Open Pit Block Sequencing Optimization: A Mathematical Model and Solution Technique

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Abstract: This study presents a comprehensive mathematical formulation model for a short-term open-pit mine block sequencing problem, which considers nearly all relevant technical aspects in open pit mining. The proposed model aims to obtain the optimum extraction sequences of the original-size (smallest) blocks over short-time intervals and in the presence of real-life constraints, including precedence relationship, machine capacity, grade requirements, processing demands and stockpile management. A hybrid branch-and-bound and simulated annealing algorithm is developed to solve the problem. Computational experiments show that the proposed methodology is promising to provide quantitative recommendations for mine planning and scheduling engineers.

Keywords: Mine Optimization, Block Sequencing, Hybrid Heuristic, Simulated Annealing

1. Introduction

Open-pit mining is a type of surface mining which is used to extract near-surface minerals. In mining optimization process, a deposit of interest is divided into thousands of three-dimensional (3D) rectangular cubes called a block model. In a pre-optimization stage, a set of attributes is estimated for each block and it is assumed that the attributes are homogeneously distributed and given for each block.

According to the specification of blocks, a destination is assigned to each individual block. In an open pit mine, a destination can be a mineral processing plant, a stockpile or a waste dump. The destination assignment can be done thorough a static procedure such that the accepted ranges of attributes are defined for each destination and eligible blocks are assigned to this destination. This static approach suffers from ignoring blending, processing demands consideration and grade-tonnage distribution of mined blocks (Asad and Topal 2011; Johnson et al. 2011). In a more flexible approach, which is called dynamic destination assignment, destinations of blocks are dynamically determined while the processing plants requirements, grade-tonnage distribution and economical parameters are considered.

This article addresses a real-world problem which is called open-pit mine block sequencing (MBS) problem. The MBS problem can be defined as specifying the sequence in which blocks should be removed from the mine and allocated to the appropriate destination. The key components of the MBS model are: blocks, processing circuits, stockpiles, excavators, time horizons and mining rules.

The considered stockpiles in this study are blending and mixing stockpiles used to blend material with a particular characteristic with other material to improve recovery of processing. A time horizon in this article is about three to six months, in which blocks are sequenced for weekly or fortnight periods. The main mining rules are: minimum required space for excavators; the extraction direction; excavator's working territory; drop-cut considerations; waste extraction priority; and the number of active benches.

The literature review demonstrates that there are two main gaps in the previous studies in the area of the block sequencing problem. The first gap is to model the MBS for a short term horizon and the second gap is the lack of efficient solution approaches which are suitable for this problem. Several studies have addressed the MBS problem over the life of mine. Caccetta and Hill (2003) presented an MIP model of the long term MBS problem and developed a branch and cut algorithm to solve the problem. Chicoisne et al. (2012) developed an efficient heuristic approach to solve a real-world MBS case. Lamghari and Dimitrakopoulos (2012) presented a tabu search metaheuristic for an extended MBS problem with the consideration of metal uncertainty. Espinoza et al. (2013) presented a library of benchmark MBS instances called MineLib to mining community. Lambert and Newman (2014) employed a tailored Lagrangian relaxation in the MBS formulation. Lambert et al. (2014) concluded a tutorial of fundamental MBS mathematical formulation models. Shishvan and Sattarvand (2015) developed an ant colony optimization metaheuristic to solve an extended MBS problem for a Copper-Gold mine. Lamghari et al. (2015) developed a two-phase approach to solve MBS, in which the first phase is to generate the initial solution by a series of LP models and the second phase is to apply a variable neighbourhood search procedure to improve the initial solution. Liu and Kozan (2016) developed two state-of-the-art graph-based algorithms to efficiently solve large-scale benchmark MBS instances from MineLib. To see more related papers in the area of long term MBS problem, readers are referred to see review papers by Osanloo et al. (2008), Newman et al. (2010) and Kozan and Liu (2011).

Compared to the sequencing over life of mine, following main considerations should be simultaneously taken into account for a short time horizon (Whittle 2011):

- operational mining rules should be considered in the model;
- the destination of mined material should be determined dynamically and the blending should be allowed;
- stockpiles should be included in the production circuit as they are used to feed the process circuits in some periods; and
- Block extraction should be determined at the level of the original block size to keep the resolution and accuracy of grade estimation.

Eivazy and Askari-Nasab (2012) developed a model based on the aggregated blocks for MBS problem and used the TOMLAB/CPLEX package in which branch-and-cut algorithm is implemented to solved the problem. In the aggregation approach several blocks are combined and larger units (aggregates) are created (Ramazan 2007). An aggregate acts as a single large-size block, therefore a homogenous grade is assigned to it. The aggregation approach decreases the size of the problem as well as the complexity of the problem in terms of precedence relations. However, assigning a unique grade to a large-size aggregate (which is a combination of several blocks) decreases the resolution and the granularity of the grade estimation (Cullenbine et al. 2011). The aggregation method is used in most of commercial mine planning software packages (Runge PincockMinarco Limited 2013; Huang et al. 2009; Minemap Pty. Ltd 2013; Minemax Pty Ltd 2012). When the solution of aggregation approach is decomposed to the original blocks, the obtained solution may not be optimal and sometimes it may be even infeasible (Boland et al. 2007). L'Heureux et al. (2013) developed a MIP model for short-term production optimization in open-pit mines where the excavator movements have been integrated into the block sequencing problem. They used the ILOG CPLEX to solve the problem and pointed out that this problem cannot be solved in reasonable time by standard solvers. Kumral (2013, 2012) formulated block sequencing problem and considered block destination as a decision variable in MIP formulation of the problem and applied a simulated annealing metaheuristic algorithm to solve this problem. Mousavi et al. (2014) developed a MIP model to optimize block sequencing over a short time horizon. Numerical investigations indicated that the industry-scale MBS instances are intractable for standard MIP solvers. Groeneveld and Topal (2011) discussed the application of stockpiles in mining operations and proposed a binning approach in which each virtual bin has a maximum and a minimum grade limits on the entrance ore mineral. Singh et al. (2013) developed an optimization tool for medium-term rail scheduling for iron ore mining in Western Australia. As discussed, considering blending stockpile in mining operation introduces non-linearity in the MIP models of optimization problems. Kozan and Liu (2015) proposed a new multi-resource multi-stage mine scheduling model for optimizing the open-pit drilling, blasting and loading operations.

This paper aims to formulate a more comprehensive model to involve nearly all relevant technical and practical aspects of the MBS problem and develops an efficient hybrid heuristic algorithm to solve the problem. The proposed MBS model simultaneously optimizes the block extraction, blending, grade control, stockpiling and destination determination. The proposed model is presented at the operational level to keep the processing selectivity and can be applied for open-pit hard rock mining (e.g., iron, copper, gold). As the industry-scale instances of the problem cannot be solved by standard solver in reasonable time, a hybrid heuristic algorithm includes simulated annealing (SA), branch-and-bound (B&B), and large neighbourhood search (LNS) is proposed.

