth. I analyse how these withdrawal plans affect the various investment strategies in our analysis and benefit levels over the retirement phase.

5.2.3 Mortality

There has been a significant increase in human longevity in the last century. In both developed and developing countries, people are living longer than before. Findings by Oeppen and Vaupel (2002) reveal that female life expectancy in the last century has been an increasing steadily by almost three months every year. With increasing life expectation is increasing longevity risk associated with retirement planning. Longevity risk refers to the uncertainty of the age of death and the likelihood of a retiree outliving her wealth. Of relevance to this study are the mortality characteristics of individuals between retirement age 65, and the terminal age 100. According to the American Life Table 2011, provided in the National Vital Statistics Report (2014)[1], a couple both aged 65 have an average life expectation of 19.1 years. Independently, males have a life expectation of 17.6 years while the female counterparts have up to 20.3 years life expectation in retirement. The median life expectation is around 86 years whilst a tenth of the population at 65 years will grow to age 96.

Population ageing is not an American issue alone; many developed countries are facing similar ageing populations, with Baby Boomers reaching retirement. An analysis of the Australian Life Tables 2010-2012 (2013) shows the conditional survival probabilities for retirees rising beyond the life expectancy at birth values. Whilst the life table estimates the male and female life expectancies at birth as 80 and 84 years respectively, life expectation conditional to surviving to age 65 is 84 and 87 for males and females respectively. A significant observation is that one in every ten females in retirement is expected to live until age 96. This requires a retiree who wants to be 90% certain that her superannuation savings last as long as she lives to plan her consumption and savings for up to 31 years. These are entirely different from the expectation of life at birth ages and hence requires retirees to plan for longer horizons. These observations slightly exceed the life expectations in the US.

I assume retirees live until their median life expectation in retirement, hence planning for a 20-year retirement horizon. The balance of their retirement wealth after this age

[1] http://www.cdc.gov/nchs/data/nvsr/nvsr61/nvsr61_06.pdf

may continue to meet retirement needs if they are alive or bequest needs otherwise.

5.2.4 Asset Allocation

Bengen (1997) advises that if the future market follows the trends of behaviour in the past, then a retirement portfolio should contain 50-75% stock allocation. Milevsky and Kyrychenko (2008) demonstrate that investors with downside protection in the form of longevity puts assume 5% to 30% more risk exposure than the investors without such protection. This evidence shows that in the presence of insurance for asset allocation in retirement investment, plan participants defy the conventional approach to investment, where investors' allocation to risky assets decreases with increasing age. Other studies by Milevsky (2001) and Ameriks et al. (2001) demonstrate through simulation, the need for holding a substantial stock allocation in retirement portfolio. Cooley et al. (2001) propose that at least 50% of a retirement portfolio should be invested in stocks and their findings show increased sustainability of the pension fund as it tilts more towards stocks. They show that the presence of bonds is mainly to restrain portfolio volatility and provide liquidity to cover an investor's living expenses. Hubbard (2006) considers different withdrawal rates given various portfolio allocation strategies and fixed withdrawal periods, and emphasises the benefits of holding a stock dominated portfolio.

In contrast to these suggestions, I find the lifecycle strategy, which decreases in stock-like investments with increasing age, to be a dominating post-retirement strategy in many markets. In the US, the lifecycle accounts for 77% of retirement plans. In Australia, lifecycle funds are increasing rapidly, expected to catch up or surpass the US in the next decade with large funds such as QSuper switching to the lifecycle strategy as its default investment option (QSuper, 2014). The lifecycle strategy was implemented with the aim of avoiding insufficient diversification as well as to avoid investment choices that may be age-inappropriate. While it undoubtedly achieves these aims, the relevance of having investments following a predetermined glide path solely dependent on age is simplistic. Other relevant factors such as account balance, gender and marital status influence expectations regarding income and life expectation in retirement.

I analyse eight different asset allocation strategies which I fully discuss in Methodology. There are four different Static Strategic Asset Allocation strategies ranging from a Conservative approach, with total investment in bills and bonds to an Aggressive approach where a retiree completely indulges in stocks. There is the target date funds approach which I refer to as the Lifecycle approach where the retiree reduces her allocation to stocks as she grows older and a Reverse Lifecycle approach with increasing allocation to stocks. Finally, there are two Partial Lifecycle strategies which are hybrids of the static and the target date fund approaches.

5.3 Methodology

5.3.1 Simulation Set-up

I use historical daily returns data on US stock, bonds and bills between 31 January, 1928 and 31 January, 2013, spanning a period of 85 years. I may subsequently refer to stocks as growth assets and bonds and bills as conservative assets through the remainder of this study. From the data, I am able to obtain four independent non-overlapping 20-year holding period observations within our dataset. The initial cohort of retirees begins their investment horizon in 1928 until 1948, when they attain age 85. There are 66 overlapping 20-year cohorts, with the final cohort beginning in 1993 and ending in 2013. While the use of overlapping returns has been used in previous studies (see Bengen, 2001, 2004; Cooley et al., 2001; Hubbard, 2006), I believe this is insufficient to draw a reliable conclusion.

To cater for the insufficient data, I use block bootstrap resampling to generate 20-year return time series. The method involves sampling blocks of consecutive values of the original returns time series, say X_{i+1},...,X_{i+b} where $0 \leq b \leq N - b$ is chosen in some random way; and placed one after the other in an attempt to reproduce the 20-year time series. Where b denotes block length and N, the length of the time series. Specifically, I use a 'moving block' bootstrap technique which allows for block overlap making the $i_{j's}$ independent and uniformly distributed on the values $1, ..., N - b$. Since the return matrices hold rows of the different asset class returns, I am able to preserve the cross-correlation between the asset returns as well as correlation within the various asset returns within each block. By using historical rolling returns, I am able to ascertain through historical evidence the success or failure of the various retirement investment plans. With this information, I am able to adequately cater for current retirees through making informed decisions about future expectations. Kunsch (1989) explains in details how and why such a block bootstrap works in his seminal paper. More recent use of this methodology is in works by (Basu, Chen, & Clements, 2014).

I employ a block bootstrap resampling of 5-year length based on the return vectors for the three asset classes in the dataset. I randomly resample the return vectors with replacement from the empirical return distribution to generate 20-year asset class return vectors. I repeat the procedure 10,000 times, creating 10,000 replica return time series. I allow for monthly income drawdowns from our wealth levels and this serves as retirement income for the plan participant. The portfolio is immediately rebalanced after the monthly withdrawal to its target allocation. The level of income is chosen according to a percentage of wealth approach which is adjusted annually for inflation. Our simulations are based on retirees choosing one of eight asset allocation strategies at retirement age and holding that strategy for the remainder of their lives.

The retirement investment plans analysed in this study are:

◦ The static Strategic Asset Allocation plan (SAA): This involves setting target allocations for various asset classes in retiree's retirement portfolio and periodically rebalancing the portfolio back to the original allocations when they deviate from the initial settings due to differing returns from various assets and periodic withdrawals made from the portfolio. This is a typical 'buy and hold' strategy and the choice of target allocations depends on the retiree's risk tolerance level and investment objectives as well as investment time horizon. The SAA is based on modern portfolio theory's rationality of diversifying among various assets to increase overall portfolio returns. I investigate four SAA allocations: the Conservative, Balanced, Growth and Aggressive allocations, the choice of which depends on the retiree preferences.

- *Conservative*: The conservative plan allocates the retirees entire wealth to conservative assets, thus holding an equal allocation to bonds and bills. This set allocation is held through rebalancing after returns and income drawdowns for the entire investment horizon.
- *Balanced*: The balanced plan allocates the retirees wealth equally between growth and conservative assets, holding a static 50% in stocks and the remaining 50% in bonds and bills. This is rebalanced monthly throughout the 20-year horizon
- *Growth*: The growth plan allocates 70% of the retiree's wealth to stocks and the remaining is shared between the bonds and bills. This set allocation is held

by the retiree throughout the investment horizon regardless of the nature of returns.

- *Aggressive*: The aggressive plan allocates all the retiree's wealth to stocks at retirement. The retiree thus maintains a 100% allocation to growth assets throughout her horizon with periodic rebalancing to the target allocation.

◦ *Lifecycle Strategy (LC)*: This strategy begins with an equal allocation between growth and conservative assets, with the growth assets gradually reduced to 0% at the end of the investment horizon. As the allocation to growth assets is reduced, the allocation to conservative assets is increased, with the retiree switching to a full investment in conservative assets by the end of the horizon.

◦ *Reverse Lifecycle (RLC)*: This approach works in an opposite method to the Lifecycle strategy. The retiree begins investment with a full allocation to conservative assets at retirement and gradually increases investment in growth assets till the investment is fully invested in growth assets at the final year in retirement. This is against the conventional wisdom of increasing investment in conservative assets with increasing age.

