Kozan, Erhan & Liu, Shi Qiang
(2017)
An operational-level multi-stage mine production timetabling model for optimally synchronising drilling, blasting and excavating operations.
International Journal of Mining, Reclamation and Environment, *31*(7), pp. 457-474.

https://doi.org/10.1080/17480930.2016.1160818

An operational-level multi-stage mine production timetabling model for optimally synchronising drilling, blasting and excavating operations

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Abstract

This paper proposes a new multi-stage mine production timetabling (MMPT) model to optimise open-pit mine production operations including drilling, blasting and excavating under real-time mining constraints. The MMPT problem is formulated as a mixed integer programming model and can be optimally solved for small-size MMPT instances by IBM ILOG-CPLEX. Due to NP-hardness, an improved shifting-bottleneck-procedure algorithm based on the extended disjunctive graph is developed to solve large-size MMPT instances in an effective and efficient way. Extensive computational experiments are presented to validate the proposed algorithm that is able to efficiently obtain the near-optimal operational timetable of mining equipment units. The advantages are indicated by sensitivity analysis under various real-life scenarios. The proposed MMPT methodology is promising to be implemented as a tool for mining industry because it is straightforwardly modelled as a standard scheduling model, efficiently solved by the heuristic algorithm, and flexibly expanded by adopting additional industrial constraints.

Keywords: open-pit mining; drilling; blasting; excavating; heuristics; disjunctive graph.

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1. Introduction

Nowadays, mining activities take place all over the world and become a major source of a country's natural wealth and economic income, especially for Australia. Due to abundance of mineral resources and mining booms in 2000-2010, "the mining industry's contribution to total goods exported from Australia is increased from 32% in 2004 to 51% in 2009" [1]. Australia's economy growth is mainly dominated by mining industry which is the fastest growing sector of the economy accounting for 13.5 percent of total GDP in 2014 [2].

Mining methods are mainly divided into two groups: surface mining and underground mining. The time horizons to operate a large mine can be quite varied. Surface mining comprises a wide range of mining methods including open pit mining, strip mining, auger mining, mountaintop removal mining, and etc. This paper is mainly concerned about open pit mining. As a standard convention, they are usually classified into long-term (strategic-level), mid-term (tactical-level) and short-term (operational-level) time horizons. In a long-term or strategic-level time horizon, mining industry practitioners are interested in making strategic decisions whether to explore a mine and how to determine the mine contour to maximise the value of an orebody in its mine life (several years). In a mid-term or tactical-level time horizon, mining industry practitioners commonly need to meet production targets by sequencing the selected blocks that are to be mined over periods (months) for satisfying mid-term demands at the tactical level. A mid-term time or tactical-level horizon typically has a time frame of several months in which the length of one tactical period is one month (or one fortnight). From a tactical viewpoint, mine practitioners should focus on satisfying the production targets and demands over tactical periods subject to critical resources' capacity constraints. In comparison, short-term horizons usually have no more than one month ahead and are subject to the tactical targets. The decisions made in a short-term horizon are operational decision making, such as allocation of mining equipment units, timetables of mining equipment units (e.g., drills and excavators), maintenance scheduling, shift scheduling, etc. As such, these operational decisions tend to be of the very specific focus on narrow entities (e.g., excavating operations) over relatively short time intervals (days or hours). At the operational level, mine practitioners should aim to minimise the makespan (throughput) or minimise the idle times of mining equipment units at various operational stages.

However, most papers in the literature mining optimisation dealt with the problems only at the strategic and tactical level. In this sense, this paper would be a ground breaking to optimise the operational open-pit mine production process using a multi-stage mine production timetabling methodology. To justify the contribution of this work, in terms of this time-horizon classification, a related brief literature review on open-pit mining optimisation problems is presented in the following.

In a long-term or strategic-level time horizon, the vital objective is to determine the ultimate pit limit or orebody contour that yields the maximum total value based on the exploration block model with estimated geological information. In mining community, this fundamental problem type is called *Ultimate Pit Limit* (UPIT) or *Mine Design Planning* (MDP). As pioneers, Lerchs and Grossmann [3] presented to the mining community the well-known Lerchs-Grossmann approach for long-term open-pit mine design. Caccetta and Giannini [4] and Underwood and Tolwinski [5] proposed several mathematical theorems to reinforce the Lerchs-Grossmann approach. Hochbaum and Chen [6] presented a push-relabel heuristic algorithm for MDP based on the network flow graph theory. Epstein et al. [7] extended the long-term open-pit MDP model to design an underground and open-pit sharing copper deposit by a capacitated multi-commodity network flow formulation. Asad and Dimitrakopoulos [8] implemented a parametric maximum flow algorithm to solve an extend MDP problem with uncertain supply and demand. Meagher et al. [9] presented a comprehensive review on the open pit mine design, pushbacks and the gap problems to mining community. Topal and Ramazan [10] developed a linear programming model based on the network analysis to efficiently optimise the strategic mine plan with the objective of maximise the net present value. After the determination of the ultimate pit contour, the next widely-studied mining optimisation problem type is to determine when the selected blocks should be extracted over time periods so that the total net present value is maximised. In mining community, this problem type is called *Mine Production Scheduling* (MPS) [11,12], or *Open Pit Block Sequencing* (OPBS) [13], or *Constrained Pit Limit* (CPIT) [14,15]. For convenience, the term “MBS” is used to call this problem type throughout the paper. The following leading papers contribute to MBS in the mining optimisation literature. Caccetta and Hill [12] proposed a branch-and-cut algorithm with LP relaxation to solve MBS. Due to software commercialisation and confidentiality agreements, they only summarised some important features and thus full details of all aspects of their proposed algorithm were not provided in this paper. Ramazan [16] proposed a method to aggregate a subset of blocks as branched trees, which are able to reduce the number of integer variables and number of constraints required within the MIP formulation. Boland et al. [17] developed a LP-based relaxation approach to solve large-size MBS instances. Bley et al. [18] relaxed the MIP formulation by adding inequalities through combining the precedence and production constraints. Many researchers indicated that the MIP formulation is computationally intractable for large-size MBS instances, thus leading to the development of numerous heuristic algorithms. Kumral and Dowd [19] developed a simulated annealing metaheuristic combined with Lagrangian relaxation for solving MBS. Ferland et al. [20] modelled the MBS problem as a resource-constrained project scheduling problem, which was solved by a particle swarm optimisation metaheuristic algorithm. Myburgh and Deb [21] reported an application of an evolutionary algorithm in which an initial feasible sequence of blocks represented as a chromosome is iteratively improved by genetic operators such as crossover and mutation. Cullenbine et al. [22] developed a sliding-time-window heuristic for MBS under a decomposition framework. Chicoisne et al. [11] developed an advanced heuristic algorithm based on topological sorting techniques to efficiently solve large-size MBS instances. Lamghari et al. [23] developed a hybrid approach based on linear programming and variable neighbourhood search for solving MBS efficiently. Liu and Kozan [15] developed two advanced graph-based algorithms without relying on any MIP optimiser to solve large-scale benchmark instances from MineLib [14]. Jélvez et al. [24] recently reported a block-aggregation heuristic algorithm for MBS and also validated its performance based on benchmark instances from MineLib. Nanjari and Golosinski [25] extended the MBS model by considering time value of money and mining restrictions. Mousavi et al. [26,27] developed an extended model to optimise the block sequencing by incorporating more real-life constraints such as blending, stockpiling and processing requirements. Nowadays, the MDP and MBS optimisation methodologies based on the 2D (3D) block graphic interfaces have been commonly developed by most off-the-shelf commercial mining software packages and implemented in Australian mine sites (e.g., Whittle Gemcom on strategic mine planning; XPAC on mine block sequencing).

After the determination of ultimate pit limit at the strategic level and the block sequence over mid-term periods at the tactical level, mining industry practitioners need to determine how and when different mining resource units with non-identical operating capacities (e.g., Drills, Mobile Processing Units, or Excavators) at various operational stages should be allocated to perform specific operations (e.g., Drilling, Blasting or Excavating operations) over a short time interval in a synchronised way. As the main resource at blasting stage, it is noted that Mobile Processing Units (MPUs) provide the service of adding explosive, exploding and subsequent marking on the blasted blocks. According to above literation review and recent comprehensive literature survey [28,29] on the applications of Operations Research approaches to mining industry, state-of-the-art multi-stage timetabling methodologies have not been applied to mine production at the operational level. In Australia, mining companies are keen to minimise the operational costs and maximise productivity of mining equipment by the adoption of advanced mining management software. For example, it was recently reported by Stringer [30] that "Rio Tinto, one of the world's biggest mining companies, spent $370 million on its 730-person technology and innovation unit in 2013 according to its annual report. Australia's mining industry alone spends about A$4 billion ($3.7 billion) a year on research and development according to the country's Bureau of Statistics". In this context, the requirement from Australia's mining industry has incentivised the researchers in mining optimisation to develop more advanced and industry-oriented optimisation methodologies for mining industry [31–37]. To meet the industrial requirement, a new operational-level *Multi-stage Mine Production Timetabling* (MMPT) model is proposed for open-pit mining industry to optimise drilling, blasting and excavating operations. As a contribution to mining industry, this paper is the pioneering study to develop a multi-stage mine production timetabling model to optimise drilling, blasting and excavating operations at the operational level. This paper also fills the gap between theory and practice due to the fact that the advanced multi-stage scheduling theory has not been widely studied and applied to mining industry yet.