The reminder of this paper is organized as follows: Section 2 describes the block sequencing problem and presents the mathematical formulation. The proposed solution approaches are presented in Section 3. Section 4 discusses computational experiments. A detailed application of proposed methodology for a real mine operation are given in Section 5. Finally, Section 6 contains the conclusion of this study and recommends future work in this field.

2. The MBS problem definition and formulation

The MBS problem is defined to determine the sequence in which blocks should be extracted such that stockpiling costs including rehandling and holding costs are minimized and all physical and tactical constraints such as precedence relationships, mining capacity, processing demands and grade requirements are satisfied. In addition, the solution of MBS should obtain the optimum material flow from mine to processes and stockpiles, and from stockpiles to processes.

Indices

t: Time period index, $t = 1,2,\dots,T$.

i: Block index, $i = 1,2,\dots,I$.

m: Machine index (e.g., excavator, shovel, loader), $m = 1,2,\dots,M$.

d: Destination index, $d = 1,2,\dots,D$.

ρ: Processing (mineral processing plant or mill) index, $\rho = 1,2,\dots,P$.

s: Stockpile index, $s = 1,2,\dots,S$.

w: Waste dump index, $w = 1,2,\dots,W$.

α: Attribute (grade) index, $\alpha = 1,2,\dots,\mathcal{A}$.

Parameters

v_i: Volume of block i (cubic meter).

b_i: Tonnage of block i (tonne).

g_i^α: Percentage of attribute α of block i.

Γ_{p_i}: Set of immediate predecessors of block i.

Γ_{af_i}: Set of adjacent blocks in side f of block i.

Γ_{dc_i}: Set of blocks which should be extracted consecutively as block i, if block i is a drop-cut.

β_i: Swell factor of block i (%).

f_i: Fillability (fill factor) of block i (%).

E_m: Extraction capacity of machine m (cubic meters).

λ_m^t: Effectiveness of machine m in period t *(%).*

I_m: Blocks which are eligible to be extracted by machine m.

M_m^t: Minimum required mining production in period t.

M_ρ^{min}: Minimum capacity of processing ρ.

M_ρ^{max}: Maximum capacity of processing ρ.

$\varphi_{\alpha d}^{min}$: Minimum attribute α requirement at destination d.

$\varphi_{\alpha d}^{max}$: Maximum attribute α requirement at destination d.

ϕ_s: Safety inventory level of stockpile s.

M_s: Storage capacity of stockpile s.

S_ρ: Set of stockpiles which feed processing ρ.

I_s^0: Initial inventory of stockpile s.

c_h^t: Inventory holding cost of one tonne of material in period t.

$c_r^{s\rho}$: Rehandling cost for one tonne of material transferred from stockpile s to processing ρ.

c_p^i: Cost of processing waste block i in processing ρ.

c_w^i: Cost of sending ore block i to the waste dump w.

c_{dc}^i: Drop-cut cost for block i if block i is extracted by drop-cut.

Decision variables

$$x_{imd}^t = \begin{cases} 1 & \text{if block } i \text{ is extracted by machine } m \text{ in period } t \text{ and sent to destination } d\ (\rho, s, or\ w). \\ 0 & \text{otherwise.} \end{cases}$$

$z_{s\rho}^t$: Amount of material transferred from stockpile s to processing ρ in period t.

$$y_{if}^t = \begin{cases} 1 & \text{if block } i \text{ is extracted from side } f \text{ in period } t. \\ 0 & \text{otherwise.} \end{cases}$$

The objective

The objective of the MBS problem is to minimize the total cost, which includes rehandling and holding costs, misclassification and drop-cut costs. The misclassification cost is monitored to ensure that material is assigned to the right destination. Finally, a drop-cut cost is considered in order to give priority to the side cut extraction, unless a new working bench is required to be opened**.**

Minimize

$$\sum_{t=1}^{T}\sum_{s=1}^{S}\sum_{\rho=1}^{P} c_r^{s\rho} z_{s\rho}^{t} +$$
$$\sum_{t=1}^{T}\sum_{s=1}^{S} c_h^{t}\left(I_s^{0}+\sum_{r=1}^{t}\sum_{m=1}^{M}\sum_{i=1}^{I} x_{ims}^{r} b_i - \sum_{r=1}^{t}\sum_{\rho=1}^{P} z_{s\rho}^{r}\right) +$$
$$\sum_{t=1}^{T}\sum_{d=1,d\neq w}^{D}\sum_{m=1}^{M}\sum_{i=1}^{I} x_{imd}^{t} c_p^{i} + \sum_{t=1}^{T}\sum_{w=1}^{W}\sum_{m=1}^{M}\sum_{i=1}^{I} x_{imw}^{t} c_w^{i} + \sum_{i=1}^{I}\sum_{t=1}^{T} y_{i5}^{t} c_{dc}^{i} \quad (1)$$

A drop-cut is a condition such that a block is extracted while all the adjacent blocks have not been extracted yet, shown in Figure 1.a. Contrary to drop-cut, side-cut is performed when the excavator is located in a same bench as the block, shown in Figure 1.b.

Figure 1. a) drop cut, b) side cut.

Constraints

Constraint (2) enforces that the top-down precedence relations must be satisfied.

$$\sum_{r=1}^{t}\sum_{m=1}^{M}\sum_{d=1}^{D} x_{jmd}^{r} - \sum_{m=1}^{M}\sum_{d=1}^{D} x_{imd}^{t} \geq 0 \qquad \forall \{ i,j \in I \mid j \neq i;\ j \in \Gamma_{p_i}\};\ t = 1,2,\ldots,T. \quad (2)$$

Constraints (3) and (4) satisfy drop-cut and precedence relationships exist in a bench. In a real-life mining operation a block can be extracted from either one of four adjacent sides or by a drop-cut. Therefore, totally there are five constraints of which at least one should be satisfied. In-bench precedence relationships are considered in order to provide enough space before extracting a given block.

$$\sum_{m=1}^{M}\sum_{d=1}^{D} x_{imd}^{t} - \sum_{r=1}^{t}\sum_{m=1}^{M}\sum_{d=1}^{D} x_{kmd}^{r} \leq \left(1 - y_{if}^{t}\right) \quad \forall \{\, i, k \in I \mid k \neq i;\ k \in \Gamma_{af_i});\ f = 1,2,\dots,4;\ t = 1,2,\dots,T. \tag{3}$$

$$x_{imd}^{t} \leq \sum_{f=1}^{5} y_{if}^{t} \quad \forall\, i = 1,2,\dots,I; m = 1,2,\dots,M; d = 1,2,\dots,D; t = 1,2,\dots,T. \tag{4}$$