◦ *Partial Lifecycle (PLC)*: This strategy is a variant of the Lifecycle strategy; the retiree holds a fixed allocation to stocks throughout the investment horizon. A constant fraction of the portfolio, 50% in this case, is held in growth assets. The remaining 50% is invested fully in growth assets at the onset and reduces to a full investment in conservative assets at the end of the horizon. Thus the retiree holds the 50% allocation to growth assets and invests the remaining balance according to the Lifecycle plan, decreasing allocation to growth assets with increasing age. This investment strategy therefore begins with a 100% stock investment, reducing to an equal allocation to growth assets and conservative assets in the final investment period.

◦ *Partial Reverse Lifecycle (PRLC)*: This is another variant of the Lifecycle strategy, with the retiree having a fixed allocation to bonds/bills which is held throughout the investment horizon. This constant fraction of the conservative asset is held at the beginning of the investment horizon, 50% in this case. The remaining 50% is invested fully in conservative assets at the onset and reduces to a full investment in growth assets at the end of the horizon. Thus the retiree holds the 50% allocation to conservative assets and invests the remaining balance according to the Reverse

Lifecycle plan, increasing allocation to growth assets with increasing age. This investment strategy therefore begins with a 100% bonds/bills investment and reduces to an equal allocation to growth assets and conservative assets in the final investment period.

5.3.2 Analysis

Firstly, I compare the wealth accumulated in retirement by the alternative investment strategies after 20 years in retirement. At retirement, retirees' have life expectancies of up to 20 years for both sexes in Australia and up to 19 years for both sexes in the US. For the eight investment strategies, I compare their performances based on their ending portfolio values. These wealth levels are contingent on the retiree surviving 20 years in retirement and making monthly income withdrawals of 4% of her wealth levels to meet consumption[2]. From the wealth estimates of our iterations, I rank the probabilities of strategies outperforming or underperforming competing strategies.

Next, I calculate the chances of ruin for retirees based on the choice of any of the eight alternative investment strategies. This is the probability that the retiree ends with a negative balance after the 20-year investment horizon. The ruin probability is calculated on the assumption of the retiree surviving the investment period and making regular drawdowns from her wealth. Upon exhaustion of the retiree's wealth, she may borrow to meet her income needs and has no more wealth to invest. I compare the investment strategies based on their ruin probabilities to determine which of the plans has a better chance of being sustainable in the long term. Retirees seek adequacy and sustainability of their investments to be comfortable in retirement and will prefer a strategy with a higher chance of lasting as long as they expect to live.

Besides the probability of ruin, I estimate the extent of ruin for the alternative withdrawal plans. This is the measure of the size of shortfall for the various strategies when ruin occurs. As the retiree is unable to meet her consumption needs upon portfolio ruin, I calculate the extent of shortfall based on the timing of ruin of retiree's wealth. I measure the size of the portfolio shortfall as the number of years in ruin relative to the length of the investment horizon needed to meet the retiree's life expectancy. I define shortfall years as the final year in the investment horizon less the ruin year. Justification for this approach is expounded in works by Butt and Deng

[2]Withdrawal levels of 3%, 5%, 6% and 7% are also analysed, I however present results of the 4% withdrawal level which serves as our benchmark drawdown level.

(2012).

Campbell and Viceira (2002) suggest that long term investors consume out of wealth and derive their utility from consumption rather than from wealth. Arnott et al. (2013) further echo that for retirement purposes it makes sense to gauge the success of a portfolio in terms of annuitised income rather than notional portfolio gains or losses. I consider a shortfall measure which is based on the retiree meeting a certain level of consumption. I compute income levels that the retiree is able to generate from her level of terminal wealth at the end of the investment horizon. I compare the annuity streams provided by terminal wealth values to see how they compare to a level annuity purchased at retirement date. I calculate benefit shortfall as the difference between what retirees receive from their guaranteed income and what they will receive if they annuitised at retirement age. I calculate the probability and extent of the benefit shortfall when it occurs.

Finally, I go further from analysing preferences based on wealth levels and guaranteed income from different wealth levels to comparing strategies based on gains and losses. I incorporate the concept of loss aversion, which is the tendency for individuals to be more sensitive to reduction in their wellbeing than to increases. Retirees set for themselves wealth thresholds from which they are able to obtain income adequate for a comfortable retirement. I refer to a utility function approach as motivated by Kahneman and Tversky's (1979) prospect theory. The prospect theory utility has a kink at the origin, with the slope of the loss function being steeper than the slope of the gain function. I compare a retiree's choice of alternative investment strategies for different wealth thresholds.

5.4 Results

Retirees choose one of the eight post-retirement strategies at retirement age 65 and hold this strategy for the entire investment horizon of 20 years. They make monthly withdrawals based on 4% of the starting wealth withdrawal level which is annually adjusted for inflation. I summarise the statistics of the ending wealth in Table 5.1.

Table 5.1 shows the summary statistics of the terminal wealth of the eight investment strategies. The highest average terminal wealth values are realised for the Aggressive Static Asset Allocation, the Growth and the Partial Lifecycle strategies and this

Table 5.1: Summary Statistics

This table presents the summary statistics for the eight post-retirement strategies analysed. I show the ending portfolio averages, the standard deviation, the minimum and maximum values as well as different percentile distributions of the terminal wealth.

	Average	**SD**	**25th**	**50th**	**75th**	**90th**
Conservative	450,904	356,245	213,611	361,287	587,875	894,454
Balanced	1,253,532	919,305	632,527	1,033,728	1,651,107	2,399,011
Growth	1,744,026	1,471,331	719,188	1,379,486	2,371,598	3,633,884
Aggressive	2,433,408	2,529,092	721,675	1,745,053	3,379,009	5,600,133
LC	764,448	490,969	425,794	665,688	993,614	1,398,461
PLC	1,738,056	1,509,418	676,425	1,404,855	2,410,207	3,678,148
PRLC	671,098	442,019	366,384	560,683	852,087	1,241,063
RLC	1,358,473	1,006,345	650,558	1,122,558	1,824,826	2,660,604

is accompanied by the highest standard deviations. The Reverse Lifecycle strategy generates a higher average ending wealth level than the Balanced strategy and this is accompanied by a higher standard deviation. The Conservative strategy has the lowest average terminal wealth, followed by the Partial Reverse LC and LC, with respectively increasing standard deviations of terminal wealth as expected. These strategies provide the most certainty in terms of ending wealth levels at a 4% withdrawal rate as they have the lowest standard deviations. Whilst all wealth estimates at the 25th percentile are positive, at the 25th percentile, again the stock dominated portfolios result in higher terminal wealth values compared to the conservative strategies. A similar trend is observed at the 50th percentile. At the 90th percentile, the Aggressive strategy is significantly higher than all remaining strategies. The Partial LC and Growth strategies dominate the remaining strategies with significantly high wealth estimates, trailed by the Reverse LC and Balanced strategies. The Conservative strategy, which has no stock allocation, ends with the lowest wealth estimates at the 90th percentile.

The varying terminal wealth levels associated with the Aggressive and Growth strategies, as well as the Partial LC and to some extent in the Reverse LC strategies are a depiction of the influence of stocks at different levels in the retiree's portfolio. The impact of stocks is significant in the portfolio dynamics as I find extreme differences in the stock and bond/bills dominated portfolios. Because resampling is done in blocks and with replacements, a period of poor stock returns such as the GFC can appear multiple times in a data sample affecting the wealth levels of stock dominated portfolios. In the same sense, vectors of good returns may also appear several times in the same time series sample enhancing returns in the stock-dominated portfolio.

This helps to capture a wider range of future possibilities which are derived from empirical historical data and explain the wide standard deviations in wealth estimates.

Higher stock levels are good for the right tail of the distribution, providing the possibility of extremely high terminal wealth for retirees. On the left side, however, the variability in returns acts against retirees as it makes it increasingly difficult for retirement planning compared to balanced and conservative investment strategies. If a retiree is interested in a portfolio that is able to provide a high wealth level which can generate a sustainable level of consumption in retirement, a stock dominated strategy gives a higher chance of meeting this need. Whilst the implementation of high stock portfolios in retirement remains a sensitive subject for any investor or adviser due to the uncertainty of the stock markets, it is important to outline the benefits of such a strategy based on historical successes. The extent to which the strategies outperform each other based on the size of their terminal wealth is discussed in Table 5.2.

Table 5.2: Comparing Performance of Alternative Strategies

I compare the chances of each of the eight strategies outperforming competing strategies. The probabilities represent the chances of the rows outperforming the columns (columns underperforming the rows). My performance measure is based on terminal wealth levels based on strategy selected for retirees in 10,000 simulations. Overall measures the chance of a strategy being dominant among the remaining strategies in a simulation.