The remainder of this paper is organised as follows. In Section 2, we define the MMPT problem and develop a mathematical programming model of MMPT. In Section 3, a heuristic algorithm based on an extended disjunctive graph is developed to efficiently solve MMPT. In Section 4, computational experiments are reported based on the real-world case study. Finally, we conclude the contribution and significance of this research in the last section.

2. MMPT Formulation

The geometry of a typical open-pit mine is depicted in Figure 1 and described in the following. According to the SME Mining Handbook [38], main considerations in the design of open-pit mine geometry are to accommodate mining equipment (e.g., shovels and trucks). Benches, which are divided into working benches and inactive benches, are the most critical geometry features of an open pit. In the operational mine production process, working benches are under being excavated while inactive benches are the remainder of working benches left to maintain pit-slop stability. Usually, the height of a bench is about 15 metres for an iron ore mine.

In a block model generated for exploration and mine design at the strategic level, the smallest element is termed as **a strategic-level block** (or a block unit in some literature), of which its size is 10*10*15 (10 meters in width, 10 meters in length and 15 meters in height) in this study. Note that the size of a block may be a little bit different (e.g., 20*20*12 or 15*15*15) in different mine sites with various ore types. The smaller a block's size is, the more accurate a block's material property is. Ultimate pit limit is the contour of an open pit that maximises the long-term value based on a strategic-level 2D or 3D block model in which the slope constraints (i.e., precedence relationships between blocks) are defined and the values of blocks are estimated. Nested pit shells (pits within pits) are usually generated by repeating the mine design planning process while adjusting the values of key parameters (e.g., either ore price or mining cost) in a specific (updated) block model in a rolling time horizon. A pushback is an incremental expansion of a pit outline and regarded as an aggregation of several nested pit shells. Pushbacks are often selected by balancing the strip ratios (the waste volume : the ore volume) throughout the mining phases. For example, a 10:1 strip ratio implies that excavating one cubic meter of ore would require removing ten cubic meters of waste during a mining phase. To meet grade control targets, blocks with different grade properties in different benchers or different pushbacks will be selected to be mined in each tactical-level period. A parcel is the set of strategic-level blocks which are both on the same bench and in the same nested pit shell. To provide enough working space for large mining equipment (e.g., shovel or excavator), some blocks have to be aggregated at the operational level. Consequently, **an operational-level mining job** is defined as an aggregate subset of blocks in the same grade group on the same bench in the same pushback so that the blocks in a mining job can be drilled, blasted, excavated at the same production rate. Note that the term “a mining job” is particularly defined in our MMPT model; and thus it is important to be distinguished from the term “a block” that is used as the smallest unit of a strategic-level block model.

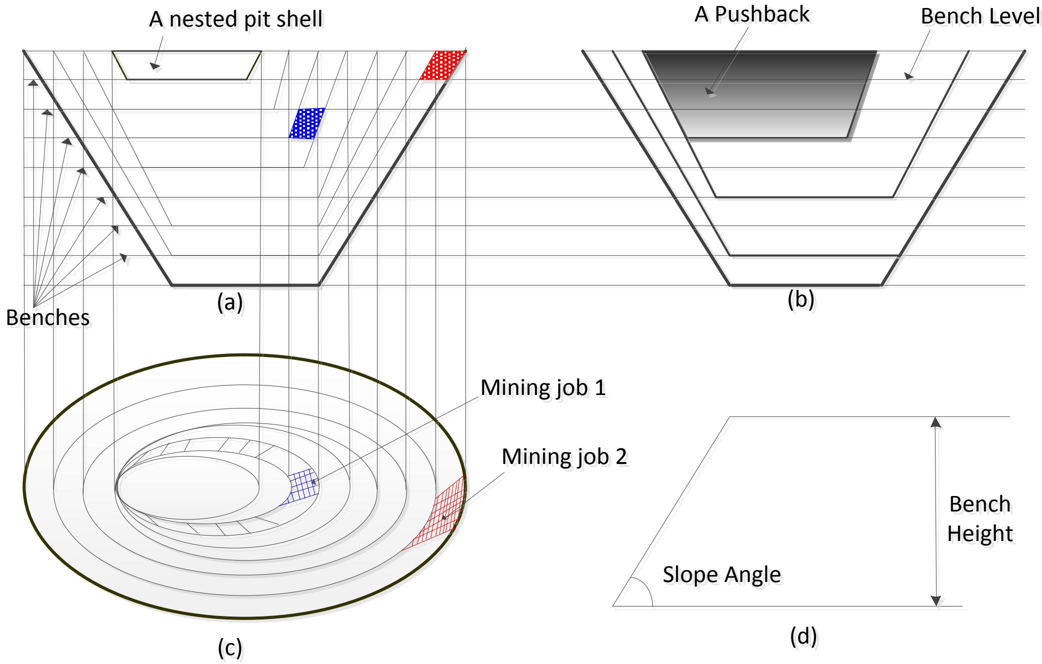


Figure 1: Schematic open-pit mine geometry: a-b) cross-section views of an open-pit with several benches; c) plan view of an open-pit; d) a bench geometry.

To comprehend the relationship among strategic-level MDP, tactical-level MBS and operational-level MMPT, the first important thing is to understand the operational mining jobs, each of which is an aggregate subset of smallest elements (i.e., blocks termed in the strategic-level block model) and such an aggregation is based on the output of tactical MBS model. The difference between a block and a mining job should be recognised, because in some real-world operational schedules from our cooperated mine site, the term “block” is always misused as a mining job (e.g., 60m*60m*15m) which is actually equivalent to 36 strategic-level blocks (36 times 10m*10m*15m). Moreover, the mining jobs in our MMPT model are usefully independent and accessible to be drilled, blasted and excavated in an operational-level time window, because various mining jobs are on different benches or in different pushbacks or from different pits.

With the cooperation with a large Australian iron ore mine site, the flow process of some critical operational stages is analysed as follows. In an operational time horizon, a mining job needs to be processed through several stages such as drilling (for blasting purpose), blasting and excavating. In the drilling stage, the blocks in a mining job are drilled for the purpose to collect the samples or to load the explosives to fragment the rock. The critical resource in the drilling stage is drill equipment. Note that the holes in this drilling stage are not for geostatistical modelling at the strategic exploration stage. Instead, the drilling samples (especially for metallic ores) are tested for achieving better blasting effects at the operational level. Collected ore samples are tested for ore properties such as ingredients and density. To achieve a good fragmentation during blasting, the sampling results are used to determine blasting pattern such as the strength and volume of explosives. At the excavating stage, blasted blocks in each mining job are extracted by excavators (shovels or front-end-loaders) and then simultaneously delivered to various destinations (mills, waste dumps or stockpiles) by a fleet of mine trucks. Note that the mine truck haulage optimisation model is beyond the scope of this paper and will be reported in another paper.

Based on the above analysis, a generic MMPT mixed integer programming model is developed to optimise the drilling, blasting and excavating operations at the operational level.

Indices and Parameters:

I number of independent mining jobs.

i index of a mining job, $i = 1, \ldots, I$.

K number of operational stages.

k index of an operational stage, $k = 1, \ldots, K$.

L_k number of equipment units used at stage k.

l index of an equipment unit at stage k, $l = 1, \ldots, L_k$.

r_{lk} ready time of equipment unit l at stage k.

s_{ilk} setup time for mining job i by equipment unit l at stage k.

Ω_{ik} workload for mining job i at stage k; for example, the workload of a mining job at drilling stage is the total drilling metres of holes; the workload of a mining job at blasting stage is the surface measured in square metres; the workload of a mining job at excavating stage is the volume measured in cubic metres.

θ_{lk} operating capacity of equipment unit l at stage k; for example, at drilling stage the operating capacity of a drill equipment unit is 50 meters per hour. Thus, if a mining job's total drilling workload is 1200 metres, then the operational time of this mining job at drilling stage is calculated as 24 hours if this drill equipment unit is assigned to this mining job's drilling operation.

ω_{ik} workload weighting factor (e.g., harness, density or swell factor) of mining job i at stage k, determined by its material property (e.g., high-grade, low-grade, or waste).

U a constant large value.

Decision Variables:

C_{ik} completion time of mining job i at stage k;
it is a semi-continuous timing variable; $i = 1, \dots, I;\ k = 1, \dots, K$.

x_{ilk} 1, if job i is assigned to a machine unit l at stage k;
0, otherwise; it is a binary variable; $i = 1, \dots, I;\ k = 1, \dots, K;\ l = 1, \dots, L_k$.

$y_{i'ilk}$ 1, if mining job i' precedes mining job i on the l^{th} equipment unit at stage k;
0, otherwise; it is a binary variable; $i, i' = 1, \dots, I | i \neq i';\ k = 1, \dots, K;\ l = 1, \dots, L_k$.