Constraint (5) ensures that if block i is extracted as a drop-cut, then a set of pre-determined blocks should be extracted in the same time period. This mining rule ensures that when a new bench is opened, enough space for mining machinery is provided.

$$\sum_{r=1}^{t}\sum_{m=1}^{M}\sum_{d=1}^{D} x_{lmd}^{r} - y_{i5}^{t} \geq\ 0 \quad \forall \{\, i, l \in I \mid l \neq i;\ l\ \in \Gamma_{dc_i});\ t = 1,2,\dots,T. \tag{5}$$

Constraint (6) ensures that each block can be extracted no more than once. However, some blocks may remain as un-mined blocks.

$$\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{d=1}^{D} x_{imd}^{t} \leq 1 \quad \forall\, i = 1,2,\dots,I. \tag{6}$$

The machine capacity is controlled by Constraint (7):

$$\sum_{i=1}^{I}\sum_{d=1}^{D} x_{imd}^{t}\, v_i \beta_i / f_i \leq E_m \lambda_m^t \quad \forall\, m = 1,2,\dots,M;\ t = 1,2,\dots,T. \tag{7}$$

Constraint (8) deals with the working territory for each excavator. As the MBS problem is solved for a short term periods, long distance machine movement is not practical. Therefore, the excavator territory is limited with the specific area.

$$\sum_{t=1}^{T}\sum_{i\in\{I-I^m\},}^{I}\sum_{d=1}^{D} x_{imd}^{t} \leq 0 \quad \forall\, m = 1,2,\dots,M. \tag{8}$$

Constraint (9) ensures that minimum production capacity is satisfied. Indeed, this constraint forces the model to extract waste material after satisfying processing requirements.

$$\sum_{m=1}^{M}\sum_{d=1}^{D}\sum_{i=1}^{I}x_{imd}^{t}b_i \geq M_m^t \qquad \forall\, t = 1,2,\dots,T. \tag{9}$$

Constraints (10) and (11) satisfy the minimum and maximum processing capacities.

$$\sum_{m=1}^{M}\sum_{i=1}^{I}x_{im\rho}^{t}\,b_i + \sum_{s=1}^{S_\rho}z_{s\rho}^{t} \leq\ M_\rho^{max} \quad \forall\, t = 1,2,\dots,T;\ \rho = 1,2,\dots,P. \tag{10}$$

$$\sum_{m=1}^{M}\sum_{i=1}^{I}x_{im\rho}^{t}\,b_i + \sum_{s=1}^{S_\rho}z_{s\rho}^{t} \geq\ M_\rho^{min} \quad \forall\, t = 1,2,\dots,T;\ \rho = 1,2,\dots,P. \tag{11}$$

Constraint (12) ensures that the stockpile inventory stays below the capacity of stockpile. Constraints (13) and (14) state that the material flow from stockpile to processing should be less than the stockpile inventory while the safety level is kept.

$$I_s^t \leq M_s \qquad \forall\, t = 1,2,\dots,T;\ s = 1,2,\dots,S. \tag{12}$$

$$z_{s\rho}^{t} \leq I_s^{t-1} -\ \phi_s \qquad \forall\, t = 2,3,\dots,T;\ s = 1,2,\dots,S;\ \rho = 1,2,\dots,P. \tag{13}$$

$$z_{s\rho}^{1} \leq I_s^0\ + \sum_{i=1}^{I}\sum_{m=1}^{M}x_{ims}^{1}b_i - \phi_s \quad \forall\, s = 1,2,\dots,S;\ \rho = 1,2,\dots,P. \tag{14}$$

Here, I_s^t is the inventory of the s^{th} stockpile at the end of period t which can be written as an ordinary inventory balance equation:

$$I_s^t = I_s^0 + \sum_{r=1}^{t}\sum_{m=1}^{M}\sum_{i=1}^{I}x_{ims}^{r}b_i -\ \sum_{r=1}^{t}\sum_{\rho=1}^{P}z_{s\rho}^{r} \quad \forall\, t = 2,3,\dots,T;\ s = 1,2,\dots,S. \tag{15}$$

Constraints (16) and (17) satisfy the lower and upper bounds on required grades at processing circuit. In other words, the ore content of the material sent to processing in period *t* must be between minimum and maximum required ore content.

$$\sum_{m=1}^{M}\sum_{i=1}^{I}(g_i^{\alpha}-g_{\alpha\rho}^{max})x_{im\rho}^{t}b_i+\sum_{s=1}^{S_\rho}(g_s^{\alpha}-g_{\alpha\rho}^{max})z_{s\rho}^{t}\leq 0\ \forall\, t=1,2,\dots,T;\rho=1,2,\dots,P;\ \alpha=1,2,\dots,\mathcal{A}. \quad (16)$$

$$\sum_{m=1}^{M}\sum_{i=1}^{I}(g_i^{\alpha}-g_{\alpha\rho}^{min})x_{im\rho}^{t}b_i+\sum_{s=1}^{S_\rho}(g_s^{\alpha}-g_{\alpha\rho}^{min})z_{s\rho}^{t}\geq 0\ \ \forall\, t=1,2,\dots,T;\rho=1,2,\dots,P;\ \alpha=1,2,\dots,\mathcal{A}. \quad (17)$$

Here, g_s^{α} represents the percentage of attribute α in stockpile s. In the case that all material is stocked in the stockpile and homogenized, these constraints state that the average grade of ore production of a period should be within the minimum and maximum limits, so that the constructed stockpile contains appropriate ore material to be used in next periods.

Constraints (18) and (19) ensure that the average percentage of attribute α in the feed of stockpile s, is within the predefined acceptable range.

$$\sum_{i=1}^{I}\sum_{m=1}^{M}(g_i^{\alpha}-g_{\alpha s}^{max})x_{ims}^{t}b_i\leq\ 0\ \ \forall s=1,\dots,S;\forall\, t=1,2,\dots,T;\alpha=1,\dots,\mathcal{A}. \quad (18)$$

$$\sum_{i=1}^{I}\sum_{m=1}^{M}(\,g_i^{\alpha}-g_{\alpha s}^{min})x_{ims}^{t}b_i\geq 0\ \ \forall s=1,..,S;\forall\, t=1,2,..,T;\alpha=1,\dots,\mathcal{A}. \quad (19)$$

Similar to the last two constraints, Constraints (20) and (21) are applied to control material flow to the right waste dump.

$$\sum_{i=1}^{I}\sum_{m=1}^{M}(g_i^{\alpha}-g_{\alpha w}^{max})x_{imw}^{t}b_i\leq\ 0\ \ \forall w=1,..,W;\forall\, t=1,2,..,T;\alpha=1,\dots,\mathcal{A}. \quad (20)$$

$$\sum_{i=1}^{I}\sum_{m=1}^{M}(g_i^{\alpha} - g_{\alpha w}^{min})x_{imw}^{t}b_i \geq 0 \;\; \forall w = 1,..,W; \forall\, t = 1,2,..,T; \alpha = 1,\dots,\mathcal{A}. \quad (21)$$