	Cons	Bal	Gro	Agg	RC	PRC	PRRC	RRC	**Overall**
Conservative	0.00	0.08	0.12	0.16	0.14	0.15	0.10	0.10	**0.03**
Balanced	0.92	0.00	0.22	0.25	0.93	0.29	0.89	0.38	**0.14**
Growth	0.88	0.78	0.00	0.27	0.87	0.53	0.86	0.82	**0.18**
Aggressive	0.84	0.75	0.73	0.00	0.82	0.75	0.82	0.78	**0.20**
LC	0.86	0.07	0.13	0.18	0.00	0.16	0.63	0.13	**0.08**
PLC	0.85	0.71	0.47	0.25	0.84	0.00	0.82	0.71	**0.17**
PRLC	0.90	0.11	0.14	0.18	0.37	0.18	0.00	0.12	**0.07**
RLC	0.90	0.62	0.18	0.22	0.87	0.29	0.88	0.00	**0.14**

I compare the chances of a strategy outperforming other competing strategies based on the levels of terminal wealth after 20 years of investment. The Aggressive strategy, which has an all stock allocation, significantly dominates the remaining strategies. It dominates the Growth strategy by 73%, the lowest probability of dominance observed. This level increases further with other strategies, ranging between 75% compared to the Partial LC and Balanced strategies to 84% for the Conservative strategy. The Aggressive strategy hence becomes the most dominating strategy among the eight competing investment strategies with an overall chance of providing higher wealth levels in 20% of the total number of simulations.

Although the Growth strategy is dominated by the Aggressive Strategy, it generates higher wealth estimates than the Partial LC in over half of the simulations. It also performs significantly well against the Conservative, Balanced, Partial Reverse LC and the Reverse LC strategies, dominating in over 78% in all simulations. It also performs better than the LC in 88% of the simulations, which is the highest level of dominance over any strategy. The Growth strategy has 18% chance of dominating competing investment strategies overall and is the strategy with the second highest ending portfolio wealth levels.

The Partial LC strategy which retains 50% of the investor's wealth in stocks whilst the remaining 50% is invested in typical lifecycle style is the third highest dominating strategy in terms of the terminal wealth levels after 20 years of investment in retirement. Overall, the PLC has a 17% chance of dominating the other strategies in our total simulations at 4% withdrawal rate, closing trailing the Growth strategy. This strategy is dominated by the Aggressive strategy 3 times in every 4 simulations and up to 1 in 2 simulations by the Growth strategy. It however dominates the remaining strategies with probabilities ranging between 71% for Balanced and Reverse LC to 85% for the Conservative strategy.

The Reverse LC which is in reverse to conventional lifecycle investment approach (Lifecycle) and increases stock investment as retiree ages and retirement horizon decreases performs better than the LC 87% of the time. This is a significant result considering the high numbers of retirees who default into a lifecycle style investments and leave their portfolio as such in retirement. The Reverse LC also dominates the Partial Reverse LC, the Conservative and the Balanced strategies. The Balanced strategy performs better than the Conservative, the LC and the Partial Reverse LC with significant probabilities of 90%, 93% and 89% respectively but performs poorly against the four remaining strategies. The lifecycle strategy in retirement, LC, which follows the conventional approach of decreasing stock levels with increasing age generates very ordinary terminal wealth estimates compared to the other strategies. Whilst it dominates the Conservative strategy in 86% and the Partial Reverse LC at 63% of the simulations, it falls short to all other remaining strategies. The LC has an 8% chance of dominating competing strategies overall. The Conservative strategy, which has no stocks in the retiree's portfolio, performs poorly compared to the remaining strategies that include stocks, with as little as 3% chance of dominating the other investment strategies overall.

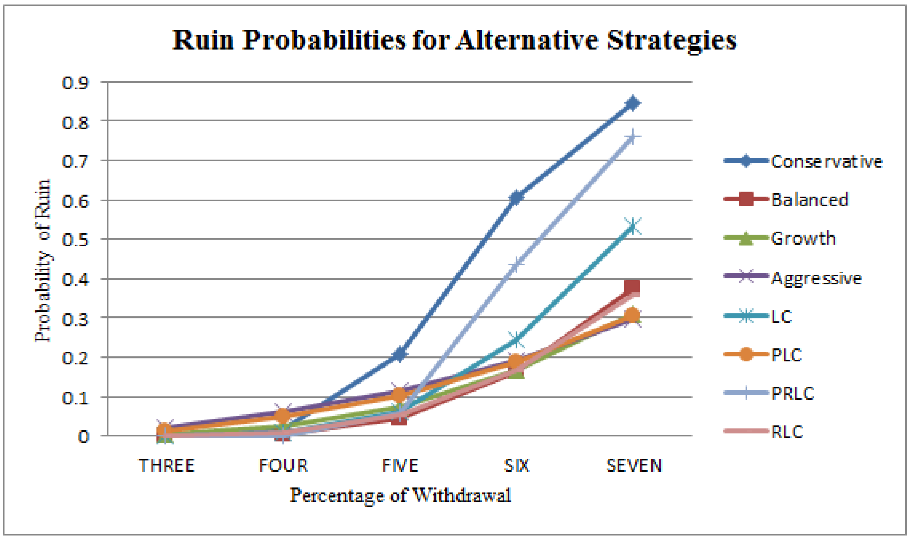
While this analysis is solely based on the terminal wealth levels, I assume retirees, irrespective of their risk aversion levels and other preferences will always prefer a higher level of terminal wealth to a lower level. In the Further Analysis section, I test the impact of varying standard deviation at various wealth levels on retiree's choice of strategy, that is, the trade-off between risk and return. The Conservative strategy which avoids stock investment in retiree's portfolio turns out to underperform all competing strategies. The inclusion of stock in the retirement portfolio improves the portfolio performance, with portfolio performance increasing with increasing stock levels. The Balanced strategy performs better than the Conservative, the Growth, better than the Balanced, and the Aggressive better than the Growth strategy. I find improvement in the retirement wealth levels to have a direct relation with stock levels in the portfolio. The Aggressive strategy and the Partial LC Strategies which both begin with 100% invested in stocks and the Growth strategy with 70% stock investment dominate the remaining strategies in more than half of the total simulations. The desire for high levels of terminal wealth makes stock investment an attractive venture for retiree willing to take some level of risk.

5.4.1 Ruin Probabilities

I analyse the performance of the different investment strategies with respect to their chances of not meeting retirees' needs in terms of regular income withdrawals and a positive wealth level at age 85 or both. I first analyse the probabilities of ruin and later the extent of ruin should it occur. The extent of ruin in this case is quantified by the length of time, in years, the retiree is unable to meet her income needs. When ruin occurs a retiree may borrow to meet her consumption but cease to have any investments. The shortfall years are appropriate as retirees need to borrow over the ruin years to meet their consumption needs and the number of years portrays the extent of ruin suffered. I analyse different withdrawal rates ranging between 3%-7% of initial wealth which I adjust annually for inflation in subsequent withdrawals. I use the widely accepted 4% of initial wealth as our baseline withdrawal level for comparison purposes. I show the ruin probabilities of the various investment strategies in Figure 5.1.

Figure 5.1: Ruin Probabilities for Alternative Strategies

This figure shows the instances of ruin in the simulations for the different investment strategies and the probabilities of ruin for the alternative investment strategies for withdrawal rates ranging between 3%-7% of initial wealth and subsequent withdrawal levels adjusted for inflation.



At low withdrawal rates of 3% and 4% of initial wealth, I find all strategies to have low probabilities of ruin. The Aggressive strategy has the highest ruin probability, increasing from 2% to 6% at 4% withdrawal level. The Growth and Partial LC strategies are the only other strategies that experience ruin at 3% withdrawal. The Balanced, the LC, the Partial Reverse LC and Reverse LC have lower than 1% chance of ruin at 3% withdrawal level and up to 99% of simulations not going to ruin at 4% withdrawal level. At 5% withdrawal rate, I find a steep increase in the ruin probabilities of the Conservative portfolio, soaring to 21%. With the exception of the Aggressive strategy which has ruin chances of 12% and the Partial LC with 10%, all remaining strategies have ruin probabilities less than 10%.

At the high withdrawal rates of 6%-7% of initial wealth, I find very pronounced differences in the ruin patterns for the different investment strategies. These differences are mainly seen for varying stock levels of the various strategies. Whilst the instances of ruin increases for all strategies, the rate of increase is higher for the Conservative, Partial Reverse LC and LC strategies compared to the remaining strategies. The ruin probability decreases as I increase our allocation to stock, from Balanced, through Growth to Aggressive strategies. At 7% of annual withdrawal, the Conservative and Partial Reverse LC ruin probabilities exceed three quarters of the total simulations whilst the LC has a ruin probability of 53%. The remaining strategies have ruin probabilities less than 40% with the Aggressive strategy having the lowest ruin probabilities of 30%.