MMPT MIP Model

Objective:

$$Minimise\ \ (max_i\, C_{iK}) \tag{1}$$

Equation (1) defines the objective function of minimising the makespan which is equivalent to maximising the efficiency of mining equipment units at various stages. This is because the smaller completion times of mining equipment units leads to the earlier release times for the use in the next timetabling horizon.

Subject to:

$$C_{ik} \geq \sum_{l=1}^{L_k} x_{ilk}\, r_{lk} + \sum_{l=1}^{L_k} (s_{ilk} + \frac{\omega_{ik}\Omega_{ik}}{\theta_{lk}}) x_{ilk}, \quad i = 1, \dots, I;\ k = 1 \tag{2}$$

Equation (2) requires that the completion time of the first operation of each mining job should be greater than or equal to the ready time of the assigned equipment unit plus its setup time and processing time at the first stage depending on the assignment of an equipment unit.

$$C_{ik} \geq C_{i,k-1} + \sum_{l=1}^{L_k} (s_{ilk} + \frac{\omega_{ik}\Omega_{ik}}{\theta_{lk}}) x_{ilk}, \quad i = 1, \dots, I;\ k = 2, \dots, K \tag{3}$$

Equation (3) satisfies the processing route of each mining job through several stages, that is, the completion time of a mining job at a stage should be greater than or equal to its completion time at the immediate previous stage plus its setup time and processing time at the current stage.

$$\sum_{l=1}^{L_k} x_{ilk} = 1,$$
$$i = 1, \ldots, I;\ k = 1, \ldots, K \tag{4}$$

Equation (4) restricts that one equipment unit at each stage is allocated to only one mining job at a time.

$$\sum_{l=1}^{L_k} y_{i'ilk} \leq 1,$$
$$i, i' = 1, \ldots, I | i \neq i';\ k = 1, \ldots, K \tag{5}$$

$$\sum_{l=1}^{L_k} y_{ii'lk} \leq 1,$$
$$i, i' = 1, \ldots, I | i \neq i';\ k = 1, \ldots, K \tag{6}$$

$$\sum_{l=1}^{L_k} y_{i'ilk} + \sum_{l=1}^{L_k} y_{ii'lk} \leq 1,$$
$$i, i' = 1, \ldots, I | i \neq i';\ k = 1, \ldots, K \tag{7}$$

$$y_{i'ilk} + y_{ii'lk} \geq x_{ilk} + x_{i'lk} - 1,$$
$$i, i' = 1, \ldots, I | i \neq i';\ k = 1, \ldots, K;\ l = 1, \ldots, L_k \tag{8}$$

Equations (5-8) guarantee that the sequencing relationship between each pair of mining jobs on any assigned equipment unit at each stage should be exclusive.

$$C_{ik} \geq C_{i'k} + \sum_{l=1}^{L_k} (s_{ilk} + \frac{\omega_{ik}\Omega_{ik}}{\theta_{lk}}) x_{ilk} + \left(\sum_{l=1}^{L_k} y_{i'ilk} - 1\right) * U,$$
$$i, i' = 1, \ldots, I | i \neq i';\ k = 1, \ldots, K; \tag{9}$$

$$C_{i'k} \geq C_{ik} + \sum_{l=1}^{L_k} (s_{i'lk} + \frac{\omega_{i'k}\Omega_{i'k}}{\theta_{lk}}) x_{i'lk} + \left(\sum_{l=1}^{L_k} y_{ii'lk} - 1\right) * U,$$
$$i, i' = 1, \ldots, I | i \neq i';\ k = 1, \ldots, K; \tag{10}$$

Equations (9-10) satisfy the timing relationship between each pair of mining jobs on the assigned equipment unit at each stage. Here, the implication on assignment decisions of different resource units at a stage is illustrated. For example, the processing time of such a drilling operation is 12 ($\omega_{ik}\Omega_{ik}/\theta_{lk}$=1.2*200/20=12) hours. On the other hand, if the capacity ($\theta_{l'k}$) of another drill equipment (indexed by l') at drilling stage (indexed by k) is 25 m/hr, then the model may choose it and thus the drilling time may be reduced to 9.6 ($\omega_{ik}\Omega_{ik}/\theta_{l'k}$=1.2*200/25=9.6) hours. The processing time depends on the workload (Ω_{ik}) of an operation, the workload weighting factor (ω_{ik}) related to ore properties, and the operating capacity of assigned equipment unit (θ_{lk}). In addition, the implementation of equipment units with different operating capacities is due to real-world mining requirements. For instance, a large excavator processes the corresponding excavating operation in a shorter extraction time. However, a small front-end loader is also needed for a limited-space working bench.

The mathematical formulation model is essential to ensure that critical constraints are satisfied. Although a MIP exact solver (e.g., IBM ILOG-CPLEX) can exactly solve the MMPT MIP model for small-size instances, it is intractable (time-consuming and memory-demanding) for solving industry-scale instances due to strong NP-hardness. This implies that no polynomial-time algorithms could be developed to exactly solve this problem type and it is better to take efforts in developing an approximation algorithm [39]. In practice, mining industry practitioners need to make a rapid response in real-world scenarios. Such requirements inspire us to develop an efficient shifting-bottleneck-procedure (SBP)-based algorithm to efficiently solve MMPT by taking advantage of MMPT's properties in terms of an extended disjunctive graph [40–45].

3. Heuristic Approach

In this section, an improved shifting-bottleneck-procedure (SBP) algorithm based on an extended disjunctive graph is developed to solve MMPT.

Extended Disjunctive graph for MMPT

An extended activity-on-the-vertex *disjunctive graph* $DG = (V, C, E)$ particularly devised for MMPT is shown in Figure 2.

In $DG = (V, C, E)$, the set of all vertices is defined as V and the number of potential actual vertices (operations) is equal to $|V| = I \times \sum_{k=1}^{K} L_k$, where I is the number of mining jobs, K is the number of operational stages, L_k is the number of equipment units at stage k. In addition, two virtual vertices (source and sink) with zero processing time and setup time are added to V. Each directed arc has a distance value that equals the processing time plus the setup time of the predecessor of this arc. In $DG = (V, C, E)$, C is the set of conjunctive arcs which denote the fixed precedence relationships between each pair of potential operations belonging to the same mining job as well as the directed arcs for connecting the virtual source (sink) vertex with the first (last) operation of each mining job. In $DG = (V, C, E)$, E is the set of disjunctive arcs, each of which represents the undecided sequencing status between any pair of potential operations if they are performed by the same equipment unit at a stage.