Constraints (22) and (23) state the type of decision variables:

$$x_{imd}^{t} \; and \; y_{if}^{t} \in \{0,1\} \quad \forall\, i = 1,2,\dots,I;\; m = 1,2,\dots,M;\; d = 1,2,\dots,D;\; t = 1,2,\dots,T; f = 1,2,\dots,5. \quad (22)$$

$$z_{s\rho}^{t} \geq 0 \;\; \forall\, s = 1,2,\dots,S;\; \rho = 1,2,\dots,P; t = 1,2,\dots,T. \quad (23)$$

Supplementary material 1

3. Solution Approach

To solve the MBS problem, simulated annealing (SA), large neighbourhood search (LNS) and branch-and-bound (B&B) are hybridized. In the proposed hybrid heuristic, a partial neighbourhood solution (PNS) is constructed in each iteration of the SA. Then a full neighbourhood solution (NS) is achieved by assigning destinations to the blocks using the B&B algorithm.

3.1. Simulated annealing

Simulated annealing (SA) was introduced by Kirkpatrick et al. (1983) and independently by (Černý 1985) for solving the combinational optimization problems. The key element in an efficient simulated annealing is the cooling schedule which decreases the temperature from an initial temperature to the final temperature. For the MBS problem, a cooling schedule is developed in which the number of internal iterations (number of iterations executed at each temperature) increases as the temperature decreases. Temperature and number of internal iterations are updated at each iteration using following equations:

$$\tau_k = \alpha\tau_{k-1} \mid \tau_k \geq \mathcal{T}_f, \quad k = 1,2, \dots, K. \tag{24}$$

$$\mathrm{H}^{\tau_k} = \mathrm{H}^{\tau_{k-1}} + \Delta \tag{25}$$

Here, α is a constant number (less than one), τ_k is the temperature at iteration k, $\mathcal{T}_f$ is the final temperature, K is the number of iterations, H^{τ_k} is equal to the number of internal iterations at temperature τ_k and Δ is a constant number. As the iteration of algorithm changes from k to $k+1$, the cooling mechanism reduces temperature by α% and increases number of internal iterations by Δ.

3.2. Large neighbourhood search

Large neighbourhood search (LNS) was proposed and applied by Shaw (1998). Unlike the common neighbourhood search, LNS uses a destroy-and-repair mechanism to define a new solution. According to this mechanism, a part of the solution is destroyed and then a repair technique rebuilds the solution (Pisinger and Ropke 2010).

The proposed LNS for the MBS problem destroys the assigned destinations and the new destinations are allocated to blocks by branch-and-bound method. The LNS can improve the quality of the MBS solution by covering two drawbacks of the common local search. Firstly, there are some blocks which should be extracted in a certain time period due to the precedence relationships. Since these blocks are fixed in a period, they are not considered in the neighbourhood search. Secondly, as blending is allowed, a decision for a given block should be made taking into account other blocks' destinations.

3.3. Neighborhood structure for MBS

The following two main movements are defined in the structure of MBS neighbourhood search: move between periods and move between destinations.

In a move between periods the assigned extraction period of one or several blocks may be altered while a move between destinations is made for reassigning the destination of some blocks.

Move between Periods

To select the most attractive time periods and blocks, the following strategies are applied:

- select time periods in order to compensate processing shortage:

$F^t: w \rightarrow (0,1)$ is a function which assigns a priority weight w to period t. Therefore, each period is mapped with a weight such that $\sum_{t=1}^{T} w_t = 1$.

The weights are assigned such that if t' has the maximum shortage of processing feed, then $w_t < w_{t'} < 1 \;\; \forall\, t \in \{\, t1, t2, \dots, T\,\}$. After assigning weights to the periods, a random weighted sampling method is performed to identify a time period which accepts new blocks from other time periods. This period is called $t_{\_in}$ and the period from which some blocks are transferred to $t_{\_in}$ is named $t_{\_out}$. The $t_{\_out}$ is selected as follows:

$$t_{\_out} = \begin{cases} t_{\_in} - 1, & if\ SLB^{t_{\_in}-1} \geq SLB^{t_{\_in}+1}\ and\ t_{\_out} \neq T; \\ t_{\_in} + 1, & otherwise; \end{cases} \qquad (26)$$

Where $SLB^{t_{\_in}-1}$ shows the number of blocks assigned to the stockpiles in period $t_{\_in} - 1$.

- select time periods in order to postpone stocking material

To minimize stockpiling holding costs, material should be stocked in the stockpile as late as possible. Therefore, if some stockpile-labelled blocks are transferred from period t (as $t_{\_out}$) to period $t+1$ (as $t_{\_in}$) then the objective value may be improved. The periods are prioritized such that the period which has the maximum stockpile-labelled blocks has the highest priority to be selected as $t_{\_out}$. In this case $t_{\_out} \neq T$ and $t_{\_in} = t_{\_out} + 1$.

- select the most attractive blocks

Suppose that (ϖ_i, i) represents block i with weight ϖ_i. A function F^b is defined such that $F^b\colon \varpi \rightarrow (0,1)$. F^b is a discrete function which maps a priority weight between zero and one to all blocks, to be extracted in $t_{_in}$and $t_{\_out}$, according to the following rules:

For block i labelled to be extracted in time period $t_{\_out}$, a weight is assigned based on the number of stockpile-labelled blocks in the $\mathcal{L}_{si}^{t\_out}$ (if $t_{\_out} < t_{\_in}$, otherwise $\mathcal{L}_{pi}^{t\_out}$), Where $\mathcal{L}_{si}^{t\_out}$ denotes successors of block i which are extracted in time $t_{\_out}$ in solution x and $\mathcal{L}_{pi}^{t\_out}$ represents predecessors of block i which are mined in time $t_{\_out}$ in solution x. The weights are assigned to blocks such that a block which has the maximum number of stockpile-labelled blocks in $\mathcal{L}_{si}^{t\_out}$ (if $t_{\_out} > t_{\_in}$, $\mathcal{L}_{pi}^{t\_out}$) has the largest weight.

For block j labelled to be extracted in time period $t_{\_out}$, a weight is assigned based on the number of processing-labelled blocks in the $\mathcal{L}_{pi}^{t\_in}$ (if $t_{\_out} < t_{\_in}$, otherwise $\mathcal{L}_{si}^{t\_out}$), such that a block that has the minimum number of processing-labelled blocks has the largest weight.