At low rates of withdrawal, the conservative portfolios are sustainable and hardly run to ruin as they have little variability in wealth levels and can squarely meet the low rates of withdrawal. Whilst the instances of ruin are generally low, I still observe ruin to be higher for the stock dominated portfolios. As the rate of withdrawal is increased, ruin chances for conservative strategies significantly increase whilst the remaining strategies observe ruin but at relatively lower levels. At high rates of withdrawal, I find the ruin probabilities to increase with decreasing stock levels. With a high observed number of ruin in the Conservative and Partial Reverse LC strategies, I find this number to decrease through Balanced, Growth to the Aggressive strategies. The Reverse LC has lower ruin chances compared to the LC whilst the Partial LC also runs to ruin on more occasions than the Partial Reverse LC. In addition to calculating the ruin probabilities, I compute the extent of portfolio shortfall should ruin occur. This answers the question of how large the potential shortfall may be or how bad is the portfolio ruin. For each of the investment strategies, I measure the mean shortfall years, the maximum shortfall years and the number of events with ruin years exceeding the mean shortfall. The mean shortfall years, which is the average number of years a retiree potentially faces ruin is our measure of shortfall severity. I show the analysis of shortfall for the different strategies for the different withdrawal rates in Table 5.3

TABLE 5.3: Extent of Portfolio Ruin

This table presents the shortfall analysis with regard to portfolio ruin. I compute the Mean Shortfall (in years), Exceeding Shortfall, which is the number of events resulting in ruin levels exceeding the Mean Shortfall and the Maximum Shortfall levels for the range of initial withdrawal levels.

					Investment	**Plans**			
		Conservative	**Balanced**	**Growth**	**Aggressive**	**LC**	**PLC**	**PRLC**	**RLC**
	Mean Shortfall (Years)	2	2	2	4	1	3	0	2
3%	Exceeding Shortfall (Number)	1	0	15	91	0	57	0	1
	Maximum Shortfall (Number)	3	2	8	12	1	11	0	4
	Mean Shortfall (Years)	1	2	3	5	1	4	1	2
4%	Exceeding Shortfall (Number)	75	28	117	291	33	231	7	48
	Maximum Shortfall (Number)	4	5	11	13	4	13	1	7
	Mean Shortfall (Years)	2	2	4	5	2	5	1	3
5%	Exceeding Shortfall (Number)	860	193	336	531	286	464	210	220
	Maximum Shortfall (Number)	7	7	11	13	7	13	5	8
	Mean Shortfall (Years)	3	3	4	5	3	5	2	3
6%	Exceeding Shortfall (Number)	2950	725	757	873	1032	833	2074	761
	Maximum Shortfall (Number)	9	9	13	14	9	14	7	10
	Mean Shortfall (Years)	4	4	5	6	3	6	4	4
7%	Exceeding Shortfall (Number)	4645	1677	1427	1384	2370	1414	4128	1632
	Maximum Shortfall (Number)	10	11	13	15	11	15	9	12

At low rates of withdrawal between 3% and 4%, I find the stock dominated portfolios to experience portfolio ruin more often than the less volatile and conservative strategies. The Aggressive, Growth and Partial LC strategies have high maximum shortfall years as well as ruin years exceeding the mean. The Partial Reverse LC does not experience ruin at 3% but has shortfall of up to one year at 4%. At 4% withdrawal, I find over 200 events of the Aggressive and Partial LC facing ruin years exceeding 5 years and 4 years respectively. The Conservative, LC and the Partial Reverse LC, the more conservative strategies have mean shortfalls of 1 year whilst the Balanced and Reverse LC also have low mean shortfalls of 2 years, with 5 and 48 iterations exceeding their means.

At higher rates of withdrawal, as expected, I find the different strategies to increase in mean shortfall years as well as the maximum shortfall years. Of significance is the sharp increase in the numbers exceeding the mean shortfall for the conservative strategies over the more aggressive strategies. At 7% withdrawal rate, the Conservative and Partial Reverse LC strategies, both with mean shortfalls of 4 years, have over 4,000 events exceeding this mean shortfall. The Aggressive and Partial LC on the other hand, with a slightly higher mean shortfall of 6 years, have 1,400 events exceeding this shortfall level. At an equivalent level of 4 years shortfall, only 1,900 of the iterations of the Aggressive Strategy and Partial LC exceed this level compared to 4,000 of the more conservative strategies.

At all rates of withdrawal, the stock dominated portfolios have higher mean shortfall years compared to the more defensive strategies. This means when portfolio ruin occurs it is more pronounced for the aggressive investor than the defensive. The maximum shortfall years are also higher for the stock dominated portfolios than the balanced and conservative portfolios. Therefore a retiree may experience potentially severe shortfall by investing in an aggressive strategy, as these strategies exhibit higher mean shortfall years and higher maximum shortfall years. However, the probability of facing the high levels of shortfall is relatively low for the aggressive investor. The retiree who chooses a more defensive strategy, for the lower mean average shortfall and lower maximum shortfall years has a higher chance of ruin although at a lower extent. The number of ruins increases at a higher rate for the conservative strategies than the stock dominated portfolios with increasing withdrawal rates. Although the average ruin levels are low for the conservative strategies, the number of retirees facing ruin at this and higher levels are exceedingly high with ruin probabilities increasing exponentially as withdrawal rates increase.

5.4.2 Income Shortfall Analysis

I compare guaranteed income levels that retirees are able to purchase at the end of the investment horizon with their terminal wealth. This is a fixed income amount a retiree generates from the terminal wealth after the investment horizon level which will last as long as she lives. I compare this to an annuity income available to a retiree who fully annuitises with a Single Premium Immediate Annuity (SPIA) at retirement which pays a fixed amount of monthly income for life.

The SPIA is calculated with rates made available on CNN Money which provides annuity income estimates based on individual's age, gender, residence and invested premium. Retirees purchase this annuity with their terminal wealth at age 85. The income generated from this annuity purchased at retirement age 65, with the starting wealth of $500,00 is $32,500 which is 6.5% of starting wealth. I estimate the income shortfall as the difference between the retirees' income from her terminal wealth and the expected income from a life annuity purchased at retirement. Retirees' Shortfall Probability is calculated as:

$$SP(C_t) = P(C_t < z) \tag{5.1}$$

The probability that the guaranteed periodic income received by the retiree from her terminal wealth, C_t, is less than the benchmark z, in this case, the life annuity. In the absence of any government pensions, retirees that run into ruin have no guaranteed income after their investment horizon ends. The annuity used in this comparison is a fixed whole life annuity which the investor purchases at retirement age 65. The income received from the annuity is adjusted for inflation and compared to the income levels that are obtained from the terminal wealth of retirees who choose to invest in any of the eight strategies.

TABLE 5.4: Income Shortfall Probabilities

The probability that income generated from the terminal wealth of individuals after the investment horizon being less than the expected income from a life annuity purchased at retirement age.

Rate	**Conservative**	**Balanced**	**Growth**	**Aggressive**	**LC**	**PLC**	**PRLC**	**RLC**
3%	40%	7%	8%	12%	13%	11%	13%	7%
4%	67%	17%	17%	19%	33%	19%	43%	17%
5%	85%	34%	27%	27%	57%	29%	71%	32%
6%	93%	54%	40%	36%	76%	40%	87%	50%
7%	97%	70%	54%	46%	89%	51%	95%	68%

From Table 5.4, for the Conservative strategy, I find large shortfall probabilities ranging between 40%-97% for our range of withdrawals. The Partial Reverse LC is the next strategy with highest levels of shortfall ranging between 13% for a 3% initial wealth withdrawal rate to 95% for a 7% withdrawal rate. The Reverse LC has lower shortfall probabilities at all levels compared to the LC strategy. At low rates of withdrawal, the Reverse LC performs up to 6 percentage points better than the LC whilst this difference increases to 21 percentage points at the highest withdrawal rate of 7%. The Balance strategy does a modest job, with shortfall probabilities ranging between 7% and 70% for the range of withdrawal rates analysed. The Growth, Aggressive and Partial LC respectively provide the lowest shortfall probabilities compared to the benchmark annuity.

At the baseline withdrawal rate of 4%, I find the Conservative strategy to have the highest shortfall probability among the alternative investment strategies, falling below the annuity income threshold 67% of our simulations. The Partial Reverse LC falls below the benchmark in 43% of our simulations. These two represent the strategy with the highest non-risky asset dominance. Whilst the LC falls below the annuity benchmark in 33% of our simulations, I find its inverse, the RLC strategy, to fall below the benchmark by 17%. The Balanced and Growth strategies also fall short of the annuity level by 17% and this is the lowest shortfalls probability observed. Overall, I find the instances of benefit shortfall to be highest for the conservative strategies. The Balanced, Growth and Reverse LC portfolios perform better at low withdrawal levels than the more aggressive portfolios which begin with all stock allocation. The reverse is evident at high levels of withdrawal. The Reverse LC strategy significantly and consistently outperforms the LC strategy whilst the Partial LC also consistently performs better than the Partial Reverse LC, the more conservative variant.