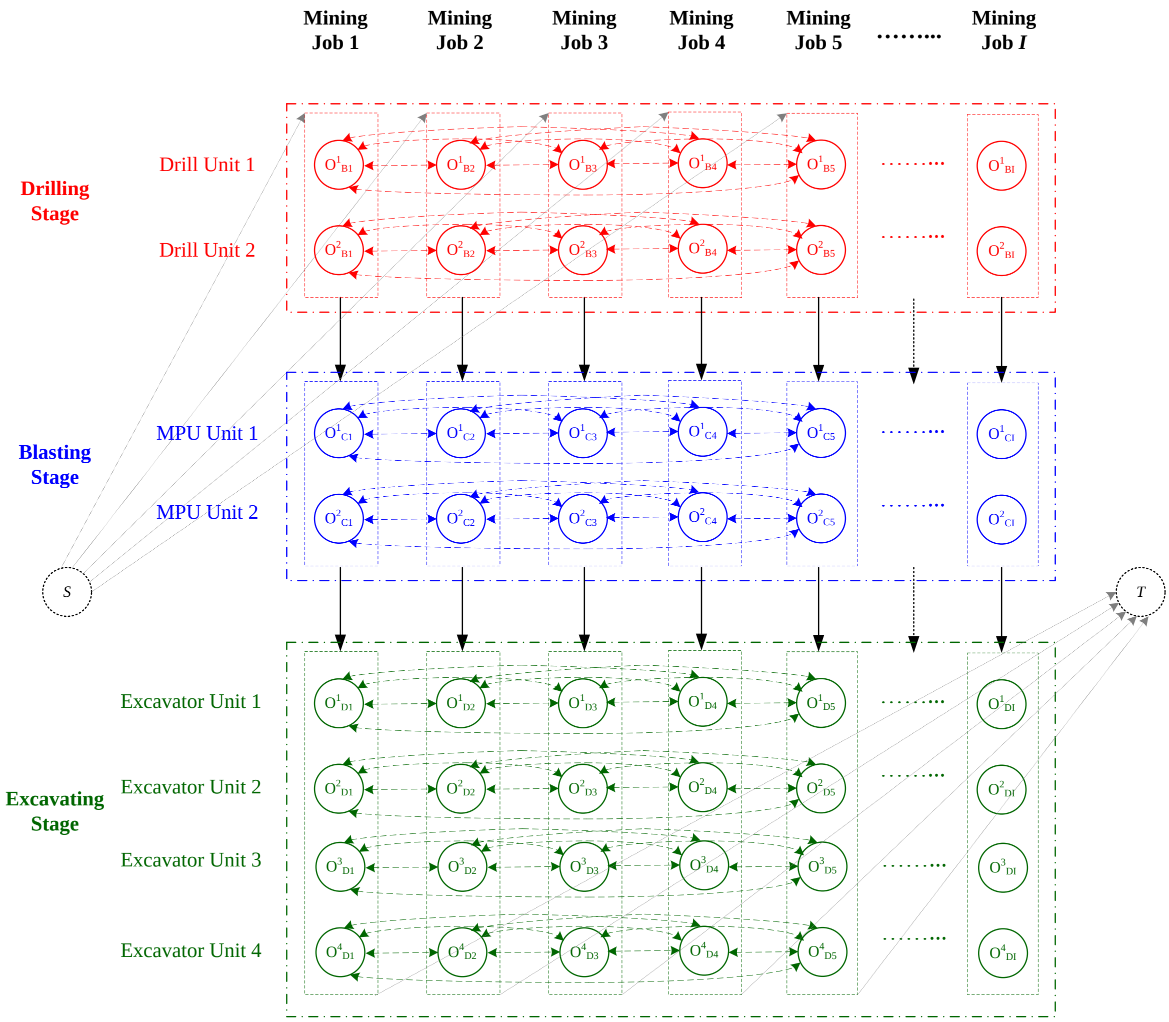


Figure 2: An extended disjunctive graph for 3-stage MMPT

On each stage in the extended disjunctive graph, the vertices surrounded by a dot-line rectangle represent the potential operations that are processed by an equipment unit at a stage. For example, operations O_{B3}^1 and O_{B3}^2 are highlighted by superscripts 1 and 2 for indicating Unit 1 and Unit 2 of equipment type B for Mining Job 3. Note that the relationship between operations O_{B3}^1 and O_{B3}^2 are exclusive, that is, only one of them is existing because it is supposed in our MMPT model that one mining job is allocated to only one equipment unit at each stage.

A conjunctive arc (e.g., $O_{B3}^{1} \rightarrow O_{C3}^{2}$) connects two existing actual vertices O_{B3}^{1} and O_{C3}^{2}, which implies that operation O_{B3}^{1} should be processed before operation O_{C3}^{2}. For example, for Mining Job 3 that has a fixed processing route through drilling, blasting and excavating stages, the subset of conjunctive arcs for three operations of this mining job may be listed as: $O_{B3}^{1} \rightarrow O_{C3}^{2} \rightarrow O_{D3}^{4}$, if O_{B3}^{1} is an existing operation that is associated with equipment unit 1 of equipment type B (i.e., Drill Unit 1) at the drilling stage for Mining Job 3; O_{C3}^{2} is an existing operation that is associated with equipment unit 2 of equipment type C (i.e., MPU Unit 2) at the blasting stage for Mining Job 3; O_{D3}^{4} is an existing operation that is associated with equipment unit 4 of equipment type D (i.e., Excavator Unit 4) at the excavating stage for Mining Job 3.

The number of disjunctive arcs on a single-equipment-unit stage $k|L_k = 1$ is $n_k \times (n_k - 1)$, where $n_k|n_k \leq I$ is the number of mining jobs that are actually processed on stage k. The properties of disjunctive arcs on a multiple-equipment-units stage $k|L_k > 1$ is much more complicated, as the number of disjunctive arcs is equal to $n_k \times (n_k - 1) \times {L_k}^2$ in this case. With the above analysis, the set of total disjunctive arcs E can be divided into K subsets, i.e., $E = \{E_1, E_2, \dots, E_K\}$, where $E_k \subseteq E|k = 1,2, \dots, K$ denotes the subset of disjunctive arcs at stage k. In a sense, the determination of a MMPT schedule is transformed to decide the direction for each pair of disjunctive arcs and discarding the redundant arcs in $E = \bigcup_{k=1}^{K} E_k$. After such a determination, let $E_k^{\wedge}$ denote the subset of *directed disjunctive arcs* at stage k and let $E^{\wedge} = \bigcup_{k=1}^{K} E_k^{\wedge}$ denote a corresponding compound set of directed disjunctive arcs at all stages. Then, the *directed disjunctive graph* denoted by $DDG = (V, C, E^{\wedge})$ represents a MMPT schedule. A feasible MMPT schedule requires that $DDG = (V, C, E^{\wedge})$ should be acyclic while satisfying all of the constraints.

Based on the above extended disjunctive graph, the detailed procedure of an improved SBP algorithm for MMPT is presented as follows.

Improved SBP Algorithm for MMPT

Step 1: Set the initial *partial directed disjunctive graph* $PDDG = (V, C)$ which contains all of conjunctive arcs but no disjunctive arcs.

Step 2: Initialise the list of unscheduled stages Ω ($|\Omega| = K$) and the list of scheduled stages as empty: $\Omega^{\wedge} \leftarrow \phi$.

Step 3: While $\Omega \neq \phi$:

- 3.1: Determine the topological sequence of vertices based on the current $PDDG$.
  - 3.1.1: Based on the current *PDDG*, set the list of current immediate predecessors and successors of each vertex.
  - 3.1.2: Compute the in-count value (i.e., the number of immediate predecessors) of each vertex in the graph.
  - 3.1.3: Decrease the in-count value for each of the immediate successor of the selected vertex by one after determining the current topological vertex.
  - 3.1.4: If none of the undetermined vertices have a zero in-count value, stop running algorithm as the given graph model is cyclic (infeasible); else, select any of the undetermined vertices having a zero in-count value and put this vertex as the next candidate in the topological order.
  - 3.1.5: Repeat Steps 3.1.1-3.1.4 until all vertices are determined.
- 3.2: Compute the head time (i.e., the longest distance from the virtual source to a vertex) and the tail time (i.e., the longest distance from a vertex to the virtual sink) of each vertex in terms of the topological sequence.
- 3.3: Generate a single-equipment-unit scheduling subproblem with release dates (head times) and delivery times (tail times) of mining jobs at stage k, if $L_k = 1$; otherwise, generate a multiple-equipment-unit scheduling subproblem with release dates and delivery times.
- 3.4: For each subproblem generated above:
  - 3.4.1: If it is a single-equipment-unit scheduling subproblem, then solve it by the following procedure:
    - 3.4.1.1: Initialise Π that is the list of unscheduled mining jobs and $\Pi^{\wedge} \leftarrow \phi$ that is the list of scheduled mining jobs in the subproblem.
    - 3.4.1.2: Set the current time point: $t \leftarrow \min_{i \in \Pi} r_i$, where r_i is the release date (head time) of mining job i in the subproblem.
    - 3.4.1.3: At the current time point t, set the starting time $e_{i^*} = t$ of a selected mining job i^*, under the condition $r_{i^*} \leq t$ and $i^* \leftarrow \text{argmax}_{i \in \Pi} q_i$, where q_i is the delivery time (tail time) of mining job i;
    - 3.4.1.4: Update $\Pi \leftarrow \Pi - i^*$, $\Pi^{\wedge} \leftarrow \Pi^{\wedge} \cup i^*$ and $t \leftarrow max(e_{i^*} + p_{i^*}, min_{i \in \Pi} r_i)$ where p_{i^*} is the processing time of selected mining job i^* in the subproblem.
    - 3.4.1.5: If $\Pi = \phi$, go to Step 3.4; otherwise, go to Step 3.4.1.2.

3.4.2: Else if it is a multiple-equipment-unit scheduling subproblem, then solve it by the following procedure:

3.4.2.1: Initialise Π that is the list of unscheduled mining jobs and $\Pi^{\wedge} \leftarrow \phi$ that is the list of scheduled mining jobs in the subproblem.

3.4.2.2: Initialise the available times of all machines (equipment units) in the subproblem: $\alpha_l = 0 | l = 1, \dots, L_k$.

3.4.2.3: Choose the earliest available equipment unit $l^* \leftarrow argmin_{l=1,\dots,L_k} \alpha_l$.

3.4.2.4: Set a list of candidate mining jobs: $\Gamma \leftarrow \Gamma \cup i$ if $r_i \geq \alpha_{l^*} | i \in \Pi$.

3.4.2.5: If $\Gamma = \phi$, set the selected job: $i^* \leftarrow argmax_{i \in \Pi} q_i$ and its starting time: $e_{i^*} = \alpha_{l^*}$; else, set the selected job: $i^* \leftarrow argmin_{i \in \Gamma} r_i$ and its starting time: $e_{i^*} = r_{i^*}$.

3.4.2.6: Update $\Pi \leftarrow \Pi - i^*$, $\Pi^{\wedge} \leftarrow \Pi^{\wedge} \cup i^*$ and $\alpha_{l^*} \leftarrow e_{i^*} + p_{i^*}$.

3.4.2.7: If $\Pi = \phi$, go to Step 3.4; otherwise, go to Step 3.4.2.3.

3.5: Based on the obtained solutions of decomposed subproblems, determine the critical stage: $k^* \leftarrow argmax_{k \in \Omega} M_k$, where M_k is the makespan of one subproblem at stage k.