Move between destinations

Since moves between periods are *k-exchange* moves, in which k blocks are swapped between periods, the solution space for move between destinations is very large. Performing move-between destinations can be more complicated when there are several processing circuits, stockpiles and average grade requirements constraints are applied. Therefore, a sub-problem is defined and called destination assignment sub-problem (DA) and solved in associated with large neighbourhood search. Rules for applying DA are given below:

- Only the transferred blocks (blocks moved from a period to another period) are taken into account for DA.
- With a certain probability, a large neighbourhood search (LNS) is applied to reset the destination of all stockpile-labelled and processing-labelled blocks.
- A block which is labelled as a waste block keeps its destination unless it has a positive economic value.
- The DA is solved only for $t_{\_in}$ and $t_{\_out}$, unless LNS is applied. When LNS is hybridized, DA is solved for $t_{\_in}$, $t_{\_out}$ and other time periods which are greater than $t_{\_in}$ and $t_{\_out}$.

The B&B algorithm is proposed to solve the DA, because the conducted computational experiments show that B&B can obtain a high quality solution for DA quickly.

3.4. Evaluation of a neighbourhood solution

The evaluation process is performed by checking the constraints of the MBS model to be satisfied. However, to enhance the chance of obtaining a feasible solution, considerations are taken into account for precedence relationships and machine capacity. The other constraints are checked after performing a move.

Precedence relationships consideration

Let assume that the current solution is x and the move will construct a neighbourhood solution $x' \in N(x)$. If transferring block i from $t_{\_out}$ to $t_{\_in}$ makes the new solution x', then:

If $t_{\_in} > t_{\_out}$:

$$x_{imd}^{t\_out} = 0,\ x_{imd}^{t\_in} = 1,\ x_{jmd}^{t\_out} = 0\ ,\ and\ x_{jmd}^{t\_in} = 1 \quad \forall\, j \in \mathcal{L}_{si}^{t}, m \in \{1, 2, \dots, M\}, and\ d \in \{1,2, \dots, D\},$$

Otherwise:

$$x_{imd}^{t\_out} = 0, x_{imd}^{t\_in} = 1;\ \ x_{kmd}^{t\_out} = 0\ , x_{kmd}^{t\_in} = 1\ \ \forall\, k \in \mathcal{L}_{pi}^{t}, m \in \{1, 2, \dots, M\}, and\ d \in \{1,2, \dots, D\},$$

Machine capacity constraint consideration

To obtain a feasible solution in terms of machine capacity the following constraints should be satisfied:

$$\mathcal{M}^{out} + \mathcal{M}^{t\_in} \leq \ \mathcal{M}_{c}^{t\_in} \tag{27}$$

$$\mathcal{M}^{in} + \mathcal{M}^{t\_out} \leq \ \mathcal{M}_{c}^{t\_out} \tag{28}$$

Where,

$\mathcal{M}^{out}$: Tonnage of material sent from period $t_{\_out}$ to $t_{\_in}$.

$\mathcal{M}^{t\_in}$: Tonnage of material extracted in period $t_{_in}$in solution *x* (the transferred blocks should be ignored at this stage).

$\mathcal{M}_{c}^{t\_in}$: Machine capacity in period $t_{\_in}$.

$\mathcal{M}^{in}$: Tonnage of material transferred from period $t_{\_in}$ to $t_{\_out}$.

$\mathcal{M}^{t_{\_out}}$: Tonnage of material extracted in period $t_{\_out}$ in solution x.

$\mathcal{M}_c^{t_{\_out}}$: Machine capacity in period $t_{\_out}$.

When a decision is made to transfer some blocks from $t_{\_out}$ to $t_{\_in}$, the Equations (27) and (28) should be solved. The $\mathcal{M}^{out}$ can be calculated because the blocks which are sent to the $t_{\_in}$ are known. $\mathcal{M}_c^{t_{\_in}}$ and $\mathcal{M}_c^{t_{\_out}}$ are given parameters.

4. Computational experiments

The MBS model is tested by using several realistic instances reported in Table 1. Two different studies are presented. Firstly, the solution of proposed hybrid SA and B&B (SA_B&B) is compared with the CPLEX solution for those instances which CPLEX obtained at least one solution. Secondly, a comparison is performed between the obtained solution by SA_B&B, and hybrid SA_B&B and LNS (SA_B&B_LNS). The results of both algorithms as well as the best obtained objective value by CPLEX are reported in Table 1.

Following parameters are considered for heuristic technique: initial temperature for SA=0.95, final temperature=1E-8, maximum run time for B&B=2 seconds, probability of applying LNS=0.2. In addition to the final temperature, a 2-hour time limit is considered to stop the SA in all instances except instances 10, 11, and 22-24 in which a 6-hour time limit is applied.

A column labelled as deviation represents the deviation between CPLEX solution and the proposed heuristic solution. Where the deviation is negative, the objective value of our heuristic is better than CPLEX solution. Furthermore, each instance is run 15 times in order to observe the variance of achieved solutions. The standard deviation and coefficient of variation (CV%) are calculated and summarized in Table 1. CV% is a ratio of standard deviation to the mean and represents the dispersion of the obtained results.

As it is shown in Table 1 both SA_B&B and SA_LNS_B&B obtained good solutions for all instances. The reported deviations show that the SA_LNS_B&B reaches the optimum solutions for instances with known optimum values. In addition, only in a few cases there are positive deviations which are less than 1%. As the size of the instances increase the trend of the deviations goes negative which means that the quality of SA_LNS_B&B is better than CPLEX solution. Moreover, for industry-scale cases such as instances 22-24, which CPLEX is unable to find a solution, good solutions are obtained by the SA_LNS_B&B. Finally, the outcomes show that the SA_B&B can also provide good solutions. However, the quality of the solution is not as good as the SA_LNS_B&B. Nevertheless, still the deviations for all instances are less than 3 %.

The reported CV% for both proposed heuristics shows that in many cases the value of CV% is zero. This means that in all 15 runs, the proposed heuristic converges to a same solution. However, due to the direction of neighbourhood search, a positive amount of CV% can be observed in some cases.

Table 1. Results of computational experiment.

5. Case Study

The proposed model and solution approaches are implemented in the real iron ore mine case study. A case study, extraction of 2500 blocks over 6 months is used and detail is given in Table 2. A time period is considered as two weeks and the problem is solved for 12 time periods (6 months). Blocks are rectangular and each one is 20m*20m*12 m.

Table 2. Case study: Extraction 2500 blocks over 12 time periods.

The solution algorithm runs about 6 hours and results are discussed here. The material flow, reported in Table 3, shows that a high percentage of processing demands (89%) is provided by direct feed. Furthermore, it is observed that in the first period in which the minimum demands of P1 (270 kt) and P2 (570 kt) are not satisfied by run-of-mine material, rehandling provides the maximum demands of P1 and P2. However, there is a similar situation in period 10, but rehandling provides the minimum demands of P1 and P2. This happens because keeping a tonne of material in the stockpile from first period to last period costs $1.2(0.1*12) which is more than the $1 rehandling cost. Therefore, the difference between direct feed and maximum demand is rehandled. However, for period 10, holding a tonne of material is always cheaper than rehandling.