5.4.3 Severity of Income Shortfall

I measure the severity of shortfall as the size of benefit shortfall over the investment horizon. Should benefit shortfall occur, I analyse the severity of the shortfall and how it disperses on average throughout the investment horizon for the different strategies. The size of the benefit shortfall is calculated with reference to the income level provided by our benchmark annuity income. To calculate the size and extent of shortfall, I consider both the probability of shortfall and the average size of shortfall for each of the strategies considered. This is similar to the metric used in Dus et al. (2005)

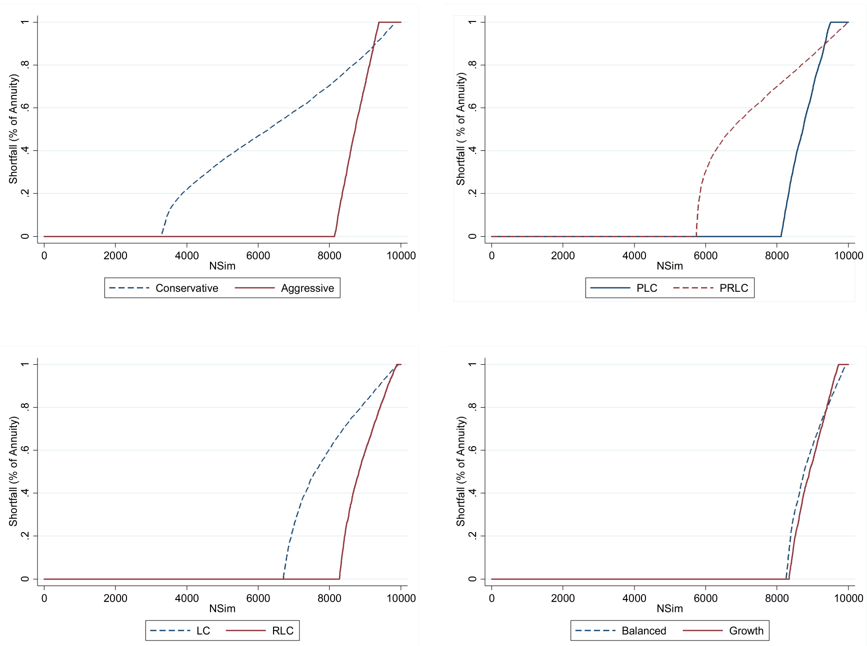
which they refer to as the Shortfall Expectation (S.E.).

$$SE(B_t) = E(max(z - B_t, 0) \tag{5.2}$$

Where z refers to the benchmark annuity income level and B_t refers to the benefit income level derived from retiree's terminal wealth. The Shortfall Expectation is therefore the sum of losses weighted by their probabilities. I compare the various strategies based on their Shortfall Expectations in Figure 5.2.

FIGURE 5.2: Extent of Income Shortfall

I show the extent of shortfall for the alternative strategies relative to the benchmark income level. I show the number of hypothethical retirees and the different extents of shortfall for competing strategies. The strategies with higher stock allocations are represented by the red solid lines and the blue dashed lines for the lower stock allocated strategies.



I compare the income provided by the terminal wealth resulting from the different investment strategies to the annuity and evaluate how much these incomes fall short of the benchmark. There is no income shortfall observed for up to 3,200 of our 10,000 simulations, after which I find the Conservative strategy to experience benefit shortfall. The shortfall level rises gradually with up to the next 3,000 lives facing benefit shortfalls of up to 50% of the benchmark annuity income. The shortfall level steadily increases, reaching 100% for about 2% of the cohort. This is the highest level of benefit shortfall experienced among the alternative investment strategies and is generally the proportion of simulations that run into ruin and need to borrow to meet consumption. The Aggressive strategy on the other hand has over 80% of the simulations giving income levels that are equal to or greater than the benchmark income. The remaining simulations which are exposed to shortfall show a steep rise in the shortfall levels, with about 600 of the 10,000 simulations ending with the maximum shortfall levels of 100% short of the benchmark.

The Partial Reverse LC has 4,300 lives experiencing benefit shortfall. Of this number, the level of shortfall increases steadily for the next 1,000 lives and steeply over the remaining lives. Less than 1% of the simulated lives experience a total benefit shortfall as opposed to over 500 lives experiencing total shortfall for the Partial LC strategy. The Partial LC plan, like the Aggressive strategy, has benefits equal or exceeding the benchmark in over 80% of our simulations. The remaining face shortfall levels rising sharply up to 50% of the annuity benchmark for the next 1,000 lives and 100% for the next 500 lives. Comparing the LC to the Reverse LC, the former has a third of simulated lives facing benefit shortfalls while the latter has less than a fifth of total simulations facing benefit shortfall. The rate of increase in shortfall levels is similar for both strategies but the Reverse LC is slightly steeper than the LC. At 50% of the benchmark shortfall, there are over 2,000 lives experiencing shortfall for the LC strategy whilst the Reverse strategy has about half this number of lives experiencing shortfall. About 1% of both strategies face total benefit ruin. The Growth and Balanced strategies have very little variation in terms of shortfall to the benchmark. Shortfall occurs in less than 20% of the simulations for both strategies. The extent of shortfall rises steeply to 100% among the first 1,000 lives to experience shortfall in both strategies, with the Growth strategy having a steeper rise in shortfall than the Balanced strategy.

Measuring the severity of shortfall based on how much income from the various strategies falls short of the benchmark annuity income, there is clear evidence of outperformance of the benchmark by stock dominating portfolios. The Aggressive, Growth, Partial LC, the Reverse LC as well as the Balanced strategies have up to 80% of terminal wealth levels providing wealth levels which are equal or higher than the benchmark income. The steep shortfall curve shows that although severity of benefit shortfall for the stock dominating strategies reaches 100% in all cases, a lower number of lives are affected, reducing the probability of extreme shortfall. The defensive strategies provide less protection for retirees who hope to meet a level of income generated from their terminal wealth. Shortfall is more prevalent for these strategies, with the concave shortfall curve showing that many lives experience increasing shortfall levels although the total shortfall level is reached only by a few lives.

5.4.4 Further Analysis

Whilst the stock dominated strategies are beneficial for strong upside potential, they appear to be very volatile, affecting the extent of ruin on the downside. The more conservative strategies are more stable and have lower mean ruin levels. To take this into perspective, I consider investor's preference of strategy not only based on return (ending portfolio wealth) and the risk associated with this level of return (the standard deviation of ending portfolio wealth). I use a utility-of-terminal wealth approach assuming individuals have preferences over wealth at retirement. Since the individual's ultimate concern is that her retirement wealth would be able to provide income which is equal to or exceeding a sustainable income benchmark, I use a utility measure that accounts for a threshold level of wealth. This measure is in line with work by Poterba, Rauh, Venti, and Wise (2007) and is represented as:

$$U(W) = \frac{(|W - K|)^{1-\gamma}}{1-\gamma} if W > K \tag{5.3}$$

$$U(W) = -h\frac{(|W - K|)^{1-\gamma}}{1-\gamma} if W < K \tag{5.4}$$

Where
W is wealth,
γ is the relative risk aversion parameter,
h is the loss aversion parameter and K is the threshold wealth for sustenance.

The utility function is consistent with Kahneman and Tversky's (1979) prospect theory with agents framing their choices in terms of gains and losses or deviations relative to a threshold level rather than levels of wealth. The utility's value function is generally concave on the region of gains and convex in the region of losses. If the retirement wealth is above the referenced wealth level, the constant relative risk aversion (CRRA) utility applies. If the retirement wealth falls below the referenced wealth level, the CRRA utility is adjusted to incorporate the loss aversion parameter

h, allowing for steeper convexity as the individuals are more sensitive to losses and hence heavily punish such outcomes.

I calculate the utility levels for each of our 10,000 hypothetical lives, $U(Wi)$, and I derive the probability weighted expected utility as the expected utility for the strategy.

$$E[U(W)] = \sum_{i=1}^{N}(U(W_i)) \tag{5.5}$$

where N is the total number of simulated return paths. Based on simulations, the expected utilities from different strategies are compared at different threshold levels and ranked in Table 5.5.

TABLE 5.5: Ranking of Alternative Strategies based on Utility of Terminal Wealth

This table presents the ranking of alternative strategies for different RRA levels in the first column and different wealth threshold levels in the subsequent columns. We rank the eight alternative investment strategies based on the utilities of terminal wealth. The strategies are represented by numbers 1-8 as shown below the table.