3.6: Update $\Omega^{\wedge} \leftarrow \Omega^{\wedge} \cup k^*$, $\Omega \leftarrow \Omega - k^*$ and $PDDG = (V, C, \bigcup_{k \in \Omega^{\wedge}} E_k^{\wedge})$.

4. Case Study

The proposed MMPT methodology was applied in an industrial project with the cooperation of an Australian mining company. In this section, a case study established based on the real-world mining data is conducted to justify the applicability of the MMPT methodology.

Data Analysis in Case Study

In this case study, a number of varied-size mining jobs (i.e., from 5 mining jobs to 55 mining jobs in different time windows) with different combinations of equipment units at various stages under different MMPT instances will be scheduled. Each mining job will be processed consecutively through several stages such as drilling, blasting and excavating. Note that all time elements are measured in hours in this MMPT case study. The critical equipment at drilling stage is drill equipment with two available units in this case study. The potential drilling time of each mining job is determined by the total drilling meter and the operating rate of a drill. The critical resource unit at blasting stage is *Mobile Processing Unit* (MPU) with two available units. A blasting operation actually consists of several tasks including sampling, explosive adding, exploding, clearing and marking. The sampling times are dependent on the sizes of mining jobs and regarded as various setup times at a blasting stage. At blasting stage, the blasted areas are not allowed to be entered immediately due to safety requirements and the need of subsequent clearing and marking for blasted blocks. Thus, the processing time of a blasting operation is not only dependent on the surface size (in square metres) of each mining job but also on safety requirements. At excavating stage, excavators are critical equipment with two to eight available non-identical excavators (shovels or front-end-loaders) that have different extraction rates. The excavating time depends on the volume (m^3) of a mining job and the extraction rate (m^3/hr) of the allocated excavator. The input data items of a MMPT instance are presented in Tables 1-2, in which some technical factors are obtained from our industry partners and referred to the SME mining engineering handbook [46].

Table 1: Data of resources at drilling, blasting and excavating stages in a MMPT instance*

Stage Name	Equipment Type	Measurement	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	…
Drilling	Drills	m/hr	50	60	NA	NA	NA	……
Blasting	MPUs	m^2/hr	300	450	NA	NA	NA	……
Excavating	Excavators	m^3/hr	600	700	800	900	1000	……

*NA: Not-Available

Table 2: Data of mining jobs in a MMPT instance with 55 mining jobs*

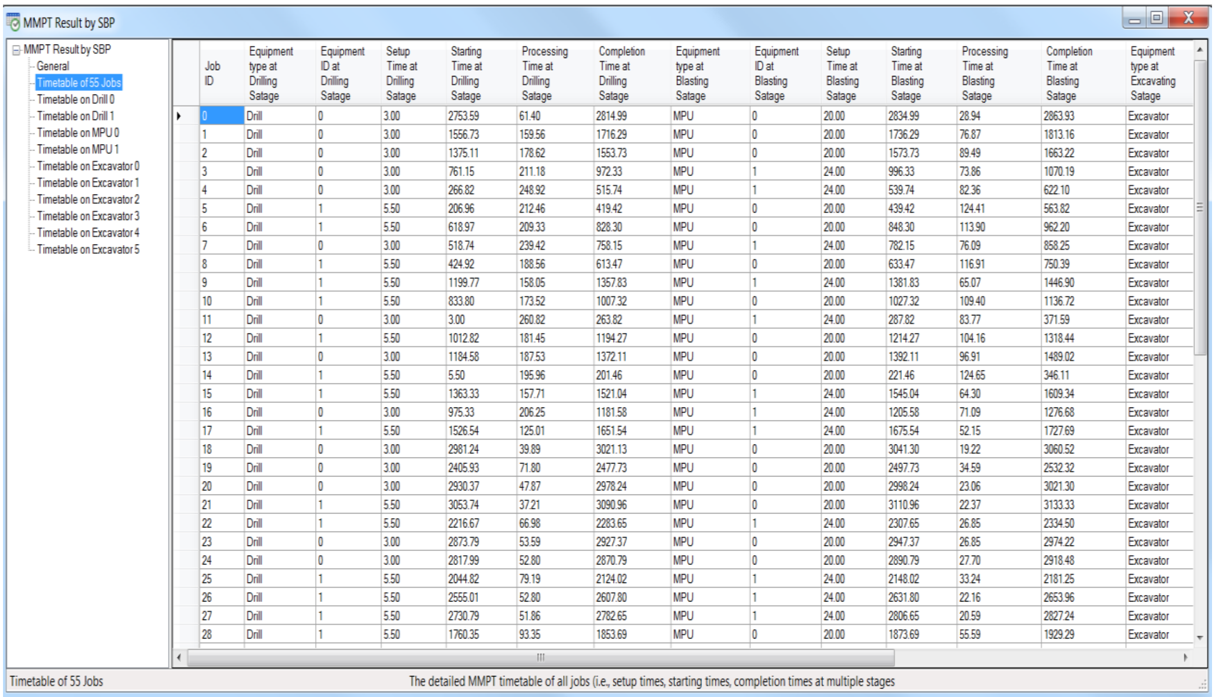
Mining Job ID	Tonnes (t)	Drilling Metre (m)	Surface (m^2)	Volume (m^3)	Number of Blocks	Material Property** (grade)
1	97656	768	2170	32552	22	Hg
2	175780	1382	3906	58593	39	Lg
3	117187	921	2604	39062	26	Hg
4	259440	1994	5765	86480	58	Lg
5	466992	3590	10377	155664	104	Hg
6	311328	2393	6918	103776	69	W
7	302019	2233	6712	100673	67	Lg
8	543635	4019	12081	181212	121	Hg
9	362423	2679	8054	120808	81	Hg
10	373895	2640	8309	124632	83	Hg
11	673010	4752	14956	224337	150	W
12	448673	3168	9971	149558	100	W
13	416945	3112	9266	138982	93	Lg
14	750501	5601	16678	250167	167	Hg
15	500334	3734	11119	166778	111	Hg
16	419868	3187	9331	139956	94	Lg
17	755762	5736	16795	251921	168	Hg
18	503842	3824	11197	167947	112	Hg

19	384422	3140	8543	128141	86	Lg
20	691960	5652	15377	230653	154	Lg
21	461307	3768	10251	153769	103	Hg
22	385224	2993	8561	128408	86	Hg
23	693402	5387	15409	231134	154	Hg
24	462268	3591	10273	154089	103	Lg
…	……	……	……	……	……	……
…	……	……	……	……	……	……
55	392486	2813	8722	130829	87	Hg

**Due to confidentiality agreement, values of some data items are relatively modified in the case study.*
***Hg: high grade; Lg: low grade; W: waste.*

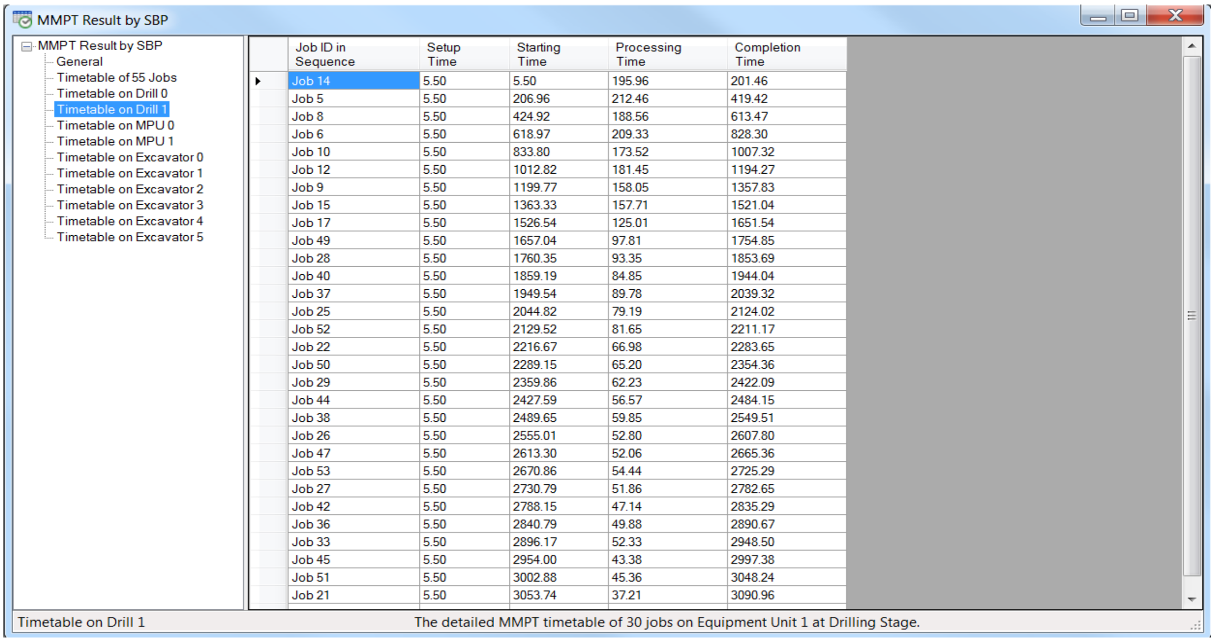
Solution representation of one MMPT instance

The proposed MMPT methodology was coded in Visual C#. After inputting the data of one MMPT instance from an Excel Workbook file and then running either of two solution approaches (either ILOG-CPLEX or the improved SBP algorithm), the MMPT timetable is obtained and saved to an output file into an Excel Workbook. For instance, the solution representation of a 55-job 2-drill 2-MPU 5-excavator MMPT result (see Instance MMPT_47 in Table 4) is shown in Figure 3-5, in which Figure 3 displays the timetable of all 55 mining jobs through drilling, blasting and excavating stages; Figure 4 shows the timetable of 30 mining operations by Drill Unit 1 at drilling stage; Figure 5 shows the timetable of 10 mining operations by Excavator Unit 5 at excavating stage.