As can be seen in Table 3, the model tries to mine stockpile-labelled blocks as late as possible. The results show that most material is sent to the stockpiles in periods 5 to 9 and not too much in the last three periods. Investigation of the extraction periods, precedence relations and ramp points shows that the available blocks for the last three periods are mostly waste. Therefore, a large amount of waste is extracted in these three periods, especially in period 11.

Supplementary material 2

The results for mined waste material, columns W1 and W2, shows that the stripping ratio (ratio of extracted waste to mined ore in a period) is not constant. Therefore, defining the stripping ratio by a mining engineer is not required because it can be obtained more accurately by the model, which is based on the accessibility of ore blocks and the processing requirements.

Table 3. Material flow and stockpile inventory.

The results of Fe% and Al2O3% in run-of-mine material and final processing feed are shown in Figure 2. Figure 2.a and Figure 2.b give the Fe% and Al2O3% of blocks assigned to P1 and P2 in each period. The green solid circles in these figures show the average grade of Fe% and Al2O3% provided by run-of-mine material for each period. Unlike the static approach, currently used by the mine, the dynamic allocation assigns some blocks with Fe<62 and Al2O3>2.1 to P2. Furthermore, observation shows that the average grade of directed run-of-mine material may not satisfy mill requirements (e.g., period 10 for P2). This shortage is compensated for by stockpile-provided feed. Figure 2.c and Figure 2.d present the final achieved average grades of Fe% and Al2O3% in processing feed provided by the mine and the stockpile. These figures confirm that grade requirements are completely satisfied for both processing circuits.

The percentages of dispatched material to P1 & S1 and P2 & S2 for both static and dynamic approaches are given in Figure 3. Applying dynamic allocation can allocate about 20% more material to P2 and S2, compared to the static approach, while all constraints are fulfilled. We assumed that processing at P2 is cheaper than P1. Therefore, processing cost can be decreased by applying dynamic destination allocation.

a) Fe% of blocks assigned to P1 and P2 | b) Al2O3% of blocks assigned to P1 and P2

c) Average grade of Fe in processing feed | d) Average grade of Al2O3 in processing feed

Figure 2. Average grade of bi-weekly production in processing feed.

Figure 3. Percentage of dispatched material in static and dynamic allocation

5.1. Sensitivity Analysis

Sensitivity analysis is performed to observe the changes in the solution. Several scenarios for target grades, processing demands and stockpiling costs are conducted.

The grade bound for P1 and P2 are gradually relaxed and the results for objective value are reported in Figure 4. As observed in this figure, the objective value decreases as the grade bounds are relaxed. This is logical because in large ranges of the accepted grade, ore blocks can be sent directly to the processing and both rehandling and holding costs are reduced. Since the misclassification and drop-cut costs (terms three to five in the objective function) are zero, the values for these costs have not been reported. Drop-cut cost is zero because there are some blocks without predecessor and extractions are always side cut (no drop-cut is needed to open a bench). Misclassification cost is zero as well, since the cost of the processing a waste block or dumping an ore block is high compared to stockpiling costs. Additionally, the ore and waste cut-off grade is considered as 50% for Fe, because in the current mine operation blocks with Fe% less than 50 are not processed at any processing circuits (although in some mines it may not be a true cut-off grade).

Figure 4. Objective value for several grade bounds.

Figure 5 shows the objective values for several cases which are run under different processing demands. As can be observed, the objective value can decrease by increasing the maximum capacity of the process, because more material can be directly transferred to process instead of storing in the stockpile. On the other hand, increasing the minimum process demand may enhance the objective value, since the model is forced to compensate for the process deficit by rehandling from stockpiles. For those cases which have no bar in Figure 5, no feasible solution is found, because of lack of enough ore blocks.

Figure 5. Objective value for different processing demands.

To verify the effect of rehandling and holding cost, the problem is solved under different ratios of rehandling and holding costs such that rehandling cost is kept constant as $1 in all cases and holding cost is gradually increased.

The model is solved under scenarios 1-4 and the amount of rehandled material is reported in Figure 6. This figure shows as unit holding costs increase, the amount of rehandled material also increases. This happens because keeping material in the stockpile is more expensive than rehandling it. Therefore, the model tries to send more material to the processes and satisfies maximum process demand, especially in early periods.

Figure 6. Rehandled tonnage with different holding costs.

To observe the effect of the grade distribution of the input blocks on MBS solution, the grades of blocks are randomly changed (the location of blocks remained same). Then the model is solved for the new grade configuration. Figure 7 shows the values of objective function, rehandling and holding costs.

Figure 7. Objective values for five cases with different grade distributions.

The results show that depends on the grade distribution of input data, different objective values may be obtained. If ore blocks are accessible in all periods, then objective value decreases (e.g., C2). If the precedence relationships prevent access to ore blocks in some periods, then rehandling cost enhances (C1).

The implementation of the model on real case study demonstrates that the proposed model can be successfully applied in open-pit mines, especially in mines that run-of-mine ore material can be directly sent to process. However, because the input data may not be correctly matched with the model requirements, some boundary effects may happen. For example, in the presented case study the excavators are unable to extract all blocks because they are not available full time (the average effectiveness of excavators for this case study is 0.36). In another scenario, excavator may finish extraction earlier than the end of time horizon, so they will be idle at the late periods. To solve this weakness, a practical way is to apply rolling horizon strategy. In a rolling horizon strategy the model is solved for longer time horizon, but only the solution of the first couple of periods is considered. Then more blocks are brought into the input data and model is solved again. This strategy can be useful, as in the mining industry the input data are regularly updated (e.g., additional drilling information, machine break down).

6. Conclusion and future works

In this paper, a new and more comprehensive mathematical programming model for short-term mine block sequencing is developed by tackling nearly all relevant physical and technical aspects in open-pit mining. The model is verified based on the smallest mining unit (block) to take the advantages of granularity of the grade estimation. Therefore, no aggregation or clustering is needed in the proposed MBS solution techniques. In addition, the stockpiling process is included into the model to reflect the mining and processing operations over short intervals (e.g., days). The proposed model also considers in-site mining rules, dynamic destination assignment, blending and stockpiling. The ability of exact MIP solvers such as IBM ILOG-CPLEX to solve the MBS problem is investigated. According to the computational experiments, the real-life instances of the problem are intractable for CPLEX. The second main part of this paper is developing a hybrid heuristic approach to solve the large-size instances. In the proposed hybrid heuristic, the best dispatching of ore blocks is determined by applying B&B algorithm applied at the each iteration of SA. Moreover, in an alternative version of proposed heuristic, LNS is hybridized to improve the quality of solution. To test the efficiency of the proposed heuristic, several case studies are run by our proposed heuristic and CPLEX optimizer. Computational experiments validate that the proposed heuristics can obtain promising results. For those cases of which the optimum solutions are obtained by CPLEX, the heuristic also find the same solutions. For other instances, our heuristic is so competitive especially when LNS is applied as the deviation from CPLEX is less than 1% on average.