RRA\THRESHOLD	$0	$500,000	$700,000	$900,000
2	2,8,3,6,4,5,7,1	3,8,2,4,6,5,7,1	3,4,6,8,2,5,7,1	4,3,6,8,2,5,7,1
3	2,8,7,5,3,6,4,1	3,8,2,4,6,5,7,1	3,4,6,8,2,5,7,1	4,3,6,8,2,5,7,1
5	8,7,6,5,4,3,2,1	3,8,2,4,6,5,7,1	3,4,6,8,2,5,7,1	4,3,6,8,2,5,7,1

1 ↦	Conservative
2 ↦	Balanced
3 ↦	Growth
4 ↦	Aggressive
5 ↦	LC
6 ↦	PLC
7 ↦	PRLC
8 ↦	RLC

With zero wealth threshold, a retire will want to avoid ruin and her average utility is entirely based on her wealth level and the risk associated with such outcome. I find retirees to prefer the balanced strategies, that is, the Balanced and Reverse LC plans, to the more conservative and aggressive strategies at low risk averse levels. These two strategies begin investment with an equal allocation to stock and bonds and the latter ends with a full allocation to stocks. These strategies not only have low chances of ending in negative balances but also very high wealth levels. The Conservative and Partial Reverse LC strategies, although with lower standard deviations, end in lower wealth balances and have chances of not meeting the threshold. These two represent the least preferred strategies at low risk aversion level of 2. At high levels of risk aversion, the Reverse LC is the dominant investment strategy among the alternatives. The strategy is attractive to high risk averse retirees as it provides low risk (ruin probability) for its high level of terminal wealth. The Partial Reverse LC dominates the Conservative strategy at all levels of the risk parameter and becomes increasingly attractive with increasing risk aversion, becoming the second ranking alternative at high risk aversion levels. Although the average wealth level for this strategy only exceeds the wealth level provided by the Conservative strategy, a wealth threshold of zero makes the wealth size less relevant with its superior ruin levels. The Balanced and Conservative strategies are the least preferred strategies at high levels of risk aversion.

When I include thresholds of different levels other than zero, I find systematic changes in the preferences of retirees based on their terminal wealth distributions. At a moderate threshold of $500,000, I find the Growth strategy to dominate the alternative strategies. The Reverse LC strategy is the next preferred of the remaining strategies; next in ranks is the Balanced strategy. When I increase the threshold to $700,000, the Growth strategy continues to dominate, the Aggressive strategy however becomes increasingly attractive and is the second ranked strategy. The Partial LC is the third preferred strategy, increasing in ranks from the previous threshold. At a high threshold of $900,000, I find the Aggressive strategy to top the alternative strategies. The Growth strategy is moved into second place, while the Partial LC and the Reverse LC strategies are the subsequently ranked strategies respectively as the threshold is increased.

The Balanced strategy is less attractive to retirees as threshold level increases. This strategy is however preferred to the LC and the Conservative strategies at all threshold levels. The Conservative strategy is the least preferred strategy at all threshold and risk aversion levels in terms of the utility-of-terminal wealth. The LC only ranks higher than the Balanced strategy and the more aggressive portfolios, thus the Reverse LC, Growth, Aggressive and Partial LC, when there is zero threshold and the retiree has a risk aversion level exceeding 2. With the inclusion of a threshold level, the LC is dominated by these strategies. At all threshold levels, the Reverse LC ranks higher than the LC. The Conservative is the least preferred strategy even at zero threshold.

Using utility-of-terminal wealth, The Balanced strategy which is static at 50/50 stock to bonds/bills is the preferred strategy at zero threshold. The more aggressive strategies with higher stock levels are next in preference while the conservative strategies with low or no stock levels are the least preferred strategies. When I include a threshold which provides a minimum income level needed for sustenance, the Balanced strategy is no longer the preferred strategy. I find the Growth strategy to be the preferred alternative at moderate to increasing thresholds. The Aggressive and remaining strategies with substantial stock components are preferred to the Balanced and conservative strategies. At high wealth threshold levels, the Aggressive strategy, which has a 100% allocation to stock, is dominant. The remaining strategies with substantial stock allocation are preferred to the Balanced, with the latter preferred to the conservative strategies.

5.5 Conclusion

Investment managers seek to grow their clients' portfolios in order to meet their income and expense needs in retirement. These needs may include meeting a minimum sustenance level and living a comfortable lifestyle in retirement. Our analyses show that the dependence on the lifecycle style investment creating an illusion of security in retirement should be given a second look. The lifecycle strategy, where investment managers apply a holistic approach of decreasing retiree's investment in stocks or risky assets with increasing age is found to underperform several hybrid strategies which are more aggressive or increasing in stocks. Utility-of-terminal wealth analysis including loss aversion shows the lifecycle style strategy to underperform more aggressive static strategic asset allocation models as well as balanced and investments strategies with periodic increases in stock given different wealth thresholds.

Whilst increasing stock levels of portfolios significantly increases the wealth output for the right tail of the distribution, there exists a chance of ruin which results from the overreliance on positive returns of volatile assets. At a 4% level of annual withdrawal, this chance of ruin is up to 6% for an all stock portfolio, giving the retiree a 94% chance of having a positive ending portfolio with a good potential of this level being very high. For retirees interested in high terminal wealth and willing to take up a 6% less certainty in having a positive outcome, they are rewarded with high terminal wealth levels. Comparing this to the Conservative strategy, which is an all bonds/bills allocation, the retiree faces up to 2% chance of ruin but up to 6 times lower the average wealth level. The Aggressive strategy dominates the Conservative in 84% of our simulations.

At the widely accepted retirement wealth withdrawal level of 4%, I find a more aggressive strategy such as the Reverse Lifecycle to dominate the Lifecycle by 87% based on the distribution of terminal wealth levels. Terminal wealth estimates of stock dominating portfolios emphatically dominate the remaining strategies, with strategies starting with 100% investment in defensive assets being the worst performers, followed by the Lifecycle strategy. In terms of meeting a benchmark income level, again I find the stock dominating portfolios to outperform the benchmark in up to 80% of our simulations. The shortfall, which is observed in a fifth of the simulations, although severe in up to the 5th percentile, the probability of severe shortfall is low. The defensive strategies however, more often are unable to meet the set benchmark income with up to 60% of the Conservative strategy holders and 30% of the LC respectively failing to meet the income benchmark.

Lastly, considering the utility-of-terminal wealth, I rank the alternative strategies based on retirees' preference to meet certain wealth thresholds in retirement. The utility function enables retirees to set for themselves a minimum wealth threshold level and penalise themselves for wealth levels below this threshold. When retirees are only interested in ending in positive wealth and the only risk is to guard against portfolio ruin, the Balanced strategy which holds 50% in growth assets and the remaining in defensive assets is the preferred strategy. Retirees place more weight on the Balanced strategy for the high certainty in retaining positive wealth levels coupled with the high wealth values. The Reverse Lifecycle strategy, which increases allocation to stock over time, is the second preferred strategy. At wealth threshold of \$500,000, the Growth strategy is the dominant investment strategy among the alternatives with the Reverse Lifecycle coming in second. The Growth strategy continues to dominate the remaining alternatives at a threshold of \$700,000, closely trailed by the Aggressive strategy. At thresholds beyond this level, the Aggressive strategy is the preferred investment style with the Growth coming in as the second preferred strategy. At all thresholds, strategies with substantial stock allocation are preferred to strategies with low stock allocation. The Reverse Lifecycle strategy that is increasing in stock is preferred to the Lifecycle style strategy decreasing in stock at all wealth threshold levels.

I believe that the Lifecycle strategy with decreasing allocation to stock with increasing age needs to be improved. As Baby Boomers move from the accumulation phase to the decumulation phase and with the change in Defined Benefits to Defined Contribution plans, it is necessary to develop more sophisticated models that will help investors meet their needs in retirement. In this era of market uncertainty, such a model should be able to yield return to grow a retiree's wealth and provide her with adequate income to live comfortably and should be tailored to the needs of the individual investor.

The failure of the Lifecycle strategy to compete with alternative strategies illustrates the need to grow wealth in retirement. There is the perception of bonds as an absolute safe asset which helps to protect retiree's wealth position from unexpected shocks. This is however detrimental to the retiree's investment as the stress of periodic income withdrawal demands that the retiree's portfolio be managed and increased even in retirement. The possibility of recovering the portfolio after a long period of poor returns is also severely limited for the Lifecycle strategy as all stock is sold out while reverse Lifecycle strategies are able to take advantage of the markets when they recover, decreasing the chances of ruin for such strategies.

With this finding of high and increasing equity allocations performing better in retirement, the present challenge is to model products that will be attractive to retirees. Financial planners and advisors are reluctant to make such suggestions to retirees because of the perceived inherent risk. This is a major limitation to this finding as its application has practical difficulties and behavioural biases on the part of retirees to overcome.
Secondly, the use of historical returns to mimic future returns assumes that future returns will not be significantly different from past returns. Although this is an adequate representation, current low interest rates tell a different story. While the simulations incorporate previous market crisis and acknowlwdges the possibilities of these recurring in future, future interest rates and asset returns may vary significantly from what has been observed in the past. Long periods of low interest rates, low returns and low yields may significantly impact on the results of this study and more importantly on the investment decisions of retirees.

Appendix

5.A Portfolio Ruin and Income Shortfall Analysis

Table 5.A.1: Portfolio Ruin Count

The count of occurences of portfolio ruin for alternative strategies at various income withdrawal rates.