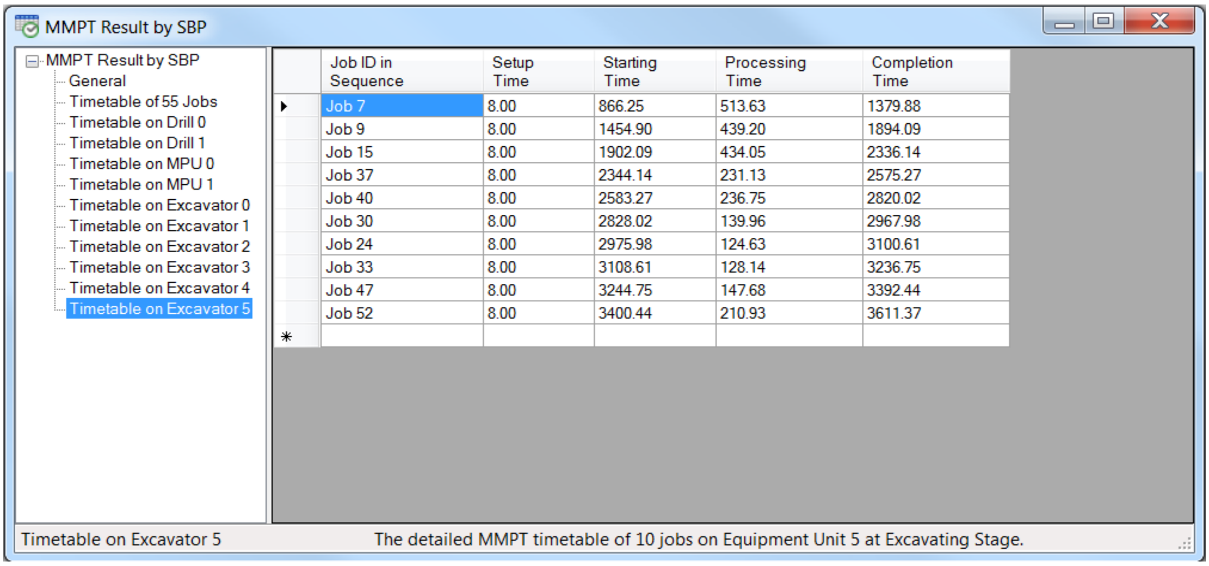
Job ID	Equipment type at Drilling Satage	Equipment ID at Drilling Satage	Setup Time at Drilling Satage	Starting Time at Drilling Satage	Processing Time at Drilling Satage	Completion Time at Drilling Satage	Equipment type at Blasting Satage	Equipment ID at Blasting Satage	Setup Time at Blasting Satage	Starting Time at Blasting Satage	Processing Time at Blasting Satage	Completion Time at Blasting Satage	Equipment type at Excavating Satage
0	Drill	0	3.00	2753.59	61.40	2814.99	MPU	0	20.00	2834.99	28.94	2863.93	Excavator
1	Drill	0	3.00	1556.73	159.56	1716.29	MPU	0	20.00	1736.29	76.87	1813.16	Excavator
2	Drill	0	3.00	1375.11	178.62	1553.73	MPU	0	20.00	1573.73	89.49	1663.22	Excavator
3	Drill	0	3.00	761.15	211.18	972.33	MPU	1	24.00	996.33	73.86	1070.19	Excavator
4	Drill	0	3.00	266.82	248.92	515.74	MPU	1	24.00	539.74	82.36	622.10	Excavator
5	Drill	1	5.50	206.96	212.46	419.42	MPU	0	20.00	439.42	124.41	563.82	Excavator
6	Drill	1	5.50	618.97	209.33	828.30	MPU	0	20.00	848.30	113.90	962.20	Excavator
7	Drill	0	3.00	518.74	239.42	758.15	MPU	1	24.00	782.15	76.09	858.25	Excavator
8	Drill	1	5.50	424.92	188.56	613.47	MPU	0	20.00	633.47	116.91	750.39	Excavator
9	Drill	1	5.50	1199.77	158.05	1357.83	MPU	1	24.00	1381.83	65.07	1446.90	Excavator
10	Drill	1	5.50	833.80	173.52	1007.32	MPU	0	20.00	1027.32	109.40	1136.72	Excavator
11	Drill	0	3.00	3.00	260.82	263.82	MPU	1	24.00	287.82	83.77	371.59	Excavator
12	Drill	1	5.50	1012.82	181.45	1194.27	MPU	0	20.00	1214.27	104.16	1318.44	Excavator
13	Drill	0	3.00	1184.58	187.53	1372.11	MPU	0	20.00	1392.11	96.91	1489.02	Excavator
14	Drill	1	5.50	5.50	195.96	201.46	MPU	0	20.00	221.46	124.65	346.11	Excavator
15	Drill	1	5.50	1363.33	157.71	1521.04	MPU	1	24.00	1545.04	64.30	1609.34	Excavator
16	Drill	0	3.00	975.33	206.25	1181.58	MPU	1	24.00	1205.58	71.09	1276.68	Excavator
17	Drill	1	5.50	1526.54	125.01	1651.54	MPU	1	24.00	1675.54	52.15	1727.69	Excavator
18	Drill	0	3.00	2981.24	39.89	3021.13	MPU	0	20.00	3041.30	19.22	3060.52	Excavator
19	Drill	0	3.00	2405.93	71.80	2477.73	MPU	0	20.00	2497.73	34.59	2532.32	Excavator
20	Drill	0	3.00	2930.37	47.87	2978.24	MPU	0	20.00	2998.24	23.06	3021.30	Excavator
21	Drill	1	5.50	3053.74	37.21	3090.96	MPU	0	20.00	3110.96	22.37	3133.33	Excavator
22	Drill	1	5.50	2216.67	66.98	2283.65	MPU	1	24.00	2307.65	26.85	2334.50	Excavator
23	Drill	0	3.00	2873.79	53.59	2927.37	MPU	0	20.00	2947.37	26.85	2974.22	Excavator
24	Drill	0	3.00	2817.99	52.80	2870.79	MPU	0	20.00	2890.79	27.70	2918.48	Excavator
25	Drill	1	5.50	2044.82	79.19	2124.02	MPU	1	24.00	2148.02	33.24	2181.25	Excavator
26	Drill	1	5.50	2555.01	52.80	2607.80	MPU	1	24.00	2631.80	22.16	2653.96	Excavator
27	Drill	1	5.50	2730.79	51.86	2782.65	MPU	1	24.00	2806.65	20.59	2827.24	Excavator
28	Drill	1	5.50	1760.35	93.35	1853.69	MPU	0	20.00	1873.69	55.59	1929.29	Excavator

Figure 3: Timetable of 55 jobs under one MMPT solution



Job ID in Sequence	Setup Time	Starting Time	Processing Time	Completion Time
Job 14	5.50	5.50	195.96	201.46
Job 5	5.50	206.96	212.46	419.42
Job 8	5.50	424.92	188.56	613.47
Job 6	5.50	618.97	209.33	828.30
Job 10	5.50	833.80	173.52	1007.32
Job 12	5.50	1012.82	181.45	1194.27
Job 9	5.50	1199.77	158.05	1357.83
Job 15	5.50	1363.33	157.71	1521.04
Job 17	5.50	1526.54	125.01	1651.54
Job 49	5.50	1657.04	97.81	1754.85
Job 28	5.50	1760.35	93.35	1853.69
Job 40	5.50	1859.19	84.85	1944.04
Job 37	5.50	1949.54	89.78	2039.32
Job 25	5.50	2044.82	79.19	2124.02
Job 52	5.50	2129.52	81.65	2211.17
Job 22	5.50	2216.67	66.98	2283.65
Job 50	5.50	2289.15	65.20	2354.36
Job 29	5.50	2359.86	62.23	2422.09
Job 44	5.50	2427.59	56.57	2484.15
Job 38	5.50	2489.65	59.85	2549.51
Job 26	5.50	2555.01	52.80	2607.80
Job 47	5.50	2613.30	52.06	2665.36
Job 53	5.50	2670.86	54.44	2725.29
Job 27	5.50	2730.79	51.86	2782.65
Job 42	5.50	2788.15	47.14	2835.29
Job 36	5.50	2840.79	49.88	2890.67
Job 33	5.50	2896.17	52.33	2948.50
Job 45	5.50	2954.00	43.38	2997.38
Job 51	5.50	3002.88	45.36	3048.24
Job 21	5.50	3053.74	37.21	3090.96