The following research directions are proposed for future work. First, it is proposed to develop more advanced metaheuristics for solving the MBS problem without relying on any MIP solver. Second, for evaluating the performance of the heuristics in a better way, a lower bounding method based on a lagrangian relaxation will be developed. Third, a goal programming approach will be proposed when dynamic blending, processing and stockpiling requirements need to be satisfied. Last, the proposed MBS methodology will be incorporated with operational-level multi-stage mine production scheduling models (e.g., drilling, blasting, excavating, hauling and railing) within a demand-responsive mines-to-ports supply chain scheduling system (Kozan and Liu 2012; Liu and Kozan 2011, 2012).

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References

Asad, M. W. A., and Topal, E. 2011. Net present value maximization model for optimum cut-off grade policy of open pit mining operations. *Journal of the Southern African Institute of Mining and Metallurgy* 111:741-750.

Boland, N., Dumitrescu, I., Froyland, G., and Gleixner, A. M. 2007. LP-based disaggregation approaches to solving the open pit mining production scheduling problem with block processing selectivity. *Computers & Operations Research* 36:1064-1089.

Caccetta, L., and Hill, S. P. 2003. An application of branch and cut to open pit mine scheduling. *Journal of Global Optimization* 27:349-365.

Černý, V. 1985. Thermodynamical approach to the traveling salesman problem: An efficient simulation algorithm. *Journal of Optimization Theory and Applications* 45 (1):41-51.

Chicoisne, R., Espinoza, D., Goycoolea, M., Moreno, E., and Rubio, E. 2012. A new algorithm for the open-pit mine production scheduling problem. *Operations Research* 60 (3):517-528.

Cullenbine, C., Wood, R. K., and Newman, A. 2011. A sliding time window heuristic for open pit mine block sequencing. *Optimization Letters* 5:365–377.

Eivazy, H., and Askari-Nasab, H. 2012. A mixed integer linear programming model for short-term open pit mine production scheduling. *Mining Technology* 121 (2):97-108.

Espinoza, D., Goycoolea, M., Moreno, E., and Newman, A. 2013. MineLib: a library of open pit mining problems. *Annals of operations research* 206 (1):93-114.

Groeneveld, B., and Topal, E. 2011. Flexible open-pit mine design under uncertainty. *Journal of Mining Science* 47 (2):212-226.

Huang, Z., Cai, W., and Banfield, A. F. *A new short- and medium-term production scheduling tool – MineSight Schedule Optimizer (MSSO)* 2009. Available from http://www.minesight.com/.

Johnson, P. V., Evatt, G., Duck, P., and Howell, S. 2011. The determination of a dynamic cut-off grade for the mining industry. In *Electrical Engineering and Applied Computing*, edited by Ao, S. I. and Gelman, L.: Springer Netherlands.
Kirkpatrick, S., Gelatt, C. D., and Vecchi, M. P. 1983. Optimization by simulated annealing. *Science* 220 (4598):671-680.
Kozan, E., and Liu, S. Q. 2011. Operations research for mining: a classification and literature review. *ASOR Bulletin* 30 (1):2-23.
Kozan, E., and Liu, S. Q. 2012. A demand-responsive decision support system for coal transportation. *Decision Support Systems* 54 (1):665-680.
Kozan, E., and Liu, S. Q. 2015. A new open-pit multi-stage mine production timetabling model for drilling, blasting and excavating operations. *Mining Technology*:1743286315Y. 0000000031.
Kumral, M. 2012. Production planning of mines: Optimisation of block sequencing and destination. *International Journal of Mining, Reclamation and Environment* 26 (2):93–103.
Kumral, M. 2013. Optimizing ore–waste discrimination and block sequencing through simulated annealing. *Applied Soft Computing* 13 (8):3737-3744.
L'Heureux, G., Gamache, M., and Soumis, F. 2013. Mixed integer programming model for short term planning in open-pit mines. *Mining Technology* 122 (2):101-109.
Lambert, W., and Newman, A. 2014. Tailored Lagrangian Relaxation for the open pit block sequencing problem. *Annals of operations research* 222 (1):419-438.
Lambert, W. B., Brickey, A., Newman, A. M., and Eurek, K. 2014. Open-pit block-sequencing formulations: A tutorial. *Interfaces* 44 (2):127-142.
Lamghari, A., and Dimitrakopoulos, R. 2012. A diversified Tabu search approach for the open-pit mine production scheduling problem with metal uncertainty. *European Journal of Operational Research* 222 (3):642-652.
Lamghari, A., Dimitrakopoulos, R., and Ferland, J. 2015. A hybrid method based on linear programming and variable neighborhood descent for scheduling production in open-pit mines. *Journal of Global Optimization* 63 (3):555-582.
Liu, S. Q., and Kozan, E. 2011. An interactive planning and scheduling framework for optimising pits-to-crushers operations. *Industrial Engineering & Management Systems* 11 (1):94-102.
Liu, S. Q., and Kozan, E. 2012. A hybrid shifting bottleneck procedure algorithm for the parallel-machine job-shop scheduling problem. *Journal of the operational research society* 63 (2):168-182.
Liu, S. Q., and Kozan, E. 2016. New graph-based algorithms to efficiently solve large scale open pit mining optimisation problems. *Expert Systems with Applications* 43:59-65.
Minemap Pty. Ltd. *http://www.minemap.com.au/* 2013.
Minemax Pty Ltd. *http://www.minemax.com/* 2012.
Mousavi, A., Kozan, E., and Liu, S. Q. 2014. Integrated approach to optimize open-pit mine block sequencing. In *Industrial Engineering Non-Traditional Applications in International Settings*, edited by Bidanda, B., Sabuncuoglu, I. and Kara, B. Y.: CRC Press.
Newman, A. M., Rubio, E., Caro, R., and Eurek, K. 2010. A review of operations research in mine planning. *Interfaces* 40 (3):222-245.
Osanloo, M., Gholamnejad, J., and Karimi, B. 2008. Long-term open pit mine production planning: a review of models and algorithms. *International Journal of Mining, Reclamation and Environment* 22 (1):3-35.

Pisinger, D., and Ropke, S. 2010. Large neighborhood search. In *Handbook of metaheuristics*, edited by Gendreau, M. and Potvin, J.-Y.: Springer US.

Ramazan, S. 2007. The new fundamental tree algorithm for production scheduling of open pit mines. *European Journal of Operational Research* 177 (2):1153-1166.

Runge PincockMinarco Limited. *http://www.rpmglobal.com/mining-software* 2013.