	Conservative	Balanced	Growth	Aggressive	RC	PRC	PRRC	RRC
3 PERCENT	2	1	34	207	1	140	0	2
4 PERCENT	167	66	271	614	73	494	12	112
5 PERCENT	2081	447	745	1163	636	1031	568	529
6 PERCENT	6062	1623	1690	1913	2445	1898	4345	1679
7 PERCENT	8462	3732	3079	2986	5336	3058	7622	3588

Table 5.A.2: Income Shortfall Count

The count of occurences where simulated retirees' income level generated from the terminal wealth after 20-year investment horizon are less than the expected income from the SPIA purchased at retirement age 65 for 10,000 simulations.

	Conservative	Balanced	Growth	Aggressive	RC	PRC	PRRC	RRC
3 PERCENT	3953	654	834	1185	1298	1091	1344	734
4 PERCENT	6712	1743	1660	1859	3292	1885	4261	1720
5 PERCENT	8470	3368	2741	2675	5670	2869	7071	3165
6 PERCENT	9344	5353	4031	3579	7599	3952	8692	4981
7 PERCENT	9741	7006	5391	4645	8868	5109	9473	6754

Chapter 6

Conclusions and Future Work

6.1 Summary of Research

This thesis explores an important phase in retirement planning that is otherwise less discussed, the decumulation phase. In particular, I analyse investment strategies available to retirees in their retirement drawdown years. My first study explores a retiree's withdrawal strategy choice when she has access to pensions that depend on her available wealth level, such as the Means-Tested Age Pension in Australia. Previous studies in this context have made reference to the CRRA utility, which assumes EIS to be the inverse of RRA and have largely ignored the impact of pensions on retirement drawdown choice. These studies find the Percentage of Wealth strategy, where the retiree consumes a fixed percentage of her available wealth at all drawdown periods to be the dominant strategy. Recent studies have shown the relationship between RRA and EIS to be different from directly inverse with individuals preferring early resolution to late resolution of uncertainty. This study models retiree's preferences based on Epstein-Zin utility, which enables a disentangling of the two preference parameters and incorporates the impact of pensions. I find that expected utility decreases as RRA levels increase for any given level of EIS. I find that the decrease in utility for increasing RRA is steep for the Annuity, decreasing in steepness for the Fixed, Percentage of Wealth through to the Life Expectancy strategy in that order. Retirees' preference for phased withdrawals over annuities is an exhibition of RRA levels higher than the inverse of EIS and this is consistent with prediction by recursive preferences.

For the range of portfolios considered in the analyses, I find that increasing the level of equities, from conservative through balanced to a growth portfolio increases the chances of sustainability for the long horizon investment and vice versa. Whilst the chance of ruin is minimised for the Fixed withdrawal strategy, the average withdrawals and terminal values for all the alternative phased withdrawal strategies increase with increasing equity allocation. This finding is in agreement with works such as Cooley et al. (2001) who assert that a retirement fund should have at least 50% allocation to equity to be sustainable. Asset allocation does not change the order of preference for the alternative strategies. The Life Expectancy withdrawal plan, where the retiree consumes a fraction of wealth proportional to her remaining life expectation is the preferred drawdown choice across a broad range of RRA and EIS parameters, with the Percentage of Wealth strategy being the second preferred strategy and the Fixed strategy being the least preferred phased strategy; preferred only to the life annuity.

In the presence of pensions, I find the Life Expectancy withdrawal to dominate the other withdrawal strategies, with the Percentage of Wealth withdrawal being preferred at low RRA and high EIS. Overall, Life Expectancy withdrawal dominates for a wide range of RRA and EIS levels, and the second preferred strategy is the Percentage of Wealth. The Fixed withdrawal strategy is next in the ranks, with the Annuity being the least preferred strategy. In the absence of pension income however, the Life Expectancy withdrawal continues to dominate the other withdrawal strategies with the Fixed withdrawal strategy preferred at low risk averse levels. The Fixed withdrawal also becomes the second preferred strategy after the Life Expectancy for a broad range of RRA and EIS. The absence of pensions shifts plan dominance to the Fixed Withdrawal over the Percentage of Wealth withdrawal. I find the inclusion of a pension model such as the means-tested Age Pension included in this study to serve as a buffer for retirees' withdrawals, increasing with less disposable wealth and vice versa. This makes high volatile wealth dependent strategies such as the Percentage of Wealth attractive. The absence of pensions reveals the extent of volatility and in many cases, the low levels of withdrawals for retirees on this strategy when returns are low. On the other hand, the absence of pensions shows the stability from the Fixed withdrawal, which is useful for retirement planning, making it more attractive for retirees compared to the Percentage of Wealth strategy. This finding makes the presence of pensions a significant determinant of retiree's choice of drawdown strategy in retirement.

Again, I find that retirees' choice of alternative withdrawal plan depends on their decision to bequeath. Retirees choose annuities over phased withdrawals only if they have no bequest motives. When retirees have no bequest motives and derive all their utility from their withdrawals, they prefer stability in consumption over varying and volatile consumption. Annuities dominate the phased withdrawal strategies, with the

Fixed withdrawal being the most attractive phased withdrawal strategy. The presence of pensions make the Fixed withdrawal more attractive for moderate to high risk averse retirees as it increases the average withdrawal level with minimum change in standard deviation and the chance of ruin is eliminated by the pensions. The two least preferred strategies at high risk averse levels, the Life Expectancy and Percentage of Wealth withdrawals have high deviations about the mean consumption level even in the early years of retirement. In the absence of bequest motives however, the Life Expectancy and the Percentage of Wealth withdrawals are the dominant phased withdrawal strategies for the long term.

The second study, set in the Australian context, examines the benefits of including deferred annuities in a retirement decumulation plan. While deferred annuities are unavailable in Australia for taxation reasons, their introduction in the annuity market and legislation regarding this is imminent. Previous studies on deferred annuities have drawn from the US and Germany among other well-developed annuity markets. While these studies tout the benefits of the deferred annuity, they are silent about the appropriate length of the deferral period or when these annuities should be purchased. This study analyses this gap in the presence of retirees possessing bequest motives or capital preservation needs and having HARA preferences. I find that risk averse retirees derive more utility in purchasing deferred annuities over immediate annuitisation and self annuitisation in the decumulation phase.

Low risk averse retirees will choose self-annuitisation strategy with downside constraints over a simple percentage of wealth withdrawal, allowing consumption to vary over a broad range of values yet ensuring that consumption remains above a sustenance level. Compared to the benefits of annuitising, I find retirees to prefer purchasing deferred annuities at retirement age to provide higher and more utility-enhancing consumption to any of the two self-annuitising strategies discussed in the study. I also find that even at high levels of risk aversion, the retiree is better off purchasing deferred annuities than self-annuitisation if she is interested in the possibility of obtaining superior consumption levels leading to superior terminal utilities. In the case of annuity deferral periods, I find that risk averse retirees prefer a shorter timeframe to a longer one. At age 65, a retiree prefers a 10-year deferred annuity to a 15-year deferred annuity, and a 15-year deferred annuity to a 20-year deferred. Retirees prefer to purchase deferred annuities with longer deferral periods only at retirement age 65 and derive less utility when they purchase annuities with deferral periods exceeding 10 years later in retirement. For two strategies that both begin payments at age 85, a 20-year deferred annuity purchased at age 65 is preferred to a 15-year deferred annuity purchased at age 70. Since there is no benefit should the retiree die during the deferral period, retirees are reluctant to purchase annuities with lengthy deferred periods late in retirement. Instead, retirees choose a short deferral period where they believe their conditional survival probabilities are still high enough to survive and receive the annuity payouts.

I find that the option to delay annuitisation exists for retirees of all relative risk aversion levels within the range used in this essay. My analysis shows that while risk averse retirees choose to annuitise in retirement, the timing of annuitisation depends on their level of risk aversion. The low risk averse retirees choose to annuitise later than the retirement age of 65. They gain utility in the expectation of stochastic improvements in their wealth levels through investing assets in the money market for the first few years in retirement. Since their consumption in the years before the annuity payments begin is a direct function of wealth level, the low risk averse retiree will bet on increasing consumption through the money market in the early years of retirement while they still have high survival probabilities. There is less option to delay annuitisation, however, for more risk averse retirees. They prefer early annuitisation to guarantee their income instead of betting on the equities market in anticipation of equity premium. The option to delay annuitisation is more valuable in the case of no exogenous pension income than when government pensions are available. In the absence of pension income, retirees prefer to self-annuitise longer with the possibility of increasing their wealth before eventual partial annuitisation later in retirement. Low to medium risk averse retirees will prefer annuitising at age 70 or age 75 to annuitising at age 65. I find the presence of pension income to be an incentive for retirees at all risk levels to transfer risk to life companies and benefit from the longevity protection provided by the pension income and the increased consumption from annuitisation.

Finally, when retirees have no bequest motives, they prefer annuitising earlier in retirement to later. Annuities of varying deferral lengths at retirement age 65 are preferred to annuitising at age 70, while the latter is preferred to annuitising at age 75. At low utility thresholds, the self-annuitisation strategy performs better than annuitisation at age 75 although this is reversed for high thresholds. The certainty of the government pensions makes self-annuitisation attractive as retirees are able to increase their consumption without losing utility from lowering bequest levels.