Figure 4: Timetable of Drill Unit 1 under one MMPT solution



Job ID in Sequence	Setup Time	Starting Time	Processing Time	Completion Time
Job 7	8.00	866.25	513.63	1379.88
Job 9	8.00	1454.90	439.20	1894.09
Job 15	8.00	1902.09	434.05	2336.14
Job 37	8.00	2344.14	231.13	2575.27
Job 40	8.00	2583.27	236.75	2820.02
Job 30	8.00	2828.02	139.96	2967.98
Job 24	8.00	2975.98	124.63	3100.61
Job 33	8.00	3108.61	128.14	3236.75
Job 47	8.00	3244.75	147.68	3392.44
Job 52	8.00	3400.44	210.93	3611.37

Figure 5: Timetable of Excavator Unit 5 under one MMPT solution

Solution analysis of small-size MMPT instances by CPLEX

Table 3 presents the optimal results (except Instance MMPT_21) of small-size MMPT instances solved by ILOG-CPLEX. In Table 3, the first column (*Instance ID*) indicates the index of each MMPT instance. The second column (*MMPT Problem Size*) defines the problem size (e.g., number of mining job; number of drills at drilling stage; number of MPUs at blasting stage; number of excavators at excavating stage). The third column (*MMPT MIP Model Size*) shows the complexity of MMPT MIP models for various instances (e.g., number of variables; number of constraints; number of nodes in CPLEX modelling). The last three columns respectively give the objective function value, the ILOG-CPLEX CPU time and solution status. Note that for most of small-size MMPT instances, the solution status of ILOG-CPLEX is *optimal*. For the last instance (i.e., MMPT-21) in Table 3, however, the CPLEX solution status is still *feasible* even after running over 3600 seconds with evaluation of over 10 million nodes. Computational results in Table 3 validate that only small-size MMPT instances can be exactly solved by the MIP solver in a reasonable CPU time.

Table 3: Computational results of small-size MMPT instances by CPLEX*

Instance ID	MMPT Problem Size	MMPT MIP Model Size	Objective Value	CPU Time (s)	Solution Status
MMPT_1	5Jobs_2Drills_1MPUs_2Exs	141 Vars; 255 Cons; 111 Nodes	1745.41	0.219	Optimal
MMPT_2	5Jobs_2Drills_1MPUs_3Exs	166 Vars; 275 Cons; 948 Nodes	1312.78	0.265	Optimal
MMPT_3	5Jobs_2Drills_2MPUs_2Exs	166 Vars; 275 Cons; 123 Nodes	1709.03	0.203	Optimal
MMPT_4	5Jobs_2Drills_2MPUs_3Exs	191 Vars; 295 Cons; 2423 Nodes	1271.18	0.203	Optimal
MMPT_5	6Jobs_2Drills_1MPUs_2Exs	199 Vars; 372 Cons; 6735 Nodes	2160.63	1.123	Optimal
MMPT_6	6Jobs_2Drills_1MPUs_3Exs	235 Vars; 402 Cons; 6190 Nodes	1531.69	1.077	Optimal
MMPT_7	6Jobs_2Drills_2MPUs_2Exs	235 Vars; 402 Cons; 2447 Nodes	2136.05	0.375	Optimal
MMPT_8	6Jobs_2Drills_2MPUs_3Exs	271 Vars; 432 Cons; 6587 Nodes	1474.15	1.076	Optimal
MMPT_9	7Jobs_2Drills_1MPUs_2Exs	267 Vars; 511 Cons; 11809 Nodes	2537.71	2.808	Optimal
MMPT_10	7Jobs_2Drills_1MPUs_3Exs	316 Vars; 553 Cons; 16906 Nodes	1765.15	2.528	Optimal
MMPT_11	7Jobs_2Drills_2MPUs_2Exs	316 Vars; 553 Cons; 9395 Nodes	2530.98	3.229	Optimal
MMPT_12	7Jobs_2Drills_2MPUs_3Exs	365 Vars; 595 Cons; 17004 Nodes	1743.79	2.902	Optimal
MMPT_13	8Jobs_2Drills_1MPUs_2Exs	345 Vars; 672 Cons; 55393 Nodes	2921.68	15.880	Optimal
MMPT_14	8Jobs_2Drills_1MPUs_3Exs	409 Vars; 728 Cons; 96339 Nodes	1998.30	13.167	Optimal
MMPT_15	8Jobs_2Drills_2MPUs_2Exs	409 Vars; 728 Cons; 21392 Nodes	2920.05	10.561	Optimal
MMPT_16	8Jobs_2Drills_2MPUs_3Exs	473 Vars; 784 Cons; 83695 Nodes	1963.09	11.654	Optimal
MMPT_17	9Jobs_2Drills_1MPUs_2Exs	433 Vars; 855 Cons; 192341 Nodes	3355.12	105.192	Optimal
MMPT_18	9Jobs_2Drills_1MPUs_3Exs	514 Vars; 927 Cons; 963948 Nodes	2246.87	168.216	Optimal
MMPT_19	9Jobs_2Drills_2MPUs_2Exs	514 Vars; 927 Cons; 138463 Nodes	3338.78	67.814	Optimal
MMPT_20	9Jobs_2Drills_2MPUs_3Exs	595 Vars; 999 Cons; 290322 Nodes	2222.70	54.975	Optimal
MMPT_21	10Jobs_2Drills_2MPUs_3Exs	731 Vars; 1240 Cons; 10012612 Nodes	2434.49	3600.052	Feasible

*Exs: Excavators; Vars: Variables; Cons: Constraints.

Solution analysis of large-size MMPT instances by SBP

To evaluate the solution quality, a tight lower bound (*LB*) computed by the following equation is used in the third column (*Lower Bound*).

$$k^* = arg\max_{k=1,\ldots K}\left(\sum_{i=1}^{I}\left(\min_{l=1,\ldots,L_k}\left(\frac{\omega_{ik}\Omega_{ik}}{\theta_{lk}} + s_{ilk}\right)\right)\right)$$

$$LB = \frac{\sum_{i=1}^{I}\left(\min_{l=1,\ldots,L_{k^*}}\left(\frac{\omega_{ik^*}\Omega_{ik^*}}{\theta_{lk^*}}+s_{ilk^*}\right)\right)}{L_{k^*}} + min_{i=1,\ldots,I}\left(\sum_{k=1|k\neq k^*}^{K}\left(\min_{l=1,\ldots,L_k}\left(\frac{\omega_{ik}\Omega_{ik}}{\theta_{lk}} + s_{ilk}\right)\right)\right)$$

Computational results of 27 large-size MMPT instances (i.e., Instances MMPT_22 to MMPT_48) obtained by the improved SBP algorithm are presented in Table 4.

Table 4: Computational results of large-size MMPT instances by SBP

Instance ID	MMPT Problem Size	Lower Bound	Objective Value	CPU Time (s)	Deviation (%)
MMPT_22	15Jobs_2Drills_1MPUs_3Exs	3766.98	4048.97	0.093	6.96
MMPT_23	15Jobs_2Drills_1MPUs_4Exs	2788.63	3084.39	0.089	9.59
MMPT_24	15Jobs_2Drills_1MPUs_5Exs	2240.56	2679.31	0.065	16.38
MMPT_25	20Jobs_2Drills_2MPUs_3Exs	4394.01	4631.55	0.215	5.13
MMPT_26	20Jobs_2Drills_2MPUs_4Exs	3204.54	3401.87	0.225	5.80
MMPT_27	20Jobs_2Drills_2MPUs_5Exs	2538.17	2819.58	0.185	9.98
MMPT_28	25Jobs_2Drills_2MPUs_3Exs	4663.63	4858.23	0.282	4.01
MMPT_29	25Jobs_2Drills_2MPUs_4Exs	3384.84	3613.93	0.281	6.34
MMPT_30	25Jobs_2Drills_2MPUs_5Exs	2668.39	2950.23	0.286	9.55
MMPT_31	30Jobs_2Drills_2MPUs_3Exs	5057.72	5435.36	0.432	6.95
MMPT_32	30Jobs_2Drills_2MPUs_4Exs	3648.13	3947.69	0.412	7.59
MMPT_33	30Jobs_2Drills_2MPUs_5Exs	2858.36	3321.36	0.415	13.94
MMPT_34	35Jobs_2Drills_2MPUs_3Exs	5447.14	5910.02	0.731	7.83
MMPT_35	35Jobs_2Drills_2MPUs_4Exs	3908.30	4390.31	0.735	10.98
MMPT_36	35Jobs_2Drills_2MPUs_5Exs	3046.08	3451.97	0.736	11.76
MMPT_37	40Jobs_2Drills_2MPUs_3Exs	5786.69	6180.21	1.209	6.37
MMPT_38	40Jobs_2Drills_2MPUs_4Exs	4135.22	4629.67	1.189	10.68
MMPT_39	40Jobs_2Drills_2MPUs_5Exs	3209.86	3588.94	1.184	10.56
MMPT_40	45Jobs_2Drills_2MPUs_3Exs	6176.98	6646.90	1.235	7.07
MMPT_41	45Jobs_2Drills_2MPUs_4Exs	4395.97	4929.96	2.216	10.83
MMPT_42	45Jobs_2Drills_2MPUs_5Exs	3398.01	3853.75	2.212	11.83
MMPT_43	50Jobs_2Drills_2MPUs_4Exs	4647.66	5219.69	2.313	10.96
MMPT_44	50Jobs_2Drills_2MPUs_5Exs	3579.62	4133.86	3.309	13.41
MMPT_45	50Jobs_2Drills_2MPUs_6Exs	3119.51	3532.44	3.328	11.69
MMPT_46	55Jobs_2Drills_2MPUs_5Exs	3753.48	4361.50	3.306	13.94
MMPT_47	55Jobs_2Drills_2MPUs_6Exs	3264.39	3744.44	3.318	12.82
MMPT_48	55Jobs_2Drills_2MPUs_7Exs	2915.04	3305.12	3.311	11.80