Shaw, P. 1998. Using constraint programming and local search methods to solve vehicle routing problems. In *Principles and Practice of Constraint Programming—CP98*, edited by Maher, M. and Puget, J.-F.: Springer Berlin Heidelberg.

Shishvan, M. S., and Sattarvand, J. 2015. Long term production planning of open pit mines by ant colony optimization. *European Journal of Operational Research* 240 (3):825-836.

Singh, G., García-Flores, R., Ernst, A., Welgama, P., Zhang, M., and Munday, K. 2013. Medium-term rail scheduling for an iron ore mining company. *Interfaces* 44 (2):222-240.

Whittle, D. 2011. Chapter 10.2: Open-pit planning and design. In *SME Mining Engineering Handbook (3rd Edition)*, edited by Darling, P.: Society for Mining, Metallurgy, and Exploration (SME).

Table 1. Results of computational experiments

Instance	Characteristics (#I, #M, #D, #T)	Best integer solution by CPLEX × 10^5 (a)	SA_B&B					SA_LNS_B&B				
			Average objective value of runs × 10^5 (b)	Minimum objective value of runs × 10^5	Standard deviation × 10^3	CV %	Deviation % (1-a/b)*100	Average objective value of runs × 10^5 (c)	Minimum objective value of runs × 10^5	Standard deviation × 10^3	CV %	Deviation % (1-a/c)*100
Ins1	(25,2,4,2)	0.54*	0.54	0.54	0	0	0.00	0.54	0.543	0	0	0
Ins2	(50, 2, 4, 2)	2.02*	2.02	2.02	0	0	0.00	2.02	2.02	0	0	0
Ins3	(75,2,4,2)	1.31*	1.32	1.31	0.36	0.28	0.76	1.31	1.31	0	0	0
Ins4	(100 ,2,4,2)	1.39	1.4	1.4	0	0	0.71	1.4	1.39	0.17	0.12	0.71
Ins5	(180,2,4,2)	2.15*	2.17	2.16	0.79	0.37	0.92	2.15	2.15	0	0	0
Ins6	(242,2,4,2)	2.62	2.7	2.66	2.91	1.08	2.96	2.62	2.61	0.49	0.19	0
Ins7	(372,2,4,3)	4.13	4.16	4.08	5.12	1.23	0.72	4.03	3.94	4.30	1.07	-2.49
Ins8	(412,2,4,3)	3.82	3.83	3.82	0.50	0.13	0.26	3.82	3.82	0.23	0.06	0
Ins9	(515,4,4,4)	5.56	5.47	5.36	8.98	1.64	-1.65	5.38	5.3	8.02	1.49	-3.34
Ins10	(620,4,4,4)	-	15.4	1.52	21.2	1.37	-	1.55	1.52	1.65	1.07	-
Ins11	(900,4,4,6)	-	15.8	15.4	33.97	2.15	-	15.8	15.4	33.9	2.15	-
Ins12	(25,2,6,2)	0.91*	0.91	0.91	0	0	0.00	0.91	0.913	0	0	0
Ins13	(50,2,6,2)	2.09*	2.12	2.09	2.32	1.1	1.42	2.09	2.09	0	0	0
Ins14	(75,2,6,2)	2.05*	2.06	2.05	0.78	0.38	0.49	2.05	2.05	0	0	0
Ins15	(100,2,6,2)	1.91	1.94	1.92	2.01	1.04	1.55	1.91	1.91	0	0.07	0
Ins16	(180,2,6,2)	2.01	2.07	2.04	1.35	0.65	2.90	2.02	2.01	1.52	0.75	0.49
Ins17	(242,2,6,2)	3.49	3.52	3.5	0.49	0.14	0.85	3.51	3.49	1.14	0.32	0.57
Ins18	(372,2,6,3)	5.07	4.93	4.92	1.79	0.36	-2.84	4.93	4.92	1.18	0.24	-2.85
Ins19	(412,2,6,3)	6.61	6.74	6.65	5.11	0.76	1.93	6.66	6.61	2.47	0.37	0.75
Ins20	(515,4,6,4)	6.69	6.61	6.43	19.9	3	-1.23	6.58	6.43	17.4	2.65	-1.67
Ins21	(620,4,6,4)	8.21	7.81	7.71	9.68	1.24	-5.12	7.82	7.72	8.29	1.06	-5.00
Ins22	(900,4,6,6)	-	17.9	17.7	13.7	0.77	-	17.9	17.8	10.73	0.60	-
Ins23	(1200,4,6,6)	-	13.3	13.2	8.22	0.62	-	13.3	13.2	6.35	0.47	-
Ins24	(2500,6,6,12)	-	23.8	21.6	96.6	4.06	-	24.0	22.2	126.1	5.25	-

*The optimal solution is obtained by CPLEX.

Table 2. Case study: Extraction 2500 blocks over 12 time periods

	Excavators							
	E1	E2	E3	E4	E5	E6	E7	E8
Units	1	1	1	1	1	1	3	2
Capacity (kt/day)	52.2	41.6	27.5	33.8	11.5	10.8	79.1	29.2

	Processing circuits	
	P1	P2
Target feed range	270-300 kt	570-600 kt
Target %Fe range	56.5-57.5%	63-63.5%
Target % Al2O3 range	no limitation	< 2.1%
	Waste dumps	
	W1	W2
Capacity	unlimited	unlimited
Target %S	no limitation	>0.2
	Stockpiles	
	S1	S2
Initial inventory	900 kt	1500 kt
Safety level	300kt	600 kt
Capacity	unlimited	unlimited
Target Fe% range	>50	>50
Target Al2O3% range	no limitation	< 2.1
Rehandlin Cost ($/tonne)	1	1
Holding Cost ($/tonne)	0.1	0.1

Table 3. Material flow and stockpile inventory

Time period	Dispatched run-of-mine material (kt)						Stockpiles to processing flow (kt)		Final processing feed (kt)		Stockpile inventory (kt)	
	P1	P2	S1	S2	W1	W2	S1 to P1	S2 to P2	P1	P2	S1	S2
1	198	566	39	0	1790	519	102	34	300	600	837	1470
2	300	600	238	82	1800	285	0	0	300	600	1080	1550
3	200	600	0	0	1930	537	70	0	270	600	1010	1550
4	295	600	0	169	1630	503	0	0	295	600	1010	1720
5	300	600	134	324	1220	647	0	0	300	600	1140	2040
6	300	600	147	383	1290	522	0	0	300	600	1290	2420
7	300	600	253	508	1330	239	0	0	300	600	1540	2930
8	300	600	327	618	1150	199	0	0	300	600	1870	3550
9	300	600	81	192	1860	187	0	0	300	600	1950	3740
10	147	448	0	0	2460	179	123	122	270	570	1820	3620
11	18	153	0	0	3050	0	252	417	270	570	1570	3200
12	277	599	0	350	2020	0	0	0	277	599	1570	3550