The third study examines the performance of the commonly nominated default pension plan, the lifecycle fund against other alternative investment strategies in the decumulation phase. I compare the terminal wealth levels of the alternative strategies after 20 years of investment with periodic income drawdowns of rates between 3% and 7% per annum. I also examine the chances of ruin and the extent of ruin for the alternative strategies and rank them accordingly. I find under different shortfall measures that the lifecycle fund in retirement consistently underperforms other investment plans which hold substantial stock allocations as well as plans that increase in stock over time. The Lifecycle plan only outperforms the all-defensive fund, comprised of bills and bonds. I perform further analysis using utility functions motivated by prospect theory, including the influence of loss aversion, in which retirees penalise themselves for not attaining certain wealth thresholds.

Whilst increasing stock levels of portfolios significantly increases the wealth output for the right tail of the distribution, it creates a chance of ruin which results from the overreliance on positive returns of volatile assets. At a 4% level of annual withdrawal, this chance of ruin is up to 6% for an all stock portfolio. For retirees interested in high terminal wealth and willing to take up a 6% less certainty in having a positive outcome, they are rewarded with high terminal wealth levels. Comparing this to the Conservative strategy, which is an all bonds/bills allocation, the retiree faces up to 2% chance of ruin but up to 6 times lower the average wealth level. The Aggressive strategy dominates the Conservative in 84% of our simulations based on terminal wealth levels after 20 years investment in retirement. The Reverse Lifecycle strategy which has increasing allocation to stock over time dominates the Lifecycle by 87%. Terminal wealth estimates of stock dominating portfolios emphatically dominate the remaining strategies, with strategies starting with 100% investment in defensive assets being the worst performers, followed by the Lifecycle strategy. In terms of meeting a benchmark income level, which in this case is a fixed life annuity purchased at retirement age 65, again I find the stock dominating portfolios to outperform the benchmark in up to 80% of our simulations. The shortfall, which is observed in a fifth of the simulations, although severe in up to the 5th percentile, has a lower probability of occurring relative to strategies with lower stock allocations. The defensive strategies however, more often are unable to meet the set benchmark income with up to 60% of the Conservative strategy holders and 30% of the Lifecycle respectively failing to meet the income benchmark.

Lastly, considering the utility-of-terminal wealth, I rank the alternative strategies based on retirees' preference to meet certain wealth thresholds in retirement. The utility function enables retirees to set for themselves a minimum wealth threshold level and penalise themselves for wealth levels below this threshold. The setting of thresholds may be useful when the retiree hopes to generate certain income levels in retirement or meet aged-care costs later in life. When retirees are only interested in ending in positive wealth and the only risk is to guard against portfolio ruin, the Balanced strategy which holds 50% in growth assets and the remaining in defensive assets is the preferred strategy. The Reverse Lifecycle is the second preferred strategy. At a wealth threshold of $500,000, the Growth strategy which has 70% of retiree's investment allocated to stock is the dominant strategy. This strategy dominates at the moderately high threshold of $700,000; at wealth thresholds exceeding this level, the Aggressive strategy, which is an all-stock portfolio is preferred. The Growth strategy becomes the second preferred strategy at high wealth thresholds. At all wealth threshold levels, strategies with substantial stock allocation are preferred to strategies with low stock allocation. The Reverse Lifecycle strategy that is increasing in stock is preferred to the Lifecycle style strategy decreasing in stock at all threshold levels in this study.

6.2 Policy Implications

The choice of drawdown strategy has several implications for governments, policy makers and retirees as a whole. With the decrease in mortality and constantly improving life expectancy due to improved health systems and increased awareness, planning for a longer retirement horizons has become particularly important. Governments face the task of deciding on policies such as compulsory annuitisation or appropriate mandatory drawdown rules for retirees to ensure that they do not outlive their savings and depend solely on government support. According to Financial Services Inquiry's (FSI) interim report,[1] the (Australian) retirement phase of superannuation is underdeveloped and does not meet the risk management needs of many retirees. The inquiry wants to explore whether rules should be introduced to encourage retirees to buy retirement income products, introduce a default 'investments' option for retirees or mandate the use of certain retirement income products such as annuities (Yeates, 2014)

The impact of pensions on retirement drawdown choice is also relevant for governments and policy makers as full government-funded policies such as the Age Pension are increasingly becoming unsustainable. The need for regulations regarding pension

[1]The third government review into Australia's financial system lead by former Commonwealth Bank chief, Mr David Murray. It is 16 years since Mr Stan Wallis AC led a similar inquiry.

allocation should be openly discussed as it directly affects retirement planning and the retiree's choice of drawdown strategy. This study makes an important contribution in this respect, specifically, with regard to the retirees' preference for the Life Expectancy over a wide range of preference parameters and reasonable utility expectations. This means the Australia system may be able to learn from the United States, who uses this strategy as a 'default' drawdown strategy in defined contribution plans including Individual Retirement Accounts (IRAs).

The findings of the second study also present some important policy implications. Firstly, despite the finding that deferred annuities are preferred to immediate and self-annuitisation, delaying the purchase of deferred annuities is utility enhancing and public policies regarding decumulation legislations need to consider this. Compulsory annuitisation even with deferred annuities may lead to substantial utility losses if not implemented at the right time. However, mandatory annuitising after a period of phased withdrawal may be appealing for a wide range of risk averse individuals. While compulsory immediate annuitisation is perceived to give poor value, annuitising after age 85 is better value. In this respect, the promotion of deferred annuities such as those discussed in this study and beginning payment late in retirement will be of immense value to retirees and provide an effective decumulation option in retirement.

Secondly, the introduction of deferred annuities on the Australian market is necessary for innovative and sustainable decumulation plans and provides a utility enhancing alternative to self-annuitisation options. It also serves the interest of retirees who have substantial bequest needs as it allows them to receive the benefits of longevity insurance through annuitisation without necessarily fully annuitising all their available wealth.

Financial planners and investment managers look to help their clients meet their objective of living a comfortable retirement; avoiding portfolio ruin and drawing down a sustainable income level from their wealth periodically. Based on this premise, I argue that the popular default strategy, the Lifecycle plan, is flawed. For several reasons discussed in this thesis, many retirees do not exercise their investment options and rely on the default strategy as an endorsement by the plan provider. Using a dataset of daily market returns on stocks, bills and bonds spanning the last 85 years, there is strong evidence to show that a lifecycle style investment underperforms several alternative investment plans under common shortfall measures. Default plans in retirement savings accounts therefore need to be improved; the conventional wisdom supporting the lifecycle style investment as being suitable in the retirement phase should be looked at more critically for the benefit of the majority of retirees.

6.3 Limitations and Avenues for Future Research

A number of issues regarding this thesis require further attention. Firstly, I make extensive references to life annuities throughout this thesis. While the annuity pricing estimation may be accurate in the Australian context, I acknowledge the actuarially unfair market due to adverse selection as well as mortality heterogeneity may result in some variations when this study is implemented in different regions and populations. Also, as expressed in Section 3.5, I assume the annuitisation decision is made at retirement and compare this to other options that are selected at retirement age by the retiree. Future studies could price annuities at all ages in retirement, giving the retiree the choice of annuitising at any given age. Furthermore, the assumption of the female survival probabilities in the computing of annuity rates may be 'unfair' to males as they possess lower survival probabilities, have lower life expectations and may be entitled to higher annuity income levels. Alternatively, future studies may use unisex life tables which use the average of survival expectations for both sexes, albeit with the risk of underestimating the length of female life expectation.

Secondly, I assume a retiree's portfolio is made up of her savings in the DC plan and make no provision for wealth outside retirement fund. Future research may include the role of savings outside the retirement plan as this may have some influence on a retiree's drawdown plan and ultimately her asset allocation. Home ownership, ownership of other assets and income from alternative sources may also influence individual investment decisions. In the context of bequest motives, wealth outside retirement plans may play a significant role in meeting this need; making the DC plan savings insensitive to the retiree's desire to leave an estate.

Finally, the role of human capital in the allocation of retiree's investable assets is not considered in this thesis and may be an avenue for further study. For the purpose of this thesis, I assume individuals retire at age 65. While people work well beyond this age, others work part-time or casually after age 65 which may impact their decisions on asset allocation and investments. Human capital refers to the net present value of an individual's future labour income. Labour income is assumed to be a bond-like asset which is fairly stable over time and may be significant in an individual's asset allocation selection. Young plan members are likely to have high significant human capital and lower human capital as they age; and compensate for this bond-like asset by holding high levels of stock at younger ages and decreasing the allocation to stocks over time. This seems to be an appealing justification for the lifecycle style in pension plans. The riskiness of such human capital and its correlation with the financial asset is still being explored in economic literature. There are also no quantitative studies exploring the human capital effect in the decumulation framework and this remains an avenue for future research.

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