The average deviation away from lower bound values is less than 10%, implying that high-quality MMPT solutions could be obtained in few seconds by the improved SBP algorithm.

Sensitivity analysis under different usages of equipment units

Based on sensitivity analysis of MMPT results as shown in Figure 6, the proposed methodology is able to quantitatively evaluate the usage performance of equipment units at each stage and then identify the bottleneck stage.

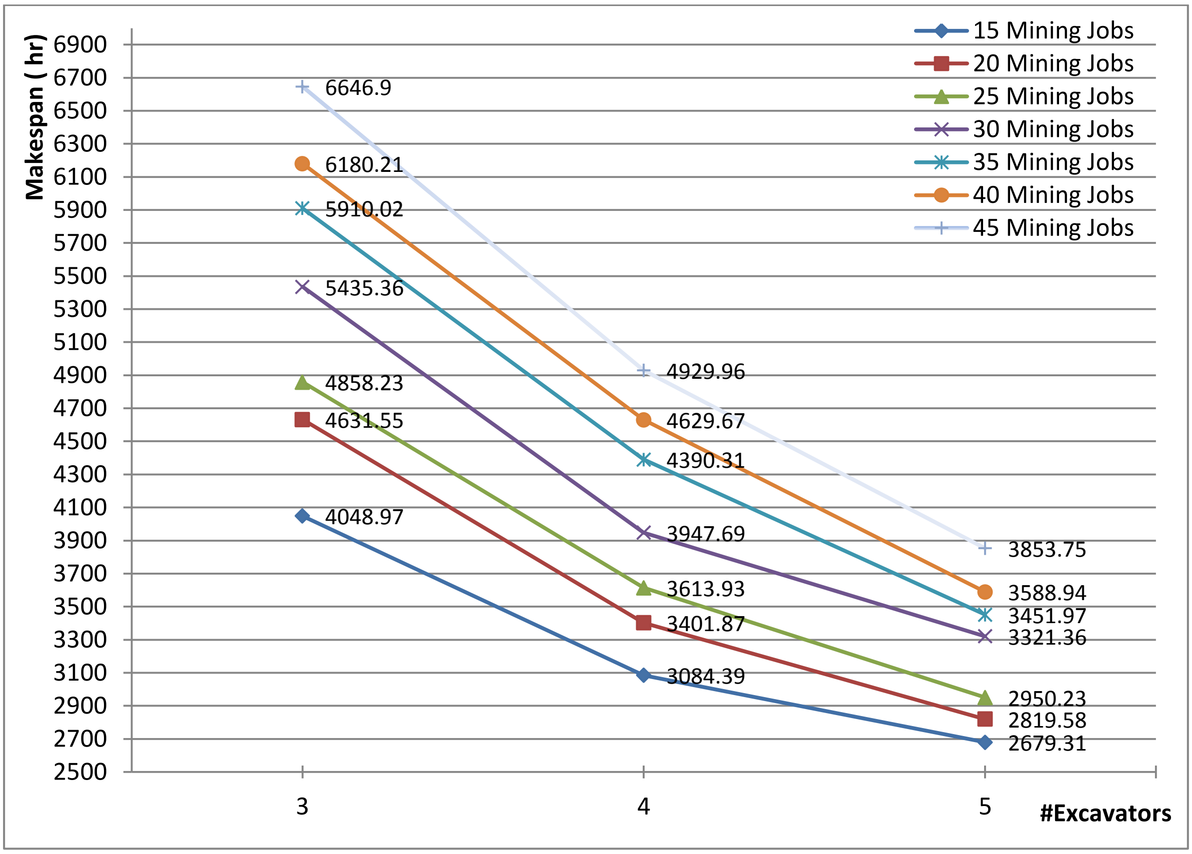


Figure 6: Sensitivity analysis of MMPT results under different usages of excavators

In practice, knowing the availability and effectiveness of mining equipment units at each mining stage at all times is essential to running a smooth operation and reducing the cost of mining resources (i.e., mining equipment units and labours). For example, comparing between Instances MMPT_40 (with 45 Mining Jobs; 2 Drills; 2 MPUs; 3 Excavators) and MMPT_42 (with 45 Mining Jobs; 2 Drills; 2 MPUs; 5 Excavators), if the number of excavators is increased from 3 to 5 while other input data remain the same, the makespan can be significantly reduced from 6646.90 to 3853.75. Due to huge expenditure on mining equipment, the implementation of the proposed methodology would bring significant benefits to mining industry practitioners on optimal decision making about whether to further improve productivity by purchasing additional equipment units at the bottleneck stage or to cut the operational cost by decreasing the number of redundant equipment units at some non-bottleneck stages.

Sensitivity analysis for identifying the bottleneck resource type

In mine production systems, it is important to identify the bottleneck resource type. The proposed MMPT methodology is able to determine the bottleneck resource type, as reflected in Step 3.5 of the proposed SBP algorithm. For instance, the first bottleneck stage in a list of scheduled stages (denoted by $\Omega^{\wedge}$) of this case study is the drilling stage. Then, this means that the drill equipment for drilling operations is the most critical resource type. After identifying the bottleneck resource, the efficiency of a short-term mine production timetable is able to be significantly improved if the number of resource units for the identified bottleneck stage is augmented. Due to the importance of identifying bottleneck resource type, the MMPT methodology can be implemented as a useful decision-making tool for mining industry practitioners on decision making whether it is needed to purchase additional resource units to significantly improve production efficiency.

5. Conclusion

This paper presents a pioneering study to propose a multi-stage mine production timetabling problem to optimise drilling, blasting and excavating operations at the operational level. A generic MMPT problem is defined and formulated using mixed integer programming (MIP). Due to strong NP-hardness, the MMPT MIP model is intractable to be solved by exact MIP solver (e.g., ILOG-CPLEX) for large industry-scale instances. Thus, based on an extended disjunctive graph, an improved SBP algorithm is developed to obtain the high-quality solution in an efficient and effective way. The optimality performance of the proposed algorithm is evaluated by extensive computational experiments. A real-world case study is presented for illustrating the implementation of the proposed MMPT methodology to iron ore mining industry. A sensitivity analysis of MMPT results under different usages of mining equipment units as well as for identifying the bottleneck resource type is conducted to provide better decision making in mining implementation.

In practice, the availability of mining resources is essential due to huge expenditure. For instance, a large excavator used in open-pit iron ore mining industry costs several millions. With the application of the proposed MMPT methodology, mining industry practitioners are able to algorithmically generate the near-optimal operational timetable of mining equipment units at various operational stages and to bring the following potential benefits in practice:

- quantitatively evaluate the usage performance of mining equipment units at each stage, which is shown in Figure 6 by a sensitivity analysis of MMPT results under different usages of equipment units;
- identify the bottleneck stage in terms of mining jobs versus mining equipment units at each stage, which is validated by a sensitivity analysis of MMPT results for identifying the bottleneck resource type; and
- maximise the productivity of the whole mining process in an integrated way, which is achieved by the MMPT's objective function of minimising the makespan.

Therefore, the potential implementations of the proposed MMET methodology at a mine site include the rapid generation of the operational-level mine production timetable by the SBP algorithm; the intelligent decision making on the best number of mining equipment units; and the identification of the critical resource type at a bottleneck stage.

Regarding future research directions, to extend the proposed methodology, more research works need to be conducted to investigate important uncertain characteristics such as equipment maintenance activities, arrivals of new mining tasks and unexpected breakdown events by the development of dynamic-version MMPT models. Moreover, the proposed MMPT model will be integrated in a whole mine supply chain optimisation model for iron ore mining industry [47,48].

Acknowledgements

The authors would like to acknowledge the support of CRC ORE, established and supported by the Australian Government's Cooperative Research Centres Programme.

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